Computer algebra independent integration tests

Summer 2022 edition

1-Algebraic-functions/1.2-Trinomial-products/1.2.2-Quartic/41-1.2.2.4-f-x-^m-d+e-x^2-^q-a+b-x^2+c-x^4-^p

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [413]. This is test number [41].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (413)	0.00 (0)
Mathematica	98.31 (406)	1.69 (7)
Maple	91.04 (376)	8.96 (37)
Fricas	69.01 (285)	30.99 (128)
Giac	64.65 (267)	35.35 (146)
Mupad	52.78 (218)	47.22 (195)
Maxima	41.89 (173)	58.11 (240)
Sympy	30.02 (124)	69.98 (289)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

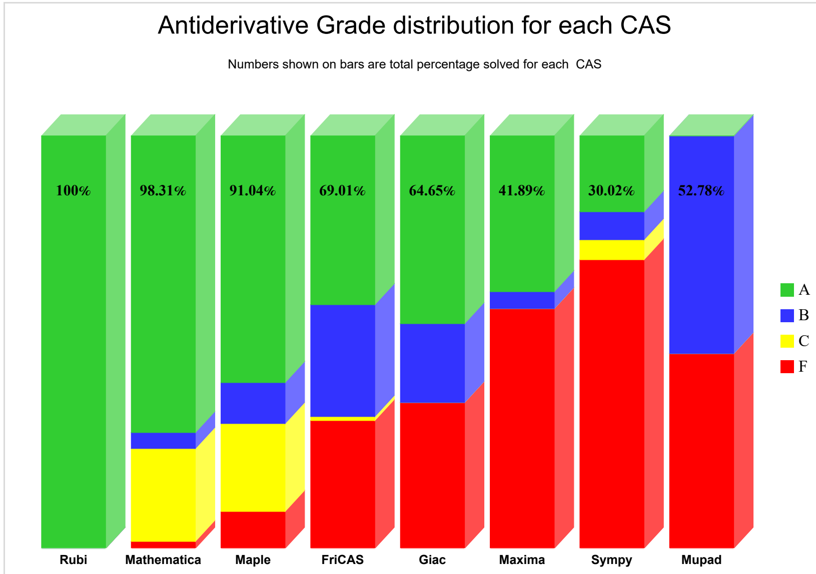
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

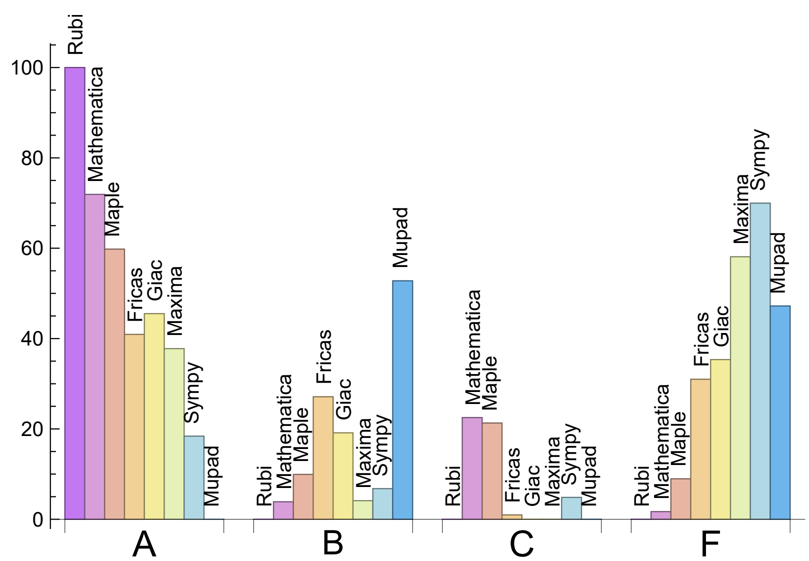
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	71.91	3.87	22.52	1.69
Maple	59.81	9.93	21.31	8.96
Giac	45.52	19.13	0.00	35.35
Fricas	40.92	27.12	0.97	30.99
Maxima	37.77	4.12	0.00	58.11
Sympy	18.40	6.78	4.84	69.98
Mupad	N/A	52.78	0.00	47.22

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	7	100.00 %	0.00 %	0.00 %
Maple	37	100.00 %	0.00 %	0.00 %
Fricas	128	53.12 %	16.41 %	30.47 %
Giac	146	78.08 %	11.64 %	10.27 %
Maxima	240	83.33 %	0.00 %	16.67 %
Sympy	289	68.51 %	31.49 %	0.00 %
Mupad	195	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

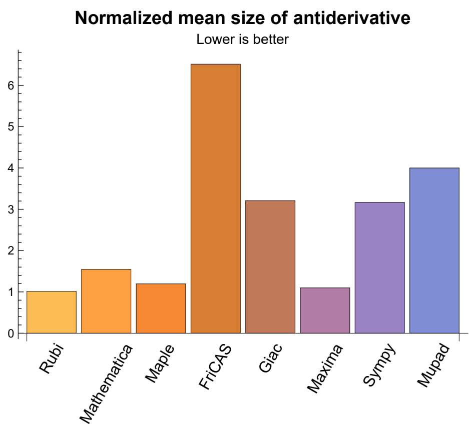
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

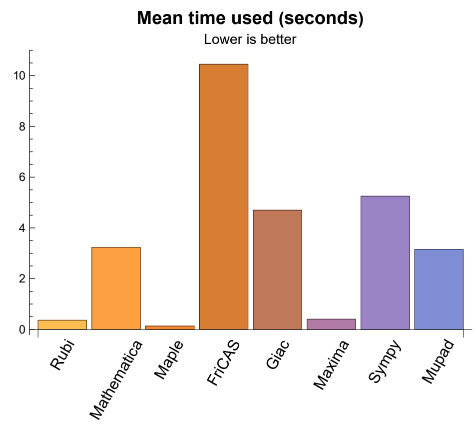
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.36	229.98	1.01	189.00	1.00
Mathematica	3.23	454.89	1.55	158.50	0.96
Maple	0.13	261.87	1.20	175.50	0.92
Maxima	0.40	134.08	1.10	110.00	0.96
Fricas	10.45	1969.65	6.51	290.00	2.47
Sympy	5.25	527.89	3.17	114.00	1.16
Giac	4.70	826.24	3.21	144.00	1.07
Mupad	3.15	928.74	4.00	169.00	1.95

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
1
2
3
4
5
6
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
2
4
6
8
10
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

{}

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 225, 227, 394, 395, 396, 397, 398, 399}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

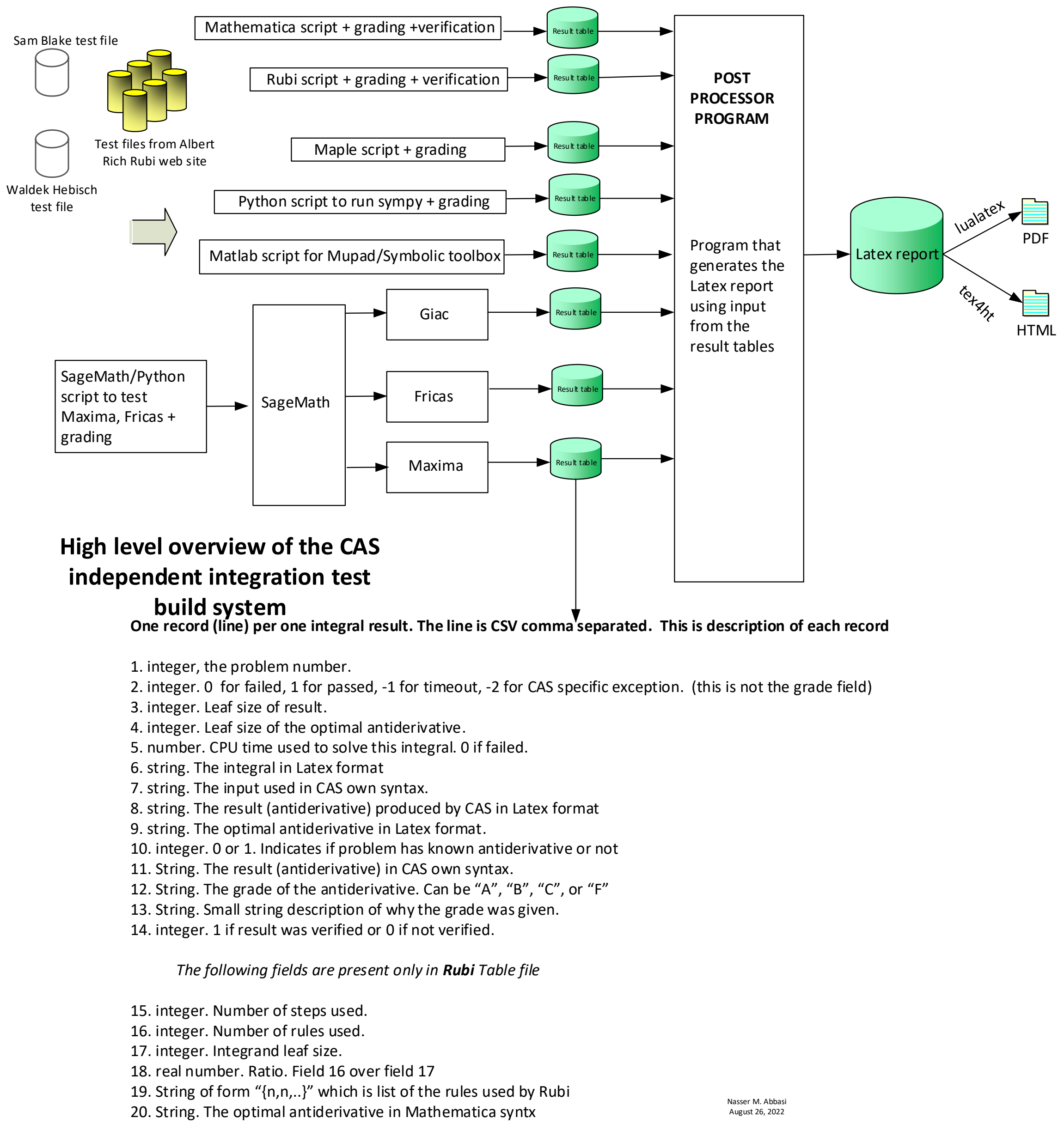
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 55, 57, 59, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 262, 263, 264, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 332, 333, 334, 335, 336, 342, 343, 344, 345, 346, 347, 354, 355, 358, 359, 360, 376, 377, 378, 379, 380, 386, 387, 388, 391, 392, 393, 401, 402, 403, 404, 405, 406, 413 }

B grade: { 56, 58, 60, 66, 68, 361, 362, 364, 365, 366, 371, 372, 373, 374, 375, 385 }

C grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 224, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 356, 357, 363, 367, 368, 369, 370, 381, 382, 383, 384, 389, 390, 394, 395, 396, 397, 398, 399 }

F grade: { 400, 407, 408, 409, 410, 411, 412 }

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 127, 128, 129, 130, 131, 132, 133, 134, 135, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 171, 172, 173, 174, 175, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 222, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 261, 263, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 332, 333, 335, 336, 376 }

B grade: { 55, 56, 58, 60, 65, 87, 88, 125, 126, 136, 169, 170, 176, 195, 196, 220, 221, 262, 264, 311, 312, 313, 314, 315, 322, 323, 324, 325, 326, 334, 342, 343, 344, 345, 346, 347, 377, 378, 379, 380, 385 }

C grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 259, 260, 265, 266, 310, 316, 317, 318, 319, 320, 321, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 413 }

F grade: { 90, 91, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

2.1.4 Maxima

A grade: { 1, 2, 3, 4, 5, 6, 7, 12, 13, 14, 20, 24, 25, 26, 32, 33, 37, 38, 44, 45, 46, 47, 48, 49, 55, 57, 59, 61, 62, 63, 64, 65, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 93, 94, 95, 96, 97, 98, 99, 100, 101, 137, 138, 139, 140, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 196, 197, 198, 220, 221, 222, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294 }

B grade: { 8, 9, 10, 11, 21, 22, 23, 34, 35, 36, 56, 58, 60, 66, 68, 70, 92 }

C grade: { }

F grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 141, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 21, 22, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 88, 89, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 156, 157, 158, 159, 160, 161, 162, 169, 170, 171, 172, 173, 174, 175, 181, 182, 183, 184, 185, 186, 187, 188, 194, 195, 198, 229, 230, 231, 232, 233, 234, 235, 236, 244, 245, 246, 247, 248, 249, 263, 264, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 296, 297, 298, 299, 311, 312, 313, 314, 315, 324, 336 }

B grade: { 55, 56, 58, 60, 65, 66, 68, 70, 87, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 196, 197, 220, 221, 222, 237, 238, 239, 240, 241, 242, 243, 250, 252, 253, 254, 255, 256, 257, 258, 303, 304, 305, 306, 307, 308, 332, 333, 334, 335, 342, 343, 344, 345, 346, 347, 355, 356, 357, 358, 361, 362, 363, 364, 365, 366, 368, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 395, 396, 397, 413 }

C grade: { 259, 260, 265, 266 }

F grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 251, 261, 262, 295, 300, 301, 302, 309, 310, 316, 317, 318, 319, 320, 321, 322, 323, 325, 326, 327, 328, 329, 330, 331, 337, 338, 339, 340, 341, 348, 349, 350, 351, 352, 353, 354, 359, 360, 367, 369, 370, 371, 394, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 20, 23, 24, 25, 26, 32, 33, 34, 35, 36, 37, 38, 44, 45, 46, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 95, 96, 97, 98, 99, 100, 101, 137, 138, 139, 140, 141, 273, 274, 275, 276, 277, 278, 279, 281, 282, 284, 285, 288, 289, 290, 291, 292, 293 }

B grade: { 21, 22, 47, 48, 49, 55, 56, 58, 60, 65, 66, 68, 70, 92, 103, 104, 114, 115, 128, 129, 220, 221, 222, 280, 283, 286, 287, 294 }

C grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54 }

F grade: { 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 93, 94, 102, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 130, 131, 132, 133, 134, 135, 136, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 20, 21, 22, 23, 24, 32, 33, 34, 35, 44, 45, 46, 47, 48, 49, 57, 59, 61, 62, 63, 64, 67, 69, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 112, 113, 114, 115, 116, 117, 124, 125, 127, 128, 129, 130, 131, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 156, 158, 159, 160, 161, 169, 170, 171, 173, 181, 182, 183, 184, 185, 194, 195, 196, 197, 198, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 315, 334, 336, 347, 357, 361, 362, 370, 371, 372, 373, 386, 387, 388, 394, 413 }

B grade: { 13, 14, 25, 26, 36, 37, 38, 55, 56, 58, 60, 65, 66, 68, 70, 87, 88, 89, 93, 94, 107, 108, 109, 110, 111, 118, 119, 120, 121, 122, 123, 126, 132, 133, 134, 135, 136, 147, 148, 149, 150, 157, 162, 174, 175, 186, 187, 188, 220, 221, 222, 303, 304, 305, 306, 307, 308, 309, 343, 344, 345, 354, 355, 356, 358, 360, 367, 368, 369, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

C grade: { }

F grade: { 15, 16, 17, 18, 19, 27, 28, 29, 30, 31, 39, 40, 41, 42, 43, 50, 51, 52, 53, 54, 90, 91, 151, 152, 153, 154, 155, 163, 164, 165, 166, 167, 168, 172, 176, 177, 178, 179, 180, 189, 190, 191, 192, 193, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 310, 311, 312, 313, 314, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 337, 338, 339, 340, 341, 342, 346, 348, 349, 350, 351, 352, 353, 359, 363, 364, 365, 366, 374, 375, 389, 390, 391, 392, 393, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412 }

2.1.8 Mupad

A grade: { }

B grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 18, 20, 21, 22, 23, 24, 25, 26, 30, 32, 33, 34, 35, 36, 37, 38, 42, 44, 45, 46, 47, 48, 49, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 76, 78, 80, 82, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 158, 171, 172, 173, 184, 185, 186, 195, 196, 220, 221, 222, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 354, 355, 356, 357, 358, 359, 360, 367, 368, 369, 370, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385 }

C grade: { }

F grade: { 15, 16, 17, 19, 27, 28, 29, 31, 39, 40, 41, 43, 50, 51, 52, 54, 75, 77, 79, 81, 83, 84, 85, 86, 87, 88, 89, 90, 91, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 187, 188, 189, 190, 191, 192, 193, 194, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 223, 224, 225, 226, 227, 228, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 361, 362, 363, 364, 365, 366, 371, 372, 373, 374, 375, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	131	124	151	131	125
N.S.	1	1.00	1.00	0.85	0.88	0.83	1.01	0.88	0.84
time (sec)	N/A	0.143	0.003	0.123	0.292	0.409	0.015	3.695	0.306

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	149	126	131	124	155	131	125
N.S.	1	1.00	1.00	0.85	0.88	0.83	1.04	0.88	0.84
time (sec)	N/A	0.061	0.003	0.125	0.270	0.334	0.015	6.341	0.069

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	146	125	130	122	150	130	124
N.S.	1	1.00	1.64	1.40	1.46	1.37	1.69	1.46	1.39
time (sec)	N/A	0.052	0.003	0.132	0.281	0.341	0.015	3.637	0.071

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	122	127	120	148	127	121
N.S.	1	1.00	1.00	0.87	0.90	0.85	1.05	0.90	0.86
time (sec)	N/A	0.049	0.003	0.138	0.274	0.335	0.015	4.129	0.070

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	131	122	150	131	122
N.S.	1	1.00	1.00	0.87	0.92	0.86	1.06	0.92	0.86
time (sec)	N/A	0.068	0.006	0.109	0.268	0.377	0.075	4.148	0.109

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	139	122	127	126	143	127	121
N.S.	1	1.00	1.00	0.88	0.91	0.91	1.03	0.91	0.87
time (sec)	N/A	0.047	0.006	0.131	0.282	0.367	0.067	4.192	0.094

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	142	123	131	130	150	142	122
N.S.	1	1.00	1.00	0.87	0.92	0.92	1.06	1.00	0.86
time (sec)	N/A	0.074	0.006	0.141	0.283	0.369	0.087	4.333	0.072

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	54	53	102	48	97	54	42
N.S.	1	1.00	0.81	0.79	1.52	0.72	1.45	0.81	0.63
time (sec)	N/A	0.033	0.073	0.157	0.493	0.370	3.249	4.326	0.370

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	46	93	43	70	45	37
N.S.	1	1.00	0.96	0.90	1.82	0.84	1.37	0.88	0.73
time (sec)	N/A	0.019	0.061	0.131	0.482	0.351	2.501	2.901	0.392

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	40	34	67	34	53	38	32
N.S.	1	1.00	0.91	0.77	1.52	0.77	1.20	0.86	0.73
time (sec)	N/A	0.013	0.062	0.125	0.493	0.357	1.603	3.283	0.137

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	67	49	99	56	83	76	45
N.S.	1	1.00	1.16	0.84	1.71	0.97	1.43	1.31	0.78
time (sec)	N/A	0.034	0.100	0.260	0.487	0.354	7.858	3.176	0.146

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	67	61	88	72	83	91	51
N.S.	1	1.00	1.14	1.03	1.49	1.22	1.41	1.54	0.86
time (sec)	N/A	0.033	0.112	0.270	0.496	0.348	3.764	3.860	0.787

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	70	75	91	72	76	129	56
N.S.	1	1.00	1.11	1.19	1.44	1.14	1.21	2.05	0.89
time (sec)	N/A	0.033	0.163	0.270	0.502	0.363	2.988	4.083	0.423

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	52	59	59	63	116	43
N.S.	1	1.00	1.03	0.90	1.02	1.02	1.09	2.00	0.74
time (sec)	N/A	0.029	0.154	0.204	0.500	0.388	2.899	4.243	0.681

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	82	192	0	0	78	0	-1
N.S.	1	1.00	0.39	0.92	0.00	0.00	0.38	0.00	-0.00
time (sec)	N/A	0.079	4.496	0.155	0.000	0.000	1.106	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	68	180	0	0	78	0	-1
N.S.	1	1.00	0.35	0.94	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.065	3.888	0.145	0.000	0.000	1.022	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	48	168	0	0	76	0	-1
N.S.	1	1.00	0.27	0.95	0.00	0.00	0.43	0.00	-0.01
time (sec)	N/A	0.043	3.464	0.125	0.000	0.000	0.944	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	108	167	0	0	78	0	61
N.S.	1	1.00	0.63	0.98	0.00	0.00	0.46	0.00	0.36
time (sec)	N/A	0.045	3.416	0.151	0.000	0.000	1.107	0.000	0.410

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	98	170	0	0	83	0	-1
N.S.	1	1.00	0.51	0.89	0.00	0.00	0.43	0.00	-0.01
time (sec)	N/A	0.059	7.827	0.141	0.000	0.000	1.177	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	64	73	127	58	131	80	52
N.S.	1	1.00	0.77	0.88	1.53	0.70	1.58	0.96	0.63
time (sec)	N/A	0.042	0.102	0.139	0.496	0.339	8.971	3.617	0.319

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	59	58	118	53	124	71	47
N.S.	1	1.00	0.88	0.87	1.76	0.79	1.85	1.06	0.70
time (sec)	N/A	0.025	0.087	0.131	0.481	0.385	7.972	4.133	0.430

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	54	46	95	48	109	57	42
N.S.	1	1.00	0.90	0.77	1.58	0.80	1.82	0.95	0.70
time (sec)	N/A	0.021	0.085	0.129	0.496	0.384	4.249	4.850	0.177

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	77	75	138	67	114	90	55
N.S.	1	1.00	0.99	0.96	1.77	0.86	1.46	1.15	0.71
time (sec)	N/A	0.049	0.132	0.259	0.488	0.350	16.339	4.035	0.180

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	76	75	122	78	114	102	64
N.S.	1	1.00	0.94	0.93	1.51	0.96	1.41	1.26	0.79
time (sec)	N/A	0.048	0.147	0.280	0.487	0.365	5.436	4.685	0.765

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	73	123	82	133	146	71
N.S.	1	1.00	0.94	0.85	1.43	0.95	1.55	1.70	0.83
time (sec)	N/A	0.050	0.189	0.273	0.490	0.402	6.473	2.731	0.552

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	79	73	112	82	148	158	82
N.S.	1	1.00	0.96	0.89	1.37	1.00	1.80	1.93	1.00
time (sec)	N/A	0.048	0.204	0.267	0.505	0.361	6.581	4.811	0.946

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	74	216	0	0	160	0	-1
N.S.	1	1.00	0.31	0.92	0.00	0.00	0.68	0.00	-0.00
time (sec)	N/A	0.088	6.945	0.141	0.000	0.000	1.926	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	68	204	0	0	160	0	-1
N.S.	1	1.00	0.31	0.93	0.00	0.00	0.73	0.00	-0.00
time (sec)	N/A	0.080	5.652	0.137	0.000	0.000	1.770	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	49	192	0	0	158	0	-1
N.S.	1	1.00	0.25	0.97	0.00	0.00	0.80	0.00	-0.01
time (sec)	N/A	0.055	4.888	0.128	0.000	0.000	1.650	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	125	192	0	0	160	0	48
N.S.	1	1.00	0.63	0.96	0.00	0.00	0.80	0.00	0.24
time (sec)	N/A	0.058	4.778	0.155	0.000	0.000	1.930	0.000	0.533

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	124	192	0	0	163	0	-1
N.S.	1	1.00	0.62	0.96	0.00	0.00	0.81	0.00	-0.00
time (sec)	N/A	0.058	9.404	0.140	0.000	0.000	1.981	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	49	51	104	43	85	46	38
N.S.	1	1.00	0.73	0.76	1.55	0.64	1.27	0.69	0.57
time (sec)	N/A	0.038	0.074	0.135	0.498	0.339	3.914	4.127	0.590

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	39	76	34	66	37	32
N.S.	1	1.00	0.78	0.76	1.49	0.67	1.29	0.73	0.63
time (sec)	N/A	0.026	0.078	0.137	0.489	0.348	2.704	5.044	0.309

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	39	32	65	33	53	33	27
N.S.	1	1.00	1.11	0.91	1.86	0.94	1.51	0.94	0.77
time (sec)	N/A	0.017	0.067	0.128	0.487	0.363	1.997	4.221	0.491

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	28	20	42	26	22	26	19
N.S.	1	1.00	1.17	0.83	1.75	1.08	0.92	1.08	0.79
time (sec)	N/A	0.011	0.067	0.132	0.487	0.332	0.947	3.693	0.294

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	48	30	67	41	31	61	30
N.S.	1	1.00	1.26	0.79	1.76	1.08	0.82	1.61	0.79
time (sec)	N/A	0.026	0.080	0.245	0.500	0.358	2.928	3.615	0.615

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	46	31	47	47	31	66	31
N.S.	1	1.00	1.10	0.74	1.12	1.12	0.74	1.57	0.74
time (sec)	N/A	0.025	0.088	0.181	0.491	0.374	1.760	4.778	0.329

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	55	43	59	50	88	114	43
N.S.	1	1.00	0.95	0.74	1.02	0.86	1.52	1.97	0.74
time (sec)	N/A	0.033	0.138	0.199	0.505	0.343	5.662	5.760	0.694

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	74	168	0	0	75	0	-1
N.S.	1	1.00	0.40	0.91	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.056	10.028	0.128	0.000	0.000	1.169	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	66	155	0	0	75	0	-1
N.S.	1	1.00	0.40	0.93	0.00	0.00	0.45	0.00	-0.01
time (sec)	N/A	0.046	10.034	0.131	0.000	0.000	1.083	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	48	146	0	0	73	0	-1
N.S.	1	1.00	0.31	0.94	0.00	0.00	0.47	0.00	-0.01
time (sec)	N/A	0.033	10.035	0.129	0.000	0.000	0.767	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	81	158	0	0	75	0	48
N.S.	1	1.00	0.47	0.91	0.00	0.00	0.43	0.00	0.28
time (sec)	N/A	0.044	10.124	0.136	0.000	0.000	0.853	0.000	0.504

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	97	170	0	0	80	0	-1
N.S.	1	1.00	0.51	0.90	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.062	10.101	0.135	0.000	0.000	0.973	0.000	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	49	50	89	62	66	45	97
N.S.	1	1.00	0.84	0.86	1.53	1.07	1.14	0.78	1.67
time (sec)	N/A	0.031	0.111	0.143	0.483	0.382	6.497	3.866	1.108

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	40	37	63	58	48	39	89
N.S.	1	1.00	0.89	0.82	1.40	1.29	1.07	0.87	1.98
time (sec)	N/A	0.026	0.118	0.155	0.487	0.374	5.372	3.567	0.894

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	39	34	54	52	39	33	82
N.S.	1	1.00	1.11	0.97	1.54	1.49	1.11	0.94	2.34
time (sec)	N/A	0.017	0.108	0.148	0.507	0.384	4.416	3.322	0.841

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	23	22	31	31	16	16
N.S.	1	1.00	1.00	1.15	1.10	1.55	1.55	0.80	0.80
time (sec)	N/A	0.010	0.098	0.128	0.484	0.347	3.208	3.785	0.164

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	52	40	56	61	212	61	40
N.S.	1	1.00	1.13	0.87	1.22	1.33	4.61	1.33	0.87
time (sec)	N/A	0.028	0.130	0.203	0.484	0.346	8.372	4.310	0.475

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	47	68	77	228	82	47
N.S.	1	1.00	0.91	0.72	1.05	1.18	3.51	1.26	0.72
time (sec)	N/A	0.037	0.140	0.200	0.517	0.357	5.561	6.059	0.537

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	70	168	0	0	75	0	-1
N.S.	1	1.00	0.36	0.86	0.00	0.00	0.38	0.00	-0.01
time (sec)	N/A	0.058	10.031	0.139	0.000	0.000	2.417	0.000	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	68	168	0	0	75	0	-1
N.S.	1	1.00	0.38	0.95	0.00	0.00	0.42	0.00	-0.01
time (sec)	N/A	0.050	10.026	0.138	0.000	0.000	2.236	0.000	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	66	168	0	0	73	0	-1
N.S.	1	1.00	0.37	0.93	0.00	0.00	0.41	0.00	-0.01
time (sec)	N/A	0.042	10.023	0.133	0.000	0.000	2.126	0.000	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	108	180	0	0	75	0	48
N.S.	1	1.00	0.55	0.92	0.00	0.00	0.38	0.00	0.24
time (sec)	N/A	0.057	10.087	0.180	0.000	0.000	3.013	0.000	0.456

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	C	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	119	192	0	0	80	0	-1
N.S.	1	1.00	0.56	0.90	0.00	0.00	0.37	0.00	-0.00
time (sec)	N/A	0.077	10.101	0.135	0.000	0.000	3.447	0.000	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	189	2295	405	1495	21612	3752	1539
N.S.	1	1.00	0.70	8.53	1.51	5.56	80.34	13.95	5.72
time (sec)	N/A	0.108	0.708	0.036	0.339	0.376	3.438	3.966	1.777

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	153	130	140	127	134	143	121
N.S.	1	1.00	2.43	2.06	2.22	2.02	2.13	2.27	1.92
time (sec)	N/A	0.127	0.020	0.087	0.271	0.337	0.026	4.029	0.094

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	153	130	140	127	141	144	123
N.S.	1	1.00	1.00	0.85	0.92	0.83	0.92	0.94	0.80
time (sec)	N/A	0.076	0.015	0.079	0.297	0.367	0.017	4.397	0.121

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	151	130	140	127	136	144	123
N.S.	1	1.00	3.36	2.89	3.11	2.82	3.02	3.20	2.73
time (sec)	N/A	0.081	0.013	0.079	0.268	0.355	0.025	4.986	0.078

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	153	130	140	127	139	144	123
N.S.	1	1.00	1.00	0.85	0.92	0.83	0.91	0.94	0.80
time (sec)	N/A	0.056	0.012	0.079	0.280	0.349	0.016	4.171	0.081

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	149	130	140	127	133	144	123
N.S.	1	1.00	5.14	4.48	4.83	4.38	4.59	4.97	4.24
time (sec)	N/A	0.030	0.008	0.081	0.274	0.369	0.030	4.320	0.077

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	143	127	136	124	134	141	120
N.S.	1	1.00	1.00	0.89	0.95	0.87	0.94	0.99	0.84
time (sec)	N/A	0.046	0.011	0.122	0.280	0.347	0.025	3.682	0.079

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	149	132	141	123	131	145	121
N.S.	1	1.00	1.60	1.42	1.52	1.32	1.41	1.56	1.30
time (sec)	N/A	0.033	0.019	0.022	0.277	0.557	0.111	3.510	0.129

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	141	129	136	128	124	139	119
N.S.	1	1.00	1.00	0.91	0.96	0.91	0.88	0.99	0.84
time (sec)	N/A	0.052	0.019	0.040	0.282	0.442	0.105	3.750	0.081

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	147	130	141	134	131	156	120
N.S.	1	1.00	1.00	0.88	0.96	0.91	0.89	1.06	0.82
time (sec)	N/A	0.091	0.026	0.026	0.279	0.427	0.142	3.572	0.084

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	122	1121	192	759	11387	1848	1483
N.S.	1	1.00	0.60	5.52	0.95	3.74	56.09	9.10	7.31
time (sec)	N/A	0.049	0.076	0.024	0.307	0.600	2.750	3.874	1.251

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	85	29	61	61	76	61	61
N.S.	1	1.00	2.50	0.85	1.79	1.79	2.24	1.79	1.79
time (sec)	N/A	0.029	0.002	0.062	0.277	0.495	0.012	3.702	0.062

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	61	61	75	61	61
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73
time (sec)	N/A	0.019	0.002	0.061	0.281	0.377	0.011	3.988	0.059

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	83	20	61	61	75	61	61
N.S.	1	1.00	3.61	0.87	2.65	2.65	3.26	2.65	2.65
time (sec)	N/A	0.014	0.001	0.034	0.270	0.448	0.012	3.688	0.060

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	83	62	61	61	75	61	61
N.S.	1	1.00	1.00	0.75	0.73	0.73	0.90	0.73	0.73
time (sec)	N/A	0.015	0.001	0.060	0.275	0.378	0.011	4.048	0.059

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	10	61	61	71	76	61
N.S.	1	1.00	1.00	0.91	5.55	5.55	6.45	6.91	5.55
time (sec)	N/A	0.002	0.001	0.036	0.284	0.421	0.012	4.300	0.058

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	58	57	57	68	57	57
N.S.	1	1.00	1.00	0.79	0.78	0.78	0.93	0.78	0.78
time (sec)	N/A	0.013	0.001	0.073	0.285	0.642	0.013	3.813	0.058

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	59	62	58	75	62	58
N.S.	1	1.00	1.00	0.74	0.78	0.72	0.94	0.78	0.72
time (sec)	N/A	0.021	0.003	0.015	0.283	0.399	0.036	3.874	0.061

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	60	59	62	66	59	59
N.S.	1	1.00	1.00	0.82	0.81	0.85	0.90	0.81	0.81
time (sec)	N/A	0.015	0.003	0.023	0.270	0.418	0.032	5.976	0.060

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	80	61	62	64	75	69	60
N.S.	1	1.00	1.00	0.76	0.78	0.80	0.94	0.86	0.75
time (sec)	N/A	0.024	0.002	0.029	0.284	0.345	0.032	5.063	0.061

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	80	91	57	133	90	101	-1
N.S.	1	1.00	0.55	0.63	0.39	0.92	0.62	0.70	-0.01
time (sec)	N/A	0.056	0.036	0.132	0.478	0.384	0.184	5.120	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	51	55	33	31	27	42	103
N.S.	1	1.00	0.61	0.66	0.40	0.37	0.33	0.51	1.24
time (sec)	N/A	0.046	0.015	0.122	0.271	0.363	0.123	5.056	0.876

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	69	62	35	102	82	59	-1
N.S.	1	1.00	0.71	0.64	0.36	1.05	0.85	0.61	-0.01
time (sec)	N/A	0.030	0.020	0.129	0.476	0.452	0.140	5.171	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	92	54	57	36	34	26	61	83
N.S.	1	1.00	0.59	0.62	0.39	0.37	0.28	0.66	0.90
time (sec)	N/A	0.047	0.015	0.122	0.282	0.547	0.392	4.973	0.766

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	72	67	38	110	82	62	-1
N.S.	1	1.00	0.71	0.66	0.38	1.09	0.81	0.61	-0.01
time (sec)	N/A	0.040	0.022	0.141	0.499	0.355	0.170	3.628	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	70	78	50	56	41	131	125
N.S.	1	1.00	0.51	0.57	0.36	0.41	0.30	0.96	0.91
time (sec)	N/A	0.063	0.024	0.131	0.269	0.448	0.396	4.072	0.810

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	108	188	126	308	0	101	-1
N.S.	1	1.00	0.71	1.23	0.82	2.01	0.00	0.66	-0.01
time (sec)	N/A	0.076	0.043	0.048	0.491	0.516	0.000	4.214	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	45	38	66	43	0	40	48
N.S.	1	1.00	0.58	0.49	0.86	0.56	0.00	0.52	0.62
time (sec)	N/A	0.045	0.013	0.026	0.292	0.369	0.000	5.577	0.179

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	108	188	125	308	0	101	-1
N.S.	1	1.00	0.69	1.21	0.80	1.97	0.00	0.65	-0.01
time (sec)	N/A	0.055	0.037	0.042	0.492	0.437	0.000	4.069	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	92	132	89	120	0	107	-1
N.S.	1	1.00	0.57	0.82	0.55	0.75	0.00	0.66	-0.01
time (sec)	N/A	0.082	0.028	0.042	0.280	0.355	0.000	4.561	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	124	206	135	340	0	116	-1
N.S.	1	1.00	0.65	1.08	0.71	1.79	0.00	0.61	-0.01
time (sec)	N/A	0.121	0.048	0.051	0.491	0.340	0.000	3.629	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	130	249	139	212	0	188	-1
N.S.	1	1.00	0.58	1.12	0.62	0.95	0.00	0.84	-0.00
time (sec)	N/A	0.122	0.048	0.054	0.283	0.368	0.000	3.530	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	400	400	160	1099	494	866	0	2213	-1
N.S.	1	1.00	0.40	2.75	1.24	2.16	0.00	5.53	-0.00
time (sec)	N/A	0.159	0.448	0.036	0.313	0.370	0.000	3.650	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	112	495	246	390	0	1013	-1
N.S.	1	1.00	0.41	1.79	0.89	1.41	0.00	3.67	-0.00
time (sec)	N/A	0.100	0.135	0.017	0.290	0.376	0.000	5.421	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	86	131	78	98	0	269	-1
N.S.	1	1.00	0.56	0.86	0.51	0.64	0.00	1.76	-0.01
time (sec)	N/A	0.048	0.060	0.015	0.287	0.386	0.000	2.787	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	78	0	0	0	0	0	-1
N.S.	1	1.00	0.58	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.054	0.078	0.009	0.000	0.000	0.000	0.000	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	101	0	0	0	0	0	-1
N.S.	1	1.00	0.66	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.118	0.006	0.000	0.000	0.000	0.000	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	25	40	86	47	155	32	59
N.S.	1	1.00	0.74	1.18	2.53	1.38	4.56	0.94	1.74
time (sec)	N/A	0.019	0.007	0.044	0.302	0.383	4.119	4.595	0.142

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	45	62	135	92	0	196	108
N.S.	1	1.00	0.52	0.72	1.57	1.07	0.00	2.28	1.26
time (sec)	N/A	0.058	0.048	0.032	0.280	0.374	0.000	3.440	0.171

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	68	99	196	140	0	331	169
N.S.	1	1.00	0.53	0.77	1.53	1.09	0.00	2.59	1.32
time (sec)	N/A	0.084	0.060	0.040	0.291	0.357	0.000	3.992	0.202

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	226	166	166	202	193	169
N.S.	1	1.00	1.00	1.36	1.00	1.00	1.22	1.16	1.02
time (sec)	N/A	0.252	0.035	0.080	0.274	0.349	0.023	4.390	0.076

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	166	226	166	166	204	193	169
N.S.	1	1.00	1.00	1.36	1.00	1.00	1.23	1.16	1.02
time (sec)	N/A	0.096	0.035	0.083	0.275	0.337	0.022	3.667	0.096

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	154	226	166	166	199	193	169
N.S.	1	1.00	0.93	1.36	1.00	1.00	1.20	1.16	1.02
time (sec)	N/A	0.182	0.040	0.089	0.280	0.326	0.022	3.046	0.048

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	161	223	163	163	199	189	165
N.S.	1	1.00	1.00	1.39	1.01	1.01	1.24	1.17	1.02
time (sec)	N/A	0.073	0.032	0.089	0.284	0.348	0.022	4.131	0.049

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	162	191	167	164	199	193	166
N.S.	1	1.00	1.00	1.18	1.03	1.01	1.23	1.19	1.02
time (sec)	N/A	0.138	0.040	0.020	0.286	0.346	0.125	5.277	0.103

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	156	186	162	168	185	185	163
N.S.	1	1.00	1.00	1.19	1.04	1.08	1.19	1.19	1.04
time (sec)	N/A	0.067	0.059	0.022	0.279	0.335	0.123	4.839	0.051

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	162	188	167	170	197	212	166
N.S.	1	1.00	1.00	1.16	1.03	1.05	1.22	1.31	1.02
time (sec)	N/A	0.139	0.049	0.024	0.290	0.354	0.180	4.213	0.056

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	126	136	0	421	0	126	1343
N.S.	1	1.00	0.95	1.02	0.00	3.17	0.00	0.95	10.10
time (sec)	N/A	0.140	0.046	0.088	0.000	0.390	0.000	4.581	0.458

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	93	98	0	312	434	91	979
N.S.	1	1.00	0.96	1.01	0.00	3.22	4.47	0.94	10.09
time (sec)	N/A	0.079	0.049	0.055	0.000	0.374	71.093	3.397	0.646

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	65	0	219	287	67	606
N.S.	1	1.00	1.00	0.92	0.00	3.08	4.04	0.94	8.54
time (sec)	N/A	0.048	0.033	0.056	0.000	0.373	5.040	5.400	0.498

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	128	76	0	249	0	78	2424
N.S.	1	1.00	1.64	0.97	0.00	3.19	0.00	1.00	31.08
time (sec)	N/A	0.092	0.070	0.039	0.000	0.374	0.000	5.883	4.476

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	186	126	0	385	0	124	2500
N.S.	1	1.00	1.66	1.12	0.00	3.44	0.00	1.11	22.32
time (sec)	N/A	0.156	0.145	0.046	0.000	0.482	0.000	5.457	4.855

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	327	289	0	5140	0	4391	2500
N.S.	1	1.00	1.25	1.11	0.00	19.69	0.00	16.82	9.58
time (sec)	N/A	0.890	0.270	0.087	0.000	2.943	0.000	4.802	1.594

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	251	212	0	2632	0	3179	2500
N.S.	1	1.00	1.21	1.02	0.00	12.65	0.00	15.28	12.02
time (sec)	N/A	0.346	0.105	0.055	0.000	0.750	0.000	8.693	1.256

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	173	164	0	1569	0	1400	2500
N.S.	1	1.00	1.01	0.95	0.00	9.12	0.00	8.14	14.53
time (sec)	N/A	0.143	0.062	0.035	0.000	0.522	0.000	5.873	0.997

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	206	173	0	2914	0	2805	2500
N.S.	1	1.00	1.09	0.92	0.00	15.42	0.00	14.84	13.23
time (sec)	N/A	0.249	0.188	0.051	0.000	0.886	0.000	4.281	1.354

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	267	232	0	5442	0	2870	2500
N.S.	1	1.00	0.99	0.86	0.00	20.08	0.00	10.59	9.23
time (sec)	N/A	0.443	0.215	0.059	0.000	3.239	0.000	5.137	2.187

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	208	282	0	1323	0	239	2282
N.S.	1	1.00	0.98	1.33	0.00	6.24	0.00	1.13	10.76
time (sec)	N/A	0.253	0.193	0.099	0.000	0.441	0.000	4.877	0.834

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	160	211	0	849	0	194	1527
N.S.	1	1.00	1.09	1.44	0.00	5.78	0.00	1.32	10.39
time (sec)	N/A	0.118	0.129	0.083	0.000	0.404	0.000	4.766	1.217

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	111	126	0	538	394	120	283
N.S.	1	1.00	1.04	1.18	0.00	5.03	3.68	1.12	2.64
time (sec)	N/A	0.072	0.055	0.047	0.000	0.356	4.280	5.103	0.323

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	101	95	0	474	374	102	264
N.S.	1	1.00	1.07	1.01	0.00	5.04	3.98	1.09	2.81
time (sec)	N/A	0.059	0.052	0.043	0.000	0.381	2.349	4.647	0.301

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	243	212	0	1014	0	201	2500
N.S.	1	1.00	1.62	1.41	0.00	6.76	0.00	1.34	16.67
time (sec)	N/A	0.216	0.229	0.072	0.000	0.733	0.000	3.489	7.884

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	379	300	0	1635	0	250	2500
N.S.	1	1.00	1.70	1.35	0.00	7.33	0.00	1.12	11.21
time (sec)	N/A	0.280	0.376	0.077	0.000	1.339	0.000	5.438	9.088

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	455	447	0	7252	0	5681	2500
N.S.	1	1.00	1.07	1.05	0.00	17.06	0.00	13.37	5.88
time (sec)	N/A	2.622	0.775	0.067	0.000	8.301	0.000	6.507	4.358

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	362	359	0	4658	0	4538	2500
N.S.	1	1.00	1.08	1.07	0.00	13.86	0.00	13.51	7.44
time (sec)	N/A	1.215	0.569	0.067	0.000	2.502	0.000	7.123	5.184

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	298	290	0	3467	0	3776	2500
N.S.	1	1.00	1.08	1.05	0.00	12.56	0.00	13.68	9.06
time (sec)	N/A	0.389	0.428	0.072	0.000	1.220	0.000	6.525	4.411

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	304	464	0	4885	0	4426	2500
N.S.	1	1.00	1.04	1.58	0.00	16.67	0.00	15.11	8.53
time (sec)	N/A	0.569	0.504	0.092	0.000	3.262	0.000	6.800	4.839

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	389	389	382	379	0	7583	0	5408	2500
N.S.	1	1.00	0.98	0.97	0.00	19.49	0.00	13.90	6.43
time (sec)	N/A	0.834	0.693	0.103	0.000	9.282	0.000	6.331	5.380

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	522	522	487	484	0	10190	0	6327	2500
N.S.	1	1.00	0.93	0.93	0.00	19.52	0.00	12.12	4.79
time (sec)	N/A	0.935	0.796	0.094	0.000	21.185	0.000	8.807	5.698

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	435	624	0	3196	0	598	2500
N.S.	1	1.00	1.19	1.71	0.00	8.76	0.00	1.64	6.85
time (sec)	N/A	0.966	0.447	0.135	0.000	0.621	0.000	5.355	4.660

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	354	495	0	2167	0	466	2500
N.S.	1	1.00	1.39	1.95	0.00	8.53	0.00	1.83	9.84
time (sec)	N/A	0.274	0.318	0.109	0.000	0.482	0.000	7.576	5.151

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	261	343	0	1378	0	318	593
N.S.	1	1.00	1.79	2.35	0.00	9.44	0.00	2.18	4.06
time (sec)	N/A	0.099	0.185	0.067	0.000	0.396	0.000	7.418	0.694

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	233	303	0	1369	0	268	625
N.S.	1	1.00	1.26	1.64	0.00	7.40	0.00	1.45	3.38
time (sec)	N/A	0.171	0.167	0.069	0.000	0.402	0.000	4.387	0.680

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	172	273	0	1226	789	228	587
N.S.	1	1.00	1.01	1.61	0.00	7.21	4.64	1.34	3.45
time (sec)	N/A	0.115	0.142	0.069	0.000	0.390	204.271	7.244	0.660

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	142	147	0	1109	661	208	517
N.S.	1	1.00	1.02	1.06	0.00	7.98	4.76	1.50	3.72
time (sec)	N/A	0.089	0.091	0.075	0.000	0.393	163.740	5.898	0.587

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	396	442	0	2494	0	421	2500
N.S.	1	1.00	1.57	1.75	0.00	9.90	0.00	1.67	9.92
time (sec)	N/A	0.358	0.433	0.101	0.000	2.082	0.000	5.672	11.572

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	363	363	642	616	0	3956	0	648	2500
N.S.	1	1.00	1.77	1.70	0.00	10.90	0.00	1.79	6.89
time (sec)	N/A	0.492	0.942	0.114	0.000	4.302	0.000	5.298	15.906

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	554	554	644	710	0	9636	0	3987	2500
N.S.	1	1.00	1.16	1.28	0.00	17.39	0.00	7.20	4.51
time (sec)	N/A	7.636	1.512	0.085	0.000	19.290	0.000	7.755	5.047

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	543	591	0	7060	0	7578	2500
N.S.	1	1.00	1.18	1.28	0.00	15.31	0.00	16.44	5.42
time (sec)	N/A	3.232	1.299	0.076	0.000	6.162	0.000	8.374	3.951

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	380	380	447	495	0	5650	0	3164	2500
N.S.	1	1.00	1.18	1.30	0.00	14.87	0.00	8.33	6.58
time (sec)	N/A	0.938	1.081	0.075	0.000	4.311	0.000	6.499	3.487

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	438	438	436	524	0	7270	0	7267	2500
N.S.	1	1.00	1.00	1.20	0.00	16.60	0.00	16.59	5.71
time (sec)	N/A	0.685	1.021	0.076	0.000	8.306	0.000	6.995	3.916

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	516	1552	0	9909	0	4613	2500
N.S.	1	1.00	1.12	3.37	0.00	21.54	0.00	10.03	5.43
time (sec)	N/A	0.850	1.389	0.222	0.000	22.367	0.000	6.596	4.614

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	17	17	17	19	17
N.S.	1	1.00	1.00	0.72	0.68	0.68	0.68	0.76	0.68
time (sec)	N/A	0.012	0.005	0.025	0.280	0.353	0.051	4.191	0.057

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	18	25	17	17	19	17
N.S.	1	1.00	1.00	0.72	1.00	0.68	0.68	0.76	0.68
time (sec)	N/A	0.019	0.005	0.018	0.274	0.344	0.037	4.977	0.029

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	30	30	37	30	32
N.S.	1	1.00	1.00	0.84	0.81	0.81	1.00	0.81	0.86
time (sec)	N/A	0.022	0.009	0.021	0.497	0.358	0.039	4.558	0.212

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	31	53	30	37	30	32
N.S.	1	1.00	1.00	0.84	1.43	0.81	1.00	0.81	0.86
time (sec)	N/A	0.028	0.005	0.017	0.474	0.355	0.038	3.726	0.034

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	41	0	47	44	38	41
N.S.	1	1.00	1.00	0.91	0.00	1.04	0.98	0.84	0.91
time (sec)	N/A	0.030	0.017	0.019	0.000	0.369	0.051	4.292	0.052

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	69	91	104	61	0	102	102
N.S.	1	1.00	0.68	0.89	1.02	0.60	0.00	1.00	1.00
time (sec)	N/A	0.053	0.105	0.120	0.270	0.347	0.000	4.016	0.490

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	74	87	56	0	88	85
N.S.	1	1.00	0.79	0.91	1.07	0.69	0.00	1.09	1.05
time (sec)	N/A	0.038	0.083	0.105	0.276	0.367	0.000	4.430	0.433

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	59	57	70	51	0	74	67
N.S.	1	1.00	0.80	0.77	0.95	0.69	0.00	1.00	0.91
time (sec)	N/A	0.030	0.082	0.105	0.301	0.351	0.000	4.370	0.287

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	88	85	89	95	0	98	86
N.S.	1	1.00	0.94	0.90	0.95	1.01	0.00	1.04	0.91
time (sec)	N/A	0.054	0.124	0.302	0.488	0.381	0.000	4.340	0.426

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	91	104	89	112	0	138	84
N.S.	1	1.00	0.94	1.07	0.92	1.15	0.00	1.42	0.87
time (sec)	N/A	0.055	0.149	0.286	0.486	0.349	0.000	3.486	0.878

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	91	121	106	112	0	169	-1
N.S.	1	1.00	0.92	1.22	1.07	1.13	0.00	1.71	-0.01
time (sec)	N/A	0.054	0.177	0.280	0.488	0.346	0.000	4.217	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	118	99	90	0	189	-1
N.S.	1	1.00	0.78	1.31	1.10	1.00	0.00	2.10	-0.01
time (sec)	N/A	0.044	0.201	0.201	0.523	0.348	0.000	3.495	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	80	135	116	95	0	233	-1
N.S.	1	1.00	0.72	1.22	1.05	0.86	0.00	2.10	-0.01
time (sec)	N/A	0.056	0.234	0.203	0.495	0.362	0.000	3.541	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	80	152	133	100	0	255	-1
N.S.	1	1.00	0.61	1.15	1.01	0.76	0.00	1.93	-0.01
time (sec)	N/A	0.072	0.285	0.219	0.483	0.365	0.000	3.193	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	237	260	0	0	0	0	-1
N.S.	1	1.00	0.74	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.189	5.933	0.131	0.000	0.000	0.000	0.000	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	234	243	0	0	0	0	-1
N.S.	1	1.00	0.77	0.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.139	5.133	0.048	0.000	0.000	0.000	0.000	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	229	226	0	0	0	0	-1
N.S.	1	1.00	0.82	0.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.084	4.683	0.049	0.000	0.000	0.000	0.000	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	284	284	231	225	0	0	0	0	-1
N.S.	1	1.00	0.81	0.79	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.088	4.508	0.052	0.000	0.000	0.000	0.000	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	237	228	0	0	0	0	-1
N.S.	1	1.00	0.78	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.118	9.471	0.061	0.000	0.000	0.000	0.000	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	79	138	135	71	0	207	-1
N.S.	1	1.00	0.62	1.09	1.06	0.56	0.00	1.63	-0.01
time (sec)	N/A	0.062	0.170	0.116	0.273	0.333	0.000	3.217	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	74	121	118	66	0	179	-1
N.S.	1	1.00	0.70	1.14	1.11	0.62	0.00	1.69	-0.01
time (sec)	N/A	0.046	0.136	0.115	0.272	0.341	0.000	3.755	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	69	104	101	61	0	151	127
N.S.	1	1.00	0.70	1.05	1.02	0.62	0.00	1.53	1.28
time (sec)	N/A	0.038	0.134	0.111	0.279	0.404	0.000	3.831	0.530

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	99	117	120	106	0	113	-1
N.S.	1	1.00	0.83	0.98	1.01	0.89	0.00	0.95	-0.01
time (sec)	N/A	0.067	0.193	0.251	0.486	0.350	0.000	4.289	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	102	117	120	122	0	153	-1
N.S.	1	1.00	0.84	0.96	0.98	1.00	0.00	1.25	-0.01
time (sec)	N/A	0.069	0.225	0.287	0.505	0.396	0.000	5.221	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	101	117	137	122	0	190	-1
N.S.	1	1.00	0.80	0.92	1.08	0.96	0.00	1.50	-0.01
time (sec)	N/A	0.071	0.242	0.270	0.488	0.365	0.000	5.655	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	101	117	154	122	0	227	-1
N.S.	1	1.00	0.80	0.92	1.21	0.96	0.00	1.79	-0.01
time (sec)	N/A	0.070	0.329	0.281	0.490	0.359	0.000	3.817	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	356	356	249	294	0	0	0	0	-1
N.S.	1	1.00	0.70	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.171	9.024	0.066	0.000	0.000	0.000	0.000	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	244	277	0	0	0	0	-1
N.S.	1	1.00	0.74	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.152	7.464	0.051	0.000	0.000	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	308	308	239	260	0	0	0	0	-1
N.S.	1	1.00	0.78	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.109	6.568	0.058	0.000	0.000	0.000	0.000	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	235	260	0	0	0	0	-1
N.S.	1	1.00	0.75	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.110	6.300	0.059	0.000	0.000	0.000	0.000	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	247	260	0	0	0	0	-1
N.S.	1	1.00	0.79	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.122	10.172	0.061	0.000	0.000	0.000	0.000	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	331	331	244	259	0	0	0	0	-1
N.S.	1	1.00	0.74	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.145	10.202	0.066	0.000	0.000	0.000	0.000	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	135	282	0	315	0	138	-1
N.S.	1	1.00	0.88	1.84	0.00	2.06	0.00	0.90	-0.01
time (sec)	N/A	0.136	0.391	0.076	0.000	0.384	0.000	4.759	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	104	176	0	233	0	98	-1
N.S.	1	1.00	1.04	1.76	0.00	2.33	0.00	0.98	-0.01
time (sec)	N/A	0.061	0.316	0.069	0.000	0.400	0.000	5.270	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	78	94	0	178	0	69	92
N.S.	1	1.00	1.03	1.24	0.00	2.34	0.00	0.91	1.21
time (sec)	N/A	0.042	0.274	0.049	0.000	0.378	0.000	4.058	1.046

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	76	0	517	0	0	81
N.S.	1	1.00	0.94	0.84	0.00	5.74	0.00	0.00	0.90
time (sec)	N/A	0.062	0.231	0.040	0.000	0.445	0.000	0.000	0.760

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	81	105	0	197	0	124	103
N.S.	1	1.00	1.01	1.31	0.00	2.46	0.00	1.55	1.29
time (sec)	N/A	0.052	0.322	0.058	0.000	0.401	0.000	3.704	0.776

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	145	194	0	255	0	339	-1
N.S.	1	1.00	1.17	1.56	0.00	2.06	0.00	2.73	-0.01
time (sec)	N/A	0.096	0.536	0.066	0.000	0.439	0.000	3.508	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	186	307	0	339	0	571	-1
N.S.	1	1.00	1.05	1.73	0.00	1.92	0.00	3.23	-0.01
time (sec)	N/A	0.158	0.747	0.086	0.000	0.568	0.000	3.192	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	403	403	532	815	0	0	0	0	-1
N.S.	1	1.00	1.32	2.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.182	11.362	0.043	0.000	0.000	0.000	0.000	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	479	607	0	0	0	0	-1
N.S.	1	1.00	1.43	1.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.097	10.870	0.040	0.000	0.000	0.000	0.000	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	302	362	0	0	0	0	-1
N.S.	1	1.00	1.07	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.053	10.142	0.025	0.000	0.000	0.000	0.000	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	448	386	0	0	0	0	-1
N.S.	1	1.00	1.44	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.083	10.665	0.055	0.000	0.000	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	373	656	0	0	0	0	-1
N.S.	1	1.00	0.99	1.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.143	10.423	0.051	0.000	0.000	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	64	87	90	56	0	60	-1
N.S.	1	1.00	0.65	0.89	0.92	0.57	0.00	0.61	-0.01
time (sec)	N/A	0.053	0.111	0.104	0.303	0.363	0.000	3.968	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	70	73	51	0	53	-1
N.S.	1	1.00	0.77	0.91	0.95	0.66	0.00	0.69	-0.01
time (sec)	N/A	0.042	0.098	0.099	0.277	0.360	0.000	4.154	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	53	56	46	0	46	-1
N.S.	1	1.00	0.96	0.95	1.00	0.82	0.00	0.82	-0.02
time (sec)	N/A	0.029	0.091	0.099	0.270	0.368	0.000	4.030	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	47	36	39	39	0	39	35
N.S.	1	1.00	0.96	0.73	0.80	0.80	0.00	0.80	0.71
time (sec)	N/A	0.020	0.083	0.102	0.283	0.414	0.000	3.802	0.523

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	63	52	58	75	0	78	56
N.S.	1	1.00	0.91	0.75	0.84	1.09	0.00	1.13	0.81
time (sec)	N/A	0.038	0.105	0.250	0.537	0.365	0.000	3.838	1.009

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	58	49	51	78	0	101	83
N.S.	1	1.00	0.94	0.79	0.82	1.26	0.00	1.63	1.34
time (sec)	N/A	0.031	0.128	0.194	0.492	0.398	0.000	3.531	0.663

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	63	66	68	83	0	145	-1
N.S.	1	1.00	0.76	0.80	0.82	1.00	0.00	1.75	-0.01
time (sec)	N/A	0.042	0.159	0.208	0.487	0.354	0.000	3.724	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	75	83	85	90	0	167	-1
N.S.	1	1.00	0.72	0.80	0.82	0.87	0.00	1.61	-0.01
time (sec)	N/A	0.054	0.204	0.193	0.492	0.351	0.000	3.631	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	229	226	0	0	0	0	-1
N.S.	1	1.00	0.77	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.119	10.163	0.061	0.000	0.000	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	222	208	0	0	0	0	-1
N.S.	1	1.00	0.82	0.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.082	10.157	0.050	0.000	0.000	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	159	194	0	0	0	0	-1
N.S.	1	1.00	0.62	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.059	10.072	0.036	0.000	0.000	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	278	278	224	211	0	0	0	0	-1
N.S.	1	1.00	0.81	0.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.086	10.177	0.049	0.000	0.000	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	302	302	237	228	0	0	0	0	-1
N.S.	1	1.00	0.78	0.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.117	10.181	0.058	0.000	0.000	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	59	91	73	86	0	52	-1
N.S.	1	1.00	0.77	1.18	0.95	1.12	0.00	0.68	-0.01
time (sec)	N/A	0.041	0.179	0.112	0.273	0.380	0.000	4.426	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	54	95	56	81	0	46	52
N.S.	1	1.00	0.96	1.70	1.00	1.45	0.00	0.82	0.93
time (sec)	N/A	0.028	0.141	0.112	0.289	0.398	0.000	5.425	0.312

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	44	32	46	0	21	21
N.S.	1	1.00	1.00	1.76	1.28	1.84	0.00	0.84	0.84
time (sec)	N/A	0.013	0.144	0.037	0.278	0.349	0.000	6.330	0.238

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	62	67	65	107	0	78	-1
N.S.	1	1.00	0.94	1.02	0.98	1.62	0.00	1.18	-0.02
time (sec)	N/A	0.037	0.201	0.203	0.512	0.343	0.000	3.606	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	70	84	82	124	0	122	-1
N.S.	1	1.00	0.78	0.93	0.91	1.38	0.00	1.36	-0.01
time (sec)	N/A	0.048	0.232	0.201	0.500	0.406	0.000	3.466	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	307	307	219	240	0	0	0	0	-1
N.S.	1	1.00	0.71	0.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.112	10.162	0.072	0.000	0.000	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	219	240	0	0	0	0	-1
N.S.	1	1.00	0.77	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.089	10.162	0.061	0.000	0.000	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	219	240	0	0	0	0	-1
N.S.	1	1.00	0.78	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.082	10.200	0.048	0.000	0.000	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	228	257	0	0	0	0	-1
N.S.	1	1.00	0.74	0.83	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.117	10.160	0.076	0.000	0.000	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F(-2)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	234	274	0	0	0	0	-1
N.S.	1	1.00	0.72	0.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.145	10.172	0.065	0.000	0.000	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	430	0	0	0	0	0	-1
N.S.	1	1.00	1.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	10.653	0.011	0.000	0.000	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	386	0	0	0	0	0	-1
N.S.	1	1.00	1.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.224	15.520	0.009	0.000	0.000	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	386	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	10.476	0.010	0.000	0.000	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	370	0	0	0	0	0	-1
N.S.	1	1.00	1.25	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.220	10.571	0.008	0.000	0.000	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	567	0	0	0	0	0	-1
N.S.	1	1.00	1.90	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.240	10.938	0.010	0.000	0.000	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	299	490	0	0	0	0	0	-1
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	15.782	0.009	0.000	0.000	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	487	0	0	0	0	0	-1
N.S.	1	1.00	1.64	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.245	10.800	0.011	0.000	0.000	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	447	0	0	0	0	0	-1
N.S.	1	1.00	1.51	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	10.682	0.012	0.000	0.000	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	354	0	0	0	0	0	-1
N.S.	1	1.00	1.19	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	10.404	0.007	0.000	0.000	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	242	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.233	15.143	0.010	0.000	0.000	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	241	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	10.132	0.012	0.000	0.000	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	356	0	0	0	0	0	-1
N.S.	1	1.00	1.21	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.233	10.422	0.011	0.000	0.000	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	375	0	0	0	0	0	-1
N.S.	1	1.00	1.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.251	10.502	0.011	0.000	0.000	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	397	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.250	15.523	0.010	0.000	0.000	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	395	0	0	0	0	0	-1
N.S.	1	1.00	1.31	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	10.472	0.019	0.000	0.000	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	301	301	460	0	0	0	0	0	-1
N.S.	1	1.00	1.53	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.249	10.655	0.015	0.000	0.000	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	191	1935	438	1376	11266	2816	769
N.S.	1	1.00	0.79	7.96	1.80	5.66	46.36	11.59	3.16
time (sec)	N/A	0.115	0.770	0.030	0.309	0.389	1.618	5.868	1.058

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	117	783	248	578	4068	1178	429
N.S.	1	1.00	0.75	5.05	1.60	3.73	26.25	7.60	2.77
time (sec)	N/A	0.065	0.445	0.017	0.294	0.414	0.856	3.826	0.598

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	59	82	113	179	1015	350	171
N.S.	1	1.00	0.71	0.99	1.36	2.16	12.23	4.22	2.06
time (sec)	N/A	0.033	0.072	0.017	0.309	0.383	0.389	3.330	0.339

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	156	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.330	0.016	0.000	0.000	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	392	358	160	0	0	0	0	0	-1
N.S.	1	0.91	0.41	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.775	0.566	0.006	0.000	0.000	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	466	0	0	0	0	0	-1
N.S.	1	1.00	1.46	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	1.761	0.006	0.000	0.000	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	0.859	0.006	0.000	0.000	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	267	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	1.592	0.008	0.000	0.000	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	307	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	10.262	0.004	0.000	0.000	0.000	0.000	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	134	103	118	273	0	121	181
N.S.	1	1.00	1.00	0.77	0.88	2.04	0.00	0.90	1.35
time (sec)	N/A	0.116	0.042	0.176	0.498	3.269	0.000	5.690	0.873

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	99	92	104	211	0	105	166
N.S.	1	1.00	0.84	0.78	0.88	1.79	0.00	0.89	1.41
time (sec)	N/A	0.098	0.065	0.171	0.491	1.389	0.000	4.385	0.722

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	77	80	89	172	0	90	138
N.S.	1	1.00	0.73	0.76	0.85	1.64	0.00	0.86	1.31
time (sec)	N/A	0.092	0.024	0.177	0.496	0.949	0.000	3.819	0.989

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	66	72	81	147	0	86	944
N.S.	1	1.00	0.69	0.75	0.84	1.53	0.00	0.90	9.83
time (sec)	N/A	0.060	0.028	0.169	0.498	0.507	0.000	4.577	1.936

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	67	75	82	150	0	85	328
N.S.	1	1.00	0.70	0.78	0.85	1.56	0.00	0.89	3.42
time (sec)	N/A	0.042	0.026	0.191	0.532	0.601	0.000	3.105	1.020

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	134	90	99	199	0	102	527
N.S.	1	1.00	1.18	0.79	0.87	1.75	0.00	0.89	4.62
time (sec)	N/A	0.084	0.042	0.187	0.500	4.720	0.000	3.581	0.961

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	169	107	119	275	0	132	820
N.S.	1	1.00	1.31	0.83	0.92	2.13	0.00	1.02	6.36
time (sec)	N/A	0.095	0.067	0.185	0.496	22.839	0.000	3.920	1.381

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	209	127	143	350	0	168	1017
N.S.	1	1.00	1.34	0.81	0.92	2.24	0.00	1.08	6.52
time (sec)	N/A	0.119	0.063	0.182	0.486	54.530	0.000	5.416	1.868

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	344	279	291	4154	0	363	2500
N.S.	1	1.00	0.96	0.78	0.81	11.57	0.00	1.01	6.96
time (sec)	N/A	0.226	0.258	0.203	0.503	5.531	0.000	2.999	2.056

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	373	263	287	4110	0	333	2500
N.S.	1	1.00	1.08	0.76	0.83	11.91	0.00	0.97	7.25
time (sec)	N/A	0.194	0.165	0.183	0.506	1.134	0.000	3.642	1.827

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	233	254	266	3790	0	327	2500
N.S.	1	1.00	0.69	0.76	0.79	11.28	0.00	0.97	7.44
time (sec)	N/A	0.181	0.103	0.188	0.505	0.586	0.000	3.652	2.199

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	337	337	232	246	274	3651	0	336	2500
N.S.	1	1.00	0.69	0.73	0.81	10.83	0.00	1.00	7.42
time (sec)	N/A	0.172	0.085	0.179	0.501	0.443	0.000	4.004	1.589

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	234	253	266	3842	0	339	2500
N.S.	1	1.00	0.70	0.75	0.79	11.43	0.00	1.01	7.44
time (sec)	N/A	0.170	0.097	0.155	0.493	0.903	0.000	3.608	1.672

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	389	266	290	4120	0	348	2500
N.S.	1	1.00	1.12	0.76	0.83	11.84	0.00	1.00	7.18
time (sec)	N/A	0.191	0.170	0.191	0.497	1.961	0.000	3.261	1.999

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	367	284	296	4228	0	364	2500
N.S.	1	1.00	1.02	0.79	0.82	11.74	0.00	1.01	6.94
time (sec)	N/A	0.195	0.286	0.181	0.513	5.607	0.000	4.137	2.257

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	135	160	214	548	0	251	305
N.S.	1	1.00	0.80	0.95	1.27	3.24	0.00	1.49	1.80
time (sec)	N/A	0.238	0.143	0.227	0.498	9.525	0.000	3.619	1.302

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	142	146	191	460	0	223	647
N.S.	1	1.00	0.95	0.97	1.27	3.07	0.00	1.49	4.31
time (sec)	N/A	0.163	0.075	0.186	0.489	4.635	0.000	4.382	1.487

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	153	120	136	187	484	0	220	528
N.S.	1	0.99	0.77	0.88	1.21	3.12	0.00	1.42	3.41
time (sec)	N/A	0.160	0.106	0.178	0.504	1.997	0.000	3.220	1.520

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	148	114	132	178	475	0	188	527
N.S.	1	0.99	0.77	0.89	1.19	3.19	0.00	1.26	3.54
time (sec)	N/A	0.119	0.099	0.200	0.511	1.929	0.000	3.150	1.411

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	117	146	187	433	0	199	649
N.S.	1	1.00	0.77	0.97	1.24	2.87	0.00	1.32	4.30
time (sec)	N/A	0.118	0.093	0.201	0.511	4.452	0.000	3.770	1.493

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	241	171	220	671	0	279	1082
N.S.	1	1.00	1.15	0.82	1.05	3.21	0.00	1.33	5.18
time (sec)	N/A	0.159	0.120	0.201	0.496	72.445	0.000	5.193	2.577

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	248	181	269	863	0	344	1337
N.S.	1	1.00	1.05	0.77	1.14	3.66	0.00	1.46	5.67
time (sec)	N/A	0.177	0.287	0.204	0.505	210.296	0.000	3.097	2.942

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	278	209	320	0	0	350	1545
N.S.	1	1.00	1.05	0.79	1.21	0.00	0.00	1.32	5.83
time (sec)	N/A	0.217	0.269	0.206	0.516	0.000	0.000	4.049	3.477

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	712	712	431	334	487	9273	0	581	2500
N.S.	1	1.00	0.61	0.47	0.68	13.02	0.00	0.82	3.51
time (sec)	N/A	0.438	0.202	0.200	0.525	10.104	0.000	3.893	2.864

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	687	687	428	339	462	9239	0	595	2500
N.S.	1	1.00	0.62	0.49	0.67	13.45	0.00	0.87	3.64
time (sec)	N/A	0.383	0.248	0.208	0.506	7.143	0.000	3.571	2.825

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	423	327	470	9091	0	586	2500
N.S.	1	1.00	0.62	0.48	0.69	13.27	0.00	0.86	3.65
time (sec)	N/A	0.389	0.180	0.208	0.506	6.223	0.000	4.419	4.873

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	685	685	428	339	458	9185	0	603	2500
N.S.	1	1.00	0.62	0.49	0.67	13.41	0.00	0.88	3.65
time (sec)	N/A	0.364	0.186	0.234	0.533	6.787	0.000	3.887	2.867

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	689	689	429	334	489	9294	0	603	2500
N.S.	1	1.00	0.62	0.48	0.71	13.49	0.00	0.88	3.63
time (sec)	N/A	0.377	0.174	0.166	0.520	12.504	0.000	4.462	2.732

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	745	745	499	355	506	9600	0	639	2500
N.S.	1	1.00	0.67	0.48	0.68	12.89	0.00	0.86	3.36
time (sec)	N/A	0.497	0.247	0.198	0.521	28.269	0.000	4.201	5.161

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	751	751	513	355	527	9816	0	628	2500
N.S.	1	1.00	0.68	0.47	0.70	13.07	0.00	0.84	3.33
time (sec)	N/A	0.436	0.260	0.232	0.538	63.536	0.000	5.360	5.219

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	40	110	0	31	0	0	-1
N.S.	1	1.00	0.57	1.57	0.00	0.44	0.00	0.00	-0.01
time (sec)	N/A	0.037	10.088	0.202	0.000	0.129	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	36	112	0	55	0	0	-1
N.S.	1	1.00	0.51	1.60	0.00	0.79	0.00	0.00	-0.01
time (sec)	N/A	0.041	10.091	0.198	0.000	0.124	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	46	96	0	0	0	0	-1
N.S.	1	1.00	0.46	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	10.100	0.145	0.000	0.000	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	37	143	0	0	0	0	-1
N.S.	1	1.00	0.61	2.34	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.026	10.074	0.155	0.000	0.000	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	54	99	0	17	0	0	-1
N.S.	1	1.00	0.48	0.88	0.00	0.15	0.00	0.00	-0.01
time (sec)	N/A	0.042	10.068	0.161	0.000	0.096	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	134	0	17	0	0	-1
N.S.	1	1.00	0.61	2.35	0.00	0.30	0.00	0.00	-0.02
time (sec)	N/A	0.035	10.073	0.154	0.000	0.081	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	60	168	0	74	0	0	-1
N.S.	1	1.00	0.81	2.27	0.00	1.00	0.00	0.00	-0.01
time (sec)	N/A	0.040	10.082	0.192	0.000	0.100	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	56	115	0	73	0	0	-1
N.S.	1	1.00	0.76	1.55	0.00	0.99	0.00	0.00	-0.01
time (sec)	N/A	0.041	10.080	0.198	0.000	0.107	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	120	159	124	206	0	156	-1
N.S.	1	1.00	0.49	0.65	0.51	0.85	0.00	0.64	-0.00
time (sec)	N/A	0.086	0.124	0.124	0.295	0.379	0.000	4.421	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	56	51	50	50	0	68	-1
N.S.	1	1.00	0.52	0.47	0.46	0.46	0.00	0.63	-0.01
time (sec)	N/A	0.074	0.027	0.117	0.267	0.341	0.000	3.980	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	96	119	81	155	0	109	-1
N.S.	1	1.00	0.54	0.67	0.46	0.87	0.00	0.61	-0.01
time (sec)	N/A	0.050	0.092	0.131	0.270	0.376	0.000	2.784	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	83	80	45	123	0	84	-1
N.S.	1	1.00	0.55	0.53	0.30	0.81	0.00	0.55	-0.01
time (sec)	N/A	0.063	0.061	0.125	0.284	0.383	0.000	3.477	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	93	130	59	134	0	116	-1
N.S.	1	1.00	0.53	0.73	0.33	0.76	0.00	0.66	-0.01
time (sec)	N/A	0.063	0.106	0.125	0.273	0.366	0.000	3.624	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	90	133	83	141	0	100	-1
N.S.	1	1.00	0.51	0.75	0.47	0.80	0.00	0.56	-0.01
time (sec)	N/A	0.082	0.100	0.137	0.278	0.371	0.000	4.547	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	72	73	72	77	76	79	73
N.S.	1	1.00	0.92	0.94	0.92	0.99	0.97	1.01	0.94
time (sec)	N/A	0.091	0.017	0.171	0.275	0.342	0.012	3.846	0.038

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	78	73	72	77	82	79	73
N.S.	1	1.00	1.00	0.94	0.92	0.99	1.05	1.01	0.94
time (sec)	N/A	0.043	0.011	0.145	0.286	0.350	0.012	3.518	0.028

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	73	72	77	76	79	73
N.S.	1	1.00	0.96	0.97	0.96	1.03	1.01	1.05	0.97
time (sec)	N/A	0.093	0.016	0.141	0.293	0.334	0.012	3.028	0.029

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	73	70	69	74	78	76	70
N.S.	1	1.00	1.00	0.96	0.95	1.01	1.07	1.04	0.96
time (sec)	N/A	0.032	0.011	0.107	0.302	0.374	0.013	2.906	0.029

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	77	73	75	73	79	70
N.S.	1	1.00	1.00	1.04	0.99	1.01	0.99	1.07	0.95
time (sec)	N/A	0.065	0.014	0.108	0.279	0.333	0.061	3.113	0.034

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	71	75	69	78	73	74	70
N.S.	1	1.00	1.00	1.06	0.97	1.10	1.03	1.04	0.99
time (sec)	N/A	0.033	0.022	0.118	0.299	0.346	0.059	5.144	0.033

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	71	74	73	83	71	97	70
N.S.	1	1.00	0.96	1.00	0.99	1.12	0.96	1.31	0.95
time (sec)	N/A	0.064	0.029	0.115	0.286	0.368	0.115	6.219	0.038

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	165	159	154	415	320	160	251
N.S.	1	1.00	0.98	0.95	0.92	2.47	1.90	0.95	1.49
time (sec)	N/A	0.158	0.090	0.132	0.521	0.367	0.672	4.389	0.325

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	133	122	122	341	189	125	179
N.S.	1	1.00	0.99	0.90	0.90	2.53	1.40	0.93	1.33
time (sec)	N/A	0.105	0.053	0.137	0.504	0.387	0.709	4.156	0.322

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	102	90	90	300	162	91	95
N.S.	1	1.00	0.96	0.85	0.85	2.83	1.53	0.86	0.90
time (sec)	N/A	0.072	0.043	0.131	0.524	0.353	0.606	5.892	0.342

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	88	79	74	266	153	75	77
N.S.	1	1.05	1.06	0.95	0.89	3.20	1.84	0.90	0.93
time (sec)	N/A	0.059	0.038	0.126	0.499	0.345	0.430	4.109	0.357

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	86	89	85	81	274	155	83	81
N.S.	1	0.97	1.00	0.96	0.91	3.08	1.74	0.93	0.91
time (sec)	N/A	0.075	0.040	0.144	0.479	0.378	0.608	4.408	0.367

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	105	94	101	340	167	94	98
N.S.	1	1.00	0.99	0.89	0.95	3.21	1.58	0.89	0.92
time (sec)	N/A	0.092	0.044	0.134	0.493	0.346	0.808	3.663	0.357

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	135	122	135	389	284	131	128
N.S.	1	1.00	0.99	0.90	0.99	2.86	2.09	0.96	0.94
time (sec)	N/A	0.166	0.061	0.133	0.493	0.374	1.094	3.513	0.383

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	166	149	165	458	328	164	156
N.S.	1	1.00	0.99	0.89	0.99	2.74	1.96	0.98	0.93
time (sec)	N/A	0.213	0.064	0.132	0.516	0.373	1.347	4.411	0.403

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	170	151	165	488	235	160	223
N.S.	1	1.00	0.98	0.87	0.95	2.82	1.36	0.92	1.29
time (sec)	N/A	0.213	0.074	0.138	0.511	0.372	2.022	4.469	0.354

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	141	120	132	450	212	125	137
N.S.	1	1.00	0.99	0.84	0.92	3.15	1.48	0.87	0.96
time (sec)	N/A	0.139	0.059	0.144	0.491	0.369	1.873	4.675	0.344

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	122	106	114	415	201	107	118
N.S.	1	1.00	0.98	0.85	0.92	3.35	1.62	0.86	0.95
time (sec)	N/A	0.091	0.069	0.162	0.515	0.362	1.374	4.893	0.386

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	119	110	107	110	389	196	101	112
N.S.	1	1.03	0.96	0.93	0.96	3.38	1.70	0.88	0.97
time (sec)	N/A	0.077	0.065	0.134	0.509	0.359	0.785	3.228	0.379

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	124	124	111	121	428	202	110	118
N.S.	1	0.98	0.98	0.87	0.95	3.37	1.59	0.87	0.93
time (sec)	N/A	0.131	0.093	0.159	0.517	0.384	1.099	3.299	0.390

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	141	124	143	491	214	128	138
N.S.	1	1.00	0.99	0.87	1.01	3.46	1.51	0.90	0.97
time (sec)	N/A	0.139	0.057	0.135	0.503	0.372	1.422	4.456	0.398

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	173	151	175	534	330	164	168
N.S.	1	1.00	1.01	0.88	1.02	3.12	1.93	0.96	0.98
time (sec)	N/A	0.239	0.077	0.136	0.507	0.364	1.949	4.014	0.412

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	228	216	0	0	0	236	2500
N.S.	1	1.00	0.99	0.94	0.00	0.00	0.00	1.03	10.87
time (sec)	N/A	0.333	0.162	0.256	0.000	0.000	0.000	4.166	69.941

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	186	172	0	609	0	194	2304
N.S.	1	1.00	0.98	0.91	0.00	3.22	0.00	1.03	12.19
time (sec)	N/A	0.217	0.122	0.180	0.000	117.401	0.000	5.482	15.207

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	139	137	0	421	0	157	1853
N.S.	1	1.00	0.88	0.87	0.00	2.66	0.00	0.99	11.73
time (sec)	N/A	0.170	0.070	0.227	0.000	33.105	0.000	4.524	11.051

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	114	112	0	325	0	133	2500
N.S.	1	1.00	0.86	0.85	0.00	2.46	0.00	1.01	18.94
time (sec)	N/A	0.105	0.046	0.161	0.000	11.051	0.000	5.531	9.751

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	112	113	0	329	0	134	2434
N.S.	1	1.00	0.84	0.85	0.00	2.47	0.00	1.01	18.30
time (sec)	N/A	0.084	0.046	0.197	0.000	7.889	0.000	6.404	8.707

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	242	166	0	0	0	172	2500
N.S.	1	1.00	1.45	0.99	0.00	0.00	0.00	1.03	14.97
time (sec)	N/A	0.202	0.204	0.217	0.000	0.000	0.000	7.255	17.199

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	331	215	0	0	0	237	2500
N.S.	1	1.00	1.61	1.05	0.00	0.00	0.00	1.16	12.20
time (sec)	N/A	0.267	0.217	0.204	0.000	0.000	0.000	4.371	62.948

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	426	284	0	0	0	332	2500
N.S.	1	1.00	1.59	1.06	0.00	0.00	0.00	1.24	9.33
time (sec)	N/A	0.366	0.274	0.234	0.000	0.000	0.000	4.452	144.755

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	463	413	0	24206	0	12506	2500
N.S.	1	1.00	1.20	1.07	0.00	62.55	0.00	32.32	6.46
time (sec)	N/A	2.767	0.397	0.235	0.000	162.321	0.000	13.667	7.134

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	385	331	0	19849	0	11030	2500
N.S.	1	1.00	1.19	1.02	0.00	61.45	0.00	34.15	7.74
time (sec)	N/A	0.945	0.338	0.209	0.000	25.984	0.000	9.917	6.446

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	323	264	0	15257	0	8658	2500
N.S.	1	1.00	1.15	0.94	0.00	54.49	0.00	30.92	8.93
time (sec)	N/A	0.615	0.217	0.184	0.000	2.600	0.000	6.545	5.800

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	251	251	277	213	0	11974	0	6921	2500
N.S.	1	1.00	1.10	0.85	0.00	47.71	0.00	27.57	9.96
time (sec)	N/A	0.290	0.323	0.201	0.000	1.294	0.000	6.348	4.959

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	274	215	0	15733	0	7650	2500
N.S.	1	1.00	1.08	0.85	0.00	61.94	0.00	30.12	9.84
time (sec)	N/A	0.359	0.161	0.114	0.000	13.063	0.000	5.890	5.614

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	340	276	0	20327	0	10058	2500
N.S.	1	1.00	1.14	0.93	0.00	68.21	0.00	33.75	8.39
time (sec)	N/A	0.624	0.261	0.214	0.000	163.302	0.000	7.535	5.890

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	410	349	0	0	0	12268	2500
N.S.	1	1.00	1.18	1.00	0.00	0.00	0.00	35.25	7.18
time (sec)	N/A	1.062	0.359	0.216	0.000	0.000	0.000	7.604	6.726

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	866	866	215	264	0	0	0	0	2500
N.S.	1	1.00	0.25	0.30	0.00	0.00	0.00	0.00	2.89
time (sec)	N/A	1.767	0.253	0.167	0.000	0.000	0.000	0.000	6.836

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	259	1013	0	1501	0	0	-1
N.S.	1	1.00	0.95	3.72	0.00	5.52	0.00	0.00	-0.00
time (sec)	N/A	0.384	0.961	0.172	0.000	98.959	0.000	0.000	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	200	868	0	1247	0	0	-1
N.S.	1	1.00	0.96	4.17	0.00	6.00	0.00	0.00	-0.00
time (sec)	N/A	0.212	0.598	0.157	0.000	11.533	0.000	0.000	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	153	757	0	1074	0	0	-1
N.S.	1	1.00	0.91	4.51	0.00	6.39	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.433	0.158	0.000	1.029	0.000	0.000	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	174	858	0	2407	0	0	-1
N.S.	1	1.00	0.94	4.61	0.00	12.94	0.00	0.00	-0.01
time (sec)	N/A	0.168	0.493	0.130	0.000	42.788	0.000	0.000	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	158	1004	0	1130	0	216	-1
N.S.	1	1.00	0.44	2.78	0.00	3.13	0.00	0.60	-0.00
time (sec)	N/A	0.353	0.666	0.162	0.000	0.628	0.000	4.630	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	424	619	209	528	0	0	0	0	-1
N.S.	1	1.46	0.49	1.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	5.529	0.166	0.000	0.000	0.000	0.000	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	591	204	509	0	0	0	0	-1
N.S.	1	1.42	0.49	1.22	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	4.921	0.128	0.000	0.000	0.000	0.000	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	381	470	127	341	0	0	0	0	-1
N.S.	1	1.23	0.33	0.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.133	9.737	0.135	0.000	0.000	0.000	0.000	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	208	511	0	0	0	0	-1
N.S.	1	1.00	0.52	1.28	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.143	10.135	0.142	0.000	0.000	0.000	0.000	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	154	448	0	0	0	0	-1
N.S.	1	1.00	0.43	1.24	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.123	10.126	0.207	0.000	0.000	0.000	0.000	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	546	546	224	549	0	0	0	0	-1
N.S.	1	1.00	0.41	1.01	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	10.158	0.139	0.000	0.000	0.000	0.000	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	482	482	545	1985	0	0	0	0	-1
N.S.	1	1.00	1.13	4.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.732	10.661	0.174	0.000	0.000	0.000	0.000	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	345	1663	0	0	0	0	-1
N.S.	1	1.00	0.96	4.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.452	1.983	0.164	0.000	0.000	0.000	0.000	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	260	1411	0	1573	0	0	-1
N.S.	1	1.00	0.97	5.25	0.00	5.85	0.00	0.00	-0.00
time (sec)	N/A	0.290	1.085	0.191	0.000	115.144	0.000	0.000	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	350	350	258	1613	0	0	0	0	-1
N.S.	1	1.00	0.74	4.61	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.364	0.939	0.136	0.000	0.000	0.000	0.000	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	562	562	230	1789	0	0	0	0	-1
N.S.	1	1.00	0.41	3.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.601	1.130	0.192	0.000	0.000	0.000	0.000	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	463	875	214	547	0	0	0	0	-1
N.S.	1	1.89	0.46	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	7.271	0.145	0.000	0.000	0.000	0.000	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	428	602	209	377	0	0	0	0	-1
N.S.	1	1.41	0.49	0.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.219	10.123	0.171	0.000	0.000	0.000	0.000	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	722	722	213	528	0	0	0	0	-1
N.S.	1	1.00	0.30	0.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	10.158	0.153	0.000	0.000	0.000	0.000	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	625	625	219	530	0	0	0	0	-1
N.S.	1	1.00	0.35	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.235	10.167	0.151	0.000	0.000	0.000	0.000	0.000

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	553	553	224	549	0	0	0	0	-1
N.S.	1	1.00	0.41	0.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.273	10.164	0.144	0.000	0.000	0.000	0.000	0.000

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	178	266	0	1364	0	0	-1
N.S.	1	1.00	1.03	1.54	0.00	7.88	0.00	0.00	-0.01
time (sec)	N/A	0.209	0.659	0.161	0.000	12.613	0.000	0.000	0.000

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	146	204	0	1100	0	0	-1
N.S.	1	1.00	1.07	1.49	0.00	8.03	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.442	0.128	0.000	1.029	0.000	0.000	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	96	165	0	367	0	75	-1
N.S.	1	1.00	1.12	1.92	0.00	4.27	0.00	0.87	-0.01
time (sec)	N/A	0.057	0.305	0.122	0.000	0.414	0.000	4.192	0.000

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	144	207	0	1121	0	0	-1
N.S.	1	1.00	1.04	1.50	0.00	8.12	0.00	0.00	-0.01
time (sec)	N/A	0.133	0.475	0.133	0.000	0.530	0.000	0.000	0.000

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	163	275	0	1486	0	208	-1
N.S.	1	1.00	0.75	1.26	0.00	6.82	0.00	0.95	-0.00
time (sec)	N/A	0.179	0.884	0.155	0.000	0.794	0.000	3.458	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	127	222	0	0	0	0	-1
N.S.	1	1.00	0.30	0.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.117	10.155	0.115	0.000	0.000	0.000	0.000	0.000

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	99	134	0	0	0	0	-1
N.S.	1	1.00	0.40	0.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.080	10.111	0.136	0.000	0.000	0.000	0.000	0.000

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	80	70	0	0	0	0	-1
N.S.	1	1.00	0.33	0.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.070	10.042	0.124	0.000	0.000	0.000	0.000	0.000

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	147	178	0	0	0	0	-1
N.S.	1	1.00	0.37	0.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.218	10.157	0.140	0.000	0.000	0.000	0.000	0.000

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	219	260	0	0	0	0	-1
N.S.	1	1.00	0.52	0.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.328	10.132	0.134	0.000	0.000	0.000	0.000	0.000

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	251	696	0	4905	0	0	-1
N.S.	1	1.00	1.06	2.95	0.00	20.78	0.00	0.00	-0.00
time (sec)	N/A	0.311	0.967	0.136	0.000	38.183	0.000	0.000	0.000

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	180	542	0	1399	0	458	-1
N.S.	1	1.00	1.08	3.25	0.00	8.38	0.00	2.74	-0.01
time (sec)	N/A	0.193	0.654	0.141	0.000	0.604	0.000	4.114	0.000

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	172	499	0	1367	0	441	-1
N.S.	1	1.00	1.08	3.14	0.00	8.60	0.00	2.77	-0.01
time (sec)	N/A	0.124	0.708	0.136	0.000	0.652	0.000	2.939	0.000

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	177	454	0	1377	0	454	-1
N.S.	1	1.00	1.07	2.73	0.00	8.30	0.00	2.73	-0.01
time (sec)	N/A	0.113	0.621	0.129	0.000	0.635	0.000	3.075	0.000

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	247	563	0	4917	0	0	-1
N.S.	1	1.00	0.93	2.12	0.00	18.48	0.00	0.00	-0.00
time (sec)	N/A	0.243	1.124	0.336	0.000	2.480	0.000	0.000	0.000

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	419	419	363	764	0	6590	0	762	-1
N.S.	1	1.00	0.87	1.82	0.00	15.73	0.00	1.82	-0.00
time (sec)	N/A	0.353	1.713	0.412	0.000	5.418	0.000	5.004	0.000

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	566	199	603	0	0	0	0	-1
N.S.	1	1.26	0.44	1.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	10.200	0.135	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	503	199	586	0	0	0	0	-1
N.S.	1	1.19	0.47	1.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.169	9.645	0.141	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	501	199	561	0	0	0	0	-1
N.S.	1	1.19	0.47	1.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.156	7.134	0.138	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	503	199	536	0	0	0	0	-1
N.S.	1	1.19	0.47	1.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.162	7.028	0.157	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	422	501	199	366	0	0	0	0	-1
N.S.	1	1.19	0.47	0.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.154	10.111	0.143	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	468	644	211	553	0	0	0	0	-1
N.S.	1	1.38	0.45	1.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	10.161	0.150	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	475	457	0	0	0	928	2500
N.S.	1	1.00	1.17	1.13	0.00	0.00	0.00	2.29	6.16
time (sec)	N/A	5.439	1.438	0.168	0.000	0.000	0.000	4.070	2.476

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	383	335	0	4208	0	745	2500
N.S.	1	1.00	1.18	1.03	0.00	12.99	0.00	2.30	7.72
time (sec)	N/A	2.493	1.058	0.148	0.000	103.298	0.000	3.632	1.994

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	350	273	0	2452	0	619	2500
N.S.	1	1.00	1.20	0.93	0.00	8.40	0.00	2.12	8.56
time (sec)	N/A	2.434	0.941	0.139	0.000	29.463	0.000	4.236	2.342

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	256	177	0	1037	0	228	717
N.S.	1	1.00	1.27	0.88	0.00	5.13	0.00	1.13	3.55
time (sec)	N/A	0.244	0.573	0.118	0.000	6.191	0.000	3.962	1.723

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	274	299	0	3169	0	717	2500
N.S.	1	1.00	0.98	1.06	0.00	11.28	0.00	2.55	8.90
time (sec)	N/A	0.831	0.673	0.147	0.000	49.977	0.000	3.946	6.875

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	382	370	348	404	0	0	0	0	2500
N.S.	1	0.97	0.91	1.06	0.00	0.00	0.00	0.00	6.54
time (sec)	N/A	2.756	1.186	0.161	0.000	0.000	0.000	0.000	5.460

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	552	552	445	597	0	0	0	1055	2500
N.S.	1	1.00	0.81	1.08	0.00	0.00	0.00	1.91	4.53
time (sec)	N/A	2.815	1.848	0.168	0.000	0.000	0.000	5.380	7.300

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	390	390	10915	294	0	3310	0	53	-1
N.S.	1	1.00	27.99	0.75	0.00	8.49	0.00	0.14	-0.00
time (sec)	N/A	2.176	16.332	0.162	0.000	15.596	0.000	5.439	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	324	324	4331	222	0	1663	0	27	-1
N.S.	1	1.00	13.37	0.69	0.00	5.13	0.00	0.08	-0.00
time (sec)	N/A	1.016	16.033	0.127	0.000	2.660	0.000	3.849	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	250	161	0	1007	0	0	-1
N.S.	1	1.00	1.04	0.67	0.00	4.20	0.00	0.00	-0.00
time (sec)	N/A	0.233	8.662	0.124	0.000	0.868	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	291	4644	270	0	2435	0	0	-1
N.S.	1	1.00	15.96	0.93	0.00	8.37	0.00	0.00	-0.00
time (sec)	N/A	0.490	16.249	0.157	0.000	1.642	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	7777	318	0	4132	0	0	-1
N.S.	1	1.00	20.85	0.85	0.00	11.08	0.00	0.00	-0.00
time (sec)	N/A	1.715	16.307	0.153	0.000	6.467	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	512	10933	417	0	5811	0	0	-1
N.S.	1	1.00	21.35	0.81	0.00	11.35	0.00	0.00	-0.00
time (sec)	N/A	3.501	16.409	0.157	0.000	15.802	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	460	460	501	444	0	0	0	857	2500
N.S.	1	1.00	1.09	0.97	0.00	0.00	0.00	1.86	5.43
time (sec)	N/A	3.583	1.601	0.145	0.000	0.000	0.000	4.964	3.413

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	327	327	373	277	0	4378	0	649	2500
N.S.	1	1.00	1.14	0.85	0.00	13.39	0.00	1.98	7.65
time (sec)	N/A	0.957	1.137	0.148	0.000	68.190	0.000	7.193	4.129

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	379	381	0	0	0	827	2500
N.S.	1	1.00	1.10	1.10	0.00	0.00	0.00	2.39	7.23
time (sec)	N/A	1.096	1.267	0.129	0.000	0.000	0.000	7.133	7.673

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	417	416	427	535	0	0	0	433	2500
N.S.	1	1.00	1.02	1.28	0.00	0.00	0.00	1.04	6.00
time (sec)	N/A	2.274	1.549	0.157	0.000	0.000	0.000	5.538	6.097

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	595	595	18689	459	0	0	0	104	-1
N.S.	1	1.00	31.41	0.77	0.00	0.00	0.00	0.17	-0.00
time (sec)	N/A	2.383	16.354	0.162	0.000	0.000	0.000	6.188	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	491	14032	347	0	5968	0	56	-1
N.S.	1	1.00	28.58	0.71	0.00	12.15	0.00	0.11	-0.00
time (sec)	N/A	1.199	16.199	0.162	0.000	52.900	0.000	4.504	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	9290	215	0	3855	0	27	-1
N.S.	1	1.00	19.08	0.44	0.00	7.92	0.00	0.06	-0.00
time (sec)	N/A	1.070	16.129	0.127	0.000	15.373	0.000	7.769	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	432	7789	351	0	4096	0	0	-1
N.S.	1	1.66	29.96	1.35	0.00	15.75	0.00	0.00	-0.00
time (sec)	N/A	0.583	16.256	0.161	0.000	8.466	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	F	B	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	9321	478	0	7833	0	0	-1
N.S.	1	1.00	17.82	0.91	0.00	14.98	0.00	0.00	-0.00
time (sec)	N/A	1.800	16.309	0.161	0.000	32.028	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	346	463	0	3615	0	4637	917
N.S.	1	1.00	1.23	1.65	0.00	12.86	0.00	16.50	3.26
time (sec)	N/A	4.878	0.851	0.159	0.000	3.727	0.000	7.630	1.450

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	285	383	0	2053	0	4060	776
N.S.	1	1.00	1.24	1.67	0.00	8.97	0.00	17.73	3.39
time (sec)	N/A	1.205	0.577	0.147	0.000	1.490	0.000	6.046	1.311

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	169	360	0	871	0	591	649
N.S.	1	1.00	0.93	1.98	0.00	4.79	0.00	3.25	3.57
time (sec)	N/A	0.176	0.237	0.133	0.000	0.810	0.000	5.099	1.288

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	264	475	0	1232	0	3639	669
N.S.	1	1.00	1.10	1.97	0.00	5.11	0.00	15.10	2.78
time (sec)	N/A	1.034	0.573	0.137	0.000	2.646	0.000	8.100	1.298

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	307	580	0	2799	0	1675	825
N.S.	1	1.00	1.06	2.00	0.00	9.65	0.00	5.78	2.84
time (sec)	N/A	1.523	1.538	0.166	0.000	7.203	0.000	6.678	1.410

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	588	226	0	2860	0	1709	1024
N.S.	1	1.00	1.81	0.70	0.00	8.80	0.00	5.26	3.15
time (sec)	N/A	3.679	0.644	0.152	0.000	1.344	0.000	7.539	1.300

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	412	175	0	1430	0	3580	870
N.S.	1	1.00	1.57	0.67	0.00	5.44	0.00	13.61	3.31
time (sec)	N/A	1.300	0.431	0.129	0.000	0.726	0.000	5.381	1.272

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	155	130	0	759	0	641	989
N.S.	1	1.00	0.70	0.59	0.00	3.45	0.00	2.91	4.50
time (sec)	N/A	0.156	0.117	0.125	0.000	0.454	0.000	5.621	1.268

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	265	265	471	212	0	1998	0	3965	1234
N.S.	1	1.00	1.78	0.80	0.00	7.54	0.00	14.96	4.66
time (sec)	N/A	0.495	0.455	0.147	0.000	0.558	0.000	9.248	1.205

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	199	160	0	290	0	209	383
N.S.	1	1.00	2.07	1.67	0.00	3.02	0.00	2.18	3.99
time (sec)	N/A	0.137	0.419	0.574	0.000	0.376	0.000	4.804	1.500

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	479	479	461	375	0	9162	0	105	-1
N.S.	1	1.00	0.96	0.78	0.00	19.13	0.00	0.22	-0.00
time (sec)	N/A	1.249	11.287	0.159	0.000	135.038	0.000	6.417	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	355	267	0	7362	0	55	-1
N.S.	1	1.00	0.97	0.73	0.00	20.11	0.00	0.15	-0.00
time (sec)	N/A	0.815	10.733	0.156	0.000	55.004	0.000	6.627	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	292	200	0	5585	0	27	-1
N.S.	1	1.00	0.98	0.67	0.00	18.74	0.00	0.09	-0.00
time (sec)	N/A	0.508	10.452	0.119	0.000	9.716	0.000	7.411	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	250	161	0	3417	0	0	-1
N.S.	1	1.00	1.04	0.67	0.00	14.24	0.00	0.00	-0.00
time (sec)	N/A	0.210	5.207	0.122	0.000	2.782	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	179	151	0	4577	0	0	-1
N.S.	1	1.00	0.74	0.62	0.00	18.84	0.00	0.00	-0.00
time (sec)	N/A	0.125	0.126	0.121	0.000	3.688	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	271	197	0	6466	0	0	-1
N.S.	1	1.00	0.97	0.70	0.00	23.09	0.00	0.00	-0.00
time (sec)	N/A	0.444	10.822	0.153	0.000	13.250	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	320	248	0	8220	0	0	-1
N.S.	1	1.00	0.94	0.73	0.00	24.11	0.00	0.00	-0.00
time (sec)	N/A	0.497	10.534	0.144	0.000	62.843	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	443	443	383	349	0	10039	0	0	-1
N.S.	1	1.00	0.86	0.79	0.00	22.66	0.00	0.00	-0.00
time (sec)	N/A	1.042	11.252	0.148	0.000	157.674	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	350	507	2281	369	0	0	0	75	-1
N.S.	1	1.45	6.52	1.05	0.00	0.00	0.00	0.21	-0.00
time (sec)	N/A	2.855	20.340	0.125	0.000	0.000	0.000	5.521	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	360	360	2162	284	0	14319	0	0	-1
N.S.	1	1.00	6.01	0.79	0.00	39.78	0.00	0.00	-0.00
time (sec)	N/A	0.841	17.756	0.141	0.000	165.057	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	333	333	2119	246	0	13948	0	0	-1
N.S.	1	1.00	6.36	0.74	0.00	41.89	0.00	0.00	-0.00
time (sec)	N/A	0.483	16.500	0.145	0.000	116.395	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	B	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	341	341	2061	240	0	17065	0	0	-1
N.S.	1	1.00	6.04	0.70	0.00	50.04	0.00	0.00	-0.00
time (sec)	N/A	0.557	15.377	0.118	0.000	225.020	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	462	2158	329	0	0	0	0	-1
N.S.	1	1.36	6.37	0.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	2.007	16.565	0.169	0.000	0.000	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	419	647	2218	431	0	0	0	0	-1
N.S.	1	1.54	5.29	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	3.949	16.161	0.194	0.000	0.000	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	243	243	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.441	0.194	0.024	0.000	0.000	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	272	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.648	1.012	0.053	0.000	0.000	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	211	0	0	0	0	0	-1
N.S.	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.381	0.430	0.043	0.000	0.000	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	183	0	0	0	0	0	-1
N.S.	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.228	0.255	0.038	0.000	0.000	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	168	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.262	0.257	0.036	0.000	0.000	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	218	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.373	0.605	0.029	0.000	0.000	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	259	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.478	0.357	0.044	0.000	0.000	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.408	1.711	0.043	0.000	0.000	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	0.315	0.037	0.000	0.000	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.189	0.038	0.000	0.000	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.025	0.036	0.000	0.000	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	0.328	0.049	0.000	0.000	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	328	328	0	0	0	0	0	0	-1
N.S.	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	1.284	0.022	0.000	0.000	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	58	101	0	120	0	42	-1
N.S.	1	1.10	1.45	2.52	0.00	3.00	0.00	1.05	-0.02
time (sec)	N/A	0.049	0.469	0.131	0.000	0.366	0.000	5.062	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [267] had the largest ratio of [37]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	3	2	1.00	20	0.100
2	A	2	1	1.00	20	0.050
3	A	4	3	1.00	18	0.167
4	A	2	1	1.00	17	0.059
5	A	3	2	1.00	20	0.100
6	A	2	1	1.00	20	0.050
7	A	3	2	1.00	20	0.100
8	A	5	5	1.00	20	0.250
9	A	4	4	1.00	20	0.200
10	A	4	4	1.00	18	0.222
11	A	7	7	1.00	20	0.350
12	A	7	7	1.00	20	0.350
13	A	7	7	1.00	20	0.350
14	A	6	6	1.00	20	0.300
15	A	6	5	1.00	20	0.250
16	A	5	5	1.00	20	0.250
17	A	4	4	1.00	17	0.235
18	A	4	4	1.00	20	0.200
19	A	5	5	1.00	20	0.250
20	A	6	5	1.00	20	0.250
21	A	5	4	1.00	20	0.200
22	A	5	4	1.00	18	0.222
23	A	8	7	1.00	20	0.350
24	A	8	8	1.00	20	0.400
25	A	8	7	1.00	20	0.350

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	8	8	1.00	20	0.400
27	A	7	5	1.00	20	0.250
28	A	6	5	1.00	20	0.250
29	A	5	4	1.00	17	0.235
30	A	5	5	1.00	20	0.250
31	A	5	4	1.00	20	0.200
32	A	5	4	1.00	20	0.200
33	A	4	4	1.00	20	0.200
34	A	3	3	1.00	20	0.150
35	A	3	3	1.00	18	0.167
36	A	6	6	1.00	20	0.300
37	A	5	5	1.00	20	0.250
38	A	6	6	1.00	20	0.300
39	A	5	4	1.00	20	0.200
40	A	4	4	1.00	20	0.200
41	A	3	3	1.00	17	0.176
42	A	4	4	1.00	20	0.200
43	A	5	4	1.00	20	0.200
44	A	4	4	1.00	20	0.200
45	A	4	4	1.00	20	0.200
46	A	3	3	1.00	20	0.150
47	A	2	2	1.00	18	0.111
48	A	6	6	1.00	20	0.300
49	A	6	6	1.00	20	0.300
50	A	5	5	1.00	20	0.250
51	A	4	4	1.00	20	0.200
52	A	4	4	1.00	17	0.235
53	A	5	5	1.00	20	0.250
54	A	6	5	1.00	20	0.250
55	A	3	2	1.00	25	0.080
56	A	4	3	1.00	23	0.130
57	A	3	2	1.00	23	0.087
58	A	4	3	1.00	23	0.130
59	A	3	2	1.00	23	0.087
60	A	4	3	1.00	21	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	3	2	1.00	20	0.100
62	A	5	4	1.00	23	0.174
63	A	3	2	1.00	23	0.087
64	A	4	3	1.00	23	0.130
65	A	3	2	1.00	23	0.087
66	A	4	3	1.00	21	0.143
67	A	3	2	1.00	21	0.095
68	A	4	3	1.00	21	0.143
69	A	3	2	1.00	21	0.095
70	A	2	2	1.00	19	0.105
71	A	3	2	1.00	18	0.111
72	A	4	3	1.00	21	0.143
73	A	3	2	1.00	21	0.095
74	A	4	3	1.00	21	0.143
75	A	4	4	1.00	33	0.121
76	A	4	4	1.00	31	0.129
77	A	3	3	1.00	30	0.100
78	A	4	3	1.00	33	0.091
79	A	3	3	1.00	33	0.091
80	A	4	3	1.00	33	0.091
81	A	4	4	1.00	33	0.121
82	A	3	3	1.00	31	0.097
83	A	4	4	1.00	30	0.133
84	A	4	3	1.00	33	0.091
85	A	5	4	1.00	33	0.121
86	A	4	3	1.00	33	0.091
87	A	3	2	1.00	35	0.057
88	A	3	2	1.00	35	0.057
89	A	3	2	1.00	35	0.057
90	A	3	3	1.00	35	0.086
91	A	3	3	1.00	35	0.086
92	A	2	2	1.00	29	0.069
93	A	5	4	1.00	31	0.129
94	A	5	4	1.00	31	0.129
95	A	3	2	1.00	25	0.080

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	2	1	1.00	25	0.040
97	A	3	2	1.00	23	0.087
98	A	2	1	1.00	22	0.045
99	A	3	2	1.00	25	0.080
100	A	2	1	1.00	25	0.040
101	A	3	2	1.00	25	0.080
102	A	7	6	1.00	25	0.240
103	A	6	6	1.00	25	0.240
104	A	5	5	1.00	23	0.217
105	A	7	6	1.00	25	0.240
106	A	7	6	1.00	25	0.240
107	A	5	3	1.00	25	0.120
108	A	4	3	1.00	25	0.120
109	A	3	2	1.00	22	0.091
110	A	4	3	1.00	25	0.120
111	A	5	3	1.00	25	0.120
112	A	7	7	1.00	25	0.280
113	A	6	6	1.00	25	0.240
114	A	4	4	1.00	25	0.160
115	A	4	4	1.00	23	0.174
116	A	8	7	1.00	25	0.280
117	A	8	7	1.00	25	0.280
118	A	6	4	1.00	25	0.160
119	A	5	4	1.00	25	0.160
120	A	4	3	1.00	25	0.120
121	A	4	3	1.00	22	0.136
122	A	5	4	1.00	25	0.160
123	A	6	4	1.00	25	0.160
124	A	8	7	1.00	25	0.280
125	A	7	6	1.00	25	0.240
126	A	5	5	1.00	25	0.200
127	A	5	5	1.00	25	0.200
128	A	5	5	1.00	25	0.200
129	A	5	5	1.00	23	0.217
130	A	9	7	1.00	25	0.280

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	9	7	1.00	25	0.280
132	A	7	4	1.00	25	0.160
133	A	6	4	1.00	25	0.160
134	A	5	3	1.00	25	0.120
135	A	5	4	1.00	25	0.160
136	A	5	3	1.00	22	0.136
137	A	4	3	1.00	21	0.143
138	A	5	4	1.00	22	0.182
139	A	5	5	1.00	17	0.294
140	A	6	6	1.00	18	0.333
141	A	5	5	1.00	22	0.227
142	A	6	6	1.00	25	0.240
143	A	5	5	1.00	25	0.200
144	A	5	5	1.00	23	0.217
145	A	7	6	1.00	25	0.240
146	A	7	6	1.00	25	0.240
147	A	7	6	1.00	25	0.240
148	A	5	5	1.00	25	0.200
149	A	6	6	1.00	25	0.240
150	A	7	6	1.00	25	0.240
151	A	6	5	1.00	25	0.200
152	A	5	5	1.00	25	0.200
153	A	4	4	1.00	22	0.182
154	A	4	4	1.00	25	0.160
155	A	5	5	1.00	25	0.200
156	A	7	6	1.00	25	0.240
157	A	6	5	1.00	25	0.200
158	A	6	5	1.00	23	0.217
159	A	8	6	1.00	25	0.240
160	A	8	7	1.00	25	0.280
161	A	8	6	1.00	25	0.240
162	A	8	7	1.00	25	0.280
163	A	7	5	1.00	25	0.200
164	A	6	5	1.00	25	0.200
165	A	5	4	1.00	22	0.182

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	5	5	1.00	25	0.200
167	A	5	4	1.00	25	0.160
168	A	6	5	1.00	25	0.200
169	A	5	5	1.00	27	0.185
170	A	4	4	1.00	27	0.148
171	A	4	4	1.00	25	0.160
172	A	6	5	1.00	27	0.185
173	A	4	4	1.00	27	0.148
174	A	5	5	1.00	27	0.185
175	A	6	5	1.00	27	0.185
176	A	5	4	1.00	27	0.148
177	A	4	4	1.00	27	0.148
178	A	3	3	1.00	24	0.125
179	A	4	4	1.00	27	0.148
180	A	5	4	1.00	27	0.148
181	A	6	5	1.00	25	0.200
182	A	5	5	1.00	25	0.200
183	A	4	4	1.00	25	0.160
184	A	4	4	1.00	23	0.174
185	A	6	5	1.00	25	0.200
186	A	4	4	1.00	25	0.160
187	A	5	5	1.00	25	0.200
188	A	6	5	1.00	25	0.200
189	A	5	4	1.00	25	0.160
190	A	4	4	1.00	25	0.160
191	A	3	3	1.00	22	0.136
192	A	4	4	1.00	25	0.160
193	A	5	4	1.00	25	0.160
194	A	5	5	1.00	25	0.200
195	A	4	4	1.00	25	0.160
196	A	2	2	1.00	23	0.087
197	A	5	5	1.00	25	0.200
198	A	5	5	1.00	25	0.200
199	A	5	5	1.00	25	0.200
200	A	4	4	1.00	25	0.160

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	4	4	1.00	22	0.182
202	A	5	5	1.00	25	0.200
203	A	6	5	1.00	25	0.200
204	A	6	3	1.00	31	0.097
205	A	6	3	1.00	31	0.097
206	A	6	3	1.00	31	0.097
207	A	6	3	1.00	31	0.097
208	A	6	3	1.00	31	0.097
209	A	6	3	1.00	31	0.097
210	A	6	3	1.00	31	0.097
211	A	6	3	1.00	31	0.097
212	A	6	3	1.00	31	0.097
213	A	6	3	1.00	31	0.097
214	A	6	3	1.00	31	0.097
215	A	6	3	1.00	31	0.097
216	A	6	3	1.00	31	0.097
217	A	6	3	1.00	31	0.097
218	A	6	3	1.00	31	0.097
219	A	6	3	1.00	31	0.097
220	A	2	1	1.00	27	0.037
221	A	2	1	1.00	27	0.037
222	A	2	1	1.00	25	0.040
223	A	3	2	1.00	27	0.074
224	A	4	3	0.91	27	0.111
225	A	6	3	1.00	29	0.103
226	A	6	3	1.00	29	0.103
227	A	6	3	1.00	29	0.103
228	A	6	3	1.00	29	0.103
229	A	6	5	1.00	22	0.227
230	A	6	5	1.00	22	0.227
231	A	6	5	1.00	22	0.227
232	A	6	5	1.00	22	0.227
233	A	6	6	1.00	20	0.300
234	A	6	5	1.00	22	0.227
235	A	6	5	1.00	22	0.227

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	6	5	1.00	22	0.227
237	A	12	8	1.00	22	0.364
238	A	12	8	1.00	22	0.364
239	A	12	8	1.00	22	0.364
240	A	12	8	1.00	22	0.364
241	A	12	8	1.00	19	0.421
242	A	12	8	1.00	22	0.364
243	A	12	8	1.00	22	0.364
244	A	7	6	1.00	22	0.273
245	A	7	6	1.00	22	0.273
246	A	7	6	0.99	22	0.273
247	A	7	6	0.99	22	0.273
248	A	7	6	1.00	20	0.300
249	A	8	6	1.00	22	0.273
250	A	8	6	1.00	22	0.273
251	A	8	6	1.00	22	0.273
252	A	24	11	1.00	22	0.500
253	A	23	10	1.00	22	0.454
254	A	23	10	1.00	22	0.454
255	A	23	10	1.00	22	0.454
256	A	22	9	1.00	19	0.474
257	A	22	9	1.00	22	0.409
258	A	22	9	1.00	22	0.409
259	A	4	4	1.00	20	0.200
260	A	4	4	1.00	22	0.182
261	A	6	6	1.00	22	0.273
262	A	3	3	1.00	24	0.125
263	A	7	7	1.00	20	0.350
264	A	4	4	1.00	22	0.182
265	A	4	4	1.00	22	0.182
266	A	4	4	1.00	24	0.167
267	A	6	6	1.00	37	0.162
268	A	4	3	1.00	35	0.086
269	A	5	5	1.00	34	0.147
270	A	6	6	1.00	37	0.162

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	5	5	1.00	37	0.135
272	A	6	6	1.00	37	0.162
273	A	3	2	1.00	25	0.080
274	A	2	1	1.00	25	0.040
275	A	3	2	1.00	23	0.087
276	A	2	1	1.00	22	0.045
277	A	3	2	1.00	25	0.080
278	A	2	1	1.00	25	0.040
279	A	3	2	1.00	25	0.080
280	A	4	3	1.00	25	0.120
281	A	4	3	1.00	25	0.120
282	A	4	3	1.00	25	0.120
283	A	3	3	1.05	22	0.136
284	A	3	3	0.97	25	0.120
285	A	4	3	1.00	25	0.120
286	A	4	3	1.00	25	0.120
287	A	4	3	1.00	25	0.120
288	A	5	4	1.00	25	0.160
289	A	5	4	1.00	25	0.160
290	A	4	4	1.00	25	0.160
291	A	3	3	1.03	22	0.136
292	A	4	4	0.98	25	0.160
293	A	5	3	1.00	25	0.120
294	A	5	4	1.00	25	0.160
295	A	7	6	1.00	27	0.222
296	A	7	6	1.00	27	0.222
297	A	7	6	1.00	27	0.222
298	A	7	6	1.00	27	0.222
299	A	7	7	1.00	25	0.280
300	A	7	6	1.00	27	0.222
301	A	7	6	1.00	27	0.222
302	A	7	6	1.00	27	0.222
303	A	6	3	1.00	27	0.111
304	A	6	3	1.00	27	0.111
305	A	6	3	1.00	27	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	6	3	1.00	27	0.111
307	A	6	3	1.00	24	0.125
308	A	6	3	1.00	27	0.111
309	A	6	3	1.00	27	0.111
310	A	19	12	1.00	31	0.387
311	A	8	7	1.00	29	0.241
312	A	7	6	1.00	29	0.207
313	A	7	6	1.00	27	0.222
314	A	9	6	1.00	29	0.207
315	A	21	8	1.00	29	0.276
316	A	17	9	1.46	29	0.310
317	A	13	8	1.42	29	0.276
318	A	7	6	1.23	26	0.231
319	A	8	7	1.00	29	0.241
320	A	7	6	1.00	29	0.207
321	A	13	9	1.00	29	0.310
322	A	9	7	1.00	29	0.241
323	A	8	6	1.00	29	0.207
324	A	8	7	1.00	27	0.259
325	A	14	8	1.00	29	0.276
326	A	24	9	1.00	29	0.310
327	A	19	9	1.89	29	0.310
328	A	12	7	1.41	26	0.269
329	A	13	10	1.00	29	0.345
330	A	13	9	1.00	29	0.310
331	A	15	9	1.00	29	0.310
332	A	7	6	1.00	29	0.207
333	A	6	5	1.00	29	0.172
334	A	3	3	1.00	27	0.111
335	A	7	4	1.00	29	0.138
336	A	10	5	1.00	29	0.172
337	A	4	4	1.00	29	0.138
338	A	3	3	1.00	29	0.103
339	A	3	3	1.00	26	0.115
340	A	6	6	1.00	29	0.207

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	7	7	1.00	29	0.241
342	A	7	6	1.00	29	0.207
343	A	5	5	1.00	29	0.172
344	A	5	5	1.00	29	0.172
345	A	5	5	1.00	27	0.185
346	A	11	6	1.00	29	0.207
347	A	15	7	1.00	29	0.241
348	A	10	8	1.26	29	0.276
349	A	8	7	1.19	29	0.241
350	A	8	7	1.19	29	0.241
351	A	8	7	1.19	29	0.241
352	A	8	7	1.19	26	0.269
353	A	15	10	1.38	29	0.345
354	A	7	5	1.00	29	0.172
355	A	7	5	1.00	29	0.172
356	A	6	5	1.00	29	0.172
357	A	5	4	1.00	27	0.148
358	A	8	6	1.00	29	0.207
359	A	10	7	0.97	29	0.241
360	A	13	7	1.00	29	0.241
361	A	10	7	1.00	29	0.241
362	A	9	6	1.00	29	0.207
363	A	11	6	1.00	26	0.231
364	A	8	5	1.00	29	0.172
365	A	12	7	1.00	29	0.241
366	A	15	7	1.00	29	0.241
367	A	7	5	1.00	29	0.172
368	A	6	5	1.00	27	0.185
369	A	8	6	1.00	29	0.207
370	A	10	7	1.00	29	0.241
371	A	17	9	1.00	29	0.310
372	A	16	8	1.00	29	0.276
373	A	13	7	1.00	26	0.269
374	A	16	8	1.66	29	0.276
375	A	19	10	1.00	29	0.345

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	7	5	1.00	29	0.172
377	A	6	5	1.00	29	0.172
378	A	5	4	1.00	27	0.148
379	A	8	6	1.00	29	0.207
380	A	8	6	1.00	29	0.207
381	A	9	6	1.00	29	0.207
382	A	8	5	1.00	29	0.172
383	A	9	5	1.00	26	0.192
384	A	8	5	1.00	29	0.172
385	A	8	6	1.00	25	0.240
386	A	17	7	1.00	29	0.241
387	A	13	7	1.00	29	0.241
388	A	10	6	1.00	29	0.207
389	A	6	3	1.00	29	0.103
390	A	5	3	1.00	26	0.115
391	A	9	5	1.00	29	0.172
392	A	11	6	1.00	29	0.207
393	A	14	6	1.00	29	0.207
394	A	14	7	1.45	29	0.241
395	A	8	5	1.00	29	0.172
396	A	8	5	1.00	29	0.172
397	A	8	5	1.00	26	0.192
398	A	12	8	1.36	29	0.276
399	A	15	8	1.54	29	0.276
400	A	6	3	1.00	29	0.103
401	A	5	3	1.00	27	0.111
402	A	5	3	1.00	27	0.111
403	A	5	3	1.00	27	0.111
404	A	5	3	1.00	25	0.120
405	A	8	5	1.00	27	0.185
406	A	9	5	1.00	27	0.185
407	A	12	8	1.00	27	0.296
408	A	10	6	1.00	27	0.222
409	A	6	3	1.00	27	0.111
410	A	5	3	1.00	24	0.125

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	10	6	1.00	27	0.222
412	A	12	6	1.00	27	0.222
413	A	5	5	1.10	28	0.179

Chapter 3

Listing of integrals

Local contents

3.1 $\int x^3(d+ex^2)\left(a+cx^4\right)^5\,dx$

Optimal. Leaf size=149

$$\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{2}$$

```
[Out] 1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d*
x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d*x
^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

Rubi [A]
time = 0.14, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1266, 780}

$$\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}c^5dx^{24}+\frac{1}{26}c^5ex^{26}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^3\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx=\frac{1}{2}\text{Subst}\left(\int x(d+ex)\left(a+cx^2\right)^5\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int\left(a^5dx+a^5ex^2+5a^4cdx^3+5a^4cex^4+10a^3c^2dx^5+10a^3c^2ex^6\right.\right.$$

$$=\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3$$

Mathematica [A]
time = 0.00, size = 149, normalized size = 1.00

$$\frac{1}{4}a^5dx^4+\frac{1}{6}a^5ex^6+\frac{5}{8}a^4cdx^8+\frac{1}{2}a^4cex^{10}+\frac{5}{6}a^3c^2dx^{12}+\frac{5}{7}a^3c^2ex^{14}+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{9}a^2c^3ex^{18}+\frac{1}{4}ac^4dx^{20}+\frac{5}{22}ac^4ex^{22}+\frac{1}{24}c^5dx^{24}+\frac{1}{26}c^5ex^{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (5*a^4*c*d*x^8)/8 + (a^4*c*e*x^10)/2 + (5*a
^3*c^2*d*x^12)/6 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*d*x^16)/8 + (5*a^2*c^3
*e*x^18)/9 + (a*c^4*d*x^20)/4 + (5*a*c^4*e*x^22)/22 + (c^5*d*x^24)/24 + (c^
5*e*x^26)/26
```

Maple [A]
time = 0.12, size = 126, normalized size = 0.85

method	result
gosper	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x^{18}+$
default	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x^{18}+$
norman	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x^{18}+$
risch	$\frac{1}{4}a^5d\,x^4+\frac{1}{6}a^5e\,x^6+\frac{5}{8}a^4cd\,x^8+\frac{1}{2}a^4ce\,x^{10}+\frac{5}{6}a^3c^2d\,x^{12}+\frac{5}{7}a^3c^2e\,x^{14}+\frac{5}{8}a^2c^3d\,x^{16}+\frac{5}{9}a^2c^3e\,x^{18}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*a^5*d*x^4+1/6*a^5*e*x^6+5/8*a^4*c*d*x^8+1/2*a^4*c*e*x^10+5/6*a^3*c^2*d*
x^12+5/7*a^3*c^2*e*x^14+5/8*a^2*c^3*d*x^16+5/9*a^2*c^3*e*x^18+1/4*a*c^4*d*x
^20+5/22*a*c^4*e*x^22+1/24*c^5*d*x^24+1/26*c^5*e*x^26
```

Maxima [A]
time = 0.29, size = 131, normalized size = 0.88

$$\frac{1}{26}\,c^5x^{26}e+\frac{1}{24}\,c^5dx^{24}+\frac{5}{22}\,ac^4x^{22}e+\frac{1}{4}\,ac^4dx^{20}+\frac{5}{9}\,a^2c^3x^{18}e+\frac{5}{8}\,a^2c^3dx^{16}+\frac{5}{7}\,a^3c^2x^{14}e+\frac{5}{6}\,a^3c^2dx^{12}+\frac{1}{2}\,a^4cx^{10}e+\frac{5}{8}\,a^4cdx^8+\frac{1}{6}\,a^5x^6e+\frac{1}{4}\,a^5dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/26*c^5*x^26*e + 1/24*c^5*d*x^24 + 5/22*a*c^4*x^22*e + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*x^18*e + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*x^14*e + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*x^10*e + 5/8*a^4*c*d*x^8 + 1/6*a^5*x^6*e + 1/4*a^5*d*x^4
```

Fricas [A]
time = 0.41, size = 124, normalized size = 0.83

$$\frac{1}{24}\,c^5dx^{24}+\frac{1}{4}\,ac^4dx^{20}+\frac{5}{8}\,a^2c^3dx^{16}+\frac{5}{6}\,a^3c^2dx^{12}+\frac{5}{8}\,a^4cdx^8+\frac{1}{4}\,a^5dx^4+\frac{1}{18018}\,\left(693\,c^5x^{26}+4095\,ac^4x^{22}+10010\,a^2c^3x^{18}+12870\,a^3c^2x^{14}+9009\,a^4cx^{10}+3003\,a^5x^6\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/24*c^5*d*x^24 + 1/4*a*c^4*d*x^20 + 5/8*a^2*c^3*d*x^16 + 5/6*a^3*c^2*d*x^1
2 + 5/8*a^4*c*d*x^8 + 1/4*a^5*d*x^4 + 1/18018*(693*c^5*x^26 + 4095*a*c^4*x^
22 + 10010*a^2*c^3*x^18 + 12870*a^3*c^2*x^14 + 9009*a^4*c*x^10 + 3003*a^5*x
^6)*e
```

Sympy [A]
time = 0.02, size = 151, normalized size = 1.01

$$\frac{a^5dx^4}{4}+\frac{a^5ex^6}{6}+\frac{5a^4cdx^8}{8}+\frac{a^4cex^{10}}{2}+\frac{5a^3c^2dx^{12}}{6}+\frac{5a^3c^2ex^{14}}{7}+\frac{5a^2c^3dx^{16}}{8}+\frac{5a^2c^3ex^{18}}{9}+\frac{ac^4dx^{20}}{4}+\frac{5ac^4ex^{22}}{22}+\frac{c^5dx^{24}}{24}+\frac{c^5ex^{26}}{26}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**4/4 + a**5*e*x**6/6 + 5*a**4*c*d*x**8/8 + a**4*c*e*x**10/2 + 5*a*
*3*c**2*d*x**12/6 + 5*a**3*c**2*e*x**14/7 + 5*a**2*c**3*d*x**16/8 + 5*a**2*
c**3*e*x**18/9 + a*c**4*d*x**20/4 + 5*a*c**4*e*x**22/22 + c**5*d*x**24/24 +
 c**5*e*x**26/26
```

Giac [A]
time = 3.70, size = 131, normalized size = 0.88

$$\frac{1}{26}c^5x^{26}e+\frac{1}{24}c^5dx^{24}+\frac{5}{22}ac^4x^{22}e+\frac{1}{4}ac^4dx^{20}+\frac{5}{9}a^2c^3x^{18}e+\frac{5}{8}a^2c^3dx^{16}+\frac{5}{7}a^3c^2x^{14}e+\frac{5}{6}a^3c^2dx^{12}+\frac{1}{2}a^4cx^{10}e+\frac{5}{8}a^4cdx^8+\frac{1}{6}a^5x^6e+\frac{1}{4}a^5dx^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/26*c^5*x^26*e + 1/24*c^5*d*x^24 + 5/22*a*c^4*x^22*e + 1/4*a*c^4*d*x^20 +
5/9*a^2*c^3*x^18*e + 5/8*a^2*c^3*d*x^16 + 5/7*a^3*c^2*x^14*e + 5/6*a^3*c^2*
d*x^12 + 1/2*a^4*c*x^10*e + 5/8*a^4*c*d*x^8 + 1/6*a^5*x^6*e + 1/4*a^5*d*x^4
```

Mupad [B]
time = 0.31, size = 125, normalized size = 0.84

$$\frac{e\,a^5\,x^6}{6}+\frac{d\,a^5\,x^4}{4}+\frac{e\,a^4\,c\,x^{10}}{2}+\frac{5\,d\,a^4\,c\,x^8}{8}+\frac{5\,e\,a^3\,c^2\,x^{14}}{7}+\frac{5\,d\,a^3\,c^2\,x^{12}}{6}+\frac{5\,e\,a^2\,c^3\,x^{18}}{9}+\frac{5\,d\,a^2\,c^3\,x^{16}}{8}+\frac{5\,e\,a\,c^4\,x^{22}}{22}+\frac{d\,a\,c^4\,x^{20}}{4}+\frac{e\,c^5\,x^{26}}{26}+\frac{d\,c^5\,x^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^4)/4 + (a^5*e*x^6)/6 + (c^5*d*x^24)/24 + (c^5*e*x^26)/26 + (5*a^3*
c^2*d*x^12)/6 + (5*a^2*c^3*d*x^16)/8 + (5*a^3*c^2*e*x^14)/7 + (5*a^2*c^3*e*
x^18)/9 + (5*a^4*c*d*x^8)/8 + (a*c^4*d*x^20)/4 + (a^4*c*e*x^10)/2 + (5*a*c^
4*e*x^22)/22
```

3.2 $\int x^2(d+ex^2)\left(a+cx^4\right)^5\,dx$

Optimal. Leaf size=149

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4e$$

```
[Out] 1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*d
*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a*c
^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

Rubi [A]
time = 0.06, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = $\{1276\}$

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(d+ex^2\right)\left(a+cx^4\right)^5\,dx &= \int \left(a^5dx^2+a^5ex^4+5a^4cdx^6+5a^4cex^8+10a^3c^2dx^{10}+10a^3c^2ex^{12}+10a^2c^3\right.\\ &= \frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c\end{aligned}$$

Mathematica [A]
time = 0.00, size = 149, normalized size = 1.00

$$\frac{1}{3}a^5dx^3+\frac{1}{5}a^5ex^5+\frac{5}{7}a^4cdx^7+\frac{5}{9}a^4cex^9+\frac{10}{11}a^3c^2dx^{11}+\frac{10}{13}a^3c^2ex^{13}+\frac{2}{3}a^2c^3dx^{15}+\frac{10}{17}a^2c^3ex^{17}+\frac{5}{19}ac^4dx^{19}+\frac{5}{21}ac^4ex^{21}+\frac{1}{23}c^5dx^{23}+\frac{1}{25}c^5ex^{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (5*a^4*c*d*x^7)/7 + (5*a^4*c*e*x^9)/9 + (10
*a^3*c^2*d*x^11)/11 + (10*a^3*c^2*e*x^13)/13 + (2*a^2*c^3*d*x^15)/3 + (10*a
^2*c^3*e*x^17)/17 + (5*a*c^4*d*x^19)/19 + (5*a*c^4*e*x^21)/21 + (c^5*d*x^23
)/23 + (c^5*e*x^25)/25
```

Maple [A]
time = 0.12, size = 126, normalized size = 0.85

method	result
gosper	$\frac{1}{3}a^5 d\,x^3+\frac{1}{5}a^5 e\,x^5+\frac{5}{7}a^4 cd\,x^7+\frac{5}{9}a^4 ce\,x^9+\frac{10}{11}a^3c^2 d\,x^{11}+\frac{10}{13}a^3c^2 e\,x^{13}+\frac{2}{3}a^2c^3 d\,x^{15}+\frac{10}{17}a^2c^3 e\,x^{17}$
default	$\frac{1}{3}a^5 d\,x^3+\frac{1}{5}a^5 e\,x^5+\frac{5}{7}a^4 cd\,x^7+\frac{5}{9}a^4 ce\,x^9+\frac{10}{11}a^3c^2 d\,x^{11}+\frac{10}{13}a^3c^2 e\,x^{13}+\frac{2}{3}a^2c^3 d\,x^{15}+\frac{10}{17}a^2c^3 e\,x^{17}$
norman	$\frac{1}{3}a^5 d\,x^3+\frac{1}{5}a^5 e\,x^5+\frac{5}{7}a^4 cd\,x^7+\frac{5}{9}a^4 ce\,x^9+\frac{10}{11}a^3c^2 d\,x^{11}+\frac{10}{13}a^3c^2 e\,x^{13}+\frac{2}{3}a^2c^3 d\,x^{15}+\frac{10}{17}a^2c^3 e\,x^{17}$
risch	$\frac{1}{3}a^5 d\,x^3+\frac{1}{5}a^5 e\,x^5+\frac{5}{7}a^4 cd\,x^7+\frac{5}{9}a^4 ce\,x^9+\frac{10}{11}a^3c^2 d\,x^{11}+\frac{10}{13}a^3c^2 e\,x^{13}+\frac{2}{3}a^2c^3 d\,x^{15}+\frac{10}{17}a^2c^3 e\,x^{17}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*a^5*d*x^3+1/5*a^5*e*x^5+5/7*a^4*c*d*x^7+5/9*a^4*c*e*x^9+10/11*a^3*c^2*d
*x^11+10/13*a^3*c^2*e*x^13+2/3*a^2*c^3*d*x^15+10/17*a^2*c^3*e*x^17+5/19*a*c
^4*d*x^19+5/21*a*c^4*e*x^21+1/23*c^5*d*x^23+1/25*c^5*e*x^25
```

Maxima [A]
time = 0.27, size = 131, normalized size = 0.88

$$\frac{1}{25}\,c^5x^{25}e+\frac{1}{23}\,c^5dx^{23}+\frac{5}{21}\,ac^4x^{21}e+\frac{5}{19}\,ac^4dx^{19}+\frac{10}{17}\,a^2c^3x^{17}e+\frac{2}{3}\,a^2c^3dx^{15}+\frac{10}{13}\,a^3c^2x^{13}e+\frac{10}{11}\,a^3c^2dx^{11}+\frac{5}{9}\,a^4cx^9e+\frac{5}{7}\,a^4cdx^7+\frac{1}{5}\,a^5x^5e+\frac{1}{3}\,a^5dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/25*c^5*x^25*e + 1/23*c^5*d*x^23 + 5/21*a*c^4*x^21*e + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*x^17*e + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*x^13*e + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*x^9*e + 5/7*a^4*c*d*x^7 + 1/5*a^5*x^5*e + 1/3*a^5
*d*x^3
```

Fricas [A]
time = 0.33, size = 124, normalized size = 0.83

$$\frac{1}{23}\,c^5dx^{23}+\frac{5}{19}\,ac^4dx^{19}+\frac{2}{3}\,a^2c^3dx^{15}+\frac{10}{11}\,a^3c^2dx^{11}+\frac{5}{7}\,a^4cdx^7+\frac{1}{3}\,a^5dx^3+\frac{1}{348075}\,\left(13923\,c^5x^{25}+82875\,ac^4x^{21}+204750\,a^2c^3x^{17}+267750\,a^3c^2x^{13}+193375\,a^4cx^9+69615\,a^5x^5\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/23*c^5*d*x^23 + 5/19*a*c^4*d*x^19 + 2/3*a^2*c^3*d*x^15 + 10/11*a^3*c^2*d*
x^11 + 5/7*a^4*c*d*x^7 + 1/3*a^5*d*x^3 + 1/348075*(13923*c^5*x^25 + 82875*a
*c^4*x^21 + 204750*a^2*c^3*x^17 + 267750*a^3*c^2*x^13 + 193375*a^4*c*x^9 +
69615*a^5*x^5)*e
```

Sympy [A]
time = 0.01, size = 155, normalized size = 1.04

$$\frac{a^5dx^3}{3}+\frac{a^5ex^5}{5}+\frac{5a^4cdx^7}{7}+\frac{5a^4cex^9}{9}+\frac{10a^3c^2dx^{11}}{11}+\frac{10a^3c^2ex^{13}}{13}+\frac{2a^2c^3dx^{15}}{3}+\frac{10a^2c^3ex^{17}}{17}+\frac{5ac^4dx^{19}}{19}+\frac{5ac^4ex^{21}}{21}+\frac{c^5dx^{23}}{23}+\frac{c^5ex^{25}}{25}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**3/3 + a**5*e*x**5/5 + 5*a**4*c*d*x**7/7 + 5*a**4*c*e*x**9/9 + 10*
a**3*c**2*d*x**11/11 + 10*a**3*c**2*e*x**13/13 + 2*a**2*c**3*d*x**15/3 + 10
*a**2*c**3*e*x**17/17 + 5*a*c**4*d*x**19/19 + 5*a*c**4*e*x**21/21 + c**5*d*
x**23/23 + c**5*e*x**25/25
```

Giac [A]
time = 6.34, size = 131, normalized size = 0.88

$$\frac{1}{25}\,c^5x^{25}e+\frac{1}{23}\,c^5dx^{23}+\frac{5}{21}\,ac^4x^{21}e+\frac{5}{19}\,ac^4dx^{19}+\frac{10}{17}\,a^2c^3x^{17}e+\frac{2}{3}\,a^2c^3dx^{15}+\frac{10}{13}\,a^3c^2x^{13}e+\frac{10}{11}\,a^3c^2dx^{11}+\frac{5}{9}\,a^4cx^9e+\frac{5}{7}\,a^4cdx^7+\frac{1}{5}\,a^5x^5e+\frac{1}{3}\,a^5dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/25*c^5*x^25*e + 1/23*c^5*d*x^23 + 5/21*a*c^4*x^21*e + 5/19*a*c^4*d*x^19 +
 10/17*a^2*c^3*x^17*e + 2/3*a^2*c^3*d*x^15 + 10/13*a^3*c^2*x^13*e + 10/11*a
^3*c^2*d*x^11 + 5/9*a^4*c*x^9*e + 5/7*a^4*c*d*x^7 + 1/5*a^5*x^5*e + 1/3*a^5
*d*x^3
```

Mupad [B]
time = 0.07, size = 125, normalized size = 0.84

$$\frac{e\,a^5\,x^5}{5}+\frac{d\,a^5\,x^3}{3}+\frac{5\,e\,a^4\,c\,x^9}{9}+\frac{5\,d\,a^4\,c\,x^7}{7}+\frac{10\,e\,a^3\,c^2\,x^{13}}{13}+\frac{10\,d\,a^3\,c^2\,x^{11}}{11}+\frac{10\,e\,a^2\,c^3\,x^{17}}{17}+\frac{2\,d\,a^2\,c^3\,x^{15}}{3}+\frac{5\,e\,a\,c^4\,x^{21}}{21}+\frac{5\,d\,a\,c^4\,x^{19}}{19}+\frac{e\,c^5\,x^{25}}{25}+\frac{d\,c^5\,x^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^3)/3 + (a^5*e*x^5)/5 + (c^5*d*x^23)/23 + (c^5*e*x^25)/25 + (10*a^3
*c^2*d*x^11)/11 + (2*a^2*c^3*d*x^15)/3 + (10*a^3*c^2*e*x^13)/13 + (10*a^2*c
^3*e*x^17)/17 + (5*a^4*c*d*x^7)/7 + (5*a*c^4*d*x^19)/19 + (5*a^4*c*e*x^9)/9
 + (5*a*c^4*e*x^21)/21
```

3.3 $\int x(d+ex^2)\left(a+cx^4\right)^5\,dx$

Optimal. Leaf size=89

$$\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{1}{22}c^5dx^{22}+\frac{e(a+cx^4)^6}{24c}$$

```
[Out] 1/2*a^5*d*x^2+5/6*a^4*c*d*x^6+a^3*c^2*d*x^10+5/7*a^2*c^3*d*x^14+5/18*a*c^4*
d*x^18+1/22*c^5*d*x^22+1/24*e*(c*x^4+a)^6/c
```

Rubi [A]
time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1262, 655, 200}

$$\frac{1}{2}a^5dx^2+\frac{5}{6}a^4cdx^6+a^3c^2dx^{10}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{18}ac^4dx^{18}+\frac{e(a+cx^4)^6}{24c}+\frac{1}{22}c^5dx^{22}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (5*a^4*c*d*x^6)/6 + a^3*c^2*d*x^10 + (5*a^2*c^3*d*x^14)/7 +
 (5*a*c^4*d*x^18)/18 + (c^5*d*x^22)/22 + (e*(a + c*x^4)^6)/(24*c)
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\int x(d+ex^2)\left(a+cx^4\right)^5\,dx = \frac{1}{2}\text{Subst}\left(\int (d+ex)\left(a+cx^2\right)^5\,dx, x, x^2\right)$$
$$= \frac{e(a+cx^4)^6}{24c} + \frac{1}{2}d\text{Subst}\left(\int \left(a+cx^2\right)^5\,dx, x, x^2\right)$$
$$= \frac{e(a+cx^4)^6}{24c} + \frac{1}{2}d\text{Subst}\left(\int \left(a^5+5a^4cx^2+10a^3c^2x^4+10a^2c^3x^6+5ac^4x^8+c\right.\right.$$
$$= \frac{1}{2}a^5dx^2 + \frac{5}{6}a^4cdx^6 + a^3c^2dx^{10} + \frac{5}{7}a^2c^3dx^{14} + \frac{5}{18}ac^4dx^{18} + \frac{1}{22}c^5dx^{22} + \frac{e(a+}{24}$$

Mathematica [A]
time = 0.00, size = 146, normalized size = 1.64

$$\frac{1}{2}a^5dx^2+\frac{1}{4}a^5ex^4+\frac{5}{6}a^4cdx^6+\frac{5}{8}a^4cex^8+a^3c^2dx^{10}+\frac{5}{6}a^3c^2ex^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}a^2c^3ex^{16}+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}ac^4ex^{20}+\frac{1}{22}c^5dx^{22}+\frac{1}{24}c^5ex^{24}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (5*a^4*c*d*x^6)/6 + (5*a^4*c*e*x^8)/8 + a^3
*c^2*d*x^10 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*d*x^14)/7 + (5*a^2*c^3*e*x^
16)/8 + (5*a*c^4*d*x^18)/18 + (a*c^4*e*x^20)/4 + (c^5*d*x^22)/22 + (c^5*e*x
^24)/24
```

Maple [A]
time = 0.13, size = 125, normalized size = 1.40

method	result
gosper	$\frac{5}{8}eca^4x^8+a^3c^2dx^{10}+\frac{5}{6}ea^3c^2x^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}ea^2c^3x^{16}+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}eac^4x^{20}+\frac{1}{22}c^5dx^{22}$
default	$\frac{5}{8}eca^4x^8+a^3c^2dx^{10}+\frac{5}{6}ea^3c^2x^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}ea^2c^3x^{16}+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}eac^4x^{20}+\frac{1}{22}c^5dx^{22}$
norman	$\frac{5}{8}eca^4x^8+a^3c^2dx^{10}+\frac{5}{6}ea^3c^2x^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}ea^2c^3x^{16}+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}eac^4x^{20}+\frac{1}{22}c^5dx^{22}$
risch	$\frac{5}{8}eca^4x^8+a^3c^2dx^{10}+\frac{5}{6}ea^3c^2x^{12}+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{8}ea^2c^3x^{16}+\frac{5}{18}ac^4dx^{18}+\frac{1}{4}eac^4x^{20}+\frac{1}{22}c^5dx^{22}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 5/8*e*c*a^4*x^8+a^3*c^2*d*x^10+5/6*e*a^3*c^2*x^12+5/7*a^2*c^3*d*x^14+5/8*e*
a^2*c^3*x^16+5/18*a*c^4*d*x^18+1/4*e*a*c^4*x^20+1/22*c^5*d*x^22+1/24*e*c^5*
x^24+1/2*a^5*d*x^2+1/4*e*a^5*x^4+5/6*a^4*c*d*x^6
```

Maxima [A]
time = 0.28, size = 130, normalized size = 1.46

$$\frac{1}{24}c^5x^{24}e+\frac{1}{22}c^5dx^{22}+\frac{1}{4}ac^4x^{20}e+\frac{5}{18}ac^4dx^{18}+\frac{5}{8}a^2c^3x^{16}e+\frac{5}{7}a^2c^3dx^{14}+\frac{5}{6}a^3c^2x^{12}e+a^3c^2dx^{10}+\frac{5}{8}a^4cx^8e+\frac{5}{6}a^4cdx^6+\frac{1}{4}a^5x^4e+\frac{1}{2}a^5dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/24*c^5*x^24*e + 1/22*c^5*d*x^22 + 1/4*a*c^4*x^20*e + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*x^16*e + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*x^12*e + a^3*c^2*d*x^
10 + 5/8*a^4*c*x^8*e + 5/6*a^4*c*d*x^6 + 1/4*a^5*x^4*e + 1/2*a^5*d*x^2
```

Fricas [A]
time = 0.34, size = 122, normalized size = 1.37

$$\frac{1}{22}\,c^5dx^{22}+\frac{5}{18}\,ac^4dx^{18}+\frac{5}{7}\,a^2c^3dx^{14}+a^3c^2dx^{10}+\frac{5}{6}\,a^4cdx^6+\frac{1}{2}\,a^5dx^2+\frac{1}{24}\,\left(c^5x^{24}+6\,ac^4x^{20}+15\,a^2c^3x^{16}+20\,a^3c^2x^{12}+15\,a^4cx^8+6\,a^5x^4\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/22*c^5*d*x^22 + 5/18*a*c^4*d*x^18 + 5/7*a^2*c^3*d*x^14 + a^3*c^2*d*x^10 +
 5/6*a^4*c*d*x^6 + 1/2*a^5*d*x^2 + 1/24*(c^5*x^24 + 6*a*c^4*x^20 + 15*a^2*c
^3*x^16 + 20*a^3*c^2*x^12 + 15*a^4*c*x^8 + 6*a^5*x^4)*e
```

Sympy [A]
time = 0.02, size = 150, normalized size = 1.69

$$\frac{a^5dx^2}{2}+\frac{a^5ex^4}{4}+\frac{5a^4cdx^6}{6}+\frac{5a^4cex^8}{8}+a^3c^2dx^{10}+\frac{5a^3c^2ex^{12}}{6}+\frac{5a^2c^3dx^{14}}{7}+\frac{5a^2c^3ex^{16}}{8}+\frac{5ac^4dx^{18}}{18}+\frac{ac^4ex^{20}}{4}+\frac{c^5dx^{22}}{22}+\frac{c^5ex^{24}}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x**2/2 + a**5*e*x**4/4 + 5*a**4*c*d*x**6/6 + 5*a**4*c*e*x**8/8 + a**
3*c**2*d*x**10 + 5*a**3*c**2*e*x**12/6 + 5*a**2*c**3*d*x**14/7 + 5*a**2*c**
3*e*x**16/8 + 5*a*c**4*d*x**18/18 + a*c**4*e*x**20/4 + c**5*d*x**22/22 + c*
*5*e*x**24/24
```

Giac [A]
time = 3.64, size = 130, normalized size = 1.46

$$\frac{1}{24}\,c^5x^{24}e+\frac{1}{22}\,c^5dx^{22}+\frac{1}{4}\,ac^4x^{20}e+\frac{5}{18}\,ac^4dx^{18}+\frac{5}{8}\,a^2c^3x^{16}e+\frac{5}{7}\,a^2c^3dx^{14}+\frac{5}{6}\,a^3c^2x^{12}e+a^3c^2dx^{10}+\frac{5}{8}\,a^4cx^8e+\frac{5}{6}\,a^4cdx^6+\frac{1}{4}\,a^5x^4e+\frac{1}{2}\,a^5dx^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/24*c^5*x^24*e + 1/22*c^5*d*x^22 + 1/4*a*c^4*x^20*e + 5/18*a*c^4*d*x^18 +
5/8*a^2*c^3*x^16*e + 5/7*a^2*c^3*d*x^14 + 5/6*a^3*c^2*x^12*e + a^3*c^2*d*x^
10 + 5/8*a^4*c*x^8*e + 5/6*a^4*c*d*x^6 + 1/4*a^5*x^4*e + 1/2*a^5*d*x^2
```

Mupad [B]
time = 0.07, size = 124, normalized size = 1.39

$$\frac{e\,a^5\,x^4}{4}+\frac{d\,a^5\,x^2}{2}+\frac{5\,e\,a^4\,c\,x^8}{8}+\frac{5\,d\,a^4\,c\,x^6}{6}+\frac{5\,e\,a^3\,c^2\,x^{12}}{6}+d\,a^3\,c^2\,x^{10}+\frac{5\,e\,a^2\,c^3\,x^{16}}{8}+\frac{5\,d\,a^2\,c^3\,x^{14}}{7}+\frac{e\,a\,c^4\,x^{20}}{4}+\frac{5\,d\,a\,c^4\,x^{18}}{18}+\frac{e\,c^5\,x^{24}}{24}+\frac{d\,c^5\,x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*d*x^2)/2 + (a^5*e*x^4)/4 + (c^5*d*x^22)/22 + (c^5*e*x^24)/24 + a^3*c^2
*d*x^10 + (5*a^2*c^3*d*x^14)/7 + (5*a^3*c^2*e*x^12)/6 + (5*a^2*c^3*e*x^16)/
8 + (5*a^4*c*d*x^6)/6 + (5*a*c^4*d*x^18)/18 + (5*a^4*c*e*x^8)/8 + (a*c^4*e*
x^20)/4
```

3.4 $\int (d+ex^2)(a+cx^4)^5\,dx$

Optimal. Leaf size=141

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}$$

```
[Out] a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11*
a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+5/
19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

Rubi [A]
time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {1168}

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5dx^{21}+\frac{1}{23}c^5ex^{23}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e
}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)(a+cx^4)^5\,dx &= \int \left(a^5d+a^5ex^2+5a^4cdx^4+5a^4cex^6+10a^3c^2dx^8+10a^3c^2ex^{10}+10a^2c^3dx^{12}\right.\\ &= a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 141, normalized size = 1.00

$$a^5dx+\frac{1}{3}a^5ex^3+a^4cdx^5+\frac{5}{7}a^4cex^7+\frac{10}{9}a^3c^2dx^9+\frac{10}{11}a^3c^2ex^{11}+\frac{10}{13}a^2c^3dx^{13}+\frac{2}{3}a^2c^3ex^{15}+\frac{5}{17}ac^4dx^{17}+\frac{5}{19}ac^4ex^{19}+\frac{1}{21}c^5dx^{21}+\frac{1}{23}c^5ex^{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(a + c*x^4)^5,x]
```

```
[Out] a^5*d*x + (a^5*e*x^3)/3 + a^4*c*d*x^5 + (5*a^4*c*e*x^7)/7 + (10*a^3*c^2*d*x
^9)/9 + (10*a^3*c^2*e*x^11)/11 + (10*a^2*c^3*d*x^13)/13 + (2*a^2*c^3*e*x^15
)/3 + (5*a*c^4*d*x^17)/17 + (5*a*c^4*e*x^19)/19 + (c^5*d*x^21)/21 + (c^5*e*
x^23)/23
```

Maple [A]
time = 0.14, size = 122, normalized size = 0.87

method	result
gosper	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}+\frac{5}{17}a$
default	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}+\frac{5}{17}a$
norman	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}+\frac{5}{17}a$
risch	$a^5dx+\frac{1}{3}a^5e\,x^3+a^4cd\,x^5+\frac{5}{7}a^4ce\,x^7+\frac{10}{9}a^3c^2d\,x^9+\frac{10}{11}a^3c^2e\,x^{11}+\frac{10}{13}a^2c^3d\,x^{13}+\frac{2}{3}a^2c^3e\,x^{15}+\frac{5}{17}a$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5,x,method=_RETURNVERBOSE)
```

```
[Out] a^5*d*x+1/3*a^5*e*x^3+a^4*c*d*x^5+5/7*a^4*c*e*x^7+10/9*a^3*c^2*d*x^9+10/11*
a^3*c^2*e*x^11+10/13*a^2*c^3*d*x^13+2/3*a^2*c^3*e*x^15+5/17*a*c^4*d*x^17+5/
19*a*c^4*e*x^19+1/21*c^5*d*x^21+1/23*c^5*e*x^23
```

Maxima [A]
time = 0.27, size = 127, normalized size = 0.90

$$\frac{1}{23}\,c^5x^{23}e+\frac{1}{21}\,c^5dx^{21}+\frac{5}{19}\,ac^4x^{19}e+\frac{5}{17}\,ac^4dx^{17}+\frac{2}{3}\,a^2c^3x^{15}e+\frac{10}{13}\,a^2c^3dx^{13}+\frac{10}{11}\,a^3c^2x^{11}e+\frac{10}{9}\,a^3c^2dx^9+\frac{5}{7}\,a^4cx^7e+a^4cdx^5+\frac{1}{3}\,a^5x^3e+a^5dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="maxima")
```

```
[Out] 1/23*c^5*x^23*e + 1/21*c^5*d*x^21 + 5/19*a*c^4*x^19*e + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*x^15*e + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*x^11*e + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*x^7*e + a^4*c*d*x^5 + 1/3*a^5*x^3*e + a^5*d*x
```

Fricas [A]
time = 0.33, size = 120, normalized size = 0.85

$$\frac{1}{21}\,c^5dx^{21}+\frac{5}{17}\,ac^4dx^{17}+\frac{10}{13}\,a^2c^3dx^{13}+\frac{10}{9}\,a^3c^2dx^9+a^4cdx^5+a^5dx+\frac{1}{100947}\,\left(4389\,c^5x^{23}+26565\,ac^4x^{19}+67298\,a^2c^3x^{15}+91770\,a^3c^2x^{11}+72105\,a^4cx^7+33649\,a^5x^3\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="fricas")
```

```
[Out] 1/21*c^5*d*x^21 + 5/17*a*c^4*d*x^17 + 10/13*a^2*c^3*d*x^13 + 10/9*a^3*c^2*d
*x^9 + a^4*c*d*x^5 + a^5*d*x + 1/100947*(4389*c^5*x^23 + 26565*a*c^4*x^19 +
 67298*a^2*c^3*x^15 + 91770*a^3*c^2*x^11 + 72105*a^4*c*x^7 + 33649*a^5*x^3)
*e
```

Sympy [A]
time = 0.01, size = 148, normalized size = 1.05

$$a^5dx + \frac{a^5ex^3}{3} + a^4cdx^5 + \frac{5a^4cex^7}{7} + \frac{10a^3c^2dx^9}{9} + \frac{10a^3c^2ex^{11}}{11} + \frac{10a^2c^3dx^{13}}{13} + \frac{2a^2c^3ex^{15}}{3} + \frac{5ac^4dx^{17}}{17} + \frac{5ac^4ex^{19}}{19} + \frac{c^5dx^{21}}{21} + \frac{c^5ex^{23}}{23}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5,x)
```

```
[Out] a**5*d*x + a**5*e*x**3/3 + a**4*c*d*x**5 + 5*a**4*c*e*x**7/7 + 10*a**3*c**2
*d*x**9/9 + 10*a**3*c**2*e*x**11/11 + 10*a**2*c**3*d*x**13/13 + 2*a**2*c**3
*e*x**15/3 + 5*a*c**4*d*x**17/17 + 5*a*c**4*e*x**19/19 + c**5*d*x**21/21 +
c**5*e*x**23/23
```

Giac [A]
time = 4.13, size = 127, normalized size = 0.90

$$\frac{1}{23}c^5x^{23}e + \frac{1}{21}c^5dx^{21} + \frac{5}{19}ac^4x^{19}e + \frac{5}{17}ac^4dx^{17} + \frac{2}{3}a^2c^3x^{15}e + \frac{10}{13}a^2c^3dx^{13} + \frac{10}{11}a^3c^2x^{11}e + \frac{10}{9}a^3c^2dx^9 + \frac{5}{7}a^4cx^7e + a^4cdx^5 + \frac{1}{3}a^5x^3e + a^5dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5,x, algorithm="giac")
```

```
[Out] 1/23*c^5*x^23*e + 1/21*c^5*d*x^21 + 5/19*a*c^4*x^19*e + 5/17*a*c^4*d*x^17 +
 2/3*a^2*c^3*x^15*e + 10/13*a^2*c^3*d*x^13 + 10/11*a^3*c^2*x^11*e + 10/9*a^
3*c^2*d*x^9 + 5/7*a^4*c*x^7*e + a^4*c*d*x^5 + 1/3*a^5*x^3*e + a^5*d*x
```

Mupad [B]
time = 0.07, size = 121, normalized size = 0.86

$$\frac{e\,a^5\,x^3}{3} + d\,a^5\,x + \frac{5\,e\,a^4\,c\,x^7}{7} + d\,a^4\,c\,x^5 + \frac{10\,e\,a^3\,c^2\,x^{11}}{11} + \frac{10\,d\,a^3\,c^2\,x^9}{9} + \frac{2\,e\,a^2\,c^3\,x^{15}}{3} + \frac{10\,d\,a^2\,c^3\,x^{13}}{13} + \frac{5\,e\,a\,c^4\,x^{19}}{19} + \frac{5\,d\,a\,c^4\,x^{17}}{17} + \frac{e\,c^5\,x^{23}}{23} + \frac{d\,c^5\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + c*x^4)^5*(d + e*x^2),x)
```

```
[Out] (a^5*e*x^3)/3 + (c^5*d*x^21)/21 + (c^5*e*x^23)/23 + a^5*d*x + (10*a^3*c^2*d
*x^9)/9 + (10*a^2*c^3*d*x^13)/13 + (10*a^3*c^2*e*x^11)/11 + (2*a^2*c^3*e*x^
15)/3 + a^4*c*d*x^5 + (5*a*c^4*d*x^17)/17 + (5*a^4*c*e*x^7)/7 + (5*a*c^4*e*
x^19)/19
```

3.5 $\int \frac{(d+ex^2)(a+cx^4)^5}{x} \, dx$

Optimal. Leaf size=142

$$\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+$$

```
[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*x
^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*x^
18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

Rubi [A]
time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1266, 780}

$$a^5d\log(x)+\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(d+ex^2)(a+cx^4)^5}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)(a+cx^2)^5}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(a^5e+\frac{a^5d}{x}+5a^4cdx+5a^4cex^2+10a^3c^2dx^3+10a^3c^2ex^4+10a^2c^3\right.\right.$$

$$= \frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}$$

Mathematica [A]
time = 0.01, size = 142, normalized size = 1.00

$$\frac{1}{2}a^5ex^2+\frac{5}{4}a^4cdx^4+\frac{5}{6}a^4cex^6+\frac{5}{4}a^3c^2dx^8+a^3c^2ex^{10}+\frac{5}{6}a^2c^3dx^{12}+\frac{5}{7}a^2c^3ex^{14}+\frac{5}{16}ac^4dx^{16}+\frac{5}{18}ac^4ex^{18}+\frac{1}{20}c^5dx^{20}+\frac{1}{22}c^5ex^{22}+a^5d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x,x]
```

```
[Out] (a^5*e*x^2)/2 + (5*a^4*c*d*x^4)/4 + (5*a^4*c*e*x^6)/6 + (5*a^3*c^2*d*x^8)/4
 + a^3*c^2*e*x^10 + (5*a^2*c^3*d*x^12)/6 + (5*a^2*c^3*e*x^14)/7 + (5*a*c^4*
d*x^16)/16 + (5*a*c^4*e*x^18)/18 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*
d*Log[x]
```

Maple [A]
time = 0.11, size = 123, normalized size = 0.87

method	result
default	$\frac{a^5 e x^2}{2}+\frac{5a^4 cd x^4}{4}+\frac{5a^4 ce x^6}{6}+\frac{5a^3c^2d x^8}{4}+a^3c^2e x^{10}+\frac{5a^2c^3d x^{12}}{6}+\frac{5a^2c^3e x^{14}}{7}+\frac{5a c^4 d x^{16}}{16}+\frac{5a c^4 e x^{18}}{18}+$
norman	$\frac{a^5 e x^2}{2}+\frac{5a^4 cd x^4}{4}+\frac{5a^4 ce x^6}{6}+\frac{5a^3c^2d x^8}{4}+a^3c^2e x^{10}+\frac{5a^2c^3d x^{12}}{6}+\frac{5a^2c^3e x^{14}}{7}+\frac{5a c^4 d x^{16}}{16}+\frac{5a c^4 e x^{18}}{18}+$
risch	$\frac{a^5 e x^2}{2}+\frac{5a^4 cd x^4}{4}+\frac{5a^4 ce x^6}{6}+\frac{5a^3c^2d x^8}{4}+a^3c^2e x^{10}+\frac{5a^2c^3d x^{12}}{6}+\frac{5a^2c^3e x^{14}}{7}+\frac{5a c^4 d x^{16}}{16}+\frac{5a c^4 e x^{18}}{18}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*a^5*e*x^2+5/4*a^4*c*d*x^4+5/6*a^4*c*e*x^6+5/4*a^3*c^2*d*x^8+a^3*c^2*e*x
^10+5/6*a^2*c^3*d*x^12+5/7*a^2*c^3*e*x^14+5/16*a*c^4*d*x^16+5/18*a*c^4*e*x^
18+1/20*c^5*d*x^20+1/22*c^5*e*x^22+a^5*d*ln(x)
```

Maxima [A]
time = 0.27, size = 131, normalized size = 0.92

$$\frac{1}{22}\,c^5x^{22}e+\frac{1}{20}\,c^5dx^{20}+\frac{5}{18}\,ac^4x^{18}e+\frac{5}{16}\,ac^4dx^{16}+\frac{5}{7}\,a^2c^3x^{14}e+\frac{5}{6}\,a^2c^3dx^{12}+a^3c^2x^{10}e+\frac{5}{4}\,a^3c^2dx^8+\frac{5}{6}\,a^4cx^6e+\frac{5}{4}\,a^4cdx^4+\frac{1}{2}\,a^5x^2e+\frac{1}{2}\,a^5d\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*c^5*x^22*e + 1/20*c^5*d*x^20 + 5/18*a*c^4*x^18*e + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*x^14*e + 5/6*a^2*c^3*d*x^12 + a^3*c^2*x^10*e + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*x^6*e + 5/4*a^4*c*d*x^4 + 1/2*a^5*x^2*e + 1/2*a^5*d*log(x^2)
```

Fricas [A]
time = 0.38, size = 122, normalized size = 0.86

$$\frac{1}{20}\,c^5dx^{20}+\frac{5}{16}\,ac^4dx^{16}+\frac{5}{6}\,a^2c^3dx^{12}+\frac{5}{4}\,a^3c^2dx^8+\frac{5}{4}\,a^4cdx^4+a^5d\log\left(x\right)+\frac{1}{1386}\,\left(63\,c^5x^{22}+385\,ac^4x^{18}+990\,a^2c^3x^{14}+1386\,a^3c^2x^{10}+1155\,a^4cx^6+693\,a^5x^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="fricas")
```

```
[Out] 1/20*c^5*d*x^20 + 5/16*a*c^4*d*x^16 + 5/6*a^2*c^3*d*x^12 + 5/4*a^3*c^2*d*x^
8 + 5/4*a^4*c*d*x^4 + a^5*d*log(x) + 1/1386*(63*c^5*x^22 + 385*a*c^4*x^18 +
 990*a^2*c^3*x^14 + 1386*a^3*c^2*x^10 + 1155*a^4*c*x^6 + 693*a^5*x^2)*e
```

Sympy [A]
time = 0.08, size = 150, normalized size = 1.06

$$a^5 d\log{(x)} + \frac{a^5ex^2}{2} + \frac{5a^4cdx^4}{4} + \frac{5a^4cex^6}{6} + \frac{5a^3c^2dx^8}{4} + a^3c^2ex^{10} + \frac{5a^2c^3dx^{12}}{6} + \frac{5a^2c^3ex^{14}}{7} + \frac{5ac^4dx^{16}}{16} + \frac{5ac^4ex^{18}}{18} + \frac{c^5dx^{20}}{20} + \frac{c^5ex^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x,x)
```

```
[Out] a**5*d*log(x) + a**5*e*x**2/2 + 5*a**4*c*d*x**4/4 + 5*a**4*c*e*x**6/6 + 5*a
**3*c**2*d*x**8/4 + a**3*c**2*e*x**10 + 5*a**2*c**3*d*x**12/6 + 5*a**2*c**3
*e*x**14/7 + 5*a*c**4*d*x**16/16 + 5*a*c**4*e*x**18/18 + c**5*d*x**20/20 +
c**5*e*x**22/22
```

Giac [A]
time = 4.15, size = 131, normalized size = 0.92

$$\frac{1}{22}c^5x^{22}e + \frac{1}{20}c^5dx^{20} + \frac{5}{18}ac^4x^{18}e + \frac{5}{16}ac^4dx^{16} + \frac{5}{7}a^2c^3x^{14}e + \frac{5}{6}a^2c^3dx^{12} + a^3c^2x^{10}e + \frac{5}{4}a^3c^2dx^8 + \frac{5}{6}a^4cx^6e + \frac{5}{4}a^4cdx^4 + \frac{1}{2}a^5x^2e + \frac{1}{2}a^5d\log{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*c^5*x^22*e + 1/20*c^5*d*x^20 + 5/18*a*c^4*x^18*e + 5/16*a*c^4*d*x^16 +
 5/7*a^2*c^3*x^14*e + 5/6*a^2*c^3*d*x^12 + a^3*c^2*x^10*e + 5/4*a^3*c^2*d*x
^8 + 5/6*a^4*c*x^6*e + 5/4*a^4*c*d*x^4 + 1/2*a^5*x^2*e + 1/2*a^5*d*log(x^2)
```

Mupad [B]
time = 0.11, size = 122, normalized size = 0.86

$$\frac{a^5\,e\,x^2}{2} + \frac{c^5\,d\,x^{20}}{20} + \frac{c^5\,e\,x^{22}}{22} + a^5\,d\,\ln{(x)} + \frac{5\,a^3\,c^2\,d\,x^8}{4} + \frac{5\,a^2\,c^3\,d\,x^{12}}{6} + a^3\,c^2\,e\,x^{10} + \frac{5\,a^2\,c^3\,e\,x^{14}}{7} + \frac{5\,a^4\,c\,d\,x^4}{4} + \frac{5\,a\,c^4\,d\,x^{16}}{16} + \frac{5\,a^4\,c\,e\,x^6}{6} + \frac{5\,a\,c^4\,e\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x,x)
```

```
[Out] (a^5*e*x^2)/2 + (c^5*d*x^20)/20 + (c^5*e*x^22)/22 + a^5*d*log(x) + (5*a^3*c
^2*d*x^8)/4 + (5*a^2*c^3*d*x^12)/6 + a^3*c^2*e*x^10 + (5*a^2*c^3*e*x^14)/7
+ (5*a^4*c*d*x^4)/4 + (5*a*c^4*d*x^16)/16 + (5*a^4*c*e*x^6)/6 + (5*a*c^4*e*
x^18)/18
```

3.6 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^2} \, dx$

Optimal. Leaf size=139

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+$$

```
[Out] -a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c^
2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a*c
^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

Rubi [A]
time = 0.05, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {1276}

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^{19}+\frac{1}{21}c^5ex^{21}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

Rule 1276

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, q}, x] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(a+cx^4)^5}{x^2} \, dx &= \int \left(a^5e+\frac{a^5d}{x^2}+5a^4cdx^2+5a^4cex^4+10a^3c^2dx^6+10a^3c^2ex^8+10a^2c^3dx^{10}+\right.\\ &= -\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 139, normalized size = 1.00

$$-\frac{a^5d}{x}+a^5ex+\frac{5}{3}a^4cdx^3+a^4cex^5+\frac{10}{7}a^3c^2dx^7+\frac{10}{9}a^3c^2ex^9+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{13}a^2c^3ex^{13}+\frac{1}{3}ac^4dx^{15}+\frac{5}{17}ac^4ex^{17}+\frac{1}{19}c^5dx^{19}+\frac{1}{21}c^5ex^{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^2,x]
```

```
[Out] -((a^5*d)/x) + a^5*e*x + (5*a^4*c*d*x^3)/3 + a^4*c*e*x^5 + (10*a^3*c^2*d*x^
7)/7 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*d*x^11)/11 + (10*a^2*c^3*e*x^13)/
13 + (a*c^4*d*x^15)/3 + (5*a*c^4*e*x^17)/17 + (c^5*d*x^19)/19 + (c^5*e*x^21
)/21
```

Maple [A]
time = 0.13, size = 122, normalized size = 0.88

method	result
default	$-\frac{a^5 d}{x} + a^5 e x + \frac{5a^4 c d x^3}{3} + a^4 c e x^5 + \frac{10a^3 c^2 d x^7}{7} + \frac{10a^3 c^2 e x^9}{9} + \frac{10a^2 c^3 d x^{11}}{11} + \frac{10a^2 c^3 e x^{13}}{13} + \frac{a c^4 d x^{15}}{3} + \frac{5a c^4 e x}{17}$
risch	$-\frac{a^5 d}{x} + a^5 e x + \frac{5a^4 c d x^3}{3} + a^4 c e x^5 + \frac{10a^3 c^2 d x^7}{7} + \frac{10a^3 c^2 e x^9}{9} + \frac{10a^2 c^3 d x^{11}}{11} + \frac{10a^2 c^3 e x^{13}}{13} + \frac{a c^4 d x^{15}}{3} + \frac{5a c^4 e x}{17}$
norman	$\frac{-d a^5 + a^5 e x^2 + \frac{5}{3} a^4 c d x^4 + a^4 c e x^6 + \frac{10}{7} a^3 c^2 d x^8 + \frac{10}{9} a^3 c^2 e x^{10} + \frac{10}{11} a^2 c^3 d x^{12} + \frac{10}{13} a^2 c^3 e x^{14} + \frac{1}{3} a c^4 d x^{16} + \frac{5}{17} a c^4 e x^{18} + \frac{1}{19} c^5 d x^{20} + \frac{1}{21} c^5}{x}$
gosper	$-\frac{-138567 c^5 e x^{22} - 153153 c^5 d x^{20} - 855855 a c^4 e x^{18} - 969969 a c^4 d x^{16} - 2238390 a^2 c^3 e x^{14} - 2645370 a^2 c^3 d x^{12} - 3233230 a^3 c^2 e x^{10} - 41}{2909907 x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -a^5*d/x+a^5*e*x+5/3*a^4*c*d*x^3+a^4*c*e*x^5+10/7*a^3*c^2*d*x^7+10/9*a^3*c^
2*e*x^9+10/11*a^2*c^3*d*x^11+10/13*a^2*c^3*e*x^13+1/3*a*c^4*d*x^15+5/17*a*c
^4*e*x^17+1/19*c^5*d*x^19+1/21*c^5*e*x^21
```

Maxima [A]
time = 0.28, size = 127, normalized size = 0.91

$$\frac{1}{21} c^5 x^{21} e + \frac{1}{19} c^5 d x^{19} + \frac{5}{17} a c^4 x^{17} e + \frac{1}{3} a c^4 d x^{15} + \frac{10}{13} a^2 c^3 x^{13} e + \frac{10}{11} a^2 c^3 d x^{11} + \frac{10}{9} a^3 c^2 x^9 e + \frac{10}{7} a^3 c^2 d x^7 + a^4 c x^5 e + \frac{5}{3} a^4 c d x^3 + a^5 x e - \frac{a^5 d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*c^5*x^21*e + 1/19*c^5*d*x^19 + 5/17*a*c^4*x^17*e + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*x^13*e + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*x^9*e + 10/7*a^3
*c^2*d*x^7 + a^4*c*x^5*e + 5/3*a^4*c*d*x^3 + a^5*x*e - a^5*d/x
```

Fricas [A]
time = 0.37, size = 126, normalized size = 0.91

$$\frac{153153 c^5 d x^{20} + 969969 a c^4 d x^{16} + 2645370 a^2 c^3 d x^{12} + 4157010 a^3 c^2 d x^8 + 4849845 a^4 c d x^4 - 2909907 a^5 d + 209 (663 c^5 x^{22} + 4095 a c^4 x^{18} + 10710 a^2 c^3 x^{14} + 15470 a^3 c^2 x^{10} + 13923 a^4 c x^6 + 13923 a^5 x^2) e}{2909907 x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/2909907*(153153*c^5*d*x^20 + 969969*a*c^4*d*x^16 + 2645370*a^2*c^3*d*x^12
 + 4157010*a^3*c^2*d*x^8 + 4849845*a^4*c*d*x^4 - 2909907*a^5*d + 209*(663*c
^5*x^22 + 4095*a*c^4*x^18 + 10710*a^2*c^3*x^14 + 15470*a^3*c^2*x^10 + 13923
*a^4*c*x^6 + 13923*a^5*x^2)*e)/x
```

Sympy [A]
time = 0.07, size = 143, normalized size = 1.03

$$-\frac{a^5d}{x}+a^5ex+\frac{5a^4cdx^3}{3}+a^4cex^5+\frac{10a^3c^2dx^7}{7}+\frac{10a^3c^2ex^9}{9}+\frac{10a^2c^3dx^{11}}{11}+\frac{10a^2c^3ex^{13}}{13}+\frac{ac^4dx^{15}}{3}+\frac{5ac^4ex^{17}}{17}+\frac{c^5dx^{19}}{19}+\frac{c^5ex^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**2,x)
```

```
[Out] -a**5*d/x + a**5*e*x + 5*a**4*c*d*x**3/3 + a**4*c*e*x**5 + 10*a**3*c**2*d*x
**7/7 + 10*a**3*c**2*e*x**9/9 + 10*a**2*c**3*d*x**11/11 + 10*a**2*c**3*e*x*
*13/13 + a*c**4*d*x**15/3 + 5*a*c**4*e*x**17/17 + c**5*d*x**19/19 + c**5*e*
x**21/21
```

Giac [A]
time = 4.19, size = 127, normalized size = 0.91

$$\frac{1}{21}c^5x^{21}e+\frac{1}{19}c^5dx^{19}+\frac{5}{17}ac^4x^{17}e+\frac{1}{3}ac^4dx^{15}+\frac{10}{13}a^2c^3x^{13}e+\frac{10}{11}a^2c^3dx^{11}+\frac{10}{9}a^3c^2x^9e+\frac{10}{7}a^3c^2dx^7+a^4cx^5e+\frac{5}{3}a^4cdx^3+a^5xe-\frac{a^5d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*c^5*x^21*e + 1/19*c^5*d*x^19 + 5/17*a*c^4*x^17*e + 1/3*a*c^4*d*x^15 +
10/13*a^2*c^3*x^13*e + 10/11*a^2*c^3*d*x^11 + 10/9*a^3*c^2*x^9*e + 10/7*a^3
*c^2*d*x^7 + a^4*c*x^5*e + 5/3*a^4*c*d*x^3 + a^5*x*e - a^5*d/x
```

Mupad [B]
time = 0.09, size = 121, normalized size = 0.87

$$\frac{c^5\,d\,x^{19}}{19}-\frac{a^5\,d}{x}+\frac{c^5\,e\,x^{21}}{21}+a^5\,e\,x+\frac{10\,a^3\,c^2\,d\,x^7}{7}+\frac{10\,a^2\,c^3\,d\,x^{11}}{11}+\frac{10\,a^3\,c^2\,e\,x^9}{9}+\frac{10\,a^2\,c^3\,e\,x^{13}}{13}+\frac{5\,a^4\,c\,d\,x^3}{3}+\frac{a\,c^4\,d\,x^{15}}{3}+a^4\,c\,e\,x^5+\frac{5\,a\,c^4\,e\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x^2,x)
```

```
[Out] (c^5*d*x^19)/19 - (a^5*d)/x + (c^5*e*x^21)/21 + a^5*e*x + (10*a^3*c^2*d*x^7
)/7 + (10*a^2*c^3*d*x^11)/11 + (10*a^3*c^2*e*x^9)/9 + (10*a^2*c^3*e*x^13)/1
3 + (5*a^4*c*d*x^3)/3 + (a*c^4*d*x^15)/3 + a^4*c*e*x^5 + (5*a*c^4*e*x^17)/1
7
```

3.7 $\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx$

Optimal. Leaf size=142

$$-\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{2}$$

```
[Out] -1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c^
2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*x^
16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

Rubi [A]

time = 0.07, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {1266, 780}

$$-\frac{a^5d}{2x^2}+a^5e\log(x)+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\frac{5}{14}ac^4dx^{14}+\frac{5}{16}ac^4ex^{16}+\frac{1}{18}c^5dx^{18}+\frac{1}{20}c^5ex^{20}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -1/2*(a^5*d)/x^2 + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

Rule 780

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + c*x^2)^p, x], x] /; F
reeQ[{a, c, e, f, g, m}, x] && IGtQ[p, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)(a+cx^4)^5}{x^3}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)(a+cx^2)^5}{x^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(5a^4cd+\frac{a^5d}{x^2}+\frac{a^5e}{x}+5a^4cex+10a^3c^2dx^2+10a^3c^2ex^3+10a^2c^3d\right.\right.\\ &= -\frac{a^5d}{2x^2}+\frac{5}{2}a^4cdx^2+\frac{5}{4}a^4cex^4+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^3c^2ex^8+a^2c^3dx^{10}+\frac{5}{6}a^2c^3ex^{12}+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 142, normalized size = 1.00

$$-\frac{a^5 d}{2 x^2}+\frac{5}{2} a^4 c d x^2+\frac{5}{4} a^4 c e x^4+\frac{5}{3} a^3 c^2 d x^6+\frac{5}{4} a^3 c^2 e x^8+a^2 c^3 d x^{10}+\frac{5}{6} a^2 c^3 e x^{12}+\frac{5}{14} a c^4 d x^{14}+\frac{5}{16} a c^4 e x^{16}+\frac{1}{18} c^5 d x^{18}+\frac{1}{20} c^5 e x^{20}+a^5 e \log (x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + c*x^4)^5)/x^3,x]
```

```
[Out] -1/2*(a^5*d)/x^2 + (5*a^4*c*d*x^2)/2 + (5*a^4*c*e*x^4)/4 + (5*a^3*c^2*d*x^6
)/3 + (5*a^3*c^2*e*x^8)/4 + a^2*c^3*d*x^10 + (5*a^2*c^3*e*x^12)/6 + (5*a*c^
4*d*x^14)/14 + (5*a*c^4*e*x^16)/16 + (c^5*d*x^18)/18 + (c^5*e*x^20)/20 + a^
5*e*Log[x]
```

Maple [A]
time = 0.14, size = 123, normalized size = 0.87

method	result
default	$-\frac{a^5 d}{2x^2}+\frac{5a^4 cd\,x^2}{2}+\frac{5a^4 ce\,x^4}{4}+\frac{5a^3 c^2 d\,x^6}{3}+\frac{5a^3 c^2 e\,x^8}{4}+a^2 c^3 d\,x^{10}+\frac{5a^2 c^3 e\,x^{12}}{6}+\frac{5a\,c^4 d\,x^{14}}{14}+\frac{5a\,c^4 e\,x^{16}}{16}+\frac{c^5 d}{18}$
risch	$-\frac{a^5 d}{2x^2}+\frac{5a^4 cd\,x^2}{2}+\frac{5a^4 ce\,x^4}{4}+\frac{5a^3 c^2 d\,x^6}{3}+\frac{5a^3 c^2 e\,x^8}{4}+a^2 c^3 d\,x^{10}+\frac{5a^2 c^3 e\,x^{12}}{6}+\frac{5a\,c^4 d\,x^{14}}{14}+\frac{5a\,c^4 e\,x^{16}}{16}+\frac{c^5 d}{18}$
norman	$\frac{a^2 c^3 d\,x^{12}-\frac{1}{2} d\,a^5+\frac{1}{18} c^5 d\,x^{20}+\frac{1}{20} c^5 e\,x^{22}+\frac{5}{14} a\,c^4 d\,x^{16}+\frac{5}{16} a\,c^4 e\,x^{18}+\frac{5}{6} a^2 c^3 e\,x^{14}+\frac{5}{3} a^3 c^2 d\,x^8+\frac{5}{4} a^3 c^2 e\,x^{10}+\frac{5}{2} a^4 cd\,x^4+\frac{5}{4} a^4 ce\,x^6}{x^2}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+a)^5/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*a^5*d/x^2+5/2*a^4*c*d*x^2+5/4*a^4*c*e*x^4+5/3*a^3*c^2*d*x^6+5/4*a^3*c^
2*e*x^8+a^2*c^3*d*x^10+5/6*a^2*c^3*e*x^12+5/14*a*c^4*d*x^14+5/16*a*c^4*e*x^
16+1/18*c^5*d*x^18+1/20*c^5*e*x^20+a^5*e*ln(x)
```

Maxima [A]
time = 0.28, size = 131, normalized size = 0.92

$$\frac{1}{20}\,c^5 x^{20} e+\frac{1}{18}\,c^5 d x^{18}+\frac{5}{16}\,a c^4 x^{16} e+\frac{5}{14}\,a c^4 d x^{14}+\frac{5}{6}\,a^2 c^3 x^{12} e+a^2 c^3 d x^{10}+\frac{5}{4}\,a^3 c^2 x^8 e+\frac{5}{3}\,a^3 c^2 d x^6+\frac{5}{4}\,a^4 c x^4 e+\frac{5}{2}\,a^4 c d x^2+\frac{1}{2}\,a^5 e \log \left(x^2\right)-\frac{a^5 d}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*c^5*x^20*e + 1/18*c^5*d*x^18 + 5/16*a*c^4*x^16*e + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*x^12*e + a^2*c^3*d*x^10 + 5/4*a^3*c^2*x^8*e + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*x^4*e + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*a^5*d/x^2
```

Fricas [A]
time = 0.37, size = 130, normalized size = 0.92

$$\frac{280\,c^5 d x^{20}+1800\,a c^4 d x^{16}+5040\,a^2 c^3 d x^{12}+8400\,a^3 c^2 d x^8+12600\,a^4 c d x^4+5040\,a^5 x^2 e \log (x)-2520\,a^5 d+21\,(12\,c^5 x^{22}+75\,a c^4 x^{18}+200\,a^2 c^3 x^{14}+300\,a^3 c^2 x^{10}+300\,a^4 c x^6) e}{5040\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(280*c^5*d*x^20 + 1800*a*c^4*d*x^16 + 5040*a^2*c^3*d*x^12 + 8400*a^3
*c^2*d*x^8 + 12600*a^4*c*d*x^4 + 5040*a^5*x^2*e*log(x) - 2520*a^5*d + 21*(1
2*c^5*x^22 + 75*a*c^4*x^18 + 200*a^2*c^3*x^14 + 300*a^3*c^2*x^10 + 300*a^4*
c*x^6)*e)/x^2
```

Sympy [A]
time = 0.09, size = 150, normalized size = 1.06

$$-\frac{a^5d}{2x^2}+a^5e\log{(x)}+\frac{5a^4cdx^2}{2}+\frac{5a^4cex^4}{4}+\frac{5a^3c^2dx^6}{3}+\frac{5a^3c^2ex^8}{4}+a^2c^3dx^{10}+\frac{5a^2c^3ex^{12}}{6}+\frac{5ac^4dx^{14}}{14}+\frac{5ac^4ex^{16}}{16}+\frac{c^5dx^{18}}{18}+\frac{c^5ex^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+a)**5/x**3,x)
```

```
[Out] -a**5*d/(2*x**2) + a**5*e*log(x) + 5*a**4*c*d*x**2/2 + 5*a**4*c*e*x**4/4 +
5*a**3*c**2*d*x**6/3 + 5*a**3*c**2*e*x**8/4 + a**2*c**3*d*x**10 + 5*a**2*c*
*3*e*x**12/6 + 5*a*c**4*d*x**14/14 + 5*a*c**4*e*x**16/16 + c**5*d*x**18/18
+ c**5*e*x**20/20
```

Giac [A]
time = 4.33, size = 142, normalized size = 1.00

$$\frac{1}{20}c^5x^{20}e+\frac{1}{18}c^5dx^{18}+\frac{5}{16}ac^4x^{16}e+\frac{5}{14}ac^4dx^{14}+\frac{5}{6}a^2c^3x^{12}e+a^2c^3dx^{10}+\frac{5}{4}a^3c^2x^8e+\frac{5}{3}a^3c^2dx^6+\frac{5}{4}a^4cx^4e+\frac{5}{2}a^4cdx^2+\frac{1}{2}a^5e\log{(x^2)}-\frac{a^5x^2e+a^5d}{2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+a)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*c^5*x^20*e + 1/18*c^5*d*x^18 + 5/16*a*c^4*x^16*e + 5/14*a*c^4*d*x^14 +
 5/6*a^2*c^3*x^12*e + a^2*c^3*d*x^10 + 5/4*a^3*c^2*x^8*e + 5/3*a^3*c^2*d*x^
6 + 5/4*a^4*c*x^4*e + 5/2*a^4*c*d*x^2 + 1/2*a^5*e*log(x^2) - 1/2*(a^5*x^2*e
 + a^5*d)/x^2
```

Mupad [B]
time = 0.07, size = 122, normalized size = 0.86

$$\frac{c^5\,d\,x^{18}}{18}-\frac{a^5\,d}{2\,x^2}+\frac{c^5\,e\,x^{20}}{20}+a^5\,e\,\ln{(x)}+\frac{5\,a^3\,c^2\,d\,x^6}{3}+a^2\,c^3\,d\,x^{10}+\frac{5\,a^3\,c^2\,e\,x^8}{4}+\frac{5\,a^2\,c^3\,e\,x^{12}}{6}+\frac{5\,a^4\,c\,d\,x^2}{2}+\frac{5\,a\,c^4\,d\,x^{14}}{14}+\frac{5\,a^4\,c\,e\,x^4}{4}+\frac{5\,a\,c^4\,e\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((a + c*x^4)^5*(d + e*x^2))/x^3,x)
```

```
[Out] (c^5*d*x^18)/18 - (a^5*d)/(2*x^2) + (c^5*e*x^20)/20 + a^5*e*log(x) + (5*a^3
*c^2*d*x^6)/3 + a^2*c^3*d*x^10 + (5*a^3*c^2*e*x^8)/4 + (5*a^2*c^3*e*x^12)/6
 + (5*a^4*c*d*x^2)/2 + (5*a*c^4*d*x^14)/14 + (5*a^4*c*e*x^4)/4 + (5*a*c^4*e
*x^16)/16
```

3.8 $\int x^5(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=67

$$-\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4\left(5+x^4\right)^{3/2}-\frac{1}{4}\left(4-x^2\right)\left(5+x^4\right)^{3/2}-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 3/10*x^4*(x^4+5)^(3/2)-1/4*(-x^2+4)*(x^4+5)^(3/2)-25/8*arcsinh(1/5*x^2*5^(1
/2))-5/8*x^2*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1266, 847, 794, 201, 221}

$$\frac{3}{10}\left(x^4+5\right)^{3/2}x^4-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{8}\sqrt{x^4+5}\,x^2-\frac{1}{4}\left(4-x^2\right)\left(x^4+5\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-5*x^2*Sqrt[5 + x^4])/8 + (3*x^4*(5 + x^4)^(3/2))/10 - ((4 - x^2)*(5 + x^4
)^(3/2))/4 - (25*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^5(2+3x^2)\sqrt{5+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{3}{10}x^4(5+x^4)^{3/2}+\frac{1}{10}\text{Subst}\left(\int x(-30+10x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{3}{10}x^4(5+x^4)^{3/2}-\frac{1}{4}(4-x^2)\,(5+x^4)^{3/2}-\frac{5}{4}\text{Subst}\left(\int\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4(5+x^4)^{3/2}-\frac{1}{4}(4-x^2)\,(5+x^4)^{3/2}-\frac{25}{8}\text{Subst}\left(\int \frac{}{\sqrt{}}\right.\\ &= -\frac{5}{8}x^2\sqrt{5+x^4}+\frac{3}{10}x^4(5+x^4)^{3/2}-\frac{1}{4}(4-x^2)\,(5+x^4)^{3/2}-\frac{25}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right.\end{aligned}$$

Mathematica [A]
time = 0.07, size = 54, normalized size = 0.81

$$\frac{1}{40}\sqrt{5+x^4}\left(-200+25x^2+20x^4+10x^6+12x^8\right)-\frac{25}{8}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (Sqrt[5 + x^4]*(-200 + 25*x^2 + 20*x^4 + 10*x^6 + 12*x^8))/40 - (25*ArcTanh
[x^2/Sqrt[5 + x^4]])/8
```

Maple [A]
time = 0.16, size = 53, normalized size = 0.79

method	result
risch	$\frac{(12x^8+10x^6+20x^4+25x^2-200)\sqrt{x^4+5}}{40}-\frac{25\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$
trager	$\left(\frac{3}{10}x^8+\frac{1}{4}x^6+\frac{1}{2}x^4+\frac{5}{8}x^2-5\right)\sqrt{x^4+5}-\frac{25\ln\left(x^2+\sqrt{x^4+5}\right)}{8}$
default	$\frac{(x^4+5)^{\frac{3}{2}}(3x^4-10)}{10}+\frac{x^2(x^4+5)^{\frac{3}{2}}}{4}-\frac{5x^2\sqrt{x^4+5}}{8}-\frac{25\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$
elliptic	$\frac{3x^8\sqrt{x^4+5}}{10}+\frac{x^4\sqrt{x^4+5}}{2}-5\sqrt{x^4+5}+\frac{x^6\sqrt{x^4+5}}{4}+\frac{5x^2\sqrt{x^4+5}}{8}-\frac{25\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$
meijerg	$-\frac{75\sqrt{5}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(1+\frac{x^4}{5}\right)^{\frac{3}{2}}\left(-\frac{3x^4}{5}+2\right)}{15}\right)}{8\sqrt{\pi}}-\frac{25\left(-\frac{\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{6x^4}{5}+3\right)\sqrt{1+\frac{x^4}{5}}}{30}+\frac{\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}\right.}{4\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*(x^4+5)^(3/2)*(3*x^4-10)+1/4*x^2*(x^4+5)^(3/2)-5/8*x^2*(x^4+5)^(1/2)-2
5/8*arcsinh(1/5*x^2*5^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(50) = 100$.

time = 0.49, size = 102, normalized size = 1.52

$$\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}-\frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}}-\frac{25\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2(x^4+5)}{x^4}-\frac{(x^4+5)^2}{x^8}-1\right)}-\frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{25}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3/2) - 25/8*(sqrt(x^4 + 5)/x^2 + (x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 25/16*log(sqrt(x
^4 + 5)/x^2 + 1) + 25/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]

time = 0.37, size = 48, normalized size = 0.72

$$\frac{1}{40}\left(12\,x^8+10\,x^6+20\,x^4+25\,x^2-200\right)\sqrt{x^4+5}+\frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/40*(12*x^8 + 10*x^6 + 20*x^4 + 25*x^2 - 200)*sqrt(x^4 + 5) + 25/8*log(-x^
2 + sqrt(x^4 + 5))
```

Sympy [A]
time = 3.25, size = 97, normalized size = 1.45

$$\frac{x^{10}}{4\sqrt{x^4+5}}+\frac{3x^8\sqrt{x^4+5}}{10}+\frac{15x^6}{8\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{25x^2}{8\sqrt{x^4+5}}-5\sqrt{x^4+5}-\frac{25\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 15*x**6/(8*sqrt(x**4
+ 5)) + x**4*sqrt(x**4 + 5)/2 + 25*x**2/(8*sqrt(x**4 + 5)) - 5*sqrt(x**4 +
5) - 25*asinh(sqrt(5)*x**2/5)/8
```

Giac [A]
time = 4.33, size = 54, normalized size = 0.81

$$\frac{1}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}-\frac{5}{2}\left(x^4+5\right)^{\frac{3}{2}}+\frac{25}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) - 5/2*(x^4 + 5)^(3
/2) + 25/8*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.37, size = 42, normalized size = 0.63

$$\sqrt{x^4+5}\,\left(\frac{3\,x^8}{10}+\frac{x^6}{4}+\frac{x^4}{2}+\frac{5\,x^2}{8}-5\right)-\frac{25\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((5*x^2)/8 + x^4/2 + x^6/4 + (3*x^8)/10 - 5) - (25*asinh((5
^(1/2)*x^2)/5))/8
```

3.9 $\int x^3(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=51

$$-\frac{15}{16}x^2\sqrt{5+x^4}+\frac{1}{24}(8+9x^2)\left(5+x^4\right)^{3/2}-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 1/24*(9*x^2+8)*(x^4+5)^(3/2)-75/16*arcsinh(1/5*x^2*5^(1/2))-15/16*x^2*(x^4+
5)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {1266, 794, 201, 221}

$$-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{16}\sqrt{x^4+5}\,x^2+\frac{1}{24}(9x^2+8)\left(x^4+5\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (-15*x^2*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 - (75*ArcSinh
[x^2/Sqrt[5]])/16
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(2+3x^2\right)\sqrt{5+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x(2+3x)\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= \frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}-\frac{15}{8}\text{Subst}\left(\int\sqrt{5+x^2}\,dx,x,x^2\right)\\ &= -\frac{15}{16}x^2\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}-\frac{75}{16}\text{Subst}\left(\int\frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= -\frac{15}{16}x^2\sqrt{5+x^4}+\frac{1}{24}\left(8+9x^2\right)\left(5+x^4\right)^{3/2}-\frac{75}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A]
time = 0.06, size = 49, normalized size = 0.96

$$\frac{1}{48}\sqrt{5+x^4}\left(80+45x^2+16x^4+18x^6\right)-\frac{75}{16}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(80 + 45*x^2 + 16*x^4 + 18*x^6))/48 - (75*ArcTanh[x^2/Sqrt[5 + x^4]])/16
```

Maple [A]
time = 0.13, size = 46, normalized size = 0.90

method	result
risch	$\frac{\left(18x^6+16x^4+45x^2+80\right)\sqrt{x^4+5}}{48}-\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$
trager	$\left(\frac{3}{8}x^6+\frac{1}{3}x^4+\frac{15}{16}x^2+\frac{5}{3}\right)\sqrt{x^4+5}-\frac{75\ln\left(x^2+\sqrt{x^4+5}\right)}{16}$
default	$\frac{3x^2\left(x^4+5\right)^{\frac{3}{2}}}{8}-\frac{15x^2\sqrt{x^4+5}}{16}-\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{\left(x^4+5\right)^{\frac{3}{2}}}{3}$
elliptic	$\frac{3x^6\sqrt{x^4+5}}{8}+\frac{15x^2\sqrt{x^4+5}}{16}-\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{5\sqrt{x^4+5}}{3}$

meijerg	$-\frac{75\left(-\frac{\sqrt{\pi}\,x^2\sqrt{5}\,\left(\frac{6x^4}{5}+3\right)\sqrt{1+\frac{x^4}{5}}}{30}+\frac{\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}\right)}{8\sqrt{\pi}}-\frac{5\sqrt{5}\,\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\,\left(2+\frac{2x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{3}\right)}{4\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^2*(x^4+5)^(3/2)-15/16*x^2*(x^4+5)^(1/2)-75/16*arcsinh(1/5*x^2*5^(1/2)
)+1/3*(x^4+5)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(40) = 80$.
time = 0.48, size = 93, normalized size = 1.82

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-\frac{75\left(\frac{\sqrt{x^4+5}}{x^2}+\frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\,(x^4+5)}{x^4}-\frac{(x^4+5)^2}{x^8}-1\right)}-\frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{75}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 75/16*(sqrt(x^4 + 5)/x^2 + (x^4 + 5)^(3/2)/x^6)/(2*(x
^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) - 75/32*log(sqrt(x^4 + 5)/x^2 + 1) + 75/
32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.35, size = 43, normalized size = 0.84

$$\frac{1}{48}\left(18\,x^6+16\,x^4+45\,x^2+80\right)\sqrt{x^4+5}+\frac{75}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 + 45*x^2 + 80)*sqrt(x^4 + 5) + 75/16*log(-x^2 + sqrt(
x^4 + 5))
```

Sympy [A]
time = 2.50, size = 70, normalized size = 1.37

$$\frac{3x^{10}}{8\sqrt{x^4+5}}+\frac{45x^6}{16\sqrt{x^4+5}}+\frac{75x^2}{16\sqrt{x^4+5}}+\frac{(x^4+5)^{\frac{3}{2}}}{3}-\frac{75\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 45*x**6/(16*sqrt(x**4 + 5)) + 75*x**2/(16*sqrt
(x**4 + 5)) + (x**4 + 5)**(3/2)/3 - 75*asinh(sqrt(5)*x**2/5)/16
```

Giac [A]
time = 2.90, size = 45, normalized size = 0.88

$$\frac{3}{16}\left(2x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}+\frac{75}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 3/16*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 1/3*(x^4 + 5)^(3/2) + 75/16*log(-x^2 +
 sqrt(x^4 + 5))
```

Mupad [B]
time = 0.39, size = 37, normalized size = 0.73

$$\sqrt{x^4+5}\,\left(\frac{3x^6}{8}+\frac{x^4}{3}+\frac{15x^2}{16}+\frac{5}{3}\right)-\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((15*x^2)/16 + x^4/3 + (3*x^6)/8 + 5/3) - (75*asinh((5^(1/2
)*x^2)/5))/16
```

3.10 $\int x(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=44

$$\frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}\left(5+x^4\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^2*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}}=0.222$, Rules used = {1262, 655, 201, 221}

$$\frac{1}{2}\left(x^4+5\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\sqrt{x^4+5}\,x^2$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x^2*Sqrt[5 + x^4])/2 + (5 + x^4)^(3/2)/2 + (5*ArcSinh[x^2/Sqrt[5]])/2
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int x(2+3x^2)\sqrt{5+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int (2+3x)\sqrt{5+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}\left(5+x^4\right)^{3/2}+\text{Subst}\left(\int \sqrt{5+x^2}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}\left(5+x^4\right)^{3/2}+\frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{1}{2}x^2\sqrt{5+x^4}+\frac{1}{2}\left(5+x^4\right)^{3/2}+\frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 40, normalized size = 0.91

$$\frac{1}{2}\sqrt{5+x^4}\left(5+x^2+x^4\right)+\frac{5}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(5 + x^2 + x^4))/2 + (5*ArcTanh[x^2/Sqrt[5 + x^4]])/2
```

Maple [A]
time = 0.12, size = 34, normalized size = 0.77

method	result	size
risch	$\frac{(x^4+x^2+5)\sqrt{x^4+5}}{2}+\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
default	$\frac{(x^4+5)^{\frac{3}{2}}}{2}+\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+\frac{x^2\sqrt{x^4+5}}{2}$	34
trager	$\left(\frac{1}{2}x^4+\frac{1}{2}x^2+\frac{5}{2}\right)\sqrt{x^4+5}-\frac{5\ln\left(x^2-\sqrt{x^4+5}\right)}{2}$	38
elliptic	$\frac{x^4\sqrt{x^4+5}}{2}+\frac{5\sqrt{x^4+5}}{2}+\frac{x^2\sqrt{x^4+5}}{2}+\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	46
meijerg	$-\frac{15\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{2\sqrt{\pi}\left(2+\frac{2x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{3}\right)}{8\sqrt{\pi}}-\frac{5\left(-\frac{2\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{5}-2\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)\right)}{4\sqrt{\pi}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^4+5)^(3/2)+5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^2*(x^4+5)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(33) = 66$.
time = 0.49, size = 67, normalized size = 1.52

$$\frac{1}{2}\left(x^{4}+5\right)^{\frac{3}{2}}+\frac{5\sqrt{x^{4}+5}}{2\,x^{2}\left(\frac{x^{4}+5}{x^{4}}-1\right)}+\frac{5}{4}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}+1\right)-\frac{5}{4}\log\left(\frac{\sqrt{x^{4}+5}}{x^{2}}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 5/4*log
(sqrt(x^4 + 5)/x^2 + 1) - 5/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.36, size = 34, normalized size = 0.77

$$\frac{1}{2}\left(x^{4}+x^{2}+5\right)\sqrt{x^{4}+5}-\frac{5}{2}\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 + x^2 + 5)*sqrt(x^4 + 5) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A]
time = 1.60, size = 53, normalized size = 1.20

$$\frac{x^{6}}{2\sqrt{x^{4}+5}}+\frac{5x^{2}}{2\sqrt{x^{4}+5}}+\frac{\left(x^{4}+5\right)^{\frac{3}{2}}}{2}+\frac{5\operatorname{asinh}\left(\frac{\sqrt{5}\,x^{2}}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 +
 5*asinh(sqrt(5)*x**2/5)/2
```

Giac [A]
time = 3.28, size = 38, normalized size = 0.86

$$\frac{1}{2}\sqrt{x^{4}+5}\,x^{2}+\frac{1}{2}\left(x^{4}+5\right)^{\frac{3}{2}}-\frac{5}{2}\log\left(-x^{2}+\sqrt{x^{4}+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] 1/2*sqrt(x^4 + 5)*x^2 + 1/2*(x^4 + 5)^(3/2) - 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.14, size = 32, normalized size = 0.73

$$\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}+\sqrt{x^4+5}\,\left(\frac{x^4}{2}+\frac{x^2}{2}+\frac{5}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
[Out] (5*asinh((5^(1/2)*x^2)/5))/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 5/2)
```

3.11 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx$

Optimal. Leaf size=58

$$\frac{1}{4}(4+3x^2)\sqrt{5+x^4}+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] 15/4*arcsinh(1/5*x^2*5^(1/2))-arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/
4*(3*x^2+4)*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1266, 829, 858, 221, 272, 65, 213}

$$-\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}\sqrt{x^4+5}\,(3x^2+4)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 + (15*ArcSinh[x^2/Sqrt[5]])/4 - Sqrt[5]*ArcTa
nh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{1}{4}\text{Subst}\left(\int \frac{20+15x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right) + 5\text{Subst}\left(\int \frac{1}{x\sqrt{5+}}\right.$$
$$= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{5}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$
$$= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + 5\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+}\right.$$
$$= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} + \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

Mathematica [A]
time = 0.10, size = 67, normalized size = 1.16

$$\frac{1}{4}\left((4+3x^2)\sqrt{5+x^4} + 15\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right) + 8\sqrt{5}\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x,x]
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4] + 15*ArcTanh[x^2/Sqrt[5 + x^4]] + 8*Sqrt[5]*ArcT
anh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/4
```

Maple [A]
time = 0.26, size = 49, normalized size = 0.84

method	result
default	$\frac{3x^2\sqrt{x^4+5}}{4} + \frac{15\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4} + \sqrt{x^4+5} - \sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
elliptic	$\frac{3x^2\sqrt{x^4+5}}{4} + \frac{15\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4} + \sqrt{x^4+5} - \sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)$
trager	$\left(\frac{3x^2}{4}+1\right)\sqrt{x^4+5} + \frac{15\ln\left(-x^2-\sqrt{x^4+5}\right)}{4} + \text{RootOf}\left(\_Z^2-5\right)\ln\left(-\frac{\text{RootOf}\left(\_Z^2-5\right)-\sqrt{x^4+5}}{x^2}\right.$

meijerg	$-\frac{\sqrt{5}\left(4\sqrt{\pi}-4\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}+4\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-2(2-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}\right)}{4\sqrt{\pi}}-\frac{15\left(-\frac{2\sqrt{\pi}\,x^2\sqrt{}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x^2*(x^4+5)^(1/2)+15/4*arcsinh(1/5*x^2*5^(1/2))+(x^4+5)^(1/2)-5^(1/2)*a
rctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(46) = 92$.
time = 0.49, size = 99, normalized size = 1.71

$$\frac{1}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/8*log(sqrt(x
^4 + 5)/x^2 + 1) - 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.35, size = 56, normalized size = 0.97

$$\frac{1}{4}\sqrt{x^4+5}\left(3x^2+4\right)+\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
 - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A]
time = 7.86, size = 83, normalized size = 1.43

$$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}+\frac{\sqrt{5}\log\left(x^4\right)}{2}-\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)+\frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + s
qrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 15*asinh(sqrt(5)*x
**2/5)/4
```

Giac [A]
time = 3.18, size = 76, normalized size = 1.31

$$\frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right)+\sqrt{5}\,\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)-\frac{15}{4}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5)
)/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.15, size = 45, normalized size = 0.78

$$\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}-\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)+\sqrt{x^4+5}\,\left(\frac{3\,x^2}{4}+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x,x)
```

```
[Out] (15*asinh((5^(1/2)*x^2)/5))/4 - 5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5)
+ (x^4 + 5)^(1/2)*((3*x^2)/4 + 1)
```

3.12 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx$

Optimal. Leaf size=59

$$-\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] arcsinh(1/5*x^2*5^(1/2))-3/2*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/2
*(-3*x^2+2)*(x^4+5)^(1/2)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {1266, 827, 858, 221, 272, 65, 213}

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,(2-3x^2)}{2x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] -1/2*((2 - 3*x^2)*Sqrt[5 + x^4])/x^2 + ArcSinh[x^2/Sqrt[5]] - (3*Sqrt[5]*Ar
cTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-30-4x}{x\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \frac{15}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right) + \text{Subst}\left(\int \frac{1}{\sqrt{5+x}}\right.$$
$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)$$
$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{2}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right.$$
$$= -\frac{(2-3x^2)\sqrt{5+x^4}}{2x^2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

Mathematica [A]
time = 0.11, size = 67, normalized size = 1.14

$$\frac{(-2+3x^2)\sqrt{5+x^4}}{2x^2} + \tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right) + 3\sqrt{5}\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^3,x]
```

```
[Out] ((-2 + 3*x^2)*Sqrt[5 + x^4])/(2*x^2) + ArcTanh[x^2/Sqrt[5 + x^4]] + 3*Sqrt[
5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]]
```

Maple [A]
time = 0.27, size = 61, normalized size = 1.03

method	result
risch	$-\frac{\sqrt{x^4+5}}{x^2}+\frac{3\sqrt{x^4+5}}{2}+\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{3\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
elliptic	$-\frac{\sqrt{x^4+5}}{x^2}+\frac{3\sqrt{x^4+5}}{2}+\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{3\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
default	$-\frac{(x^4+5)^{\frac{3}{2}}}{5x^2}+\frac{x^2\sqrt{x^4+5}}{5}+\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{3\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$

trager	$\frac{(3x^2-2)\sqrt{x^4+5}}{2x^2}+\ln\left(-x^2-\sqrt{x^4+5}\right)-\frac{3\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right)}{2}$
meijerg	$-\frac{\frac{4\sqrt{\pi}\,\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{x^2}-4\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4\sqrt{\pi}}-\frac{3\sqrt{5}\left(4\sqrt{\pi}-4\sqrt{\pi}\,\sqrt{1+\frac{x^4}{5}}+4\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right.\right.}{8\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/5/x^2*(x^4+5)^(3/2)+1/5*x^2*(x^4+5)^(1/2)+arcsinh(1/5*x^2*5^(1/2))+3/2*(
x^4+5)^(1/2)-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [A]
time = 0.50, size = 88, normalized size = 1.49

$$\frac{3}{4}\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*l
og(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.35, size = 72, normalized size = 1.22

$$\frac{3\sqrt{5}\,x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-2\,x^2\log\left(-x^2+\sqrt{x^4+5}\right)-2\,x^2+\sqrt{x^4+5}\,(3\,x^2-2)}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2*log(-x^2 + s
qrt(x^4 + 5)) - 2*x^2 + sqrt(x^4 + 5)*(3*x^2 - 2))/x^2
```

Sympy [A]
time = 3.76, size = 83, normalized size = 1.41

$$-\frac{x^2}{\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}}{2}+\frac{3\sqrt{5}\,\log\left(x^4\right)}{4}-\frac{3\sqrt{5}\,\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}+\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)-\frac{5}{x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**3,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*log(x**4)/4 - 3*sqrt(
5)*log(sqrt(x**4/5 + 1) + 1)/2 + asinh(sqrt(5)*x**2/5) - 5/(x**2*sqrt(x**4
+ 5))
```

Giac [A]
time = 3.86, size = 91, normalized size = 1.54

$$\frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}+\frac{10}{\left(x^2-\sqrt{x^4+5}\right)^2-5}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) + 3/2*sqrt(x^4 + 5) + 10/((x^2 - sqrt(x^4 + 5))^2 - 5) - log(-x^2 +
sqrt(x^4 + 5))
```

Mupad [B]
time = 0.79, size = 51, normalized size = 0.86

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{\sqrt{x^4+5}}{x^2}+\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,3\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^3,x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*3i)/
2 + (3*(x^4 + 5)^(1/2))/2 - (x^4 + 5)^(1/2)/x^2
```

3.13 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx$

Optimal. Leaf size=63

$$-\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/10*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2
)-1/2*(3*x^2+1)*(x^4+5)^(1/2)/x^4
```

Rubi [A]
time = 0.03, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1266, 825, 858, 221, 272, 65, 213}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\sqrt{x^4+5}\,(3x^2+1)}{2x^4}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
```

```
[Out] -1/2*((1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh
[Sqrt[5 + x^4]/Sqrt[5]]/(2*Sqrt[5])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^3}\,dx,x,x^2\right)$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} - \frac{1}{40}\text{Subst}\left(\int \frac{-20-60x}{x\sqrt{5+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+}}\right.$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{1}{2}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x}\right.$$
$$= -\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Mathematica [A]
time = 0.16, size = 70, normalized size = 1.11

$$-\frac{(1+3x^2)\sqrt{5+x^4}}{2x^4} + \frac{3}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right) + \frac{\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^5,x]
[Out] -1/2*((1 + 3*x^2)*Sqrt[5 + x^4])/x^4 + (3*ArcTanh[x^2/Sqrt[5 + x^4]])/2 + A
rcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]]/Sqrt[5]
```

Maple [A]
time = 0.27, size = 75, normalized size = 1.19

method	result
elliptic	$\frac{3\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2} - \frac{\sqrt{x^4+5}}{2x^4} - \frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10} - \frac{3\sqrt{x^4+5}}{2x^2}$
risch	$-\frac{3x^6+x^4+15x^2+5}{2x^4\sqrt{x^4+5}} + \frac{3\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2} - \frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}$

trager	$-\frac{(3x^2+1)\sqrt{x^4+5}}{2x^4}-\frac{3\ln\left(x^2-\sqrt{x^4+5}\right)}{2}+\frac{\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\mathrm{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{10}$
default	$-\frac{(x^4+5)^{\frac{3}{2}}}{10x^4}+\frac{\sqrt{x^4+5}}{10}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}-\frac{3(x^4+5)^{\frac{3}{2}}}{10x^2}+\frac{3x^2\sqrt{x^4+5}}{10}+\frac{3\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$
meijerg	$-\frac{\sqrt{5}\left(-\frac{5\sqrt{\pi}\left(\frac{4x^4}{5}+8\right)}{4x^4}+\frac{10\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}}{x^4}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-(-2\ln(2)-1+4\ln(x)-\ln(5))\sqrt{\pi}+\frac{10\sqrt{\pi}}{x^4}\right.}{20\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/10/x^4*(x^4+5)^(3/2)+1/10*(x^4+5)^(1/2)-1/10*5^(1/2)*arctanh(5^(1/2)/(x^
4+5)^(1/2))-3/10/x^2*(x^4+5)^(3/2)+3/10*x^2*(x^4+5)^(1/2)+3/2*arcsinh(1/5*x
^2*5^(1/2))
```

Maxima [A]
time = 0.50, size = 91, normalized size = 1.44

$$\frac{1}{20}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{2x^2}-\frac{\sqrt{x^4+5}}{2x^4}+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] 1/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/
2*sqrt(x^4 + 5)/x^2 - 1/2*sqrt(x^4 + 5)/x^4 + 3/4*log(sqrt(x^4 + 5)/x^2 + 1
) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.36, size = 72, normalized size = 1.14

$$\frac{\sqrt{5}\,x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-15\,x^4\log\left(-x^2+\sqrt{x^4+5}\right)-15\,x^4-5\sqrt{x^4+5}\,(3\,x^2+1)}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/10*(sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^4*log(-x^2 + s
qrt(x^4 + 5)) - 15*x^4 - 5*sqrt(x^4 + 5)*(3*x^2 + 1))/x^4
```

Sympy [**A**]
time = 2.99, size = 76, normalized size = 1.21

$$-\frac{3x^2}{2\sqrt{x^4+5}} - \frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10} + \frac{3\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} - \frac{\sqrt{1+\frac{5}{x^4}}}{2x^2} - \frac{15}{2x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**5,x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) - sqrt(5)*asinh(sqrt(5)/x**2)/10 + 3*asinh(sqrt(
5)*x**2/5)/2 - sqrt(1 + 5/x**4)/(2*x**2) - 15/(2*x**2*sqrt(x**4 + 5))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(49) = 98.
time = 4.08, size = 129, normalized size = 2.05

$$\frac{1}{10}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{\left(x^2-\sqrt{x^4+5}\right)^3 + 15\left(x^2-\sqrt{x^4+5}\right)^2 + 5\,x^2 - 5\sqrt{x^4+5} - 75}{\left(\left(x^2-\sqrt{x^4+5}\right)^2 - 5\right)^2} - \frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + ((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5
*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2 - 3/2*log(-x^2 + sqrt(
x^4 + 5))
```

Mupad [**B**]
time = 0.42, size = 56, normalized size = 0.89

$$\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2} - \frac{3\sqrt{x^4+5}}{2\,x^2} - \frac{\sqrt{x^4+5}}{2\,x^4} + \frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,1\mathrm{i}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^5,x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5
)*1i)/10 - (3*(x^4 + 5)^(1/2))/(2*x^2) - (x^4 + 5)^(1/2)/(2*x^4)
```

3.14 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx$

Optimal. Leaf size=58

$$-\frac{3\sqrt{5+x^4}}{4x^4}-\frac{(5+x^4)^{3/2}}{15x^6}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{4\sqrt{5}}$$

```
[Out] -1/15*(x^4+5)^(3/2)/x^6-3/20*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-3/4
*(x^4+5)^(1/2)/x^4
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1266, 821, 272, 43, 65, 213}

$$-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{4\sqrt{5}}-\frac{(x^4+5)^{3/2}}{15x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] (-3*Sqrt[5 + x^4])/(4*x^4) - (5 + x^4)^(3/2)/(15*x^6) - (3*ArcTanh[Sqrt[5 +
 x^4]/Sqrt[5]])/(4*Sqrt[5])
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Dist[d*(n/(b*(m + 1))), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && NeQ[b*c - a*d, 0] && ILtQ[m, -1] &&  !IntegerQ[n] && GtQ[n, 0]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
```

```
(LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^7}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{5+x^2}}{x^4}\,dx, x, x^2\right)\\
&= -\frac{(5+x^4)^{3/2}}{15x^6}+\frac{3}{2}\text{Subst}\left(\int \frac{\sqrt{5+x^2}}{x^3}\,dx, x, x^2\right)\\
&= -\frac{(5+x^4)^{3/2}}{15x^6}+\frac{3}{4}\text{Subst}\left(\int \frac{\sqrt{5+x}}{x^2}\,dx, x, x^4\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4}-\frac{(5+x^4)^{3/2}}{15x^6}+\frac{3}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4}-\frac{(5+x^4)^{3/2}}{15x^6}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{3\sqrt{5+x^4}}{4x^4}-\frac{(5+x^4)^{3/2}}{15x^6}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{4\sqrt{5}}
\end{aligned}
$$

Mathematica [A]

time = 0.15, size = 60, normalized size = 1.03

$$-\frac{\sqrt{5+x^4}\,(20+45x^2+4x^4)}{60x^6}+\frac{3\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^7,x]
```

```
[Out] -1/60*(Sqrt[5 + x^4]*(20 + 45*x^2 + 4*x^4))/x^6 + (3*ArcTanh[(x^2 - Sqrt[5
+ x^4])/Sqrt[5]])/(2*Sqrt[5])
```

Maple [A]
time = 0.20, size = 52, normalized size = 0.90

method	result
default	$-\frac{3(x^4+5)^{\frac{3}{2}}}{20x^4}+\frac{3\sqrt{x^4+5}}{20}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}-\frac{(x^4+5)^{\frac{3}{2}}}{15x^6}$
risch	$-\frac{4x^8+45x^6+40x^4+225x^2+100}{60x^6\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}$
elliptic	$-\frac{3\sqrt{x^4+5}}{4x^4}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{20}-\frac{\sqrt{x^4+5}}{15x^2}-\frac{\sqrt{x^4+5}}{3x^6}$
trager	$-\frac{(4x^4+45x^2+20)\sqrt{x^4+5}}{60x^6}+\frac{3\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-5\right)-\sqrt{x^4+5}}{x^2}\right)}{20}$
meijerg	$-\frac{\sqrt{5}\left(1+\frac{x^4}{5}\right)^{\frac{3}{2}}}{3x^6}-\frac{3\sqrt{5}\left(-\frac{5\sqrt{\pi}\left(\frac{4x^4}{5}+8\right)}{4x^4}+\frac{10\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}}{x^4}+2\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-(-2\ln(2)-1+4\ln(x)-\right.}{40\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -3/20/x^4*(x^4+5)^(3/2)+3/20*(x^4+5)^(1/2)-3/20*5^(1/2)*arctanh(5^(1/2)/(x^
4+5)^(1/2))-1/15*(x^4+5)^(3/2)/x^6
```

Maxima [A]
time = 0.50, size = 59, normalized size = 1.02

$$\frac{3}{40}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{4\,x^4}-\frac{(x^4+5)^{\frac{3}{2}}}{15\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 3/40*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 3/
4*sqrt(x^4 + 5)/x^4 - 1/15*(x^4 + 5)^(3/2)/x^6
```

Fricas [A]
time = 0.39, size = 59, normalized size = 1.02

$$\frac{9\sqrt{5}\,x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-4\,x^6-(4\,x^4+45\,x^2+20)\sqrt{x^4+5}}{60\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/60*(9*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 4*x^6 - (4*x^4 +
45*x^2 + 20)*sqrt(x^4 + 5))/x^6
```

Sympy [A]
time = 2.90, size = 63, normalized size = 1.09

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{15}-\frac{3\sqrt{5}\,\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{20}-\frac{3\sqrt{1+\frac{5}{x^4}}}{4x^2}-\frac{\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**7,x)
```

```
[Out] -sqrt(1 + 5/x**4)/15 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/20 - 3*sqrt(1 + 5/x**4
)/(4*x**2) - sqrt(1 + 5/x**4)/(3*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 116 vs. 2(43) = 86.
time = 4.24, size = 116, normalized size = 2.00

$$\frac{3}{20}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{9\left(x^2-\sqrt{x^4+5}\right)^5+12\left(x^2-\sqrt{x^4+5}\right)^4-225\,x^2+225\sqrt{x^4+5}+100}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] 3/20*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 1/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 12*(x^2 - sqrt(x^4 + 5))^4 - 225
*x^2 + 225*sqrt(x^4 + 5) + 100)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3
```

Mupad [B]
time = 0.68, size = 43, normalized size = 0.74

$$-\frac{3\,\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{20}-\frac{3\,\sqrt{x^4+5}}{4\,x^4}-\frac{{\left(x^4+5\right)}^{3/2}}{15\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^7,x)
```

```
[Out] - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/20 - (3*(x^4 + 5)^(1/2))/(
4*x^4) - (x^4 + 5)^(3/2)/(15*x^6)
```

3.15 $\int x^4(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=208

$$\frac{20}{21}x\sqrt{5+x^4}+\frac{2}{3}x^3\sqrt{5+x^4}-\frac{10x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4}+\frac{10\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}}{\sqrt{5+x^4}}$$

```
[Out] 20/21*x*(x^4+5)^(1/2)+2/3*x^3*(x^4+5)^(1/2)+1/21*x^5*(7*x^2+6)*(x^4+5)^(1/2
)-10*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+10*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))
^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4)
)),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
-5/21*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(
3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2
1+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1288, 1294, 1212, 226, 1210}

$$-\frac{5\sqrt[4]{5}\left(21+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{21\sqrt{x^4+5}}+\frac{10\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{20}{21}\sqrt{x^4+5}\,x+\frac{2}{3}\sqrt{x^4+5}\,x^3-\frac{10\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{1}{21}(7x^2+6)\sqrt{x^4+5}\,x^5$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (20*x*Sqrt[5 + x^4])/21 + (2*x^3*Sqrt[5 + x^4])/3 - (10*x*Sqrt[5 + x^4])/(S
qrt[5] + x^2) + (x^5*(6 + 7*x^2)*Sqrt[5 + x^4])/21 + (10*5^(1/4)*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2]
)/Sqrt[5 + x^4] - (5*5^(1/4)*(21 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4
)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(21*Sqrt[5 + x^4]
)
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
```

```
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\int x^4(2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4} + \frac{10}{63}\int \frac{x^4(18+21x^2)}{\sqrt{5+x^4}}\,dx$$

$$= \frac{2}{3}x^3\sqrt{5+x^4} + \frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4} - \frac{2}{63}\int \frac{x^2(315-90x^2)}{\sqrt{5+x^4}}\,dx$$

$$= \frac{20}{21}x\sqrt{5+x^4} + \frac{2}{3}x^3\sqrt{5+x^4} + \frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4} + \frac{2}{189}\int \frac{-450-9}{\sqrt{5+}}$$

$$= \frac{20}{21}x\sqrt{5+x^4} + \frac{2}{3}x^3\sqrt{5+x^4} + \frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4} + \left(10\sqrt{5}\right)\int \frac{1-}{\sqrt{5}}$$

$$= \frac{20}{21}x\sqrt{5+x^4} + \frac{2}{3}x^3\sqrt{5+x^4} - \frac{10x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{1}{21}x^5(6+7x^2)\sqrt{5+x^4} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.50, size = 82, normalized size = 0.39

$$\frac{1}{21}x\left(6(5+x^4)^{3/2} + 7x^2(5+x^4)^{3/2} - 30\sqrt{5}\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right) - 35\sqrt{5}\,x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x*(6*(5 + x^4)^(3/2) + 7*x^2*(5 + x^4)^(3/2) - 30*Sqrt[5]*Hypergeometric2F
1[-1/2, 1/4, 5/4, -1/5*x^4] - 35*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4, 7
/4, -1/5*x^4]))/21
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 192, normalized size = 0.92

method	result
meijerg	$\frac{3\sqrt{5}\,x^7 \operatorname{hypergeom}\left([-\frac{1}{2},\frac{7}{4}],[\frac{11}{4}],-\frac{x^4}{5}\right)}{7} + \frac{2\sqrt{5}\,x^5 \operatorname{hypergeom}\left([-\frac{1}{2},\frac{5}{4}],[\frac{9}{4}],-\frac{x^4}{5}\right)}{5}$
risch	$\frac{x(7x^6+6x^4+14x^2+20)\sqrt{x^4+5}}{21} - \frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\text{El}\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{x^7\sqrt{x^4+5}}{3} + \frac{2x^3\sqrt{x^4+5}}{3} - \frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\text{El}\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

elliptic	$\frac{x^7\sqrt{x^4+5}}{3}+\frac{2x^3\sqrt{x^4+5}}{3}-\frac{2i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\text{Ellip}\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^7*(x^4+5)^(1/2)+2/3*x^3*(x^4+5)^(1/2)-2*I/(I*5^(1/2))^(1/2)*(25-5*I*5
^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x
*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
+2/7*x^5*(x^4+5)^(1/2)+20/21*x*(x^4+5)^(1/2)-4/21*5^(1/2)/(I*5^(1/2))^(1/2)
*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Ellipt
icF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.11, size = 78, normalized size = 0.38

$$\frac{3\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\\frac{9}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
[Out] int(x^4*(x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

3.16 $\int x^2(2+3x^2)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=192

$$\frac{10}{7}x\sqrt{5+x^4}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{35}x^3\left(14+15x^2\right)\sqrt{5+x^4}-\frac{4\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\right.\right.}{\sqrt{5+x^4}}$$

```
[Out] 10/7*x*(x^4+5)^(1/2)+1/35*x^3*(15*x^2+14)*(x^4+5)^(1/2)+4*x*(x^4+5)^(1/2)/(
x^2+5^(1/2))-4*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(
1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^
(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/7*5^(1/4)*(cos(2*arc
tan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*a
rctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(14-5*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x
^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1288, 1294, 1212, 226, 1210}

$$\frac{\sqrt[4]{5}\left(14-5\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{10}{7}\sqrt{x^4+5}\,x+\frac{4\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}+\frac{1}{35}\left(15x^2+14\right)\sqrt{x^4+5}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (10*x*Sqrt[5 + x^4])/7 + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x^3*(14 + 1
5*x^2)*Sqrt[5 + x^4])/35 - (4*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*
(14 - 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
F[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\begin{aligned}\int x^2(2+3x^2)\sqrt{5+x^4}\,dx &= \frac{1}{35}x^3(14+15x^2)\sqrt{5+x^4}+\frac{2}{7}\int\frac{x^2(14+15x^2)}{\sqrt{5+x^4}}\,dx\\ &= \frac{10}{7}x\sqrt{5+x^4}+\frac{1}{35}x^3(14+15x^2)\sqrt{5+x^4}-\frac{2}{21}\int\frac{75-42x^2}{\sqrt{5+x^4}}\,dx\\ &= \frac{10}{7}x\sqrt{5+x^4}+\frac{1}{35}x^3(14+15x^2)\sqrt{5+x^4}-\left(4\sqrt{5}\right)\int\frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{7}\Big(\\ &= \frac{10}{7}x\sqrt{5+x^4}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{35}x^3(14+15x^2)\sqrt{5+x^4}-\frac{4\sqrt[4]{5}\left(\sqrt{5}+x\right.}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.89, size = 68, normalized size = 0.35

$$\frac{1}{21}x\left(9(5+x^4)^{3/2}-45\sqrt{5}\ {}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+14\sqrt{5}\,x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (x*(9*(5 + x^4)^(3/2) - 45*Sqrt[5]*Hypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x
^4] + 14*Sqrt[5]*x^2*Hypergeometric2F1[-1/2, 3/4, 7/4, -1/5*x^4]))/21
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 180, normalized size = 0.94

method	result
meijerg	$\frac{3\sqrt{5}\,x^5\,\mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{5}{4}\right],\left[\frac{9}{4}\right],-\frac{x^4}{5}\right)}{5}+\frac{2\sqrt{5}\,x^3\,\mathrm{hypergeom}\left(\left[-\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{x^4}{5}\right)}{3}$
risch	$\frac{x(15x^4+14x^2+50)\sqrt{x^4+5}}{35}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{10x\sqrt{x^4+5}}{7}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{10x\sqrt{x^4+5}}{7}-\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*x^5*(x^4+5)^(1/2)+10/7*x*(x^4+5)^(1/2)-2/7*5^(1/2)/(I*5^(1/2))^(1/2)*(2
5-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF
(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)+2/5*x^3*(x^4+5)^(1/2)+4/5*I/(I*5^(1/2))
^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*
(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^
(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 1.02, size = 78, normalized size = 0.41

$$\frac{3\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*
exp_polar(I*pi)/5)/(2*gamma(7/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^2\,\sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] int(x^2*(x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

3.17 $\int (2+3x^2)\sqrt{5+x^4}\,dx$

Optimal. Leaf size=176

$$\frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{1}{15}x(10+9x^2)\sqrt{5+x^4}-\frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\sqrt[4]{5}\,(9}{}$$

```
[Out] 1/15*x*(9*x^2+10)*(x^4+5)^(1/2)+6*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-6*5^(1/4)*(
cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Elliptic
E(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(
1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/3*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(
1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/
2*2^(1/2))*(x^2+5^(1/2))*(9+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4
+5)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1191, 1212, 226, 1210}

$$\frac{\sqrt[4]{5}\left(9+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{1}{15}(9x^2+10)\sqrt{x^4+5}\,x+\frac{6\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] (6*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (x*(10 + 9*x^2)*Sqrt[5 + x^4])/15 - (
6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*Arc
Tan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(9 + 2*Sqrt[5])*(Sqrt[5] + x
^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/
(3*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int (2+3x^2)\sqrt{5+x^4}\,dx = \frac{1}{15}x(10+9x^2)\sqrt{5+x^4} + \frac{1}{15}\int \frac{100+90x^2}{\sqrt{5+x^4}}\,dx$$

$$= \frac{1}{15}x(10+9x^2)\sqrt{5+x^4} - \left(6\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \frac{1}{3}\left(2\left(10+9\sqrt{5}\right)\right)\int -$$

$$= \frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{1}{15}x(10+9x^2)\sqrt{5+x^4} - \frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}}{\sqrt{5+x^4}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.46, size = 48, normalized size = 0.27

$$\sqrt{5}\,x\left(2\,{}_2F_1\left(-\frac{1}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+x^2\,{}_2F_1\left(-\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)*Sqrt[5 + x^4],x]
```

```
[Out] Sqrt[5]*x*(2*Hypergeometric2F1[-1/2, 1/4, 5/4, -1/5*x^4] + x^2*Hypergeometr
ic2F1[-1/2, 3/4, 7/4, -1/5*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 168, normalized size = 0.95

method	result
meijerg	$2\sqrt{5}\, x \,\text{hypergeom}\left(\left[-\frac{1}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], -\frac{x^4}{5}\right) + \sqrt{5}\, x^3 \,\text{hypergeom}\left(\left[-\frac{1}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{x^4}{5}\right)$
risch	$\frac{x(9x^2+10)\sqrt{x^4+5}}{15} + \frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right) - \text{EllipticE}\left(\frac{x\sqrt{5}}{}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{3x^3\sqrt{x^4+5}}{5} + \frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right) - \text{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$\frac{3x^3\sqrt{x^4+5}}{5} + \frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right) - \text{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*x^3*(x^4+5)^(1/2)+6/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))
^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+2/3*x*(x^4+5)^(1/2)
+4/15*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*
x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.94, size = 76, normalized size = 0.43

$$\frac{3\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp
_polar(I*pi)/5)/(2*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{x^4+5}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 5)^(1/2)*(3*x^2 + 2),x)
```

```
[Out] int((x^4 + 5)^(1/2)*(3*x^2 + 2), x)
```

3.18 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx$

Optimal. Leaf size=171

$$-\frac{(2-x^2)\sqrt{5+x^4}}{x}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{4\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\sqrt[4]{5}\left(2+\sqrt{\cdots}\right.}{\cdots}$$

```
[Out] -(-x^2+2)*(x^4+5)^(1/2)/x+4*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-4*5^(1/4)*(cos(2*
arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(
2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^
2)^(1/2)/(x^4+5)^(1/2)+5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))
*(2+5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1286, 1212, 226, 1210}

$$\frac{\sqrt[4]{5}\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{4\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{4\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{(2-x^2)\sqrt{x^4+5}}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```

```
[Out] -(((2 - x^2)*Sqrt[5 + x^4])/x) + (4*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (4*5
^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan
[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + (5^(1/4)*(2 + Sqrt[5])*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4]
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^2}\,dx &= -\frac{(2-x^2)\sqrt{5+x^4}}{x}-\frac{2}{3}\int \frac{-15-6x^2}{\sqrt{5+x^4}}\,dx\\ &= -\frac{(2-x^2)\sqrt{5+x^4}}{x}-\left(4\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx+\left(2\left(5+2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\\ &= -\frac{(2-x^2)\sqrt{5+x^4}}{x}+\frac{4x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{4\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\Big(2}{\sqrt{5+x^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 3.42, size = 108, normalized size = 0.63

$$\frac{-10+5x^2-2x^4+x^6-4(-1)^{3/4}\sqrt[4]{5}\,x\sqrt{5+x^4}\,E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)-2\sqrt[4]{-5}\left(-2i+\sqrt{5}\right)x\sqrt{5+x^4}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)}{x\sqrt{5+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^2,x]
```

```
[Out] (-10 + 5*x^2 - 2*x^4 + x^6 - 4*(-1)^(3/4)*5^(1/4)*x*Sqrt[5 + x^4]*EllipticE
[I*ArcSinh[(-1/5)^(1/4)*x], -1] - 2*(-5)^(1/4)*(-2*I + Sqrt[5])*x*Sqrt[5 +
x^4]*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/(x*Sqrt[5 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 167, normalized size = 0.98

method	result
meijerg	$-\frac{2\sqrt{5}\,\text{hypergeom}\left(\left[-\frac{1}{2},-\frac{1}{4}\right],\left[\frac{3}{4}\right],-\frac{x^4}{5}\right)}{x}+3\sqrt{5}\,x\,\text{hypergeom}\left(\left[-\frac{1}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],-\frac{x^4}{5}\right)$
default	$x\sqrt{x^4+5}+\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{x}+-$
elliptic	$x\sqrt{x^4+5}+\frac{2\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{x}+-$
risch	$\frac{x^6-2x^4+5x^2-10}{x\sqrt{x^4+5}}+\frac{4i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\text{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(x^4+5)^(1/2)+2/5*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^
(1/2),I)-2*(x^4+5)^(1/2)/x+4/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/
2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(
1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.11, size = 78, normalized size = 0.46

$$\frac{3\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**2,x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(
4*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_po
lar(I*pi)/5)/(2*x*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^2, x)
```

Mupad [B]
time = 0.41, size = 61, normalized size = 0.36

$$\frac{3\,x\,\sqrt{x^4+5}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{\sqrt{\frac{x^4}{5}+1}}+\frac{2\,\sqrt{x^4+5}\;{}_2\mathrm{F}_1\left(-\frac{1}{2},-\frac{1}{4};\ \frac{3}{4};\ -\frac{5}{x^4}\right)}{x\,\sqrt{\frac{5}{x^4}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^2,x)
```

```
[Out] (3*x*(x^4 + 5)^(1/2)*hypergeom([-1/2, 1/4], 5/4, -x^4/5))/(x^4/5 + 1)^(1/2)
 + (2*(x^4 + 5)^(1/2)*hypergeom([-1/2, -1/4], 3/4, -5/x^4))/(x*(5/x^4 + 1)^
(1/2))
```

3.19 $\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx$

Optimal. Leaf size=192

$$-\frac{6\sqrt{5+x^4}}{x}-\frac{(2-9x^2)\sqrt{5+x^4}}{3x^3}+\frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}$$

```
[Out] -6*(x^4+5)^(1/2)/x-1/3*(-9*x^2+2)*(x^4+5)^(1/2)/x^3+6*x*(x^4+5)^(1/2)/(x^2+
5^(1/2))-6*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*
x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2
))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/15*(cos(2*arctan(1/5*x*5
^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x
*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+9*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^
2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1286, 1296, 1212, 226, 1210}

$$\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt[4]{5}\sqrt{x^4+5}}-\frac{6\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{6\sqrt{x^4+5}}{x}+\frac{6\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}-\frac{(2-9x^2)\sqrt{x^4+5}}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

```
[Out] (-6*Sqrt[5 + x^4])/x - ((2 - 9*x^2)*Sqrt[5 + x^4])/(3*x^3) + (6*x*Sqrt[5 +
x^4])/(Sqrt[5] + x^2) - (6*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] + ((2 + 9*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(3*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
```

```
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{5+x^4}}{x^4}\,dx = -\frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} - \frac{2}{3}\int \frac{-45-2x^2}{x^2\sqrt{5+x^4}}\,dx$$
$$= -\frac{6\sqrt{5+x^4}}{x} - \frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} + \frac{2}{15}\int \frac{10+45x^2}{\sqrt{5+x^4}}\,dx$$
$$= -\frac{6\sqrt{5+x^4}}{x} - \frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} - \left(6\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx + \frac{1}{3}\Big(2\Big(2+9\sqrt{}$$
$$= -\frac{6\sqrt{5+x^4}}{x} - \frac{(2-9x^2)\sqrt{5+x^4}}{3x^3} + \frac{6x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{6\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5}{(\sqrt{}}}}{}$$

Mathematica [C] Result contains complex when optimal does not.

time = 7.83, size = 98, normalized size = 0.51

$$\frac{1}{15}\left(-\frac{5(10+45x^2+2x^4+9x^6)}{x^3\sqrt{5+x^4}}-90(-1)^{3/4}\sqrt[4]{5}\,E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)+2\sqrt[4]{-5}\left(45i-2\sqrt{5}\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[5 + x^4])/x^4,x]
```

```
[Out] ((-5*(10 + 45*x^2 + 2*x^4 + 9*x^6))/(x^3*Sqrt[5 + x^4]) - 90*(-1)^(3/4)*5^(
1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + 2*(-5)^(1/4)*(45*I - 2*Sqrt
[5])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/15
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 170, normalized size = 0.89

method	result
meijerg	$-\frac{2\sqrt{5}\ \mathrm{hypergeom}\left(\left[-\frac{3}{4},-\frac{1}{2}\right],\left[\frac{1}{4}\right],-\frac{x^4}{5}\right)}{3x^3}-\frac{3\sqrt{5}\ \mathrm{hypergeom}\left(\left[-\frac{1}{2},-\frac{1}{4}\right],\left[\frac{3}{4}\right],-\frac{x^4}{5}\right)}{x}$
default	$-\frac{3\sqrt{x^4+5}}{x}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{3\sqrt{x^4+5}}{x}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
risch	$-\frac{9x^6+2x^4+45x^2+10}{3x^3\sqrt{x^4+5}}+\frac{6i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}}{}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -3*(x^4+5)^(1/2)/x+6/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5
*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1
/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))-2/3*(x^4+5)^(1/2)/x^3+
4/75*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x
^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.18, size = 83, normalized size = 0.43

$$\frac{3\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(1/2)/x**4,x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(
4*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, -1/2), (1/4,), x**4*exp_
polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(1/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5)*(3*x^2 + 2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{x^4+5}\,\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^4,x)
```

```
[Out] int(((x^4 + 5)^(1/2)*(3*x^2 + 2))/x^4, x)
```

3.20 $\int x^5(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=83

$$-\frac{25}{16}x^2\sqrt{5+x^4}-\frac{5}{24}x^2\left(5+x^4\right)^{3/2}+\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -5/24*x^2*(x^4+5)^(3/2)+3/14*x^4*(x^4+5)^(5/2)-1/42*(-7*x^2+18)*(x^4+5)^(5/
2)-125/16*arcsinh(1/5*x^2*5^(1/2))-25/16*x^2*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1266, 847, 794, 201, 221}

$$\frac{3}{14}\left(x^4+5\right)^{5/2}x^4-\frac{125}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{5}{24}\left(x^4+5\right)^{3/2}x^2-\frac{25}{16}\sqrt{x^4+5}\,x^2-\frac{1}{42}\left(18-7x^2\right)\left(x^4+5\right)^{5/2}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-25*x^2*Sqrt[5 + x^4])/16 - (5*x^2*(5 + x^4)^(3/2))/24 + (3*x^4*(5 + x^4)^
(5/2))/14 - ((18 - 7*x^2)*(5 + x^4)^(5/2))/42 - (125*ArcSinh[x^2/Sqrt[5]])/
16
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int x^5\left(2+3x^2\right)\left(5+x^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$=\frac{3}{14}x^4\left(5+x^4\right)^{5/2}+\frac{1}{14}\text{Subst}\left(\int x(-30+14x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)$$
$$=\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{5}{6}\text{Subst}\left(\int\left(5+x^2\right)^{3/2}\,dx\right.$$
$$=-\frac{5}{24}x^2\left(5+x^4\right)^{3/2}+\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)\left(5+x^4\right)^{5/2}-\frac{25}{8}\text{Su}$$
$$=-\frac{25}{16}x^2\sqrt{5+x^4}-\frac{5}{24}x^2\left(5+x^4\right)^{3/2}+\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)(5$$
$$=-\frac{25}{16}x^2\sqrt{5+x^4}-\frac{5}{24}x^2\left(5+x^4\right)^{3/2}+\frac{3}{14}x^4\left(5+x^4\right)^{5/2}-\frac{1}{42}\left(18-7x^2\right)(5$$

Mathematica [A]
time = 0.10, size = 64, normalized size = 0.77

$$\frac{1}{336}\sqrt{5+x^4}\left(-3600+525x^2+360x^4+490x^6+576x^8+56x^{10}+72x^{12}\right)-\frac{125}{16}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(-3600 + 525*x^2 + 360*x^4 + 490*x^6 + 576*x^8 + 56*x^10 + 7
2*x^12))/336 - (125*ArcTanh[x^2/Sqrt[5 + x^4]])/16
```

Maple [A]
time = 0.14, size = 73, normalized size = 0.88

method	result
risch	$\frac{(72x^{12}+56x^{10}+576x^8+490x^6+360x^4+525x^2-3600)\sqrt{x^4+5}}{336}-\frac{125\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$
trager	$\left(\frac{3}{14}x^{12}+\frac{1}{6}x^{10}+\frac{12}{7}x^8+\frac{35}{24}x^6+\frac{15}{14}x^4+\frac{25}{16}x^2-\frac{75}{7}\right)\sqrt{x^4+5}-\frac{125\ln\left(x^2+\sqrt{x^4+5}\right)}{16}$
default	$\frac{3\sqrt{x^4+5}\,(x^4-2)(x^8+10x^4+25)}{14}+\frac{x^{10}\sqrt{x^4+5}}{6}+\frac{35x^6\sqrt{x^4+5}}{24}+\frac{25x^2\sqrt{x^4+5}}{16}-\frac{125\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$
elliptic	$\frac{3x^{12}\sqrt{x^4+5}}{14}+\frac{12x^8\sqrt{x^4+5}}{7}+\frac{15x^4\sqrt{x^4+5}}{14}-\frac{75\sqrt{x^4+5}}{7}+\frac{x^{10}\sqrt{x^4+5}}{6}+\frac{35x^6\sqrt{x^4+5}}{24}+25x$
meijerg	$\frac{1125\sqrt{5}\left(\frac{16\sqrt{\pi}}{105}-\frac{2\sqrt{\pi}\left(-\frac{4}{25}x^{12}-\frac{32}{25}x^8-\frac{4}{5}x^4+8\right)\sqrt{1+\frac{x^4}{5}}}{105}\right)}{16\sqrt{\pi}}+\frac{\frac{25\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{8}{25}x^8+\frac{14}{5}x^4+3\right)\sqrt{1+\frac{x^4}{5}}}{48}-\frac{125\sqrt{\pi}}{}}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/14*(x^4+5)^(1/2)*(x^4-2)*(x^8+10*x^4+25)+1/6*x^10*(x^4+5)^(1/2)+35/24*x^6
*(x^4+5)^(1/2)+25/16*x^2*(x^4+5)^(1/2)-125/16*arcsinh(1/5*x^2*5^(1/2))
```

Maxima [A]
time = 0.50, size = 127, normalized size = 1.53

$$\frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}}-\frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{48\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)}-\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{125}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) - 3/2*(x^4 + 5)^(5/2) - 125/48*(3*sqrt(x^4 + 5)/x^2 -
8*(x^4 + 5)^(3/2)/x^6 - 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 +
 5)^2/x^8 + (x^4 + 5)^3/x^12 - 1) - 125/32*log(sqrt(x^4 + 5)/x^2 + 1) + 125
/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.34, size = 58, normalized size = 0.70

$$\frac{1}{336}\left(72\,x^{12}+56\,x^{10}+576\,x^8+490\,x^6+360\,x^4+525\,x^2-3600\right)\sqrt{x^4+5}+\frac{125}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/336*(72*x^12 + 56*x^10 + 576*x^8 + 490*x^6 + 360*x^4 + 525*x^2 - 3600)*sq
rt(x^4 + 5) + 125/16*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A]
time = 8.97, size = 131, normalized size = 1.58

$$\frac{x^{14}}{6\sqrt{x^4+5}}+\frac{3x^{12}\sqrt{x^4+5}}{14}+\frac{55x^{10}}{24\sqrt{x^4+5}}+\frac{12x^8\sqrt{x^4+5}}{7}+\frac{425x^6}{48\sqrt{x^4+5}}+\frac{15x^4\sqrt{x^4+5}}{14}+\frac{125x^2}{16\sqrt{x^4+5}}-\frac{75\sqrt{x^4+5}}{7}-\frac{125\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(6*sqrt(x**4 + 5)) + 3*x**12*sqrt(x**4 + 5)/14 + 55*x**10/(24*sqrt(x*
*4 + 5)) + 12*x**8*sqrt(x**4 + 5)/7 + 425*x**6/(48*sqrt(x**4 + 5)) + 15*x**
4*sqrt(x**4 + 5)/14 + 125*x**2/(16*sqrt(x**4 + 5)) - 75*sqrt(x**4 + 5)/7 -
125*asinh(sqrt(5)*x**2/5)/16
```

Giac [A]
time = 3.62, size = 80, normalized size = 0.96

$$\frac{3}{14}\left(x^4+5\right)^{\frac{7}{2}}+\frac{1}{48}\left(2\left(4x^4+5\right)x^4-75\right)\sqrt{x^4+5}\,x^2+\frac{5}{8}\left(2x^4+5\right)\sqrt{x^4+5}\,x^2-\frac{3}{2}\left(x^4+5\right)^{\frac{5}{2}}+\frac{125}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 3/14*(x^4 + 5)^(7/2) + 1/48*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 5/
8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 - 3/2*(x^4 + 5)^(5/2) + 125/16*log(-x^2 + s
qrt(x^4 + 5))
```

Mupad [B]
time = 0.32, size = 52, normalized size = 0.63

$$\sqrt{x^4+5}\left(\frac{3x^{12}}{14}+\frac{x^{10}}{6}+\frac{12x^8}{7}+\frac{35x^6}{24}+\frac{15x^4}{14}+\frac{25x^2}{16}-\frac{75}{7}\right)-\frac{125\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((25*x^2)/16 + (15*x^4)/14 + (35*x^6)/24 + (12*x^8)/7 + x^1
0/6 + (3*x^12)/14 - 75/7) - (125*asinh((5^(1/2)*x^2)/5))/16
```

3.21 $\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=67

$$-\frac{75}{32}x^2\sqrt{5+x^4}-\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}\left(4+5x^2\right)\left(5+x^4\right)^{5/2}-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -5/16*x^2*(x^4+5)^(3/2)+1/20*(5*x^2+4)*(x^4+5)^(5/2)-375/32*arcsinh(1/5*x^2
*5^(1/2))-75/32*x^2*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}=0.200$, Rules used = {1266, 794, 201, 221}

$$-\frac{375}{32}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{20}\left(5x^2+4\right)\left(x^4+5\right)^{5/2}-\frac{5}{16}x^2\left(x^4+5\right)^{3/2}-\frac{75}{32}x^2\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (-75*x^2*Sqrt[5 + x^4])/32 - (5*x^2*(5 + x^4)^(3/2))/16 + ((4 + 5*x^2)*(5 +
 x^4)^(5/2))/20 - (375*ArcSinh[x^2/Sqrt[5]])/32
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^3(2+3x^2)\left(5+x^4\right)^{3/2}\,dx &= \frac{1}{2}\text{Subst}\left(\int x(2+3x)\left(5+x^2\right)^{3/2}\,dx, x, x^2\right)\\ &= \frac{1}{20}(4+5x^2)\left(5+x^4\right)^{5/2}-\frac{5}{4}\text{Subst}\left(\int \left(5+x^2\right)^{3/2}\,dx, x, x^2\right)\\ &= -\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}(4+5x^2)\left(5+x^4\right)^{5/2}-\frac{75}{16}\text{Subst}\left(\int \sqrt{5+x^2}\,dx,\right.\\ &= -\frac{75}{32}x^2\sqrt{5+x^4}-\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}(4+5x^2)\left(5+x^4\right)^{5/2}-\frac{375}{32}\text{Subs}\\ &= -\frac{75}{32}x^2\sqrt{5+x^4}-\frac{5}{16}x^2\left(5+x^4\right)^{3/2}+\frac{1}{20}(4+5x^2)\left(5+x^4\right)^{5/2}-\frac{375}{32}\text{sinh}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 59, normalized size = 0.88

$$\frac{1}{160}\sqrt{5+x^4}\left(800+375x^2+320x^4+350x^6+32x^8+40x^{10}\right)-\frac{375}{32}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
[Out] (Sqrt[5 + x^4]*(800 + 375*x^2 + 320*x^4 + 350*x^6 + 32*x^8 + 40*x^10))/160
- (375*ArcTanh[x^2/Sqrt[5 + x^4]])/32
```

Maple [A]
time = 0.13, size = 58, normalized size = 0.87

method	result
risch	$\frac{(40x^{10}+32x^8+350x^6+320x^4+375x^2+800)\sqrt{x^4+5}}{160}-\frac{375\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}$
trager	$\left(\frac{1}{4}x^{10}+\frac{1}{5}x^8+\frac{35}{16}x^6+2x^4+\frac{75}{32}x^2+5\right)\sqrt{x^4+5}-\frac{375\ln\left(x^2+\sqrt{x^4+5}\right)}{32}$
default	$\frac{x^{10}\sqrt{x^4+5}}{4}+\frac{35x^6\sqrt{x^4+5}}{16}+\frac{75x^2\sqrt{x^4+5}}{32}-\frac{375\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}+\frac{(x^4+5)^{\frac{5}{2}}}{5}$

elliptic	$\frac{x^{10}\sqrt{x^4+5}}{4}+\frac{35x^6\sqrt{x^4+5}}{16}+\frac{75x^2\sqrt{x^4+5}}{32}-\frac{375\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}+\frac{x^8\sqrt{x^4+5}}{5}+2x^4\sqrt{x^4+5}+$
meijerg	$\frac{\frac{25\sqrt{\pi}\,x^2\sqrt{5}\,\left(\frac{8}{25}x^8+\frac{14}{5}x^4+3\right)\sqrt{1+\frac{x^4}{5}}}{32}-\frac{375\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{32}}{\sqrt{\pi}}+\frac{75\sqrt{5}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\,\left(\frac{2}{25}x^8+\frac{4}{5}x^4+2\right)\sqrt{1+}}{15}\right.}{8\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^10*(x^4+5)^(1/2)+35/16*x^6*(x^4+5)^(1/2)+75/32*x^2*(x^4+5)^(1/2)-375/
32*arcsinh(1/5*x^2*5^(1/2))+1/5*(x^4+5)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 118 vs. $2(52) = 104$.
time = 0.48, size = 118, normalized size = 1.76

$$\frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}}-\frac{125\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{8\left(x^4+5\right)^{\frac{3}{2}}}{x^6}-\frac{3\left(x^4+5\right)^{\frac{5}{2}}}{x^{10}}\right)}{32\left(\frac{3\left(x^4+5\right)}{x^4}-\frac{3\left(x^4+5\right)^2}{x^8}+\frac{\left(x^4+5\right)^3}{x^{12}}-1\right)}-\frac{375}{64}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{375}{64}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*(x^4 + 5)^(5/2) - 125/32*(3*sqrt(x^4 + 5)/x^2 - 8*(x^4 + 5)^(3/2)/x^6 -
 3*(x^4 + 5)^(5/2)/x^10)/(3*(x^4 + 5)/x^4 - 3*(x^4 + 5)^2/x^8 + (x^4 + 5)^3
/x^12 - 1) - 375/64*log(sqrt(x^4 + 5)/x^2 + 1) + 375/64*log(sqrt(x^4 + 5)/x
^2 - 1)
```

Fricas [A]
time = 0.39, size = 53, normalized size = 0.79

$$\frac{1}{160}\left(40\,x^{10}+32\,x^8+350\,x^6+320\,x^4+375\,x^2+800\right)\sqrt{x^4+5}+\frac{375}{32}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/160*(40*x^10 + 32*x^8 + 350*x^6 + 320*x^4 + 375*x^2 + 800)*sqrt(x^4 + 5)
+ 375/32*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(61) = 122$.
time = 7.97, size = 124, normalized size = 1.85

$$\frac{x^{14}}{4\sqrt{x^4+5}}+\frac{55x^{10}}{16\sqrt{x^4+5}}+\frac{x^8\sqrt{x^4+5}}{5}+\frac{425x^6}{32\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{375x^2}{32\sqrt{x^4+5}}+\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{3}-\frac{10\sqrt{x^4+5}}{3}-\frac{375\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**14/(4*sqrt(x**4 + 5)) + 55*x**10/(16*sqrt(x**4 + 5)) + x**8*sqrt(x**4 +
5)/5 + 425*x**6/(32*sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/3 + 375*x**2/(32*
sqrt(x**4 + 5)) + 5*(x**4 + 5)**(3/2)/3 - 10*sqrt(x**4 + 5)/3 - 375*asinh(s
qrt(5)*x**2/5)/32
```

Giac [A]
time = 4.13, size = 71, normalized size = 1.06

$$\frac{1}{32}\left(2\left(4x^4+5\right)x^4-75\right)\sqrt{x^4+5}\,x^2+\frac{15}{16}\left(2x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{1}{5}\left(x^4+5\right)^{\frac{5}{2}}+\frac{375}{32}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/32*(2*(4*x^4 + 5)*x^4 - 75)*sqrt(x^4 + 5)*x^2 + 15/16*(2*x^4 + 5)*sqrt(x^
4 + 5)*x^2 + 1/5*(x^4 + 5)^(5/2) + 375/32*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.43, size = 47, normalized size = 0.70

$$\sqrt{x^4+5}\left(\frac{x^{10}}{4}+\frac{x^8}{5}+\frac{35x^6}{16}+2x^4+\frac{75x^2}{32}+5\right)-\frac{375\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{32}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((75*x^2)/32 + 2*x^4 + (35*x^6)/16 + x^8/5 + x^10/4 + 5) -
(375*asinh((5^(1/2)*x^2)/5))/32
```

3.22 $\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=60

$$\frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 1/4*x^2*(x^4+5)^(3/2)+3/10*(x^4+5)^(5/2)+75/8*arcsinh(1/5*x^2*5^(1/2))+15/8
*x^2*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1262, 655, 201, 221}

$$\frac{3}{10}\left(x^4+5\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{4}x^2\left(x^4+5\right)^{3/2}+\frac{15}{8}x^2\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (15*x^2*Sqrt[5 + x^4])/8 + (x^2*(5 + x^4)^(3/2))/4 + (3*(5 + x^4)^(5/2))/10
 + (75*ArcSinh[x^2/Sqrt[5]])/8
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
```

```
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int x(2+3x^2)\left(5+x^4\right)^{3/2}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int(2+3x)\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{3}{10}\left(5+x^4\right)^{5/2}+\mathrm{Subst}\left(\int\left(5+x^2\right)^{3/2}\,dx,x,x^2\right)\\
&= \frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{15}{4}\mathrm{Subst}\left(\int\sqrt{5+x^2}\,dx,x,x^2\right)\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\mathrm{Subst}\left(\int\frac{1}{\sqrt{5+x^2}}\right.\\
&= \frac{15}{8}x^2\sqrt{5+x^4}+\frac{1}{4}x^2\left(5+x^4\right)^{3/2}+\frac{3}{10}\left(5+x^4\right)^{5/2}+\frac{75}{8}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 54, normalized size = 0.90

$$\frac{1}{40}\sqrt{5+x^4}\left(300+125x^2+120x^4+10x^6+12x^8\right)+\frac{75}{8}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[5 + x^4]*(300 + 125*x^2 + 120*x^4 + 10*x^6 + 12*x^8))/40 + (75*ArcTan
h[x^2/Sqrt[5 + x^4]])/8
```

Maple [A]
time = 0.13, size = 46, normalized size = 0.77

method	result
risch	$\frac{(12x^8+10x^6+120x^4+125x^2+300)\sqrt{x^4+5}}{40}+\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$
default	$\frac{3(x^4+5)^{\frac{5}{2}}}{10}+\frac{x^6\sqrt{x^4+5}}{4}+\frac{25x^2\sqrt{x^4+5}}{8}+\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$
trager	$\left(\frac{3}{10}x^8+\frac{1}{4}x^6+3x^4+\frac{25}{8}x^2+\frac{15}{2}\right)\sqrt{x^4+5}-\frac{75\ln\left(x^2-\sqrt{x^4+5}\right)}{8}$
elliptic	$\frac{3x^8\sqrt{x^4+5}}{10}+3x^4\sqrt{x^4+5}+\frac{15\sqrt{x^4+5}}{2}+\frac{x^6\sqrt{x^4+5}}{4}+\frac{25x^2\sqrt{x^4+5}}{8}+\frac{75\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}$

meijerg	$\frac{225\sqrt{5}\left(-\frac{8\sqrt{\pi}}{15}+\frac{4\sqrt{\pi}\left(\frac{2}{25}x^8+\frac{4}{5}x^4+2\right)\sqrt{1+\frac{x^4}{5}}}{15}\right)}{16\sqrt{\pi}}+\frac{5\sqrt{\pi}\,x^2\sqrt{5}\left(\frac{x^4}{20}+\frac{5}{8}\right)\sqrt{1+\frac{x^4}{5}}+\frac{75\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{8}}{\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/10*(x^4+5)^(5/2)+1/4*x^6*(x^4+5)^(1/2)+25/8*x^2*(x^4+5)^(1/2)+75/8*arcsin
h(1/5*x^2*5^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(45) = 90.
time = 0.50, size = 95, normalized size = 1.58

$$\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{25\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{8\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{75}{16}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5)^(5/2) + 25/8*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 + 5)^(3/2)/x^6)/(
2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 75/16*log(sqrt(x^4 + 5)/x^2 + 1) -
 75/16*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.38, size = 48, normalized size = 0.80

$$\frac{1}{40}\left(12\,x^8+10\,x^6+120\,x^4+125\,x^2+300\right)\sqrt{x^4+5}-\frac{75}{8}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/40*(12*x^8 + 10*x^6 + 120*x^4 + 125*x^2 + 300)*sqrt(x^4 + 5) - 75/8*log(-
x^2 + sqrt(x^4 + 5))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 109 vs. 2(54) = 108.
time = 4.25, size = 109, normalized size = 1.82

$$\frac{x^{10}}{4\sqrt{x^4+5}}+\frac{3x^8\sqrt{x^4+5}}{10}+\frac{35x^6}{8\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{125x^2}{8\sqrt{x^4+5}}+\frac{5(x^4+5)^{\frac{3}{2}}}{2}-5\sqrt{x^4+5}+\frac{75\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] x**10/(4*sqrt(x**4 + 5)) + 3*x**8*sqrt(x**4 + 5)/10 + 35*x**6/(8*sqrt(x**4
+ 5)) + x**4*sqrt(x**4 + 5)/2 + 125*x**2/(8*sqrt(x**4 + 5)) + 5*(x**4 + 5)*
*(3/2)/2 - 5*sqrt(x**4 + 5) + 75*asinh(sqrt(5)*x**2/5)/8
```

Giac [A]
time = 4.85, size = 57, normalized size = 0.95

$$\frac{1}{8}\left(2\,x^4+5\right)\sqrt{x^4+5}\,x^2+\frac{3}{10}\left(x^4+5\right)^{\frac{5}{2}}+\frac{5}{2}\,\sqrt{x^4+5}\,x^2-\frac{75}{8}\,\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*x^4 + 5)*sqrt(x^4 + 5)*x^2 + 3/10*(x^4 + 5)^(5/2) + 5/2*sqrt(x^4 + 5
)*x^2 - 75/8*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.18, size = 42, normalized size = 0.70

$$\frac{75\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{8}+\sqrt{x^4+5}\,\left(\frac{3\,x^8}{10}+\frac{x^6}{4}+3\,x^4+\frac{25\,x^2}{8}+\frac{15}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] (75*asinh((5^(1/2)*x^2)/5))/8 + (x^4 + 5)^(1/2)*((25*x^2)/8 + 3*x^4 + x^6/4
 + (3*x^8)/10 + 15/2)
```

3.23 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx$

Optimal. Leaf size=78

$$\frac{5}{16}(16+9x^2)\sqrt{5+x^4}+\frac{1}{24}(8+9x^2)(5+x^4)^{3/2}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] 1/24*(9*x^2+8)*(x^4+5)^(3/2)+225/16*arcsinh(1/5*x^2*5^(1/2))-5*arctanh(1/5*
(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+5/16*(9*x^2+16)*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1266, 829, 858, 221, 272, 65, 213}

$$-5\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{24}(9x^2+8)(x^4+5)^{3/2}+\frac{5}{16}(9x^2+16)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (5*(16 + 9*x^2)*Sqrt[5 + x^4])/16 + ((8 + 9*x^2)*(5 + x^4)^(3/2))/24 + (225
*ArcSinh[x^2/Sqrt[5]])/16 - 5*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x}\,dx, x, x^2\right)$$

$$= \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{1}{8}\text{Subst}\left(\int \frac{(40+45x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{1}{16}\text{Subst}\left(\int \frac{400+225x}{x\sqrt{5+x^2}}\right.$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\right.$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{25}{2}$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + 25$$

$$= \frac{5}{16}(16+9x^2)\sqrt{5+x^4} + \frac{1}{24}(8+9x^2)(5+x^4)^{3/2} + \frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - 5\sqrt{}$$

Mathematica [A]
time = 0.13, size = 77, normalized size = 0.99

$$\frac{1}{48}\left(\sqrt{5+x^4}\,(320+225x^2+16x^4+18x^6) + 675\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right) + 480\sqrt{5}\,\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x,x]
```

```
[Out] (Sqrt[5 + x^4]*(320 + 225*x^2 + 16*x^4 + 18*x^6) + 675*ArcTanh[x^2/Sqrt[5 +
 x^4]] + 480*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/48
```

Maple [A]
time = 0.26, size = 75, normalized size = 0.96

method	result
trager	$\left(\frac{3}{8}x^6+\frac{1}{3}x^4+\frac{75}{16}x^2+\frac{20}{3}\right)\sqrt{x^4+5} - 5\,\text{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\text{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right) - \frac{225\ln}{}$
default	$\frac{3x^6\sqrt{x^4+5}}{8} + \frac{75x^2\sqrt{x^4+5}}{16} + \frac{225\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16} + \frac{x^4\sqrt{x^4+5}}{3} + \frac{20\sqrt{x^4+5}}{3} - 5\sqrt{5}\,\text{arctanh}\Big($

elliptic	$\frac{3x^6\sqrt{x^4+5}}{8}+\frac{75x^2\sqrt{x^4+5}}{16}+\frac{225\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{x^4\sqrt{x^4+5}}{3}+\frac{20\sqrt{x^4+5}}{3}-5\sqrt{5}\ \mathrm{arctanh}\ ($
meijerg	$\frac{15\sqrt{5}\left(-\frac{32\sqrt{\pi}}{9}+\frac{2\sqrt{\pi}\left(\frac{4x^4}{5}+16\right)\sqrt{1+\frac{x^4}{5}}}{9}-\frac{8\sqrt{\pi}\ \ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)}{3}+\frac{4\left(\frac{8}{3}-2\ln(2)+4\ln(x)-\ln(5)\right)\sqrt{\pi}}{3}\right)}{8\sqrt{\pi}}+\frac{15\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^6*(x^4+5)^(1/2)+75/16*x^2*(x^4+5)^(1/2)+225/16*arcsinh(1/5*x^2*5^(1/2
))+1/3*x^4*(x^4+5)^(1/2)+20/3*(x^4+5)^(1/2)-5*5^(1/2)*arctanh(5^(1/2)/(x^4+
5)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(62) = 124$.
time = 0.49, size = 138, normalized size = 1.77

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}+\frac{5}{2}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+5\sqrt{x^4+5}+\frac{75\left(\frac{3\sqrt{x^4+5}}{x^2}-\frac{5\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) + 5/2*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) +
 sqrt(x^4 + 5))) + 5*sqrt(x^4 + 5) + 75/16*(3*sqrt(x^4 + 5)/x^2 - 5*(x^4 +
5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(x^4
 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.35, size = 67, normalized size = 0.86

$$\frac{1}{48}\left(18\,x^6+16\,x^4+225\,x^2+320\right)\sqrt{x^4+5}+5\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 + 225*x^2 + 320)*sqrt(x^4 + 5) + 5*sqrt(5)*log(-(sqrt
(5) - sqrt(x^4 + 5))/x^2) - 225/16*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A]
time = 16.34, size = 114, normalized size = 1.46

$$\frac{3x^{10}}{8\sqrt{x^4+5}}+\frac{105x^6}{16\sqrt{x^4+5}}+\frac{375x^2}{16\sqrt{x^4+5}}+\frac{(x^4+5)^{\frac{3}{2}}}{3}+5\sqrt{x^4+5}+\frac{5\sqrt{5}\log(x^4)}{2}-5\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)+\frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x,x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) + 105*x**6/(16*sqrt(x**4 + 5)) + 375*x**2/(16*sq
rt(x**4 + 5)) + (x**4 + 5)**(3/2)/3 + 5*sqrt(x**4 + 5) + 5*sqrt(5)*log(x**4
)/2 - 5*sqrt(5)*log(sqrt(x**4/5 + 1) + 1) + 225*asinh(sqrt(5)*x**2/5)/16
```

Giac [A]
time = 4.03, size = 90, normalized size = 1.15

$$\frac{1}{48}\sqrt{x^4+5}\left((2(9x^2+8)x^2+225)x^2+320\right)+5\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 + 225)*x^2 + 320) + 5*sqrt(5)*log(-(
x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) - 225/16*lo
g(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.18, size = 55, normalized size = 0.71

$$\frac{225\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{16}-5\sqrt{5}\operatorname{atanh}\left(\frac{\sqrt{5}\sqrt{x^4+5}}{5}\right)+\sqrt{x^4+5}\left(\frac{3x^6}{8}+\frac{x^4}{3}+\frac{75x^2}{16}+\frac{20}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x,x)
```

```
[Out] (225*asinh((5^(1/2)*x^2)/5))/16 - 5*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))
/5) + (x^4 + 5)^(1/2)*((75*x^2)/16 + x^4/3 + (3*x^6)/8 + 20/3)
```

3.24 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=81

$$\frac{3}{2}(5+x^2)\sqrt{5+x^4}-\frac{(2-x^2)(5+x^4)^{3/2}}{2x^2}+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] -1/2*(-x^2+2)*(x^4+5)^(3/2)/x^2+15/2*arcsinh(1/5*x^2*5^(1/2))-15/2*arctanh(
1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+3/2*(x^2+5)*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$,
Rules used = {1266, 827, 829, 858, 221, 272, 65, 213}

$$-\frac{15}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-x^2)(x^4+5)^{3/2}}{2x^2}+\frac{3}{2}(x^2+5)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(5 + x^2)*Sqrt[5 + x^4])/2 - ((2 - x^2)*(5 + x^4)^(3/2))/(2*x^2) + (15*A
rcSinh[x^2/Sqrt[5]])/2 - (15*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 829

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Dist[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), Int[(d + e*x)^
m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*d
*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x]
, x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p,
0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILt
Q[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{(-30-12x)\sqrt{5+x^2}}{x}\,dx, x, x^2\right)$$
$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} - \frac{1}{8}\text{Subst}\left(\int \frac{-300-60x}{x\sqrt{5+x^2}}\,dx, x, x$$
$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2$$
$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{75}{4}\text{Subst}\Big($$
$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{75}{2}\text{Subst}\Big($$
$$= \frac{3}{2}(5+x^2)\sqrt{5+x^4} - \frac{(2-x^2)(5+x^4)^{3/2}}{2x^2} + \frac{15}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{15}{2}\sqrt{5}\,\tan$$

Mathematica [A]
time = 0.15, size = 76, normalized size = 0.94

$$\frac{1}{2}\left(\frac{\sqrt{5+x^4}\,(-10+20x^2+x^4+x^6)}{x^2} + 15\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right) + 30\sqrt{5}\,\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^3,x]
[Out] ((Sqrt[5 + x^4]*(-10 + 20*x^2 + x^4 + x^6))/x^2 + 15*ArcTanh[x^2/Sqrt[5 + x
^4]] + 30*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/2
```

Maple [A]
time = 0.28, size = 75, normalized size = 0.93

method	result
trager	$\frac{(x^6+x^4+20x^2-10)\sqrt{x^4+5}}{2x^2} + \frac{15\,\text{RootOf}(\_Z^2-5)\ln\left(\frac{\sqrt{x^4+5}-\text{RootOf}(\_Z^2-5)}{x^2}\right)}{2} + \frac{15\ln\left(-x^2-\sqrt{x^4+5}\right)}{2}$
default	$\frac{x^2\sqrt{x^4+5}}{2} + \frac{15\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2} - \frac{5\sqrt{x^4+5}}{x^2} + \frac{x^4\sqrt{x^4+5}}{2} + 10\sqrt{x^4+5} - \frac{15\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4}}\right.}{2}$

elliptic	$\frac{x^2\sqrt{x^4+5}}{2}+\frac{15\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{5\sqrt{x^4+5}}{x^2}+\frac{x^4\sqrt{x^4+5}}{2}+10\sqrt{x^4+5}-\frac{15\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+}}\right.}{2}$
risch	$-\frac{5\sqrt{x^4+5}}{x^2}+\frac{\sqrt{x^4+5}\,(x^4-10)}{2}+\frac{x^2\sqrt{x^4+5}}{2}+\frac{15\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}+15\sqrt{x^4+5}-\frac{15\sqrt{5}\,\text{arctanh}\left(\right.}{2}$
meijerg	$\frac{-\frac{5\sqrt{\pi}\,\sqrt{5}\,\left(-\frac{x^4}{10}+1\right)\sqrt{1+\frac{x^4}{5}}}{x^2}+\frac{15\sqrt{\pi}\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}}{\sqrt{\pi}}+\frac{45\sqrt{5}\left(-\frac{32\sqrt{\pi}}{9}+\frac{2\sqrt{\pi}\,\left(\frac{4x^4}{5}+16\right)\sqrt{1+\frac{x^4}{5}}}{9}-\frac{8\sqrt{}}{}\right.}{16\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2*(x^4+5)^(1/2)+15/2*arcsinh(1/5*x^2*5^(1/2))-5*(x^4+5)^(1/2)/x^2+1/2
*x^4*(x^4+5)^(1/2)+10*(x^4+5)^(1/2)-15/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1
/2))
```

Maxima [A]
time = 0.49, size = 122, normalized size = 1.51

$$\frac{1}{2}\,(x^4+5)^{\frac{3}{2}}+\frac{15}{4}\,\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{15}{2}\,\sqrt{x^4+5}-\frac{5\,\sqrt{x^4+5}}{x^2}+\frac{5\,\sqrt{x^4+5}}{2\,x^2\left(\frac{x^4+5}{x^4}-1\right)}+\frac{15}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{15}{4}\,\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) + 15/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5)
+ sqrt(x^4 + 5))) + 15/2*sqrt(x^4 + 5) - 5*sqrt(x^4 + 5)/x^2 + 5/2*sqrt(x^4
 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) + 15/4*log(sqrt(x^4 + 5)/x^2 + 1) - 15/4*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.37, size = 78, normalized size = 0.96

$$\frac{15\,\sqrt{5}\,x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-15\,x^2\log\left(-x^2+\sqrt{x^4+5}\right)-10\,x^2+(x^6+x^4+20\,x^2-10)\sqrt{x^4+5}}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/2*(15*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 15*x^2*log(-x^2 +
 sqrt(x^4 + 5)) - 10*x^2 + (x^6 + x^4 + 20*x^2 - 10)*sqrt(x^4 + 5))/x^2
```

Sympy [A]
time = 5.44, size = 114, normalized size = 1.41

$$\frac{x^6}{2\sqrt{x^4+5}} - \frac{5x^2}{2\sqrt{x^4+5}} + \frac{(x^4+5)^{\frac{3}{2}}}{2} + \frac{15\sqrt{x^4+5}}{2} + \frac{15\sqrt{5}\log(x^4)}{4} - \frac{15\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2} + \frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} - \frac{25}{x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**3,x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) - 5*x**2/(2*sqrt(x**4 + 5)) + (x**4 + 5)**(3/2)/2 +
 15*sqrt(x**4 + 5)/2 + 15*sqrt(5)*log(x**4)/4 - 15*sqrt(5)*log(sqrt(x**4/5
+ 1) + 1)/2 + 15*asinh(sqrt(5)*x**2/5)/2 - 25/(x**2*sqrt(x**4 + 5))
```

Giac [A]
time = 4.69, size = 102, normalized size = 1.26

$$\frac{1}{2}\sqrt{x^4+5}\left((x^2+1)x^2+20\right) + \frac{15}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right) + \frac{50}{\left(x^2-\sqrt{x^4+5}\right)^2-5} - \frac{15}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 + 20) + 15/2*sqrt(5)*log(-(x^2 + sqrt(5) -
 sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 50/((x^2 - sqrt(x^4 + 5)
)^2 - 5) - 15/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.76, size = 64, normalized size = 0.79

$$\frac{15\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2} + \sqrt{x^4+5}\left(\frac{x^4}{2}+\frac{x^2}{2}+10\right) - \frac{5\sqrt{x^4+5}}{x^2} + \frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\sqrt{x^4+5}\,1i}{5}\right)15i}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^3,x)
```

```
[Out] (15*asinh((5^(1/2)*x^2)/5))/2 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/
5)*15i)/2 + (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 + 10) - (5*(x^4 + 5)^(1/2))/x^2
```

3.25 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=86

$$-\frac{3(15-2x^2)\sqrt{5+x^4}}{4x^2}-\frac{(2-3x^2)(5+x^4)^{3/2}}{4x^4}+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] -1/4*(-3*x^2+2)*(x^4+5)^(3/2)/x^4+45/4*arcsinh(1/5*x^2*5^(1/2))-3/2*arctanh
(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-3/4*(-2*x^2+15)*(x^4+5)^(1/2)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1266, 827, 858, 221, 272, 65, 213}

$$-\frac{3}{2}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(2-3x^2)(x^4+5)^{3/2}}{4x^4}-\frac{3(15-2x^2)\sqrt{x^4+5}}{4x^2}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(15 - 2*x^2)*Sqrt[5 + x^4])/(4*x^2) - ((2 - 3*x^2)*(5 + x^4)^(3/2))/(4*
x^4) + (45*ArcSinh[x^2/Sqrt[5]])/4 - (3*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[
5]])/2
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\,(5+x^4)^{3/2}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\,(5+x^2)^{3/2}}{x^3}\,dx, x, x^2\right)$$
$$= -\frac{(2-3x^2)\,(5+x^4)^{3/2}}{4x^4} - \frac{3}{16}\text{Subst}\left(\int \frac{(-60-8x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{3(15-2x^2)\,\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\,(5+x^4)^{3/2}}{4x^4} + \frac{3}{32}\text{Subst}\left(\int \frac{80+120x}{x\sqrt{5+x^2}}\,dx,\right.$$
$$= -\frac{3(15-2x^2)\,\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\,(5+x^4)^{3/2}}{4x^4} + \frac{15}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,\right.$$
$$= -\frac{3(15-2x^2)\,\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\,(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{4}\text{Sub}$$
$$= -\frac{3(15-2x^2)\,\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\,(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{15}{2}\text{Sub}$$
$$= -\frac{3(15-2x^2)\,\sqrt{5+x^4}}{4x^2} - \frac{(2-3x^2)\,(5+x^4)^{3/2}}{4x^4} + \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{3}{2}\sqrt{5}$$

Mathematica [A]
time = 0.19, size = 81, normalized size = 0.94

$$\frac{\sqrt{5+x^4}\,(-10-30x^2+4x^4+3x^6)}{4x^4} + \frac{45}{4}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right) + 3\sqrt{5}\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^5,x]
```

```
[Out] (Sqrt[5 + x^4]*(-10 - 30*x^2 + 4*x^4 + 3*x^6))/(4*x^4) + (45*ArcTanh[x^2/Sq
rt[5 + x^4]])/4 + 3*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]]
```

Maple [A]
time = 0.27, size = 73, normalized size = 0.85

method	result
default	$\sqrt{x^4+5} - \frac{5\sqrt{x^4+5}}{2x^4} - \frac{3\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2} + \frac{3x^2\sqrt{x^4+5}}{4} + \frac{45\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4} - \frac{15\sqrt{x^4+}}{2x^2}$
elliptic	$\sqrt{x^4+5} - \frac{5\sqrt{x^4+5}}{2x^4} - \frac{3\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2} + \frac{3x^2\sqrt{x^4+5}}{4} + \frac{45\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4} - \frac{15\sqrt{x^4+}}{2x^2}$

risch	$-\frac{5(3x^6+x^4+15x^2+5)}{2x^4\sqrt{x^4+5}}+\frac{3x^2\sqrt{x^4+5}}{4}+\frac{45\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}-\frac{3\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{2}$
trager	$\frac{(3x^6+4x^4-30x^2-10)\sqrt{x^4+5}}{4x^4}+\frac{3\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-5\right)-\sqrt{x^4+5}}{x^2}\right)}{2}+\frac{45\ln\left(-x^2-\sqrt{x^4+5}\right.}{4}$
meijerg	$\frac{3\sqrt{5}\left(\frac{5\sqrt{\pi}\left(-\frac{12x^4}{5}+8\right)}{6x^4}-\frac{5\sqrt{\pi}\left(8-\frac{16x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{6x^4}-4\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+2(1-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}\right.}{8\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] (x^4+5)^(1/2)-5/2*(x^4+5)^(1/2)/x^4-3/2*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/
2))+3/4*x^2*(x^4+5)^(1/2)+45/4*arcsinh(1/5*x^2*5^(1/2))-15/2*(x^4+5)^(1/2)/
x^2
```

Maxima [A]
time = 0.49, size = 123, normalized size = 1.43

$$\frac{3}{4}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\sqrt{x^4+5}-\frac{15\sqrt{x^4+5}}{2x^2}+\frac{15\sqrt{x^4+5}}{4x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{5\sqrt{x^4+5}}{2x^4}+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + sqr
t(x^4 + 5) - 15/2*sqrt(x^4 + 5)/x^2 + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^
4 - 1)) - 5/2*sqrt(x^4 + 5)/x^4 + 45/8*log(sqrt(x^4 + 5)/x^2 + 1) - 45/8*lo
g(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.40, size = 82, normalized size = 0.95

$$\frac{6\sqrt{5}x^4\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-45x^4\log\left(-x^2+\sqrt{x^4+5}\right)-30x^4+(3x^6+4x^4-30x^2-10)\sqrt{x^4+5}}{4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/4*(6*sqrt(5)*x^4*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 45*x^4*log(-x^2 +
sqrt(x^4 + 5)) - 30*x^4 + (3*x^6 + 4*x^4 - 30*x^2 - 10)*sqrt(x^4 + 5))/x^4
```

Sympy [A]
time = 6.47, size = 133, normalized size = 1.55

$$\frac{3x^6}{4\sqrt{x^4+5}}-\frac{15x^2}{4\sqrt{x^4+5}}+\sqrt{x^4+5}+\frac{\sqrt{5}\log(x^4)}{2}-\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)-\frac{\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{2}+\frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4}-\frac{5\sqrt{1+\frac{5}{x^4}}}{2x^2}-\frac{75}{2x^2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**5,x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) - 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) + s
qrt(5)*log(x**4)/2 - sqrt(5)*log(sqrt(x**4/5 + 1) + 1) - sqrt(5)*asinh(sqrt
(5)/x**2)/2 + 45*asinh(sqrt(5)*x**2/5)/4 - 5*sqrt(1 + 5/x**4)/(2*x**2) - 75
/(2*x**2*sqrt(x**4 + 5))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(68) = 136.
time = 2.73, size = 146, normalized size = 1.70

$$\frac{1}{4}\sqrt{x^4+5}\,(3x^2+4)+\frac{3}{2}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{5\left(\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5x^2-5\sqrt{x^4+5}-75\right)}{\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}-\frac{45}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 3/2*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4
+ 5))/(x^2 - sqrt(5) - sqrt(x^4 + 5))) + 5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x
^2 - sqrt(x^4 + 5))^2 + 5*x^2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5)
)^2 - 5)^2 - 45/4*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.55, size = 71, normalized size = 0.83

$$\frac{45\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}+\sqrt{x^4+5}\,\left(\frac{3\,x^2}{4}+1\right)-\frac{15\,\sqrt{x^4+5}}{2\,x^2}-\frac{5\,\sqrt{x^4+5}}{2\,x^4}+\frac{\sqrt{5}\,\operatorname{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)\,3\mathrm{i}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^5,x)
```

```
[Out] (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/
5)*3i)/2 + (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*(x^4 + 5)^(1/2))/(2*x^2) -
 (5*(x^4 + 5)^(1/2))/(2*x^4)
```

3.26 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=82

$$-\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2}-\frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)$$

```
[Out] -1/12*(9*x^2+4)*(x^4+5)^(3/2)/x^6+arcsinh(1/5*x^2*5^(1/2))-9/4*arctanh(1/5*
(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/4*(-9*x^2+4)*(x^4+5)^(1/2)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {1266, 825, 827, 858, 221, 272, 65, 213}

$$-\frac{9}{4}\sqrt{5}\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(4-9x^2)\sqrt{x^4+5}}{4x^2}-\frac{(9x^2+4)(x^4+5)^{3/2}}{12x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] -1/4*((4 - 9*x^2)*Sqrt[5 + x^4])/x^2 - ((4 + 9*x^2)*(5 + x^4)^(3/2))/(12*x^
6) + ArcSinh[x^2/Sqrt[5]] - (9*Sqrt[5]*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/4
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 825

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + c*x^2)^p/(e^2*(m + 1)*(m
+ 2)*(c*d^2 + a*e^2)))*((d*g - e*f*(m + 2))*(c*d^2 + a*e^2) - 2*c*d^2*p*(e*
f - d*g) - e*(g*(m + 1)*(c*d^2 + a*e^2) + 2*c*d*p*(e*f - d*g))*x), x] - Dis
t[p/(e^2*(m + 1)*(m + 2)*(c*d^2 + a*e^2)), Int[(d + e*x)^(m + 2)*(a + c*x^2
)^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) - c*(2*c*d*(d*g*(2*p + 1) - e*f*
(m + 2*p + 2)) - 2*a*e^2*g*(m + 1))*x, x], x], x] /; FreeQ[{a, c, d, e, f,
g}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[p, 0] && LtQ[m, -2] && LtQ[m + 2*p,
0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 827

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) - d*g*(2*p + 1)
 + e*g*(m + 1)*x)*((a + c*x^2)^p/(e^2*(m + 1)*(m + 2*p + 2))), x] + Dist[p/
(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a + c*x^2)^(p - 1)*Simp
[g*(2*a*e + 2*a*e*m) + (g*(2*c*d + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x, x],
 x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && NeQ[c*d^2 + a*e^2, 0] && Rati
onalQ[p] && p > 0 && (LtQ[m, -1] || EqQ[p, 1] || (IntegerQ[p] &&  !Rational
Q[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && (IntegerQ[m] || IntegerQ[
p] || IntegersQ[2*m, 2*p])
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(5+x^2)^{3/2}}{x^4}\,dx, x, x^2\right)$$
$$= -\frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} - \frac{1}{40}\text{Subst}\left(\int \frac{(-40-90x)\sqrt{5+x^2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \frac{1}{80}\text{Subst}\left(\int \frac{900+80x}{x\sqrt{5+x^2}}\,dx, \right.$$
$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \frac{45}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, \right.$$
$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{8}\text{Subst}\left(\right.$$
$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) + \frac{45}{4}\text{Subst}\left(\right.$$
$$= -\frac{(4-9x^2)\sqrt{5+x^4}}{4x^2} - \frac{(4+9x^2)(5+x^4)^{3/2}}{12x^6} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{9}{4}\sqrt{5}\,\text{tanh}$$

Mathematica [A]
time = 0.20, size = 79, normalized size = 0.96

$$\frac{\sqrt{5+x^4}\,(-20-45x^2-16x^4+18x^6)}{12x^6} + \tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right) + \frac{9}{2}\sqrt{5}\,\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^7,x]
```

```
[Out] (Sqrt[5 + x^4]*(-20 - 45*x^2 - 16*x^4 + 18*x^6))/(12*x^6) + ArcTanh[x^2/Sqr
t[5 + x^4]] + (9*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/2
```

Maple [A]
time = 0.27, size = 73, normalized size = 0.89

method	result
risch	$-\frac{16x^8+45x^6+100x^4+225x^2+100}{12x^6\sqrt{x^4+5}} + \frac{3\sqrt{x^4+5}}{2} + \text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right) - \frac{9\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}$
default	$\frac{3\sqrt{x^4+5}}{2} - \frac{15\sqrt{x^4+5}}{4x^4} - \frac{9\sqrt{5}\,\text{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4} + \text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right) - \frac{4\sqrt{x^4+5}}{3x^2} - \frac{5\sqrt{x^4}}{3x^6}$

elliptic	$\frac{3\sqrt{x^4+5}}{2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{9\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{4}+\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)-\frac{4\sqrt{x^4+5}}{3x^2}-\frac{5\sqrt{x^4+}}{3x^6}$
trager	$\frac{(18x^6-16x^4-45x^2-20)\sqrt{x^4+5}}{12x^6}+\frac{9\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\sqrt{x^4+5}-\operatorname{RootOf}\left(\_Z^2-5\right)}{x^2}\right)}{4}+\ln\left(-x^2-\sqrt{x^4+5}\right.$
meijerg	$\frac{-\frac{5\sqrt{\pi}\,\sqrt{5}\,\left(\frac{4x^4}{5}+1\right)\sqrt{1+\frac{x^4}{5}}}{3x^6}+\sqrt{\pi}\,\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{\sqrt{\pi}}+\frac{9\sqrt{5}\,\left(\frac{5\sqrt{\pi}\,\left(-\frac{12x^4}{5}+8\right)}{6x^4}-\frac{5\sqrt{\pi}\,\left(8-\frac{16x^4}{5}\right)\sqrt{1+\frac{x^4}{5}}}{6x^4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(x^4+5)^(1/2)-15/4*(x^4+5)^(1/2)/x^4-9/4*5^(1/2)*arctanh(5^(1/2)/(x^4+5
)^(1/2))+arcsinh(1/5*x^2*5^(1/2))-4/3*(x^4+5)^(1/2)/x^2-5/3/x^6*(x^4+5)^(1/
2)
```

Maxima [A]
time = 0.50, size = 112, normalized size = 1.37

$$\frac{9}{8}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}-\frac{\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4x^4}-\frac{(x^4+5)^{\frac{3}{2}}}{3x^6}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 9/8*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/2
*sqrt(x^4 + 5) - sqrt(x^4 + 5)/x^2 - 15/4*sqrt(x^4 + 5)/x^4 - 1/3*(x^4 + 5)
^(3/2)/x^6 + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1
)
```

Fricas [A]
time = 0.36, size = 82, normalized size = 1.00

$$\frac{27\sqrt{5}\,x^6\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-12\,x^6\log\left(-x^2+\sqrt{x^4+5}\right)-16\,x^6+(18\,x^6-16\,x^4-45\,x^2-20)\sqrt{x^4+5}}{12\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/12*(27*sqrt(5)*x^6*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 12*x^6*log(-x^2
+ sqrt(x^4 + 5)) - 16*x^6 + (18*x^6 - 16*x^4 - 45*x^2 - 20)*sqrt(x^4 + 5))/
x^6
```

Sympy [A]
time = 6.58, size = 148, normalized size = 1.80

$$-\frac{x^2}{\sqrt{x^4+5}}-\frac{\sqrt{1+\frac{5}{x^4}}}{3}+\frac{3\sqrt{x^4+5}}{2}+\frac{3\sqrt{5}\log(x^4)}{4}-\frac{3\sqrt{5}\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{2}-\frac{3\sqrt{5}\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{4}+\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)-\frac{15\sqrt{1+\frac{5}{x^4}}}{4x^2}-\frac{5}{x^2\sqrt{x^4+5}}-\frac{5\sqrt{1+\frac{5}{x^4}}}{3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**7,x)
```

```
[Out] -x**2/sqrt(x**4 + 5) - sqrt(1 + 5/x**4)/3 + 3*sqrt(x**4 + 5)/2 + 3*sqrt(5)*
log(x**4)/4 - 3*sqrt(5)*log(sqrt(x**4/5 + 1) + 1)/2 - 3*sqrt(5)*asinh(sqrt(
5)/x**2)/4 + asinh(sqrt(5)*x**2/5) - 15*sqrt(1 + 5/x**4)/(4*x**2) - 5/(x**2
*sqrt(x**4 + 5)) - 5*sqrt(1 + 5/x**4)/(3*x**4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 158 vs. 2(66) = 132.
time = 4.81, size = 158, normalized size = 1.93

$$\frac{9}{4}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{3}{2}\sqrt{x^4+5}+\frac{5\left(9\left(x^2-\sqrt{x^4+5}\right)^5+24\left(x^2-\sqrt{x^4+5}\right)^4-120\left(x^2-\sqrt{x^4+5}\right)^2-225x^2+225\sqrt{x^4+5}+400\right)}{6\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^3}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 9/4*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) + 3/2*sqrt(x^4 + 5) + 5/6*(9*(x^2 - sqrt(x^4 + 5))^5 + 24*(x^2 - sqr
t(x^4 + 5))^4 - 120*(x^2 - sqrt(x^4 + 5))^2 - 225*x^2 + 225*sqrt(x^4 + 5) +
 400)/((x^2 - sqrt(x^4 + 5))^2 - 5)^3 - log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.95, size = 82, normalized size = 1.00

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}+\sqrt{x^4+5}\left(\frac{2}{3\,x^2}-\frac{5}{3\,x^6}\right)-\frac{2\sqrt{x^4+5}}{x^2}-\frac{15\sqrt{x^4+5}}{4\,x^4}+\frac{\sqrt{5}\operatorname{atan}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}\,1\mathrm{i}}{5}\right)9\mathrm{i}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^7,x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (5^(1/2)*atan((5^(1/2)*(x^4 + 5)^(1/2)*1i)/5)*9i)/
4 + (3*(x^4 + 5)^(1/2))/2 + (x^4 + 5)^(1/2)*(2/(3*x^2) - 5/(3*x^6)) - (2*(x
^4 + 5)^(1/2))/x^2 - (15*(x^4 + 5)^(1/2))/(4*x^4)
```

3.27 $\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=235

$$\frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}-\frac{300x\sqrt{5+x^4}}{13\left(\sqrt{5}+x^2\right)}+\frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5\left(26+33x^2\right)\left(5+x^4\right)^{3/2}$$

```
[Out] 1/143*x^5*(33*x^2+26)*(x^4+5)^(3/2)+200/77*x*(x^4+5)^(1/2)+20/13*x^3*(x^4+5
)^(1/2)+10/1001*x^5*(77*x^2+78)*(x^4+5)^(1/2)-300/13*x*(x^4+5)^(1/2)/(x^2+5
^(1/2))+300/13*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(
1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^
(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-50/1001*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(231+26*5^(1/2))*((x^4
+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1288, 1294, 1212, 226, 1210}

$$-\frac{50\sqrt[4]{5}\left(231+26\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{1001\sqrt{x^4+5}}+\frac{300\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{13\sqrt{x^4+5}}+\frac{200}{77}\sqrt{x^4+5}\,x+\frac{20}{13}\sqrt{x^4+5}\,x^3-\frac{300\sqrt{x^4+5}\,x}{13\left(x^2+\sqrt{5}\right)}+\frac{1}{143}\left(33x^2+26\right)\left(x^4+5\right)^{3/2}x^5+\frac{10(77x^2+78)\sqrt{x^4+5}\,x^5}{1001}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (200*x*Sqrt[5 + x^4])/77 + (20*x^3*Sqrt[5 + x^4])/13 - (300*x*Sqrt[5 + x^4]
)/(13*(Sqrt[5] + x^2)) + (10*x^5*(78 + 77*x^2)*Sqrt[5 + x^4])/1001 + (x^5*(
26 + 33*x^2)*(5 + x^4)^(3/2))/143 + (300*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(13*Sqrt[5 + x
^4]) - (50*5^(1/4)*(231 + 26*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[
5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(1001*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\int x^4(2+3x^2)\left(5+x^4\right)^{3/2}dx = \frac{1}{143}x^5(26+33x^2)\left(5+x^4\right)^{3/2}+\frac{30}{143}\int x^4(26+33x^2)\sqrt{5+x^4}\,dx$$

$$= \frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5(26+33x^2)\left(5+x^4\right)^{3/2}+\frac{100\int\frac{x^4(234+}{\sqrt{5+}}}{3003}$$

$$= \frac{20}{13}x^3\sqrt{5+x^4}+\frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5(26+33x^2)\left(5+x^4\right)^{3/}$$

$$= \frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}+\frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5(26-$$

$$= \frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}+\frac{10x^5(78+77x^2)\sqrt{5+x^4}}{1001}+\frac{1}{143}x^5(26-$$

$$= \frac{200}{77}x\sqrt{5+x^4}+\frac{20}{13}x^3\sqrt{5+x^4}-\frac{300x\sqrt{5+x^4}}{13\left(\sqrt{5}+x^2\right)}+\frac{10x^5(78+77x^2)\sqrt{5+}}{1001}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.94, size = 74, normalized size = 0.31

$$\frac{1}{143}x\left(\left(26+33x^2\right)\left(5+x^4\right)^{5/2}-650\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)-825\sqrt{5}\,x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x*((26 + 33*x^2)*(5 + x^4)^(5/2) - 650*Sqrt[5]*Hypergeometric2F1[-3/2, 1/4
, 5/4, -1/5*x^4] - 825*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -1/5*x
^4]))/143
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 216, normalized size = 0.92

method	result
meijerg	$\frac{15\sqrt{5}\,x^7\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{7}{4}\right],\left[\frac{11}{4}\right],-\frac{x^4}{5}\right)}{7}+2\sqrt{5}\,x^5\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{5}{4}\right],\left[\frac{9}{4}\right],-\frac{x^4}{5}\right)$

risch	$\frac{x(231x^{10}+182x^8+1925x^6+1690x^4+1540x^2+2600)\sqrt{x^4+5}}{1001} - \frac{60i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\text{Elliptic}\right.}{13\sqrt{i\sqrt{5}}}$
default	$\frac{3x^{11}\sqrt{x^4+5}}{13} + \frac{25x^7\sqrt{x^4+5}}{13} + \frac{20x^3\sqrt{x^4+5}}{13} - \frac{60i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\text{EllipticF}\right.}{13\sqrt{i\sqrt{5}}}$
elliptic	$\frac{3x^{11}\sqrt{x^4+5}}{13} + \frac{25x^7\sqrt{x^4+5}}{13} + \frac{20x^3\sqrt{x^4+5}}{13} - \frac{60i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\text{EllipticF}\right.}{13\sqrt{i\sqrt{5}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/13*x^11*(x^4+5)^(1/2)+25/13*x^7*(x^4+5)^(1/2)+20/13*x^3*(x^4+5)^(1/2)-60/
13*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2
)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5
*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+2/11*x^9*(x^4+5)^(1/2)+130/77*x^5*(x^4+5)^
(1/2)+200/77*x*(x^4+5)^(1/2)-40/77*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2
)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/
2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.93, size = 160, normalized size = 0.68

$$\frac{3\sqrt{5}\,x^{11}\Gamma\left(\frac{11}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{11}{4}\\ \frac{15}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{15}{4}\right)}+\frac{\sqrt{5}\,x^{9}\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{13}{4}\right)}+\frac{15\sqrt{5}\,x^{7}\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{5\sqrt{5}\,x^{5}\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**11*gamma(11/4)*hyper((-1/2, 11/4), (15/4,), x**4*exp_polar(I*p
i)/5)/(4*gamma(15/4)) + sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,),
 x**4*exp_polar(I*pi)/5)/(2*gamma(13/4)) + 15*sqrt(5)*x**7*gamma(7/4)*hyper
((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(11/4)) + 5*sqrt(5)*
x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(2*gamma
(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,{\left(x^4+5\right)}^{3/2}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] int(x^4*(x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.28 $\int x^2(2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=219

$$\frac{300}{77}x\sqrt{5+x^4}+\frac{40x\sqrt{5+x^4}}{3\left(\sqrt{5}+x^2\right)}+\frac{2}{231}x^3\left(154+135x^2\right)\sqrt{5+x^4}+\frac{1}{99}x^3\left(22+27x^2\right)\left(5+x^4\right)^{3/2}-\frac{40\sqrt[4]{5}\left(\sqrt{5}\right.}{}$$

```
[Out] 1/99*x^3*(27*x^2+22)*(x^4+5)^(3/2)+300/77*x*(x^4+5)^(1/2)+2/231*x^3*(135*x^
2+154)*(x^4+5)^(1/2)+40/3*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-40/3*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))
^2)^(1/2)/(x^4+5)^(1/2)+10/231*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/
2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*
2^(1/2))*(154-45*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^
4+5)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {1288, 1294, 1212, 226, 1210}

$$\frac{10\sqrt[4]{5}\left(154-45\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{231\sqrt{x^4+5}}-\frac{40\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}+\frac{300}{77}\sqrt{x^4+5}\,x+\frac{40\sqrt{x^4+5}\,x}{3\left(x^2+\sqrt{5}\right)}+\frac{1}{99}\left(27x^2+22\right)\left(x^4+5\right)^{3/2}x^3+\frac{2}{231}\left(135x^2+154\right)\sqrt{x^4+5}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (300*x*Sqrt[5 + x^4])/77 + (40*x*Sqrt[5 + x^4])/(3*(Sqrt[5] + x^2)) + (2*x^
3*(154 + 135*x^2)*Sqrt[5 + x^4])/231 + (x^3*(22 + 27*x^2)*(5 + x^4)^(3/2))/
99 - (40*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Elliptic
E[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4]) + (10*5^(1/4)*(154 - 45*Sqrt
[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x
/5^(1/4)], 1/2])/(231*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1288

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((c*d*(m + 4*p + 3) + c*e*(4*p
 + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p + 3))), x] + Dist[4*a*(p/((4*p +
 m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a + c*x^4)^(p - 1)*Simp[d*(m + 4*p +
3) + e*(4*p + m + 1)*x^2, x], x], x] /; FreeQ[{a, c, d, e, f, m}, x] && GtQ
[p, 0] && NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (I
ntegerQ[p] || IntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\int x^2(2+3x^2)(5+x^4)^{3/2}\,dx = \frac{1}{99}x^3(22+27x^2)(5+x^4)^{3/2} + \frac{10}{33}\int x^2(22+27x^2)\sqrt{5+x^4}\,dx$$
$$= \frac{2}{231}x^3(154+135x^2)\sqrt{5+x^4} + \frac{1}{99}x^3(22+27x^2)(5+x^4)^{3/2} + \frac{20}{231}\int \frac{x^{2}}{}$$
$$= \frac{300}{77}x\sqrt{5+x^4} + \frac{2}{231}x^3(154+135x^2)\sqrt{5+x^4} + \frac{1}{99}x^3(22+27x^2)(5+$$
$$= \frac{300}{77}x\sqrt{5+x^4} + \frac{2}{231}x^3(154+135x^2)\sqrt{5+x^4} + \frac{1}{99}x^3(22+27x^2)(5+$$
$$= \frac{300}{77}x\sqrt{5+x^4} + \frac{40x\sqrt{5+x^4}}{3\left(\sqrt{5}+x^2\right)} + \frac{2}{231}x^3(154+135x^2)\sqrt{5+x^4} + \frac{1}{99}x^3($$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.65, size = 68, normalized size = 0.31

$$\frac{1}{33}x\left(9(5+x^4)^{5/2} - 225\sqrt{5}\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right) + 110\sqrt{5}\,x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (x*(9*(5 + x^4)^(5/2) - 225*Sqrt[5]*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*
x^4] + 110*Sqrt[5]*x^2*Hypergeometric2F1[-3/2, 3/4, 7/4, -1/5*x^4]))/33
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 204, normalized size = 0.93

method	result
meijerg	$3\sqrt{5}\,x^5\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{5}{4}\right],\left[\frac{9}{4}\right],-\frac{x^4}{5}\right) + \frac{10\sqrt{5}\,x^3\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{x^4}{5}\right)}{3}$
risch	$\frac{x(189x^8+154x^6+1755x^4+1694x^2+2700)\sqrt{x^4+5}}{693} + \frac{8i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}}{}\right.\right.}{3\sqrt{i\sqrt{5}}\,\sqrt{x^4+}}$
default	$\frac{3x^9\sqrt{x^4+5}}{11} + \frac{195x^5\sqrt{x^4+5}}{77} + \frac{300x\sqrt{x^4+5}}{77} - \frac{60\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\mathrm{Elliptic}}{77\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

elliptic	$\frac{3x^9\sqrt{x^4+5}}{11}+\frac{195x^5\sqrt{x^4+5}}{77}+\frac{300x\sqrt{x^4+5}}{77}-\frac{60\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\text{EllipticF}}{77\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/11*x^9*(x^4+5)^(1/2)+195/77*x^5*(x^4+5)^(1/2)+300/77*x*(x^4+5)^(1/2)-60/7
7*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)
^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)+2/9*x^7*(
x^4+5)^(1/2)+22/9*x^3*(x^4+5)^(1/2)+8/3*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)
*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/
2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.77, size = 160, normalized size = 0.73

$$\frac{3\sqrt{5}\,x^9\Gamma\left(\frac{9}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{9}{4}\\ \frac{13}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{13}{4}\right)}+\frac{\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{11}{4}\right)}+\frac{15\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{5\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**9*gamma(9/4)*hyper((-1/2, 9/4), (13/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(13/4)) + sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x*
*4*exp_polar(I*pi)/5)/(2*gamma(11/4)) + 15*sqrt(5)*x**5*gamma(5/4)*hyper((-
1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(9/4)) + 5*sqrt(5)*x**3*
gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(7/4)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,{\left(x^4+5\right)}^{3/2}\,\left(3\,x^2+2\right) \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^4 + 5)^(3/2)*(3*x^2 + 2),x)
[Out] int(x^2*(x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.29 $\int (2+3x^2)\left(5+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=197

$$\frac{20x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}-\frac{20\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\Big(}{\sqrt{5+x^4}}$$

```
[Out] 1/21*x*(7*x^2+6)*(x^4+5)^(3/2)+2/7*x*(7*x^2+10)*(x^4+5)^(1/2)+20*x*(x^4+5)^
(1/2)/(x^2+5^(1/2))-20*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2
*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))
*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+10/7*5^(1/4)*(
cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Elliptic
F(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(7+2*5^(1/2))*((x
^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1191, 1212, 226, 1210}

$$\frac{10\sqrt[4]{5}\left(7+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{20\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{1}{21}x\left(7x^2+6\right)\left(x^4+5\right)^{3/2}+\frac{2}{7}x\left(7x^2+10\right)\sqrt{x^4+5}+\frac{20x\sqrt{x^4+5}}{x^2+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] (20*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (2*x*(10 + 7*x^2)*Sqrt[5 + x^4])/7 +
 (x*(6 + 7*x^2)*(5 + x^4)^(3/2))/21 - (20*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
 x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
 + (10*5^(1/4)*(7 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^
2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int (2+3x^2)\left(5+x^4\right)^{3/2}\,dx = \frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}+\frac{1}{21}\int \left(180+210x^2\right)\sqrt{5+x^4}\,dx$$

$$= \frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}+\frac{1}{315}\int \frac{9000+6300x^2}{\sqrt{5+x^4}}$$

$$= \frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}-\left(20\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}$$

$$20\sqrt[4]{5}$$

$$= \frac{20x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{2}{7}x\left(10+7x^2\right)\sqrt{5+x^4}+\frac{1}{21}x\left(6+7x^2\right)\left(5+x^4\right)^{3/2}-$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.89, size = 49, normalized size = 0.25

$$5\sqrt{5}\,x\left(2\,{}_2F_1\left(-\frac{3}{2},\frac{1}{4};\frac{5}{4};-\frac{x^4}{5}\right)+x^2\,{}_2F_1\left(-\frac{3}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)*(5 + x^4)^(3/2),x]
```

```
[Out] 5*Sqrt[5]*x*(2*Hypergeometric2F1[-3/2, 1/4, 5/4, -1/5*x^4] + x^2*Hypergeome
tric2F1[-3/2, 3/4, 7/4, -1/5*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 192, normalized size = 0.97

method	result
meijerg	$10\sqrt{5}\, x \,\text{hypergeom}\left(\left[-\frac{3}{2}, \frac{1}{4}\right], \left[\frac{5}{4}\right], -\frac{x^4}{5}\right) + 5\sqrt{5}\, x^3 \,\text{hypergeom}\left(\left[-\frac{3}{2}, \frac{3}{4}\right], \left[\frac{7}{4}\right], -\frac{x^4}{5}\right)$
risch	$\frac{x(7x^6+6x^4+77x^2+90)\sqrt{x^4+5}}{21} + \frac{4i\sqrt{25-5i\sqrt{5}\, x^2}\, \sqrt{25+5i\sqrt{5}\, x^2}\, \left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right) - \text{Ellip}\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{x^7\sqrt{x^4+5}}{3} + \frac{11x^3\sqrt{x^4+5}}{3} + \frac{4i\sqrt{25-5i\sqrt{5}\, x^2}\, \sqrt{25+5i\sqrt{5}\, x^2}\, \left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right) - \text{Ell}\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$\frac{x^7\sqrt{x^4+5}}{3} + \frac{11x^3\sqrt{x^4+5}}{3} + \frac{4i\sqrt{25-5i\sqrt{5}\, x^2}\, \sqrt{25+5i\sqrt{5}\, x^2}\, \left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5}, i\right) - \text{Ell}\right.}{\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^7*(x^4+5)^(1/2)+11/3*x^3*(x^4+5)^(1/2)+4*I/(I*5^(1/2))^(1/2)*(25-5*I*
5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*
x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
)+2/7*x^5*(x^4+5)^(1/2)+30/7*x*(x^4+5)^(1/2)+8/7*5^(1/2)/(I*5^(1/2))^(1/2)*
(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Ellipti
cF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.65, size = 158, normalized size = 0.80

$$\frac{3\sqrt{5}\,x^{7}\Gamma\left(\frac{7}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}\,x^{5}\Gamma\left(\frac{5}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{9}{4}\right)}+\frac{15\sqrt{5}\,x^{3}\Gamma\left(\frac{3}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{5\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^{4}e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((-1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/
5)/(4*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**
4*exp_polar(I*pi)/5)/(2*gamma(9/4)) + 15*sqrt(5)*x**3*gamma(3/4)*hyper((-1/
2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(7/4)) + 5*sqrt(5)*x*gamma
(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp_polar(I*pi)/5)/(2*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(x^4+5\right)^{3/2}\,\left(3\,x^2+2\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4 + 5)^(3/2)*(3*x^2 + 2),x)
```

```
[Out] int((x^4 + 5)^(3/2)*(3*x^2 + 2), x)
```

3.30 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=199

$$\frac{24x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{6}{35}x(25+14x^2)\sqrt{5+x^4}-\frac{(14-3x^2)(5+x^4)^{3/2}}{7x}-\frac{24\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\right.}{\sqrt{5+x^4}}$$

```
[Out] -1/7*(-3*x^2+14)*(x^4+5)^(3/2)/x+6/35*x*(14*x^2+25)*(x^4+5)^(1/2)+24*x*(x^4
+5)^(1/2)/(x^2+5^(1/2))-24*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/c
os(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1
/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+6/7*5^(1/4
)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ellip
ticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(14+5*5^(1/2))
*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1286, 1191, 1212, 226, 1210}

$$\frac{6\sqrt[4]{5}\left(14+5\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{7\sqrt{x^4+5}}-\frac{24\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{(14-3x^2)(x^4+5)^{3/2}}{7x}+\frac{6}{35}x(14x^2+25)\sqrt{x^4+5}+\frac{24x\sqrt{x^4+5}}{x^2+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

```
[Out] (24*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) + (6*x*(25 + 14*x^2)*Sqrt[5 + x^4])/35
 - ((14 - 3*x^2)*(5 + x^4)^(3/2))/(7*x) - (24*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[
(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 +
x^4] + (6*5^(1/4)*(14 + 5*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5]
+ x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(7*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1191

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*(
d*(4*p + 3) + e*(4*p + 1)*x^2)*((a + c*x^4)^p/((4*p + 1)*(4*p + 3))), x] +
Dist[2*(p/((4*p + 1)*(4*p + 3))), Int[Simp[2*a*d*(4*p + 3) + (2*a*e*(4*p +
1))*x^2, x]*(a + c*x^4)^(p - 1), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*
```

```
d^2 + a*e^2, 0] && GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^2}\,dx = -\frac{(14-3x^2)(5+x^4)^{3/2}}{7x} - \frac{6}{7}\int (-15-14x^2)\sqrt{5+x^4}\,dx$$

$$= \frac{6}{35}x(25+14x^2)\sqrt{5+x^4} - \frac{(14-3x^2)(5+x^4)^{3/2}}{7x} - \frac{2}{35}\int \frac{-750-420x^2}{\sqrt{5+x^4}}\,dx$$

$$= \frac{6}{35}x(25+14x^2)\sqrt{5+x^4} - \frac{(14-3x^2)(5+x^4)^{3/2}}{7x} - \left(24\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}$$

$$24\sqrt[4]{5}$$

$$= \frac{24x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{6}{35}x(25+14x^2)\sqrt{5+x^4} - \frac{(14-3x^2)(5+x^4)^{3/2}}{7x} -$$

Mathematica [C] Result contains complex when optimal does not.

time = 4.78, size = 125, normalized size = 0.63

$$\frac{-1750+1125x^2-280x^4+300x^6+14x^8+15x^{10}-840(-1)^{3/4}\sqrt[4]{5}\,x\sqrt{5+x^4}\,E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)+60\sqrt[4]{-5}\left(14i-5\sqrt{5}\right)x\sqrt{5+x^4}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)}{35x\sqrt{5+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^2,x]
```

```
[Out] (-1750 + 1125*x^2 - 280*x^4 + 300*x^6 + 14*x^8 + 15*x^10 - 840*(-1)^(3/4)*5
^(1/4)*x*Sqrt[5 + x^4]*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + 60*(-5)^(
1/4)*(14*I - 5*Sqrt[5])*x*Sqrt[5 + x^4]*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x]
, -1])/(35*x*Sqrt[5 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 192, normalized size = 0.96

method	result
meijerg	$-\frac{10\sqrt{5}\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},-\frac{1}{4}\right],\left[\frac{3}{4}\right],-\frac{x^4}{5}\right)}{x}+15\sqrt{5}\,x\,\mathrm{hypergeom}\left(\left[-\frac{3}{2},\frac{1}{4}\right],\left[\frac{5}{4}\right],-\frac{x^4}{5}\right)$
risch	$\frac{15x^{10}+14x^8+300x^6-280x^4+1125x^2-1750}{35x\sqrt{x^4+5}}+\frac{24i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{45x\sqrt{x^4+5}}{7}+\frac{12\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$\frac{3x^5\sqrt{x^4+5}}{7}+\frac{45x\sqrt{x^4+5}}{7}+\frac{12\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{7\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*x^5*(x^4+5)^(1/2)+45/7*x*(x^4+5)^(1/2)+12/7*5^(1/2)/(I*5^(1/2))^(1/2)*(
25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Elliptic
F(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-10*(x^4+5)^(1/2)/x+2/5*x^3*(x^4+5)^(1/
2)+24/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)
^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-Elliptic
E(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^2, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.93, size = 160, normalized size = 0.80

$$\frac{3\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{7}{4}\right)}+\frac{15\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{5}{4}\right)}+\frac{5\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right)\,{}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**2,x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((-1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*
exp_polar(I*pi)/5)/(2*gamma(7/4)) + 15*sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/
4), (5/4,), x**4*exp_polar(I*pi)/5)/(4*gamma(5/4)) + 5*sqrt(5)*gamma(-1/4)*
hyper((-1/2, -1/4), (3/4,), x**4*exp_polar(I*pi)/5)/(2*x*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^2,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Mupad [B]
time = 0.53, size = 48, normalized size = 0.24

$$15\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{4};\ \frac{5}{4};\ -\frac{x^4}{5}\right)+\frac{2\left(x^4+5\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},-\frac{5}{4};\ -\frac{1}{4};\ -\frac{5}{x^4}\right)}{5\,x\left(\frac{5}{x^4}+1\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^2,x)
```

```
[Out] 15*5^(1/2)*x*hypergeom([-3/2, 1/4], 5/4, -x^4/5) + (2*(x^4 + 5)^(3/2)*hyper
geom([-3/2, -5/4], -1/4, -5/x^4))/(5*x*(5/x^4 + 1)^(3/2))
```

3.31 $\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4} \, dx$

Optimal. Leaf size=201

$$-\frac{2(27-2x^2)\sqrt{5+x^4}}{3x}+\frac{36x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3}-\frac{36\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\right.}{\sqrt{5+x^4}}$$

```
[Out] -1/15*(-9*x^2+10)*(x^4+5)^(3/2)/x^3-2/3*(-2*x^2+27)*(x^4+5)^(1/2)/x+36*x*(x
^4+5)^(1/2)/(x^2+5^(1/2))-36*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)
/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^
(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+2/3*5^(1
/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ell
ipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(27+2*5^(1/2
))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1286, 1212, 226, 1210}

$$\frac{2\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{3\sqrt{x^4+5}}-\frac{36\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}-\frac{2(27-2x^2)\sqrt{x^4+5}}{3x}+\frac{36x\sqrt{x^4+5}}{x^2+\sqrt{5}}-\frac{(10-9x^2)(x^4+5)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```

```
[Out] (-2*(27 - 2*x^2)*Sqrt[5 + x^4])/(3*x) + (36*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2
) - ((10 - 9*x^2)*(5 + x^4)^(3/2))/(15*x^3) - (36*5^(1/4)*(Sqrt[5] + x^2)*S
qrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[
5 + x^4] + (2*5^(1/4)*(27 + 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt
[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(3*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1286

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[(f*x)^(m + 1)*(a + c*x^4)^p*((d*(m + 4*p + 3) + e*(m + 1)*
x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[4*(p/(f^2*(m + 1)*(m + 4*p + 3))
), Int[(f*x)^(m + 2)*(a + c*x^4)^(p - 1)*(a*e*(m + 1) - c*d*(m + 4*p + 3)*x
^2), x], x] /; FreeQ[{a, c, d, e, f}, x] && GtQ[p, 0] && LtQ[m, -1] && m +
4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)(5+x^4)^{3/2}}{x^4}\,dx = -\frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \frac{2}{5}\int \frac{(-45-10x^2)\sqrt{5+x^4}}{x^2}\,dx$$

$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} + \frac{4}{15}\int \frac{50+135x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \left(36\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2(27-2x^2)\sqrt{5+x^4}}{3x} + \frac{36x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{(10-9x^2)(5+x^4)^{3/2}}{15x^3} - \frac{36\sqrt[4]{5}\left(\sqrt{}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.40, size = 124, normalized size = 0.62

$$\frac{-250-1125x^2-180x^6+10x^8+9x^{10}-540(-1)^{3/4}\sqrt[4]{5}\,x^3\sqrt{5+x^4}\,E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)+20\sqrt[4]{-5}\left(27i-2\sqrt{5}\right)x^3\sqrt{5+x^4}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)}{15x^3\sqrt{5+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(5 + x^4)^(3/2))/x^4,x]
```

```
[Out] (-250 - 1125*x^2 - 180*x^6 + 10*x^8 + 9*x^10 - 540*(-1)^(3/4)*5^(1/4)*x^3*S
qrt[5 + x^4]*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + 20*(-5)^(1/4)*(27*I
 - 2*Sqrt[5])*x^3*Sqrt[5 + x^4]*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/(
15*x^3*Sqrt[5 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 192, normalized size = 0.96

method	result
meijerg	$-\frac{10\sqrt{5}\ \mathrm{hypergeom}\left(\left[-\frac{3}{2},-\frac{3}{4}\right],\left[\frac{1}{4}\right],-\frac{x^4}{5}\right)}{3x^3}-\frac{15\sqrt{5}\ \mathrm{hypergeom}\left(\left[-\frac{3}{2},-\frac{1}{4}\right],\left[\frac{3}{4}\right],-\frac{x^4}{5}\right)}{x}$
risch	$\frac{9x^{10}+10x^8-180x^6-1125x^2-250}{15x^3\sqrt{x^4+5}}+\frac{36i\sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{Elli}\right.}{5\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}$
default	$-\frac{15\sqrt{x^4+5}}{x}+\frac{3x^3\sqrt{x^4+5}}{5}+\frac{36i\sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)\right.}{5\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}$
elliptic	$-\frac{15\sqrt{x^4+5}}{x}+\frac{3x^3\sqrt{x^4+5}}{5}+\frac{36i\sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)\right.}{5\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -15*(x^4+5)^(1/2)/x+3/5*x^3*(x^4+5)^(1/2)+36/5*I/(I*5^(1/2))^(1/2)*(25-5*I*
5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*
x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
)-10/3*(x^4+5)^(1/2)/x^3+2/3*x*(x^4+5)^(1/2)+8/15*5^(1/2)/(I*5^(1/2))^(1/2)
*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*Ellipt
icF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 2*x^4 + 15*x^2 + 10)*sqrt(x^4 + 5)/x^4, x)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.98, size = 163, normalized size = 0.81

$$\frac{3\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},\frac{1}{4}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2\Gamma\left(\frac{5}{4}\right)}+\frac{15\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{2},-\frac{1}{4}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{4x\Gamma\left(\frac{3}{4}\right)}+\frac{5\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},-\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{2x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5)**(3/2)/x**4,x)
```

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((-1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5
)/(4*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((-1/2, 1/4), (5/4,), x**4*exp
_polar(I*pi)/5)/(2*gamma(5/4)) + 15*sqrt(5)*gamma(-1/4)*hyper((-1/2, -1/4),
 (3/4,), x**4*exp_polar(I*pi)/5)/(4*x*gamma(3/4)) + 5*sqrt(5)*gamma(-3/4)*h
yper((-3/4, -1/2), (1/4,), x**4*exp_polar(I*pi)/5)/(2*x**3*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5)^(3/2)/x^4,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(x^4+5\right)}^{3/2}\,\left(3\,x^2+2\right)}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^4,x)
```

```
[Out] int(((x^4 + 5)^(3/2)*(3*x^2 + 2))/x^4, x)
```

3.32 $\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=67

$$\frac{1}{3}x^4\sqrt{5+x^4}+\frac{3}{8}x^6\sqrt{5+x^4}-\frac{5}{48}(32+27x^2)\sqrt{5+x^4}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 225/16*arcsinh(1/5*x^2*5^(1/2))+1/3*x^4*(x^4+5)^(1/2)+3/8*x^6*(x^4+5)^(1/2)
-5/48*(27*x^2+32)*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1266, 847, 794, 221}

$$\frac{1}{3}\sqrt{x^4+5}\,x^4+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{3}{8}\sqrt{x^4+5}\,x^6-\frac{5}{48}(27x^2+32)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/3 + (3*x^6*Sqrt[5 + x^4])/8 - (5*(32 + 27*x^2)*Sqrt[5 +
 x^4])/48 + (225*ArcSinh[x^2/Sqrt[5]])/16
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7(2+3x^2)}{\sqrt{5+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(2+3x)}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{3}{8}x^6\sqrt{5+x^4}+\frac{1}{8}\text{Subst}\left(\int \frac{x^2(-45+8x)}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^4\sqrt{5+x^4}+\frac{3}{8}x^6\sqrt{5+x^4}+\frac{1}{24}\text{Subst}\left(\int \frac{(-80-135x)x}{\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{3}x^4\sqrt{5+x^4}+\frac{3}{8}x^6\sqrt{5+x^4}-\frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4}+\frac{225}{16}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,\right.\\ &= \frac{1}{3}x^4\sqrt{5+x^4}+\frac{3}{8}x^6\sqrt{5+x^4}-\frac{5}{48}\left(32+27x^2\right)\sqrt{5+x^4}+\frac{225}{16}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A]
time = 0.07, size = 49, normalized size = 0.73

$$\frac{1}{48}\sqrt{5+x^4}\left(-160-135x^2+16x^4+18x^6\right)+\frac{225}{16}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(-160 - 135*x^2 + 16*x^4 + 18*x^6))/48 + (225*ArcTanh[x^2/Sq
rt[5 + x^4]])/16
```

Maple [A]
time = 0.14, size = 51, normalized size = 0.76

method	result	s
risch	$\frac{\left(18x^6+16x^4-135x^2-160\right)\sqrt{x^4+5}}{48}+\frac{225\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}$	3

trager	$\left(\frac{3}{8}x^6+\frac{1}{3}x^4-\frac{45}{16}x^2-\frac{10}{3}\right)\sqrt{x^4+5}+\frac{225\ln\left(x^2+\sqrt{x^4+5}\right)}{16}$
default	$\frac{3x^6\sqrt{x^4+5}}{8}-\frac{45x^2\sqrt{x^4+5}}{16}+\frac{225\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{\sqrt{x^4+5}\,(x^4-10)}{3}$
elliptic	$\frac{3x^6\sqrt{x^4+5}}{8}-\frac{45x^2\sqrt{x^4+5}}{16}+\frac{225\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}+\frac{x^4\sqrt{x^4+5}}{3}-\frac{10\sqrt{x^4+5}}{3}$
meijerg	$\frac{-\frac{3\sqrt{\pi}\,x^2\sqrt{5}\,(-2x^4+15)\sqrt{1+\frac{x^4}{5}}}{16}+\frac{225\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{16}}{\sqrt{\pi}}+\frac{5\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-\frac{4x^4}{5}+8\right)\sqrt{1+\frac{x^4}{5}}}{6}\right)}{2\sqrt{\pi}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^6*(x^4+5)^(1/2)-45/16*x^2*(x^4+5)^(1/2)+225/16*arcsinh(1/5*x^2*5^(1/2
))+1/3*(x^4+5)^(1/2)*(x^4-10)
```

Maxima [A]
time = 0.50, size = 104, normalized size = 1.55

$$\frac{1}{3}\left(x^4+5\right)^{\frac{3}{2}}-5\sqrt{x^4+5}-\frac{75\left(\frac{5\sqrt{x^4+5}}{x^2}-\frac{3\left(x^4+5\right)^{\frac{3}{2}}}{x^6}\right)}{16\left(\frac{2\left(x^4+5\right)}{x^4}-\frac{\left(x^4+5\right)^2}{x^8}-1\right)}+\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{225}{32}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(x^4 + 5)^(3/2) - 5*sqrt(x^4 + 5) - 75/16*(5*sqrt(x^4 + 5)/x^2 - 3*(x^4
 + 5)^(3/2)/x^6)/(2*(x^4 + 5)/x^4 - (x^4 + 5)^2/x^8 - 1) + 225/32*log(sqrt(
x^4 + 5)/x^2 + 1) - 225/32*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.34, size = 43, normalized size = 0.64

$$\frac{1}{48}\left(18\,x^6+16\,x^4-135\,x^2-160\right)\sqrt{x^4+5}-\frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/48*(18*x^6 + 16*x^4 - 135*x^2 - 160)*sqrt(x^4 + 5) - 225/16*log(-x^2 + sq
rt(x^4 + 5))
```

Sympy [A]
time = 3.91, size = 85, normalized size = 1.27

$$\frac{3x^{10}}{8\sqrt{x^4+5}} - \frac{15x^6}{16\sqrt{x^4+5}} + \frac{x^4\sqrt{x^4+5}}{3} - \frac{225x^2}{16\sqrt{x^4+5}} - \frac{10\sqrt{x^4+5}}{3} + \frac{225\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**10/(8*sqrt(x**4 + 5)) - 15*x**6/(16*sqrt(x**4 + 5)) + x**4*sqrt(x**4 +
 5)/3 - 225*x**2/(16*sqrt(x**4 + 5)) - 10*sqrt(x**4 + 5)/3 + 225*asinh(sqrt
(5)*x**2/5)/16
```

Giac [A]
time = 4.13, size = 46, normalized size = 0.69

$$\frac{1}{48}\sqrt{x^4+5}\left(\left(2\left(9x^2+8\right)x^2-135\right)x^2-160\right) - \frac{225}{16}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(x^4 + 5)*((2*(9*x^2 + 8)*x^2 - 135)*x^2 - 160) - 225/16*log(-x^2
+ sqrt(x^4 + 5))
```

Mupad [B]
time = 0.59, size = 38, normalized size = 0.57

$$\frac{225\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{16} - \sqrt{x^4+5}\left(-\frac{3x^6}{8} - \frac{x^4}{3} + \frac{45x^2}{16} + \frac{10}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] (225*asinh((5^(1/2)*x^2)/5))/16 - (x^4 + 5)^(1/2)*((45*x^2)/16 - x^4/3 - (3
*x^6)/8 + 10/3)
```

3.33 $\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}} \, dx$

Optimal. Leaf size=51

$$\frac{1}{2}x^4\sqrt{5+x^4} - \frac{1}{2}(10-x^2)\sqrt{5+x^4} - \frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -5/2*arcsinh(1/5*x^2*5^(1/2))+1/2*x^4*(x^4+5)^(1/2)-1/2*(-x^2+10)*(x^4+5)^(
1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1266, 847, 794, 221}

$$\frac{1}{2}\sqrt{x^4+5}\,x^4 - \frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right) - \frac{1}{2}(10-x^2)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x^4*Sqrt[5 + x^4])/2 - ((10 - x^2)*Sqrt[5 + x^4])/2 - (5*ArcSinh[x^2/Sqrt[
5]])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 847

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + c*x^2)^(p + 1)/(c*(m + 2*p + 2))
), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)*(a + c*x^2)^p*Simp[
c*d*f*(m + 2*p + 2) - a*e*g*m + c*(e*f*(m + 2*p + 2) + d*g*m)*x, x], x], x]
 /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 + a*e^2, 0] && GtQ[m, 0] &
& NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p]
```

```
) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(2+3x^2)}{\sqrt{5+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}x^4\sqrt{5+x^4} + \frac{1}{6}\text{Subst}\left(\int \frac{x(-30+6x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}x^4\sqrt{5+x^4} - \frac{1}{2}(10-x^2)\sqrt{5+x^4} - \frac{5}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}x^4\sqrt{5+x^4} - \frac{1}{2}(10-x^2)\sqrt{5+x^4} - \frac{5}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A]
time = 0.08, size = 40, normalized size = 0.78

$$\frac{1}{2}\sqrt{5+x^4}\left(-10+x^2+x^4\right) - \frac{5}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] (Sqrt[5 + x^4]*(-10 + x^2 + x^4))/2 - (5*ArcTanh[x^2/Sqrt[5 + x^4]])/2
```

Maple [A]
time = 0.14, size = 39, normalized size = 0.76

method	result	size
risch	$\frac{(x^4+x^2-10)\sqrt{x^4+5}}{2} - \frac{5\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	30
trager	$\left(\frac{1}{2}x^4+\frac{1}{2}x^2-5\right)\sqrt{x^4+5} - \frac{5\ln\left(x^2+\sqrt{x^4+5}\right)}{2}$	36
default	$\frac{\sqrt{x^4+5}\,(x^4-10)}{2} + \frac{x^2\sqrt{x^4+5}}{2} - \frac{5\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	39

elliptic	$\frac{x^4\sqrt{x^4+5}}{2}-5\sqrt{x^4+5}+\frac{x^2\sqrt{x^4+5}}{2}-\frac{5\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	46
meijerg	$\frac{15\sqrt{5}\left(\frac{4\sqrt{\pi}}{3}-\frac{\sqrt{\pi}\left(-\frac{4x^4}{5}+8\right)\sqrt{1+\frac{x^4}{5}}}{6}\right)}{4\sqrt{\pi}}+\frac{\frac{\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{2}-\frac{5\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}}{\sqrt{\pi}}$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^4+5)^(1/2)*(x^4-10)+1/2*x^2*(x^4+5)^(1/2)-5/2*arcsinh(1/5*x^2*5^(1/2
))
```

Maxima [A]
time = 0.49, size = 76, normalized size = 1.49

$$\frac{1}{2}\left(x^4+5\right)^{\frac{3}{2}}-\frac{15}{2}\sqrt{x^4+5}+\frac{5\sqrt{x^4+5}}{2x^2\left(\frac{x^4+5}{x^4}-1\right)}-\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{5}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(x^4 + 5)^(3/2) - 15/2*sqrt(x^4 + 5) + 5/2*sqrt(x^4 + 5)/(x^2*((x^4 + 5
)/x^4 - 1)) - 5/4*log(sqrt(x^4 + 5)/x^2 + 1) + 5/4*log(sqrt(x^4 + 5)/x^2 -
1)
```

Fricas [A]
time = 0.35, size = 34, normalized size = 0.67

$$\frac{1}{2}\left(x^4+x^2-10\right)\sqrt{x^4+5}+\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(x^4 + x^2 - 10)*sqrt(x^4 + 5) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A]
time = 2.70, size = 66, normalized size = 1.29

$$\frac{x^6}{2\sqrt{x^4+5}}+\frac{x^4\sqrt{x^4+5}}{2}+\frac{5x^2}{2\sqrt{x^4+5}}-5\sqrt{x^4+5}-\frac{5\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] x**6/(2*sqrt(x**4 + 5)) + x**4*sqrt(x**4 + 5)/2 + 5*x**2/(2*sqrt(x**4 + 5))
 - 5*sqrt(x**4 + 5) - 5*asinh(sqrt(5)*x**2/5)/2
```

Giac [A]
time = 5.04, size = 37, normalized size = 0.73

$$\frac{1}{2}\sqrt{x^4+5}\left((x^2+1)x^2-10\right)+\frac{5}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^4 + 5)*((x^2 + 1)*x^2 - 10) + 5/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.31, size = 32, normalized size = 0.63

$$\sqrt{x^4+5}\left(\frac{x^4}{2}+\frac{x^2}{2}-5\right)-\frac{5\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] (x^4 + 5)^(1/2)*(x^2/2 + x^4/2 - 5) - (5*asinh((5^(1/2)*x^2)/5))/2
```

3.34 $\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}} \, dx$

Optimal. Leaf size=35

$$\frac{1}{4}(4+3x^2)\sqrt{5+x^4} - \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -15/4*arcsinh(1/5*x^2*5^(1/2))+1/4*(3*x^2+4)*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {1266, 794, 221}

$$\frac{1}{4}(3x^2+4)\sqrt{x^4+5} - \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcSinh[x^2/Sqrt[5]])/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^3(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} - \frac{15}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{1}{4}(4+3x^2)\sqrt{5+x^4} - \frac{15}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A]
time = 0.07, size = 39, normalized size = 1.11

$$\frac{1}{4}(4+3x^2)\sqrt{5+x^4} - \frac{15}{4}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[5 + x^4],x]
[Out] ((4 + 3*x^2)*Sqrt[5 + x^4])/4 - (15*ArcTanh[x^2/Sqrt[5 + x^4]])/4
```

Maple [A]
time = 0.13, size = 32, normalized size = 0.91

method	result	size
risch	$-\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{(3x^2+4)\sqrt{x^4+5}}{4}$	29
trager	$\left(\frac{3x^2}{4}+1\right)\sqrt{x^4+5}-\frac{15\ln\left(x^2+\sqrt{x^4+5}\right)}{4}$	31
default	$\frac{3x^2\sqrt{x^4+5}}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}$	32
elliptic	$\frac{3x^2\sqrt{x^4+5}}{4}-\frac{15\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\sqrt{x^4+5}$	32
meijerg	$\frac{\frac{3\sqrt{\pi}\,x^2\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{4}-\frac{15\sqrt{\pi}\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}}{\sqrt{\pi}}+\frac{\sqrt{5}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+\frac{x^4}{5}}\right)}{2\sqrt{\pi}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
[Out] 3/4*x^2*(x^4+5)^(1/2)-15/4*arcsinh(1/5*x^2*5^(1/2))+(x^4+5)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(28) = 56$.
time = 0.49, size = 65, normalized size = 1.86

$$\sqrt{x^4+5} + \frac{15\sqrt{x^4+5}}{4\,x^2\left(\frac{x^4+5}{x^4}-1\right)} - \frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right) + \frac{15}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + 5) + 15/4*sqrt(x^4 + 5)/(x^2*((x^4 + 5)/x^4 - 1)) - 15/8*log(sqr
t(x^4 + 5)/x^2 + 1) + 15/8*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.36, size = 33, normalized size = 0.94

$$\frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right) + \frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A]
time = 2.00, size = 53, normalized size = 1.51

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{15x^2}{4\sqrt{x^4+5}} + \sqrt{x^4+5} - \frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + 15*x**2/(4*sqrt(x**4 + 5)) + sqrt(x**4 + 5) - 1
5*asinh(sqrt(5)*x**2/5)/4
```

Giac [A]
time = 4.22, size = 33, normalized size = 0.94

$$\frac{1}{4}\sqrt{x^4+5}\left(3\,x^2+4\right) + \frac{15}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(x^4 + 5)*(3*x^2 + 4) + 15/4*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.49, size = 27, normalized size = 0.77

$$\sqrt{x^4+5}\,\left(\frac{3\,x^2}{4}+1\right)-\frac{15\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (15*asinh((5^(1/2)*x^2)/5))/4
```

3.35 $\int \frac{x(2+3x^2)}{\sqrt{5+x^4}} \, dx$

Optimal. Leaf size=24

$$\frac{3\sqrt{5+x^4}}{2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {1262, 655, 221}

$$\frac{3\sqrt{x^4+5}}{2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcSinh[x^2/Sqrt[5]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(2+3x^2)}{\sqrt{5+x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{3\sqrt{5+x^4}}{2} + \text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= \frac{3\sqrt{5+x^4}}{2} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A]
time = 0.07, size = 28, normalized size = 1.17

$$\frac{3\sqrt{5+x^4}}{2} + \tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (3*Sqrt[5 + x^4])/2 + ArcTanh[x^2/Sqrt[5 + x^4]]
```

Maple [A]
time = 0.13, size = 20, normalized size = 0.83

method	result	size
default	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right) + \frac{3\sqrt{x^4+5}}{2}$	20
risch	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right) + \frac{3\sqrt{x^4+5}}{2}$	20
elliptic	$\operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right) + \frac{3\sqrt{x^4+5}}{2}$	20
trager	$\frac{3\sqrt{x^4+5}}{2} - \ln\left(x^2 - \sqrt{x^4+5}\right)$	27
meijerg	$\frac{3\sqrt{5}\left(-2\sqrt{\pi}+2\sqrt{\pi}\,\sqrt{1+\frac{x^4}{5}}\right)}{4\sqrt{\pi}} + \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] arcsinh(1/5*x^2*5^(1/2))+3/2*(x^4+5)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.
time = 0.49, size = 42, normalized size = 1.75

$$\frac{3}{2}\sqrt{x^4+5}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/2*sqrt(x^4 + 5) + 1/2*log(sqrt(x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/
x^2 - 1)
```

Fricas [A]
time = 0.33, size = 26, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

Sympy [A]
time = 0.95, size = 22, normalized size = 0.92

$$\frac{3\sqrt{x^4+5}}{2}+\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(x**4 + 5)/2 + asinh(sqrt(5)*x**2/5)
```

Giac [A]
time = 3.69, size = 26, normalized size = 1.08

$$\frac{3}{2}\sqrt{x^4+5}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 3/2*sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.29, size = 19, normalized size = 0.79

$$\operatorname{asinh}\left(\frac{\sqrt{5}\ x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
[Out] asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2
```

3.36 $\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=38

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}$$

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/5*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1266, 858, 221, 272, 65, 213}

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (3*ArcSinh[x^2/Sqrt[5]])/2 - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/Sqrt[5]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 858

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\sqrt{5+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx,x,x^2\right)+\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\frac{1}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)+\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 48, normalized size = 1.26

$$\frac{3}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)+\frac{2\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[5 + x^4]),x]
```

```
[Out] (3*ArcTanh[x^2/Sqrt[5 + x^4]])/2 + (2*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]
])/Sqrt[5]
```

Maple [A]
time = 0.24, size = 30, normalized size = 0.79

method	result	size
default	$\frac{3\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$	30
elliptic	$\frac{3\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}-\frac{\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{5}$	30
trager	$\frac{\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(-\frac{\mathrm{RootOf}\left(\_Z^2-5\right)-\sqrt{x^4+5}}{x^2}\right)}{5}+\frac{3\ln\left(-x^2-\sqrt{x^4+5}\right)}{2}$	50
meijerg	$\frac{\sqrt{5}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+(-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}\right)}{10\sqrt{\pi}}+\frac{3\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	58

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))-1/5*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(30) = 60$.
time = 0.50, size = 67, normalized size = 1.76

$$\frac{1}{10}\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) + 3/
4*log(sqrt(x^4 + 5)/x^2 + 1) - 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.36, size = 41, normalized size = 1.08

$$\frac{1}{5}\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-\frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 3/2*log(-x^2 + sqrt(x^4 +
 5))
```

Sympy [A]
time = 2.93, size = 31, normalized size = 0.82

$$-\frac{\sqrt{5}\,\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{5}+\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(5)*asinh(sqrt(5)/x**2)/5 + 3*asinh(sqrt(5)*x**2/5)/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(30) = 60.
time = 3.61, size = 61, normalized size = 1.61

$$\frac{1}{5}\sqrt{5}\,\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)-\frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 1/5*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
+ 5))) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.61, size = 30, normalized size = 0.79

$$\frac{3\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(x^4 + 5)^(1/2)),x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))
/5
```

3.37 $\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=42

$$-\frac{\sqrt{5+x^4}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}$$

```
[Out] -3/10*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/5*(x^4+5)^(1/2)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1266, 821, 272, 65, 213}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{2\sqrt{5}}-\frac{\sqrt{x^4+5}}{5x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
```

```
[Out] -1/5*Sqrt[5 + x^4]/x^2 - (3*ArcTanh[Sqrt[5 + x^4]/Sqrt[5]])/(2*Sqrt[5])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^3\sqrt{5+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\ &= -\frac{\sqrt{5+x^4}}{5x^2}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\ &= -\frac{\sqrt{5+x^4}}{5x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{2\sqrt{5}}\end{aligned}$$

Mathematica [A]
time = 0.09, size = 46, normalized size = 1.10

$$-\frac{\sqrt{5+x^4}}{5x^2}+\frac{3\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[5 + x^4]),x]
[Out] -1/5*Sqrt[5 + x^4]/x^2 + (3*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sqrt[5]])/Sqrt[5]
```

Maple [A]
time = 0.18, size = 31, normalized size = 0.74

method	result	size
default	$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}$	31
risch	$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}$	31
elliptic	$-\frac{\sqrt{x^4+5}}{5x^2}-\frac{3\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{10}$	31
trager	$-\frac{\sqrt{x^4+5}}{5x^2}+\frac{3\,\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-5\right)-\sqrt{x^4+5}}{x^2}\right)}{10}$	44
meijerg	$-\frac{\sqrt{5}\,\sqrt{1+\frac{x^4}{5}}}{5x^2}+\frac{3\sqrt{5}\left(-2\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+(-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}\right)}{20\sqrt{\pi}}$	64

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*(x^4+5)^(1/2)/x^2-3/10*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))
```

Maxima [A]
time = 0.49, size = 47, normalized size = 1.12

$$\frac{3}{20}\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{\sqrt{x^4+5}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/20*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) - 1/
5*sqrt(x^4 + 5)/x^2
```

Fricas [A]
time = 0.37, size = 47, normalized size = 1.12

$$\frac{3\sqrt{5}\,x^2\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)-2\,x^2-2\sqrt{x^4+5}}{10\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(3*sqrt(5)*x^2*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) - 2*x^2 - 2*sqrt(x^
4 + 5))/x^2
```

Sympy [A]
time = 1.76, size = 31, normalized size = 0.74

$$-\frac{\sqrt{1+\frac{5}{x^4}}}{5}-\frac{3\sqrt{5}\,\operatorname{asinh}\left(\frac{\sqrt{5}}{x^2}\right)}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(1/2),x)
```

```
[Out] -sqrt(1 + 5/x**4)/5 - 3*sqrt(5)*asinh(sqrt(5)/x**2)/10
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(31) = 62$.
time = 4.78, size = 66, normalized size = 1.57

$$\frac{3}{10}\sqrt{5}\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{2}{\left(x^2-\sqrt{x^4+5}\right)^2-5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] 3/10*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) + 2/((x^2 - sqrt(x^4 + 5))^2 - 5)
```

Mupad [B]
time = 0.33, size = 31, normalized size = 0.74

$$-\frac{3\sqrt{5}\,\operatorname{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{10}-\frac{\sqrt{x^4+5}}{5\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(1/2)),x)
```

```
[Out] - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/10 - (x^4 + 5)^(1/2)/(5*x^
2)
```

3.38 $\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=58

$$-\frac{\sqrt{5+x^4}}{10x^4}-\frac{3\sqrt{5+x^4}}{10x^2}+\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

```
[Out] 1/50*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)-1/10*(x^4+5)^(1/2)/x^4-3/10
*(x^4+5)^(1/2)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1266, 849, 821, 272, 65, 213}

$$-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}-\frac{3\sqrt{x^4+5}}{10x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] -1/10*Sqrt[5 + x^4]/x^4 - (3*Sqrt[5 + x^4])/(10*x^2) + ArcTanh[Sqrt[5 + x^4
]/Sqrt[5]]/(10*Sqrt[5])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 849

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1)/(
(m + 1)*(c*d^2 + a*e^2))), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x^5\sqrt{5+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^3\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{1}{20}\text{Subst}\left(\int \frac{-30+2x}{x^2\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{20}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} - \frac{1}{10}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= -\frac{\sqrt{5+x^4}}{10x^4} - \frac{3\sqrt{5+x^4}}{10x^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}
\end{aligned}
$$

Mathematica [A]

time = 0.14, size = 55, normalized size = 0.95

$$\frac{1}{50}\left(-\frac{5(1+3x^2)\sqrt{5+x^4}}{x^4}-2\sqrt{5}\,\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[5 + x^4]),x]
```

```
[Out] ((-5*(1 + 3*x^2)*Sqrt[5 + x^4])/x^4 - 2*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4
])/Sqrt[5]])/50
```

Maple [A]
time = 0.20, size = 43, normalized size = 0.74

method	result
default	$-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}-\frac{3\sqrt{x^4+5}}{10x^2}$
elliptic	$-\frac{\sqrt{x^4+5}}{10x^4}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}-\frac{3\sqrt{x^4+5}}{10x^2}$
risch	$-\frac{3x^6+x^4+15x^2+5}{10x^4\sqrt{x^4+5}}+\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$
trager	$-\frac{(3x^2+1)\sqrt{x^4+5}}{10x^4}-\frac{\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(-\frac{\operatorname{RootOf}\left(\_Z^2-5\right)-\sqrt{x^4+5}}{x^2}\right)}{50}$
meijerg	$\frac{\sqrt{5}\left(\frac{5\sqrt{\pi}\left(\frac{4x^4}{5}+8\right)}{8x^4}-\frac{5\sqrt{\pi}\sqrt{1+\frac{x^4}{5}}}{x^4}+\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)-\frac{(1-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}}{2}-\frac{5\sqrt{\pi}}{x^4}\right)}{50\sqrt{\pi}}-\frac{3\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^5/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/10*(x^4+5)^(1/2)/x^4+1/50*5^(1/2)*arctanh(5^(1/2)/(x^4+5)^(1/2))-3/10*(x
^4+5)^(1/2)/x^2
```

Maxima [A]
time = 0.50, size = 59, normalized size = 1.02

$$-\frac{1}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)-\frac{3\sqrt{x^4+5}}{10\,x^2}-\frac{\sqrt{x^4+5}}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(5) + sqrt(x^4 + 5))) -
3/10*sqrt(x^4 + 5)/x^2 - 1/10*sqrt(x^4 + 5)/x^4
```

Fricas [A]
time = 0.34, size = 50, normalized size = 0.86

$$\frac{\sqrt{5}\,x^4\log\left(\frac{\sqrt{5}+\sqrt{x^4+5}}{x^2}\right)-15\,x^4-5\,\sqrt{x^4+5}\,(3\,x^2+1)}{50\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/50*(sqrt(5)*x^4*log((sqrt(5) + sqrt(x^4 + 5))/x^2) - 15*x^4 - 5*sqrt(x^4
+ 5)*(3*x^2 + 1))/x^4
```

Sympy [A]
time = 5.66, size = 88, normalized size = 1.52

$$\frac{\sqrt{5}\left(-\frac{\log\left(\sqrt{\frac{x^4}{5}+1}-1\right)}{4}+\frac{\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{4}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}+1\right)}-\frac{1}{4\left(\sqrt{\frac{x^4}{5}+1}-1\right)}\right)}{25}-\frac{3\sqrt{5}\,\sqrt{5x^4+25}}{50x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**5/(x**4+5)**(1/2),x)
```

```
[Out] sqrt(5)*(-log(sqrt(x**4/5 + 1) - 1)/4 + log(sqrt(x**4/5 + 1) + 1)/4 - 1/(4*
(sqrt(x**4/5 + 1) + 1)) - 1/(4*(sqrt(x**4/5 + 1) - 1)))/25 - 3*sqrt(5)*sqrt
(5*x**4 + 25)/(50*x**2)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(43) = 86.
time = 5.76, size = 114, normalized size = 1.97

$$-\frac{1}{50}\sqrt{5}\,\log\left(-\frac{x^2+\sqrt{5}-\sqrt{x^4+5}}{x^2-\sqrt{5}-\sqrt{x^4+5}}\right)+\frac{\left(x^2-\sqrt{x^4+5}\right)^3+15\left(x^2-\sqrt{x^4+5}\right)^2+5\,x^2-5\,\sqrt{x^4+5}-75}{5\left(\left(x^2-\sqrt{x^4+5}\right)^2-5\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] -1/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^
4 + 5))) + 1/5*((x^2 - sqrt(x^4 + 5))^3 + 15*(x^2 - sqrt(x^4 + 5))^2 + 5*x^
2 - 5*sqrt(x^4 + 5) - 75)/((x^2 - sqrt(x^4 + 5))^2 - 5)^2
```

Mupad [B]
time = 0.69, size = 43, normalized size = 0.74

$$\frac{\sqrt{5}\,\operatorname{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{50}-\frac{3\,\sqrt{x^4+5}}{10\,x^2}-\frac{\sqrt{x^4+5}}{10\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^5*(x^4 + 5)^(1/2)),x)
```

```
[Out] (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50 - (3*(x^4 + 5)^(1/2))/(10*x
^2) - (x^4 + 5)^(1/2)/(10*x^4)
```

3.39 $\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=185

$$\frac{2}{3}x\sqrt{5+x^4}+\frac{3}{5}x^3\sqrt{5+x^4}-\frac{9x\sqrt{5+x^4}}{\sqrt{5}+x^2}+\frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}-\frac{\sqrt[4]{5}\cdots}{\cdots}$$

```
[Out] 2/3*x*(x^4+5)^(1/2)+3/5*x^3*(x^4+5)^(1/2)-9*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+9
*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4))
)*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5
)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/6*5^(1/4)*(cos(2*arctan(1/5*x*5^(3
/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^
(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(27+2*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)
^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1294, 1212, 226, 1210}

$$-\frac{\sqrt[4]{5}\left(27+2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{6\sqrt{x^4+5}}+\frac{9\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{2}{3}\sqrt{x^4+5}\,x+\frac{3}{5}\sqrt{x^4+5}\,x^3-\frac{9\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (2*x*Sqrt[5 + x^4])/3 + (3*x^3*Sqrt[5 + x^4])/5 - (9*x*Sqrt[5 + x^4])/(Sqrt
[5] + x^2) + (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*E
llipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4] - (5^(1/4)*(27 + 2*Sqrt[5
])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5
^(1/4)], 1/2])/(6*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
```

```
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\begin{aligned}\int \frac{x^4(2+3x^2)}{\sqrt{5+x^4}}\,dx &= \frac{3}{5}x^3\sqrt{5+x^4} - \frac{1}{5}\int \frac{x^2(45-10x^2)}{\sqrt{5+x^4}}\,dx\\ &= \frac{2}{3}x\sqrt{5+x^4} + \frac{3}{5}x^3\sqrt{5+x^4} + \frac{1}{15}\int \frac{-50-135x^2}{\sqrt{5+x^4}}\,dx\\ &= \frac{2}{3}x\sqrt{5+x^4} + \frac{3}{5}x^3\sqrt{5+x^4} + \left(9\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx - \frac{1}{3}\left(10+27\sqrt{5}\right)\int \frac{1}{\sqrt{5+x}}\\ &= \frac{2}{3}x\sqrt{5+x^4} + \frac{3}{5}x^3\sqrt{5+x^4} - \frac{9x\sqrt{5+x^4}}{\sqrt{5}+x^2} + \frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\Big(}{\sqrt{5+x^4}}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 74, normalized size = 0.40

$$\frac{1}{15}x\left(\left(10+9x^2\right)\sqrt{5+x^4} - 10\sqrt{5}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right) - 9\sqrt{5}\,x^2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] (x*((10 + 9*x^2)*Sqrt[5 + x^4] - 10*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4
, -1/5*x^4] - 9*Sqrt[5]*x^2*Hypergeometric2F1[1/2, 3/4, 7/4, -1/5*x^4]))/15
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 168, normalized size = 0.91

method	result
meijerg	$\frac{3\sqrt{5}\, x^7 \operatorname{hypergeom}\left([\frac{1}{2},\frac{7}{4}],[\frac{11}{4}],-\frac{x^4}{5}\right)}{35}+\frac{2\sqrt{5}\, x^5 \operatorname{hypergeom}\left([\frac{1}{2},\frac{5}{4}],[\frac{9}{4}],-\frac{x^4}{5}\right)}{25}$
risch	$\frac{x(9x^2+10)\sqrt{x^4+5}}{15}-\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\operatorname{EllipticE}\left(\frac{x\sqrt{5}}{}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{3x^3\sqrt{x^4+5}}{5}-\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\operatorname{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$\frac{3x^3\sqrt{x^4+5}}{5}-\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\operatorname{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*x^3*(x^4+5)^(1/2)-9/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))
^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+2/3*x*(x^4+5)^(1/2)
-2/15*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*
x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.17, size = 75, normalized size = 0.41

$$\frac{3\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{7}{4}\\ \frac{11}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((1/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5
)/(20*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4
*exp_polar(I*pi)/5)/(10*gamma(9/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(3\,x^2+2\right)}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)
```

3.40 $\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=166

$$x\sqrt{5+x^4}+\frac{2x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{2\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(\sqrt{5}\right.}{}$$

```
[Out] x*(x^4+5)^(1/2)+2*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-2*5^(1/4)*(cos(2*arctan(1/5
*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1
/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(
x^4+5)^(1/2)+1/2*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arcta
n(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(2-5^
(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1294, 1212, 226, 1210}

$$\frac{\sqrt[4]{5}\left(2-\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}}-\frac{2\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\sqrt{x^4+5}\,x+\frac{2\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] x*Sqrt[5 + x^4] + (2*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (2*5^(1/4)*(Sqrt[5]
 + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/
2])/Sqrt[5 + x^4] + (5^(1/4)*(2 - Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(
Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2(2+3x^2)}{\sqrt{5+x^4}}\,dx &= x\sqrt{5+x^4}-\frac{1}{3}\int \frac{15-6x^2}{\sqrt{5+x^4}}\,dx \\
&= x\sqrt{5+x^4}-\left(2\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\left(5-2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx \\
&= x\sqrt{5+x^4}+\frac{2x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{2\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right.}{\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 66, normalized size = 0.40

$$
x\sqrt{5+x^4}-\sqrt{5}\,x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+\frac{2x^3\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)}{3\sqrt{5}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(2 + 3*x^2))/Sqrt[5 + x^4],x]
```

```
[Out] x*Sqrt[5 + x^4] - Sqrt[5]*x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] + (2
*x^3*Hypergeometric2F1[1/2, 3/4, 7/4, -1/5*x^4])/(3*Sqrt[5])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 155, normalized size = 0.93

method	result
meijerg	$\frac{3\sqrt{5}\, x^5 \operatorname{hypergeom}\left([\frac{1}{2},\frac{5}{4}],[\frac{9}{4}],-\frac{x^4}{5}\right)}{25}+\frac{2\sqrt{5}\, x^3 \operatorname{hypergeom}\left([\frac{1}{2},\frac{3}{4}],[\frac{7}{4}],-\frac{x^4}{5}\right)}{15}$
default	$x\sqrt{x^4+5}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2i\sqrt{25-5i\sqrt{5}}}{}$
risch	$x\sqrt{x^4+5}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2i\sqrt{25-5i\sqrt{5}}}{}$
elliptic	$x\sqrt{x^4+5}-\frac{\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2i\sqrt{25-5i\sqrt{5}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(x^4+5)^(1/2)-1/5*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^
(1/2),I)+2/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)
*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-Ell
ipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 1.08, size = 75, normalized size = 0.45

$$\frac{3\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{5}{4}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{2},\frac{3}{4}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((1/2, 5/4), (9/4,), x**4*exp_polar(I*pi)/5)
/(20*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*e
xp_polar(I*pi)/5)/(10*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(3\,x^2+2\right)}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(1/2),x)
```

```
[Out] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(1/2), x)
```

3.41 $\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=155

$$\frac{3x\sqrt{5+x^4}}{\sqrt{5}+x^2}-\frac{3\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}}+\frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+x^2\right)\sqrt{\frac{5}{\left(\sqrt{5}\right.}}}{2\sqrt[4]{5}\,\sqrt{}}$$

```
[Out] 3*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-3*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^
(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1
/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/1
0*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ellip
ticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+3*5^(1/2))*
((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {1212, 226, 1210}

$$\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt[4]{5}\,\sqrt{x^4+5}}-\frac{3\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{x^4+5}}+\frac{3\sqrt{x^4+5}\,x}{x^2+\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/Sqrt[5 + x^4],x]
```

```
[Out] (3*x*Sqrt[5 + x^4])/(Sqrt[5] + x^2) - (3*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 +
x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/Sqrt[5 + x^4]
+ ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*Ellipt
icF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
```

```
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\int \frac{2+3x^2}{\sqrt{5+x^4}}\,dx = -\left(\left(3\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx\right)+\left(2+3\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= \frac{3x\sqrt{5+x^4}}{\sqrt{5}+x^2} - \frac{3\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{\sqrt{5+x^4}} + \frac{\left(2+3\sqrt{5}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.04, size = 48, normalized size = 0.31

$$\frac{x\left(2\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+x^2\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-\frac{x^4}{5}\right)\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/Sqrt[5 + x^4],x]
[Out] (x*(2*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] + x^2*Hypergeometric2F1[1/
2, 3/4, 7/4, -1/5*x^4]))/Sqrt[5]
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 146, normalized size = 0.94

method	result
meijerg	$\frac{2\sqrt{5}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{x^4}{5}\right)}{5}+\frac{\sqrt{5}\,x^3\,\mathrm{hypergeom}\left(\left[\frac{1}{2},\frac{3}{4}\right],\left[\frac{7}{4}\right],-\frac{x^4}{5}\right)}{5}$
default	$\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2\sqrt{}}{}$

elliptic	$\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\text{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)\right)}{5\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{2\sqrt{5}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)

[Out] 3/5*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+2/25*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="maxima")

[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [C] Result contains complex when optimal does not.
time = 0.77, size = 73, normalized size = 0.47

$$\frac{3\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{2},\frac{3}{4}\\\frac{7}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right)\,{}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{1}{2}\\\frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{10\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)/(x**4+5)**(1/2),x)

```
[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((1/2, 3/4), (7/4,), x**4*exp_polar(I*pi)/5)
/(20*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_
polar(I*pi)/5)/(10*gamma(5/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2+2}{\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4 + 5)^(1/2),x)
```

```
[Out] int((3*x^2 + 2)/(x^4 + 5)^(1/2), x)
```

3.42 $\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=173

$$-\frac{2\sqrt{5+x^4}}{5x}+\frac{2x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}-\frac{2\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}}+\frac{\left(2+3\sqrt{5}\right)\left(\sqrt{5}+\right.}{}$$

```
[Out] -2/5*(x^4+5)^(1/2)/x+2/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-2/5*5^(1/4)*(cos(2*a
rctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2
*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2
)^(1/2)/(x^4+5)^(1/2)+1/10*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arc
tan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^
2+5^(1/2))*(2+3*5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/4)/(x^4+5)^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1296, 1212, 226, 1210}

$$\frac{\left(2+3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\ 5^{3/4}\sqrt{x^4+5}}-\frac{2\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Sqrt[5 + x^4])/(5*x) + (2*x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)) - (2*(Sq
rt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)
], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) + ((2 + 3*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5
 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(2*5^(3/4)*
Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{2+3x^2}{x^2\sqrt{5+x^4}}\,dx = -\frac{2\sqrt{5+x^4}}{5x} - \frac{1}{5}\int \frac{-15-2x^2}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2\sqrt{5+x^4}}{5x} - \frac{2\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}} + \frac{1}{5}\left(15+2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx$$

$$= -\frac{2\sqrt{5+x^4}}{5x} + \frac{2x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)} - \frac{2\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)\middle|\frac{1}{2}\right.}{5^{3/4}\sqrt{5+x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 81, normalized size = 0.47

$$\frac{1}{5}\left(-\frac{2\sqrt{5+x^4}}{x} - 2(-1)^{3/4}\sqrt[4]{5}\,E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right) - \sqrt[4]{-5}\left(-2i+3\sqrt{5}\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^2*Sqrt[5 + x^4]),x]
```

[Out] ((-2*Sqrt[5 + x^4])/x - 2*(-1)^(3/4)*5^(1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] - (-5)^(1/4)*(-2*I + 3*Sqrt[5])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/5

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 158, normalized size = 0.91

method	result
meijerg	$-\frac{2\sqrt{5}\ \mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{4}\right],-\frac{x^4}{5}\right)}{5x}+\frac{3\sqrt{5}\ x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{1}{2}\right],\left[\frac{5}{4}\right],-\frac{x^4}{5}\right)}{5}$
default	$\frac{3\sqrt{5}\ \sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2i\sqrt{25-5i\sqrt{5}}}{}$
risch	$\frac{3\sqrt{5}\ \sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2i\sqrt{25-5i\sqrt{5}}}{}$
elliptic	$\frac{3\sqrt{5}\ \sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)}{25\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{5x}+\frac{2i\sqrt{25-5i\sqrt{5}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2+2)/x^2/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)

[Out] 3/25*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-2/5*(x^4+5)^(1/2)/x+2/25*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="maxima")

[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.85, size = 75, normalized size = 0.43

$$\frac{3\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\begin{matrix}\frac{1}{4},\frac{1}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 1/2), (5/4,), x**4*exp_polar(I*pi)/5)/(2
0*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_pol
ar(I*pi)/5)/(10*x*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^2), x)
```

Mupad [B]
time = 0.50, size = 48, normalized size = 0.28

$$\frac{3\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{1}{2};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{5}-\frac{2\sqrt{\frac{5}{x^4}+1}\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{3}{4};\ \frac{7}{4};\ -\frac{5}{x^4}\right)}{3\,x\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^2*(x^4 + 5)^(1/2)),x)
```

```
[Out] (3*5^(1/2)*x*hypergeom([1/4, 1/2], 5/4, -x^4/5))/5 - (2*(5/x^4 + 1)^(1/2)*h
ypergeom([1/2, 3/4], 7/4, -5/x^4))/(3*x*(x^4 + 5)^(1/2))
```

3.43 $\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx$

Optimal. Leaf size=189

$$-\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{3x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}-\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}}-\frac{\left(2-9\sqrt{\;}\right.}{}$$

```
[Out] -2/15*(x^4+5)^(1/2)/x^3-3/5*(x^4+5)^(1/2)/x+3/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2
))-3/5*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^
(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(
(x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/150*(cos(2*arctan(1/5*x*5^(3
/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^
(3/4))),1/2*2^(1/2))*(2-9*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^
(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1296, 1212, 226, 1210}

$$-\frac{\left(2-9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{30\sqrt[4]{5}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}}{5x}-\frac{2\sqrt{x^4+5}}{15x^3}+\frac{3\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```

```
[Out] (-2*Sqrt[5 + x^4])/(15*x^3) - (3*Sqrt[5 + x^4])/(5*x) + (3*x*Sqrt[5 + x^4])
/(5*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]
*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 9*Sqr
t[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[
x/5^(1/4)], 1/2])/(30*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^4\sqrt{5+x^4}}\,dx &= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{1}{15}\int \frac{-45+2x^2}{x^2\sqrt{5+x^4}}\,dx\\
&= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{1}{75}\int \frac{-10+45x^2}{\sqrt{5+x^4}}\,dx\\
&= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}-\frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}}+\frac{1}{15}\left(-2+9\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\
&= -\frac{2\sqrt{5+x^4}}{15x^3}-\frac{3\sqrt{5+x^4}}{5x}+\frac{3x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}-\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan\right.}{5^{3/4}\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 97, normalized size = 0.51

$$\frac{1}{75}\left(-\frac{5(10+45x^2+2x^4+9x^6)}{x^3\sqrt{5+x^4}}-45(-1)^{3/4}\sqrt[4]{5}\,E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)+\sqrt[4]{-5}\left(45i+2\sqrt{5}\right)F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^4*Sqrt[5 + x^4]),x]
```

```
[Out] ((-5*(10 + 45*x^2 + 2*x^4 + 9*x^6))/(x^3*Sqrt[5 + x^4]) - 45*(-1)^(3/4)*5^(
1/4)*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] + (-5)^(1/4)*(45*I + 2*Sqrt[5
])*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/75
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 170, normalized size = 0.90

method	result
meijerg	$-\frac{2\sqrt{5}\ \mathrm{hypergeom}\left(\left[-\frac{3}{4},\frac{1}{2}\right],\left[\frac{1}{4}\right],-\frac{x^4}{5}\right)}{15x^3}-\frac{3\sqrt{5}\ \mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{1}{2}\right],\left[\frac{3}{4}\right],-\frac{x^4}{5}\right)}{5x}$
default	$-\frac{3\sqrt{x^4+5}}{5x}+\frac{3i\sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\ \sqrt{i}}{5}\right.\right.}{25\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}$
elliptic	$-\frac{3\sqrt{x^4+5}}{5x}+\frac{3i\sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\ \sqrt{i}}{5}\right.\right.}{25\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}$
risch	$-\frac{9x^6+2x^4+45x^2+10}{15x^3\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}}{}\right.\right.}{25\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/5*(x^4+5)^(1/2)/x+3/25*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(2
5+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))
^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))-2/15*(x^4+5)^(1/2)/
x^3-2/375*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1
/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 0.97, size = 80, normalized size = 0.42

$$\frac{3\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{1}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{20x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{1}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{10x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5)**(1/2),x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 1/2), (3/4,), x**4*exp_polar(I*pi)/5)/(2
0*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 1/2), (1/4,), x**4*exp_p
olar(I*pi)/5)/(10*x**3*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2+2}{x^4\,\sqrt{x^4+5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(1/2)), x)
```

3.44 $\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}} \, dx$

Optimal. Leaf size=58

$$-\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}}+\frac{1}{4}(8+9x^2)\sqrt{5+x^4}-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] -45/4*arcsinh(1/5*x^2*5^(1/2))-1/2*x^4*(3*x^2+2)/(x^4+5)^(1/2)+1/4*(9*x^2+8
)*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1266, 833, 794, 221}

$$-\frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(3x^2+2)x^4}{2\sqrt{x^4+5}}+\frac{1}{4}(9x^2+8)\sqrt{x^4+5}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/2*(x^4*(2 + 3*x^2))/Sqrt[5 + x^4] + ((8 + 9*x^2)*Sqrt[5 + x^4])/4 - (45*
ArcSinh[x^2/Sqrt[5]])/4
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 794

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[((e*f + d*g)*(2*p + 3) + 2*e*g*(p + 1)*x)*((a + c*x^2)^(p
+ 1)/(2*c*(p + 1)*(2*p + 3))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(c*(2*p
+ 3)), Int[(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] &&  !Le
Q[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
```

```
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7(2+3x^2)}{(5+x^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(2+3x)}{(5+x^2)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{10}\text{Subst}\left(\int \frac{x(20+45x)}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{4}(8+9x^2)\sqrt{5+x^4} - \frac{45}{4}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= -\frac{x^4(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{4}(8+9x^2)\sqrt{5+x^4} - \frac{45}{4}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.11, size = 49, normalized size = 0.84

$$\frac{40+45x^2+4x^4+3x^6}{4\sqrt{5+x^4}} - \frac{45}{4}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (40 + 45*x^2 + 4*x^4 + 3*x^6)/(4*Sqrt[5 + x^4]) - (45*ArcTanh[x^2/Sqrt[5 +
x^4]])/4
```

Maple [A]
time = 0.14, size = 50, normalized size = 0.86

method	result	size
risch	$\frac{3x^6+4x^4+45x^2+40}{4\sqrt{x^4+5}} - \frac{45\,\mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}$	39

trager	$\frac{3x^6+4x^4+45x^2+40}{4\sqrt{x^4+5}}-\frac{45\ln\left(x^2+\sqrt{x^4+5}\right)}{4}$	42
default	$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{45x^2}{4\sqrt{x^4+5}}-\frac{45\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{x^4+10}{\sqrt{x^4+5}}$	50
elliptic	$\frac{3x^6}{4\sqrt{x^4+5}}+\frac{45x^2}{4\sqrt{x^4+5}}-\frac{45\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}+\frac{x^4}{\sqrt{x^4+5}}+\frac{10}{\sqrt{x^4+5}}$	57
meijerg	$\frac{\frac{3\sqrt{\pi}\,x^2\sqrt{5}\,(x^4+15)}{20\sqrt{1+\frac{x^4}{5}}}-\frac{45\sqrt{\pi}\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{4}}{\sqrt{\pi}}+\frac{\sqrt{5}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(\frac{4x^4}{5}+8\right)}{4\sqrt{1+\frac{x^4}{5}}}\right)}{\sqrt{\pi}}$	81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x^6/(x^4+5)^(1/2)+45/4*x^2/(x^4+5)^(1/2)-45/4*arcsinh(1/5*x^2*5^(1/2))+
1/(x^4+5)^(1/2)*(x^4+10)
```

Maxima [A]
time = 0.48, size = 89, normalized size = 1.53

$$\sqrt{x^4+5}-\frac{15\left(\frac{3\,(x^4+5)}{x^4}-2\right)}{4\left(\frac{\sqrt{x^4+5}}{x^2}-\frac{(x^4+5)^{\frac{3}{2}}}{x^6}\right)}+\frac{5}{\sqrt{x^4+5}}-\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)+\frac{45}{8}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] sqrt(x^4 + 5) - 15/4*(3*(x^4 + 5)/x^4 - 2)/(sqrt(x^4 + 5)/x^2 - (x^4 + 5)^(
3/2)/x^6) + 5/sqrt(x^4 + 5) - 45/8*log(sqrt(x^4 + 5)/x^2 + 1) + 45/8*log(sq
rt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.38, size = 62, normalized size = 1.07

$$\frac{30\,x^4+45\,(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)+(3\,x^6+4\,x^4+45\,x^2+40)\sqrt{x^4+5}+150}{4\,(x^4+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(30*x^4 + 45*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + (3*x^6 + 4*x^4 + 45*
x^2 + 40)*sqrt(x^4 + 5) + 150)/(x^4 + 5)
```

Sympy [A]
time = 6.50, size = 66, normalized size = 1.14

$$\frac{3x^6}{4\sqrt{x^4+5}} + \frac{x^4}{\sqrt{x^4+5}} + \frac{45x^2}{4\sqrt{x^4+5}} - \frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} + \frac{10}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**6/(4*sqrt(x**4 + 5)) + x**4/sqrt(x**4 + 5) + 45*x**2/(4*sqrt(x**4 + 5)
) - 45*asinh(sqrt(5)*x**2/5)/4 + 10/sqrt(x**4 + 5)
```

Giac [A]
time = 3.87, size = 45, normalized size = 0.78

$$\frac{((3x^2+4)x^2+45)x^2+40}{4\sqrt{x^4+5}} + \frac{45}{4}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(((3*x^2 + 4)*x^2 + 45)*x^2 + 40)/sqrt(x^4 + 5) + 45/4*log(-x^2 + sqrt(
x^4 + 5))
```

Mupad [B]
time = 1.11, size = 97, normalized size = 1.67

$$\sqrt{x^4+5}\left(\frac{3x^2}{4}+1\right) - \frac{45\operatorname{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{4} + \frac{\sqrt{5}\left(10+\sqrt{5}\,15\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)} - \frac{\sqrt{5}\left(-10+\sqrt{5}\,15\mathrm{i}\right)\sqrt{x^4+5}\,1\mathrm{i}}{20\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (x^4 + 5)^(1/2)*((3*x^2)/4 + 1) - (45*asinh((5^(1/2)*x^2)/5))/4 + (5^(1/2)*
(5^(1/2)*15i + 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) - (5^(1/2)*(
5^(1/2)*15i - 10)*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))
```

3.45 $\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=45

$$-\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}}+3\sqrt{5+x^4}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] arcsinh(1/5*x^2*5^(1/2))-1/2*x^2*(3*x^2+2)/(x^4+5)^(1/2)+3*(x^4+5)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1266, 833, 655, 221}

$$3\sqrt{x^4+5}+\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{(3x^2+2)\,x^2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/2*(x^2*(2 + 3*x^2))/Sqrt[5 + x^4] + 3*Sqrt[5 + x^4] + ArcSinh[x^2/Sqrt[5
]]
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 655

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[e*((
a + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[d, Int[(a + c*x^2)^p, x], x] /
; FreeQ[{a, c, d, e, p}, x] && NeQ[p, -1]
```

Rule 833

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m - 1)*(a + c*x^2)^(p + 1)*((a*(e*f + d*g
) - (c*d*f - a*e*g)*x)/(2*a*c*(p + 1))), x] - Dist[1/(2*a*c*(p + 1)), Int[(
d + e*x)^(m - 2)*(a + c*x^2)^(p + 1)*Simp[a*e*(e*f*(m - 1) + d*g*m) - c*d^2
*f*(2*p + 3) + e*(a*e*g*m - c*d*f*(m + 2*p + 2))*x, x], x], x] /; FreeQ[{a,
 c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] &&
(EqQ[d, 0] || (EqQ[m, 2] && EqQ[p, -3] && RationalQ[a, c, d, e, f, g]) ||
!ILtQ[m + 2*p + 3, 0])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(2+3x^2)}{(5+x^4)^{3/2}}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{x^2(2+3x)}{(5+x^2)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}} + \frac{1}{10}\mathrm{Subst}\left(\int \frac{10+30x}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \mathrm{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)\\ &= -\frac{x^2(2+3x^2)}{2\sqrt{5+x^4}} + 3\sqrt{5+x^4} + \sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)\end{aligned}$$

Mathematica [A]
time = 0.12, size = 40, normalized size = 0.89

$$\frac{30-2x^2+3x^4}{2\sqrt{5+x^4}} + \tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (30 - 2*x^2 + 3*x^4)/(2*Sqrt[5 + x^4]) + ArcTanh[x^2/Sqrt[5 + x^4]]
```

Maple [A]
time = 0.16, size = 37, normalized size = 0.82

method	result	size
risch	$\frac{3x^4-2x^2+30}{2\sqrt{x^4+5}} + \mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	32
default	$\frac{\frac{3x^4}{2}+15}{\sqrt{x^4+5}} - \frac{x^2}{\sqrt{x^4+5}} + \mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	37
trager	$\frac{3x^4-2x^2+30}{2\sqrt{x^4+5}} - \ln\left(x^2-\sqrt{x^4+5}\right)$	39
elliptic	$\frac{3x^4}{2\sqrt{x^4+5}} + \frac{15}{\sqrt{x^4+5}} - \frac{x^2}{\sqrt{x^4+5}} + \mathrm{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)$	44

meijerg	$\frac{3\sqrt{5}\left(-2\sqrt{\pi}+\frac{\sqrt{\pi}\left(\frac{4x^4}{5}+8\right)}{4\sqrt{1+\frac{x^4}{5}}}\right)}{2\sqrt{\pi}}+\frac{-\frac{\sqrt{\pi}\,x^2\sqrt{5}}{5\sqrt{1+\frac{x^4}{5}}}+\sqrt{\pi}\ \operatorname{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{\sqrt{\pi}}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2/(x^4+5)^(1/2)*(x^4+10)-x^2/(x^4+5)^(1/2)+arcsinh(1/5*x^2*5^(1/2))
```

Maxima [A]
time = 0.49, size = 63, normalized size = 1.40

$$-\frac{x^2}{\sqrt{x^4+5}}+\frac{3}{2}\sqrt{x^4+5}+\frac{15}{2\sqrt{x^4+5}}+\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{1}{2}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -x^2/sqrt(x^4 + 5) + 3/2*sqrt(x^4 + 5) + 15/2/sqrt(x^4 + 5) + 1/2*log(sqrt(
x^4 + 5)/x^2 + 1) - 1/2*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.37, size = 58, normalized size = 1.29

$$-\frac{2\,x^4+2\,(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)-(3\,x^4-2\,x^2+30)\sqrt{x^4+5}+10}{2\,(x^4+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*x^4 + 2*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) - (3*x^4 - 2*x^2 + 30)*
sqrt(x^4 + 5) + 10)/(x^4 + 5)
```

Sympy [A]
time = 5.37, size = 48, normalized size = 1.07

$$\frac{3x^4}{2\sqrt{x^4+5}}-\frac{x^2}{\sqrt{x^4+5}}+\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{15}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4/(2*sqrt(x**4 + 5)) - x**2/sqrt(x**4 + 5) + asinh(sqrt(5)*x**2/5) + 1
5/sqrt(x**4 + 5)
```

Giac [A]
time = 3.57, size = 39, normalized size = 0.87

$$\frac{(3x^2-2)x^2+30}{2\sqrt{x^4+5}}-\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*((3*x^2 - 2)*x^2 + 30)/sqrt(x^4 + 5) - log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.89, size = 89, normalized size = 1.98

$$\operatorname{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)+\frac{3\sqrt{x^4+5}}{2}-\frac{\sqrt{5}\,\left(-15+\sqrt{5}\,2\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)}+\frac{\sqrt{5}\,\left(15+\sqrt{5}\,2\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] asinh((5^(1/2)*x^2)/5) + (3*(x^4 + 5)^(1/2))/2 - (5^(1/2)*(5^(1/2)*2i - 15)
*(x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*2i + 15)*(
x^4 + 5)^(1/2)*1i)/(20*(5^(1/2)*1i + x^2))
```

3.46 $\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=35

$$\frac{-2-3x^2}{2\sqrt{5+x^4}}+\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

```
[Out] 3/2*arcsinh(1/5*x^2*5^(1/2))+1/2*(-3*x^2-2)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {1266, 792, 221}

$$\frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)-\frac{3x^2+2}{2\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/2*(2 + 3*x^2)/Sqrt[5 + x^4] + (3*ArcSinh[x^2/Sqrt[5]])/2
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 792

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x
_Symbol] :> Simp[(a*(e*f + d*g) - (c*d*f - a*e*g)*x)*((a + c*x^2)^(p + 1)/(
2*a*c*(p + 1))), x] - Dist[(a*e*g - c*d*f*(2*p + 3))/(2*a*c*(p + 1)), Int[(
a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ[p, -1]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{x^3(2+3x^2)}{(5+x^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{(5+x^2)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{2+3x^2}{2\sqrt{5+x^4}} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{5+x^2}}\,dx, x, x^2\right)$$
$$= -\frac{2+3x^2}{2\sqrt{5+x^4}} + \frac{3}{2}\sinh^{-1}\left(\frac{x^2}{\sqrt{5}}\right)$$

Mathematica [A]
time = 0.11, size = 39, normalized size = 1.11

$$\frac{-2-3x^2}{2\sqrt{5+x^4}} + \frac{3}{2}\tanh^{-1}\left(\frac{x^2}{\sqrt{5+x^4}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
[Out] (-2 - 3*x^2)/(2*Sqrt[5 + x^4]) + (3*ArcTanh[x^2/Sqrt[5 + x^4]])/2
```

Maple [A]
time = 0.15, size = 34, normalized size = 0.97

method	result	size
risch	$-\frac{3x^2+2}{2\sqrt{x^4+5}} + \frac{3\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}$	29
trager	$-\frac{3x^2+2}{2\sqrt{x^4+5}} + \frac{3\ln\left(x^2+\sqrt{x^4+5}\right)}{2}$	32
default	$-\frac{3x^2}{2\sqrt{x^4+5}} + \frac{3\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2} - \frac{1}{\sqrt{x^4+5}}$	34
elliptic	$-\frac{3x^2}{2\sqrt{x^4+5}} + \frac{3\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2} - \frac{1}{\sqrt{x^4+5}}$	34
meijerg	$\frac{-\frac{3\sqrt{\pi}\,x^2\sqrt{5}}{10\sqrt{1+\frac{x^4}{5}}}+\frac{3\sqrt{\pi}\,\text{arcsinh}\left(\frac{x^2\sqrt{5}}{5}\right)}{2}}{\sqrt{\pi}} + \frac{\sqrt{5}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}\right)}{5\sqrt{\pi}}$	67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x^2/(x^4+5)^(1/2)+3/2*arcsinh(1/5*x^2*5^(1/2))-1/(x^4+5)^(1/2)
```

Maxima [A]
time = 0.51, size = 54, normalized size = 1.54

$$-\frac{3x^2}{2\sqrt{x^4+5}}-\frac{1}{\sqrt{x^4+5}}+\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}+1\right)-\frac{3}{4}\log\left(\frac{\sqrt{x^4+5}}{x^2}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/2*x^2/sqrt(x^4 + 5) - 1/sqrt(x^4 + 5) + 3/4*log(sqrt(x^4 + 5)/x^2 + 1) -
 3/4*log(sqrt(x^4 + 5)/x^2 - 1)
```

Fricas [A]
time = 0.38, size = 52, normalized size = 1.49

$$-\frac{3x^4+3(x^4+5)\log\left(-x^2+\sqrt{x^4+5}\right)+\sqrt{x^4+5}(3x^2+2)+15}{2(x^4+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(3*x^4 + 3*(x^4 + 5)*log(-x^2 + sqrt(x^4 + 5)) + sqrt(x^4 + 5)*(3*x^2
+ 2) + 15)/(x^4 + 5)
```

Sympy [A]
time = 4.42, size = 39, normalized size = 1.11

$$-\frac{3x^2}{2\sqrt{x^4+5}}+\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}x^2}{5}\right)}{2}-\frac{1}{\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] -3*x**2/(2*sqrt(x**4 + 5)) + 3*asinh(sqrt(5)*x**2/5)/2 - 1/sqrt(x**4 + 5)
```

Giac [A]
time = 3.32, size = 33, normalized size = 0.94

$$-\frac{3x^2+2}{2\sqrt{x^4+5}}-\frac{3}{2}\log\left(-x^2+\sqrt{x^4+5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*x^2 + 2)/sqrt(x^4 + 5) - 3/2*log(-x^2 + sqrt(x^4 + 5))
```

Mupad [B]
time = 0.84, size = 82, normalized size = 2.34

$$\frac{3\,\mathrm{asinh}\left(\frac{\sqrt{5}\,x^2}{5}\right)}{2}-\frac{\sqrt{5}\,\left(2+\sqrt{5}\,3\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(-x^2+\sqrt{5}\,1\mathrm{i}\right)}+\frac{\sqrt{5}\,\left(-2+\sqrt{5}\,3\mathrm{i}\right)\,\sqrt{x^4+5}\,1\mathrm{i}}{20\,\left(x^2+\sqrt{5}\,1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (3*asinh((5^(1/2)*x^2)/5))/2 - (5^(1/2)*(5^(1/2)*3i + 2)*(x^4 + 5)^(1/2)*1i
)/(20*(5^(1/2)*1i - x^2)) + (5^(1/2)*(5^(1/2)*3i - 2)*(x^4 + 5)^(1/2)*1i)/(
20*(5^(1/2)*1i + x^2))
```

3.47 $\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}} \, dx$

Optimal. Leaf size=20

$$\frac{-15+2x^2}{10\sqrt{5+x^4}}$$

```
[Out] 1/10*(2*x^2-15)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1262, 651}

$$-\frac{15-2x^2}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/10*(15 - 2*x^2)/Sqrt[5 + x^4]
```

Rule 651

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2)^(3/2), x_Symbol] :> Simp[((-a
)*e + c*d*x)/(a*c*Sqrt[a + c*x^2]), x] /; FreeQ[{a, c, d, e}, x]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(2+3x^2)}{(5+x^4)^{3/2}} \, dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{(5+x^2)^{3/2}} \, dx, x, x^2\right) \\ &= -\frac{15-2x^2}{10\sqrt{5+x^4}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 20, normalized size = 1.00

$$\frac{-15+2x^2}{10\sqrt{5+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (-15 + 2*x^2)/(10*Sqrt[5 + x^4])
```

Maple [A]
time = 0.13, size = 23, normalized size = 1.15

method	result	size
gosper	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
trager	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
risch	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
elliptic	$\frac{2x^2-15}{10\sqrt{x^4+5}}$	17
default	$-\frac{3}{2\sqrt{x^4+5}}+\frac{x^2}{5\sqrt{x^4+5}}$	23
meijerg	$\frac{3\sqrt{5}\left(\sqrt{\pi}-\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}\right)}{10\sqrt{\pi}}+\frac{\sqrt{5}\,x^2}{25\sqrt{1+\frac{x^4}{5}}}$	45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2/(x^4+5)^(1/2)+1/5*x^2/(x^4+5)^(1/2)
```

Maxima [A]
time = 0.48, size = 22, normalized size = 1.10

$$\frac{x^2}{5\sqrt{x^4+5}}-\frac{3}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/5*x^2/sqrt(x^4 + 5) - 3/2/sqrt(x^4 + 5)
```

Fricas [A]
time = 0.35, size = 31, normalized size = 1.55

$$\frac{2\,x^4+\sqrt{x^4+5}\,(2\,x^2-15)+10}{10\,(x^4+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/10*(2*x^4 + sqrt(x^4 + 5)*(2*x^2 - 15) + 10)/(x^4 + 5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(15) = 30$.
time = 3.21, size = 31, normalized size = 1.55

$$\frac{\sqrt{5}\,x^2}{5\sqrt{5x^4+25}} - \frac{3}{2\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] sqrt(5)*x**2/(5*sqrt(5*x**4 + 25)) - 3/(2*sqrt(x**4 + 5))
```

Giac [A]
time = 3.78, size = 16, normalized size = 0.80

$$\frac{2\,x^2-15}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/10*(2*x^2 - 15)/sqrt(x^4 + 5)
```

Mupad [B]
time = 0.16, size = 16, normalized size = 0.80

$$\frac{2\,x^2-15}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] (2*x^2 - 15)/(10*(x^4 + 5)^(1/2))
```

3.48 $\int \frac{2+3x^2}{x\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=46

$$\frac{2+3x^2}{10\sqrt{5+x^4}}-\frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

```
[Out] -1/25*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/10*(3*x^2+2)/(x^4+5)^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1266, 837, 12, 272, 65, 213}

$$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{5\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*Sqrt[5 + x^4]) - ArcTanh[Sqrt[5 + x^4]/Sqrt[5]]/(5*Sqrt[5])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\,(5+x^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\,(5+x^2)^{3/2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} - \frac{1}{50}\text{Subst}\left(\int -\frac{10}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{5}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx, x, x^2\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{10}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx, x, x^4\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} + \frac{1}{5}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx, x, \sqrt{5+x^4}\right)\\
&= \frac{2+3x^2}{10\sqrt{5+x^4}} - \frac{\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{5\sqrt{5}}
\end{aligned}$$

Mathematica [A]
time = 0.13, size = 52, normalized size = 1.13

$$\frac{1}{50}\left(\frac{5(2+3x^2)}{\sqrt{5+x^4}} + 4\sqrt{5}\,\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*(5 + x^4)^(3/2)),x]
```

```
[Out] ((5*(2 + 3*x^2))/Sqrt[5 + x^4] + 4*Sqrt[5]*ArcTanh[(x^2 - Sqrt[5 + x^4])/Sq
rt[5]])/50
```

Maple [A]
time = 0.20, size = 40, normalized size = 0.87

method	result	size
risch	$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}$	35
default	$\frac{3x^2}{10\sqrt{x^4+5}}+\frac{1}{5\sqrt{x^4+5}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}$	40
elliptic	$\frac{3x^2}{10\sqrt{x^4+5}}+\frac{1}{5\sqrt{x^4+5}}-\frac{\sqrt{5}\,\operatorname{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{25}$	40
trager	$\frac{3x^2+2}{10\sqrt{x^4+5}}-\frac{\operatorname{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\operatorname{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right)}{25}$	45
meijerg	$\frac{\sqrt{5}\left(-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}-\sqrt{\pi}\,\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+\frac{(2-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}}{2}\right)}{25\sqrt{\pi}}+\frac{3\sqrt{5}\,x^2}{50\sqrt{1+\frac{x^4}{5}}}$	84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/10*x^2/(x^4+5)^(1/2)+1/5/(x^4+5)^(1/2)-1/25*5^(1/2)*arctanh(5^(1/2)/(x^4+
5)^(1/2))
```

Maxima [A]
time = 0.48, size = 56, normalized size = 1.22

$$\frac{3\,x^2}{10\,\sqrt{x^4+5}}+\frac{1}{50}\,\sqrt{5}\,\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{1}{5\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/10*x^2/sqrt(x^4 + 5) + 1/50*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqrt(
5) + sqrt(x^4 + 5))) + 1/5/sqrt(x^4 + 5)
```

Fricas [A]
time = 0.35, size = 61, normalized size = 1.33

$$\frac{15\,x^4+2\,\sqrt{5}\,(x^4+5)\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{x^2}\right)+5\,\sqrt{x^4+5}\,(3\,x^2+2)+75}{50\,(x^4+5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/50*(15*x^4 + 2*sqrt(5)*(x^4 + 5)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2) + 5*
sqrt(x^4 + 5)*(3*x^2 + 2) + 75)/(x^4 + 5)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 212 vs. 2(41) = 82.
time = 8.37, size = 212, normalized size = 4.61

$$\frac{2x^4\log(x^4)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{4x^4\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{2x^4\log(5)}{20\sqrt{5}\,x^4+100\sqrt{5}}+\frac{3x^2}{10\sqrt{x^4+5}}+\frac{4\sqrt{5}\,\sqrt{x^4+5}}{20\sqrt{5}\,x^4+100\sqrt{5}}+\frac{10\log(x^4)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{20\log\left(\sqrt{\frac{x^4}{5}+1}+1\right)}{20\sqrt{5}\,x^4+100\sqrt{5}}-\frac{10\log(5)}{20\sqrt{5}\,x^4+100\sqrt{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5)**(3/2),x)
```

```
[Out] 2*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 4*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 2*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 3*x**2/(10*sqrt(x**4 + 5)) + 4*sqrt(5)*sqrt(x**4 + 5)/(20*s
qrt(5)*x**4 + 100*sqrt(5)) + 10*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) -
 20*log(sqrt(x**4/5 + 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 10*log(5)/(
20*sqrt(5)*x**4 + 100*sqrt(5))
```

Giac [A]
time = 4.31, size = 61, normalized size = 1.33

$$\frac{1}{25}\,\sqrt{5}\,\log\left(x^2+\sqrt{5}-\sqrt{x^4+5}\right)-\frac{1}{25}\,\sqrt{5}\,\log\left(-x^2+\sqrt{5}+\sqrt{x^4+5}\right)+\frac{3\,x^2+2}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 1/25*sqrt(5)*log(x^2 + sqrt(5) - sqrt(x^4 + 5)) - 1/25*sqrt(5)*log(-x^2 + s
qrt(5) + sqrt(x^4 + 5)) + 1/10*(3*x^2 + 2)/sqrt(x^4 + 5)
```

Mupad [B]
time = 0.48, size = 40, normalized size = 0.87

$$\frac{1}{5\sqrt{x^4+5}}-\frac{\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{25}+\frac{3\,x^2}{10\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(x^4 + 5)^(3/2)),x)
```

```
[Out] 1/(5*(x^4 + 5)^(1/2)) - (5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/25 + (
3*x^2)/(10*(x^4 + 5)^(1/2))
```

3.49 $\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}$$

```
[Out] -3/50*arctanh(1/5*(x^4+5)^(1/2)*5^(1/2))*5^(1/2)+1/10*(3*x^2+2)/x^2/(x^4+5)
^(1/2)-2/25*(x^4+5)^(1/2)/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {1266, 837, 821, 272, 65, 213}

$$-\frac{3\tanh^{-1}\left(\frac{\sqrt{x^4+5}}{\sqrt{5}}\right)}{10\sqrt{5}}+\frac{3x^2+2}{10x^2\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x^2}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^2*Sqrt[5 + x^4]) - (2*Sqrt[5 + x^4])/(25*x^2) - (3*ArcTan
h[Sqrt[5 + x^4]/Sqrt[5]])/(10*Sqrt[5])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 821

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a + c*x^2)^(p + 1
)/(2*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2),
Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m,
p}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x^3\left(5+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\left(5+x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{1}{50}\text{Subst}\left(\int \frac{-20-15x}{x^2\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}+\frac{3}{10}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x^2}}\,dx,x,x^2\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}+\frac{3}{20}\text{Subst}\left(\int \frac{1}{x\sqrt{5+x}}\,dx,x,x^4\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}+\frac{3}{10}\text{Subst}\left(\int \frac{1}{-5+x^2}\,dx,x,\sqrt{5+x^4}\right)\\
&= \frac{2+3x^2}{10x^2\sqrt{5+x^4}}-\frac{2\sqrt{5+x^4}}{25x^2}-\frac{3\tanh^{-1}\left(\frac{\sqrt{5+x^4}}{\sqrt{5}}\right)}{10\sqrt{5}}
\end{aligned}
$$

Mathematica [A]
time = 0.14, size = 59, normalized size = 0.91

$$\frac{1}{50}\left(\frac{-10+15x^2-4x^4}{x^2\sqrt{5+x^4}}+6\sqrt{5}\tanh^{-1}\left(\frac{x^2-\sqrt{5+x^4}}{\sqrt{5}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*(5 + x^4)^(3/2)),x]
[Out] ((-10 + 15*x^2 - 4*x^4)/(x^2*Sqrt[5 + x^4]) + 6*Sqrt[5]*ArcTanh[(x^2 - Sqrt
[5 + x^4])/Sqrt[5]])/50
```

Maple [A]
time = 0.20, size = 47, normalized size = 0.72

method	result	size
risch	$-\frac{4x^4-15x^2+10}{50x^2\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	43
default	$-\frac{2x^4+5}{25x^2\sqrt{x^4+5}}+\frac{3}{10\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	47
elliptic	$-\frac{1}{5x^2\sqrt{x^4+5}}-\frac{2x^2}{25\sqrt{x^4+5}}+\frac{3}{10\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\mathrm{arctanh}\left(\frac{\sqrt{5}}{\sqrt{x^4+5}}\right)}{50}$	52
trager	$-\frac{4x^4-15x^2+10}{50x^2\sqrt{x^4+5}}-\frac{3\,\mathrm{RootOf}\left(\_Z^2-5\right)\ln\left(\frac{\mathrm{RootOf}\left(\_Z^2-5\right)+\sqrt{x^4+5}}{x^2}\right)}{50}$	53
meijerg	$-\frac{\sqrt{5}\left(1+\frac{2x^4}{5}\right)}{25x^2\sqrt{1+\frac{x^4}{5}}}+\frac{3\sqrt{5}\left(-\sqrt{\pi}+\frac{\sqrt{\pi}}{\sqrt{1+\frac{x^4}{5}}}-\sqrt{\pi}\ln\left(\frac{1}{2}+\frac{\sqrt{1+\frac{x^4}{5}}}{2}\right)+\frac{(2-2\ln(2)+4\ln(x)-\ln(5))\sqrt{\pi}}{2}\right)}{50\sqrt{\pi}}$	91

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/25/x^2*(2*x^4+5)/(x^4+5)^(1/2)+3/10/(x^4+5)^(1/2)-3/50*5^(1/2)*arctanh(5
^(1/2)/(x^4+5)^(1/2))
```

Maxima [A]
time = 0.52, size = 68, normalized size = 1.05

$$-\frac{x^2}{25\sqrt{x^4+5}}+\frac{3}{100}\sqrt{5}\log\left(-\frac{\sqrt{5}-\sqrt{x^4+5}}{\sqrt{5}+\sqrt{x^4+5}}\right)+\frac{3}{10\sqrt{x^4+5}}-\frac{\sqrt{x^4+5}}{25x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/25*x^2/sqrt(x^4 + 5) + 3/100*sqrt(5)*log(-(sqrt(5) - sqrt(x^4 + 5))/(sqr
t(5) + sqrt(x^4 + 5))) + 3/10/sqrt(x^4 + 5) - 1/25*sqrt(x^4 + 5)/x^2
```

Fricas [A]
time = 0.36, size = 77, normalized size = 1.18

$$-\frac{4\,x^{6}-3\,\sqrt{5}\,\left(x^{6}+5\,x^{2}\right)\log\left(-\frac{\sqrt{5}-\sqrt{x^{4}+5}}{x^{2}}\right)+20\,x^{2}+\left(4\,x^{4}-15\,x^{2}+10\right)\sqrt{x^{4}+5}}{50\,\left(x^{6}+5\,x^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/50*(4*x^6 - 3*sqrt(5)*(x^6 + 5*x^2)*log(-(sqrt(5) - sqrt(x^4 + 5))/x^2)
+ 20*x^2 + (4*x^4 - 15*x^2 + 10)*sqrt(x^4 + 5))/(x^6 + 5*x^2)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(61) = 122$.
time = 5.56, size = 228, normalized size = 3.51

$$\frac{3x^{4}\log\left(x^{4}\right)}{20\sqrt{5}\,x^{4}+100\sqrt{5}}-\frac{6x^{4}\log\left(\sqrt{\frac{x^{4}}{5}+1}+1\right)}{20\sqrt{5}\,x^{4}+100\sqrt{5}}-\frac{3x^{4}\log\left(5\right)}{20\sqrt{5}\,x^{4}+100\sqrt{5}}+\frac{6\sqrt{5}\,\sqrt{x^{4}+5}}{20\sqrt{5}\,x^{4}+100\sqrt{5}}+\frac{15\log\left(x^{4}\right)}{20\sqrt{5}\,x^{4}+100\sqrt{5}}-\frac{30\log\left(\sqrt{\frac{x^{4}}{5}+1}+1\right)}{20\sqrt{5}\,x^{4}+100\sqrt{5}}-\frac{15\log\left(5\right)}{20\sqrt{5}\,x^{4}+100\sqrt{5}}-\frac{2}{25\sqrt{1+\frac{5}{x^{4}}}}-\frac{1}{5x^{4}\sqrt{1+\frac{5}{x^{4}}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5)**(3/2),x)
```

```
[Out] 3*x**4*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 6*x**4*log(sqrt(x**4/5 +
 1) + 1)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 3*x**4*log(5)/(20*sqrt(5)*x**4 +
 100*sqrt(5)) + 6*sqrt(5)*sqrt(x**4 + 5)/(20*sqrt(5)*x**4 + 100*sqrt(5)) +
15*log(x**4)/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 30*log(sqrt(x**4/5 + 1) + 1)
/(20*sqrt(5)*x**4 + 100*sqrt(5)) - 15*log(5)/(20*sqrt(5)*x**4 + 100*sqrt(5)
) - 2/(25*sqrt(1 + 5/x**4)) - 1/(5*x**4*sqrt(1 + 5/x**4))
```

Giac [A]
time = 6.06, size = 82, normalized size = 1.26

$$\frac{3}{50}\,\sqrt{5}\,\log\left(-\frac{x^{2}+\sqrt{5}-\sqrt{x^{4}+5}}{x^{2}-\sqrt{5}-\sqrt{x^{4}+5}}\right)-\frac{2\,x^{2}-15}{50\,\sqrt{x^{4}+5}}+\frac{2}{5\,\left(\left(x^{2}-\sqrt{x^{4}+5}\right)^{2}-5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] 3/50*sqrt(5)*log(-(x^2 + sqrt(5) - sqrt(x^4 + 5))/(x^2 - sqrt(5) - sqrt(x^4
 + 5))) - 1/50*(2*x^2 - 15)/sqrt(x^4 + 5) + 2/5/((x^2 - sqrt(x^4 + 5))^2 -
5)
```

Mupad [B]
time = 0.54, size = 47, normalized size = 0.72

$$\frac{3}{10\sqrt{x^4+5}}-\frac{3\sqrt{5}\,\mathrm{atanh}\left(\frac{\sqrt{5}\,\sqrt{x^4+5}}{5}\right)}{50}-\frac{2\,x^4+5}{25\,x^2\,\sqrt{x^4+5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(x^4 + 5)^(3/2)),x)
```

```
[Out] 3/(10*(x^4 + 5)^(1/2)) - (3*5^(1/2)*atanh((5^(1/2)*(x^4 + 5)^(1/2))/5))/50
- (2*x^4 + 5)/(25*x^2*(x^4 + 5)^(1/2))
```

3.50 $\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=196

$$-\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}+\frac{9x\sqrt{5+x^4}}{2\left(\sqrt{5}+x^2\right)}-\frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{5+x^4}}$$

```
[Out] -1/10*x^3*(-2*x^2+15)/(x^4+5)^(1/2)-1/5*x*(x^4+5)^(1/2)+9/2*x*(x^4+5)^(1/2)
/(x^2+5^(1/2))-9/2*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arc
tan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^
2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/20*(cos(2*arctan
(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arct
an(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*(2+9*5^(1/2))*((x^4+5)/(x^2+5
^(1/2))^2)^(1/2)*5^(3/4)/(x^4+5)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1290, 1294, 1212, 226, 1210}

$$\frac{\left(2+9\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\sqrt[4]{5}\sqrt{x^4+5}}-\frac{9\sqrt[4]{5}\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\sqrt{x^4+5}}-\frac{1}{5}\sqrt{x^4+5}\,x+\frac{9\sqrt{x^4+5}\,x}{2\left(x^2+\sqrt{5}\right)}-\frac{(15-2x^2)\,x^3}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/10*(x^3*(15 - 2*x^2))/Sqrt[5 + x^4] - (x*Sqrt[5 + x^4])/5 + (9*x*Sqrt[5
+ x^4])/(2*(Sqrt[5] + x^2)) - (9*5^(1/4)*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sq
rt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(2*Sqrt[5 + x^4]) + ((
2 + 9*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(4*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1290

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(2+3x^2)}{(5+x^4)^{3/2}}\,dx &= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}+\frac{1}{10}\int \frac{x^2(45-6x^2)}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}-\frac{1}{30}\int \frac{-30-135x^2}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}-\frac{1}{2}\left(9\sqrt{5}\right)\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx-\frac{1}{2}\left(-2-9\sqrt{5}\right)\int \frac{1}{\sqrt{5+}}\\
&= -\frac{x^3(15-2x^2)}{10\sqrt{5+x^4}}-\frac{1}{5}x\sqrt{5+x^4}+\frac{9x\sqrt{5+x^4}}{2\left(\sqrt{5}+x^2\right)}-\frac{9\sqrt[4]{5}\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E}{2\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 70, normalized size = 0.36

$$\frac{x(-1+3x^2)}{\sqrt{5+x^4}}+\frac{x\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)}{\sqrt{5}}-\frac{3x^3\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)}{\sqrt{5}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(-1 + 3*x^2))/Sqrt[5 + x^4] + (x*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x
^4])/Sqrt[5] - (3*x^3*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4])/Sqrt[5]
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 168, normalized size = 0.86

method	result
meijerg	$\frac{3\sqrt{5}\,x^7\,\mathrm{hypergeom}\left(\left[\frac{3}{2},\frac{7}{4}\right],\left[\frac{11}{4}\right],-\frac{x^4}{5}\right)}{175}+\frac{2\sqrt{5}\,x^5\,\mathrm{hypergeom}\left(\left[\frac{5}{4},\frac{3}{2}\right],\left[\frac{9}{4}\right],-\frac{x^4}{5}\right)}{125}$
risch	$-\frac{x(3x^2+2)}{2\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{}}{5}\right.\right.}{10\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\left(\frac{3}{4}x^3+\frac{1}{2}x\right)}{\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{10\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$-\frac{3x^3}{2\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{}}{5}\right.\right.}{10\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x^3/(x^4+5)^(1/2)+9/10*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*
(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2
))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))-x/(x^4+5)^(1/2)+1
/25*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^
2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**C**] Result contains complex when optimal does not.
time = 2.42, size = 75, normalized size = 0.38

$$\frac{3\sqrt{5}\,x^7\Gamma\left(\frac{7}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{3}{2},\frac{7}{4}\\\frac{11}{4}\end{matrix}\middle|\,\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{11}{4}\right)}+\frac{\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right)\,{}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\\frac{9}{4}\end{matrix}\middle|\,\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{9}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**7*gamma(7/4)*hyper((3/2, 7/4), (11/4,), x**4*exp_polar(I*pi)/5
)/(100*gamma(11/4)) + sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**
4*exp_polar(I*pi)/5)/(50*gamma(9/4))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^4\,\left(3\,x^2+2\right)}{{\left(x^4+5\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] int((x^4*(3*x^2 + 2))/(x^4 + 5)^(3/2), x)
```

3.51 $\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{x(15-2x^2)}{10\sqrt{5+x^4}}-\frac{x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}+\frac{\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}}-\frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}\right.}{}$$

```
[Out] -1/10*x*(-2*x^2+15)/(x^4+5)^(1/2)-1/5*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+1/5*5^(
1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*El
lipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x
^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)-1/20*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1
/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2
*2^(1/2))*(2-3*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/
4)/(x^4+5)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1290, 1212, 226, 1210}

$$-\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{4\,5^{3/4}\sqrt{x^4+5}}+\frac{\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{x^4+5}}-\frac{\sqrt{x^4+5}\,x}{5\left(x^2+\sqrt{5}\right)}-\frac{(15-2x^2)\,x}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] -1/10*(x*(15 - 2*x^2))/Sqrt[5 + x^4] - (x*Sqrt[5 + x^4])/(5*(Sqrt[5] + x^2)
) + ((Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x
/5^(1/4)], 1/2])/(5^(3/4)*Sqrt[5 + x^4]) - ((2 - 3*Sqrt[5])*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(4*
5^(3/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1290

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(2+3x^2)}{(5+x^4)^{3/2}}\,dx &= -\frac{x(15-2x^2)}{10\sqrt{5+x^4}}+\frac{1}{10}\int \frac{15-2x^2}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x(15-2x^2)}{10\sqrt{5+x^4}}+\frac{\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{\sqrt{5}}+\frac{1}{10}\left(15-2\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx\\
&= -\frac{x(15-2x^2)}{10\sqrt{5+x^4}}-\frac{x\sqrt{5+x^4}}{5\left(\sqrt{5}+x^2\right)}+\frac{\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5^{3/4}\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.03, size = 68, normalized size = 0.38

$$\frac{1}{150}x\left(-\frac{225}{\sqrt{5+x^4}}+45\sqrt{5}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+4\sqrt{5}\,x^2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(2 + 3*x^2))/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(-225/Sqrt[5 + x^4] + 45*Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x
^4] + 4*Sqrt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4]))/150
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 168, normalized size = 0.95

method	result
meijerg	$\frac{3\sqrt{5}\, x^5 \operatorname{hypergeom}\left(\left[\frac{5}{4},\frac{3}{2}\right],\left[\frac{9}{4}\right],-\frac{x^4}{5}\right)}{125}+\frac{2\sqrt{5}\, x^3 \operatorname{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],-\frac{x^4}{5}\right)}{75}$
risch	$\frac{x(2x^2-15)}{10\sqrt{x^4+5}}-\frac{i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\operatorname{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{}}}{5}\right.\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\left(-\frac{1}{10}x^3+\frac{3}{4}x\right)}{\sqrt{x^4+5}}-\frac{i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\operatorname{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{}}{5}\right.\right.}{25\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$-\frac{3x}{2\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\operatorname{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}+\frac{x^3}{5\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x/(x^4+5)^(1/2)+3/50*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1
/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(
1/2))^(1/2),I)+1/5*x^3/(x^4+5)^(1/2)-1/25*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/
2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(
1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 2.24, size = 75, normalized size = 0.42

$$\frac{3\sqrt{5}\,x^5\Gamma\left(\frac{5}{4}\right){}_2F_1\left(\begin{matrix}\frac{5}{4},\frac{3}{2}\\ \frac{9}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{9}{4}\right)}+\frac{\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\begin{matrix}\frac{3}{4},\frac{3}{2}\\ \frac{7}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{7}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x**5*gamma(5/4)*hyper((5/4, 3/2), (9/4,), x**4*exp_polar(I*pi)/5)
/(100*gamma(9/4)) + sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*
exp_polar(I*pi)/5)/(50*gamma(7/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(3\,x^2+2\right)}{{\left(x^4+5\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(3/2),x)
```

```
[Out] int((x^2*(3*x^2 + 2))/(x^4 + 5)^(3/2), x)
```

3.52 $\int \frac{2+3x^2}{\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=180

$$\frac{x(2+3x^2)}{10\sqrt{5+x^4}}-\frac{3x\sqrt{5+x^4}}{10\left(\sqrt{5}+x^2\right)}+\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{5+x^4}}+\frac{\left(2-3\sqrt{5}\right)\left(\sqrt{5}\right.}{}$$

```
[Out] 1/10*x*(3*x^2+2)/(x^4+5)^(1/2)-3/10*x*(x^4+5)^(1/2)/(x^2+5^(1/2))+3/10*5^(1
/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*Ell
ipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^
2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+1/100*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1
/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2
*2^(1/2))*(2-3*5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(3/
4)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {1193, 1212, 226, 1210}

$$\frac{\left(2-3\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\sqrt[4]{5}\sqrt{x^4+5}}+\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\,5^{3/4}\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}\,x}{10\left(x^2+\sqrt{5}\right)}+\frac{\left(3x^2+2\right)x}{10\sqrt{x^4+5}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(2 + 3*x^2))/(10*Sqrt[5 + x^4]) - (3*x*Sqrt[5 + x^4])/(10*(Sqrt[5] + x^2
)) + (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTa
n[x/5^(1/4)], 1/2])/(2*5^(3/4)*Sqrt[5 + x^4]) + ((2 - 3*Sqrt[5])*(Sqrt[5] +
 x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2]
)/(20*5^(1/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
```

```
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{\left(5+x^4\right)^{3/2}}\,dx &= \frac{x(2+3x^2)}{10\sqrt{5+x^4}} - \frac{1}{10}\int \frac{-2+3x^2}{\sqrt{5+x^4}}\,dx \\
&= \frac{x(2+3x^2)}{10\sqrt{5+x^4}} + \frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{2\sqrt{5}} - \frac{1}{10}\left(-2+3\sqrt{5}\right)\int \frac{1}{\sqrt{5+x^4}}\,dx \\
&= \frac{x(2+3x^2)}{10\sqrt{5+x^4}} - \frac{3x\sqrt{5+x^4}}{10\left(\sqrt{5}+x^2\right)} + \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{2\ 5^{3/4}\sqrt{5+x^4}}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 10.02, size = 66, normalized size = 0.37

$$\frac{1}{25}x\left(\frac{5}{\sqrt{5+x^4}}+\sqrt{5}\ {}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-\frac{x^4}{5}\right)+\sqrt{5}\,x^2\,{}_2F_1\left(\frac{3}{4},\frac{3}{2};\frac{7}{4};-\frac{x^4}{5}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(5 + x^4)^(3/2),x]
```

```
[Out] (x*(5/Sqrt[5 + x^4] + Sqrt[5]*Hypergeometric2F1[1/4, 1/2, 5/4, -1/5*x^4] +
Sqrt[5]*x^2*Hypergeometric2F1[3/4, 3/2, 7/4, -1/5*x^4]))/25
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 168, normalized size = 0.93

method	result
meijerg	$\frac{2\sqrt{5}\, x \,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-\frac{x^4}{5}\right)}{25}+\frac{\sqrt{5}\, x^3 \,\mathrm{hypergeom}\left(\left[\frac{3}{4},\frac{3}{2}\right],\left[\frac{7}{4}\right],-\frac{x^4}{5}\right)}{25}$
risch	$\frac{x(3x^2+2)}{10\sqrt{x^4+5}}-\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\left(-\frac{3}{20}x^3-\frac{1}{10}x\right)}{\sqrt{x^4+5}}-\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}}{}\right.\right.}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{3x^3}{10\sqrt{x^4+5}}-\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\,\sqrt{i}}{5}\right.\right.}{50\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/10*x^3/(x^4+5)^(1/2)-3/50*I/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*
(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2
))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I))+1/5*x/(x^4+5)^(1/
2)+1/125*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/
2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [C] Result contains complex when optimal does not.
time = 2.13, size = 73, normalized size = 0.41

$$\frac{3\sqrt{5}\,x^3\Gamma\left(\frac{3}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{3}{4},\frac{3}{2}\\\frac{7}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{7}{4}\right)}+\frac{\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_2F_1\left(\left.\begin{matrix}\frac{1}{4},\frac{3}{2}\\\frac{5}{4}\end{matrix}\right|\frac{x^4e^{i\pi}}{5}\right)}{50\Gamma\left(\frac{5}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x**2+2)/(x**4+5)**(3/2),x)

[Out] 3*sqrt(5)*x**3*gamma(3/4)*hyper((3/4, 3/2), (7/4,), x**4*exp_polar(I*pi)/5)/(100*gamma(7/4)) + sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi)/5)/(50*gamma(5/4))

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((3*x^2+2)/(x^4+5)^(3/2),x, algorithm="giac")

[Out] integrate((3*x^2 + 2)/(x^4 + 5)^(3/2), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2+2}{{\left(x^4+5\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((3*x^2 + 2)/(x^4 + 5)^(3/2),x)

[Out] int((3*x^2 + 2)/(x^4 + 5)^(3/2), x)

3.53 $\int \frac{2+3x^2}{x^2(5+x^4)^{3/2}}\,dx$

Optimal. Leaf size=196

$$\frac{2+3x^2}{10x\sqrt{5+x^4}}-\frac{3\sqrt{5+x^4}}{25x}+\frac{3x\sqrt{5+x^4}}{25\left(\sqrt{5}+x^2\right)}-\frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\;5^{3/4}\sqrt{5+x^4}}+\frac{3\Big(2}{}$$

```
[Out] 1/10*(3*x^2+2)/x/(x^4+5)^(1/2)-3/25*(x^4+5)^(1/2)/x+3/25*x*(x^4+5)^(1/2)/(x
^2+5^(1/2))-3/25*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arcta
n(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(x^2+
5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)+3/100*5^(1/4)*(cos(2
*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticF(sin
(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(2+5^(1/2))*(x^2+5^(1/2))*((x^4+5)/(
x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1292, 1296, 1212, 226, 1210}

$$\frac{3\left(2+\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{20\;5^{3/4}\sqrt{x^4+5}}-\frac{3\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{5\;5^{3/4}\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}}{25x}+\frac{3\sqrt{x^4+5}\,x}{25\left(x^2+\sqrt{5}\right)}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x*Sqrt[5 + x^4]) - (3*Sqrt[5 + x^4])/(25*x) + (3*x*Sqrt[5 +
 x^4])/(25*(Sqrt[5] + x^2)) - (3*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] +
x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(5*5^(3/4)*Sqrt[5 + x^4]) + (3
*(2 + Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticF[
2*ArcTan[x/5^(1/4)], 1/2])/(20*5^(3/4)*Sqrt[5 + x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1292

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(-(f*x)^(m + 1))*(a + c*x^4)^(p + 1)*((d + e*x^2)/(4*a*f*(p
 + 1))), x] + Dist[1/(4*a*(p + 1)), Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d*
(m + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a,
c, d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || Intege
rQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x^2\left(5+x^4\right)^{3/2}}\,dx &= \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{1}{10}\int \frac{-6-3x^2}{x^2\sqrt{5+x^4}}\,dx \\
&= \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{3\sqrt{5+x^4}}{25x} + \frac{1}{50}\int \frac{15+6x^2}{\sqrt{5+x^4}}\,dx \\
&= \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{3\sqrt{5+x^4}}{25x} - \frac{3\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{5\sqrt{5}} + \frac{1}{50}\left(3\left(5+2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{5+x^4}}\,dx \\
&= \frac{2+3x^2}{10x\sqrt{5+x^4}} - \frac{3\sqrt{5+x^4}}{25x} + \frac{3x\sqrt{5+x^4}}{25\left(\sqrt{5}+x^2\right)} - \frac{3\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\Big(2}{5\,5^{3/4}\sqrt{5+x^4}}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 108, normalized size = 0.55

$$-\frac{20-15x^2+6x^4+6(-1)^{3/4}\sqrt[4]{5}\,x\sqrt{5+x^4}\,E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)+3\sqrt[4]{-5}\left(-2i+\sqrt{5}\right)x\sqrt{5+x^4}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)}{50x\sqrt{5+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^2*(5 + x^4)^(3/2)),x]
```

```
[Out] -1/50*(20 - 15*x^2 + 6*x^4 + 6*(-1)^(3/4)*5^(1/4)*x*Sqrt[5 + x^4]*EllipticE
[I*ArcSinh[(-1/5)^(1/4)*x], -1] + 3*(-5)^(1/4)*(-2*I + Sqrt[5])*x*Sqrt[5 +
x^4]*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/(x*Sqrt[5 + x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.18, size = 180, normalized size = 0.92

method	result
meijerg	$-\frac{2\sqrt{5}\,\mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{3}{2}\right],\left[\frac{3}{4}\right],-\frac{x^4}{5}\right)}{25x}+\frac{3\sqrt{5}\,x\,\mathrm{hypergeom}\left(\left[\frac{1}{4},\frac{3}{2}\right],\left[\frac{5}{4}\right],-\frac{x^4}{5}\right)}{25}$
risch	$-\frac{6x^4-15x^2+20}{50x\sqrt{x^4+5}}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{5}\ \cdots}{}\right.\right.}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
elliptic	$-\frac{2\left(\frac{1}{50}x^3-\frac{3}{20}x\right)}{\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{25x}+\frac{3i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{Ell}\cdots\right.}{125\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$
default	$\frac{3x}{10\sqrt{x^4+5}}+\frac{3\sqrt{5}\,\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}-\frac{x^3}{25\sqrt{x^4+5}}\cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/10*x/(x^4+5)^(1/2)+3/250*5^(1/2)/(I*5^(1/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(
1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*EllipticF(1/5*x*5^(1/2)*(I*5^
(1/2))^(1/2),I)-1/25*x^3/(x^4+5)^(1/2)-2/25*(x^4+5)^(1/2)/x+3/125*I/(I*5^(1
/2))^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1
/2)*(EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(
I*5^(1/2))^(1/2),I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.01, size = 75, normalized size = 0.38

$$\frac{3\sqrt{5}\,x\Gamma\left(\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}\frac{1}{4},\frac{3}{2}\\ \frac{5}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100\Gamma\left(\frac{5}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_{2}F_{1}\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x\Gamma\left(\frac{3}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*x*gamma(1/4)*hyper((1/4, 3/2), (5/4,), x**4*exp_polar(I*pi)/5)/(1
00*gamma(5/4)) + sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_po
lar(I*pi)/5)/(50*x*gamma(3/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^2), x)
```

Mupad [B]
time = 0.46, size = 48, normalized size = 0.24

$$\frac{3\,\sqrt{5}\,x\,{}_2\mathrm{F}_1\left(\frac{1}{4},\frac{3}{2};\ \frac{5}{4};\ -\frac{x^4}{5}\right)}{25}-\frac{2\,{\left(\frac{5}{x^4}+1\right)}^{3/2}\,{}_2\mathrm{F}_1\left(\frac{3}{2},\frac{7}{4};\ \frac{11}{4};\ -\frac{5}{x^4}\right)}{7\,x\,{\left(x^4+5\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^2*(x^4 + 5)^(3/2)),x)
```

```
[Out] (3*5^(1/2)*x*hypergeom([1/4, 3/2], 5/4, -x^4/5))/25 - (2*(5/x^4 + 1)^(3/2)*
hypergeom([3/2, 7/4], 11/4, -5/x^4))/(7*x*(x^4 + 5)^(3/2))
```

3.54 $\int \frac{2+3x^2}{x^4\left(5+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=214

$$\frac{2+3x^2}{10x^3\sqrt{5+x^4}}-\frac{\sqrt{5+x^4}}{15x^3}-\frac{9\sqrt{5+x^4}}{50x}+\frac{9x\sqrt{5+x^4}}{50\left(\sqrt{5}+x^2\right)}-\frac{9\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+x^4}{\left(\sqrt{5}+x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{x}{\sqrt[4]{5}}\right)\right.}{10\;5^{3/4}\sqrt{5+x^4}}$$

```
[Out] 1/10*(3*x^2+2)/x^3/(x^4+5)^(1/2)-1/15*(x^4+5)^(1/2)/x^3-9/50*(x^4+5)^(1/2)/
x+9/50*x*(x^4+5)^(1/2)/(x^2+5^(1/2))-9/50*5^(1/4)*(cos(2*arctan(1/5*x*5^(3/
4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4)))*EllipticE(sin(2*arctan(1/5*x*5^(
3/4))),1/2*2^(1/2))*(x^2+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)/(x^4+5)^(
1/2)+1/300*(cos(2*arctan(1/5*x*5^(3/4)))^2)^(1/2)/cos(2*arctan(1/5*x*5^(3/4
)))*EllipticF(sin(2*arctan(1/5*x*5^(3/4))),1/2*2^(1/2))*(27-2*5^(1/2))*(x^2
+5^(1/2))*((x^4+5)/(x^2+5^(1/2))^2)^(1/2)*5^(1/4)/(x^4+5)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {1292, 1296, 1212, 226, 1210}

$$\frac{\left(27-2\sqrt{5}\right)\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{60\;5^{3/4}\sqrt{x^4+5}}-\frac{9\left(x^2+\sqrt{5}\right)\sqrt{\frac{x^4+5}{\left(x^2+\sqrt{5}\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{x}{\sqrt[4]{5}}\right)|\frac{1}{2}\right)}{10\;5^{3/4}\sqrt{x^4+5}}-\frac{9\sqrt{x^4+5}}{50x}-\frac{\sqrt{x^4+5}}{15x^3}+\frac{9\sqrt{x^4+5}\,x}{50\left(x^2+\sqrt{5}\right)}+\frac{3x^2+2}{10\sqrt{x^4+5}\,x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

```
[Out] (2 + 3*x^2)/(10*x^3*Sqrt[5 + x^4]) - Sqrt[5 + x^4]/(15*x^3) - (9*Sqrt[5 + x
^4])/(50*x) + (9*x*Sqrt[5 + x^4])/(50*(Sqrt[5] + x^2)) - (9*(Sqrt[5] + x^2)
*Sqrt[(5 + x^4)/(Sqrt[5] + x^2)^2]*EllipticE[2*ArcTan[x/5^(1/4)], 1/2])/(10
*5^(3/4)*Sqrt[5 + x^4]) + ((27 - 2*Sqrt[5])*(Sqrt[5] + x^2)*Sqrt[(5 + x^4)/
(Sqrt[5] + x^2)^2]*EllipticF[2*ArcTan[x/5^(1/4)], 1/2])/(60*5^(3/4)*Sqrt[5
+ x^4])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1210

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + c*x^4]/(a*(1 + q^2*x^2))), x] + Simp[d*
```

```
(1 + q^2*x^2)*(Sqrt[(a + c*x^4)/(a*(1 + q^2*x^2)^2)]/(q*Sqrt[a + c*x^4]))*E
llipticE[2*ArcTan[q*x], 1/2], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, c, d, e
}, x] && PosQ[c/a]
```

Rule 1212

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + c*x^4], x], x] - Dist[e/q, I
nt[(1 - q*x^2)/Sqrt[a + c*x^4], x], x] /; NeQ[e + d*q, 0]] /; FreeQ[{a, c,
d, e}, x] && PosQ[c/a]
```

Rule 1292

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[(-(f*x)^(m + 1))*(a + c*x^4)^(p + 1)*((d + e*x^2)/(4*a*f*(p
 + 1))), x] + Dist[1/(4*a*(p + 1)), Int[(f*x)^m*(a + c*x^4)^(p + 1)*Simp[d*
(m + 4*(p + 1) + 1) + e*(m + 2*(2*p + 3) + 1)*x^2, x], x], x] /; FreeQ[{a,
c, d, e, f, m}, x] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] || Intege
rQ[m])
```

Rule 1296

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[d*(f*x)^(m + 1)*((a + c*x^4)^(p + 1)/(a*f*(m + 1))), x] + D
ist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + c*x^4)^p*(a*e*(m + 1) - c*d*(
m + 4*p + 5)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && LtQ[m, -1] &&
IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{2+3x^2}{x^4(5+x^4)^{3/2}}\,dx = \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{1}{10}\int \frac{-10-9x^2}{x^4\sqrt{5+x^4}}\,dx$$
$$= \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} + \frac{1}{150}\int \frac{135-10x^2}{x^2\sqrt{5+x^4}}\,dx$$
$$= \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} - \frac{9\sqrt{5+x^4}}{50x} - \frac{1}{750}\int \frac{50-135x^2}{\sqrt{5+x^4}}\,dx$$
$$= \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} - \frac{9\sqrt{5+x^4}}{50x} - \frac{9\int \frac{1-\frac{x^2}{\sqrt{5}}}{\sqrt{5+x^4}}\,dx}{10\sqrt{5}} - \frac{1}{150}\left(10-27\sqrt{5}\right)\int$$
$$= \frac{2+3x^2}{10x^3\sqrt{5+x^4}} - \frac{\sqrt{5+x^4}}{15x^3} - \frac{9\sqrt{5+x^4}}{50x} + \frac{9x\sqrt{5+x^4}}{50\left(\sqrt{5}+x^2\right)} - \frac{9\left(\sqrt{5}+x^2\right)\sqrt{\frac{5+}{\left(\sqrt{5}\right.}}}{10\ 5}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.10, size = 119, normalized size = 0.56

$$-\frac{20+90x^2+10x^4+27x^6+27(-1)^{3/4}\sqrt[4]{5}\,x^3\sqrt{5+x^4}\,E\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)-\sqrt[4]{-5}\left(27i+2\sqrt{5}\right)x^3\sqrt{5+x^4}\,F\left(i\sinh^{-1}\left(\sqrt[4]{-\frac{1}{5}}\,x\right)\middle|-1\right)}{150x^3\sqrt{5+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^4*(5 + x^4)^(3/2)),x]
```

```
[Out] -1/150*(20 + 90*x^2 + 10*x^4 + 27*x^6 + 27*(-1)^(3/4)*5^(1/4)*x^3*Sqrt[5 +
x^4]*EllipticE[I*ArcSinh[(-1/5)^(1/4)*x], -1] - (-5)^(1/4)*(27*I + 2*Sqrt[5
])*x^3*Sqrt[5 + x^4]*EllipticF[I*ArcSinh[(-1/5)^(1/4)*x], -1])/(x^3*Sqrt[5
+ x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 192, normalized size = 0.90

method	result
meijerg	$-\frac{2\sqrt{5}\ \mathrm{hypergeom}\left(\left[-\frac{3}{4},\frac{3}{2}\right],\left[\frac{1}{4}\right],-\frac{x^4}{5}\right)}{75x^3}-\frac{3\sqrt{5}\ \mathrm{hypergeom}\left(\left[-\frac{1}{4},\frac{3}{2}\right],\left[\frac{3}{4}\right],-\frac{x^4}{5}\right)}{25x}$
risch	$-\frac{27x^6+10x^4+90x^2+20}{150x^3\sqrt{x^4+5}}+\frac{9i\sqrt{25-5i\sqrt{5}\ x^2}\ \sqrt{25+5i\sqrt{5}\ x^2}\ \left(\mathrm{EllipticF}\left(\frac{x\sqrt{5}\ \sqrt{i\sqrt{5}}}{5},i\right)-\mathrm{EllipticE}\left(\frac{x\sqrt{}}{}\right.\right.}{250\sqrt{i\sqrt{5}}\ \sqrt{x^4+5}}$

elliptic	$-\frac{2\left(\frac{3}{100}x^3+\frac{1}{50}x\right)}{\sqrt{x^4+5}}-\frac{2\sqrt{x^4+5}}{75x^3}-\frac{3\sqrt{x^4+5}}{25x}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{5}}{}\right.\right.}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+}}$
default	$-\frac{3x^3}{50\sqrt{x^4+5}}-\frac{3\sqrt{x^4+5}}{25x}+\frac{9i\sqrt{25-5i\sqrt{5}\,x^2}\,\sqrt{25+5i\sqrt{5}\,x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{5}\,\sqrt{i\sqrt{5}}}{5},i\right)-\text{El}\right.}{250\sqrt{i\sqrt{5}}\,\sqrt{x^4+5}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/50*x^3/(x^4+5)^(1/2)-3/25*(x^4+5)^(1/2)/x+9/250*I/(I*5^(1/2))^(1/2)*(25-
5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*(EllipticF(
1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)-EllipticE(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2
),I))-1/25*x/(x^4+5)^(1/2)-2/75*(x^4+5)^(1/2)/x^3-1/375*5^(1/2)/(I*5^(1/2))
^(1/2)*(25-5*I*5^(1/2)*x^2)^(1/2)*(25+5*I*5^(1/2)*x^2)^(1/2)/(x^4+5)^(1/2)*
EllipticF(1/5*x*5^(1/2)*(I*5^(1/2))^(1/2),I)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [C] Result contains complex when optimal does not.
time = 3.45, size = 80, normalized size = 0.37

$$\frac{3\sqrt{5}\,\Gamma\left(-\frac{1}{4}\right){}_2F_1\left(\begin{matrix}-\frac{1}{4},\frac{3}{2}\\ \frac{3}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{100x\Gamma\left(\frac{3}{4}\right)}+\frac{\sqrt{5}\,\Gamma\left(-\frac{3}{4}\right){}_2F_1\left(\begin{matrix}-\frac{3}{4},\frac{3}{2}\\ \frac{1}{4}\end{matrix}\middle|\frac{x^4e^{i\pi}}{5}\right)}{50x^3\Gamma\left(\frac{1}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5)**(3/2),x)
```

```
[Out] 3*sqrt(5)*gamma(-1/4)*hyper((-1/4, 3/2), (3/4,), x**4*exp_polar(I*pi)/5)/(1
00*x*gamma(3/4)) + sqrt(5)*gamma(-3/4)*hyper((-3/4, 3/2), (1/4,), x**4*exp_
polar(I*pi)/5)/(50*x**3*gamma(1/4))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5)^(3/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^4\,{\left(x^4+5\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(3/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^4*(x^4 + 5)^(3/2)), x)
```

3.55 $\int (fx)^m \left(d+ex^2\right) \left(1+2x^2+x^4\right)^5 \, dx$

Optimal. Leaf size=269

$$\frac{d(fx)^{1+m}}{f(1+m)}+\frac{(10d+e)(fx)^{3+m}}{f^3(3+m)}+\frac{5(9d+2e)(fx)^{5+m}}{f^5(5+m)}+\frac{15(8d+3e)(fx)^{7+m}}{f^7(7+m)}+\frac{30(7d+4e)(fx)^{9+m}}{f^9(9+m)}+\frac{42(6d+}{f^{11}(}$$

```
[Out] d*(f*x)^(1+m)/f/(1+m)+(10*d+e)*(f*x)^(3+m)/f^3/(3+m)+5*(9*d+2*e)*(f*x)^(5+m
)/f^5/(5+m)+15*(8*d+3*e)*(f*x)^(7+m)/f^7/(7+m)+30*(7*d+4*e)*(f*x)^(9+m)/f^9
/(9+m)+42*(6*d+5*e)*(f*x)^(11+m)/f^11/(11+m)+42*(5*d+6*e)*(f*x)^(13+m)/f^13
/(13+m)+30*(4*d+7*e)*(f*x)^(15+m)/f^15/(15+m)+15*(3*d+8*e)*(f*x)^(17+m)/f^1
7/(17+m)+5*(2*d+9*e)*(f*x)^(19+m)/f^19/(19+m)+(d+10*e)*(f*x)^(21+m)/f^21/(2
1+m)+e*(f*x)^(23+m)/f^23/(23+m)
```

Rubi [A]

time = 0.11, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {28, 459}

$$\frac{(d+10e)(fx)^{m+21}}{f^{21}(m+21)}+\frac{5(2d+9e)(fx)^{m+19}}{f^{19}(m+19)}+\frac{15(3d+8e)(fx)^{m+17}}{f^{17}(m+17)}+\frac{30(4d+7e)(fx)^{m+15}}{f^{15}(m+15)}+\frac{42(5d+6e)(fx)^{m+13}}{f^{13}(m+13)}+\frac{42(6d+5e)(fx)^{m+11}}{f^{11}(m+11)}+\frac{30(7d+4e)(fx)^{m+9}}{f^9(m+9)}+\frac{15(8d+3e)(fx)^{m+7}}{f^7(m+7)}+\frac{5(9d+2e)(fx)^{m+5}}{f^5(m+5)}+\frac{(10d+e)(fx)^{m+3}}{f^3(m+3)}+\frac{d(fx)^{m+1}}{f(m+1)}+\frac{e(fx)^{m+23}}{f^{23}(m+23)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*(f*x)^(1 + m))/(f*(1 + m)) + ((10*d + e)*(f*x)^(3 + m))/(f^3*(3 + m)) +
(5*(9*d + 2*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (15*(8*d + 3*e)*(f*x)^(7 + m)
)/(f^7*(7 + m)) + (30*(7*d + 4*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (42*(6*d +
 5*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (42*(5*d + 6*e)*(f*x)^(13 + m))/(f^
13*(13 + m)) + (30*(4*d + 7*e)*(f*x)^(15 + m))/(f^15*(15 + m)) + (15*(3*d +
 8*e)*(f*x)^(17 + m))/(f^17*(17 + m)) + (5*(2*d + 9*e)*(f*x)^(19 + m))/(f^1
9*(19 + m)) + ((d + 10*e)*(f*x)^(21 + m))/(f^21*(21 + m)) + (e*(f*x)^(23 +
m))/(f^23*(23 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right)\left(1+2x^2+x^4\right)^5 \, dx = \int (fx)^m \left(1+x^2\right)^{10}\left(d+ex^2\right) \, dx$$
$$= \int \left(d(fx)^m + \frac{(10d+e)(fx)^{2+m}}{f^2} + \frac{5(9d+2e)(fx)^{4+m}}{f^4} + \frac{15(8d+}{}\right.$$
$$= \frac{d(fx)^{1+m}}{f(1+m)} + \frac{(10d+e)(fx)^{3+m}}{f^3(3+m)} + \frac{5(9d+2e)(fx)^{5+m}}{f^5(5+m)} + \frac{15(8d+3e}{f^7(7}$$

Mathematica [A]
time = 0.71, size = 189, normalized size = 0.70

$$x(fx)^m\left(\frac{d}{1+m}+\frac{(10d+e)x^2}{3+m}+\frac{5(9d+2e)x^4}{5+m}+\frac{15(8d+3e)x^6}{7+m}+\frac{30(7d+4e)x^8}{9+m}+\frac{42(6d+5e)x^{10}}{11+m}+\frac{42(5d+6e)x^{12}}{13+m}+\frac{30(4d+7e)x^{14}}{15+m}+\frac{15(3d+8e)x^{16}}{17+m}+\frac{5(2d+9e)x^{18}}{19+m}+\frac{(d+10e)x^{20}}{21+m}+\frac{ex^{22}}{23+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*(d/(1 + m) + ((10*d + e)*x^2)/(3 + m) + (5*(9*d + 2*e)*x^4)/(5 +
m) + (15*(8*d + 3*e)*x^6)/(7 + m) + (30*(7*d + 4*e)*x^8)/(9 + m) + (42*(6*d
 + 5*e)*x^10)/(11 + m) + (42*(5*d + 6*e)*x^12)/(13 + m) + (30*(4*d + 7*e)*x
^14)/(15 + m) + (15*(3*d + 8*e)*x^16)/(17 + m) + (5*(2*d + 9*e)*x^18)/(19 +
 m) + ((d + 10*e)*x^20)/(21 + m) + (e*x^22)/(23 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2294 vs. 2(269) = 538.
time = 0.04, size = 2295, normalized size = 8.53

method	result	size
gosper	Expression too large to display	2295
risch	Expression too large to display	2295

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (f*x)^m*(e*m^11*x^22+121*e*m^10*x^22+d*m^11*x^20+10*e*m^11*x^20+6435*e*m^9*
x^22+123*d*m^10*x^20+1230*e*m^10*x^20+197835*e*m^8*x^22+10*d*m^11*x^18+6635
*d*m^9*x^20+45*e*m^11*x^18+66350*e*m^9*x^20+3889578*e*m^7*x^22+1250*d*m^10*
x^18+206505*d*m^8*x^20+5625*e*m^10*x^18+2065050*e*m^8*x^20+51069018*e*m^6*x
^22+45*d*m^11*x^16+68430*d*m^9*x^18+4103178*d*m^7*x^20+120*e*m^11*x^16+3079
35*e*m^9*x^18+41031780*e*m^7*x^20+453714470*e*m^5*x^22+5715*d*m^10*x^16+215
8230*d*m^8*x^18+54362574*d*m^6*x^20+15240*e*m^10*x^16+9712035*e*m^8*x^18+54
3625740*e*m^6*x^20+2702025590*e*m^4*x^22+120*d*m^11*x^14+317655*d*m^9*x^16+
```

43391460*d*m^7*x^18+486687830*d*m^5*x^20+210*e*m^11*x^14+847080*e*m^9*x^16+
195261570*e*m^7*x^18+4866878300*e*m^5*x^20+10431670821*e*m^3*x^22+15480*d*m
^10*x^14+10162665*d*m^8*x^16+580855380*d*m^6*x^18+2917013970*d*m^4*x^20+270
90*e*m^10*x^14+27100440*e*m^8*x^16+2613849210*e*m^6*x^18+29170139700*e*m^4*
x^20+24372200061*e*m^2*x^22+210*d*m^11*x^12+873960*d*m^9*x^14+207024930*d*m
^7*x^16+5246766620*d*m^5*x^18+11320966021*d*m^3*x^20+252*e*m^11*x^12+152943
0*e*m^9*x^14+552066480*e*m^7*x^16+23610449790*e*m^5*x^18+113209660210*e*m^3
*x^20+29985521895*e*m*x^22+27510*d*m^10*x^12+28391400*d*m^8*x^14+2804395230
*d*m^6*x^16+31686018220*d*m^4*x^18+26560342503*d*m^2*x^20+33012*e*m^10*x^12
+49684950*e*m^8*x^14+7478387280*e*m^6*x^16+142587081990*e*m^4*x^18+26560342
5030*e*m^2*x^20+13749310575*e*x^22+252*d*m^11*x^10+1578150*d*m^9*x^12+58690
2960*d*m^7*x^14+25598865870*d*m^5*x^16+123748247730*d*m^3*x^18+32778930735*
d*m*x^20+210*e*m^11*x^10+1893780*e*m^9*x^12+1027080180*e*m^7*x^14+682636423
20*e*m^5*x^16+556867114785*e*m^3*x^18+327789307350*e*m*x^20+33516*d*m^10*x^
10+52110450*d*m^8*x^12+8059973040*d*m^6*x^14+156004908210*d*m^4*x^16+291789
582570*d*m^2*x^18+15058768725*d*x^20+27930*e*m^10*x^10+62532540*e*m^8*x^12+
14104952820*e*m^6*x^14+416013088560*e*m^4*x^16+1313053121565*e*m^2*x^18+150
587687250*e*x^20+210*d*m^11*x^8+1954260*d*m^9*x^10+1094918580*d*m^7*x^12+74
496630480*d*m^5*x^14+613938233025*d*m^3*x^16+361459164150*d*m*x^18+120*e*m^
11*x^8+1628550*e*m^9*x^10+1313902296*e*m^7*x^12+130369103340*e*m^5*x^14+163
7168621400*e*m^3*x^16+1626566238675*e*m*x^18+28350*d*m^10*x^8+65654820*d*m^
8*x^10+15277213980*d*m^6*x^12+459045550800*d*m^4*x^14+1456578341055*d*m^2*x
^16+166439022750*d*x^18+16200*e*m^10*x^8+54712350*e*m^8*x^10+18332656776*e*
m^6*x^12+803329713900*e*m^4*x^14+3884208909480*e*m^2*x^16+748975602375*e*x^
18+120*d*m^11*x^6+1680630*d*m^9*x^8+1404622296*d*m^7*x^10+143339613900*d*m^
5*x^12+1823707864920*d*m^3*x^14+1812743750475*d*m*x^16+45*e*m^11*x^6+960360
*e*m^9*x^8+1170518580*e*m^7*x^10+172007536680*e*m^5*x^12+3191488763610*e*m^
3*x^14+4833983334600*e*m*x^16+16440*d*m^10*x^6+57500730*d*m^8*x^8+199625413
68*d*m^6*x^10+895451283300*d*m^4*x^12+4360457499480*d*m^2*x^14+837090379125
*d*x^16+6165*e*m^10*x^6+32857560*e*m^8*x^8+16635451140*e*m^6*x^10+107454153
9960*e*m^4*x^12+7630800624090*e*m^2*x^14+2232241011000*e*x^16+45*d*m^11*x^4
+991080*d*m^9*x^6+1254847860*d*m^7*x^8+190744119720*d*m^5*x^10+360056778921
0*d*m^3*x^12+5458672303560*d*m*x^14+10*e*m^11*x^4+371655*e*m^9*x^6+71705592
0*e*m^7*x^8+158953433100*e*m^5*x^10+4320681347052*e*m^3*x^12+9552676531230*
e*m*x^14+6255*d*m^10*x^4+34563240*d*m^8*x^6+18217524780*d*m^6*x^8+121245419
9880*d*m^4*x^10+8695750818510*d*m^2*x^12+2529873145800*d*x^14+1390*e*m^10*x
^4+12961215*e*m^8*x^6+10410014160*e*m^6*x^8+1010378499900*e*m^4*x^10+104349
00982212*e*m^2*x^12+4427278005150*e*x^14+10*d*m^11*x^2+383535*d*m^9*x^4+770
831280*d*m^7*x^6+177985672620*d*m^5*x^8+4952725167852*d*m^3*x^10+1096992525
1950*d*m*x^12+e*m^11*x^2+85230*e*m^9*x^4+289061730*e*m^7*x^6+101706098640*e
*m^5*x^8+4127270973210*e*m^3*x^10+13163910302340*e*m*x^12+1410*d*m^10*x^2+1
3645125*d*m^8*x^4+11467698480*d*m^6*x^6+1156995210420*d*m^4*x^8+12123781647
516*d*m^2*x^10+5108397698250*d*x^12+141*e*m^10*x^2+3032250*e*m^8*x^4+430038
6930*e*m^6*x^6+661140120240*e*m^4*x^8+10103151372930*e*m^2*x^10+61300772379
00*e*x^12+d*m^11+87950*d*m^9*x^2+311564610*d*m^7*x^4+115122336720*d*m^5*x^6

```
+4828477578330*d*m^3*x^8+15456024948420*d*m*x^10+8795*e*m^9*x^2+69236580*e*
m^7*x^4+43170876270*e*m^5*x^6+2759130044760*e*m^3*x^8+12880020790350*e*m*x^
10+143*d*m^10+3194550*d*m^8*x^2+4765995990*d*m^6*x^4+770638650960*d*m^4*x^6
+12046833873270*d*m^2*x^8+7244636735700*d*x^10+319455*e*m^8*x^2+1059110220*
e*m^6*x^4+288989494110*e*m^4*x^6+6883905070440*e*m^2*x^8+6037197279750*e*x^
10+9075*d*m^9+74814180*d*m^7*x^2+49443604830*d*m^5*x^4+3314920570200*d*m^3*
x^6+15593181033150*d*m*x^8+7481418*e*m^7*x^2+10987467740*e*m^5*x^4+12430952
13825*e*m^3*x^6+8910389161800*e*m*x^8+336765*d*m^8+1180850580*d*m^6*x^2+343
967603850*d*m^4*x^4+8511631481880*d*m^2*x^6+7378796675250*d*x^8+118085058*e
*m^6*x^2+76437245300*e*m^4*x^4+3191861805705*e*m^2*x^6+4216455243000*e*x^8+
8103018*d*m^7+12740467100*d*m^5*x^2+1546183653345*d*m^3*x^4+11284114422600*
d*m*x^6+1274046710*e*m^5*x^2+343596367410*e*m^3*x^4+4231542908475*e*m*x^6+1
32426294*d*m^6+93153182700*d*m^4*x^2+4162610035755*d*m^2*x^4+5421156741000*
d*x^6+9315318270*e*m^4*x^2+925024452390*e*m^2*x...
```

Maxima [A]
time = 0.34, size = 405, normalized size = 1.51

$$\frac{f^m x^{23} e^{(m\log(x)+1)}}{m+23}+\frac{df^m x^{21} x^m}{m+21}+\frac{10\,f^m x^{21} e^{(m\log(x)+1)}}{m+21}+\frac{10\,df^m x^{19} x^m}{m+19}+\frac{45\,f^m x^{19} e^{(m\log(x)+1)}}{m+19}+\frac{45\,df^m x^{17} x^m}{m+17}+\frac{120\,f^m x^{17} e^{(m\log(x)+1)}}{m+17}+\frac{120\,df^m x^{15} x^m}{m+15}+\frac{210\,f^m x^{15} e^{(m\log(x)+1)}}{m+15}+\frac{210\,df^m x^{13} x^m}{m+13}+\frac{252\,f^m x^{13} e^{(m\log(x)+1)}}{m+13}+\frac{252\,df^m x^{11} x^m}{m+11}+\frac{210\,f^m x^{11} e^{(m\log(x)+1)}}{m+11}+\frac{210\,df^m x^9 x^m}{m+9}+\frac{120\,f^m x^9 e^{(m\log(x)+1)}}{m+9}+\frac{120\,df^m x^7 x^m}{m+7}+\frac{45\,f^m x^7 e^{(m\log(x)+1)}}{m+7}+\frac{45\,df^m x^5 x^m}{m+5}+\frac{10\,f^m x^5 e^{(m\log(x)+1)}}{m+5}+\frac{10\,df^m x^3 x^m}{m+3}+\frac{f^m x^3 e^{(m\log(x)+1)}}{m+3}+\frac{(fx)^{m+1}\,d}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] f^m*x^23*e^(m*log(x) + 1)/(m + 23) + d*f^m*x^21*x^m/(m + 21) + 10*f^m*x^21*
e^(m*log(x) + 1)/(m + 21) + 10*d*f^m*x^19*x^m/(m + 19) + 45*f^m*x^19*e^(m*l
og(x) + 1)/(m + 19) + 45*d*f^m*x^17*x^m/(m + 17) + 120*f^m*x^17*e^(m*log(x)
 + 1)/(m + 17) + 120*d*f^m*x^15*x^m/(m + 15) + 210*f^m*x^15*e^(m*log(x) + 1
)/(m + 15) + 210*d*f^m*x^13*x^m/(m + 13) + 252*f^m*x^13*e^(m*log(x) + 1)/(m
 + 13) + 252*d*f^m*x^11*x^m/(m + 11) + 210*f^m*x^11*e^(m*log(x) + 1)/(m + 1
1) + 210*d*f^m*x^9*x^m/(m + 9) + 120*f^m*x^9*e^(m*log(x) + 1)/(m + 9) + 120
*d*f^m*x^7*x^m/(m + 7) + 45*f^m*x^7*e^(m*log(x) + 1)/(m + 7) + 45*d*f^m*x^5
*x^m/(m + 5) + 10*f^m*x^5*e^(m*log(x) + 1)/(m + 5) + 10*d*f^m*x^3*x^m/(m +
3) + f^m*x^3*e^(m*log(x) + 1)/(m + 3) + (f*x)^(m + 1)*d/(f*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1495 vs. $2(280) = 560$.
time = 0.38, size = 1495, normalized size = 5.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((d*m^11 + 123*d*m^10 + 6635*d*m^9 + 206505*d*m^8 + 4103178*d*m^7 + 5436257
4*d*m^6 + 486687830*d*m^5 + 2917013970*d*m^4 + 11320966021*d*m^3 + 26560342
503*d*m^2 + 32778930735*d*m + 15058768725*d)*x^21 + 10*(d*m^11 + 125*d*m^10
```

+ 6843*d*m^9 + 215823*d*m^8 + 4339146*d*m^7 + 58085538*d*m^6 + 524676662*d
*m^5 + 3168601822*d*m^4 + 12374824773*d*m^3 + 29178958257*d*m^2 + 361459164
15*d*m + 16643902275*d)*x^19 + 45*(d*m^11 + 127*d*m^10 + 7059*d*m^9 + 22583
7*d*m^8 + 4600554*d*m^7 + 62319894*d*m^6 + 568863686*d*m^5 + 3466775738*d*m
^4 + 13643071845*d*m^3 + 32368407579*d*m^2 + 40283194455*d*m + 18602008425*
d)*x^17 + 120*(d*m^11 + 129*d*m^10 + 7283*d*m^9 + 236595*d*m^8 + 4890858*d*
m^7 + 67166442*d*m^6 + 620805254*d*m^5 + 3825379590*d*m^4 + 15197565541*d*m
^3 + 36337145829*d*m^2 + 45488935863*d*m + 21082276215*d)*x^15 + 210*(d*m^1
1 + 131*d*m^10 + 7515*d*m^9 + 248145*d*m^8 + 5213898*d*m^7 + 72748638*d*m^6
+ 682569590*d*m^5 + 4264053730*d*m^4 + 17145560901*d*m^3 + 41408337231*d*m
^2 + 52237739295*d*m + 24325703325*d)*x^13 + 252*(d*m^11 + 133*d*m^10 + 775
5*d*m^9 + 260535*d*m^8 + 5573898*d*m^7 + 79216434*d*m^6 + 756921110*d*m^5 +
4811326190*d*m^4 + 19653671301*d*m^3 + 48110244633*d*m^2 + 61333432335*d*m
+ 28748558475*d)*x^11 + 210*(d*m^11 + 135*d*m^10 + 8003*d*m^9 + 273813*d*m
^8 + 5975466*d*m^7 + 86750118*d*m^6 + 847550822*d*m^5 + 5509501002*d*m^4 +
22992750373*d*m^3 + 57365875587*d*m^2 + 74253243015*d*m + 35137127025*d)*x^
9 + 120*(d*m^11 + 137*d*m^10 + 8259*d*m^9 + 288027*d*m^8 + 6423594*d*m^7 +
95564154*d*m^6 + 959352806*d*m^5 + 6421988758*d*m^4 + 27624338085*d*m^3 + 7
0930262349*d*m^2 + 94034286855*d*m + 45176306175*d)*x^7 + 45*(d*m^11 + 139*
d*m^10 + 8523*d*m^9 + 303225*d*m^8 + 6923658*d*m^7 + 105911022*d*m^6 + 1098
746774*d*m^5 + 7643724530*d*m^4 + 34359636741*d*m^3 + 92502445239*d*m^2 + 1
28033897103*d*m + 63246828645*d)*x^5 + 10*(d*m^11 + 141*d*m^10 + 8795*d*m^9
+ 319455*d*m^8 + 7481418*d*m^7 + 118085058*d*m^6 + 1274046710*d*m^5 + 9315
318270*d*m^4 + 44632304581*d*m^3 + 130403715201*d*m^2 + 199334977695*d*m +
105411381075*d)*x^3 + (d*m^11 + 143*d*m^10 + 9075*d*m^9 + 336765*d*m^8 + 81
03018*d*m^7 + 132426294*d*m^6 + 1495875590*d*m^5 + 11641582810*d*m^4 + 6093
6676581*d*m^3 + 203363952363*d*m^2 + 387182170935*d*m + 316234143225*d)*x +
((m^11 + 121*m^10 + 6435*m^9 + 197835*m^8 + 3889578*m^7 + 51069018*m^6 + 4
53714470*m^5 + 2702025590*m^4 + 10431670821*m^3 + 24372200061*m^2 + 2998552
1895*m + 13749310575)*x^23 + 10*(m^11 + 123*m^10 + 6635*m^9 + 206505*m^8 +
4103178*m^7 + 54362574*m^6 + 486687830*m^5 + 2917013970*m^4 + 11320966021*m
^3 + 26560342503*m^2 + 32778930735*m + 15058768725)*x^21 + 45*(m^11 + 125*m
^10 + 6843*m^9 + 215823*m^8 + 4339146*m^7 + 58085538*m^6 + 524676662*m^5 +
3168601822*m^4 + 12374824773*m^3 + 29178958257*m^2 + 36145916415*m + 166439
02275)*x^19 + 120*(m^11 + 127*m^10 + 7059*m^9 + 225837*m^8 + 4600554*m^7 +
62319894*m^6 + 568863686*m^5 + 3466775738*m^4 + 13643071845*m^3 + 323684075
79*m^2 + 40283194455*m + 18602008425)*x^17 + 210*(m^11 + 129*m^10 + 7283*m^
9 + 236595*m^8 + 4890858*m^7 + 67166442*m^6 + 620805254*m^5 + 3825379590*m^
4 + 15197565541*m^3 + 36337145829*m^2 + 45488935863*m + 21082276215)*x^15 +
252*(m^11 + 131*m^10 + 7515*m^9 + 248145*m^8 + 5213898*m^7 + 72748638*m^6
+ 682569590*m^5 + 4264053730*m^4 + 17145560901*m^3 + 41408337231*m^2 + 5223
7739295*m + 24325703325)*x^13 + 210*(m^11 + 133*m^10 + 7755*m^9 + 260535*m^
8 + 5573898*m^7 + 79216434*m^6 + 756921110*m^5 + 4811326190*m^4 + 196536713
01*m^3 + 48110244633*m^2 + 61333432335*m + 28748558475)*x^11 + 120*(m^11 +
135*m^10 + 8003*m^9 + 273813*m^8 + 5975466*m^7 + 86750118*m^6 + 847550822*m

```
^5 + 5509501002*m^4 + 22992750373*m^3 + 57365875587*m^2 + 74253243015*m + 3
5137127025)*x^9 + 45*(m^11 + 137*m^10 + 8259*m^9 + 288027*m^8 + 6423594*m^7
 + 95564154*m^6 + 959352806*m^5 + 6421988758*m^4 + 27624338085*m^3 + 709302
62349*m^2 + 94034286855*m + 45176306175)*x^7 + 10*(m^11 + 139*m^10 + 8523*m
^9 + 303225*m^8 + 6923658*m^7 + 105911022*m^6 + 1098746774*m^5 + 7643724530
*m^4 + 34359636741*m^3 + 92502445239*m^2 + 128033897103*m + 63246828645)*x^
5 + (m^11 + 141*m^10 + 8795*m^9 + 319455*m^8 + 7481418*m^7 + 118085058*m^6
+ 1274046710*m^5 + 9315318270*m^4 + 44632304581*m^3 + 130403715201*m^2 + 19
9334977695*m + 105411381075)*x^3)*e)*(f*x)^m/(m^12 + 144*m^11 + 9218*m^10 +
 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1628301884*m^6 + 13137458400*m^
5 + 72578259391*m^4 + 264300628944*m^3 + 590546123298*m^2 + 703416314160*m
+ 316234143225)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 21612 vs. $2(228) = 456$.
time = 3.44, size = 21612, normalized size = 80.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Piecewise((((-d/(2*x**2) - 5*d/(2*x**4) - 15*d/(2*x**6) - 15*d/x**8 - 21*d/x
**10 - 21*d/x**12 - 15*d/x**14 - 15*d/(2*x**16) - 5*d/(2*x**18) - d/(2*x**2
0) - d/(22*x**22) + e*log(x) - 5*e/x**2 - 45*e/(4*x**4) - 20*e/x**6 - 105*e
/(4*x**8) - 126*e/(5*x**10) - 35*e/(2*x**12) - 60*e/(7*x**14) - 45*e/(16*x*
*16) - 5*e/(9*x**18) - e/(20*x**20))/f**23, Eq(m, -23)), ((d*log(x) - 5*d/x
**2 - 45*d/(4*x**4) - 20*d/x**6 - 105*d/(4*x**8) - 126*d/(5*x**10) - 35*d/(
2*x**12) - 60*d/(7*x**14) - 45*d/(16*x**16) - 5*d/(9*x**18) - d/(20*x**20)
+ e*x**2/2 + 10*e*log(x) - 45*e/(2*x**2) - 30*e/x**4 - 35*e/x**6 - 63*e/(2*
x**8) - 21*e/x**10 - 10*e/x**12 - 45*e/(14*x**14) - 5*e/(8*x**16) - e/(18*x
**18))/f**21, Eq(m, -21)), ((d*x**2/2 + 10*d*log(x) - 45*d/(2*x**2) - 30*d/
x**4 - 35*d/x**6 - 63*d/(2*x**8) - 21*d/x**10 - 10*d/x**12 - 45*d/(14*x**14
) - 5*d/(8*x**16) - d/(18*x**18) + e*x**4/4 + 5*e*x**2 + 45*e*log(x) - 60*e
/x**2 - 105*e/(2*x**4) - 42*e/x**6 - 105*e/(4*x**8) - 12*e/x**10 - 15*e/(4*
x**12) - 5*e/(7*x**14) - e/(16*x**16))/f**19, Eq(m, -19)), ((d*x**4/4 + 5*d
*x**2 + 45*d*log(x) - 60*d/x**2 - 105*d/(2*x**4) - 42*d/x**6 - 105*d/(4*x**
8) - 12*d/x**10 - 15*d/(4*x**12) - 5*d/(7*x**14) - d/(16*x**16) + e*x**6/6
+ 5*e*x**4/2 + 45*e*x**2/2 + 120*e*log(x) - 105*e/x**2 - 63*e/x**4 - 35*e/x
**6 - 15*e/x**8 - 9*e/(2*x**10) - 5*e/(6*x**12) - e/(14*x**14))/f**17, Eq(m
, -17)), ((d*x**6/6 + 5*d*x**4/2 + 45*d*x**2/2 + 120*d*log(x) - 105*d/x**2
- 63*d/x**4 - 35*d/x**6 - 15*d/x**8 - 9*d/(2*x**10) - 5*d/(6*x**12) - d/(14
*x**14) + e*x**8/8 + 5*e*x**6/3 + 45*e*x**4/4 + 60*e*x**2 + 210*e*log(x) -
126*e/x**2 - 105*e/(2*x**4) - 20*e/x**6 - 45*e/(8*x**8) - e/x**10 - e/(12*x
**12))/f**15, Eq(m, -15)), ((d*x**8/8 + 5*d*x**6/3 + 45*d*x**4/4 + 60*d*x**
```

```
2 + 210*d*log(x) - 126*d/x**2 - 105*d/(2*x**4) - 20*d/x**6 - 45*d/(8*x**8)
- d/x**10 - d/(12*x**12) + e*x**10/10 + 5*e*x**8/4 + 15*e*x**6/2 + 30*e*x**
4 + 105*e*x**2 + 252*e*log(x) - 105*e/x**2 - 30*e/x**4 - 15*e/(2*x**6) - 5*
e/(4*x**8) - e/(10*x**10))/f**13, Eq(m, -13)), ((d*x**10/10 + 5*d*x**8/4 +
15*d*x**6/2 + 30*d*x**4 + 105*d*x**2 + 252*d*log(x) - 105*d/x**2 - 30*d/x**
4 - 15*d/(2*x**6) - 5*d/(4*x**8) - d/(10*x**10) + e*x**12/12 + e*x**10 + 45
*e*x**8/8 + 20*e*x**6 + 105*e*x**4/2 + 126*e*x**2 + 210*e*log(x) - 60*e/x**
2 - 45*e/(4*x**4) - 5*e/(3*x**6) - e/(8*x**8))/f**11, Eq(m, -11)), ((d*x**1
2/12 + d*x**10 + 45*d*x**8/8 + 20*d*x**6 + 105*d*x**4/2 + 126*d*x**2 + 210*
d*log(x) - 60*d/x**2 - 45*d/(4*x**4) - 5*d/(3*x**6) - d/(8*x**8) + e*x**14/
14 + 5*e*x**12/6 + 9*e*x**10/2 + 15*e*x**8 + 35*e*x**6 + 63*e*x**4 + 105*e*
x**2 + 120*e*log(x) - 45*e/(2*x**2) - 5*e/(2*x**4) - e/(6*x**6))/f**9, Eq(m
, -9)), ((d*x**14/14 + 5*d*x**12/6 + 9*d*x**10/2 + 15*d*x**8 + 35*d*x**6 +
63*d*x**4 + 105*d*x**2 + 120*d*log(x) - 45*d/(2*x**2) - 5*d/(2*x**4) - d/(6
*x**6) + e*x**16/16 + 5*e*x**14/7 + 15*e*x**12/4 + 12*e*x**10 + 105*e*x**8/
4 + 42*e*x**6 + 105*e*x**4/2 + 60*e*x**2 + 45*e*log(x) - 5*e/x**2 - e/(4*x*
*4))/f**7, Eq(m, -7)), ((d*x**16/16 + 5*d*x**14/7 + 15*d*x**12/4 + 12*d*x**
10 + 105*d*x**8/4 + 42*d*x**6 + 105*d*x**4/2 + 60*d*x**2 + 45*d*log(x) - 5*
d/x**2 - d/(4*x**4) + e*x**18/18 + 5*e*x**16/8 + 45*e*x**14/14 + 10*e*x**12
 + 21*e*x**10 + 63*e*x**8/2 + 35*e*x**6 + 30*e*x**4 + 45*e*x**2/2 + 10*e*lo
g(x) - e/(2*x**2))/f**5, Eq(m, -5)), ((d*x**18/18 + 5*d*x**16/8 + 45*d*x**1
4/14 + 10*d*x**12 + 21*d*x**10 + 63*d*x**8/2 + 35*d*x**6 + 30*d*x**4 + 45*d
*x**2/2 + 10*d*log(x) - d/(2*x**2) + e*x**20/20 + 5*e*x**18/9 + 45*e*x**16/
16 + 60*e*x**14/7 + 35*e*x**12/2 + 126*e*x**10/5 + 105*e*x**8/4 + 20*e*x**6
 + 45*e*x**4/4 + 5*e*x**2 + e*log(x))/f**3, Eq(m, -3)), ((d*x**20/20 + 5*d*
x**18/9 + 45*d*x**16/16 + 60*d*x**14/7 + 35*d*x**12/2 + 126*d*x**10/5 + 105
*d*x**8/4 + 20*d*x**6 + 45*d*x**4/4 + 5*d*x**2 + d*log(x) + e*x**22/22 + e*
x**20/2 + 5*e*x**18/2 + 15*e*x**16/2 + 15*e*x**14 + 21*e*x**12 + 21*e*x**10
 + 15*e*x**8 + 15*e*x**6/2 + 5*e*x**4/2 + e*x**2/2)/f, Eq(m, -1)), (d*m**11
*x**21*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**
8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4
+ 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) +
10*d*m**11*x**19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8
439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 7257825
9391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234
143225) + 45*d*m**11*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34584
0*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5
 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*
m + 316234143225) + 120*d*m**11*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m*
*10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137
458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 70
3416314160*m + 316234143225) + 210*d*m**11*x**13*(f*x)**m/(m**12 + 144*m**1
1 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m
**6 + 13137458400*m**5 + 72578259391*m**4 + 264...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3752 vs. $2(280) = 560$.
time = 3.97, size = 3752, normalized size = 13.95

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*m^11*x^23*e + 121*(f*x)^m*m^10*x^23*e + (f*x)^m*d*m^11*x^21 + 10*(
f*x)^m*m^11*x^21*e + 6435*(f*x)^m*m^9*x^23*e + 123*(f*x)^m*d*m^10*x^21 + 12
30*(f*x)^m*m^10*x^21*e + 197835*(f*x)^m*m^8*x^23*e + 10*(f*x)^m*d*m^11*x^19
 + 6635*(f*x)^m*d*m^9*x^21 + 45*(f*x)^m*m^11*x^19*e + 66350*(f*x)^m*m^9*x^2
1*e + 3889578*(f*x)^m*m^7*x^23*e + 1250*(f*x)^m*d*m^10*x^19 + 206505*(f*x)^
m*d*m^8*x^21 + 5625*(f*x)^m*m^10*x^19*e + 2065050*(f*x)^m*m^8*x^21*e + 5106
9018*(f*x)^m*m^6*x^23*e + 45*(f*x)^m*d*m^11*x^17 + 68430*(f*x)^m*d*m^9*x^19
 + 4103178*(f*x)^m*d*m^7*x^21 + 120*(f*x)^m*m^11*x^17*e + 307935*(f*x)^m*m^
9*x^19*e + 41031780*(f*x)^m*m^7*x^21*e + 453714470*(f*x)^m*m^5*x^23*e + 571
5*(f*x)^m*d*m^10*x^17 + 2158230*(f*x)^m*d*m^8*x^19 + 54362574*(f*x)^m*d*m^6
*x^21 + 15240*(f*x)^m*m^10*x^17*e + 9712035*(f*x)^m*m^8*x^19*e + 543625740*
(f*x)^m*m^6*x^21*e + 2702025590*(f*x)^m*m^4*x^23*e + 120*(f*x)^m*d*m^11*x^1
5 + 317655*(f*x)^m*d*m^9*x^17 + 43391460*(f*x)^m*d*m^7*x^19 + 486687830*(f*
x)^m*d*m^5*x^21 + 210*(f*x)^m*m^11*x^15*e + 847080*(f*x)^m*m^9*x^17*e + 195
261570*(f*x)^m*m^7*x^19*e + 4866878300*(f*x)^m*m^5*x^21*e + 10431670821*(f*
x)^m*m^3*x^23*e + 15480*(f*x)^m*d*m^10*x^15 + 10162665*(f*x)^m*d*m^8*x^17 +
 580855380*(f*x)^m*d*m^6*x^19 + 2917013970*(f*x)^m*d*m^4*x^21 + 27090*(f*x)
^m*m^10*x^15*e + 27100440*(f*x)^m*m^8*x^17*e + 2613849210*(f*x)^m*m^6*x^19*
e + 29170139700*(f*x)^m*m^4*x^21*e + 24372200061*(f*x)^m*m^2*x^23*e + 210*(
f*x)^m*d*m^11*x^13 + 873960*(f*x)^m*d*m^9*x^15 + 207024930*(f*x)^m*d*m^7*x^
17 + 5246766620*(f*x)^m*d*m^5*x^19 + 11320966021*(f*x)^m*d*m^3*x^21 + 252*(
f*x)^m*m^11*x^13*e + 1529430*(f*x)^m*m^9*x^15*e + 552066480*(f*x)^m*m^7*x^1
7*e + 23610449790*(f*x)^m*m^5*x^19*e + 113209660210*(f*x)^m*m^3*x^21*e + 29
985521895*(f*x)^m*m*x^23*e + 27510*(f*x)^m*d*m^10*x^13 + 28391400*(f*x)^m*d
*m^8*x^15 + 2804395230*(f*x)^m*d*m^6*x^17 + 31686018220*(f*x)^m*d*m^4*x^19
+ 26560342503*(f*x)^m*d*m^2*x^21 + 33012*(f*x)^m*m^10*x^13*e + 49684950*(f*
x)^m*m^8*x^15*e + 7478387280*(f*x)^m*m^6*x^17*e + 142587081990*(f*x)^m*m^4*
x^19*e + 265603425030*(f*x)^m*m^2*x^21*e + 13749310575*(f*x)^m*x^23*e + 252
*(f*x)^m*d*m^11*x^11 + 1578150*(f*x)^m*d*m^9*x^13 + 586902960*(f*x)^m*d*m^7
*x^15 + 25598865870*(f*x)^m*d*m^5*x^17 + 123748247730*(f*x)^m*d*m^3*x^19 +
32778930735*(f*x)^m*d*m*x^21 + 210*(f*x)^m*m^11*x^11*e + 1893780*(f*x)^m*m^
9*x^13*e + 1027080180*(f*x)^m*m^7*x^15*e + 68263642320*(f*x)^m*m^5*x^17*e +
 556867114785*(f*x)^m*m^3*x^19*e + 327789307350*(f*x)^m*m*x^21*e + 33516*(f
*x)^m*d*m^10*x^11 + 52110450*(f*x)^m*d*m^8*x^13 + 8059973040*(f*x)^m*d*m^6*
x^15 + 156004908210*(f*x)^m*d*m^4*x^17 + 291789582570*(f*x)^m*d*m^2*x^19 +
15058768725*(f*x)^m*d*x^21 + 27930*(f*x)^m*m^10*x^11*e + 62532540*(f*x)^m*m
```

```
^8*x^13*e + 14104952820*(f*x)^m*m^6*x^15*e + 416013088560*(f*x)^m*m^4*x^17*
e + 1313053121565*(f*x)^m*m^2*x^19*e + 150587687250*(f*x)^m*x^21*e + 210*(f
*x)^m*d*m^11*x^9 + 1954260*(f*x)^m*d*m^9*x^11 + 1094918580*(f*x)^m*d*m^7*x^
13 + 74496630480*(f*x)^m*d*m^5*x^15 + 613938233025*(f*x)^m*d*m^3*x^17 + 361
459164150*(f*x)^m*d*m*x^19 + 120*(f*x)^m*m^11*x^9*e + 1628550*(f*x)^m*m^9*x
^11*e + 1313902296*(f*x)^m*m^7*x^13*e + 130369103340*(f*x)^m*m^5*x^15*e + 1
637168621400*(f*x)^m*m^3*x^17*e + 1626566238675*(f*x)^m*m*x^19*e + 28350*(f
*x)^m*d*m^10*x^9 + 65654820*(f*x)^m*d*m^8*x^11 + 15277213980*(f*x)^m*d*m^6*
x^13 + 459045550800*(f*x)^m*d*m^4*x^15 + 1456578341055*(f*x)^m*d*m^2*x^17 +
 166439022750*(f*x)^m*d*x^19 + 16200*(f*x)^m*m^10*x^9*e + 54712350*(f*x)^m*
m^8*x^11*e + 18332656776*(f*x)^m*m^6*x^13*e + 803329713900*(f*x)^m*m^4*x^15
*e + 3884208909480*(f*x)^m*m^2*x^17*e + 748975602375*(f*x)^m*x^19*e + 120*(
f*x)^m*d*m^11*x^7 + 1680630*(f*x)^m*d*m^9*x^9 + 1404622296*(f*x)^m*d*m^7*x^
11 + 143339613900*(f*x)^m*d*m^5*x^13 + 1823707864920*(f*x)^m*d*m^3*x^15 + 1
812743750475*(f*x)^m*d*m*x^17 + 45*(f*x)^m*m^11*x^7*e + 960360*(f*x)^m*m^9*
x^9*e + 1170518580*(f*x)^m*m^7*x^11*e + 172007536680*(f*x)^m*m^5*x^13*e + 3
191488763610*(f*x)^m*m^3*x^15*e + 4833983334600*(f*x)^m*m*x^17*e + 16440*(f
*x)^m*d*m^10*x^7 + 57500730*(f*x)^m*d*m^8*x^9 + 19962541368*(f*x)^m*d*m^6*x
^11 + 895451283300*(f*x)^m*d*m^4*x^13 + 4360457499480*(f*x)^m*d*m^2*x^15 +
837090379125*(f*x)^m*d*x^17 + 6165*(f*x)^m*m^10*x^7*e + 32857560*(f*x)^m*m^
8*x^9*e + 16635451140*(f*x)^m*m^6*x^11*e + 1074541539960*(f*x)^m*m^4*x^13*e
 + 7630800624090*(f*x)^m*m^2*x^15*e + 2232241011000*(f*x)^m*x^17*e + 45*(f*
x)^m*d*m^11*x^5 + 991080*(f*x)^m*d*m^9*x^7 + 1254847860*(f*x)^m*d*m^7*x^9 +
 190744119720*(f*x)^m*d*m^5*x^11 + 3600567789210*(f*x)^m*d*m^3*x^13 + 54586
72303560*(f*x)^m*d*m*x^15 + 10*(f*x)^m*m^11*x^5*e + 371655*(f*x)^m*m^9*x^7*
e + 717055920*(f*x)^m*m^7*x^9*e + 158953433100*(f*x)^m*m^5*x^11*e + 4320681
347052*(f*x)^m*m^3*x^13*e + 9552676531230*(f*x)^m*m*x^15*e + 6255*(f*x)^m*d
*m^10*x^5 + 34563240*(f*x)^m*d*m^8*x^7 + 18217524780*(f*x)^m*d*m^6*x^9 + 12
12454199880*(f*x)^m*d*m^4*x^11 + 8695750818510*(f*x)^m*d*m^2*x^13 + 2529873
145800*(f*x)^m*d*x^15 + 1390*(f*x)^m*m^10*x^5*e + 12961215*(f*x)^m*m^8*x^7*
e + 10410014160*(f*x)^m*m^6*x^9*e + 10103784999...
```

Mupad [B]
time = 1.78, size = 1539, normalized size = 5.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] (d*x*(f*x)^m*(387182170935*m + 203363952363*m^2 + 60936676581*m^3 + 1164158
2810*m^4 + 1495875590*m^5 + 132426294*m^6 + 8103018*m^7 + 336765*m^8 + 9075
*m^9 + 143*m^10 + m^11 + 316234143225))/(703416314160*m + 590546123298*m^2
+ 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 1
40529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316
```

234143225) + (e*x^23*(f*x)^m*(29985521895*m + 24372200061*m^2 + 10431670821
*m^3 + 2702025590*m^4 + 453714470*m^5 + 51069018*m^6 + 3889578*m^7 + 197835
*m^8 + 6435*m^9 + 121*m^10 + m^11 + 13749310575))/(703416314160*m + 5905461
23298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 16283018
84*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 +
m^12 + 316234143225) + (30*x^15*(f*x)^m*(4*d + 7*e)*(45488935863*m + 363371
45829*m^2 + 15197565541*m^3 + 3825379590*m^4 + 620805254*m^5 + 67166442*m^6
 + 4890858*m^7 + 236595*m^8 + 7283*m^9 + 129*m^10 + m^11 + 21082276215))/(7
03416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 131
37458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 +
9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^13*(f*x)^m*(5*d + 6*e)*
(52237739295*m + 41408337231*m^2 + 17145560901*m^3 + 4264053730*m^4 + 68256
9590*m^5 + 72748638*m^6 + 5213898*m^7 + 248145*m^8 + 7515*m^9 + 131*m^10 +
m^11 + 24325703325))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3
+ 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439
783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (30*x^
9*(f*x)^m*(7*d + 4*e)*(74253243015*m + 57365875587*m^2 + 22992750373*m^3 +
5509501002*m^4 + 847550822*m^5 + 86750118*m^6 + 5975466*m^7 + 273813*m^8 +
8003*m^9 + 135*m^10 + m^11 + 35137127025))/(703416314160*m + 590546123298*m
^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6
+ 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 +
316234143225) + (x^3*(f*x)^m*(10*d + e)*(199334977695*m + 130403715201*m^2
+ 44632304581*m^3 + 9315318270*m^4 + 1274046710*m^5 + 118085058*m^6 + 74814
18*m^7 + 319455*m^8 + 8795*m^9 + 141*m^10 + m^11 + 105411381075))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 1313745840
0*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^
10 + 144*m^11 + m^12 + 316234143225) + (5*x^19*(f*x)^m*(2*d + 9*e)*(3614591
6415*m + 29178958257*m^2 + 12374824773*m^3 + 3168601822*m^4 + 524676662*m^5
 + 58085538*m^6 + 4339146*m^7 + 215823*m^8 + 6843*m^9 + 125*m^10 + m^11 + 1
6643902275))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 725782
59391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8
+ 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (42*x^11*(f*x)
^m*(6*d + 5*e)*(61333432335*m + 48110244633*m^2 + 19653671301*m^3 + 4811326
190*m^4 + 756921110*m^5 + 79216434*m^6 + 5573898*m^7 + 260535*m^8 + 7755*m^
9 + 133*m^10 + m^11 + 28748558475))/(703416314160*m + 590546123298*m^2 + 26
4300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 14052
9312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 3162341
43225) + (15*x^7*(f*x)^m*(8*d + 3*e)*(94034286855*m + 70930262349*m^2 + 276
24338085*m^3 + 6421988758*m^4 + 959352806*m^5 + 95564154*m^6 + 6423594*m^7
+ 288027*m^8 + 8259*m^9 + 137*m^10 + m^11 + 45176306175))/(703416314160*m +
 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 +
1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144
*m^11 + m^12 + 316234143225) + (5*x^5*(f*x)^m*(9*d + 2*e)*(128033897103*m +
 92502445239*m^2 + 34359636741*m^3 + 7643724530*m^4 + 1098746774*m^5 + 1059
11022*m^6 + 6923658*m^7 + 303225*m^8 + 8523*m^9 + 139*m^10 + m^11 + 6324682

8645))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*
m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 3458
40*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (15*x^17*(f*x)^m*(3*
d + 8*e)*(40283194455*m + 32368407579*m^2 + 13643071845*m^3 + 3466775738*m^
4 + 568863686*m^5 + 62319894*m^6 + 4600554*m^7 + 225837*m^8 + 7059*m^9 + 12
7*m^10 + m^11 + 18602008425))/(703416314160*m + 590546123298*m^2 + 26430062
8944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m
^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
 + (x^21*(f*x)^m*(d + 10*e)*(32778930735*m + 26560342503*m^2 + 11320966021*
m^3 + 2917013970*m^4 + 486687830*m^5 + 54362574*m^6 + 4103178*m^7 + 206505*
m^8 + 6635*m^9 + 123*m^10 + m^11 + 15058768725))/(703416314160*m + 59054612
3298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 162830188
4*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m
^12 + 316234143225)

3.56 $\int x^5(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=63

$$\frac{1}{22}(d-e)\left(1+x^2\right)^{11}-\frac{1}{24}(2d-3e)\left(1+x^2\right)^{12}+\frac{1}{26}(d-3e)\left(1+x^2\right)^{13}+\frac{1}{28}e\left(1+x^2\right)^{14}$$

```
[Out] 1/22*(d-e)*(x^2+1)^11-1/24*(2*d-3*e)*(x^2+1)^12+1/26*(d-3*e)*(x^2+1)^13+1/2
8*e*(x^2+1)^14
```

Rubi [A]
time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 457, 77}

$$\frac{1}{26}\left(x^2+1\right)^{13}(d-3e)-\frac{1}{24}\left(x^2+1\right)^{12}(2d-3e)+\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{28}e\left(x^2+1\right)^{14}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 - ((2*d - 3*e)*(1 + x^2)^12)/24 + ((d - 3*e)*(1 +
 x^2)^13)/26 + (e*(1 + x^2)^14)/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^5\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^5\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int x^2(1+x)^{10}(d+ex)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int ((d-e)(1+x)^{10}+(-2d+3e)(1+x)^{11}+(d-3e)(1+x\right.\\ &= \frac{1}{22}(d-e)\left(1+x^2\right)^{11}-\frac{1}{24}(2d-3e)\left(1+x^2\right)^{12}+\frac{1}{26}(d-3e)\left(1+x^2\right)\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 153 vs. 2(63) = 126.
time = 0.02, size = 153, normalized size = 2.43

$$\frac{dx^6}{6}+\frac{1}{8}(10d+e)x^8+\frac{1}{2}(9d+2e)x^{10}+\frac{5}{4}(8d+3e)x^{12}+\frac{15}{7}(7d+4e)x^{14}+\frac{21}{8}(6d+5e)x^{16}+\frac{7}{3}(5d+6e)x^{18}+\frac{3}{2}(4d+7e)x^{20}+\frac{15}{22}(3d+8e)x^{22}+\frac{5}{24}(2d+9e)x^{24}+\frac{1}{26}(d+10e)x^{26}+\frac{ex^{28}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^6)/6 + ((10*d + e)*x^8)/8 + ((9*d + 2*e)*x^10)/2 + (5*(8*d + 3*e)*x^12
)/4 + (15*(7*d + 4*e)*x^14)/7 + (21*(6*d + 5*e)*x^16)/8 + (7*(5*d + 6*e)*x^
18)/3 + (3*(4*d + 7*e)*x^20)/2 + (15*(3*d + 8*e)*x^22)/22 + (5*(2*d + 9*e)*
x^24)/24 + ((d + 10*e)*x^26)/26 + (e*x^28)/28
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(55) = 110.
time = 0.09, size = 130, normalized size = 2.06

method	result
norman	$\left(10d+\frac{15e}{4}\right)x^{12}+\left(15d+\frac{60e}{7}\right)x^{14}+\left(\frac{63d}{4}+\frac{105e}{8}\right)x^{16}+\left(\frac{35d}{3}+14e\right)x^{18}+\left(6d+\frac{21e}{2}\right)x^{20}+\left(\frac{45d}{22}+\right.$
default	$\frac{e\,x^{28}}{28}+\frac{(d+10e)x^{26}}{26}+\frac{(10d+45e)x^{24}}{24}+\frac{(45d+120e)x^{22}}{22}+\frac{(120d+210e)x^{20}}{20}+\frac{(210d+252e)x^{18}}{18}+\frac{(252d+210e)x^{16}}{16}+\frac{(21}{}$
risch	$\frac{1}{28}e\,x^{28}+\frac{1}{26}x^{26}d+\frac{5}{13}x^{26}e+\frac{5}{12}x^{24}d+\frac{15}{8}x^{24}e+\frac{45}{22}x^{22}d+\frac{60}{11}e\,x^{22}+6d\,x^{20}+\frac{21}{2}e\,x^{20}+\frac{35}{3}d\,x^{18}+1$
gosper	$\frac{x^6\left(858e\,x^{22}+924d\,x^{20}+9240e\,x^{20}+10010d\,x^{18}+45045e\,x^{18}+49140d\,x^{16}+131040e\,x^{16}+144144d\,x^{14}+252252e\,x^{14}+280280d\,x^{12}+33\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/28*e*x^28+1/26*(d+10*e)*x^26+1/24*(10*d+45*e)*x^24+1/22*(45*d+120*e)*x^22
+1/20*(120*d+210*e)*x^20+1/18*(210*d+252*e)*x^18+1/16*(252*d+210*e)*x^16+1/
14*(210*d+120*e)*x^14+1/12*(120*d+45*e)*x^12+1/10*(45*d+10*e)*x^10+1/8*(10*
d+e)*x^8+1/6*x^6*d
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(59) = 118$.
time = 0.27, size = 140, normalized size = 2.22

$$\frac{1}{28}x^{28}e + \frac{1}{26}(d + 10e)x^{26} + \frac{5}{24}(2d + 9e)x^{24} + \frac{15}{22}(3d + 8e)x^{22} + \frac{3}{2}(4d + 7e)x^{20} + \frac{7}{3}(5d + 6e)x^{18} + \frac{21}{8}(6d + 5e)x^{16} + \frac{15}{7}(7d + 4e)x^{14} + \frac{5}{4}(8d + 3e)x^{12} + \frac{1}{2}(9d + 2e)x^{10} + \frac{1}{8}(10d + e)x^{8} + \frac{1}{6}dx^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*x^28*e + 1/26*(d + 10*e)*x^26 + 5/24*(2*d + 9*e)*x^24 + 15/22*(3*d + 8
*e)*x^22 + 3/2*(4*d + 7*e)*x^20 + 7/3*(5*d + 6*e)*x^18 + 21/8*(6*d + 5*e)*x
^16 + 15/7*(7*d + 4*e)*x^14 + 5/4*(8*d + 3*e)*x^12 + 1/2*(9*d + 2*e)*x^10 +
 1/8*(10*d + e)*x^8 + 1/6*d*x^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(59) = 118$.
time = 0.34, size = 127, normalized size = 2.02

$$\frac{1}{26}dx^{26} + \frac{5}{12}dx^{24} + \frac{45}{22}dx^{22} + 6dx^{20} + \frac{35}{3}dx^{18} + \frac{63}{4}dx^{16} + 15dx^{14} + 10dx^{12} + \frac{9}{2}dx^{10} + \frac{5}{4}dx^{8} + \frac{1}{6}dx^{6} + \frac{1}{8008}\left(286x^{28} + 3080x^{26} + 15015x^{24} + 43680x^{22} + 84084x^{20} + 112112x^{18} + 105105x^{16} + 68640x^{14} + 30030x^{12} + 8008x^{10} + 1001x^{8}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*d*x^26 + 5/12*d*x^24 + 45/22*d*x^22 + 6*d*x^20 + 35/3*d*x^18 + 63/4*d*
x^16 + 15*d*x^14 + 10*d*x^12 + 9/2*d*x^10 + 5/4*d*x^8 + 1/6*d*x^6 + 1/8008*
(286*x^28 + 3080*x^26 + 15015*x^24 + 43680*x^22 + 84084*x^20 + 112112*x^18
+ 105105*x^16 + 68640*x^14 + 30030*x^12 + 8008*x^10 + 1001*x^8)*e
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(51) = 102$.
time = 0.03, size = 134, normalized size = 2.13

$$\frac{dx^{6}}{6} + \frac{ex^{28}}{28} + x^{26}\left(\frac{d}{26} + \frac{5e}{13}\right) + x^{24}\cdot\left(\frac{5d}{12} + \frac{15e}{8}\right) + x^{22}\cdot\left(\frac{45d}{22} + \frac{60e}{11}\right) + x^{20}\cdot\left(6d + \frac{21e}{2}\right) + x^{18}\cdot\left(\frac{35d}{3} + 14e\right) + x^{16}\cdot\left(\frac{63d}{4} + \frac{105e}{8}\right) + x^{14}\cdot\left(15d + \frac{60e}{7}\right) + x^{12}\cdot\left(10d + \frac{15e}{4}\right) + x^{10}\cdot\left(\frac{9d}{2} + e\right) + x^{8}\cdot\left(\frac{5d}{4} + \frac{e}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**6/6 + e*x**28/28 + x**26*(d/26 + 5*e/13) + x**24*(5*d/12 + 15*e/8) + x
**22*(45*d/22 + 60*e/11) + x**20*(6*d + 21*e/2) + x**18*(35*d/3 + 14*e) + x
**16*(63*d/4 + 105*e/8) + x**14*(15*d + 60*e/7) + x**12*(10*d + 15*e/4) + x
**10*(9*d/2 + e) + x**8*(5*d/4 + e/8)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(59) = 118$.
time = 4.03, size = 143, normalized size = 2.27

$$\frac{1}{28}x^{28}e + \frac{1}{26}dx^{26} + \frac{5}{13}x^{26}e + \frac{5}{12}dx^{24} + \frac{15}{8}x^{24}e + \frac{45}{22}dx^{22} + \frac{60}{11}x^{22}e + 6dx^{20} + \frac{21}{2}x^{20}e + \frac{35}{3}dx^{18} + 14x^{18}e + \frac{63}{4}dx^{16} + \frac{105}{8}x^{16}e + 15dx^{14} + \frac{60}{7}x^{14}e + 10dx^{12} + \frac{15}{4}x^{12}e + \frac{9}{2}dx^{10} + x^{10}e + \frac{5}{4}dx^{8} + \frac{1}{8}x^{8}e + \frac{1}{6}dx^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28*e + 1/26*d*x^26 + 5/13*x^26*e + 5/12*d*x^24 + 15/8*x^24*e + 45/22
*d*x^22 + 60/11*x^22*e + 6*d*x^20 + 21/2*x^20*e + 35/3*d*x^18 + 14*x^18*e +
 63/4*d*x^16 + 105/8*x^16*e + 15*d*x^14 + 60/7*x^14*e + 10*d*x^12 + 15/4*x^
12*e + 9/2*d*x^10 + x^10*e + 5/4*d*x^8 + 1/8*x^8*e + 1/6*d*x^6
```

Mupad [B]
time = 0.09, size = 121, normalized size = 1.92

$$\frac{e\,x^{28}}{28}+\left(\frac{d}{26}+\frac{5\,e}{13}\right)\,x^{26}+\left(\frac{5\,d}{12}+\frac{15\,e}{8}\right)\,x^{24}+\left(\frac{45\,d}{22}+\frac{60\,e}{11}\right)\,x^{22}+\left(6\,d+\frac{21\,e}{2}\right)\,x^{20}+\left(\frac{35\,d}{3}+14\,e\right)\,x^{18}+\left(\frac{63\,d}{4}+\frac{105\,e}{8}\right)\,x^{16}+\left(15\,d+\frac{60\,e}{7}\right)\,x^{14}+\left(10\,d+\frac{15\,e}{4}\right)\,x^{12}+\left(\frac{9\,d}{2}+e\right)\,x^{10}+\left(\frac{5\,d}{4}+\frac{e}{8}\right)\,x^8+\frac{d\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^8*((5*d)/4 + e/8) + x^12*(10*d + (15*e)/4) + x^20*(6*d + (21*e)/2) + x^24
*((5*d)/12 + (15*e)/8) + x^18*((35*d)/3 + 14*e) + x^26*(d/26 + (5*e)/13) +
x^14*(15*d + (60*e)/7) + x^22*((45*d)/22 + (60*e)/11) + x^16*((63*d)/4 + (1
05*e)/8) + (d*x^6)/6 + (e*x^28)/28 + x^10*((9*d)/2 + e)
```

3.57 $\int x^4(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=153

$$\frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}+\frac{14}{5}(6d+5e)x^{15}+\frac{42}{17}(5d+6e)x^{17}+\frac{30}{19}(4d+7e$$

```
[Out] 1/5*d*x^5+1/7*(10*d+e)*x^7+5/9*(9*d+2*e)*x^9+15/11*(8*d+3*e)*x^11+30/13*(7*
d+4*e)*x^13+14/5*(6*d+5*e)*x^15+42/17*(5*d+6*e)*x^17+30/19*(4*d+7*e)*x^19+5
/7*(3*d+8*e)*x^21+5/23*(2*d+9*e)*x^23+1/25*(d+10*e)*x^25+1/27*e*x^27
```

Rubi [A]
time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {28, 459}

$$\frac{1}{25}x^{25}(d+10e)+\frac{5}{23}x^{23}(2d+9e)+\frac{5}{7}x^{21}(3d+8e)+\frac{30}{19}x^{19}(4d+7e)+\frac{42}{17}x^{17}(5d+6e)+\frac{14}{5}x^{15}(6d+5e)+\frac{30}{13}x^{13}(7d+4e)+\frac{15}{11}x^{11}(8d+3e)+\frac{5}{9}x^9(9d+2e)+\frac{1}{7}x^7(10d+e)+\frac{dx^5}{5}+\frac{ex^{27}}{27}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^4\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\ &= \int \left(dx^4+(10d+e)x^6+5(9d+2e)x^8+15(8d+3e)x^{10}+30(7d+4e)x\right.\\ &= \frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x\end{aligned}$$

Mathematica [A]
time = 0.02, size = 153, normalized size = 1.00

$$\frac{dx^5}{5}+\frac{1}{7}(10d+e)x^7+\frac{5}{9}(9d+2e)x^9+\frac{15}{11}(8d+3e)x^{11}+\frac{30}{13}(7d+4e)x^{13}+\frac{14}{5}(6d+5e)x^{15}+\frac{42}{17}(5d+6e)x^{17}+\frac{30}{19}(4d+7e)x^{19}+\frac{5}{7}(3d+8e)x^{21}+\frac{5}{23}(2d+9e)x^{23}+\frac{1}{25}(d+10e)x^{25}+\frac{ex^{27}}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^5)/5 + ((10*d + e)*x^7)/7 + (5*(9*d + 2*e)*x^9)/9 + (15*(8*d + 3*e)*x^
11)/11 + (30*(7*d + 4*e)*x^13)/13 + (14*(6*d + 5*e)*x^15)/5 + (42*(5*d + 6*
e)*x^17)/17 + (30*(4*d + 7*e)*x^19)/19 + (5*(3*d + 8*e)*x^21)/7 + (5*(2*d +
 9*e)*x^23)/23 + ((d + 10*e)*x^25)/25 + (e*x^27)/27
```

Maple [A]
time = 0.08, size = 130, normalized size = 0.85

method	result
norman	$\frac{e\,x^{27}}{27}+\left(\frac{210d}{17}+\frac{252e}{17}\right)x^{17}+\left(\frac{120d}{19}+\frac{210e}{19}\right)x^{19}+\left(\frac{15d}{7}+\frac{40e}{7}\right)x^{21}+\left(\frac{10d}{23}+\frac{45e}{23}\right)x^{23}+\left(\frac{d}{25}+\frac{2e}{5}\right)x^{25}+$
default	$\frac{e\,x^{27}}{27}+\frac{(d+10e)x^{25}}{25}+\frac{(10d+45e)x^{23}}{23}+\frac{(45d+120e)x^{21}}{21}+\frac{(120d+210e)x^{19}}{19}+\frac{(210d+252e)x^{17}}{17}+\frac{(252d+210e)x^{15}}{15}+$ (21
risch	$\frac{1}{27}e\,x^{27}+\frac{1}{25}x^{25}d+\frac{2}{5}e\,x^{25}+\frac{10}{23}x^{23}d+\frac{45}{23}e\,x^{23}+\frac{15}{7}x^{21}d+\frac{40}{7}e\,x^{21}+\frac{120}{19}x^{19}d+\frac{210}{19}x^{19}e+\frac{210}{17}x^{17}d-$
gosper	$x^5\,(185910725e\,x^{22}+200783583d\,x^{20}+2007835830e\,x^{20}+2182430250d\,x^{18}+9820936125e\,x^{18}+10756263375d\,x^{16}+28683369000e\,x^{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/27*e*x^27+1/25*(d+10*e)*x^25+1/23*(10*d+45*e)*x^23+1/21*(45*d+120*e)*x^21
+1/19*(120*d+210*e)*x^19+1/17*(210*d+252*e)*x^17+1/15*(252*d+210*e)*x^15+1/
13*(210*d+120*e)*x^13+1/11*(120*d+45*e)*x^11+1/9*(45*d+10*e)*x^9+1/7*(10*d+
e)*x^7+1/5*d*x^5
```

Maxima [A]
time = 0.30, size = 140, normalized size = 0.92

$$\frac{1}{27}x^{27}e+\frac{1}{25}(d+10e)x^{25}+\frac{5}{23}(2d+9e)x^{23}+\frac{5}{7}(3d+8e)x^{21}+\frac{30}{19}(4d+7e)x^{19}+\frac{42}{17}(5d+6e)x^{17}+\frac{14}{5}(6d+5e)x^{15}+\frac{30}{13}(7d+4e)x^{13}+\frac{15}{11}(8d+3e)x^{11}+\frac{5}{9}(9d+2e)x^9+\frac{1}{7}(10d+e)x^7+\frac{1}{5}dx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/27*x^27*e + 1/25*(d + 10*e)*x^25 + 5/23*(2*d + 9*e)*x^23 + 5/7*(3*d + 8*e
)*x^21 + 30/19*(4*d + 7*e)*x^19 + 42/17*(5*d + 6*e)*x^17 + 14/5*(6*d + 5*e)
*x^15 + 30/13*(7*d + 4*e)*x^13 + 15/11*(8*d + 3*e)*x^11 + 5/9*(9*d + 2*e)*x
^9 + 1/7*(10*d + e)*x^7 + 1/5*d*x^5
```

Fricas [A]
time = 0.37, size = 127, normalized size = 0.83

$$\frac{1}{25}dx^{25} + \frac{10}{23}dx^{23} + \frac{15}{7}dx^{21} + \frac{120}{19}dx^{19} + \frac{210}{17}dx^{17} + \frac{84}{5}dx^{15} + \frac{210}{13}dx^{13} + \frac{120}{11}dx^{11} + 5dx^9 + \frac{10}{7}dx^7 + \frac{1}{5}dx^5 + \frac{1}{1003917915}(37182145x^{27} + 401567166x^{25} + 1964187225x^{23} + 5736673800x^{21} + 11095934850x^{19} + 14881606740x^{17} + 14054850810x^{15} + 9266934600x^{13} + 4106936925x^{11} + 1115464350x^9 + 143416845x^7)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/25*d*x^25 + 10/23*d*x^23 + 15/7*d*x^21 + 120/19*d*x^19 + 210/17*d*x^17 +
84/5*d*x^15 + 210/13*d*x^13 + 120/11*d*x^11 + 5*d*x^9 + 10/7*d*x^7 + 1/5*d*
x^5 + 1/1003917915*(37182145*x^27 + 401567166*x^25 + 1964187225*x^23 + 5736
673800*x^21 + 11095934850*x^19 + 14881606740*x^17 + 14054850810*x^15 + 9266
934600*x^13 + 4106936925*x^11 + 1115464350*x^9 + 143416845*x^7)*e
```

Sympy [A]
time = 0.02, size = 141, normalized size = 0.92

$$\frac{dx^5}{5} + \frac{ex^{27}}{27} + x^{25}\left(\frac{d}{25} + \frac{2e}{5}\right) + x^{23}\cdot\left(\frac{10d}{23} + \frac{45e}{23}\right) + x^{21}\cdot\left(\frac{15d}{7} + \frac{40e}{7}\right) + x^{19}\cdot\left(\frac{120d}{19} + \frac{210e}{19}\right) + x^{17}\cdot\left(\frac{210d}{17} + \frac{252e}{17}\right) + x^{15}\cdot\left(\frac{84d}{5} + 14e\right) + x^{13}\cdot\left(\frac{210d}{13} + \frac{120e}{13}\right) + x^{11}\cdot\left(\frac{120d}{11} + \frac{45e}{11}\right) + x^9\cdot\left(5d + \frac{10e}{9}\right) + x^7\cdot\left(\frac{10d}{7} + \frac{e}{7}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**5/5 + e*x**27/27 + x**25*(d/25 + 2*e/5) + x**23*(10*d/23 + 45*e/23) +
x**21*(15*d/7 + 40*e/7) + x**19*(120*d/19 + 210*e/19) + x**17*(210*d/17 + 2
52*e/17) + x**15*(84*d/5 + 14*e) + x**13*(210*d/13 + 120*e/13) + x**11*(120
*d/11 + 45*e/11) + x**9*(5*d + 10*e/9) + x**7*(10*d/7 + e/7)
```

Giac [A]
time = 4.40, size = 144, normalized size = 0.94

$$\frac{1}{27}x^{27}e + \frac{1}{25}dx^{25} + \frac{2}{5}x^{25}e + \frac{10}{23}dx^{23} + \frac{45}{23}x^{23}e + \frac{15}{7}dx^{21} + \frac{40}{7}x^{21}e + \frac{120}{19}dx^{19} + \frac{210}{19}x^{19}e + \frac{210}{17}dx^{17} + \frac{252}{17}x^{17}e + \frac{84}{5}dx^{15} + 14x^{15}e + \frac{210}{13}dx^{13} + \frac{120}{13}x^{13}e + \frac{120}{11}dx^{11} + \frac{45}{11}x^{11}e + 5dx^9 + \frac{10}{9}x^9e + \frac{10}{7}dx^7 + \frac{1}{7}x^7e + \frac{1}{5}dx^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/27*x^27*e + 1/25*d*x^25 + 2/5*x^25*e + 10/23*d*x^23 + 45/23*x^23*e + 15/7
*d*x^21 + 40/7*x^21*e + 120/19*d*x^19 + 210/19*x^19*e + 210/17*d*x^17 + 252
/17*x^17*e + 84/5*d*x^15 + 14*x^15*e + 210/13*d*x^13 + 120/13*x^13*e + 120/
11*d*x^11 + 45/11*x^11*e + 5*d*x^9 + 10/9*x^9*e + 10/7*d*x^7 + 1/7*x^7*e +
1/5*d*x^5
```

Mupad [B]
time = 0.12, size = 123, normalized size = 0.80

$$\frac{e\,x^{27}}{27} + \left(\frac{d}{25} + \frac{2\,e}{5}\right)x^{25} + \left(\frac{10\,d}{23} + \frac{45\,e}{23}\right)x^{23} + \left(\frac{15\,d}{7} + \frac{40\,e}{7}\right)x^{21} + \left(\frac{120\,d}{19} + \frac{210\,e}{19}\right)x^{19} + \left(\frac{210\,d}{17} + \frac{252\,e}{17}\right)x^{17} + \left(\frac{84\,d}{5} + 14\,e\right)x^{15} + \left(\frac{210\,d}{13} + \frac{120\,e}{13}\right)x^{13} + \left(\frac{120\,d}{11} + \frac{45\,e}{11}\right)x^{11} + \left(5\,d + \frac{10\,e}{9}\right)x^9 + \left(\frac{10\,d}{7} + \frac{e}{7}\right)x^7 + \frac{d\,x^5}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^7*((10*d)/7 + e/7) + x^9*(5*d + (10*e)/9) + x^25*(d/25 + (2*e)/5) + x^21*
((15*d)/7 + (40*e)/7) + x^15*((84*d)/5 + 14*e) + x^23*((10*d)/23 + (45*e)/2
3) + x^11*((120*d)/11 + (45*e)/11) + x^13*((210*d)/13 + (120*e)/13) + x^19*
((120*d)/19 + (210*e)/19) + x^17*((210*d)/17 + (252*e)/17) + (d*x^5)/5 + (e
*x^27)/27
```

3.58 $\int x^3(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=45

$$-\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}(d-2e)\left(1+x^2\right)^{12}+\frac{1}{26}e\left(1+x^2\right)^{13}$$

```
[Out] -1/22*(d-e)*(x^2+1)^11+1/24*(d-2*e)*(x^2+1)^12+1/26*e*(x^2+1)^13
```

Rubi [A]
time = 0.08, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {28, 457, 77}

$$\frac{1}{24}\left(x^2+1\right)^{12}(d-2e)-\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{26}e\left(x^2+1\right)^{13}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -1/22*((d - e)*(1 + x^2)^11) + ((d - 2*e)*(1 + x^2)^12)/24 + (e*(1 + x^2)^1
3)/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\begin{aligned}\int x^3\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^3\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\ &= \frac{1}{2}\text{Subst}\left(\int x(1+x)^{10}(d+ex)\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left((-d+e)(1+x)^{10}+(d-2e)(1+x)^{11}+e(1+x)^{12}\right)\,dx,\right.\\ &= -\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}(d-2e)\left(1+x^2\right)^{12}+\frac{1}{26}e\left(1+x^2\right)^{13}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 151 vs. $2(45) = 90$.
time = 0.01, size = 151, normalized size = 3.36

$$\frac{dx^4}{4}+\frac{1}{6}(10d+e)x^6+\frac{5}{8}(9d+2e)x^8+\frac{3}{2}(8d+3e)x^{10}+\frac{5}{2}(7d+4e)x^{12}+3(6d+5e)x^{14}+\frac{21}{8}(5d+6e)x^{16}+\frac{5}{3}(4d+7e)x^{18}+\frac{3}{4}(3d+8e)x^{20}+\frac{5}{22}(2d+9e)x^{22}+\frac{1}{24}(d+10e)x^{24}+\frac{ex^{26}}{26}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^4)/4 + ((10*d + e)*x^6)/6 + (5*(9*d + 2*e)*x^8)/8 + (3*(8*d + 3*e)*x^1
0)/2 + (5*(7*d + 4*e)*x^12)/2 + 3*(6*d + 5*e)*x^14 + (21*(5*d + 6*e)*x^16)/
8 + (5*(4*d + 7*e)*x^18)/3 + (3*(3*d + 8*e)*x^20)/4 + (5*(2*d + 9*e)*x^22)/
22 + ((d + 10*e)*x^24)/24 + (e*x^26)/26
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(39) = 78$.
time = 0.08, size = 130, normalized size = 2.89

method	result
norman	$\left(\frac{20d}{3}+\frac{35e}{3}\right)x^{18}+\left(\frac{9d}{4}+6e\right)x^{20}+\left(\frac{5d}{11}+\frac{45e}{22}\right)x^{22}+\left(\frac{d}{24}+\frac{5e}{12}\right)x^{24}+\frac{x^{26}e}{26}+\frac{d\,x^4}{4}+\left(\frac{5d}{3}+\frac{e}{6}\right)x^6+($
default	$\frac{x^{26}e}{26}+\frac{(d+10e)x^{24}}{24}+\frac{(10d+45e)x^{22}}{22}+\frac{(45d+120e)x^{20}}{20}+\frac{(120d+210e)x^{18}}{18}+\frac{(210d+252e)x^{16}}{16}+\frac{(252d+210e)x^{14}}{14}+$
risch	$\frac{1}{26}x^{26}e+\frac{1}{24}x^{24}d+\frac{5}{12}x^{24}e+\frac{5}{11}x^{22}d+\frac{45}{22}e\,x^{22}+\frac{9}{4}d\,x^{20}+6e\,x^{20}+\frac{20}{3}d\,x^{18}+\frac{35}{3}e\,x^{18}+\frac{105}{8}d\,x^{16}+$
gosper	$\frac{x^4(132e\,x^{22}+143d\,x^{20}+1430e\,x^{20}+1560d\,x^{18}+7020e\,x^{18}+7722d\,x^{16}+20592e\,x^{16}+22880d\,x^{14}+40040e\,x^{14}+45045d\,x^{12}+54054e\,x^{12}}{3432}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/26*x^26*e+1/24*(d+10*e)*x^24+1/22*(10*d+45*e)*x^22+1/20*(45*d+120*e)*x^20
+1/18*(120*d+210*e)*x^18+1/16*(210*d+252*e)*x^16+1/14*(252*d+210*e)*x^14+1/
12*(210*d+120*e)*x^12+1/10*(120*d+45*e)*x^10+1/8*(45*d+10*e)*x^8+1/6*(10*d+
e)*x^6+1/4*d*x^4
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(42) = 84$.
time = 0.27, size = 140, normalized size = 3.11

$$\frac{1}{26}x^{26}e+\frac{1}{24}(d+10e)x^{24}+\frac{5}{22}(2d+9e)x^{22}+\frac{3}{4}(3d+8e)x^{20}+\frac{5}{3}(4d+7e)x^{18}+\frac{21}{8}(5d+6e)x^{16}+3(6d+5e)x^{14}+\frac{5}{2}(7d+4e)x^{12}+\frac{3}{2}(8d+3e)x^{10}+\frac{5}{8}(9d+2e)x^{8}+\frac{1}{6}(10d+e)x^{6}+\frac{1}{4}dx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/26*x^26*e + 1/24*(d + 10*e)*x^24 + 5/22*(2*d + 9*e)*x^22 + 3/4*(3*d + 8*e
)*x^20 + 5/3*(4*d + 7*e)*x^18 + 21/8*(5*d + 6*e)*x^16 + 3*(6*d + 5*e)*x^14
+ 5/2*(7*d + 4*e)*x^12 + 3/2*(8*d + 3*e)*x^10 + 5/8*(9*d + 2*e)*x^8 + 1/6*(
10*d + e)*x^6 + 1/4*d*x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(42) = 84$.
time = 0.35, size = 127, normalized size = 2.82

$$\frac{1}{24}dx^{24}+\frac{5}{11}dx^{22}+\frac{9}{4}dx^{20}+\frac{20}{3}dx^{18}+\frac{105}{8}dx^{16}+18dx^{14}+\frac{35}{2}dx^{12}+12dx^{10}+\frac{45}{8}dx^{8}+\frac{5}{3}dx^{6}+\frac{1}{4}dx^{4}+\frac{1}{1716}(66x^{26}+715x^{24}+3510x^{22}+10296x^{20}+20020x^{18}+27027x^{16}+25740x^{14}+17160x^{12}+7722x^{10}+2145x^{8}+286x^{6})e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/24*d*x^24 + 5/11*d*x^22 + 9/4*d*x^20 + 20/3*d*x^18 + 105/8*d*x^16 + 18*d*
x^14 + 35/2*d*x^12 + 12*d*x^10 + 45/8*d*x^8 + 5/3*d*x^6 + 1/4*d*x^4 + 1/171
6*(66*x^26 + 715*x^24 + 3510*x^22 + 10296*x^20 + 20020*x^18 + 27027*x^16 +
25740*x^14 + 17160*x^12 + 7722*x^10 + 2145*x^8 + 286*x^6)*e
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(36) = 72$.
time = 0.02, size = 136, normalized size = 3.02

$$\frac{dx^{4}}{4}+\frac{ex^{26}}{26}+x^{24}\left(\frac{d}{24}+\frac{5e}{12}\right)+x^{22}\cdot\left(\frac{5d}{11}+\frac{45e}{22}\right)+x^{20}\cdot\left(\frac{9d}{4}+6e\right)+x^{18}\cdot\left(\frac{20d}{3}+\frac{35e}{3}\right)+x^{16}\cdot\left(\frac{105d}{8}+\frac{63e}{4}\right)+x^{14}\cdot(18d+15e)+x^{12}\cdot\left(\frac{35d}{2}+10e\right)+x^{10}\cdot\left(12d+\frac{9e}{2}\right)+x^{8}\cdot\left(\frac{45d}{8}+\frac{5e}{4}\right)+x^{6}\cdot\left(\frac{5d}{3}+\frac{e}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**4/4 + e*x**26/26 + x**24*(d/24 + 5*e/12) + x**22*(5*d/11 + 45*e/22) +
x**20*(9*d/4 + 6*e) + x**18*(20*d/3 + 35*e/3) + x**16*(105*d/8 + 63*e/4) +
x**14*(18*d + 15*e) + x**12*(35*d/2 + 10*e) + x**10*(12*d + 9*e/2) + x**8*(
45*d/8 + 5*e/4) + x**6*(5*d/3 + e/6)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(42) = 84$.
time = 4.99, size = 144, normalized size = 3.20

$$\frac{1}{26}x^{26}e+\frac{1}{24}dx^{24}+\frac{5}{12}x^{24}e+\frac{5}{11}dx^{22}+\frac{45}{22}x^{22}e+\frac{9}{4}dx^{20}+6x^{20}e+\frac{20}{3}dx^{18}+\frac{35}{3}x^{18}e+\frac{105}{8}dx^{16}+\frac{63}{4}x^{16}e+18dx^{14}+15x^{14}e+\frac{35}{2}dx^{12}+10x^{12}e+12dx^{10}+\frac{9}{2}x^{10}e+\frac{45}{8}dx^{8}+\frac{5}{4}x^{8}e+\frac{5}{3}dx^{6}+\frac{1}{6}x^{6}e+\frac{1}{4}dx^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/26*x^26*e + 1/24*d*x^24 + 5/12*x^24*e + 5/11*d*x^22 + 45/22*x^22*e + 9/4*
d*x^20 + 6*x^20*e + 20/3*d*x^18 + 35/3*x^18*e + 105/8*d*x^16 + 63/4*x^16*e
+ 18*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 10*x^12*e + 12*d*x^10 + 9/2*x^10*e
+ 45/8*d*x^8 + 5/4*x^8*e + 5/3*d*x^6 + 1/6*x^6*e + 1/4*d*x^4
```

Mupad [B]
time = 0.08, size = 123, normalized size = 2.73

$$\frac{e\,x^{26}}{26}+\left(\frac{d}{24}+\frac{5\,e}{12}\right)\,x^{24}+\left(\frac{5\,d}{11}+\frac{45\,e}{22}\right)\,x^{22}+\left(\frac{9\,d}{4}+6\,e\right)\,x^{20}+\left(\frac{20\,d}{3}+\frac{35\,e}{3}\right)\,x^{18}+\left(\frac{105\,d}{8}+\frac{63\,e}{4}\right)\,x^{16}+\left(18\,d+15\,e\right)\,x^{14}+\left(\frac{35\,d}{2}+10\,e\right)\,x^{12}+\left(12\,d+\frac{9\,e}{2}\right)\,x^{10}+\left(\frac{45\,d}{8}+\frac{5\,e}{4}\right)\,x^8+\left(\frac{5\,d}{3}+\frac{e}{6}\right)\,x^6+\frac{d\,x^4}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^6*((5*d)/3 + e/6) + x^10*(12*d + (9*e)/2) + x^20*((9*d)/4 + 6*e) + x^14*(
18*d + 15*e) + x^12*((35*d)/2 + 10*e) + x^24*(d/24 + (5*e)/12) + x^8*((45*d
)/8 + (5*e)/4) + x^18*((20*d)/3 + (35*e)/3) + x^22*((5*d)/11 + (45*e)/22) +
 x^16*((105*d)/8 + (63*e)/4) + (d*x^4)/4 + (e*x^26)/26
```

3.59 $\int x^2(d+ex^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=153

$$\frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+\frac{42}{13}(6d+5e)x^{13}+\frac{14}{5}(5d+6e)x^{15}+\frac{30}{17}(4d+7e)x$$

```
[Out] 1/3*d*x^3+1/5*(10*d+e)*x^5+5/7*(9*d+2*e)*x^7+5/3*(8*d+3*e)*x^9+30/11*(7*d+4
*e)*x^11+42/13*(6*d+5*e)*x^13+14/5*(5*d+6*e)*x^15+30/17*(4*d+7*e)*x^17+15/1
9*(3*d+8*e)*x^19+5/21*(2*d+9*e)*x^21+1/23*(d+10*e)*x^23+1/25*e*x^25
```

Rubi [A]
time = 0.06, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 459}

$$\frac{1}{23}x^{23}(d+10e)+\frac{5}{21}x^{21}(2d+9e)+\frac{15}{19}x^{19}(3d+8e)+\frac{30}{17}x^{17}(4d+7e)+\frac{14}{5}x^{15}(5d+6e)+\frac{42}{13}x^{13}(6d+5e)+\frac{30}{11}x^{11}(7d+4e)+\frac{5}{3}x^9(8d+3e)+\frac{5}{7}x^7(9d+2e)+\frac{1}{5}x^5(10d+e)+\frac{dx^3}{3}+\frac{ex^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(d+ex^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^2\left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\ &= \int \left(dx^2+(10d+e)x^4+5(9d+2e)x^6+15(8d+3e)x^8+30(7d+4e)x\right.\\ &= \frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 153, normalized size = 1.00

$$\frac{dx^3}{3}+\frac{1}{5}(10d+e)x^5+\frac{5}{7}(9d+2e)x^7+\frac{5}{3}(8d+3e)x^9+\frac{30}{11}(7d+4e)x^{11}+\frac{42}{13}(6d+5e)x^{13}+\frac{14}{5}(5d+6e)x^{15}+\frac{30}{17}(4d+7e)x^{17}+\frac{15}{19}(3d+8e)x^{19}+\frac{5}{21}(2d+9e)x^{21}+\frac{1}{23}(d+10e)x^{23}+\frac{ex^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^3)/3 + ((10*d + e)*x^5)/5 + (5*(9*d + 2*e)*x^7)/7 + (5*(8*d + 3*e)*x^9
)/3 + (30*(7*d + 4*e)*x^11)/11 + (42*(6*d + 5*e)*x^13)/13 + (14*(5*d + 6*e)
*x^15)/5 + (30*(4*d + 7*e)*x^17)/17 + (15*(3*d + 8*e)*x^19)/19 + (5*(2*d +
9*e)*x^21)/21 + ((d + 10*e)*x^23)/23 + (e*x^25)/25
```

Maple [A]
time = 0.08, size = 130, normalized size = 0.85

method	result
norman	$\frac{e\,x^{25}}{25}+\left(\frac{d}{23}+\frac{10e}{23}\right)x^{23}+\left(\frac{252d}{13}+\frac{210e}{13}\right)x^{13}+\left(14d+\frac{84e}{5}\right)x^{15}+\left(\frac{120d}{17}+\frac{210e}{17}\right)x^{17}+\left(\frac{45d}{19}+\frac{120e}{19}\right)x^{19}$
default	$\frac{e\,x^{25}}{25}+\frac{(d+10e)x^{23}}{23}+\frac{(10d+45e)x^{21}}{21}+\frac{(45d+120e)x^{19}}{19}+\frac{(120d+210e)x^{17}}{17}+\frac{(210d+252e)x^{15}}{15}+\frac{(252d+210e)x^{13}}{13}+\frac{(21}{}$
risch	$\frac{1}{25}e\,x^{25}+\frac{1}{23}x^{23}d+\frac{10}{23}e\,x^{23}+\frac{10}{21}x^{21}d+\frac{15}{7}e\,x^{21}+\frac{45}{19}x^{19}d+\frac{120}{19}x^{19}e+\frac{120}{17}x^{17}d+\frac{210}{17}x^{17}e+14x^{15}d$
gosper	$x^3\left(22309287e\,x^{22}+24249225d\,x^{20}+242492250e\,x^{20}+265586750d\,x^{18}+1195140375e\,x^{18}+1320944625d\,x^{16}+3522519000e\,x^{16}+3936\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/25*e*x^25+1/23*(d+10*e)*x^23+1/21*(10*d+45*e)*x^21+1/19*(45*d+120*e)*x^19
+1/17*(120*d+210*e)*x^17+1/15*(210*d+252*e)*x^15+1/13*(252*d+210*e)*x^13+1/
11*(210*d+120*e)*x^11+1/9*(120*d+45*e)*x^9+1/7*(45*d+10*e)*x^7+1/5*(10*d+e)
*x^5+1/3*d*x^3
```

Maxima [A]
time = 0.28, size = 140, normalized size = 0.92

$$\frac{1}{25}x^{25}e+\frac{1}{23}(d+10e)x^{23}+\frac{5}{21}(2d+9e)x^{21}+\frac{15}{19}(3d+8e)x^{19}+\frac{30}{17}(4d+7e)x^{17}+\frac{14}{5}(5d+6e)x^{15}+\frac{42}{13}(6d+5e)x^{13}+\frac{30}{11}(7d+4e)x^{11}+\frac{5}{3}(8d+3e)x^9+\frac{5}{7}(9d+2e)x^7+\frac{1}{5}(10d+e)x^5+\frac{1}{3}dx^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*x^25*e + 1/23*(d + 10*e)*x^23 + 5/21*(2*d + 9*e)*x^21 + 15/19*(3*d + 8
*e)*x^19 + 30/17*(4*d + 7*e)*x^17 + 14/5*(5*d + 6*e)*x^15 + 42/13*(6*d + 5*
e)*x^13 + 30/11*(7*d + 4*e)*x^11 + 5/3*(8*d + 3*e)*x^9 + 5/7*(9*d + 2*e)*x^
7 + 1/5*(10*d + e)*x^5 + 1/3*d*x^3
```

Fricas [A]
time = 0.35, size = 127, normalized size = 0.83

$$\frac{1}{23}dx^{23}+\frac{10}{21}dx^{21}+\frac{45}{19}dx^{19}+\frac{120}{17}dx^{17}+14dx^{15}+\frac{252}{13}dx^{13}+\frac{210}{11}dx^{11}+\frac{40}{3}dx^{9}+\frac{45}{7}dx^{7}+2dx^{5}+\frac{1}{3}dx^{3}+\frac{1}{185910725}\left(7436429x^{25}+80830750x^{23}+398380125x^{21}+1174173000x^{19}+2296544250x^{17}+3123300180x^{15}+3003173250x^{13}+2028117000x^{11}+929553625x^{9}+265586750x^{7}+37182145x^{5}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*d*x^23 + 10/21*d*x^21 + 45/19*d*x^19 + 120/17*d*x^17 + 14*d*x^15 + 252
/13*d*x^13 + 210/11*d*x^11 + 40/3*d*x^9 + 45/7*d*x^7 + 2*d*x^5 + 1/3*d*x^3
+ 1/185910725*(7436429*x^25 + 80830750*x^23 + 398380125*x^21 + 1174173000*x
^19 + 2296544250*x^17 + 3123300180*x^15 + 3003173250*x^13 + 2028117000*x^11
 + 929553625*x^9 + 265586750*x^7 + 37182145*x^5)*e
```

Sympy [A]
time = 0.02, size = 139, normalized size = 0.91

$$\frac{dx^{3}}{3}+\frac{ex^{25}}{25}+x^{23}\left(\frac{d}{23}+\frac{10e}{23}\right)+x^{21}\cdot\left(\frac{10d}{21}+\frac{15e}{7}\right)+x^{19}\cdot\left(\frac{45d}{19}+\frac{120e}{19}\right)+x^{17}\cdot\left(\frac{120d}{17}+\frac{210e}{17}\right)+x^{15}\cdot\left(14d+\frac{84e}{5}\right)+x^{13}\cdot\left(\frac{252d}{13}+\frac{210e}{13}\right)+x^{11}\cdot\left(\frac{210d}{11}+\frac{120e}{11}\right)+x^{9}\cdot\left(\frac{40d}{3}+5e\right)+x^{7}\cdot\left(\frac{45d}{7}+\frac{10e}{7}\right)+x^{5}\cdot\left(2d+\frac{e}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**3/3 + e*x**25/25 + x**23*(d/23 + 10*e/23) + x**21*(10*d/21 + 15*e/7) +
 x**19*(45*d/19 + 120*e/19) + x**17*(120*d/17 + 210*e/17) + x**15*(14*d + 8
4*e/5) + x**13*(252*d/13 + 210*e/13) + x**11*(210*d/11 + 120*e/11) + x**9*(
40*d/3 + 5*e) + x**7*(45*d/7 + 10*e/7) + x**5*(2*d + e/5)
```

Giac [A]
time = 4.17, size = 144, normalized size = 0.94

$$\frac{1}{25}x^{25}e+\frac{1}{23}dx^{23}+\frac{10}{23}x^{23}e+\frac{10}{21}dx^{21}+\frac{15}{7}x^{21}e+\frac{45}{19}dx^{19}+\frac{120}{19}x^{19}e+\frac{120}{17}dx^{17}+\frac{210}{17}x^{17}e+14dx^{15}+\frac{84}{5}x^{15}e+\frac{252}{13}dx^{13}+\frac{210}{13}x^{13}e+\frac{210}{11}dx^{11}+\frac{120}{11}x^{11}e+\frac{40}{3}dx^{9}+5x^{9}e+\frac{45}{7}dx^{7}+\frac{10}{7}x^{7}e+2dx^{5}+\frac{1}{5}x^{5}e+\frac{1}{3}dx^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*x^25*e + 1/23*d*x^23 + 10/23*x^23*e + 10/21*d*x^21 + 15/7*x^21*e + 45/
19*d*x^19 + 120/19*x^19*e + 120/17*d*x^17 + 210/17*x^17*e + 14*d*x^15 + 84/
5*x^15*e + 252/13*d*x^13 + 210/13*x^13*e + 210/11*d*x^11 + 120/11*x^11*e +
40/3*d*x^9 + 5*x^9*e + 45/7*d*x^7 + 10/7*x^7*e + 2*d*x^5 + 1/5*x^5*e + 1/3*
d*x^3
```

Mupad [B]
time = 0.08, size = 123, normalized size = 0.80

$$\frac{e\,x^{25}}{25}+\left(\frac{d}{23}+\frac{10\,e}{23}\right)x^{23}+\left(\frac{10\,d}{21}+\frac{15\,e}{7}\right)x^{21}+\left(\frac{45\,d}{19}+\frac{120\,e}{19}\right)x^{19}+\left(\frac{120\,d}{17}+\frac{210\,e}{17}\right)x^{17}+\left(14\,d+\frac{84\,e}{5}\right)x^{15}+\left(\frac{252\,d}{13}+\frac{210\,e}{13}\right)x^{13}+\left(\frac{210\,d}{11}+\frac{120\,e}{11}\right)x^{11}+\left(\frac{40\,d}{3}+5\,e\right)x^{9}+\left(\frac{45\,d}{7}+\frac{10\,e}{7}\right)x^{7}+\left(2\,d+\frac{e}{5}\right)x^{5}+\frac{d\,x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)

[Out] x^5*(2*d + e/5) + x^9*((40*d)/3 + 5*e) + x^21*((10*d)/21 + (15*e)/7) + x^7*((45*d)/7 + (10*e)/7) + x^23*(d/23 + (10*e)/23) + x^15*(14*d + (84*e)/5) + x^19*((45*d)/19 + (120*e)/19) + x^11*((210*d)/11 + (120*e)/11) + x^17*((120*d)/17 + (210*e)/17) + x^13*((252*d)/13 + (210*e)/13) + (d*x^3)/3 + (e*x^25)/25

3.60 $\int x(d + ex^2)\left(1 + 2x^2 + x^4\right)^5\, dx$

Optimal. Leaf size=29

$$\frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24}e\left(1+x^2\right)^{12}$$

```
[Out] 1/22*(d-e)*(x^2+1)^11+1/24*e*(x^2+1)^12
```

Rubi [A]
time = 0.03, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 455, 45}

$$\frac{1}{22}\left(x^2+1\right)^{11}(d-e)+\frac{1}{24}e\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] ((d - e)*(1 + x^2)^11)/22 + (e*(1 + x^2)^12)/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 455

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rubi steps

$$
\begin{aligned}
\int x\left(d+e x^2\right)\left(1+2 x^2+x^4\right)^5 \,dx &= \int x\left(1+x^2\right)^{10}\left(d+e x^2\right) \,dx \\
&= \frac{1}{2} \text{Subst}\left(\int (1+x)^{10}(d+e x) \,dx, x, x^2\right) \\
&= \frac{1}{2} \text{Subst}\left(\int \left((d-e)(1+x)^{10}+e(1+x)^{11}\right) \,dx, x, x^2\right) \\
&= \frac{1}{22}(d-e)\left(1+x^2\right)^{11}+\frac{1}{24} e\left(1+x^2\right)^{12}
\end{aligned}
$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 149 vs. $2(29) = 58$.
time = 0.01, size = 149, normalized size = 5.14

$$
\frac{dx^2}{2}+\frac{1}{4}(10d+e)x^4+\frac{5}{6}(9d+2e)x^6+\frac{15}{8}(8d+3e)x^8+3(7d+4e)x^{10}+\frac{7}{2}(6d+5e)x^{12}+3(5d+6e)x^{14}+\frac{15}{8}(4d+7e)x^{16}+\frac{5}{6}(3d+8e)x^{18}+\frac{1}{4}(2d+9e)x^{20}+\frac{1}{22}(d+10e)x^{22}+\frac{ex^{24}}{24}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (d*x^2)/2 + ((10*d + e)*x^4)/4 + (5*(9*d + 2*e)*x^6)/6 + (15*(8*d + 3*e)*x^
8)/8 + 3*(7*d + 4*e)*x^10 + (7*(6*d + 5*e)*x^12)/2 + 3*(5*d + 6*e)*x^14 + (
15*(4*d + 7*e)*x^16)/8 + (5*(3*d + 8*e)*x^18)/6 + ((2*d + 9*e)*x^20)/4 + ((
d + 10*e)*x^22)/22 + (e*x^24)/24
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(25) = 50$.
time = 0.08, size = 130, normalized size = 4.48

method	result
norman	$\left(\frac{5d}{2}+\frac{20e}{3}\right)x^{18}+\left(\frac{d}{2}+\frac{9e}{4}\right)x^{20}+\left(\frac{d}{22}+\frac{5e}{11}\right)x^{22}+\frac{x^{24}e}{24}+\left(15d+\frac{45e}{8}\right)x^{8}+\left(21d+12e\right)x^{10}+\left(21d\right.$
default	$\frac{x^{24}e}{24}+\frac{(d+10e)x^{22}}{22}+\frac{(10d+45e)x^{20}}{20}+\frac{(45d+120e)x^{18}}{18}+\frac{(120d+210e)x^{16}}{16}+\frac{(210d+252e)x^{14}}{14}+\frac{(252d+210e)x^{12}}{12}+$
risch	$\frac{1}{24}x^{24}e+\frac{1}{22}x^{22}d+\frac{5}{11}e\,x^{22}+\frac{1}{2}d\,x^{20}+\frac{9}{4}e\,x^{20}+\frac{5}{2}d\,x^{18}+\frac{20}{3}e\,x^{18}+\frac{15}{2}d\,x^{16}+\frac{105}{8}e\,x^{16}+15d\,x^{14}+$
gosper	$\frac{x^2\left(11e\,x^{22}+12d\,x^{20}+120e\,x^{20}+132d\,x^{18}+594e\,x^{18}+660d\,x^{16}+1760e\,x^{16}+1980d\,x^{14}+3465e\,x^{14}+3960d\,x^{12}+4752e\,x^{12}+5544d\,x^{10}\right.}{264}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*x^24*e+1/22*(d+10*e)*x^22+1/20*(10*d+45*e)*x^20+1/18*(45*d+120*e)*x^18
+1/16*(120*d+210*e)*x^16+1/14*(210*d+252*e)*x^14+1/12*(252*d+210*e)*x^12+1/
10*(210*d+120*e)*x^10+1/8*(120*d+45*e)*x^8+1/6*(45*d+10*e)*x^6+1/4*(10*d+e)
*x^4+1/2*d*x^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(27) = 54$.
time = 0.27, size = 140, normalized size = 4.83

$$\frac{1}{24}\,x^{24}e+\frac{1}{22}\,(d+10\,e)x^{22}+\frac{1}{4}\,(2\,d+9\,e)x^{20}+\frac{5}{6}\,(3\,d+8\,e)x^{18}+\frac{15}{8}\,(4\,d+7\,e)x^{16}+3\,(5\,d+6\,e)x^{14}+\frac{7}{2}\,(6\,d+5\,e)x^{12}+3\,(7\,d+4\,e)x^{10}+\frac{15}{8}\,(8\,d+3\,e)x^{8}+\frac{5}{6}\,(9\,d+2\,e)x^{6}+\frac{1}{4}\,(10\,d+e)x^{4}+\frac{1}{2}\,dx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*x^24*e + 1/22*(d + 10*e)*x^22 + 1/4*(2*d + 9*e)*x^20 + 5/6*(3*d + 8*e)
*x^18 + 15/8*(4*d + 7*e)*x^16 + 3*(5*d + 6*e)*x^14 + 7/2*(6*d + 5*e)*x^12 +
 3*(7*d + 4*e)*x^10 + 15/8*(8*d + 3*e)*x^8 + 5/6*(9*d + 2*e)*x^6 + 1/4*(10*
d + e)*x^4 + 1/2*d*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(27) = 54$.
time = 0.37, size = 127, normalized size = 4.38

$$\frac{1}{22}\,dx^{22}+\frac{1}{2}\,dx^{20}+\frac{5}{2}\,dx^{18}+\frac{15}{2}\,dx^{16}+15\,dx^{14}+21\,dx^{12}+21\,dx^{10}+15\,dx^{8}+\frac{15}{2}\,dx^{6}+\frac{5}{2}\,dx^{4}+\frac{1}{2}\,dx^{2}+\frac{1}{264}\,(11\,x^{24}+120\,x^{22}+594\,x^{20}+1760\,x^{18}+3465\,x^{16}+4752\,x^{14}+4620\,x^{12}+3168\,x^{10}+1485\,x^{8}+440\,x^{6}+66\,x^{4})e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/22*d*x^22 + 1/2*d*x^20 + 5/2*d*x^18 + 15/2*d*x^16 + 15*d*x^14 + 21*d*x^12
 + 21*d*x^10 + 15*d*x^8 + 15/2*d*x^6 + 5/2*d*x^4 + 1/2*d*x^2 + 1/264*(11*x^
24 + 120*x^22 + 594*x^20 + 1760*x^18 + 3465*x^16 + 4752*x^14 + 4620*x^12 +
3168*x^10 + 1485*x^8 + 440*x^6 + 66*x^4)*e
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(22) = 44$.
time = 0.03, size = 133, normalized size = 4.59

$$\frac{dx^2}{2}+\frac{ex^{24}}{24}+x^{22}\left(\frac{d}{22}+\frac{5e}{11}\right)+x^{20}\left(\frac{d}{2}+\frac{9e}{4}\right)+x^{18}\cdot\left(\frac{5d}{2}+\frac{20e}{3}\right)+x^{16}\cdot\left(\frac{15d}{2}+\frac{105e}{8}\right)+x^{14}\cdot(15d+18e)+x^{12}\cdot\left(21d+\frac{35e}{2}\right)+x^{10}\cdot(21d+12e)+x^{8}\cdot\left(15d+\frac{45e}{8}\right)+x^{6}\cdot\left(\frac{15d}{2}+\frac{5e}{3}\right)+x^{4}\cdot\left(\frac{5d}{2}+\frac{e}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x**2/2 + e*x**24/24 + x**22*(d/22 + 5*e/11) + x**20*(d/2 + 9*e/4) + x**18
*(5*d/2 + 20*e/3) + x**16*(15*d/2 + 105*e/8) + x**14*(15*d + 18*e) + x**12*
(21*d + 35*e/2) + x**10*(21*d + 12*e) + x**8*(15*d + 45*e/8) + x**6*(15*d/2
 + 5*e/3) + x**4*(5*d/2 + e/4)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(27) = 54$.
time = 4.32, size = 144, normalized size = 4.97

$$\frac{1}{24}\,x^{24}e+\frac{1}{22}\,dx^{22}+\frac{5}{11}\,x^{22}e+\frac{1}{2}\,dx^{20}+\frac{9}{4}\,x^{20}e+\frac{5}{2}\,dx^{18}+\frac{20}{3}\,x^{18}e+\frac{15}{2}\,dx^{16}+\frac{105}{8}\,x^{16}e+15\,dx^{14}+18\,x^{14}e+21\,dx^{12}+\frac{35}{2}\,x^{12}e+21\,dx^{10}+12\,x^{10}e+15\,dx^{8}+\frac{45}{8}\,x^{8}e+\frac{15}{2}\,dx^{6}+\frac{5}{3}\,x^{6}e+\frac{5}{2}\,dx^{4}+\frac{1}{4}\,x^{4}e+\frac{1}{2}\,dx^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/24*x^24*e + 1/22*d*x^22 + 5/11*x^22*e + 1/2*d*x^20 + 9/4*x^20*e + 5/2*d*x
^18 + 20/3*x^18*e + 15/2*d*x^16 + 105/8*x^16*e + 15*d*x^14 + 18*x^14*e + 21
*d*x^12 + 35/2*x^12*e + 21*d*x^10 + 12*x^10*e + 15*d*x^8 + 45/8*x^8*e + 15/
2*d*x^6 + 5/3*x^6*e + 5/2*d*x^4 + 1/4*x^4*e + 1/2*d*x^2
```

Mupad [B]
time = 0.08, size = 123, normalized size = 4.24

$$\frac{e\,x^{24}}{24}+\left(\frac{d}{22}+\frac{5\,e}{11}\right)\,x^{22}+\left(\frac{d}{2}+\frac{9\,e}{4}\right)\,x^{20}+\left(\frac{5\,d}{2}+\frac{20\,e}{3}\right)\,x^{18}+\left(\frac{15\,d}{2}+\frac{105\,e}{8}\right)\,x^{16}+\left(15\,d+18\,e\right)\,x^{14}+\left(21\,d+\frac{35\,e}{2}\right)\,x^{12}+\left(21\,d+12\,e\right)\,x^{10}+\left(15\,d+\frac{45\,e}{8}\right)\,x^8+\left(\frac{15\,d}{2}+\frac{5\,e}{3}\right)\,x^6+\left(\frac{5\,d}{2}+\frac{e}{4}\right)\,x^4+\frac{d\,x^2}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^4*((5*d)/2 + e/4) + x^6*((15*d)/2 + (5*e)/3) + x^20*(d/2 + (9*e)/4) + x^1
0*(21*d + 12*e) + x^14*(15*d + 18*e) + x^18*((5*d)/2 + (20*e)/3) + x^22*(d/
22 + (5*e)/11) + x^12*(21*d + (35*e)/2) + x^8*(15*d + (45*e)/8) + x^16*((15
*d)/2 + (105*e)/8) + (d*x^2)/2 + (e*x^24)/24
```

3.61 $\int (d+ex^2)(1+2x^2+x^4)^5\,dx$

Optimal. Leaf size=143

$$dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7+\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d+5e)x^{11}+\frac{42}{13}(5d+6e)x^{13}+2(4d+7e)x^{15}+$$

```
[Out] d*x+1/3*(10*d+e)*x^3+(9*d+2*e)*x^5+15/7*(8*d+3*e)*x^7+10/3*(7*d+4*e)*x^9+42
/11*(6*d+5*e)*x^11+42/13*(5*d+6*e)*x^13+2*(4*d+7*e)*x^15+15/17*(3*d+8*e)*x^
17+5/19*(2*d+9*e)*x^19+1/21*(d+10*e)*x^21+1/23*e*x^23
```

Rubi [A]
time = 0.05, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {28, 380}

$$\frac{1}{21}x^{21}(d+10e)+\frac{5}{19}x^{19}(2d+9e)+\frac{15}{17}x^{17}(3d+8e)+2x^{15}(4d+7e)+\frac{42}{13}x^{13}(5d+6e)+\frac{42}{11}x^{11}(6d+5e)+\frac{10}{3}x^9(7d+4e)+\frac{15}{7}x^7(8d+3e)+x^5(9d+2e)+\frac{1}{3}x^3(10d+e)+dx+\frac{ex^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 380

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (d+ex^2)\left(1+2x^2+x^4\right)^5\,dx &= \int \left(1+x^2\right)^{10}\left(d+ex^2\right)\,dx\\ &= \int \left(d+(10d+e)x^2+5(9d+2e)x^4+15(8d+3e)x^6+30(7d+4e)x^8+4\right.\\ &= dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7+\frac{10}{3}(7d+4e)x^9+\frac{42}{11}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 143, normalized size = 1.00

$$dx+\frac{1}{3}(10d+e)x^3+(9d+2e)x^5+\frac{15}{7}(8d+3e)x^7+\frac{10}{3}(7d+4e)x^9+\frac{42}{11}(6d+5e)x^{11}+\frac{42}{13}(5d+6e)x^{13}+2(4d+7e)x^{15}+\frac{15}{17}(3d+8e)x^{17}+\frac{5}{19}(2d+9e)x^{19}+\frac{1}{21}(d+10e)x^{21}+\frac{ex^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] d*x + ((10*d + e)*x^3)/3 + (9*d + 2*e)*x^5 + (15*(8*d + 3*e)*x^7)/7 + (10*(
7*d + 4*e)*x^9)/3 + (42*(6*d + 5*e)*x^11)/11 + (42*(5*d + 6*e)*x^13)/13 + 2
*(4*d + 7*e)*x^15 + (15*(3*d + 8*e)*x^17)/17 + (5*(2*d + 9*e)*x^19)/19 + ((
d + 10*e)*x^21)/21 + (e*x^23)/23
```

Maple [A]
time = 0.12, size = 127, normalized size = 0.89

method	result
norman	$dx+\left(\frac{10d}{3}+\frac{e}{3}\right)x^3+(9d+2e)x^5+\left(\frac{120d}{7}+\frac{45e}{7}\right)x^7+\left(\frac{70d}{3}+\frac{40e}{3}\right)x^9+\left(\frac{252d}{11}+\frac{210e}{11}\right)x^{11}+\left(\frac{210d}{13}\right.$
default	$\frac{e\,x^{23}}{23}+\frac{(d+10e)x^{21}}{21}+\frac{(10d+45e)x^{19}}{19}+\frac{(45d+120e)x^{17}}{17}+\frac{(120d+210e)x^{15}}{15}+\frac{(210d+252e)x^{13}}{13}+\frac{(252d+210e)x^{11}}{11}+$
risch	$\frac{1}{23}e\,x^{23}+\frac{1}{21}x^{21}d+\frac{10}{21}e\,x^{21}+\frac{10}{19}x^{19}d+\frac{45}{19}x^{19}e+\frac{45}{17}x^{17}d+\frac{120}{17}x^{17}e+8x^{15}d+14x^{15}e+\frac{210}{13}x^{13}d+$
gosper	$x(969969e\,x^{22}+1062347d\,x^{20}+10623470e\,x^{20}+11741730d\,x^{18}+52837785e\,x^{18}+59053995d\,x^{16}+157477320e\,x^{16}+178474296d\,x^{14}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/23*e*x^23+1/21*(d+10*e)*x^21+1/19*(10*d+45*e)*x^19+1/17*(45*d+120*e)*x^17
+1/15*(120*d+210*e)*x^15+1/13*(210*d+252*e)*x^13+1/11*(252*d+210*e)*x^11+1/
9*(210*d+120*e)*x^9+1/7*(120*d+45*e)*x^7+1/5*(45*d+10*e)*x^5+1/3*(10*d+e)*x
^3+d*x
```

Maxima [A]
time = 0.28, size = 136, normalized size = 0.95

$$\frac{1}{23}x^{23}e+\frac{1}{21}(d+10e)x^{21}+\frac{5}{19}(2d+9e)x^{19}+\frac{15}{17}(3d+8e)x^{17}+2(4d+7e)x^{15}+\frac{42}{13}(5d+6e)x^{13}+\frac{42}{11}(6d+5e)x^{11}+\frac{10}{3}(7d+4e)x^9+\frac{15}{7}(8d+3e)x^7+(9d+2e)x^5+\frac{1}{3}(10d+e)x^3+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*x^23*e + 1/21*(d + 10*e)*x^21 + 5/19*(2*d + 9*e)*x^19 + 15/17*(3*d + 8
*e)*x^17 + 2*(4*d + 7*e)*x^15 + 42/13*(5*d + 6*e)*x^13 + 42/11*(6*d + 5*e)*
x^11 + 10/3*(7*d + 4*e)*x^9 + 15/7*(8*d + 3*e)*x^7 + (9*d + 2*e)*x^5 + 1/3*
(10*d + e)*x^3 + d*x
```

Fricas [A]
time = 0.35, size = 124, normalized size = 0.87

$$\frac{1}{21}\,dx^{21}+\frac{10}{19}\,dx^{19}+\frac{45}{17}\,dx^{17}+8\,dx^{15}+\frac{210}{13}\,dx^{13}+\frac{252}{11}\,dx^{11}+\frac{70}{3}\,dx^{9}+\frac{120}{7}\,dx^{7}+9\,dx^{5}+\frac{10}{3}\,dx^{3}+dx+\frac{1}{22309287}\,(969969\,x^{23}+10623470\,x^{21}+52837785\,x^{19}+157477320\,x^{17}+312330018\,x^{15}+432456948\,x^{13}+425904570\,x^{11}+297457160\,x^{9}+143416845\,x^{7}+44618574\,x^{5}+7436429\,x^{3})e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/21*d*x^21 + 10/19*d*x^19 + 45/17*d*x^17 + 8*d*x^15 + 210/13*d*x^13 + 252/
11*d*x^11 + 70/3*d*x^9 + 120/7*d*x^7 + 9*d*x^5 + 10/3*d*x^3 + d*x + 1/22309
287*(969969*x^23 + 10623470*x^21 + 52837785*x^19 + 157477320*x^17 + 3123300
18*x^15 + 432456948*x^13 + 425904570*x^11 + 297457160*x^9 + 143416845*x^7 +
 44618574*x^5 + 7436429*x^3)*e
```

Sympy [A]
time = 0.02, size = 134, normalized size = 0.94

$$dx+\frac{ex^{23}}{23}+x^{21}\left(\frac{d}{21}+\frac{10e}{21}\right)+x^{19}\cdot\left(\frac{10d}{19}+\frac{45e}{19}\right)+x^{17}\cdot\left(\frac{45d}{17}+\frac{120e}{17}\right)+x^{15}\cdot(8d+14e)+x^{13}\cdot\left(\frac{210d}{13}+\frac{252e}{13}\right)+x^{11}\cdot\left(\frac{252d}{11}+\frac{210e}{11}\right)+x^{9}\cdot\left(\frac{70d}{3}+\frac{40e}{3}\right)+x^{7}\cdot\left(\frac{120d}{7}+\frac{45e}{7}\right)+x^{5}\cdot(9d+2e)+x^{3}\cdot\left(\frac{10d}{3}+\frac{e}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5,x)
```

```
[Out] d*x + e*x**23/23 + x**21*(d/21 + 10*e/21) + x**19*(10*d/19 + 45*e/19) + x**
17*(45*d/17 + 120*e/17) + x**15*(8*d + 14*e) + x**13*(210*d/13 + 252*e/13)
+ x**11*(252*d/11 + 210*e/11) + x**9*(70*d/3 + 40*e/3) + x**7*(120*d/7 + 45
*e/7) + x**5*(9*d + 2*e) + x**3*(10*d/3 + e/3)
```

Giac [A]
time = 3.68, size = 141, normalized size = 0.99

$$\frac{1}{23}\,x^{23}e+\frac{1}{21}\,dx^{21}+\frac{10}{21}\,x^{21}e+\frac{10}{19}\,dx^{19}+\frac{45}{19}\,x^{19}e+\frac{45}{17}\,dx^{17}+\frac{120}{17}\,x^{17}e+8\,dx^{15}+14\,x^{15}e+\frac{210}{13}\,dx^{13}+\frac{252}{13}\,x^{13}e+\frac{252}{11}\,dx^{11}+\frac{210}{11}\,x^{11}e+\frac{70}{3}\,dx^{9}+\frac{40}{3}\,x^{9}e+\frac{120}{7}\,dx^{7}+\frac{45}{7}\,x^{7}e+9\,dx^{5}+2\,x^{5}e+\frac{10}{3}\,dx^{3}+\frac{1}{3}\,x^{3}e+dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*x^23*e + 1/21*d*x^21 + 10/21*x^21*e + 10/19*d*x^19 + 45/19*x^19*e + 45
/17*d*x^17 + 120/17*x^17*e + 8*d*x^15 + 14*x^15*e + 210/13*d*x^13 + 252/13*
x^13*e + 252/11*d*x^11 + 210/11*x^11*e + 70/3*d*x^9 + 40/3*x^9*e + 120/7*d*
x^7 + 45/7*x^7*e + 9*d*x^5 + 2*x^5*e + 10/3*d*x^3 + 1/3*x^3*e + d*x
```

Mupad [B]
time = 0.08, size = 120, normalized size = 0.84

$$\frac{e\,x^{23}}{23}+\left(\frac{d}{21}+\frac{10\,e}{21}\right)\,x^{21}+\left(\frac{10\,d}{19}+\frac{45\,e}{19}\right)\,x^{19}+\left(\frac{45\,d}{17}+\frac{120\,e}{17}\right)\,x^{17}+\left(8\,d+14\,e\right)\,x^{15}+\left(\frac{210\,d}{13}+\frac{252\,e}{13}\right)\,x^{13}+\left(\frac{252\,d}{11}+\frac{210\,e}{11}\right)\,x^{11}+\left(\frac{70\,d}{3}+\frac{40\,e}{3}\right)\,x^{9}+\left(\frac{120\,d}{7}+\frac{45\,e}{7}\right)\,x^{7}+\left(9\,d+2\,e\right)\,x^{5}+\left(\frac{10\,d}{3}+\frac{e}{3}\right)\,x^{3}+d\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5*(9*d + 2*e) + x^3*((10*d)/3 + e/3) + x^15*(8*d + 14*e) + x^21*(d/21 + (
10*e)/21) + x^19*((10*d)/19 + (45*e)/19) + x^9*((70*d)/3 + (40*e)/3) + x^7*
((120*d)/7 + (45*e)/7) + x^17*((45*d)/17 + (120*e)/17) + x^11*((252*d)/11 +
 (210*e)/11) + x^13*((210*d)/13 + (252*e)/13) + d*x + (e*x^23)/23
```

3.62 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x} dx$

Optimal. Leaf size=93

$$5dx^2+\frac{45dx^4}{4}+20dx^6+\frac{105dx^8}{4}+\frac{126dx^{10}}{5}+\frac{35dx^{12}}{2}+\frac{60dx^{14}}{7}+\frac{45dx^{16}}{16}+\frac{5dx^{18}}{9}+\frac{dx^{20}}{20}+\frac{1}{22}e(1+x^2)^{11}+d\log(x)$$

```
[Out] 5*d*x^2+45/4*d*x^4+20*d*x^6+105/4*d*x^8+126/5*d*x^10+35/2*d*x^12+60/7*d*x^1
4+45/16*d*x^16+5/9*d*x^18+1/20*d*x^20+1/22*e*(x^2+1)^11+d*ln(x)
```

Rubi [A]
time = 0.03, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {28, 457, 81, 45}

$$\frac{dx^{20}}{20}+\frac{5dx^{18}}{9}+\frac{45dx^{16}}{16}+\frac{60dx^{14}}{7}+\frac{35dx^{12}}{2}+\frac{126dx^{10}}{5}+\frac{105dx^8}{4}+20dx^6+\frac{45dx^4}{4}+5dx^2+d\log(x)+\frac{1}{22}e(x^2+1)^{11}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] 5*d*x^2 + (45*d*x^4)/4 + 20*d*x^6 + (105*d*x^8)/4 + (126*d*x^10)/5 + (35*d*
x^12)/2 + (60*d*x^14)/7 + (45*d*x^16)/16 + (5*d*x^18)/9 + (d*x^20)/20 + (e*
(1 + x^2)^11)/22 + d*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(d+ex^2)\left(1+2x^2+x^4\right)^5}{x}\,dx = \int \frac{\left(1+x^2\right)^{10}(d+ex^2)}{x}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{22}e\left(1+x^2\right)^{11} + \frac{1}{2}d\text{Subst}\left(\int \frac{(1+x)^{10}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{22}e\left(1+x^2\right)^{11} + \frac{1}{2}d\text{Subst}\left(\int \left(10+\frac{1}{x}+45x+120x^2+210x^3+252x^4\right.\right.$$
$$= 5dx^2 + \frac{45dx^4}{4} + 20dx^6 + \frac{105dx^8}{4} + \frac{126dx^{10}}{5} + \frac{35dx^{12}}{2} + \frac{60dx^{14}}{7} + \frac{45dx}{16}$$

Mathematica [**A**]
time = 0.02, size = 149, normalized size = 1.60

$$\frac{1}{2}(10d+e)x^2+\frac{5}{4}(9d+2e)x^4+\frac{5}{2}(8d+3e)x^6+\frac{15}{4}(7d+4e)x^8+\frac{21}{5}(6d+5e)x^{10}+\frac{7}{2}(5d+6e)x^{12}+\frac{15}{7}(4d+7e)x^{14}+\frac{15}{16}(3d+8e)x^{16}+\frac{5}{18}(2d+9e)x^{18}+\frac{1}{20}(d+10e)x^{20}+\frac{ex^{22}}{22}+d\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] ((10*d + e)*x^2)/2 + (5*(9*d + 2*e)*x^4)/4 + (5*(8*d + 3*e)*x^6)/2 + (15*(7
*d + 4*e)*x^8)/4 + (21*(6*d + 5*e)*x^10)/5 + (7*(5*d + 6*e)*x^12)/2 + (15*(
4*d + 7*e)*x^14)/7 + (15*(3*d + 8*e)*x^16)/16 + (5*(2*d + 9*e)*x^18)/18 + (
(d + 10*e)*x^20)/20 + (e*x^22)/22 + d*Log[x]
```

Maple [**A**]
time = 0.02, size = 132, normalized size = 1.42

method	result
norman	$\left(5d+\frac{e}{2}\right)x^2+\left(20d+\frac{15e}{2}\right)x^6+\left(\frac{d}{20}+\frac{e}{2}\right)x^{20}+\left(\frac{5d}{9}+\frac{5e}{2}\right)x^{18}+\left(\frac{35d}{2}+21e\right)x^{12}+\left(\frac{45d}{4}+\frac{5e}{2}\right)x^4+$
default	$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+\frac{126d}{5}$
risch	$\frac{e\,x^{22}}{22}+\frac{d\,x^{20}}{20}+\frac{e\,x^{20}}{2}+\frac{5d\,x^{18}}{9}+\frac{5e\,x^{18}}{2}+\frac{45d\,x^{16}}{16}+\frac{15e\,x^{16}}{2}+\frac{60d\,x^{14}}{7}+15e\,x^{14}+\frac{35d\,x^{12}}{2}+21e\,x^{12}+\frac{126d}{5}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/22*e*x^22+1/20*d*x^20+1/2*e*x^20+5/9*d*x^18+5/2*e*x^18+45/16*d*x^16+15/2*
e*x^16+60/7*d*x^14+15*e*x^14+35/2*d*x^12+21*e*x^12+126/5*d*x^10+21*e*x^10+1
05/4*d*x^8+15*e*x^8+20*x^6*d+15/2*e*x^6+45/4*d*x^4+5/2*e*x^4+5*d*x^2+1/2*e*
x^2+d*ln(x)
```

Maxima [A]
time = 0.28, size = 141, normalized size = 1.52

$$\frac{1}{22}\,x^{22}e+\frac{1}{20}\,(d+10\,e)x^{20}+\frac{5}{18}\,(2\,d+9\,e)x^{18}+\frac{15}{16}\,(3\,d+8\,e)x^{16}+\frac{15}{7}\,(4\,d+7\,e)x^{14}+\frac{7}{2}\,(5\,d+6\,e)x^{12}+\frac{21}{5}\,(6\,d+5\,e)x^{10}+\frac{15}{4}\,(7\,d+4\,e)x^{8}+\frac{5}{2}\,(8\,d+3\,e)x^{6}+\frac{5}{4}\,(9\,d+2\,e)x^{4}+\frac{1}{2}\,(10\,d+e)x^{2}+\frac{1}{2}\,d\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*x^22*e + 1/20*(d + 10*e)*x^20 + 5/18*(2*d + 9*e)*x^18 + 15/16*(3*d + 8
*e)*x^16 + 15/7*(4*d + 7*e)*x^14 + 7/2*(5*d + 6*e)*x^12 + 21/5*(6*d + 5*e)*
x^10 + 15/4*(7*d + 4*e)*x^8 + 5/2*(8*d + 3*e)*x^6 + 5/4*(9*d + 2*e)*x^4 + 1
/2*(10*d + e)*x^2 + 1/2*d*log(x^2)
```

Fricas [A]
time = 0.56, size = 123, normalized size = 1.32

$$\frac{1}{20}\,dx^{20}+\frac{5}{9}\,dx^{18}+\frac{45}{16}\,dx^{16}+\frac{60}{7}\,dx^{14}+\frac{35}{2}\,dx^{12}+\frac{126}{5}\,dx^{10}+\frac{105}{4}\,dx^{8}+20\,dx^{6}+\frac{45}{4}\,dx^{4}+5\,dx^{2}+\frac{1}{22}\,\left(x^{22}+11\,x^{20}+55\,x^{18}+165\,x^{16}+330\,x^{14}+462\,x^{12}+462\,x^{10}+330\,x^{8}+165\,x^{6}+55\,x^{4}+11\,x^{2}\right)e+d\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/20*d*x^20 + 5/9*d*x^18 + 45/16*d*x^16 + 60/7*d*x^14 + 35/2*d*x^12 + 126/5
*d*x^10 + 105/4*d*x^8 + 20*d*x^6 + 45/4*d*x^4 + 5*d*x^2 + 1/22*(x^22 + 11*x
^20 + 55*x^18 + 165*x^16 + 330*x^14 + 462*x^12 + 462*x^10 + 330*x^8 + 165*x
^6 + 55*x^4 + 11*x^2)*e + d*log(x)
```

Sympy [A]
time = 0.11, size = 131, normalized size = 1.41

$$d\log\left(x\right)+\frac{ex^{22}}{22}+x^{20}\left(\frac{d}{20}+\frac{e}{2}\right)+x^{18}\cdot\left(\frac{5d}{9}+\frac{5e}{2}\right)+x^{16}\cdot\left(\frac{45d}{16}+\frac{15e}{2}\right)+x^{14}\cdot\left(\frac{60d}{7}+15e\right)+x^{12}\cdot\left(\frac{35d}{2}+21e\right)+x^{10}\cdot\left(\frac{126d}{5}+21e\right)+x^{8}\cdot\left(\frac{105d}{4}+15e\right)+x^{6}\cdot\left(20d+\frac{15e}{2}\right)+x^{4}\cdot\left(\frac{45d}{4}+\frac{5e}{2}\right)+x^{2}\cdot\left(5d+\frac{e}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] d*log(x) + e*x**22/22 + x**20*(d/20 + e/2) + x**18*(5*d/9 + 5*e/2) + x**16*
(45*d/16 + 15*e/2) + x**14*(60*d/7 + 15*e) + x**12*(35*d/2 + 21*e) + x**10*
(126*d/5 + 21*e) + x**8*(105*d/4 + 15*e) + x**6*(20*d + 15*e/2) + x**4*(45*
d/4 + 5*e/2) + x**2*(5*d + e/2)
```

Giac [A]
time = 3.51, size = 145, normalized size = 1.56

$$\frac{1}{22}\,x^{22}e+\frac{1}{20}\,dx^{20}+\frac{1}{2}\,x^{20}e+\frac{5}{9}\,dx^{18}+\frac{5}{2}\,x^{18}e+\frac{45}{16}\,dx^{16}+\frac{15}{2}\,x^{16}e+\frac{60}{7}\,dx^{14}+15\,x^{14}e+\frac{35}{2}\,dx^{12}+21\,x^{12}e+\frac{126}{5}\,dx^{10}+21\,x^{10}e+\frac{105}{4}\,dx^{8}+15\,x^{8}e+20\,dx^{6}+\frac{15}{2}\,x^{6}e+\frac{45}{4}\,dx^{4}+\frac{5}{2}\,x^{4}e+5\,dx^{2}+\frac{1}{2}\,x^{2}e+\frac{1}{2}\,d\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22*e + 1/20*d*x^20 + 1/2*x^20*e + 5/9*d*x^18 + 5/2*x^18*e + 45/16*d*
x^16 + 15/2*x^16*e + 60/7*d*x^14 + 15*x^14*e + 35/2*d*x^12 + 21*x^12*e + 12
6/5*d*x^10 + 21*x^10*e + 105/4*d*x^8 + 15*x^8*e + 20*d*x^6 + 15/2*x^6*e + 4
5/4*d*x^4 + 5/2*x^4*e + 5*d*x^2 + 1/2*x^2*e + 1/2*d*log(x^2)
```

Mupad [B]
time = 0.13, size = 121, normalized size = 1.30

$$x^{2}\left(5\,d+\frac{e}{2}\right)+x^{18}\left(\frac{5\,d}{9}+\frac{5\,e}{2}\right)+x^{6}\left(20\,d+\frac{15\,e}{2}\right)+x^{20}\left(\frac{d}{20}+\frac{e}{2}\right)+x^{4}\left(\frac{45\,d}{4}+\frac{5\,e}{2}\right)+x^{12}\left(\frac{35\,d}{2}+21\,e\right)+x^{16}\left(\frac{45\,d}{16}+\frac{15\,e}{2}\right)+x^{14}\left(\frac{60\,d}{7}+15\,e\right)+x^{8}\left(\frac{105\,d}{4}+15\,e\right)+x^{10}\left(\frac{126\,d}{5}+21\,e\right)+\frac{e\,x^{22}}{22}+d\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x,x)
```

```
[Out] x^2*(5*d + e/2) + x^18*((5*d)/9 + (5*e)/2) + x^6*(20*d + (15*e)/2) + x^20*(
d/20 + e/2) + x^4*((45*d)/4 + (5*e)/2) + x^12*((35*d)/2 + 21*e) + x^16*((45
*d)/16 + (15*e)/2) + x^14*((60*d)/7 + 15*e) + x^8*((105*d)/4 + 15*e) + x^10
*((126*d)/5 + 21*e) + (e*x^22)/22 + d*log(x)
```

3.63 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^2}\,dx$

Optimal. Leaf size=141

$$-\frac{d}{x}+(10d+e)x+\frac{5}{3}(9d+2e)x^3+3(8d+3e)x^5+\frac{30}{7}(7d+4e)x^7+\frac{14}{3}(6d+5e)x^9+\frac{42}{11}(5d+6e)x^{11}+\frac{30}{13}(4d+7e)x^{13}+($$

```
[Out] -d/x+(10*d+e)*x+5/3*(9*d+2*e)*x^3+3*(8*d+3*e)*x^5+30/7*(7*d+4*e)*x^7+14/3*(
6*d+5*e)*x^9+42/11*(5*d+6*e)*x^11+30/13*(4*d+7*e)*x^13+(3*d+8*e)*x^15+5/17*
(2*d+9*e)*x^17+1/19*(d+10*e)*x^19+1/21*e*x^21
```

Rubi [A]
time = 0.05, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 459}

$$\frac{1}{19}x^{19}(d+10e)+\frac{5}{17}x^{17}(2d+9e)+x^{15}(3d+8e)+\frac{30}{13}x^{13}(4d+7e)+\frac{42}{11}x^{11}(5d+6e)+\frac{14}{3}x^9(6d+5e)+\frac{30}{7}x^7(7d+4e)+3x^5(8d+3e)+\frac{5}{3}x^3(9d+2e)+x(10d+e)-\frac{d}{x}+\frac{ex^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \frac{(d+ex^2)\,(1+2x^2+x^4)^5}{x^2}\,dx = \int \frac{(1+x^2)^{10}\,(d+ex^2)}{x^2}\,dx$$
$$= \int \left(10d\left(1+\frac{e}{10d}\right)+\frac{d}{x^2}+5(9d+2e)x^2+15(8d+3e)x^4+30(7d+4e)\right.$$
$$= -\frac{d}{x}+(10d+e)x+\frac{5}{3}(9d+2e)x^3+3(8d+3e)x^5+\frac{30}{7}(7d+4e)x^7+\frac{14}{3}$$

Mathematica [A]
time = 0.02, size = 141, normalized size = 1.00

$$-\frac{d}{x}+(10d+e)x+\frac{5}{3}(9d+2e)x^3+3(8d+3e)x^5+\frac{30}{7}(7d+4e)x^7+\frac{14}{3}(6d+5e)x^9+\frac{42}{11}(5d+6e)x^{11}+\frac{30}{13}(4d+7e)x^{13}+(3d+8e)x^{15}+\frac{5}{17}(2d+9e)x^{17}+\frac{1}{19}(d+10e)x^{19}+\frac{ex^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -(d/x) + (10*d + e)*x + (5*(9*d + 2*e)*x^3)/3 + 3*(8*d + 3*e)*x^5 + (30*(7*
d + 4*e)*x^7)/7 + (14*(6*d + 5*e)*x^9)/3 + (42*(5*d + 6*e)*x^11)/11 + (30*(
4*d + 7*e)*x^13)/13 + (3*d + 8*e)*x^15 + (5*(2*d + 9*e)*x^17)/17 + ((d + 10
*e)*x^19)/19 + (e*x^21)/21
```

Maple [A]
time = 0.04, size = 129, normalized size = 0.91

method	result
norman	$\frac{-d+(10d+e)x^2+\left(15d+\frac{10e}{3}\right)x^4+(24d+9e)x^6+\left(30d+\frac{120e}{7}\right)x^8+\left(28d+\frac{70e}{3}\right)x^{10}+\left(\frac{210d}{11}+\frac{252e}{11}\right)x^{12}+\left(\frac{120d}{13}+\frac{210e}{13}\right)x^{14}+(3d+8e)x^{16}+}{x}$
default	$\frac{e\,x^{21}}{21}+\frac{x^{19}d}{19}+\frac{10x^{19}e}{19}+\frac{10x^{17}d}{17}+\frac{45x^{17}e}{17}+3x^{15}d+8x^{15}e+\frac{120x^{13}d}{13}+\frac{210x^{13}e}{13}+\frac{210x^{11}d}{11}+\frac{252e\,x^{11}}{11}+28$
risch	$\frac{e\,x^{21}}{21}+\frac{x^{19}d}{19}+\frac{10x^{19}e}{19}+\frac{10x^{17}d}{17}+\frac{45x^{17}e}{17}+3x^{15}d+8x^{15}e+\frac{120x^{13}d}{13}+\frac{210x^{13}e}{13}+\frac{210x^{11}d}{11}+\frac{252e\,x^{11}}{11}+28$
gosper	$46189e\,x^{22}+51051d\,x^{20}+510510e\,x^{20}+570570d\,x^{18}+2567565e\,x^{18}+2909907d\,x^{16}+7759752e\,x^{16}+8953560d\,x^{14}+15668730e\,x^{14}+18$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/21*e*x^21+1/19*x^19*d+10/19*x^19*e+10/17*x^17*d+45/17*x^17*e+3*x^15*d+8*x
^15*e+120/13*x^13*d+210/13*x^13*e+210/11*x^11*d+252/11*e*x^11+28*d*x^9+70/3
*e*x^9+30*x^7*d+120/7*x^7*e+24*d*x^5+9*e*x^5+15*d*x^3+10/3*e*x^3+10*d*x+e*x
-d/x
```

Maxima [A]
time = 0.28, size = 136, normalized size = 0.96

$$\frac{1}{21}x^{21}e+\frac{1}{19}(d+10\,e)x^{19}+\frac{5}{17}(2\,d+9\,e)x^{17}+(3\,d+8\,e)x^{15}+\frac{30}{13}(4\,d+7\,e)x^{13}+\frac{42}{11}(5\,d+6\,e)x^{11}+\frac{14}{3}(6\,d+5\,e)x^9+\frac{30}{7}(7\,d+4\,e)x^7+3\,(8\,d+3\,e)x^5+\frac{5}{3}(9\,d+2\,e)x^3+(10\,d+e)x-\frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*x^21*e + 1/19*(d + 10*e)*x^19 + 5/17*(2*d + 9*e)*x^17 + (3*d + 8*e)*x^
15 + 30/13*(4*d + 7*e)*x^13 + 42/11*(5*d + 6*e)*x^11 + 14/3*(6*d + 5*e)*x^9
 + 30/7*(7*d + 4*e)*x^7 + 3*(8*d + 3*e)*x^5 + 5/3*(9*d + 2*e)*x^3 + (10*d +
 e)*x - d/x
```

Fricas [A]
time = 0.44, size = 128, normalized size = 0.91

$$\frac{51051\,dx^{20}+570570\,dx^{18}+2909907\,dx^{16}+8953560\,dx^{14}+18517590\,dx^{12}+27159132\,dx^{10}+29099070\,dx^{8}+23279256\,dx^{6}+14549535\,dx^{4}+9699690\,dx^{2}+\left(46189\,x^{22}+510510\,x^{20}+2567565\,x^{18}+7759752\,x^{16}+15668730\,x^{14}+22221108\,x^{12}+22632610\,x^{10}+16628040\,x^{8}+8729721\,x^{6}+3233230\,x^{4}+969969\,x^{2}\right)e-969969\,d}{969969\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/969969*(51051*d*x^20 + 570570*d*x^18 + 2909907*d*x^16 + 8953560*d*x^14 +
18517590*d*x^12 + 27159132*d*x^10 + 29099070*d*x^8 + 23279256*d*x^6 + 14549
535*d*x^4 + 9699690*d*x^2 + (46189*x^22 + 510510*x^20 + 2567565*x^18 + 7759
752*x^16 + 15668730*x^14 + 22221108*x^12 + 22632610*x^10 + 16628040*x^8 + 8
729721*x^6 + 3233230*x^4 + 969969*x^2)*e - 969969*d)/x
```

Sympy [A]
time = 0.10, size = 124, normalized size = 0.88

$$-\frac{d}{x}+\frac{ex^{21}}{21}+x^{19}\left(\frac{d}{19}+\frac{10e}{19}\right)+x^{17}\cdot\left(\frac{10d}{17}+\frac{45e}{17}\right)+x^{15}\cdot(3d+8e)+x^{13}\cdot\left(\frac{120d}{13}+\frac{210e}{13}\right)+x^{11}\cdot\left(\frac{210d}{11}+\frac{252e}{11}\right)+x^{9}\cdot\left(28d+\frac{70e}{3}\right)+x^{7}\cdot\left(30d+\frac{120e}{7}\right)+x^{5}\cdot(24d+9e)+x^{3}\cdot\left(15d+\frac{10e}{3}\right)+x(10d+e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] -d/x + e*x**21/21 + x**19*(d/19 + 10*e/19) + x**17*(10*d/17 + 45*e/17) + x*
*15*(3*d + 8*e) + x**13*(120*d/13 + 210*e/13) + x**11*(210*d/11 + 252*e/11)
 + x**9*(28*d + 70*e/3) + x**7*(30*d + 120*e/7) + x**5*(24*d + 9*e) + x**3*
(15*d + 10*e/3) + x*(10*d + e)
```

Giac [A]
time = 3.75, size = 139, normalized size = 0.99

$$\frac{1}{21}x^{21}e+\frac{1}{19}dx^{19}+\frac{10}{19}x^{19}e+\frac{10}{17}dx^{17}+\frac{45}{17}x^{17}e+3\,dx^{15}+8\,x^{15}e+\frac{120}{13}dx^{13}+\frac{210}{13}x^{13}e+\frac{210}{11}dx^{11}+\frac{252}{11}x^{11}e+28\,dx^{9}+\frac{70}{3}x^{9}e+30\,dx^{7}+\frac{120}{7}x^{7}e+24\,dx^{5}+9\,x^{5}e+15\,dx^{3}+\frac{10}{3}x^{3}e+10\,dx+xe-\frac{d}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21*e + 1/19*d*x^19 + 10/19*x^19*e + 10/17*d*x^17 + 45/17*x^17*e + 3*
d*x^15 + 8*x^15*e + 120/13*d*x^13 + 210/13*x^13*e + 210/11*d*x^11 + 252/11*
```

```
x^11*e + 28*d*x^9 + 70/3*x^9*e + 30*d*x^7 + 120/7*x^7*e + 24*d*x^5 + 9*x^5*
e + 15*d*x^3 + 10/3*x^3*e + 10*d*x + x*e - d/x
```

Mupad [B]
time = 0.08, size = 119, normalized size = 0.84

$$x^{15}\,(3\,d+8\,e)+x^3\,\left(15\,d+\frac{10\,e}{3}\right)+x^5\,(24\,d+9\,e)+x^{19}\,\left(\frac{d}{19}+\frac{10\,e}{19}\right)+x^{17}\,\left(\frac{10\,d}{17}+\frac{45\,e}{17}\right)+x^9\,\left(28\,d+\frac{70\,e}{3}\right)+x^7\,\left(30\,d+\frac{120\,e}{7}\right)+x^{13}\,\left(\frac{120\,d}{13}+\frac{210\,e}{13}\right)+x^{11}\,\left(\frac{210\,d}{11}+\frac{252\,e}{11}\right)+x\,(10\,d+e)-\frac{d}{x}+\frac{e\,x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

```
[Out] x^15*(3*d + 8*e) + x^3*(15*d + (10*e)/3) + x^5*(24*d + 9*e) + x^19*(d/19 +
(10*e)/19) + x^17*((10*d)/17 + (45*e)/17) + x^9*(28*d + (70*e)/3) + x^7*(30
*d + (120*e)/7) + x^13*((120*d)/13 + (210*e)/13) + x^11*((210*d)/11 + (252*
e)/11) + x*(10*d + e) - d/x + (e*x^21)/21
```

3.64 $\int \frac{(d+ex^2)(1+2x^2+x^4)^5}{x^3}\,dx$

Optimal. Leaf size=147

$$-\frac{d}{2x^2}+\frac{5}{2}(9d+2e)x^2+\frac{15}{4}(8d+3e)x^4+5(7d+4e)x^6+\frac{21}{4}(6d+5e)x^8+\frac{21}{5}(5d+6e)x^{10}+\frac{5}{2}(4d+7e)x^{12}+\frac{15}{14}(3d+8e)$$

```
[Out] -1/2*d/x^2+5/2*(9*d+2*e)*x^2+15/4*(8*d+3*e)*x^4+5*(7*d+4*e)*x^6+21/4*(6*d+5
*e)*x^8+21/5*(5*d+6*e)*x^10+5/2*(4*d+7*e)*x^12+15/14*(3*d+8*e)*x^14+5/16*(2
*d+9*e)*x^16+1/18*(d+10*e)*x^18+1/20*e*x^20+(10*d+e)*ln(x)
```

Rubi [A]
time = 0.09, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {28, 457, 77}

$$\frac{1}{18}x^{18}(d+10e)+\frac{5}{16}x^{16}(2d+9e)+\frac{15}{14}x^{14}(3d+8e)+\frac{5}{2}x^{12}(4d+7e)+\frac{21}{5}x^{10}(5d+6e)+\frac{21}{4}x^{8}(6d+5e)+5x^{6}(7d+4e)+\frac{15}{4}x^{4}(8d+3e)+\frac{5}{2}x^{2}(9d+2e)+(10d+e)\log(x)-\frac{d}{2x^2}+\frac{ex^{20}}{20}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*d/x^2 + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 77

```
Int[((d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_))*((e_) + (f_.)*(x_))^(p_.), x_Sym
bol] :> Int[ExpandIntegrand[(a + b*x)*(d*x)^n*(e + f*x)^p, x], x] /; FreeQ[
{a, b, d, e, f, n}, x] && IGtQ[p, 0] && (NeQ[n, -1] || EqQ[p, 1]) && NeQ[b*
e + a*f, 0] && ( !IntegerQ[n] || LtQ[9*p + 5*n, 0] || GeQ[n + p + 1, 0] ||
(GeQ[n + p + 2, 0] && RationalQ[a, b, d, e, f])) && (NeQ[n + p + 3, 0] || E
qQ[p, 1])
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
```

```
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)\left(1+2x^2+x^4\right)^5}{x^3}\,dx &= \int \frac{\left(1+x^2\right)^{10}\left(d+ex^2\right)}{x^3}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{10}(d+ex)}{x^2}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(5(9d+2e)+\frac{d}{x^2}+\frac{10d+e}{x}+15(8d+3e)x+30(7d+4e)x \right.\right. \\
&= -\frac{d}{2x^2}+\frac{5}{2}(9d+2e)x^2+\frac{15}{4}(8d+3e)x^4+5(7d+4e)x^6+\frac{21}{4}(6d+5e)x^8
\end{aligned}
$$

Mathematica [**A**]
time = 0.03, size = 147, normalized size = 1.00

$$-\frac{d}{2x^2}+\frac{5}{2}(9d+2e)x^2+\frac{15}{4}(8d+3e)x^4+5(7d+4e)x^6+\frac{21}{4}(6d+5e)x^8+\frac{21}{5}(5d+6e)x^{10}+\frac{5}{2}(4d+7e)x^{12}+\frac{15}{14}(3d+8e)x^{14}+\frac{5}{16}(2d+9e)x^{16}+\frac{1}{18}(d+10e)x^{18}+\frac{ex^{20}}{20}+(10d+e)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*d/x^2 + (5*(9*d + 2*e)*x^2)/2 + (15*(8*d + 3*e)*x^4)/4 + 5*(7*d + 4*e)
*x^6 + (21*(6*d + 5*e)*x^8)/4 + (21*(5*d + 6*e)*x^10)/5 + (5*(4*d + 7*e)*x^
12)/2 + (15*(3*d + 8*e)*x^14)/14 + (5*(2*d + 9*e)*x^16)/16 + ((d + 10*e)*x^
18)/18 + (e*x^20)/20 + (10*d + e)*Log[x]
```

Maple [**A**]
time = 0.03, size = 130, normalized size = 0.88

method	result
norman	$\frac{\left(10d+\frac{35e}{2}\right)x^{14}+\left(21d+\frac{126e}{5}\right)x^{12}+\left(30d+\frac{45e}{4}\right)x^6+(35d+20e)x^8+\left(\frac{d}{18}+\frac{5e}{9}\right)x^{20}+\left(\frac{5d}{8}+\frac{45e}{16}\right)x^{18}+\left(\frac{45d}{2}+5e\right)x^4+\left(\frac{45d}{14}+\frac{60e}{7}\right)x^{16}+\left(\right.}{x^2}$
default	$\frac{e\,x^{20}}{20}+\frac{d\,x^{18}}{18}+\frac{5e\,x^{18}}{9}+\frac{5d\,x^{16}}{8}+\frac{45e\,x^{16}}{16}+\frac{45d\,x^{14}}{14}+\frac{60e\,x^{14}}{7}+10d\,x^{12}+\frac{35e\,x^{12}}{2}+21d\,x^{10}+\frac{126e\,x^{10}}{5}+$
risch	$\frac{e\,x^{20}}{20}+\frac{d\,x^{18}}{18}+\frac{5e\,x^{18}}{9}+\frac{5d\,x^{16}}{8}+\frac{45e\,x^{16}}{16}+\frac{45d\,x^{14}}{14}+\frac{60e\,x^{14}}{7}+10d\,x^{12}+\frac{35e\,x^{12}}{2}+21d\,x^{10}+\frac{126e\,x^{10}}{5}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/20*e*x^20+1/18*d*x^18+5/9*e*x^18+5/8*d*x^16+45/16*e*x^16+45/14*d*x^14+60/
7*e*x^14+10*d*x^12+35/2*e*x^12+21*d*x^10+126/5*e*x^10+63/2*d*x^8+105/4*e*x^
```

```
8+35*x^6*d+20*e*x^6+30*d*x^4+45/4*e*x^4+45/2*d*x^2+5*e*x^2-1/2*d/x^2+(10*d+
e)*ln(x)
```

Maxima [A]
time = 0.28, size = 141, normalized size = 0.96

$$\frac{1}{20}\,x^{20}e+\frac{1}{18}\,(d+10\,e)x^{18}+\frac{5}{16}\,(2\,d+9\,e)x^{16}+\frac{15}{14}\,(3\,d+8\,e)x^{14}+\frac{5}{2}\,(4\,d+7\,e)x^{12}+\frac{21}{5}\,(5\,d+6\,e)x^{10}+\frac{21}{4}\,(6\,d+5\,e)x^{8}+5\,(7\,d+4\,e)x^{6}+\frac{15}{4}\,(8\,d+3\,e)x^{4}+\frac{5}{2}\,(9\,d+2\,e)x^{2}+\frac{1}{2}\,(10\,d+e)\log\left(x^{2}\right)-\frac{d}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*x^20*e + 1/18*(d + 10*e)*x^18 + 5/16*(2*d + 9*e)*x^16 + 15/14*(3*d + 8
*e)*x^14 + 5/2*(4*d + 7*e)*x^12 + 21/5*(5*d + 6*e)*x^10 + 21/4*(6*d + 5*e)*
x^8 + 5*(7*d + 4*e)*x^6 + 15/4*(8*d + 3*e)*x^4 + 5/2*(9*d + 2*e)*x^2 + 1/2*
(10*d + e)*log(x^2) - 1/2*d/x^2
```

Fricas [A]
time = 0.43, size = 134, normalized size = 0.91

$$\frac{280\,dx^{20}+3150\,dx^{18}+16200\,dx^{16}+50400\,dx^{14}+105840\,dx^{12}+158760\,dx^{10}+176400\,dx^{8}+151200\,dx^{6}+113400\,dx^{4}+\left(252\,x^{22}+2800\,x^{20}+14175\,x^{18}+43200\,x^{16}+88200\,x^{14}+127008\,x^{12}+132300\,x^{10}+100800\,x^{8}+56700\,x^{6}+25200\,x^{4}\right)e+5040\,\left(10\,dx^{2}+x^{2}e\right)\log\left(x\right)-2520\,d}{5040\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(280*d*x^20 + 3150*d*x^18 + 16200*d*x^16 + 50400*d*x^14 + 105840*d*x
^12 + 158760*d*x^10 + 176400*d*x^8 + 151200*d*x^6 + 113400*d*x^4 + (252*x^2
2 + 2800*x^20 + 14175*x^18 + 43200*x^16 + 88200*x^14 + 127008*x^12 + 132300
*x^10 + 100800*x^8 + 56700*x^6 + 25200*x^4)*e + 5040*(10*d*x^2 + x^2*e)*log
(x) - 2520*d)/x^2
```

Sympy [A]
time = 0.14, size = 131, normalized size = 0.89

$$-\frac{d}{2x^{2}}+\frac{ex^{20}}{20}+x^{18}\left(\frac{d}{18}+\frac{5e}{9}\right)+x^{16}\cdot\left(\frac{5d}{8}+\frac{45e}{16}\right)+x^{14}\cdot\left(\frac{45d}{14}+\frac{60e}{7}\right)+x^{12}\cdot\left(10d+\frac{35e}{2}\right)+x^{10}\cdot\left(21d+\frac{126e}{5}\right)+x^{8}\cdot\left(\frac{63d}{2}+\frac{105e}{4}\right)+x^{6}\cdot\left(35d+20e\right)+x^{4}\cdot\left(30d+\frac{45e}{4}\right)+x^{2}\cdot\left(\frac{45d}{2}+5e\right)+\left(10d+e\right)\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] -d/(2*x**2) + e*x**20/20 + x**18*(d/18 + 5*e/9) + x**16*(5*d/8 + 45*e/16) +
 x**14*(45*d/14 + 60*e/7) + x**12*(10*d + 35*e/2) + x**10*(21*d + 126*e/5)
+ x**8*(63*d/2 + 105*e/4) + x**6*(35*d + 20*e) + x**4*(30*d + 45*e/4) + x**
2*(45*d/2 + 5*e) + (10*d + e)*log(x)
```

Giac [A]
time = 3.57, size = 156, normalized size = 1.06

$$\frac{1}{20}\,x^{20}e+\frac{1}{18}\,dx^{18}+\frac{5}{9}\,x^{18}e+\frac{5}{8}\,dx^{16}+\frac{45}{16}\,x^{16}e+\frac{45}{14}\,dx^{14}+\frac{60}{7}\,x^{14}e+10\,dx^{12}+\frac{35}{2}\,x^{12}e+21\,dx^{10}+\frac{126}{5}\,x^{10}e+\frac{63}{2}\,dx^{8}+\frac{105}{4}\,x^{8}e+35\,dx^{6}+20\,x^{6}e+30\,dx^{4}+\frac{45}{4}\,x^{4}e+\frac{45}{2}\,dx^{2}+5\,x^{2}e+\frac{1}{2}\,(10\,d+e)\log\left(x^{2}\right)-\frac{10\,dx^{2}+x^{2}e+d}{2\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20*e + 1/18*d*x^18 + 5/9*x^18*e + 5/8*d*x^16 + 45/16*x^16*e + 45/14*
d*x^14 + 60/7*x^14*e + 10*d*x^12 + 35/2*x^12*e + 21*d*x^10 + 126/5*x^10*e +
 63/2*d*x^8 + 105/4*x^8*e + 35*d*x^6 + 20*x^6*e + 30*d*x^4 + 45/4*x^4*e + 4
5/2*d*x^2 + 5*x^2*e + 1/2*(10*d + e)*log(x^2) - 1/2*(10*d*x^2 + x^2*e + d)/
x^2
```

Mupad [B]
time = 0.08, size = 120, normalized size = 0.82

$$x^{18}\left(\frac{d}{18}+\frac{5\,e}{9}\right)+x^2\left(\frac{45\,d}{2}+5\,e\right)+x^{12}\left(10\,d+\frac{35\,e}{2}\right)+x^6\,(35\,d+20\,e)+x^4\left(30\,d+\frac{45\,e}{4}\right)+x^{16}\left(\frac{5\,d}{8}+\frac{45\,e}{16}\right)+x^{14}\left(\frac{45\,d}{14}+\frac{60\,e}{7}\right)+x^{10}\left(21\,d+\frac{126\,e}{5}\right)+x^8\left(\frac{63\,d}{2}+\frac{105\,e}{4}\right)-\frac{d}{2\,x^2}+\frac{e\,x^{20}}{20}+\ln(x)\,(10\,d+e)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

```
[Out] x^18*(d/18 + (5*e)/9) + x^2*((45*d)/2 + 5*e) + x^12*(10*d + (35*e)/2) + x^6
*(35*d + 20*e) + x^4*(30*d + (45*e)/4) + x^16*((5*d)/8 + (45*e)/16) + x^14*
((45*d)/14 + (60*e)/7) + x^10*(21*d + (126*e)/5) + x^8*((63*d)/2 + (105*e)/
4) - d/(2*x^2) + (e*x^20)/20 + log(x)*(10*d + e)
```

3.65 $\int (fx)^m (1+x^2)(1+2x^2+x^4)^5 \, dx$

Optimal. Leaf size=203

$$\frac{(fx)^{1+m}}{f(1+m)}+\frac{11(fx)^{3+m}}{f^3(3+m)}+\frac{55(fx)^{5+m}}{f^5(5+m)}+\frac{165(fx)^{7+m}}{f^7(7+m)}+\frac{330(fx)^{9+m}}{f^9(9+m)}+\frac{462(fx)^{11+m}}{f^{11}(11+m)}+\frac{462(fx)^{13+m}}{f^{13}(13+m)}+\frac{330(fx)^{15+m}}{f^{15}(15+m)}$$

```
[Out] (f*x)^(1+m)/f/(1+m)+11*(f*x)^(3+m)/f^3/(3+m)+55*(f*x)^(5+m)/f^5/(5+m)+165*(
f*x)^(7+m)/f^7/(7+m)+330*(f*x)^(9+m)/f^9/(9+m)+462*(f*x)^(11+m)/f^11/(11+m)
+462*(f*x)^(13+m)/f^13/(13+m)+330*(f*x)^(15+m)/f^15/(15+m)+165*(f*x)^(17+m)
/f^17/(17+m)+55*(f*x)^(19+m)/f^19/(19+m)+11*(f*x)^(21+m)/f^21/(21+m)+(f*x)^
(23+m)/f^23/(23+m)
```

Rubi [A]
time = 0.05, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {28, 276}

$$\frac{(fx)^{m+23}}{f^{23}(m+23)}+\frac{11(fx)^{m+21}}{f^{21}(m+21)}+\frac{55(fx)^{m+19}}{f^{19}(m+19)}+\frac{165(fx)^{m+17}}{f^{17}(m+17)}+\frac{330(fx)^{m+15}}{f^{15}(m+15)}+\frac{462(fx)^{m+13}}{f^{13}(m+13)}+\frac{462(fx)^{m+11}}{f^{11}(m+11)}+\frac{330(fx)^{m+9}}{f^9(m+9)}+\frac{165(fx)^{m+7}}{f^7(m+7)}+\frac{55(fx)^{m+5}}{f^5(m+5)}+\frac{11(fx)^{m+3}}{f^3(m+3)}+\frac{(fx)^{m+1}}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (f*x)^(1 + m)/(f*(1 + m)) + (11*(f*x)^(3 + m))/(f^3*(3 + m)) + (55*(f*x)^(5
 + m))/(f^5*(5 + m)) + (165*(f*x)^(7 + m))/(f^7*(7 + m)) + (330*(f*x)^(9 +
m))/(f^9*(9 + m)) + (462*(f*x)^(11 + m))/(f^11*(11 + m)) + (462*(f*x)^(13 +
 m))/(f^13*(13 + m)) + (330*(f*x)^(15 + m))/(f^15*(15 + m)) + (165*(f*x)^(1
7 + m))/(f^17*(17 + m)) + (55*(f*x)^(19 + m))/(f^19*(19 + m)) + (11*(f*x)^(
21 + m))/(f^21*(21 + m)) + (f*x)^(23 + m)/(f^23*(23 + m))
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int (fx)^m \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \int (fx)^m \left(1+x^2\right)^{11} \, dx$$
$$= \int \left((fx)^m + \frac{11(fx)^{2+m}}{f^2} + \frac{55(fx)^{4+m}}{f^4} + \frac{165(fx)^{6+m}}{f^6} + \frac{330(fx)^{8+}}{f^8}\right.$$
$$= \frac{(fx)^{1+m}}{f(1+m)} + \frac{11(fx)^{3+m}}{f^3(3+m)} + \frac{55(fx)^{5+m}}{f^5(5+m)} + \frac{165(fx)^{7+m}}{f^7(7+m)} + \frac{330(fx)^{9+}}{f^9(9+m}$$

Mathematica [A]
time = 0.08, size = 122, normalized size = 0.60

$$x(fx)^m \left(\frac{1}{1+m} + \frac{11x^2}{3+m} + \frac{55x^4}{5+m} + \frac{165x^6}{7+m} + \frac{330x^8}{9+m} + \frac{462x^{10}}{11+m} + \frac{462x^{12}}{13+m} + \frac{330x^{14}}{15+m} + \frac{165x^{16}}{17+m} + \frac{55x^{18}}{19+m} + \frac{11x^{20}}{21+m} + \frac{x^{22}}{23+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x*(f*x)^m*((1 + m)^(-1) + (11*x^2)/(3 + m) + (55*x^4)/(5 + m) + (165*x^6)/(
7 + m) + (330*x^8)/(9 + m) + (462*x^10)/(11 + m) + (462*x^12)/(13 + m) + (3
30*x^14)/(15 + m) + (165*x^16)/(17 + m) + (55*x^18)/(19 + m) + (11*x^20)/(2
1 + m) + x^22/(23 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(203) = 406$.
time = 0.02, size = 1121, normalized size = 5.52 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] (f*x)^m*(m^11*x^22+121*m^10*x^22+11*m^11*x^20+6435*m^9*x^22+1353*m^10*x^20+
197835*m^8*x^22+55*m^11*x^18+72985*m^9*x^20+3889578*m^7*x^22+6875*m^10*x^18
+2271555*m^8*x^20+51069018*m^6*x^22+165*m^11*x^16+376365*m^9*x^18+45134958*
m^7*x^20+453714470*m^5*x^22+20955*m^10*x^16+11870265*m^8*x^18+597988314*m^6
*x^20+2702025590*m^4*x^22+330*m^11*x^14+1164735*m^9*x^16+238653030*m^7*x^18
+5353566130*m^5*x^20+10431670821*m^3*x^22+42570*m^10*x^14+37263105*m^8*x^16
+3194704590*m^6*x^18+32087153670*m^4*x^20+24372200061*m^2*x^22+462*m^11*x^1
2+2403390*m^9*x^14+759091410*m^7*x^16+28857216410*m^5*x^18+124530626231*m^3
*x^20+29985521895*m*x^22+60522*m^10*x^12+78076350*m^8*x^14+10282782510*m^6*
x^16+174273100210*m^4*x^18+292163767533*m^2*x^20+13749310575*x^22+462*m^11*
x^10+3471930*m^9*x^12+1613983140*m^7*x^14+93862508190*m^5*x^16+680615362515
*m^3*x^18+360568238085*m*x^20+61446*m^10*x^10+114642990*m^8*x^12+2216492586
0*m^6*x^14+572017996770*m^4*x^16+1604842704135*m^2*x^18+165646455975*x^20+3
30*m^11*x^8+3582810*m^9*x^10+2408820876*m^7*x^12+204865733820*m^5*x^14+2251
106854425*m^3*x^16+1988025402825*m*x^18+44550*m^10*x^8+120367170*m^8*x^10+3
```

```
3609870756*m^6*x^12+1262375264700*m^4*x^14+5340787250535*m^2*x^16+915414625
125*x^18+165*m^11*x^6+2640990*m^9*x^8+2575140876*m^7*x^10+315347150580*m^5*
x^12+5015196628530*m^3*x^14+6646727085075*m*x^16+22605*m^10*x^6+90358290*m^
8*x^8+36597992508*m^6*x^10+1969992823260*m^4*x^12+11991258123570*m^2*x^14+3
069331390125*x^16+55*m^11*x^4+1362735*m^9*x^6+1971903780*m^7*x^8+3496975528
20*m^5*x^10+7921249136262*m^3*x^12+15011348834790*m*x^14+7645*m^10*x^4+4752
4455*m^8*x^6+28627538940*m^6*x^8+2222832699780*m^4*x^10+19130651800722*m^2*
x^12+6957151150950*x^14+11*m^11*x^2+468765*m^9*x^4+1059893010*m^7*x^6+27969
1771260*m^5*x^8+9079996141062*m^3*x^10+24133835554290*m*x^12+1551*m^10*x^2+
16677375*m^8*x^4+15768085410*m^6*x^6+1818135330660*m^4*x^8+22226933020446*m
^2*x^10+11238474936150*x^12+m^11+96745*m^9*x^2+380801190*m^7*x^4+1582932129
90*m^5*x^6+7587607623090*m^3*x^8+28336045738770*m*x^10+143*m^10+3514005*m^8
*x^2+5825106210*m^6*x^4+1059628145070*m^4*x^6+18930738943710*m^2*x^8+132818
34015450*x^10+9075*m^9+82295598*m^7*x^2+60431072570*m^5*x^4+4558015784025*m
^3*x^6+24503570194950*m*x^8+336765*m^8+1298935638*m^6*x^2+420404849150*m^4*
x^4+11703493287585*m^2*x^6+11595251918250*x^8+8103018*m^7+14014513810*m^5*x
^2+1889780020755*m^3*x^4+15515657331075*m*x^6+132426294*m^6+102468500970*m^
4*x^2+5087634488145*m^2*x^4+7454090518875*x^6+1495875590*m^5+490955350391*m
^3*x^2+7041864340665*m*x^4+11641582810*m^4+1434440867211*m^2*x^2+3478575575
475*x^4+60936676581*m^3+2192684754645*m*x^2+203363952363*m^2+1159525191825*
x^2+387182170935*m+316234143225)*x/(1+m)/(3+m)/(5+m)/(7+m)/(9+m)/(11+m)/(13
+m)/(15+m)/(17+m)/(19+m)/(21+m)/(23+m)
```

Maxima [A]
time = 0.31, size = 192, normalized size = 0.95

$$\frac{f^m x^{23} x^m}{m+23}+\frac{11 f^m x^{21} x^m}{m+21}+\frac{55 f^m x^{19} x^m}{m+19}+\frac{165 f^m x^{17} x^m}{m+17}+\frac{330 f^m x^{15} x^m}{m+15}+\frac{462 f^m x^{13} x^m}{m+13}+\frac{462 f^m x^{11} x^m}{m+11}+\frac{330 f^m x^9 x^m}{m+9}+\frac{165 f^m x^7 x^m}{m+7}+\frac{55 f^m x^5 x^m}{m+5}+\frac{11 f^m x^3 x^m}{m+3}+\frac{(fx)^{m+1}}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] f^m*x^23*x^m/(m + 23) + 11*f^m*x^21*x^m/(m + 21) + 55*f^m*x^19*x^m/(m + 19)
 + 165*f^m*x^17*x^m/(m + 17) + 330*f^m*x^15*x^m/(m + 15) + 462*f^m*x^13*x^m
/(m + 13) + 462*f^m*x^11*x^m/(m + 11) + 330*f^m*x^9*x^m/(m + 9) + 165*f^m*x
^7*x^m/(m + 7) + 55*f^m*x^5*x^m/(m + 5) + 11*f^m*x^3*x^m/(m + 3) + (f*x)^(m
 + 1)/(f*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 759 vs. 2(203) = 406.
time = 0.60, size = 759, normalized size = 3.74

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] ((m^11 + 121*m^10 + 6435*m^9 + 197835*m^8 + 3889578*m^7 + 51069018*m^6 + 45
3714470*m^5 + 2702025590*m^4 + 10431670821*m^3 + 24372200061*m^2 + 29985521
895*m + 13749310575)*x^23 + 11*(m^11 + 123*m^10 + 6635*m^9 + 206505*m^8 + 4
103178*m^7 + 54362574*m^6 + 486687830*m^5 + 2917013970*m^4 + 11320966021*m^
3 + 26560342503*m^2 + 32778930735*m + 15058768725)*x^21 + 55*(m^11 + 125*m^
10 + 6843*m^9 + 215823*m^8 + 4339146*m^7 + 58085538*m^6 + 524676662*m^5 + 3
168601822*m^4 + 12374824773*m^3 + 29178958257*m^2 + 36145916415*m + 1664390
2275)*x^19 + 165*(m^11 + 127*m^10 + 7059*m^9 + 225837*m^8 + 4600554*m^7 + 6
2319894*m^6 + 568863686*m^5 + 3466775738*m^4 + 13643071845*m^3 + 3236840757
9*m^2 + 40283194455*m + 18602008425)*x^17 + 330*(m^11 + 129*m^10 + 7283*m^9
 + 236595*m^8 + 4890858*m^7 + 67166442*m^6 + 620805254*m^5 + 3825379590*m^4
 + 15197565541*m^3 + 36337145829*m^2 + 45488935863*m + 21082276215)*x^15 +
462*(m^11 + 131*m^10 + 7515*m^9 + 248145*m^8 + 5213898*m^7 + 72748638*m^6 +
 682569590*m^5 + 4264053730*m^4 + 17145560901*m^3 + 41408337231*m^2 + 52237
739295*m + 24325703325)*x^13 + 462*(m^11 + 133*m^10 + 7755*m^9 + 260535*m^8
 + 5573898*m^7 + 79216434*m^6 + 756921110*m^5 + 4811326190*m^4 + 1965367130
1*m^3 + 48110244633*m^2 + 61333432335*m + 28748558475)*x^11 + 330*(m^11 + 1
35*m^10 + 8003*m^9 + 273813*m^8 + 5975466*m^7 + 86750118*m^6 + 847550822*m^
5 + 5509501002*m^4 + 22992750373*m^3 + 57365875587*m^2 + 74253243015*m + 35
137127025)*x^9 + 165*(m^11 + 137*m^10 + 8259*m^9 + 288027*m^8 + 6423594*m^7
 + 95564154*m^6 + 959352806*m^5 + 6421988758*m^4 + 27624338085*m^3 + 709302
62349*m^2 + 94034286855*m + 45176306175)*x^7 + 55*(m^11 + 139*m^10 + 8523*m
^9 + 303225*m^8 + 6923658*m^7 + 105911022*m^6 + 1098746774*m^5 + 7643724530
*m^4 + 34359636741*m^3 + 92502445239*m^2 + 128033897103*m + 63246828645)*x^
5 + 11*(m^11 + 141*m^10 + 8795*m^9 + 319455*m^8 + 7481418*m^7 + 118085058*m
^6 + 1274046710*m^5 + 9315318270*m^4 + 44632304581*m^3 + 130403715201*m^2 +
 199334977695*m + 105411381075)*x^3 + (m^11 + 143*m^10 + 9075*m^9 + 336765*
m^8 + 8103018*m^7 + 132426294*m^6 + 1495875590*m^5 + 11641582810*m^4 + 6093
6676581*m^3 + 203363952363*m^2 + 387182170935*m + 316234143225)*x)*(f*x)^m/
(m^12 + 144*m^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1
628301884*m^6 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 5905
46123298*m^2 + 703416314160*m + 316234143225)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 11387 vs. $2(177) = 354$.

time = 2.75, size = 11387, normalized size = 56.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] Piecewise(((log(x) - 11/(2*x**2) - 55/(4*x**4) - 55/(2*x**6) - 165/(4*x**8)
 - 231/(5*x**10) - 77/(2*x**12) - 165/(7*x**14) - 165/(16*x**16) - 55/(18*x
**18) - 11/(20*x**20) - 1/(22*x**22))/f**23, Eq(m, -23)), ((x**2/2 + 11*log
```

```
(x) - 55/(2*x**2) - 165/(4*x**4) - 55/x**6 - 231/(4*x**8) - 231/(5*x**10) -
 55/(2*x**12) - 165/(14*x**14) - 55/(16*x**16) - 11/(18*x**18) - 1/(20*x**2
0))/f**21, Eq(m, -21)), ((x**4/4 + 11*x**2/2 + 55*log(x) - 165/(2*x**2) - 1
65/(2*x**4) - 77/x**6 - 231/(4*x**8) - 33/x**10 - 55/(4*x**12) - 55/(14*x**
14) - 11/(16*x**16) - 1/(18*x**18))/f**19, Eq(m, -19)), ((x**6/6 + 11*x**4/
4 + 55*x**2/2 + 165*log(x) - 165/x**2 - 231/(2*x**4) - 77/x**6 - 165/(4*x**
8) - 33/(2*x**10) - 55/(12*x**12) - 11/(14*x**14) - 1/(16*x**16))/f**17, Eq
(m, -17)), ((x**8/8 + 11*x**6/6 + 55*x**4/4 + 165*x**2/2 + 330*log(x) - 231
/x**2 - 231/(2*x**4) - 55/x**6 - 165/(8*x**8) - 11/(2*x**10) - 11/(12*x**12
) - 1/(14*x**14))/f**15, Eq(m, -15)), ((x**10/10 + 11*x**8/8 + 55*x**6/6 +
165*x**4/4 + 165*x**2 + 462*log(x) - 231/x**2 - 165/(2*x**4) - 55/(2*x**6)
 - 55/(8*x**8) - 11/(10*x**10) - 1/(12*x**12))/f**13, Eq(m, -13)), ((x**12/1
2 + 11*x**10/10 + 55*x**8/8 + 55*x**6/2 + 165*x**4/2 + 231*x**2 + 462*log(x
) - 165/x**2 - 165/(4*x**4) - 55/(6*x**6) - 11/(8*x**8) - 1/(10*x**10))/f**
11, Eq(m, -11)), ((x**14/14 + 11*x**12/12 + 11*x**10/2 + 165*x**8/8 + 55*x*
*6 + 231*x**4/2 + 231*x**2 + 330*log(x) - 165/(2*x**2) - 55/(4*x**4) - 11/(
6*x**6) - 1/(8*x**8))/f**9, Eq(m, -9)), ((x**16/16 + 11*x**14/14 + 55*x**12
/12 + 33*x**10/2 + 165*x**8/4 + 77*x**6 + 231*x**4/2 + 165*x**2 + 165*log(x
) - 55/(2*x**2) - 11/(4*x**4) - 1/(6*x**6))/f**7, Eq(m, -7)), ((x**18/18 +
11*x**16/16 + 55*x**14/14 + 55*x**12/4 + 33*x**10 + 231*x**8/4 + 77*x**6 +
165*x**4/2 + 165*x**2/2 + 55*log(x) - 11/(2*x**2) - 1/(4*x**4))/f**5, Eq(m,
 -5)), ((x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2 +
 231*x**10/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 11*log(x) -
1/(2*x**2))/f**3, Eq(m, -3)), ((x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*
x**16/16 + 165*x**14/7 + 77*x**12/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2
+ 55*x**4/4 + 11*x**2/2 + log(x))/f, Eq(m, -1)), (m**11*x**23*(f*x)**m/(m**
12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
+ 590546123298*m**2 + 703416314160*m + 316234143225) + 11*m**11*x**21*(f*x)
**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14052931
2*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643006289
44*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 55*m**11*x**
19*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 +
140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26
4300628944*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 165*
m**11*x**17*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 843978
3*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*
m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31623414322
5) + 330*m**11*x**15*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9
 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m**5 + 725
78259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 703416314160*m + 31
6234143225) + 462*m**11*x**13*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + 34
5840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 13137458400*m
**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7034163141
60*m + 316234143225) + 462*m**11*x**11*(f*x)**m/(m**12 + 144*m**11 + 9218*m
```

```
**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**6 + 1313
7458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*m**2 + 7
03416314160*m + 316234143225) + 330*m**11*x**9*(f*x)**m/(m**12 + 144*m**11
+ 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 1628301884*m**
6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 590546123298*
m**2 + 703416314160*m + 316234143225) + 165*m**11*x**7*(f*x)**m/(m**12 + 14
4*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 + 162830
1884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3 + 59054
6123298*m**2 + 703416314160*m + 316234143225) + 55*m**11*x**5*(f*x)**m/(m**
12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312*m**7 +
 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 264300628944*m**3
+ 590546123298*m**2 + 703416314160*m + 316234143225) + 11*m**11*x**3*(f*x)*
*m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 140529312
*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 26430062894
4*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + m**11*x*(f*x)
**m/(m**12 + 144*m**11 + 9218*m**10 + 345840*m**9 + 8439783*m**8 + 14052931
2*m**7 + 1628301884*m**6 + 13137458400*m**5 + 72578259391*m**4 + 2643006289
44*m**3 + 590546123298*m**2 + 703416314160*m + 316234143225) + 121*m**10*x*
*23*(f*x)**m/(m**12 + 144*m**11 + 9218*m**10 + ...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1848 vs. $2(203) = 406$.
time = 3.87, size = 1848, normalized size = 9.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] ((f*x)^m*m^11*x^23 + 121*(f*x)^m*m^10*x^23 + 11*(f*x)^m*m^11*x^21 + 6435*(f
*x)^m*m^9*x^23 + 1353*(f*x)^m*m^10*x^21 + 197835*(f*x)^m*m^8*x^23 + 55*(f*x
)^m*m^11*x^19 + 72985*(f*x)^m*m^9*x^21 + 3889578*(f*x)^m*m^7*x^23 + 6875*(f
*x)^m*m^10*x^19 + 2271555*(f*x)^m*m^8*x^21 + 51069018*(f*x)^m*m^6*x^23 + 16
5*(f*x)^m*m^11*x^17 + 376365*(f*x)^m*m^9*x^19 + 45134958*(f*x)^m*m^7*x^21 +
 453714470*(f*x)^m*m^5*x^23 + 20955*(f*x)^m*m^10*x^17 + 11870265*(f*x)^m*m^
8*x^19 + 597988314*(f*x)^m*m^6*x^21 + 2702025590*(f*x)^m*m^4*x^23 + 330*(f*
x)^m*m^11*x^15 + 1164735*(f*x)^m*m^9*x^17 + 238653030*(f*x)^m*m^7*x^19 + 53
53566130*(f*x)^m*m^5*x^21 + 10431670821*(f*x)^m*m^3*x^23 + 42570*(f*x)^m*m^
10*x^15 + 37263105*(f*x)^m*m^8*x^17 + 3194704590*(f*x)^m*m^6*x^19 + 3208715
3670*(f*x)^m*m^4*x^21 + 24372200061*(f*x)^m*m^2*x^23 + 462*(f*x)^m*m^11*x^1
3 + 2403390*(f*x)^m*m^9*x^15 + 759091410*(f*x)^m*m^7*x^17 + 28857216410*(f*
x)^m*m^5*x^19 + 124530626231*(f*x)^m*m^3*x^21 + 29985521895*(f*x)^m*m*x^23
+ 60522*(f*x)^m*m^10*x^13 + 78076350*(f*x)^m*m^8*x^15 + 10282782510*(f*x)^m
*m^6*x^17 + 174273100210*(f*x)^m*m^4*x^19 + 292163767533*(f*x)^m*m^2*x^21 +
 13749310575*(f*x)^m*x^23 + 462*(f*x)^m*m^11*x^11 + 3471930*(f*x)^m*m^9*x^1
```

```
3 + 1613983140*(f*x)^m*m^7*x^15 + 93862508190*(f*x)^m*m^5*x^17 + 6806153625
15*(f*x)^m*m^3*x^19 + 360568238085*(f*x)^m*m*x^21 + 61446*(f*x)^m*m^10*x^11
 + 114642990*(f*x)^m*m^8*x^13 + 22164925860*(f*x)^m*m^6*x^15 + 572017996770
*(f*x)^m*m^4*x^17 + 1604842704135*(f*x)^m*m^2*x^19 + 165646455975*(f*x)^m*x
^21 + 330*(f*x)^m*m^11*x^9 + 3582810*(f*x)^m*m^9*x^11 + 2408820876*(f*x)^m*
m^7*x^13 + 204865733820*(f*x)^m*m^5*x^15 + 2251106854425*(f*x)^m*m^3*x^17 +
 1988025402825*(f*x)^m*m*x^19 + 44550*(f*x)^m*m^10*x^9 + 120367170*(f*x)^m*
m^8*x^11 + 33609870756*(f*x)^m*m^6*x^13 + 1262375264700*(f*x)^m*m^4*x^15 +
5340787250535*(f*x)^m*m^2*x^17 + 915414625125*(f*x)^m*x^19 + 165*(f*x)^m*m^
11*x^7 + 2640990*(f*x)^m*m^9*x^9 + 2575140876*(f*x)^m*m^7*x^11 + 3153471505
80*(f*x)^m*m^5*x^13 + 5015196628530*(f*x)^m*m^3*x^15 + 6646727085075*(f*x)^
m*m*x^17 + 22605*(f*x)^m*m^10*x^7 + 90358290*(f*x)^m*m^8*x^9 + 36597992508*
(f*x)^m*m^6*x^11 + 1969992823260*(f*x)^m*m^4*x^13 + 11991258123570*(f*x)^m*
m^2*x^15 + 3069331390125*(f*x)^m*x^17 + 55*(f*x)^m*m^11*x^5 + 1362735*(f*x)
^m*m^9*x^7 + 1971903780*(f*x)^m*m^7*x^9 + 349697552820*(f*x)^m*m^5*x^11 + 7
921249136262*(f*x)^m*m^3*x^13 + 15011348834790*(f*x)^m*m*x^15 + 7645*(f*x)^
m*m^10*x^5 + 47524455*(f*x)^m*m^8*x^7 + 28627538940*(f*x)^m*m^6*x^9 + 22228
32699780*(f*x)^m*m^4*x^11 + 19130651800722*(f*x)^m*m^2*x^13 + 6957151150950
*(f*x)^m*x^15 + 11*(f*x)^m*m^11*x^3 + 468765*(f*x)^m*m^9*x^5 + 1059893010*(
f*x)^m*m^7*x^7 + 279691771260*(f*x)^m*m^5*x^9 + 9079996141062*(f*x)^m*m^3*x
^11 + 24133835554290*(f*x)^m*m*x^13 + 1551*(f*x)^m*m^10*x^3 + 16677375*(f*x
)^m*m^8*x^5 + 15768085410*(f*x)^m*m^6*x^7 + 1818135330660*(f*x)^m*m^4*x^9 +
 22226933020446*(f*x)^m*m^2*x^11 + 11238474936150*(f*x)^m*x^13 + (f*x)^m*m^
11*x + 96745*(f*x)^m*m^9*x^3 + 380801190*(f*x)^m*m^7*x^5 + 158293212990*(f*
x)^m*m^5*x^7 + 7587607623090*(f*x)^m*m^3*x^9 + 28336045738770*(f*x)^m*m*x^1
1 + 143*(f*x)^m*m^10*x + 3514005*(f*x)^m*m^8*x^3 + 5825106210*(f*x)^m*m^6*x
^5 + 1059628145070*(f*x)^m*m^4*x^7 + 18930738943710*(f*x)^m*m^2*x^9 + 13281
834015450*(f*x)^m*x^11 + 9075*(f*x)^m*m^9*x + 82295598*(f*x)^m*m^7*x^3 + 60
431072570*(f*x)^m*m^5*x^5 + 4558015784025*(f*x)^m*m^3*x^7 + 24503570194950*
(f*x)^m*m*x^9 + 336765*(f*x)^m*m^8*x + 1298935638*(f*x)^m*m^6*x^3 + 4204048
49150*(f*x)^m*m^4*x^5 + 11703493287585*(f*x)^m*m^2*x^7 + 11595251918250*(f*
x)^m*x^9 + 8103018*(f*x)^m*m^7*x + 14014513810*(f*x)^m*m^5*x^3 + 1889780020
755*(f*x)^m*m^3*x^5 + 15515657331075*(f*x)^m*m*x^7 + 132426294*(f*x)^m*m^6*
x + 102468500970*(f*x)^m*m^4*x^3 + 5087634488145*(f*x)^m*m^2*x^5 + 74540905
18875*(f*x)^m*x^7 + 1495875590*(f*x)^m*m^5*x + 490955350391*(f*x)^m*m^3*x^3
 + 7041864340665*(f*x)^m*m*x^5 + 11641582810*(f*x)^m*m^4*x + 1434440867211*
(f*x)^m*m^2*x^3 + 3478575575475*(f*x)^m*x^5 + 60936676581*(f*x)^m*m^3*x + 2
192684754645*(f*x)^m*m*x^3 + 203363952363*(f*x)^m*m^2*x + 1159525191825*(f*
x)^m*x^3 + 387182170935*(f*x)^m*m*x + 316234143225*(f*x)^m*x)/(m^12 + 144*m
^11 + 9218*m^10 + 345840*m^9 + 8439783*m^8 + 140529312*m^7 + 1628301884*m^6
 + 13137458400*m^5 + 72578259391*m^4 + 264300628944*m^3 + 590546123298*m^2
+ 703416314160*m + 316234143225)
```

Mupad [B]

time = 1.25, size = 1483, normalized size = 7.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)*(f*x)^m*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] (x^3*(f*x)^m*(2192684754645*m + 1434440867211*m^2 + 490955350391*m^3 + 1024
68500970*m^4 + 14014513810*m^5 + 1298935638*m^6 + 82295598*m^7 + 3514005*m^
8 + 96745*m^9 + 1551*m^10 + 11*m^11 + 1159525191825))/(703416314160*m + 590
546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628
301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^1
1 + m^12 + 316234143225) + (x^19*(f*x)^m*(1988025402825*m + 1604842704135*m
^2 + 680615362515*m^3 + 174273100210*m^4 + 28857216410*m^5 + 3194704590*m^6
 + 238653030*m^7 + 11870265*m^8 + 376365*m^9 + 6875*m^10 + 55*m^11 + 915414
625125))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 7257825939
1*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 34
5840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^11*(f*x)^m*(283
36045738770*m + 22226933020446*m^2 + 9079996141062*m^3 + 2222832699780*m^4
+ 349697552820*m^5 + 36597992508*m^6 + 2575140876*m^7 + 120367170*m^8 + 358
2810*m^9 + 61446*m^10 + 462*m^11 + 13281834015450))/(703416314160*m + 59054
6123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 162830
1884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11
+ m^12 + 316234143225) + (x^21*(f*x)^m*(360568238085*m + 292163767533*m^2 +
 124530626231*m^3 + 32087153670*m^4 + 5353566130*m^5 + 597988314*m^6 + 4513
4958*m^7 + 2271555*m^8 + 72985*m^9 + 1353*m^10 + 11*m^11 + 165646455975))/(
703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13
137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 +
 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^5*(f*x)^m*(7041864340665*
m + 5087634488145*m^2 + 1889780020755*m^3 + 420404849150*m^4 + 60431072570*
m^5 + 5825106210*m^6 + 380801190*m^7 + 16677375*m^8 + 468765*m^9 + 7645*m^1
0 + 55*m^11 + 3478575575475))/(703416314160*m + 590546123298*m^2 + 26430062
8944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m
^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)
 + (x^17*(f*x)^m*(6646727085075*m + 5340787250535*m^2 + 2251106854425*m^3 +
 572017996770*m^4 + 93862508190*m^5 + 10282782510*m^6 + 759091410*m^7 + 372
63105*m^8 + 1164735*m^9 + 20955*m^10 + 165*m^11 + 3069331390125))/(70341631
4160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 1313745840
0*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^
10 + 144*m^11 + m^12 + 316234143225) + (x*(f*x)^m*(387182170935*m + 2033639
52363*m^2 + 60936676581*m^3 + 11641582810*m^4 + 1495875590*m^5 + 132426294*
m^6 + 8103018*m^7 + 336765*m^8 + 9075*m^9 + 143*m^10 + m^11 + 316234143225)
)/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 +
 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^
9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^23*(f*x)^m*(2998552189
```

5*m + 24372200061*m^2 + 10431670821*m^3 + 2702025590*m^4 + 453714470*m^5 + 51069018*m^6 + 3889578*m^7 + 197835*m^8 + 6435*m^9 + 121*m^10 + m^11 + 13749310575))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^7*(f*x)^m*(15515657331075*m + 11703493287585*m^2 + 4558015784025*m^3 + 1059628145070*m^4 + 158293212990*m^5 + 15768085410*m^6 + 1059893010*m^7 + 47524455*m^8 + 1362735*m^9 + 22605*m^10 + 165*m^11 + 7454090518875))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^15*(f*x)^m*(15011348834790*m + 11991258123570*m^2 + 5015196628530*m^3 + 1262375264700*m^4 + 204865733820*m^5 + 22164925860*m^6 + 1613983140*m^7 + 78076350*m^8 + 2403390*m^9 + 42570*m^10 + 330*m^11 + 6957151150950))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^9*(f*x)^m*(24503570194950*m + 18930738943710*m^2 + 7587607623090*m^3 + 1818135330660*m^4 + 279691771260*m^5 + 28627538940*m^6 + 1971903780*m^7 + 90358290*m^8 + 2640990*m^9 + 44550*m^10 + 330*m^11 + 11595251918250))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225) + (x^13*(f*x)^m*(24133835554290*m + 19130651800722*m^2 + 7921249136262*m^3 + 1969992823260*m^4 + 315347150580*m^5 + 33609870756*m^6 + 2408820876*m^7 + 114642990*m^8 + 3471930*m^9 + 60522*m^10 + 462*m^11 + 11238474936150))/(703416314160*m + 590546123298*m^2 + 264300628944*m^3 + 72578259391*m^4 + 13137458400*m^5 + 1628301884*m^6 + 140529312*m^7 + 8439783*m^8 + 345840*m^9 + 9218*m^10 + 144*m^11 + m^12 + 316234143225)

3.66 $\int x^5(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=34

$$\frac{1}{24}\left(1+x^2\right)^{12}-\frac{1}{13}\left(1+x^2\right)^{13}+\frac{1}{28}\left(1+x^2\right)^{14}$$

```
[Out] 1/24*(x^2+1)^12-1/13*(x^2+1)^13+1/28*(x^2+1)^14
```

Rubi [A]
time = 0.03, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 272, 45}

$$\frac{1}{28}\left(x^2+1\right)^{14}-\frac{1}{13}\left(x^2+1\right)^{13}+\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24 - (1 + x^2)^13/13 + (1 + x^2)^14/28
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int x^5 \left(1+x^2\right) \left(1+2x^2+x^4\right)^5 \, dx = \int x^5 \left(1+x^2\right)^{11} \, dx$$
$$= \frac{1}{2}\text{Subst}\left(\int x^2(1+x)^{11} \, dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left((1+x)^{11} - 2(1+x)^{12} + (1+x)^{13}\right) \, dx, x, x^2\right)$$
$$= \frac{1}{24}\left(1+x^2\right)^{12} - \frac{1}{13}\left(1+x^2\right)^{13} + \frac{1}{28}\left(1+x^2\right)^{14}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 85 vs. 2(34) = 68.
time = 0.00, size = 85, normalized size = 2.50

$$\frac{x^6}{6} + \frac{11x^8}{8} + \frac{11x^{10}}{2} + \frac{55x^{12}}{4} + \frac{165x^{14}}{7} + \frac{231x^{16}}{8} + \frac{77x^{18}}{3} + \frac{33x^{20}}{2} + \frac{15x^{22}}{2} + \frac{55x^{24}}{24} + \frac{11x^{26}}{26} + \frac{x^{28}}{28}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/
8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 +
 x^28/28
```

Maple [A]
time = 0.06, size = 29, normalized size = 0.85

method	result
default	$\frac{\left(x^2+1\right)^{12}}{24} - \frac{\left(x^2+1\right)^{13}}{13} + \frac{\left(x^2+1\right)^{14}}{28}$
norman	$\frac{1}{28}x^{28} + \frac{33}{2}x^{20} + \frac{15}{2}x^{22} + \frac{55}{24}x^{24} + \frac{11}{26}x^{26} + \frac{165}{7}x^{14} + \frac{231}{8}x^{16} + \frac{77}{3}x^{18} + \frac{11}{8}x^8 + \frac{11}{2}x^{10} + \frac{55}{4}x^{12} + \frac{1}{6}x^6$
gosper	$\frac{x^6\left(78x^{22}+924x^{20}+5005x^{18}+16380x^{16}+36036x^{14}+56056x^{12}+63063x^{10}+51480x^8+30030x^6+12012x^4+3003x^2+364\right)}{2184}$
risch	$\frac{15}{2}x^{22} + \frac{11}{26}x^{26} + \frac{55}{24}x^{24} + \frac{11}{8}x^8 + \frac{1}{2184} + \frac{1}{6}x^6 + \frac{11}{2}x^{10} + \frac{55}{4}x^{12} + \frac{165}{7}x^{14} + \frac{33}{2}x^{20} + \frac{77}{3}x^{18} + \frac{231}{8}x^{16} +$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(x^2+1)^12-1/13*(x^2+1)^13+1/28*(x^2+1)^14
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. 2(28) = 56.
time = 0.28, size = 61, normalized size = 1.79

$$\frac{1}{28}x^{28} + \frac{11}{26}x^{26} + \frac{55}{24}x^{24} + \frac{15}{2}x^{22} + \frac{33}{2}x^{20} + \frac{77}{3}x^{18} + \frac{231}{8}x^{16} + \frac{165}{7}x^{14} + \frac{55}{4}x^{12} + \frac{11}{2}x^{10} + \frac{11}{8}x^8 + \frac{1}{6}x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.
time = 0.49, size = 61, normalized size = 1.79

$$\frac{1}{28}\,x^{28}+\frac{11}{26}\,x^{26}+\frac{55}{24}\,x^{24}+\frac{15}{2}\,x^{22}+\frac{33}{2}\,x^{20}+\frac{77}{3}\,x^{18}+\frac{231}{8}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{55}{4}\,x^{12}+\frac{11}{2}\,x^{10}+\frac{11}{8}\,x^{8}+\frac{1}{6}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(24) = 48$.
time = 0.01, size = 76, normalized size = 2.24

$$\frac{x^{28}}{28}+\frac{11x^{26}}{26}+\frac{55x^{24}}{24}+\frac{15x^{22}}{2}+\frac{33x^{20}}{2}+\frac{77x^{18}}{3}+\frac{231x^{16}}{8}+\frac{165x^{14}}{7}+\frac{55x^{12}}{4}+\frac{11x^{10}}{2}+\frac{11x^{8}}{8}+\frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**28/28 + 11*x**26/26 + 55*x**24/24 + 15*x**22/2 + 33*x**20/2 + 77*x**18/3
 + 231*x**16/8 + 165*x**14/7 + 55*x**12/4 + 11*x**10/2 + 11*x**8/8 + x**6/6
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(28) = 56$.
time = 3.70, size = 61, normalized size = 1.79

$$\frac{1}{28}\,x^{28}+\frac{11}{26}\,x^{26}+\frac{55}{24}\,x^{24}+\frac{15}{2}\,x^{22}+\frac{33}{2}\,x^{20}+\frac{77}{3}\,x^{18}+\frac{231}{8}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{55}{4}\,x^{12}+\frac{11}{2}\,x^{10}+\frac{11}{8}\,x^{8}+\frac{1}{6}\,x^{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/28*x^28 + 11/26*x^26 + 55/24*x^24 + 15/2*x^22 + 33/2*x^20 + 77/3*x^18 + 2
31/8*x^16 + 165/7*x^14 + 55/4*x^12 + 11/2*x^10 + 11/8*x^8 + 1/6*x^6
```

Mupad [B]
time = 0.06, size = 61, normalized size = 1.79

$$\frac{x^{28}}{28}+\frac{11\,x^{26}}{26}+\frac{55\,x^{24}}{24}+\frac{15\,x^{22}}{2}+\frac{33\,x^{20}}{2}+\frac{77\,x^{18}}{3}+\frac{231\,x^{16}}{8}+\frac{165\,x^{14}}{7}+\frac{55\,x^{12}}{4}+\frac{11\,x^{10}}{2}+\frac{11\,x^{8}}{8}+\frac{x^{6}}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^6/6 + (11*x^8)/8 + (11*x^10)/2 + (55*x^12)/4 + (165*x^14)/7 + (231*x^16)/
8 + (77*x^18)/3 + (33*x^20)/2 + (15*x^22)/2 + (55*x^24)/24 + (11*x^26)/26 +
 x^28/28
```

3.67 $\int x^4(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=83

$$\frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}+\frac{55x^{23}}{23}+\frac{11x^{25}}{25}+\frac{x^{27}}{27}$$

```
[Out] 1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/19
*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {28, 276}

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^9}{9}+\frac{11x^7}{7}+\frac{x^5}{5}$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^4\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^4\left(1+x^2\right)^{11}\,dx\\ &= \int \left(x^4+11x^6+55x^8+165x^{10}+330x^{12}+462x^{14}+462x^{16}+330x^{18}+\right.\\ &= \frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 83, normalized size = 1.00

$$\frac{x^5}{5}+\frac{11x^7}{7}+\frac{55x^9}{9}+15x^{11}+\frac{330x^{13}}{13}+\frac{154x^{15}}{5}+\frac{462x^{17}}{17}+\frac{330x^{19}}{19}+\frac{55x^{21}}{7}+\frac{55x^{23}}{23}+\frac{11x^{25}}{25}+\frac{x^{27}}{27}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

Maple [A]
time = 0.06, size = 62, normalized size = 0.75

method	result
default	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+\frac{1}{27}x^{27}$
norman	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+\frac{1}{27}x^{27}$
risch	$\frac{1}{5}x^5+\frac{11}{7}x^7+\frac{55}{9}x^9+15x^{11}+\frac{330}{13}x^{13}+\frac{154}{5}x^{15}+\frac{462}{17}x^{17}+\frac{330}{19}x^{19}+\frac{55}{7}x^{21}+\frac{55}{23}x^{23}+\frac{11}{25}x^{25}+\frac{1}{27}x^{27}$
gosper	$\frac{x^5(16900975x^{22}+200783583x^{20}+1091215125x^{18}+3585421125x^{16}+7925667750x^{14}+12401338950x^{12}+14054850810x^{10}+1158366825}{456326325}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/5*x^5+11/7*x^7+55/9*x^9+15*x^11+330/13*x^13+154/5*x^15+462/17*x^17+330/19
*x^19+55/7*x^21+55/23*x^23+11/25*x^25+1/27*x^27
```

Maxima [A]
time = 0.28, size = 61, normalized size = 0.73

$$\frac{1}{27}\,x^{27}+\frac{11}{25}\,x^{25}+\frac{55}{23}\,x^{23}+\frac{55}{7}\,x^{21}+\frac{330}{19}\,x^{19}+\frac{462}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{330}{13}\,x^{13}+15\,x^{11}+\frac{55}{9}\,x^9+\frac{11}{7}\,x^7+\frac{1}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

Fricas [A]
time = 0.38, size = 61, normalized size = 0.73

$$\frac{1}{27}\,x^{27}+\frac{11}{25}\,x^{25}+\frac{55}{23}\,x^{23}+\frac{55}{7}\,x^{21}+\frac{330}{19}\,x^{19}+\frac{462}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{330}{13}\,x^{13}+15\,x^{11}+\frac{55}{9}\,x^9+\frac{11}{7}\,x^7+\frac{1}{5}\,x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

Sympy [A]
time = 0.01, size = 75, normalized size = 0.90

$$\frac{x^{27}}{27}+\frac{11x^{25}}{25}+\frac{55x^{23}}{23}+\frac{55x^{21}}{7}+\frac{330x^{19}}{19}+\frac{462x^{17}}{17}+\frac{154x^{15}}{5}+\frac{330x^{13}}{13}+15x^{11}+\frac{55x^{9}}{9}+\frac{11x^{7}}{7}+\frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**27/27 + 11*x**25/25 + 55*x**23/23 + 55*x**21/7 + 330*x**19/19 + 462*x**1
7/17 + 154*x**15/5 + 330*x**13/13 + 15*x**11 + 55*x**9/9 + 11*x**7/7 + x**5
/5
```

Giac [A]
time = 3.99, size = 61, normalized size = 0.73

$$\frac{1}{27}\,x^{27}+\frac{11}{25}\,x^{25}+\frac{55}{23}\,x^{23}+\frac{55}{7}\,x^{21}+\frac{330}{19}\,x^{19}+\frac{462}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{330}{13}\,x^{13}+15\,x^{11}+\frac{55}{9}\,x^{9}+\frac{11}{7}\,x^{7}+\frac{1}{5}\,x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/27*x^27 + 11/25*x^25 + 55/23*x^23 + 55/7*x^21 + 330/19*x^19 + 462/17*x^17
 + 154/5*x^15 + 330/13*x^13 + 15*x^11 + 55/9*x^9 + 11/7*x^7 + 1/5*x^5
```

Mupad [B]
time = 0.06, size = 61, normalized size = 0.73

$$\frac{x^{27}}{27}+\frac{11\,x^{25}}{25}+\frac{55\,x^{23}}{23}+\frac{55\,x^{21}}{7}+\frac{330\,x^{19}}{19}+\frac{462\,x^{17}}{17}+\frac{154\,x^{15}}{5}+\frac{330\,x^{13}}{13}+15\,x^{11}+\frac{55\,x^{9}}{9}+\frac{11\,x^{7}}{7}+\frac{x^{5}}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^5/5 + (11*x^7)/7 + (55*x^9)/9 + 15*x^11 + (330*x^13)/13 + (154*x^15)/5 +
(462*x^17)/17 + (330*x^19)/19 + (55*x^21)/7 + (55*x^23)/23 + (11*x^25)/25 +
 x^27/27
```

3.68 $\int x^3(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=23

$$-\frac{1}{24}\left(1+x^2\right)^{12}+\frac{1}{26}\left(1+x^2\right)^{13}$$

```
[Out] -1/24*(x^2+1)^12+1/26*(x^2+1)^13
```

Rubi [A]
time = 0.01, antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 272, 45}

$$\frac{1}{26}\left(x^2+1\right)^{13}-\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] -1/24*(1 + x^2)^12 + (1 + x^2)^13/26
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$
\begin{aligned}
\int x^3\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^3\left(1+x^2\right)^{11}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int x(1+x)^{11}\,dx,x,x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-(1+x)^{11}+(1+x)^{12}\right)\,dx,x,x^2\right) \\
&= -\frac{1}{24}\left(1+x^2\right)^{12}+\frac{1}{26}\left(1+x^2\right)^{13}
\end{aligned}
$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 83 vs. $2(23) = 46$.
time = 0.00, size = 83, normalized size = 3.61

$$
\frac{x^4}{4}+\frac{11x^6}{6}+\frac{55x^8}{8}+\frac{33x^{10}}{2}+\frac{55x^{12}}{2}+33x^{14}+\frac{231x^{16}}{8}+\frac{55x^{18}}{3}+\frac{33x^{20}}{4}+\frac{5x^{22}}{2}+\frac{11x^{24}}{24}+\frac{x^{26}}{26}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (23
1*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26
```

Maple [**A**]
time = 0.03, size = 20, normalized size = 0.87

method	result
default	$-\frac{\left(x^2+1\right)^{12}}{24}+\frac{\left(x^2+1\right)^{13}}{26}$
norman	$\frac{231}{8}x^{16}+\frac{55}{3}x^{18}+\frac{33}{4}x^{20}+\frac{5}{2}x^{22}+\frac{11}{24}x^{24}+\frac{1}{26}x^{26}+\frac{1}{4}x^4+\frac{11}{6}x^6+\frac{55}{8}x^8+\frac{33}{2}x^{10}+\frac{55}{2}x^{12}+33x^{14}$
gosper	$\frac{x^4\left(12x^{22}+143x^{20}+780x^{18}+2574x^{16}+5720x^{14}+9009x^{12}+10296x^{10}+8580x^8+5148x^6+2145x^4+572x^2+78\right)}{312}$
risch	$\frac{5}{2}x^{22}+\frac{1}{26}x^{26}+\frac{11}{24}x^{24}+\frac{55}{8}x^8+\frac{11}{6}x^6+\frac{1}{4}x^4-\frac{1}{312}+\frac{33}{2}x^{10}+\frac{55}{2}x^{12}+33x^{14}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/24*(x^2+1)^12+1/26*(x^2+1)^13
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.
time = 0.27, size = 61, normalized size = 2.65

$$
\frac{1}{26}x^{26}+\frac{11}{24}x^{24}+\frac{5}{2}x^{22}+\frac{33}{4}x^{20}+\frac{55}{3}x^{18}+\frac{231}{8}x^{16}+33x^{14}+\frac{55}{2}x^{12}+\frac{33}{2}x^{10}+\frac{55}{8}x^8+\frac{11}{6}x^6+\frac{1}{4}x^4
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.
time = 0.45, size = 61, normalized size = 2.65

$$\frac{1}{26}\,x^{26} + \frac{11}{24}\,x^{24} + \frac{5}{2}\,x^{22} + \frac{33}{4}\,x^{20} + \frac{55}{3}\,x^{18} + \frac{231}{8}\,x^{16} + 33\,x^{14} + \frac{55}{2}\,x^{12} + \frac{33}{2}\,x^{10} + \frac{55}{8}\,x^{8} + \frac{11}{6}\,x^{6} + \frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(15) = 30$.
time = 0.01, size = 75, normalized size = 3.26

$$\frac{x^{26}}{26} + \frac{11x^{24}}{24} + \frac{5x^{22}}{2} + \frac{33x^{20}}{4} + \frac{55x^{18}}{3} + \frac{231x^{16}}{8} + 33x^{14} + \frac{55x^{12}}{2} + \frac{33x^{10}}{2} + \frac{55x^{8}}{8} + \frac{11x^{6}}{6} + \frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**26/26 + 11*x**24/24 + 5*x**22/2 + 33*x**20/4 + 55*x**18/3 + 231*x**16/8
+ 33*x**14 + 55*x**12/2 + 33*x**10/2 + 55*x**8/8 + 11*x**6/6 + x**4/4
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(19) = 38$.
time = 3.69, size = 61, normalized size = 2.65

$$\frac{1}{26}\,x^{26} + \frac{11}{24}\,x^{24} + \frac{5}{2}\,x^{22} + \frac{33}{4}\,x^{20} + \frac{55}{3}\,x^{18} + \frac{231}{8}\,x^{16} + 33\,x^{14} + \frac{55}{2}\,x^{12} + \frac{33}{2}\,x^{10} + \frac{55}{8}\,x^{8} + \frac{11}{6}\,x^{6} + \frac{1}{4}\,x^{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/26*x^26 + 11/24*x^24 + 5/2*x^22 + 33/4*x^20 + 55/3*x^18 + 231/8*x^16 + 33
*x^14 + 55/2*x^12 + 33/2*x^10 + 55/8*x^8 + 11/6*x^6 + 1/4*x^4
```

Mupad [B]
time = 0.06, size = 61, normalized size = 2.65

$$\frac{x^{26}}{26} + \frac{11\,x^{24}}{24} + \frac{5\,x^{22}}{2} + \frac{33\,x^{20}}{4} + \frac{55\,x^{18}}{3} + \frac{231\,x^{16}}{8} + 33\,x^{14} + \frac{55\,x^{12}}{2} + \frac{33\,x^{10}}{2} + \frac{55\,x^{8}}{8} + \frac{11\,x^{6}}{6} + \frac{x^{4}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^4/4 + (11*x^6)/6 + (55*x^8)/8 + (33*x^10)/2 + (55*x^12)/2 + 33*x^14 + (23
1*x^16)/8 + (55*x^18)/3 + (33*x^20)/4 + (5*x^22)/2 + (11*x^24)/24 + x^26/26
```

3.69 $\int x^2(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=83

$$\frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165x^{19}}{19}+\frac{55x^{21}}{21}+\frac{11x^{23}}{23}+\frac{x^{25}}{25}$$

```
[Out] 1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x^
17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

Rubi [A]
time = 0.02, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.095$, Rules used = {28, 276}

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^9}{3}+\frac{55x^7}{7}+\frac{11x^5}{5}+\frac{x^3}{3}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int x^2\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x^2\left(1+x^2\right)^{11}\,dx\\ &= \int \left(x^2+11x^4+55x^6+165x^8+330x^{10}+462x^{12}+462x^{14}+330x^{16}+1\right.\\ &= \frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165}{19}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 83, normalized size = 1.00

$$\frac{x^3}{3}+\frac{11x^5}{5}+\frac{55x^7}{7}+\frac{55x^9}{3}+30x^{11}+\frac{462x^{13}}{13}+\frac{154x^{15}}{5}+\frac{330x^{17}}{17}+\frac{165x^{19}}{19}+\frac{55x^{21}}{21}+\frac{11x^{23}}{23}+\frac{x^{25}}{25}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

Maple [A]
time = 0.06, size = 62, normalized size = 0.75

method	result
default	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+\frac{1}{25}x^{25}$
norman	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+\frac{1}{25}x^{25}$
risch	$\frac{1}{3}x^3+\frac{11}{5}x^5+\frac{55}{7}x^7+\frac{55}{3}x^9+30x^{11}+\frac{462}{13}x^{13}+\frac{154}{5}x^{15}+\frac{330}{17}x^{17}+\frac{165}{19}x^{19}+\frac{55}{21}x^{21}+\frac{11}{23}x^{23}+\frac{1}{25}x^{25}$
gosper	$\frac{x^3(2028117x^{22}+24249225x^{20}+132793375x^{18}+440314875x^{16}+984233250x^{14}+1561650090x^{12}+1801903950x^{10}+1521087750x^8+92}{50702925}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^3+11/5*x^5+55/7*x^7+55/3*x^9+30*x^11+462/13*x^13+154/5*x^15+330/17*x^
17+165/19*x^19+55/21*x^21+11/23*x^23+1/25*x^25
```

Maxima [A]
time = 0.27, size = 61, normalized size = 0.73

$$\frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^9+\frac{55}{7}\,x^7+\frac{11}{5}\,x^5+\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

Fricas [A]
time = 0.38, size = 61, normalized size = 0.73

$$\frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^9+\frac{55}{7}\,x^7+\frac{11}{5}\,x^5+\frac{1}{3}\,x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

Sympy [A]
time = 0.01, size = 75, normalized size = 0.90

$$\frac{x^{25}}{25}+\frac{11x^{23}}{23}+\frac{55x^{21}}{21}+\frac{165x^{19}}{19}+\frac{330x^{17}}{17}+\frac{154x^{15}}{5}+\frac{462x^{13}}{13}+30x^{11}+\frac{55x^{9}}{3}+\frac{55x^{7}}{7}+\frac{11x^{5}}{5}+\frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**25/25 + 11*x**23/23 + 55*x**21/21 + 165*x**19/19 + 330*x**17/17 + 154*x*
*15/5 + 462*x**13/13 + 30*x**11 + 55*x**9/3 + 55*x**7/7 + 11*x**5/5 + x**3/
3
```

Giac [A]
time = 4.05, size = 61, normalized size = 0.73

$$\frac{1}{25}\,x^{25}+\frac{11}{23}\,x^{23}+\frac{55}{21}\,x^{21}+\frac{165}{19}\,x^{19}+\frac{330}{17}\,x^{17}+\frac{154}{5}\,x^{15}+\frac{462}{13}\,x^{13}+30\,x^{11}+\frac{55}{3}\,x^{9}+\frac{55}{7}\,x^{7}+\frac{11}{5}\,x^{5}+\frac{1}{3}\,x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/25*x^25 + 11/23*x^23 + 55/21*x^21 + 165/19*x^19 + 330/17*x^17 + 154/5*x^1
5 + 462/13*x^13 + 30*x^11 + 55/3*x^9 + 55/7*x^7 + 11/5*x^5 + 1/3*x^3
```

Mupad [B]
time = 0.06, size = 61, normalized size = 0.73

$$\frac{x^{25}}{25}+\frac{11\,x^{23}}{23}+\frac{55\,x^{21}}{21}+\frac{165\,x^{19}}{19}+\frac{330\,x^{17}}{17}+\frac{154\,x^{15}}{5}+\frac{462\,x^{13}}{13}+30\,x^{11}+\frac{55\,x^{9}}{3}+\frac{55\,x^{7}}{7}+\frac{11\,x^{5}}{5}+\frac{x^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^3/3 + (11*x^5)/5 + (55*x^7)/7 + (55*x^9)/3 + 30*x^11 + (462*x^13)/13 + (1
54*x^15)/5 + (330*x^17)/17 + (165*x^19)/19 + (55*x^21)/21 + (11*x^23)/23 +
x^25/25
```

3.70 $\int x(1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=11

$$\frac{1}{24}\left(1+x^2\right)^{12}$$

```
[Out] 1/24*(x^2+1)^12
```

Rubi [A]
time = 0.00, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {28, 267}

$$\frac{1}{24}\left(x^2+1\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 267

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a + b*x^n)
^(p + 1)/(b*n*(p + 1)), x] /; FreeQ[{a, b, m, n, p}, x] && EqQ[m, n - 1] &&
 NeQ[p, -1]
```

Rubi steps

$$\begin{aligned}\int x\left(1+x^2\right)\left(1+2x^2+x^4\right)^5\,dx &= \int x\left(1+x^2\right)^{11}\,dx\\ &= \frac{1}{24}\left(1+x^2\right)^{12}\end{aligned}$$

Mathematica [A]
time = 0.00, size = 11, normalized size = 1.00

$$\frac{1}{24}\left(1+x^2\right)^{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] (1 + x^2)^12/24
```

Maple [A]
time = 0.04, size = 10, normalized size = 0.91

method	result
default	$\frac{(x^2+1)^{12}}{24}$
gosper	$\frac{x^2(x^{22}+12x^{20}+66x^{18}+220x^{16}+495x^{14}+792x^{12}+924x^{10}+792x^8+495x^6+220x^4+66x^2+12)}{24}$
norman	$\frac{1}{2}x^{22}+\frac{1}{24}x^{24}+\frac{11}{4}x^4+\frac{55}{6}x^6+\frac{165}{8}x^8+33x^{10}+\frac{77}{2}x^{12}+33x^{14}+\frac{165}{8}x^{16}+\frac{55}{6}x^{18}+\frac{11}{4}x^{20}+\frac{1}{2}x^2$
risch	$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33x^{14}+\frac{77}{2}x^{12}+33x^{10}+\frac{165}{8}x^8+\frac{55}{6}x^6+\frac{11}{4}x^4+\frac{1}{2}x^2+\frac{1}{24}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] 1/24*(x^2+1)^12
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(9) = 18$.
time = 0.28, size = 61, normalized size = 5.55

$$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33\,x^{14}+\frac{77}{2}x^{12}+33\,x^{10}+\frac{165}{8}x^8+\frac{55}{6}x^6+\frac{11}{4}x^4+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(9) = 18$.
time = 0.42, size = 61, normalized size = 5.55

$$\frac{1}{24}x^{24}+\frac{1}{2}x^{22}+\frac{11}{4}x^{20}+\frac{55}{6}x^{18}+\frac{165}{8}x^{16}+33\,x^{14}+\frac{77}{2}x^{12}+33\,x^{10}+\frac{165}{8}x^8+\frac{55}{6}x^6+\frac{11}{4}x^4+\frac{1}{2}x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/24*x^24 + 1/2*x^22 + 11/4*x^20 + 55/6*x^18 + 165/8*x^16 + 33*x^14 + 77/2*
x^12 + 33*x^10 + 165/8*x^8 + 55/6*x^6 + 11/4*x^4 + 1/2*x^2
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(7) = 14$.
time = 0.01, size = 71, normalized size = 6.45

$$\frac{x^{24}}{24} + \frac{x^{22}}{2} + \frac{11x^{20}}{4} + \frac{55x^{18}}{6} + \frac{165x^{16}}{8} + 33x^{14} + \frac{77x^{12}}{2} + 33x^{10} + \frac{165x^{8}}{8} + \frac{55x^{6}}{6} + \frac{11x^{4}}{4} + \frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**24/24 + x**22/2 + 11*x**20/4 + 55*x**18/6 + 165*x**16/8 + 33*x**14 + 77*
x**12/2 + 33*x**10 + 165*x**8/8 + 55*x**6/6 + 11*x**4/4 + x**2/2
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(9) = 18$.
time = 4.30, size = 76, normalized size = 6.91

$$\frac{1}{24} \left(x^{4} + 2 x^{2}\right)^{6} + \frac{1}{4} \left(x^{4} + 2 x^{2}\right)^{5} + \frac{5}{8} \left(x^{4} + 2 x^{2}\right)^{4} + \frac{1}{4} x^{4} + \frac{5}{6} \left(x^{4} + 2 x^{2}\right)^{3} + \frac{5}{8} \left(x^{4} + 2 x^{2}\right)^{2} + \frac{1}{2} x^{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/24*(x^4 + 2*x^2)^6 + 1/4*(x^4 + 2*x^2)^5 + 5/8*(x^4 + 2*x^2)^4 + 1/4*x^4
+ 5/6*(x^4 + 2*x^2)^3 + 5/8*(x^4 + 2*x^2)^2 + 1/2*x^2
```

Mupad [B]
time = 0.06, size = 61, normalized size = 5.55

$$\frac{x^{24}}{24} + \frac{x^{22}}{2} + \frac{11\,x^{20}}{4} + \frac{55\,x^{18}}{6} + \frac{165\,x^{16}}{8} + 33\,x^{14} + \frac{77\,x^{12}}{2} + 33\,x^{10} + \frac{165\,x^{8}}{8} + \frac{55\,x^{6}}{6} + \frac{11\,x^{4}}{4} + \frac{x^{2}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x^2/2 + (11*x^4)/4 + (55*x^6)/6 + (165*x^8)/8 + 33*x^10 + (77*x^12)/2 + 33*
x^14 + (165*x^16)/8 + (55*x^18)/6 + (11*x^20)/4 + x^22/2 + x^24/24
```

3.71 $\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx$

Optimal. Leaf size=73

$$x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11}+\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+\frac{55x^{19}}{19}+\frac{11x^{21}}{21}+\frac{x^{23}}{23}$$

```
[Out] x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x^
17+55/19*x^19+11/21*x^21+1/23*x^23
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {28, 200}

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^9}{3}+\frac{165x^7}{7}+11x^5+\frac{11x^3}{3}+x$$

Antiderivative was successfully verified.

```
[In] Int[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}
\int (1+x^2)\left(1+2x^2+x^4\right)^5\,dx &= \int \left(1+x^2\right)^{11}\,dx\\
&= \int \left(1+11x^2+55x^4+165x^6+330x^8+462x^{10}+462x^{12}+330x^{14}+165x\right.\\
&= x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11}+\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 73, normalized size = 1.00

$$x+\frac{11x^3}{3}+11x^5+\frac{165x^7}{7}+\frac{110x^9}{3}+42x^{11}+\frac{462x^{13}}{13}+22x^{15}+\frac{165x^{17}}{17}+\frac{55x^{19}}{19}+\frac{11x^{21}}{21}+\frac{x^{23}}{23}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + x^2)*(1 + 2*x^2 + x^4)^5,x]
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

Maple [A]
time = 0.07, size = 58, normalized size = 0.79

method	result
default	$x+\frac{11}{3}x^{3}+11x^{5}+\frac{165}{7}x^{7}+\frac{110}{3}x^{9}+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{23}$
norman	$x+\frac{11}{3}x^{3}+11x^{5}+\frac{165}{7}x^{7}+\frac{110}{3}x^{9}+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{23}$
risch	$x+\frac{11}{3}x^{3}+11x^{5}+\frac{165}{7}x^{7}+\frac{110}{3}x^{9}+42x^{11}+\frac{462}{13}x^{13}+22x^{15}+\frac{165}{17}x^{17}+\frac{55}{19}x^{19}+\frac{11}{21}x^{21}+\frac{1}{23}x^{23}$
gosper	$\frac{x(88179x^{22}+1062347x^{20}+5870865x^{18}+19684665x^{16}+44618574x^{14}+72076158x^{12}+85180914x^{10}+74364290x^{8}+47805615x^{6}+2230}{2028117}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5,x,method=_RETURNVERBOSE)
```

```
[Out] x+11/3*x^3+11*x^5+165/7*x^7+110/3*x^9+42*x^11+462/13*x^13+22*x^15+165/17*x^
17+55/19*x^19+11/21*x^21+1/23*x^23
```

Maxima [A]
time = 0.28, size = 57, normalized size = 0.78

$$\frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^{9}+\frac{165}{7}\,x^{7}+11\,x^{5}+\frac{11}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="maxima")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

Fricas [A]
time = 0.64, size = 57, normalized size = 0.78

$$\frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^{9}+\frac{165}{7}\,x^{7}+11\,x^{5}+\frac{11}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="fricas")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

Sympy [A]
time = 0.01, size = 68, normalized size = 0.93

$$\frac{x^{23}}{23}+\frac{11x^{21}}{21}+\frac{55x^{19}}{19}+\frac{165x^{17}}{17}+22x^{15}+\frac{462x^{13}}{13}+42x^{11}+\frac{110x^{9}}{3}+\frac{165x^{7}}{7}+11x^{5}+\frac{11x^{3}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5,x)
```

```
[Out] x**23/23 + 11*x**21/21 + 55*x**19/19 + 165*x**17/17 + 22*x**15 + 462*x**13/
13 + 42*x**11 + 110*x**9/3 + 165*x**7/7 + 11*x**5 + 11*x**3/3 + x
```

Giac [A]
time = 3.81, size = 57, normalized size = 0.78

$$\frac{1}{23}\,x^{23}+\frac{11}{21}\,x^{21}+\frac{55}{19}\,x^{19}+\frac{165}{17}\,x^{17}+22\,x^{15}+\frac{462}{13}\,x^{13}+42\,x^{11}+\frac{110}{3}\,x^{9}+\frac{165}{7}\,x^{7}+11\,x^{5}+\frac{11}{3}\,x^{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5,x, algorithm="giac")
```

```
[Out] 1/23*x^23 + 11/21*x^21 + 55/19*x^19 + 165/17*x^17 + 22*x^15 + 462/13*x^13 +
 42*x^11 + 110/3*x^9 + 165/7*x^7 + 11*x^5 + 11/3*x^3 + x
```

Mupad [B]
time = 0.06, size = 57, normalized size = 0.78

$$\frac{x^{23}}{23}+\frac{11\,x^{21}}{21}+\frac{55\,x^{19}}{19}+\frac{165\,x^{17}}{17}+22\,x^{15}+\frac{462\,x^{13}}{13}+42\,x^{11}+\frac{110\,x^{9}}{3}+\frac{165\,x^{7}}{7}+11\,x^{5}+\frac{11\,x^{3}}{3}+x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2 + 1)*(2*x^2 + x^4 + 1)^5,x)
```

```
[Out] x + (11*x^3)/3 + 11*x^5 + (165*x^7)/7 + (110*x^9)/3 + 42*x^11 + (462*x^13)/
13 + 22*x^15 + (165*x^17)/17 + (55*x^19)/19 + (11*x^21)/21 + x^23/23
```

3.72 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x} \, dx$

Optimal. Leaf size=80

$$\frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\log(x)$$

```
[Out] 11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/16
*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 272, 45}

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^8}{4}+\frac{55x^6}{2}+\frac{55x^4}{4}+\frac{11x^2}{2}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x}\,dx = \int \frac{(1+x^2)^{11}}{x}\,dx$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \frac{(1+x)^{11}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\mathrm{Subst}\left(\int \left(11+\frac{1}{x}+55x+165x^2+330x^3+462x^4+462x^5+330x^6+1\right.\right.$$
$$= \frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55}{1}$$

Mathematica [A]
time = 0.00, size = 80, normalized size = 1.00

$$\frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x,x]
```

```
[Out] (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5 + (77*x^1
2)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20 + x^22/22
 + Log[x]
```

Maple [A]
time = 0.02, size = 59, normalized size = 0.74

method	result	size
default	$\frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln(x)$	59
norman	$\frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln(x)$	59
risch	$\frac{11x^2}{2}+\frac{55x^4}{4}+\frac{55x^6}{2}+\frac{165x^8}{4}+\frac{231x^{10}}{5}+\frac{77x^{12}}{2}+\frac{165x^{14}}{7}+\frac{165x^{16}}{16}+\frac{55x^{18}}{18}+\frac{11x^{20}}{20}+\frac{x^{22}}{22}+\ln(x)$	59

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x,x,method=_RETURNVERBOSE)
```

```
[Out] 11/2*x^2+55/4*x^4+55/2*x^6+165/4*x^8+231/5*x^10+77/2*x^12+165/7*x^14+165/16
*x^16+55/18*x^18+11/20*x^20+1/22*x^22+ln(x)
```

Maxima [A]
time = 0.28, size = 62, normalized size = 0.78

$$\frac{1}{22}x^{22}+\frac{11}{20}x^{20}+\frac{55}{18}x^{18}+\frac{165}{16}x^{16}+\frac{165}{7}x^{14}+\frac{77}{2}x^{12}+\frac{231}{5}x^{10}+\frac{165}{4}x^8+\frac{55}{2}x^6+\frac{55}{4}x^4+\frac{11}{2}x^2+\frac{1}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="maxima")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

Fricas [A]
time = 0.40, size = 58, normalized size = 0.72

$$\frac{1}{22}\,x^{22}+\frac{11}{20}\,x^{20}+\frac{55}{18}\,x^{18}+\frac{165}{16}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{77}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{165}{4}\,x^{8}+\frac{55}{2}\,x^{6}+\frac{55}{4}\,x^{4}+\frac{11}{2}\,x^{2}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="fricas")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + log(x)
```

Sympy [A]
time = 0.04, size = 75, normalized size = 0.94

$$\frac{x^{22}}{22}+\frac{11x^{20}}{20}+\frac{55x^{18}}{18}+\frac{165x^{16}}{16}+\frac{165x^{14}}{7}+\frac{77x^{12}}{2}+\frac{231x^{10}}{5}+\frac{165x^{8}}{4}+\frac{55x^{6}}{2}+\frac{55x^{4}}{4}+\frac{11x^{2}}{2}+\log\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x,x)
```

```
[Out] x**22/22 + 11*x**20/20 + 55*x**18/18 + 165*x**16/16 + 165*x**14/7 + 77*x**1
2/2 + 231*x**10/5 + 165*x**8/4 + 55*x**6/2 + 55*x**4/4 + 11*x**2/2 + log(x)
```

Giac [A]
time = 3.87, size = 62, normalized size = 0.78

$$\frac{1}{22}\,x^{22}+\frac{11}{20}\,x^{20}+\frac{55}{18}\,x^{18}+\frac{165}{16}\,x^{16}+\frac{165}{7}\,x^{14}+\frac{77}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{165}{4}\,x^{8}+\frac{55}{2}\,x^{6}+\frac{55}{4}\,x^{4}+\frac{11}{2}\,x^{2}+\frac{1}{2}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x,x, algorithm="giac")
```

```
[Out] 1/22*x^22 + 11/20*x^20 + 55/18*x^18 + 165/16*x^16 + 165/7*x^14 + 77/2*x^12
+ 231/5*x^10 + 165/4*x^8 + 55/2*x^6 + 55/4*x^4 + 11/2*x^2 + 1/2*log(x^2)
```

Mupad [B]
time = 0.06, size = 58, normalized size = 0.72

$$\ln\left(x\right)+\frac{11\,x^2}{2}+\frac{55\,x^4}{4}+\frac{55\,x^6}{2}+\frac{165\,x^8}{4}+\frac{231\,x^{10}}{5}+\frac{77\,x^{12}}{2}+\frac{165\,x^{14}}{7}+\frac{165\,x^{16}}{16}+\frac{55\,x^{18}}{18}+\frac{11\,x^{20}}{20}+\frac{x^{22}}{22}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x,x)
```

```
[Out] log(x) + (11*x^2)/2 + (55*x^4)/4 + (55*x^6)/2 + (165*x^8)/4 + (231*x^10)/5
+ (77*x^12)/2 + (165*x^14)/7 + (165*x^16)/16 + (55*x^18)/18 + (11*x^20)/20
+ x^22/22
```

3.73 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} \, dx$

Optimal. Leaf size=73

$$-\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$$

```
[Out] -1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+5
5/17*x^17+11/19*x^19+1/21*x^21
```

Rubi [A]
time = 0.01, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {28, 276}

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 276

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^2} \, dx &= \int \frac{(1+x^2)^{11}}{x^2} \, dx\\ &= \int \left(11+\frac{1}{x^2}+55x^2+165x^4+330x^6+462x^8+462x^{10}+330x^{12}+165x^{14}\right.\\ &= -\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\end{aligned}$$

Mathematica [A]
time = 0.00, size = 73, normalized size = 1.00

$$-\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^2,x]
```

```
[Out] -x^(-1) + 11*x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11
+ (330*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

Maple [A]
time = 0.02, size = 60, normalized size = 0.82

method	result
default	$-\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$
risch	$-\frac{1}{x}+11x+\frac{55x^3}{3}+33x^5+\frac{330x^7}{7}+\frac{154x^9}{3}+42x^{11}+\frac{330x^{13}}{13}+11x^{15}+\frac{55x^{17}}{17}+\frac{11x^{19}}{19}+\frac{x^{21}}{21}$
norman	$\frac{-1+11x^2+\frac{55}{3}x^4+33x^6+\frac{330}{7}x^8+\frac{154}{3}x^{10}+42x^{12}+\frac{330}{13}x^{14}+11x^{16}+\frac{55}{17}x^{18}+\frac{11}{19}x^{20}+\frac{1}{21}x^{22}}{x}$
gosper	$\frac{4199x^{22}+51051x^{20}+285285x^{18}+969969x^{16}+2238390x^{14}+3703518x^{12}+4526522x^{10}+4157010x^8+2909907x^6+1616615x^4+969969}{88179x}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/x+11*x+55/3*x^3+33*x^5+330/7*x^7+154/3*x^9+42*x^11+330/13*x^13+11*x^15+5
5/17*x^17+11/19*x^19+1/21*x^21
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.81

$$\frac{1}{21}\,x^{21}+\frac{11}{19}\,x^{19}+\frac{55}{17}\,x^{17}+11\,x^{15}+\frac{330}{13}\,x^{13}+42\,x^{11}+\frac{154}{3}\,x^9+\frac{330}{7}\,x^7+33\,x^5+\frac{55}{3}\,x^3+11\,x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="maxima")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

Fricas [A]
time = 0.42, size = 62, normalized size = 0.85

$$\frac{4199\,x^{22}+51051\,x^{20}+285285\,x^{18}+969969\,x^{16}+2238390\,x^{14}+3703518\,x^{12}+4526522\,x^{10}+4157010\,x^8+2909907\,x^6+1616615\,x^4+969969\,x^2-88179}{88179\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="fricas")
```

```
[Out] 1/88179*(4199*x^22 + 51051*x^20 + 285285*x^18 + 969969*x^16 + 2238390*x^14
+ 3703518*x^12 + 4526522*x^10 + 4157010*x^8 + 2909907*x^6 + 1616615*x^4 + 9
69969*x^2 - 88179)/x
```

Sympy [A]
time = 0.03, size = 66, normalized size = 0.90

$$\frac{x^{21}}{21}+\frac{11x^{19}}{19}+\frac{55x^{17}}{17}+11x^{15}+\frac{330x^{13}}{13}+42x^{11}+\frac{154x^9}{3}+\frac{330x^7}{7}+33x^5+\frac{55x^3}{3}+11x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**2,x)
```

```
[Out] x**21/21 + 11*x**19/19 + 55*x**17/17 + 11*x**15 + 330*x**13/13 + 42*x**11 +
 154*x**9/3 + 330*x**7/7 + 33*x**5 + 55*x**3/3 + 11*x - 1/x
```

Giac [A]
time = 5.98, size = 59, normalized size = 0.81

$$\frac{1}{21}\,x^{21}+\frac{11}{19}\,x^{19}+\frac{55}{17}\,x^{17}+11\,x^{15}+\frac{330}{13}\,x^{13}+42\,x^{11}+\frac{154}{3}\,x^9+\frac{330}{7}\,x^7+33\,x^5+\frac{55}{3}\,x^3+11\,x-\frac{1}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^2,x, algorithm="giac")
```

```
[Out] 1/21*x^21 + 11/19*x^19 + 55/17*x^17 + 11*x^15 + 330/13*x^13 + 42*x^11 + 154
/3*x^9 + 330/7*x^7 + 33*x^5 + 55/3*x^3 + 11*x - 1/x
```

Mupad [B]
time = 0.06, size = 59, normalized size = 0.81

$$11\,x-\frac{1}{x}+\frac{55\,x^3}{3}+33\,x^5+\frac{330\,x^7}{7}+\frac{154\,x^9}{3}+42\,x^{11}+\frac{330\,x^{13}}{13}+11\,x^{15}+\frac{55\,x^{17}}{17}+\frac{11\,x^{19}}{19}+\frac{x^{21}}{21}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^2,x)
```

```
[Out] 11*x - 1/x + (55*x^3)/3 + 33*x^5 + (330*x^7)/7 + (154*x^9)/3 + 42*x^11 + (3
30*x^13)/13 + 11*x^15 + (55*x^17)/17 + (11*x^19)/19 + x^21/21
```

3.74 $\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3} \, dx$

Optimal. Leaf size=80

$$-\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\log(x)$$

```
[Out] -1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x^
14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

Rubi [A]
time = 0.02, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {28, 272, 45}

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^8}{4}+55x^6+\frac{165x^4}{4}+\frac{55x^2}{2}-\frac{1}{2x^2}+11\log(x)$$

Antiderivative was successfully verified.

```
[In] Int[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*1/x^2 + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

Rule 28

```
Int[(u_.)*((a_) + (c_.)*(x_)^(n2_.) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :>
Dist[1/c^p, Int[u*(b/2 + c*x^n)^(2*p), x], x] /; FreeQ[{a, b, c, n}, x] &&
EqQ[n2, 2*n] && EqQ[b^2 - 4*a*c, 0] && IntegerQ[p]
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 272

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rubi steps

$$\int \frac{(1+x^2)(1+2x^2+x^4)^5}{x^3}\,dx = \int \frac{(1+x^2)^{11}}{x^3}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{(1+x)^{11}}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(55+\frac{1}{x^2}+\frac{11}{x}+165x+330x^2+462x^3+462x^4+330x^5+16\right.\right.$$
$$= -\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x}{16}$$

Mathematica [A]
time = 0.00, size = 80, normalized size = 1.00

$$-\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((1 + x^2)*(1 + 2*x^2 + x^4)^5)/x^3,x]
```

```
[Out] -1/2*1/x^2 + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (231*x^10)/5
 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x^20/20 + 11
*Log[x]
```

Maple [A]
time = 0.03, size = 61, normalized size = 0.76

method	result	siz
default	$-\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\ln(x)$	61
risch	$-\frac{1}{2x^2}+\frac{55x^2}{2}+\frac{165x^4}{4}+55x^6+\frac{231x^8}{4}+\frac{231x^{10}}{5}+\frac{55x^{12}}{2}+\frac{165x^{14}}{14}+\frac{55x^{16}}{16}+\frac{11x^{18}}{18}+\frac{x^{20}}{20}+11\ln(x)$	61
norman	$\frac{-\frac{1}{2}+\frac{55}{2}x^4+\frac{165}{4}x^6+55x^8+\frac{231}{4}x^{10}+\frac{231}{5}x^{12}+\frac{55}{2}x^{14}+\frac{165}{14}x^{16}+\frac{55}{16}x^{18}+\frac{11}{18}x^{20}+\frac{1}{20}x^{22}}{x^2}+11\ln(x)$	62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2+1)*(x^4+2*x^2+1)^5/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/x^2+55/2*x^2+165/4*x^4+55*x^6+231/4*x^8+231/5*x^10+55/2*x^12+165/14*x^
14+55/16*x^16+11/18*x^18+1/20*x^20+11*ln(x)
```

Maxima [A]
time = 0.28, size = 62, normalized size = 0.78

$$\frac{1}{20}x^{20}+\frac{11}{18}x^{18}+\frac{55}{16}x^{16}+\frac{165}{14}x^{14}+\frac{55}{2}x^{12}+\frac{231}{5}x^{10}+\frac{231}{4}x^8+55x^6+\frac{165}{4}x^4+\frac{55}{2}x^2-\frac{1}{2x^2}+\frac{11}{2}\log(x^2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="maxima")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2/x^2 + 11/2*log(x^2)
```

Fricas [A]
time = 0.35, size = 64, normalized size = 0.80

$$\frac{252\,x^{22}+3080\,x^{20}+17325\,x^{18}+59400\,x^{16}+138600\,x^{14}+232848\,x^{12}+291060\,x^{10}+277200\,x^{8}+207900\,x^{6}+138600\,x^{4}+55440\,x^{2}\log\left(x\right)-2520}{5040\,x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="fricas")
```

```
[Out] 1/5040*(252*x^22 + 3080*x^20 + 17325*x^18 + 59400*x^16 + 138600*x^14 + 2328
48*x^12 + 291060*x^10 + 277200*x^8 + 207900*x^6 + 138600*x^4 + 55440*x^2*lo
g(x) - 2520)/x^2
```

Sympy [A]
time = 0.03, size = 75, normalized size = 0.94

$$\frac{x^{20}}{20}+\frac{11x^{18}}{18}+\frac{55x^{16}}{16}+\frac{165x^{14}}{14}+\frac{55x^{12}}{2}+\frac{231x^{10}}{5}+\frac{231x^{8}}{4}+55x^{6}+\frac{165x^{4}}{4}+\frac{55x^{2}}{2}+11\log\left(x\right)-\frac{1}{2x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**2+1)*(x**4+2*x**2+1)**5/x**3,x)
```

```
[Out] x**20/20 + 11*x**18/18 + 55*x**16/16 + 165*x**14/14 + 55*x**12/2 + 231*x**1
0/5 + 231*x**8/4 + 55*x**6 + 165*x**4/4 + 55*x**2/2 + 11*log(x) - 1/(2*x**2
)
```

Giac [A]
time = 5.06, size = 69, normalized size = 0.86

$$\frac{1}{20}\,x^{20}+\frac{11}{18}\,x^{18}+\frac{55}{16}\,x^{16}+\frac{165}{14}\,x^{14}+\frac{55}{2}\,x^{12}+\frac{231}{5}\,x^{10}+\frac{231}{4}\,x^{8}+55\,x^{6}+\frac{165}{4}\,x^{4}+\frac{55}{2}\,x^{2}-\frac{11\,x^{2}+1}{2\,x^{2}}+\frac{11}{2}\,\log\left(x^{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^2+1)*(x^4+2*x^2+1)^5/x^3,x, algorithm="giac")
```

```
[Out] 1/20*x^20 + 11/18*x^18 + 55/16*x^16 + 165/14*x^14 + 55/2*x^12 + 231/5*x^10
+ 231/4*x^8 + 55*x^6 + 165/4*x^4 + 55/2*x^2 - 1/2*(11*x^2 + 1)/x^2 + 11/2*l
og(x^2)
```

Mupad [B]
time = 0.06, size = 60, normalized size = 0.75

$$11\,\ln\left(x\right)-\frac{1}{2\,x^{2}}+\frac{55\,x^{2}}{2}+\frac{165\,x^{4}}{4}+55\,x^{6}+\frac{231\,x^{8}}{4}+\frac{231\,x^{10}}{5}+\frac{55\,x^{12}}{2}+\frac{165\,x^{14}}{14}+\frac{55\,x^{16}}{16}+\frac{11\,x^{18}}{18}+\frac{x^{20}}{20}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((x^2 + 1)*(2*x^2 + x^4 + 1)^5)/x^3,x)
```

```
[Out] 11*log(x) - 1/(2*x^2) + (55*x^2)/2 + (165*x^4)/4 + 55*x^6 + (231*x^8)/4 + (
231*x^10)/5 + (55*x^12)/2 + (165*x^14)/14 + (55*x^16)/16 + (11*x^18)/18 + x
^20/20
```

3.75 $\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=145

$$\frac{(bd-ae)x(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\sqrt{a}\,(bd-ae)\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] (-a*e+b*d)*x*(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/3*e*x^3*(b*x^2+a)/b/((b*x^
2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))*a^(1/2)/b^(5/2
)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1264, 470, 327, 211}

$$-\frac{\sqrt{a}\,(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bd-ae)}{b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(a+bx^2)\,(bd-ae)}{b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((b*d - a*e)*x*(a + b*x^2))/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (e*x^3*
(a + b*x^2))/(3*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (Sqrt[a]*(b*d - a*e)*(
a + b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(b^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
```

```
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(ab+b^2x^2)\int \frac{x^2(d+ex^2)}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{((-3b^2d+3abe)(ab+b^2x^2))\int \frac{x^2}{ab+b^2x^2}\,dx}{3b^2\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{(bd-ae)x(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(a(-3b^2d+3abe)(ab+b}{3b^3\sqrt{a^2+2abx^2}}\\ &= \frac{(bd-ae)x(a+bx^2)}{b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{ex^3(a+bx^2)}{3b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\sqrt{a}\,(bd-ae)(a+bx^2)\,\mathrm{ta}}{b^{5/2}\sqrt{a^2+2abx^2+}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 80, normalized size = 0.55

$$\frac{(a+bx^2)\left(\sqrt{b}\,x(3bd-3ae+bex^2)+3\sqrt{a}\,(-bd+ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{3b^{5/2}\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] ((a + b*x^2)*(Sqrt[b]*x*(3*b*d - 3*a*e + b*e*x^2) + 3*Sqrt[a]*(-(b*d) + a*e
)*ArcTan[(Sqrt[b]*x)/Sqrt[a]]))/(3*b^(5/2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.13, size = 91, normalized size = 0.63

method	result
default	$-\frac{(b\,x^2+a)\left(-\sqrt{ab}\,be\,x^3+3\sqrt{ab}\,aex-3\sqrt{ab}\,bdx-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2e+3\arctan\left(\frac{bx}{\sqrt{ab}}\right)abd\right)}{3\sqrt{(b\,x^2+a)^2}\,b^2\sqrt{ab}}$
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,(\frac{1}{3}be\,x^3-aex+bdx)}{(b\,x^2+a)b^2}+\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{-ab}\,(ae-bd)\ln\left(-\sqrt{-ab}\,x+a\right)}{2(b\,x^2+a)b^3}-\frac{\sqrt{(b\,x^2+a)^2}\,\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(b*x^2+a)*(-(a*b)^(1/2)*b*e*x^3+3*(a*b)^(1/2)*a*e*x-3*(a*b)^(1/2)*b*d*
x-3*arctan(b*x/(a*b)^(1/2))*a^2*e+3*arctan(b*x/(a*b)^(1/2))*a*b*d)/((b*x^2+
a)^2)^(1/2)/b^2/(a*b)^(1/2)
```

Maxima [A]
time = 0.48, size = 57, normalized size = 0.39

$$-\frac{(abd-a^2e)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{bx^3e+3\,(bd-ae)x}{3\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(a*b*d - a^2*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*x^3*e + 3*(
b*d - a*e)*x)/b^2
```

Fricas [A]
time = 0.38, size = 133, normalized size = 0.92

$$\left[\frac{6\,bdx-3\,(bd-ae)\sqrt{-\frac{a}{b}}\,\log\left(\frac{bx^2+2\,bx\sqrt{-\frac{a}{b}}-a}{bx^2+a}\right)+2\,(bx^3-3\,ax)e}{6\,b^2},\frac{3\,bdx-3\,(bd-ae)\sqrt{\frac{a}{b}}\,\arctan\left(\frac{bx\sqrt{\frac{a}{b}}}{a}\right)+(bx^3-3\,ax)e}{3\,b^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(6*b*d*x - 3*(b*d - a*e)*sqrt(-a/b)*log((b*x^2 + 2*b*x*sqrt(-a/b) - a)
/(b*x^2 + a)) + 2*(b*x^3 - 3*a*x)*e)/b^2, 1/3*(3*b*d*x - 3*(b*d - a*e)*sqrt
(a/b)*arctan(b*x*sqrt(a/b)/a) + (b*x^3 - 3*a*x)*e)/b^2]
```

Sympy [A]
time = 0.18, size = 90, normalized size = 0.62

$$x\left(-\frac{ae}{b^2}+\frac{d}{b}\right)-\frac{\sqrt{-\frac{a}{b^5}}\,(ae-bd)\log\left(-b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}+\frac{\sqrt{-\frac{a}{b^5}}\,(ae-bd)\log\left(b^2\sqrt{-\frac{a}{b^5}}+x\right)}{2}+\frac{ex^3}{3b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] x*(-a*e/b**2 + d/b) - sqrt(-a/b**5)*(a*e - b*d)*log(-b**2*sqrt(-a/b**5) + x
)/2 + sqrt(-a/b**5)*(a*e - b*d)*log(b**2*sqrt(-a/b**5) + x)/2 + e*x**3/(3*b
)
```

Giac [A]
time = 5.12, size = 101, normalized size = 0.70

$$-\frac{\left(abd\,\mathrm{sgn}(bx^2+a)-a^2e\,\mathrm{sgn}(bx^2+a)\right)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}+\frac{b^2x^3e\,\mathrm{sgn}(bx^2+a)+3\,b^2dx\,\mathrm{sgn}(bx^2+a)-3\,abxe\,\mathrm{sgn}(bx^2+a)}{3\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(a*b*d*sgn(b*x^2 + a) - a^2*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(
a*b)*b^2) + 1/3*(b^2*x^3*e*sgn(b*x^2 + a) + 3*b^2*d*x*sgn(b*x^2 + a) - 3*a*
b*x*e*sgn(b*x^2 + a))/b^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(e\,x^2+d\right)}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((x^2*(d + e*x^2))/((a + b*x^2)^2)^(1/2), x)
```

3.76 $\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=83

$$\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}+\frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a)/b^2/((b*x^2+a)^2)^(1/2)+1/2*e*((b*x^2+
a)^2)^(1/2)/b^2
```

Rubi [A]
time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {1261, 654, 622, 31}

$$\frac{(a+bx^2)\,(bd-ae)\log\left(a+bx^2\right)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*b^2) + ((b*d - a*e)*(a + b*x^2)*Log[
a + b*x^2])/(2*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 622

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[(b/2 + c*x)
/Sqrt[a + b*x + c*x^2], Int[1/(b/2 + c*x), x], x] /; FreeQ[{a, b, c}, x] &&
 EqQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)$$
$$= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae)\text{Subst}\left(\int \frac{1}{\sqrt{a^2+2abx+b^2x^2}}\,dx, x, x^2\right)}{2b}$$
$$= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{((bd-ae)(ab+b^2x^2))\,\text{Subst}\left(\int \frac{1}{ab+b^2x}\,dx, x, x^2\right)}{2b\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{e\sqrt{a^2+2abx^2+b^2x^4}}{2b^2} + \frac{(bd-ae)(a+bx^2)\log(a+bx^2)}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.02, size = 51, normalized size = 0.61

$$\frac{(a+bx^2)\left(bex^2+(bd-ae)\log(a+bx^2)\right)}{2b^2\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] ((a + b*x^2)*(b*e*x^2 + (b*d - a*e)*Log[a + b*x^2]))/(2*b^2*Sqrt[(a + b*x^2
)^2])
```

Maple [A]
time = 0.12, size = 55, normalized size = 0.66

method	result	size
default	$-\frac{(b\,x^2+a)(-be\,x^2+\ln(b\,x^2+a)ae-\ln(b\,x^2+a)bd)}{2\sqrt{(b\,x^2+a)^2}\;b^2}$	55
risch	$\frac{\sqrt{(b\,x^2+a)^2}\;e\,x^2}{2(b\,x^2+a)b}-\frac{\sqrt{(b\,x^2+a)^2}\;(ae-bd)\ln(b\,x^2+a)}{2(b\,x^2+a)b^2}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] -1/2*(b*x^2+a)*(-b*e*x^2+ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d)/((b*x^2+a)^2)^(1/
2)/b^2
```

Maxima [A]
time = 0.27, size = 33, normalized size = 0.40

$$\frac{x^2 e}{2\,b}+\frac{(bd-ae)\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*x^2*e/b + 1/2*(b*d - a*e)*log(b*x^2 + a)/b^2
```

Fricas [A]
time = 0.36, size = 31, normalized size = 0.37

$$\frac{bx^2 e+(bd-ae)\log\left(bx^2+a\right)}{2\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(b*x^2*e + (b*d - a*e)*log(b*x^2 + a))/b^2
```

Sympy [A]
time = 0.12, size = 27, normalized size = 0.33

$$\frac{ex^2}{2b}-\frac{(ae-bd)\log\left(a+bx^2\right)}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
[Out] e*x**2/(2*b) - (a*e - b*d)*log(a + b*x**2)/(2*b**2)
```

Giac [A]
time = 5.06, size = 42, normalized size = 0.51

$$\frac{1}{2}\left(\frac{x^2 e}{b}+\frac{(bd-ae)\log\left(|bx^2+a|\right)}{b^2}\right)\operatorname{sgn}\left(bx^2+a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*(x^2*e/b + (b*d - a*e)*log(abs(b*x^2 + a))/b^2)*sgn(b*x^2 + a)
```

Mupad [B]
time = 0.88, size = 103, normalized size = 1.24

$$\frac{e\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,b^2}-\frac{a\,b\,e\,\ln\left(a\,b+\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{b^2}+b^2\,x^2\right)}{2\,{\left(b^2\right)}^{3/2}}+\frac{b^2\,d\,\ln\left(b^2\,x^2+a\,b\right)\,\mathrm{sign}\left(2\,b^2\,x^2+2\,a\,b\right)}{2\,{\left(b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2))/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] (e*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2))/(2*b^2) - (a*b*e*log(a*b + ((a + b*x^
2)^2)^(1/2)*(b^2)^(1/2) + b^2*x^2))/(2*(b^2)^(3/2)) + (b^2*d*log(a*b + b^2*
x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(3/2))
```

3.77 $\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=97

$$\frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] e*x*(b*x^2+a)/b/((b*x^2+a)^2)^(1/2)+(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a
^(1/2))/b^(3/2)/a^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}}=0.100$, Rules used = {1162, 396, 211}

$$\frac{(a+bx^2)\,\mathrm{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bd-ae)}{\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}+\frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*x*(a + b*x^2))/(b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(a + b
*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(Sqrt[a]*b^(3/2)*Sqrt[a^2 + 2*a*b*x^2 +
b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{d+ex^2}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(ab+b^2x^2)\int \frac{d+ex^2}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} - \frac{((-b^2d+abe)\,(ab+b^2x^2))\int \frac{1}{ab+b^2x^2}\,dx}{b^2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{ex(a+bx^2)}{b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}
$$

Mathematica [**A**]
time = 0.02, size = 69, normalized size = 0.71

$$
-\frac{(a+bx^2)\left(-\sqrt{a}\,\sqrt{b}\,ex+(-bd+ae)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{\sqrt{a}\,b^{3/2}\sqrt{(a+bx^2)^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -(((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*e*x) + (-(b*d) + a*e)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(Sqrt[a]*b^(3/2)*Sqrt[(a + b*x^2)^2]))
```

Maple [**A**]
time = 0.13, size = 62, normalized size = 0.64

method	result	si
default	$\frac{(b\,x^2+a)\left(ex\sqrt{ab}-\arctan\left(\frac{bx}{\sqrt{ab}}\right)ae+\arctan\left(\frac{bx}{\sqrt{ab}}\right)bd\right)}{\sqrt{(b\,x^2+a)^2}\,b\sqrt{ab}}$	6
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,ex}{(b\,x^2+a)b} - \frac{\sqrt{(b\,x^2+a)^2}\,(ae-bd)\ln\left(bx-\sqrt{-ab}\right)}{2(b\,x^2+a)b\sqrt{-ab}} + \frac{\sqrt{(b\,x^2+a)^2}\,(ae-bd)\ln\left(-bx-\sqrt{-ab}\right)}{2(b\,x^2+a)b\sqrt{-ab}}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (b*x^2+a)*(e*x*(a*b)^(1/2)-arctan(b*x/(a*b)^(1/2))*a*e+arctan(b*x/(a*b)^(1/
2))*b*d)/((b*x^2+a)^2)^(1/2)/b/(a*b)^(1/2)
```

Maxima [A]
time = 0.48, size = 35, normalized size = 0.36

$$\frac{xe}{b}+\frac{(bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] x*e/b + (b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b)
```

Fricas [A]
time = 0.45, size = 102, normalized size = 1.05

$$\left[\frac{2\,abxe+\sqrt{-ab}\,(bd-ae)\log\left(\frac{bx^2+2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,ab^2},\frac{abxe+\sqrt{ab}\,(bd-ae)\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*x*e + sqrt(-a*b)*(b*d - a*e)*log((b*x^2 + 2*sqrt(-a*b)*x - a)/(
b*x^2 + a)))/(a*b^2), (a*b*x*e + sqrt(a*b)*(b*d - a*e)*arctan(sqrt(a*b)*x/a
))/(a*b^2)]
```

Sympy [A]
time = 0.14, size = 82, normalized size = 0.85

$$\frac{\sqrt{-\frac{1}{ab^3}}\,(ae-bd)\log\left(-ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}-\frac{\sqrt{-\frac{1}{ab^3}}\,(ae-bd)\log\left(ab\sqrt{-\frac{1}{ab^3}}+x\right)}{2}+\frac{ex}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] sqrt(-1/(a*b**3))*(a*e - b*d)*log(-a*b*sqrt(-1/(a*b**3)) + x)/2 - sqrt(-1/(
a*b**3))*(a*e - b*d)*log(a*b*sqrt(-1/(a*b**3)) + x)/2 + e*x/b
```

Giac [A]
time = 5.17, size = 59, normalized size = 0.61

$$\frac{xe\mathrm{sgn}(bx^2+a)}{b}+\frac{(bd\mathrm{sgn}(bx^2+a)-ae\mathrm{sgn}(bx^2+a))\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] x*e*sgn(b*x^2 + a)/b + (b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x
/sqrt(a*b))/(sqrt(a*b)*b)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((d + e*x^2)/((a + b*x^2)^2)^(1/2), x)
```

3.78 $\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=92

$$\frac{d(a+bx^2)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] d*(b*x^2+a)*ln(x)/a/((b*x^2+a)^2)^(1/2)-1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a
)/a/b/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 92, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1264, 457, 78}

$$\frac{d\log(x)\,(a+bx^2)}{a\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\,(bd-ae)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] (d*(a + b*x^2)*Log[x])/(a*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*(
a + b*x^2)*Log[a + b*x^2])/(2*a*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
```

```
a*c, 0] && !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{x\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(ab+b^2x^2)\int \frac{d+ex^2}{x(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{d+ex}{x(ab+b^2x)}\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \left(\frac{d}{abx}+\frac{-bd+ae}{ab(a+bx)}\right)\,dx, x, x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{d(a+bx^2)\log(x)}{a\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2ab\sqrt{a^2+2abx^2+b^2x^4}}
\end{aligned}$$

Mathematica [A]
time = 0.01, size = 54, normalized size = 0.59

$$\frac{(a+bx^2)\left(2bd\log(x)+(-bd+ae)\log\left(a+bx^2\right)\right)}{2ab\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
[Out] ((a + b*x^2)*(2*b*d*Log[x] + (-(b*d) + a*e)*Log[a + b*x^2]))/(2*a*b*Sqrt[(a
 + b*x^2)^2])
```

Maple [A]
time = 0.12, size = 57, normalized size = 0.62

method	result	size
default	$\frac{(b\,x^2+a)\left(\ln(b\,x^2+a)\,a e-\ln(b\,x^2+a)\,b d+2 d\ln(x) b\right)}{2\sqrt{(b\,x^2+a)^2}\;ab}$	57
risch	$\frac{\sqrt{(b\,x^2+a)^2}\;d\ln(x)}{(b\,x^2+a)a}+\frac{\sqrt{(b\,x^2+a)^2}\;(ae-bd)\ln(-b\,x^2-a)}{2(b\,x^2+a)ab}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/2*(b*x^2+a)*(ln(b*x^2+a)*a*e-ln(b*x^2+a)*b*d+2*d*ln(x)*b)/((b*x^2+a)^2)^(
1/2)/a/b
```

Maxima [A]
time = 0.28, size = 36, normalized size = 0.39

$$\frac{d\log\left(x^2\right)}{2\,a}-\frac{(bd-ae)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
[Out] 1/2*d*log(x^2)/a - 1/2*(b*d - a*e)*log(b*x^2 + a)/(a*b)
```

Fricas [A]
time = 0.55, size = 34, normalized size = 0.37

$$\frac{2\,bd\log\left(x\right)-(bd-ae)\log\left(bx^2+a\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*(2*b*d*log(x) - (b*d - a*e)*log(b*x^2 + a))/(a*b)
```

Sympy [A]
time = 0.39, size = 26, normalized size = 0.28

$$\frac{d\log\left(x\right)}{a}+\frac{(ae-bd)\log\left(\frac{a}{b}+x^2\right)}{2ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x/((b*x**2+a)**2)**(1/2),x)
[Out] d*log(x)/a + (a*e - b*d)*log(a/b + x**2)/(2*a*b)
```

Giac [A]
time = 4.97, size = 61, normalized size = 0.66

$$\frac{d\log\left(x^2\right)\operatorname{sgn}(bx^2+a)}{2\,a}-\frac{(bd\operatorname{sgn}(bx^2+a)-ae\operatorname{sgn}(bx^2+a))\log\left(|bx^2+a|\right)}{2\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
[Out] 1/2*d*log(x^2)*sgn(b*x^2 + a)/a - 1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 +
 a))*log(abs(b*x^2 + a))/(a*b)
```

Mupad [B]
time = 0.77, size = 83, normalized size = 0.90

$$\frac{e\,\ln\left(b^2\,x^2+a\,b\right)\,\mathrm{sign}(2\,b^2\,x^2+2\,a\,b)}{2\,\sqrt{b^2}}-\frac{d\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}}-\frac{d\,\ln\left(\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] (e*log(a*b + b^2*x^2)*sign(2*a*b + 2*b^2*x^2))/(2*(b^2)^(1/2)) - (d*log(1/x
^2))/(2*(a^2)^(1/2)) - (d*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) + a^2 + a*b
*x^2))/(2*(a^2)^(1/2))
```

3.79 $\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=101

$$-\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -d*(b*x^2+a)/a/x/((b*x^2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/
a^(1/2))/a^(3/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1264, 464, 211}

$$-\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(bd-ae)}{a^{3/2}\sqrt{b}\,\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -((d*(a + b*x^2))/(a*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])) - ((b*d - a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(a^(3/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2
 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
```

```
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{d+ex^2}{x^2\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(ab+b^2x^2)\int \frac{d+ex^2}{x^2(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{((b^2d-abe)(ab+b^2x^2))\int \frac{1}{ab+b^2x^2}\,dx}{ab\sqrt{a^2+2abx^2+b^2x^4}}\\ &= -\frac{d(a+bx^2)}{ax\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 0.71

$$\frac{(a+bx^2)\left(-\sqrt{a}\sqrt{b}\,d+(-bdx+aex)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)\right)}{a^{3/2}\sqrt{b}\,x\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
[Out] ((a + b*x^2)*(-(Sqrt[a]*Sqrt[b]*d) + (-(b*d*x) + a*e*x)*ArcTan[(Sqrt[b]*x)/
Sqrt[a]]))/(a^(3/2)*Sqrt[b]*x*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.14, size = 67, normalized size = 0.66

method	result
default	$-\frac{(b\,x^2+a)\left(-\arctan\left(\frac{bx}{\sqrt{ab}}\right)aex+\arctan\left(\frac{bx}{\sqrt{ab}}\right)bdx+d\sqrt{ab}\right)}{\sqrt{(b\,x^2+a)^2}\,ax\sqrt{ab}}$
risch	$-\frac{\sqrt{(b\,x^2+a)^2}\,d}{(b\,x^2+a)ax}-\frac{\sqrt{(b\,x^2+a)^2}\,(ae-bd)\ln\left(-\sqrt{-ab}\,x+a\right)}{2(b\,x^2+a)\sqrt{-ab}\,a}+\frac{\sqrt{(b\,x^2+a)^2}\,(ae-bd)\ln\left(-\sqrt{-ab}\,x-a\right)}{2(b\,x^2+a)\sqrt{-ab}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -(b*x^2+a)*(-arctan(b*x/(a*b)^(1/2))*a*e*x+arctan(b*x/(a*b)^(1/2))*b*d*x+d*
(a*b)^(1/2))/((b*x^2+a)^2)^(1/2)/a/x/(a*b)^(1/2)
```

Maxima [A]
time = 0.50, size = 38, normalized size = 0.38

$$-\frac{(bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a) - d/(a*x)
```

Fricas [A]
time = 0.35, size = 110, normalized size = 1.09

$$\left[-\frac{2\,abd-(bdx-axe)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)}{2\,a^2bx},\,-\frac{abd+(bdx-axe)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)}{a^2bx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*d - (b*d*x - a*x*e)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b)*x - a
)/(b*x^2 + a)))/(a^2*b*x), -(a*b*d + (b*d*x - a*x*e)*sqrt(a*b)*arctan(sqrt(
a*b)*x/a))/(a^2*b*x)]
```

Sympy [A]
time = 0.17, size = 82, normalized size = 0.81

$$-\frac{\sqrt{-\frac{1}{a^3b}}\,(ae-bd)\log\left(-a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}+\frac{\sqrt{-\frac{1}{a^3b}}\,(ae-bd)\log\left(a^2\sqrt{-\frac{1}{a^3b}}+x\right)}{2}-\frac{d}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -sqrt(-1/(a**3*b))*(a*e - b*d)*log(-a**2*sqrt(-1/(a**3*b)) + x)/2 + sqrt(-1
/(a**3*b))*(a*e - b*d)*log(a**2*sqrt(-1/(a**3*b)) + x)/2 - d/(a*x)
```

Giac [A]
time = 3.63, size = 62, normalized size = 0.61

$$-\frac{(bd\mathrm{sgn}(bx^2+a)-ae\mathrm{sgn}(bx^2+a))\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a}-\frac{d\mathrm{sgn}(bx^2+a)}{ax}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*arctan(b*x/sqrt(a*b))/(sqrt(a*b)
*a) - d*sgn(b*x^2 + a)/(a*x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{x^2\,\sqrt{{\left(b\,x^2+a\right)}^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] int((d + e*x^2)/(x^2*((a + b*x^2)^2)^(1/2)), x)
```

3.80 $\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=137

$$-\frac{d(a+bx^2)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\,(a+bx^2)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*d*(b*x^2+a)/a/x^2/((b*x^2+a)^2)^(1/2)-(-a*e+b*d)*(b*x^2+a)*ln(x)/a^2/(
(b*x^2+a)^2)^(1/2)+1/2*(-a*e+b*d)*(b*x^2+a)*ln(b*x^2+a)/a^2/((b*x^2+a)^2)^(
1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1264, 457, 78}

$$-\frac{\log(x)\,(a+bx^2)\,(bd-ae)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\,(bd-ae)\log\left(a+bx^2\right)}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
```

```
[Out] -1/2*(d*(a + b*x^2))/(a*x^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)
*(a + b*x^2)*Log[x])/(a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*(
a + b*x^2)*Log[a + b*x^2])/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
```

```
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{x^3\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{(ab+b^2x^2)\int \frac{d+ex^2}{x^3(ab+b^2x^2)}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \frac{d+ex}{x^2(ab+b^2x)}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= \frac{(ab+b^2x^2)\,\text{Subst}\left(\int \left(\frac{d}{abx^2}+\frac{-bd+ae}{a^2bx}+\frac{bd-ae}{a^2(a+bx)}\right)\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}\\
&= -\frac{d(a+bx^2)}{2ax^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)\,(a+bx^2)\log(x)}{a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)\,(a+b
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 70, normalized size = 0.51

$$\frac{(a+bx^2)\,(-ad+2(-bd+ae)x^2\log(x)+(bd-ae)x^2\log\left(a+bx^2\right))}{2a^2x^2\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]),x]
[Out] ((a + b*x^2)*(-(a*d) + 2*(-(b*d) + a*e)*x^2*Log[x] + (b*d - a*e)*x^2*Log[a
+ b*x^2]))/(2*a^2*x^2*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.13, size = 78, normalized size = 0.57

method	result	size
default	$-\frac{(b\,x^2+a)\,(\ln(b\,x^2+a)\,ae\,x^2-\ln(b\,x^2+a)\,bd\,x^2-2\ln(x)\,ae\,x^2+2\ln(x)\,bd\,x^2+ad)}{2\sqrt{(b\,x^2+a)^2}\;a^2x^2}$	78
risch	$-\frac{\sqrt{(b\,x^2+a)^2}\;(ae-bd)\ln(b\,x^2+a)}{2(b\,x^2+a)a^2}-\frac{\sqrt{(b\,x^2+a)^2}\;d}{2(b\,x^2+a)a\,x^2}+\frac{\sqrt{(b\,x^2+a)^2}\;(ae-bd)\ln(x)}{(b\,x^2+a)a^2}$	106

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*(b*x^2+a)*(ln(b*x^2+a)*a*e*x^2-ln(b*x^2+a)*b*d*x^2-2*ln(x)*a*e*x^2+2*l
n(x)*b*d*x^2+a*d)/((b*x^2+a)^2)^(1/2)/a^2/x^2
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.36

$$\frac{(bd-ae)\log\left(bx^2+a\right)}{2\,a^2}-\frac{(bd-ae)\log\left(x^2\right)}{2\,a^2}-\frac{d}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b*d - a*e)*log(b*x^2 + a)/a^2 - 1/2*(b*d - a*e)*log(x^2)/a^2 - 1/2*d/(
a*x^2)
```

Fricas [A]
time = 0.45, size = 56, normalized size = 0.41

$$-\frac{ad-\left(bdx^2-ax^2e\right)\log\left(bx^2+a\right)+2\left(bdx^2-ax^2e\right)\log\left(x\right)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*d - (b*d*x^2 - a*x^2*e)*log(b*x^2 + a) + 2*(b*d*x^2 - a*x^2*e)*log(
x))/(a^2*x^2)
```

Sympy [A]
time = 0.40, size = 41, normalized size = 0.30

$$-\frac{d}{2ax^2}+\frac{(ae-bd)\log\left(x\right)}{a^2}-\frac{(ae-bd)\log\left(\frac{a}{b}+x^2\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**3/((b*x**2+a)**2)**(1/2),x)
```

```
[Out] -d/(2*a*x**2) + (a*e - b*d)*log(x)/a**2 - (a*e - b*d)*log(a/b + x**2)/(2*a*
*2)
```

Giac [A]
time = 4.07, size = 131, normalized size = 0.96

$$-\frac{\left(bd\mathrm{sgn}\left(bx^2+a\right)-ae\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(x^2\right)}{2\,a^2}+\frac{\left(b^2d\mathrm{sgn}\left(bx^2+a\right)-abe\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left|bx^2+a\right|\right)}{2\,a^2b}+\frac{bdx^2\mathrm{sgn}\left(bx^2+a\right)-ax^2e\mathrm{sgn}\left(bx^2+a\right)-ad\mathrm{sgn}\left(bx^2+a\right)}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(b*d*sgn(b*x^2 + a) - a*e*sgn(b*x^2 + a))*log(x^2)/a^2 + 1/2*(b^2*d*sg
n(b*x^2 + a) - a*b*e*sgn(b*x^2 + a))*log(abs(b*x^2 + a))/(a^2*b) + 1/2*(b*d
*x^2*sgn(b*x^2 + a) - a*x^2*e*sgn(b*x^2 + a) - a*d*sgn(b*x^2 + a))/(a^2*x^2
)
```

Mupad [B]
time = 0.81, size = 125, normalized size = 0.91

$$\frac{a\,b\,d\,\mathrm{atanh}\left(\frac{a^2+b\,a\,x^2}{\sqrt{a^2}\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\right)}{2\,{\left(a^2\right)}^{3/2}}-\frac{e\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a^2}}-\frac{d\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}{2\,a^2\,x^2}-\frac{e\,\ln\left(\sqrt{{\left(b\,x^2+a\right)}^2}\,\sqrt{a^2}+a^2+a\,b\,x^2\right)}{2\,\sqrt{a^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^3*((a + b*x^2)^2)^(1/2)),x)
```

```
[Out] (a*b*d*atanh((a^2 + a*b*x^2)/((a^2)^(1/2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)
)))/(2*(a^2)^(3/2)) - (e*log(1/x^2))/(2*(a^2)^(1/2)) - (d*(a^2 + b^2*x^4 +
2*a*b*x^2)^(1/2))/(2*a^2*x^2) - (e*log(((a + b*x^2)^2)^(1/2)*(a^2)^(1/2) +
a^2 + a*b*x^2))/(2*(a^2)^(1/2))
```

3.81 $\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=153

$$\frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)x}{4b^2\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd+3ae)\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*(-5*a*e+b*d)*x/a/b^2/((b*x^2+a)^2)^(1/2)-1/4*(-a*e+b*d)*x/b^2/(b*x^2+a)
/((b*x^2+a)^2)^(1/2)+1/8*(3*a*e+b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^
(3/2)/b^(5/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1264, 466, 393, 211}

$$\frac{x(bd-5ae)}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(bd-ae)}{4b^2\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(3ae+bd)}{8a^{3/2}b^{5/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - 5*a*e)*x)/(8*a*b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*e)*
x)/(4*b^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d + 3*a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 466

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
```

```
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{x^2(d+ex^2)}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(bd-ae)x}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(ab+b^2x^2)\int \frac{-b(bd-ae)-4b^2ex^2}{(ab+b^2x^2)^2}\,dx}{4b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{((bd+3ae}{8ab^2}$$

$$= \frac{(bd-5ae)x}{8ab^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd+3ae)}{8a^{3/2}b^{5/}}$$

Mathematica [A]
time = 0.04, size = 108, normalized size = 0.71

$$\frac{-\sqrt{a}\sqrt{b}\,x(3a^2e-b^2dx^2+ab(d+5ex^2))+(bd+3ae)\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{3/2}b^{5/2}\,(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (-(Sqrt[a]*Sqrt[b]*x*(3*a^2*e - b^2*d*x^2 + a*b*(d + 5*e*x^2))) + (b*d + 3*
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(3/2)*b^(5/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.05, size = 188, normalized size = 1.23

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\left(-\frac{(5ae-bd)x^3}{8ab}-\frac{(3ae+bd)x}{8b^2}\right)}{(b\,x^2+a)^3}-\frac{\sqrt{(b\,x^2+a)^2}\,(3ae+bd)\ln\left(bx+\sqrt{-ab}\right)}{16(b\,x^2+a)\sqrt{-ab}\,b^2a}+\frac{\sqrt{(b\,x^2+a)^2}\,(3ae+}{16(b\,x^2+a)\sqrt{}}$
default	$-\frac{\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2e\,x^4-\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3d\,x^4+5\sqrt{ab}\,abe\,x^3-\sqrt{ab}\,b^2d\,x^3-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2be\,x^2-2\arctan\left(\right.\right.}{8\sqrt{ab}\,b^2a\left((b\,x^2+a)^2\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-3*arctan(b*x/(a*b)^(1/2))*a*b^2*e*x^4-arctan(b*x/(a*b)^(1/2))*b^3*d*
x^4+5*(a*b)^(1/2)*a*b*e*x^3-(a*b)^(1/2)*b^2*d*x^3-6*arctan(b*x/(a*b)^(1/2))
*a^2*b*e*x^2-2*arctan(b*x/(a*b)^(1/2))*a*b^2*d*x^2+3*(a*b)^(1/2)*a^2*e*x+(a
*b)^(1/2)*a*b*d*x-3*arctan(b*x/(a*b)^(1/2))*a^3*e-arctan(b*x/(a*b)^(1/2))*a
^2*b*d)*(b*x^2+a)/(a*b)^(1/2)/b^2/a/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 126, normalized size = 0.82

$$\frac{1}{8}\,d\left(\frac{bx^3-ax}{ab^3x^4+2\,a^2b^2x^2+a^3b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}\right)-\frac{1}{8}\left(\frac{5\,bx^3+3\,ax}{b^4x^4+2\,ab^3x^2+a^2b^2}-\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,b^2}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] 1/8*d*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2 + a^3*b) + arctan(b*x/sqrt(
a*b))/(sqrt(a*b)*a*b)) - 1/8*((5*b*x^3 + 3*a*x)/(b^4*x^4 + 2*a*b^3*x^2 + a^
2*b^2) - 3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*b^2))*e
```

Fricas [A]
time = 0.52, size = 308, normalized size = 2.01

$$\left[\frac{2\,ab^3dx^3-2\,a^2b^2dx-(b^3dx^4+2\,ab^2dx^2+a^2bd+3\,(ab^2x^4+2\,a^2bx^2+a^3)e)\sqrt{-ab}\,\log\left(\frac{bx^2-2\,\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,(5\,a^2b^2x^3+3\,a^3bx)e}{16\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)},\ \frac{ab^3dx^3-a^2b^2dx+(b^3dx^4+2\,ab^2dx^2+a^2bd+3\,(ab^2x^4+2\,a^2bx^2+a^3)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-(5\,a^2b^2x^3+3\,a^3bx)e}{8\,(a^2b^5x^4+2\,a^3b^4x^2+a^4b^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [1/16*(2*a*b^3*d*x^3 - 2*a^2*b^2*d*x - (b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2*b*d
 + 3*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*e)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-a*b
)*x - a)/(b*x^2 + a)) - 2*(5*a^2*b^2*x^3 + 3*a^3*b*x)*e)/(a^2*b^5*x^4 + 2*a
^3*b^4*x^2 + a^4*b^3), 1/8*(a*b^3*d*x^3 - a^2*b^2*d*x + (b^3*d*x^4 + 2*a*b^
2*d*x^2 + a^2*b*d + 3*(a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*e)*sqrt(a*b)*arctan(s
qrt(a*b)*x/a) - (5*a^2*b^2*x^3 + 3*a^3*b*x)*e)/(a^2*b^5*x^4 + 2*a^3*b^4*x^2
 + a^4*b^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(d+ex^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x**2*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.21, size = 101, normalized size = 0.66

$$\frac{(bd+3\,ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\sqrt{ab}\,ab^2\mathrm{sgn}\left(bx^2+a\right)}+\frac{b^2dx^3-5\,abx^3e-abdx-3\,a^2xe}{8\left(bx^2+a\right)^2ab^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(b*d + 3*a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b^2*sgn(b*x^2 + a)) +
1/8*(b^2*d*x^3 - 5*a*b*x^3*e - a*b*d*x - 3*a^2*x*e)/((b*x^2 + a)^2*a*b^2*sg
n(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,\left(e\,x^2+d\right)}{{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((x^2*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.82 $\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{bd-ae}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] -1/2*e/b^2/((b*x^2+a)^2)^(1/2)+1/4*(a*e-b*d)/b^2/(b*x^2+a)/((b*x^2+a)^2)^(1
/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1261, 654, 621}

$$-\frac{bd-ae}{4b^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] -1/2*e/(b^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*b^2*(a + b*x^
2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 621

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[2*((a + b*x
+ c*x^2)^(p + 1)/((2*p + 1)*(b + 2*c*x))), x] /; FreeQ[{a, b, c, p}, x] &&
EqQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{d+ex}{(a^2+2abx+b^2x^2)^{3/2}}\,dx, x, x^2\right)$$
$$= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)\text{Subst}\left(\int \frac{1}{(a^2+2abx+b^2x^2)^{3/2}}\,dx, x, x^2\right)}{2b}$$
$$= -\frac{e}{2b^2\sqrt{a^2+2abx^2+b^2x^4}} - \frac{bd-ae}{4b^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}$$

Mathematica [A]
time = 0.01, size = 45, normalized size = 0.58

$$\frac{-ae-b(d+2ex^2)}{4b^2(a+bx^2)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
[Out] (-(a*e) - b*(d + 2*e*x^2))/(4*b^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.03, size = 38, normalized size = 0.49

method	result	size
gosper	$-\frac{(b\,x^2+a)\,(2be\,x^2+ae+bd)}{4b^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	38
default	$-\frac{(b\,x^2+a)\,(2be\,x^2+ae+bd)}{4b^2\left((b\,x^2+a)^2\right)^{\frac{3}{2}}}$	38
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(-\frac{e\,x^2}{2b}-\frac{ae+bd}{4b^2}\right)}{(b\,x^2+a)^3}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
[Out] -1/4*(b*x^2+a)*(2*b*e*x^2+a*e+b*d)/b^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.29, size = 66, normalized size = 0.86

$$-\frac{(2\,bx^2+a)e}{4\,(b^4x^4+2\,ab^3x^2+a^2b^2)} - \frac{d}{4\,(b^3x^4+2\,ab^2x^2+a^2b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*b*x^2 + a)*e/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2) - 1/4*d/(b^3*x^4 + 2
*a*b^2*x^2 + a^2*b)
```

Fricas [A]
time = 0.37, size = 43, normalized size = 0.56

$$-\frac{bd + (2\,bx^2 + a)e}{4\,(b^4x^4 + 2\,ab^3x^2 + a^2b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d + (2*b*x^2 + a)*e)/(b^4*x^4 + 2*a*b^3*x^2 + a^2*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d + ex^2)}{\left((a + bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral(x*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 5.58, size = 40, normalized size = 0.52

$$-\frac{2\,bx^2e + bd + ae}{4\,(bx^2 + a)^2b^2\mathrm{sgn}\,(bx^2 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(2*b*x^2*e + b*d + a*e)/((b*x^2 + a)^2*b^2*sgn(b*x^2 + a))
```

Mupad [B]
time = 0.18, size = 48, normalized size = 0.62

$$-\frac{\sqrt{a^2 + 2\,a\,b\,x^2 + b^2\,x^4}\,\left(2\,b\,e\,x^2 + a\,e + b\,d\right)}{4\,b^2\,\left(b\,x^2 + a\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] -((a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2)*(a*e + b*d + 2*b*e*x^2))/(4*b^2*(a + b*
x^2)^3)
```

3.83 $\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=156

$$\frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)x}{4ab\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(3bd+ae)\,(a+bx^2)\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/8*(a*e+3*b*d)*x/a^2/b/((b*x^2+a)^2)^(1/2)+1/4*(-a*e+b*d)*x/a/b/(b*x^2+a)/
((b*x^2+a)^2)^(1/2)+1/8*(a*e+3*b*d)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(
5/2)/b^(3/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 30, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {1162, 393, 205, 211}

$$\frac{x(ae+3bd)}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}}+\frac{x(bd-ae)}{4ab\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(ae+3bd)}{8a^{5/2}b^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((3*b*d + a*e)*x)/(8*a^2*b*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((b*d - a*e)*
x)/(4*a*b*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d + a*e)*(a
+ b*x^2)*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
```

```
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2))\int \frac{d+ex^2}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-ae)x}{4ab\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{((3bd+ae)\,(ab+b^2x^2))\int \frac{1}{(ab+b^2x^2)^2}\,dx}{4a\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{((3bd+\cdots}{8a^2\cdots}$$

$$= \frac{(3bd+ae)x}{8a^2b\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd-ae)x}{4ab\,(a+bx^2)\,\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(3bd+a\cdots}{8a^{5/2}b\cdots}$$

Mathematica [A]
time = 0.04, size = 108, normalized size = 0.69

$$\frac{\sqrt{a}\,\sqrt{b}\,x(-a^2e+3b^2dx^2+ab(5d+ex^2))+(3bd+ae)\,(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}\,(a+bx^2)\,\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*x*(-(a^2*e) + 3*b^2*d*x^2 + a*b*(5*d + e*x^2)) + (3*b*d +
a*e)*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*(a + b*x
^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 188, normalized size = 1.21

method	result
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(\frac{(ae+3bd)x^{3}}{8a^{2}}-\frac{(ae-5bd)x}{8ab}\right)}{\left(b\,x^{2}+a\right)^{3}}-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,(ae+3bd)\ln\left(bx+\sqrt{-ab}\right)}{16\left(b\,x^{2}+a\right)\sqrt{-ab}\,b\,a^{2}}+\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,(ae+3bd)}{16\left(b\,x^{2}+a\right)\sqrt{-}}$
default	$-\frac{\left(-\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^{2}e\,x^{4}-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^{3}d\,x^{4}-\sqrt{ab}\,abe\,x^{3}-3\sqrt{ab}\,b^{2}d\,x^{3}-2\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^{2}be\,x^{2}-6\arctan\left(\right.\right.}{8\sqrt{ab}\,b\,a^{2}\left(\left(b\,x^{2}+a\right)^{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-arctan(b*x/(a*b)^(1/2))*a*b^2*e*x^4-3*arctan(b*x/(a*b)^(1/2))*b^3*d*
x^4-(a*b)^(1/2)*a*b*e*x^3-3*(a*b)^(1/2)*b^2*d*x^3-2*arctan(b*x/(a*b)^(1/2))
*a^2*b*e*x^2-6*arctan(b*x/(a*b)^(1/2))*a*b^2*d*x^2+(a*b)^(1/2)*a^2*e*x-5*(a
*b)^(1/2)*a*b*d*x-arctan(b*x/(a*b)^(1/2))*a^3*e-3*arctan(b*x/(a*b)^(1/2))*a
^2*b*d)*(b*x^2+a)/(a*b)^(1/2)/b/a^2/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 125, normalized size = 0.80

$$\frac{1}{8}\,d\left(\frac{3\,bx^{3}+5\,ax}{a^{2}b^{2}x^{4}+2\,a^{3}bx^{2}+a^{4}}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^{2}}\right)+\frac{1}{8}\left(\frac{bx^{3}-ax}{ab^{3}x^{4}+2\,a^{2}b^{2}x^{2}+a^{3}b}+\frac{\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,ab}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*d*((3*b*x^3 + 5*a*x)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/s
qrt(a*b))/(sqrt(a*b)*a^2)) + 1/8*((b*x^3 - a*x)/(a*b^3*x^4 + 2*a^2*b^2*x^2
+ a^3*b) + arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a*b))*e
```

Fricas [A]
time = 0.44, size = 308, normalized size = 1.97

$$\left[\frac{6\,ab^{3}dx^{3}+10\,a^{2}b^{2}dx-\left(3\,b^{3}dx^{4}+6\,ab^{2}dx^{2}+3\,a^{2}bd+\left(ab^{2}x^{4}+2\,a^{2}bx^{2}+a^{3}\right)e\right)\sqrt{-ab}\,\log\left(\frac{bx^{2}-2\,\sqrt{-ab}\,x-a}{bx^{2}+a}\right)+2\left(a^{2}b^{2}x^{3}-a^{3}bx\right)e}{16\left(a^{3}b^{4}x^{4}+2\,a^{4}b^{3}x^{2}+a^{5}b^{2}\right)},\ \frac{3\,ab^{3}dx^{3}+5\,a^{2}b^{2}dx+\left(3\,b^{3}dx^{4}+6\,ab^{2}dx^{2}+3\,a^{2}bd+\left(ab^{2}x^{4}+2\,a^{2}bx^{2}+a^{3}\right)e\right)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)+\left(a^{2}b^{2}x^{3}-a^{3}bx\right)e}{8\left(a^{3}b^{4}x^{4}+2\,a^{4}b^{3}x^{2}+a^{5}b^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(6*a*b^3*d*x^3 + 10*a^2*b^2*d*x - (3*b^3*d*x^4 + 6*a*b^2*d*x^2 + 3*a^
2*b*d + (a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*e)*sqrt(-a*b)*log((b*x^2 - 2*sqrt(-
a*b)*x - a)/(b*x^2 + a)) + 2*(a^2*b^2*x^3 - a^3*b*x)*e)/(a^3*b^4*x^4 + 2*a^
4*b^3*x^2 + a^5*b^2), 1/8*(3*a*b^3*d*x^3 + 5*a^2*b^2*d*x + (3*b^3*d*x^4 + 6
```

```
*a*b^2*d*x^2 + 3*a^2*b*d + (a*b^2*x^4 + 2*a^2*b*x^2 + a^3)*e)*sqrt(a*b)*arc
tan(sqrt(a*b)*x/a) + (a^2*b^2*x^3 - a^3*b*x)*e)/(a^3*b^4*x^4 + 2*a^4*b^3*x^
2 + a^5*b^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^2}{\left((a + bx^2)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [A]
time = 4.07, size = 101, normalized size = 0.65

$$\frac{(3\,bd + ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^2b\mathrm{sgn}\left(bx^2 + a\right)} + \frac{3\,b^2dx^3 + abx^3e + 5\,abdx - a^2xe}{8\left(bx^2 + a\right)^2a^2b\mathrm{sgn}\left(bx^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(3*b*d + a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2*b*sgn(b*x^2 + a)) +
1/8*(3*b^2*d*x^3 + a*b*x^3*e + 5*a*b*d*x - a^2*x*e)/((b*x^2 + a)^2*a^2*b*sg
n(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2 + d}{{\left(a^2 + 2\,a\,b\,x^2 + b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
```

```
[Out] int((d + e*x^2)/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.84 $\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=161

$$\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(a+bx^2)\log(x)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)\log\left(a+\ldots\right.}{2a^3\sqrt{a^2+2abx^2+\ldots}}$$

```
[Out] 1/2*d/a^2/((b*x^2+a)^2)^(1/2)+1/4*(-a*e+b*d)/a/b/(b*x^2+a)/((b*x^2+a)^2)^(1
/2)+d*(b*x^2+a)*ln(x)/a^3/((b*x^2+a)^2)^(1/2)-1/2*d*(b*x^2+a)*ln(b*x^2+a)/a
^3/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {1264, 457, 78}

$$\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d\log(x)\left(a+bx^2\right)}{a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)\log\left(a+bx^2\right)}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] d/(2*a^2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (b*d - a*e)/(4*a*b*(a + b*x^2)*
Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + (d*(a + b*x^2)*Log[x])/(a^3*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2)*Log[a + b*x^2])/(2*a^3*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
```

```
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{x\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx &= \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{d+ex^2}{x(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{\left(b^2(ab+b^2x^2)\right)\operatorname{Subst}\left(\int \frac{d+ex}{x(ab+b^2x)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{\left(b^2(ab+b^2x^2)\right)\operatorname{Subst}\left(\int \left(\frac{d}{a^3b^3x}+\frac{-bd+ae}{ab^3(a+bx)^3}-\frac{d}{a^2b^2(a+bx)^2}-\frac{d}{a^3b^2(a+bx)}\right)dx,\right.}{2\sqrt{a^2+2abx^2+b^2x^4}} \\
&= \frac{d}{2a^2\sqrt{a^2+2abx^2+b^2x^4}}+\frac{bd-ae}{4ab\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{d(a+}{a^3\sqrt{a^2-}}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 92, normalized size = 0.57

$$\frac{a(3abd-a^2e+2b^2dx^2)+4bd(a+bx^2)^2\log(x)-2bd(a+bx^2)^2\log\left(a+bx^2\right)}{4a^3b\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (a*(3*a*b*d - a^2*e + 2*b^2*d*x^2) + 4*b*d*(a + b*x^2)^2*Log[x] - 2*b*d*(a
+ b*x^2)^2*Log[a + b*x^2])/(4*a^3*b*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.04, size = 132, normalized size = 0.82

method	result
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(\frac{b d\,x^2}{2a^2}-\frac{ae-3bd}{4ab}\right)}{\left(b\,x^2+a\right)^3}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\,d\ln(x)}{\left(b\,x^2+a\right)a^3}-\frac{\sqrt{\left(b\,x^2+a\right)^2}\,d\ln\left(b\,x^2+a\right)}{2\left(b\,x^2+a\right)a^3}$
default	$-\frac{\left(2\ln\left(b\,x^2+a\right)b^3d\,x^4-4\ln(x)b^3d\,x^4+4\ln\left(b\,x^2+a\right)a\,b^2d\,x^2-8\ln(x)a\,b^2d\,x^2-2b^2d\,x^2a+2\ln\left(b\,x^2+a\right)a^2bd-4\ln(x)a^2bd+a^3e-3a^2bd\right.}{4b\,a^3\left(\left(b\,x^2+a\right)^2\right)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*ln(b*x^2+a)*b^3*d*x^4-4*ln(x)*b^3*d*x^4+4*ln(b*x^2+a)*a*b^2*d*x^2-8
*ln(x)*a*b^2*d*x^2-2*b^2*d*x^2*a+2*ln(b*x^2+a)*a^2*b*d-4*ln(x)*a^2*b*d+a^3*
e-3*a^2*b*d)*(b*x^2+a)/b/a^3/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.28, size = 89, normalized size = 0.55

$$\frac{1}{4}\,d\left(\frac{2\,bx^2+3\,a}{a^2b^2x^4+2\,a^3bx^2+a^4}-\frac{2\,\log\left(bx^2+a\right)}{a^3}+\frac{4\,\log\left(x\right)}{a^3}\right)-\frac{e}{4\,\left(b^3x^4+2\,ab^2x^2+a^2b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*d*((2*b*x^2 + 3*a)/(a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)
/a^3 + 4*log(x)/a^3) - 1/4*e/(b^3*x^4 + 2*a*b^2*x^2 + a^2*b)
```

Fricas [A]
time = 0.36, size = 120, normalized size = 0.75

$$\frac{2\,ab^2dx^2+3\,a^2bd-a^3e-2\,\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(bx^2+a\right)+4\,\left(b^3dx^4+2\,ab^2dx^2+a^2bd\right)\log\left(x\right)}{4\,\left(a^3b^3x^4+2\,a^4b^2x^2+a^5b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*b^2*d*x^2 + 3*a^2*b*d - a^3*e - 2*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2
*b*d)*log(b*x^2 + a) + 4*(b^3*d*x^4 + 2*a*b^2*d*x^2 + a^2*b*d)*log(x))/(a^3
*b^3*x^4 + 2*a^4*b^2*x^2 + a^5*b)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{d+ex^2}{x\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 4.56, size = 107, normalized size = 0.66

$$\frac{d\log\left(x^2\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{d\log\left(\left|bx^2+a\right|\right)}{2\,a^3\mathrm{sgn}\left(bx^2+a\right)}+\frac{3\,b^3dx^4+8\,ab^2dx^2+6\,a^2bd-a^3e}{4\,\left(bx^2+a\right)^2a^3b\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*d*log(x^2)/(a^3*sgn(b*x^2 + a)) - 1/2*d*log(abs(b*x^2 + a))/(a^3*sgn(b*
x^2 + a)) + 1/4*(3*b^3*d*x^4 + 8*a*b^2*d*x^2 + 6*a^2*b*d - a^3*e)/((b*x^2 +
 a)^2*a^3*b*sgn(b*x^2 + a))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{x\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.85 $\int \frac{d+ex^2}{x^2(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=190

$$-\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(bd-ae)x}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3(5bd-ae)(a+}{8a^{7/2}\sqrt{b}\sqrt{a^2}}$$

```
[Out] -1/8*(-3*a*e+7*b*d)*x/a^3/((b*x^2+a)^2)^(1/2)-1/4*(-a*e+b*d)*x/a^2/(b*x^2+a
)/((b*x^2+a)^2)^(1/2)-d*(b*x^2+a)/a^3/x/((b*x^2+a)^2)^(1/2)-3/8*(-a*e+5*b*d
)*(b*x^2+a)*arctan(x*b^(1/2)/a^(1/2))/a^(7/2)/b^(1/2)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {1264, 467, 464, 211}

$$-\frac{x(bd-ae)}{4a^2(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{3(a+bx^2)\,\text{ArcTan}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)(5bd-ae)}{8a^{7/2}\sqrt{b}\sqrt{a^2+2abx^2+b^2x^4}}-\frac{x(7bd-3ae)}{8a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d(a+bx^2)}{a^3x\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -1/8*((7*b*d - 3*a*e)*x)/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((b*d - a*
e)*x)/(4*a^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))
/(a^3*x*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (3*(5*b*d - a*e)*(a + b*x^2)*Arc
Tan[(Sqrt[b]*x)/Sqrt[a]])/(8*a^(7/2)*Sqrt[b]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x^2\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{d+ex^2}{x^2(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{\left(b^2(ab+b^2x^2)\right)\int \frac{-\frac{4d}{ab}+\frac{3(bd-ae)x^2}{a^2b}}{x^2(ab+b^2x^2)^2}\,dx}{4\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(b^2(a}{}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{a^3x\sqrt{}}$$

$$= -\frac{(7bd-3ae)x}{8a^3\sqrt{a^2+2abx^2+b^2x^4}} - \frac{(bd-ae)x}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}} - \frac{}{a^3x\sqrt{}}$$

Mathematica [A]
time = 0.05, size = 124, normalized size = 0.65

$$\frac{\sqrt{a}\sqrt{b}\left(-15b^2dx^4+a^2(-8d+5ex^2)+ab(-25dx^2+3ex^4)\right)+3(-5bd+ae)x(a+bx^2)^2\tan^{-1}\left(\frac{\sqrt{b}\,x}{\sqrt{a}}\right)}{8a^{7/2}\sqrt{b}\,x\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] (Sqrt[a]*Sqrt[b]*(-15*b^2*d*x^4 + a^2*(-8*d + 5*e*x^2) + a*b*(-25*d*x^2 + 3
*e*x^4)) + 3*(-5*b*d + a*e)*x*(a + b*x^2)^2*ArcTan[(Sqrt[b]*x)/Sqrt[a]])/(8
*a^(7/2)*Sqrt[b]*x*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.05, size = 206, normalized size = 1.08

method	result
risch	$\frac{\sqrt{(b\,x^2+a)^2}\,\left(\frac{3b(ae-5bd)x^4}{8a^3}+\frac{5(ae-5bd)x^2}{8a^2}-\frac{d}{a}\right)}{(b\,x^2+a)^3 x}-\frac{3\sqrt{(b\,x^2+a)^2}\,(ae-5bd)\ln\left(-\sqrt{-ab}\,x+a\right)}{16(b\,x^2+a)\sqrt{-ab}\,a^3}+\frac{3\sqrt{(b\,x^2+a)}}{16(b}$
default	$-\frac{\left(-3\arctan\left(\frac{bx}{\sqrt{ab}}\right)a\,b^2 e\,x^5+15\arctan\left(\frac{bx}{\sqrt{ab}}\right)b^3 d\,x^5-3\sqrt{ab}\,abe\,x^4+15\sqrt{ab}\,b^2 d\,x^4-6\arctan\left(\frac{bx}{\sqrt{ab}}\right)a^2 be\,x^3+30\,\mathrm{arct}\right.}{8x\sqrt{ab}\,a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8*(-3*arctan(b*x/(a*b)^(1/2))*a*b^2*e*x^5+15*arctan(b*x/(a*b)^(1/2))*b^3
*d*x^5-3*(a*b)^(1/2)*a*b*e*x^4+15*(a*b)^(1/2)*b^2*d*x^4-6*arctan(b*x/(a*b)^
(1/2))*a^2*b*e*x^3+30*arctan(b*x/(a*b)^(1/2))*a*b^2*d*x^3-5*(a*b)^(1/2)*a^2
*e*x^2+25*(a*b)^(1/2)*a*b*d*x^2-3*arctan(b*x/(a*b)^(1/2))*a^3*e*x+15*arctan
(b*x/(a*b)^(1/2))*a^2*b*d*x+8*(a*b)^(1/2)*a^2*d)*(b*x^2+a)/x/(a*b)^(1/2)/a^
3/((b*x^2+a)^2)^(3/2)
```

Maxima [A]
time = 0.49, size = 135, normalized size = 0.71

$$-\frac{1}{8}\,d\left(\frac{15\,b^2x^4+25\,abx^2+8\,a^2}{a^3b^2x^5+2\,a^4bx^3+a^5x}+\frac{15\,b\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^3}\right)+\frac{1}{8}\left(\frac{3\,bx^3+5\,ax}{a^2b^2x^4+2\,a^3bx^2+a^4}+\frac{3\,\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{\sqrt{ab}\,a^2}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/8*d*((15*b^2*x^4 + 25*a*b*x^2 + 8*a^2)/(a^3*b^2*x^5 + 2*a^4*b*x^3 + a^5*
x) + 15*b*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3)) + 1/8*((3*b*x^3 + 5*a*x)/(
a^2*b^2*x^4 + 2*a^3*b*x^2 + a^4) + 3*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^2))
*e
```

Fricas [A]
time = 0.34, size = 340, normalized size = 1.79

$$\left[-\frac{30\,ab^3dx^4+50\,a^2b^2dx^2+16\,a^3bd-3\,(5\,b^3dx^5+10\,ab^2dx^3+5\,a^2bdx-(ab^2x^5+2\,a^2bx^3+a^3x)e)\sqrt{-ab}\,\log\left(\frac{bx^2-2\sqrt{-ab}\,x-a}{bx^2+a}\right)-2\,(3\,a^2b^2x^4+5\,a^3bx^2)e}{16\,(a^4b^3x^5+2\,a^5b^2x^3+a^6bx)},\ -\frac{15\,ab^3dx^4+25\,a^2b^2dx^2+8\,a^3bd+3\,(5\,b^3dx^5+10\,ab^2dx^3+5\,a^2bdx-(ab^2x^5+2\,a^2bx^3+a^3x)e)\sqrt{ab}\,\arctan\left(\frac{\sqrt{ab}\,x}{a}\right)-(3\,a^2b^2x^4+5\,a^3bx^2)e}{8\,(a^4b^3x^5+2\,a^5b^2x^3+a^6bx)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] [-1/16*(30*a*b^3*d*x^4 + 50*a^2*b^2*d*x^2 + 16*a^3*b*d - 3*(5*b^3*d*x^5 + 1
0*a*b^2*d*x^3 + 5*a^2*b*d*x - (a*b^2*x^5 + 2*a^2*b*x^3 + a^3*x)*e)*sqrt(-a*
b)*log((b*x^2 - 2*sqrt(-a*b)*x - a)/(b*x^2 + a)) - 2*(3*a^2*b^2*x^4 + 5*a^3
*b*x^2)*e)/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x), -1/8*(15*a*b^3*d*x^4 +
25*a^2*b^2*d*x^2 + 8*a^3*b*d + 3*(5*b^3*d*x^5 + 10*a*b^2*d*x^3 + 5*a^2*b*d*
x - (a*b^2*x^5 + 2*a^2*b*x^3 + a^3*x)*e)*sqrt(a*b)*arctan(sqrt(a*b)*x/a) -
(3*a^2*b^2*x^4 + 5*a^3*b*x^2)*e)/(a^4*b^3*x^5 + 2*a^5*b^2*x^3 + a^6*b*x)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{x^2\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**2/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**2*((a + b*x**2)**2)**(3/2)), x)
```

Giac [**A**]
time = 3.63, size = 116, normalized size = 0.61

$$-\frac{3\,(5\,bd-ae)\arctan\left(\frac{bx}{\sqrt{ab}}\right)}{8\,\sqrt{ab}\,a^3\mathrm{sgn}\left(bx^2+a\right)}-\frac{d}{a^3x\mathrm{sgn}\left(bx^2+a\right)}-\frac{7\,b^2dx^3-3\,abx^3e+9\,abdx-5\,a^2xe}{8\,\left(bx^2+a\right)^2a^3\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^2/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -3/8*(5*b*d - a*e)*arctan(b*x/sqrt(a*b))/(sqrt(a*b)*a^3*sgn(b*x^2 + a)) - d
/(a^3*x*sgn(b*x^2 + a)) - 1/8*(7*b^2*d*x^3 - 3*a*b*x^3*e + 9*a*b*d*x - 5*a^
2*x*e)/((b*x^2 + a)^2*a^3*sgn(b*x^2 + a))
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{e\,x^2+d}{x^2\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x^2*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.86 $\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}-\frac{(3bd-ae)\left(a+\right.}{a^4\sqrt{a^2+2ab}}$$

```
[Out] 1/2*(a*e-2*b*d)/a^3/((b*x^2+a)^2)^(1/2)+1/4*(a*e-b*d)/a^2/(b*x^2+a)/((b*x^2
+a)^2)^(1/2)-1/2*d*(b*x^2+a)/a^3/x^2/((b*x^2+a)^2)^(1/2)-(-a*e+3*b*d)*(b*x^
2+a)*ln(x)/a^4/((b*x^2+a)^2)^(1/2)+1/2*(-a*e+3*b*d)*(b*x^2+a)*ln(b*x^2+a)/a
^4/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1264, 457, 78}

$$-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\log(x)\left(a+bx^2\right)(3bd-ae)}{a^4\sqrt{a^2+2abx^2+b^2x^4}}+\frac{\left(a+bx^2\right)(3bd-ae)\log\left(a+bx^2\right)}{2a^4\sqrt{a^2+2abx^2+b^2x^4}}-\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{d\left(a+bx^2\right)}{2a^3x^2\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
```

```
[Out] -1/2*(2*b*d - a*e)/(a^3*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (b*d - a*e)/(4*a
^2*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - (d*(a + b*x^2))/(2*a^3*x^
2*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) - ((3*b*d - a*e)*(a + b*x^2)*Log[x])/(a^
4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]) + ((3*b*d - a*e)*(a + b*x^2)*Log[a + b*x
^2])/(2*a^4*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{d+ex^2}{x^3\left(a^2+2abx^2+b^2x^4\right)^{3/2}}\,dx = \frac{\left(b^2\left(ab+b^2x^2\right)\right)\int \frac{d+ex^2}{x^3\left(ab+b^2x^2\right)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \frac{d+ex}{x^2\left(ab+b^2x\right)^3}\,dx,x,x^2\right)}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= \frac{\left(b^2\left(ab+b^2x^2\right)\right)\operatorname{Subst}\left(\int \left(\frac{d}{a^3b^3x^2}+\frac{-3bd+ae}{a^4b^3x}+\frac{bd-ae}{a^2b^2(a+bx)^3}+\frac{2bd-ae}{a^3b^2(a+bx)^2}+\frac{3}{a^4}\right.\right.}{2\sqrt{a^2+2abx^2+b^2x^4}}$$
$$= -\frac{2bd-ae}{2a^3\sqrt{a^2+2abx^2+b^2x^4}}-\frac{bd-ae}{4a^2\left(a+bx^2\right)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{}{2a^3x^2}$$

Mathematica [A]
time = 0.05, size = 130, normalized size = 0.58

$$\frac{a\left(-6b^2dx^4+a^2\left(-2d+3ex^2\right)+ab\left(-9dx^2+2ex^4\right)\right)+4(-3bd+ae)x^2\left(a+bx^2\right)^2\log(x)+2(3bd-ae)x^2\left(a+bx^2\right)^2\log\left(a+bx^2\right)}{4a^4x^2\left(a+bx^2\right)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(x^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2)),x]
[Out] (a*(-6*b^2*d*x^4 + a^2*(-2*d + 3*e*x^2) + a*b*(-9*d*x^2 + 2*e*x^4)) + 4*(-3
*b*d + a*e)*x^2*(a + b*x^2)^2*Log[x] + 2*(3*b*d - a*e)*x^2*(a + b*x^2)^2*Lo
g[a + b*x^2])/(4*a^4*x^2*(a + b*x^2)*Sqrt[(a + b*x^2)^2])
```

Maple [A]
time = 0.05, size = 249, normalized size = 1.12

method	result
risch	$\frac{\sqrt{\left(b\,x^2+a\right)^2}\,\left(\frac{b(ae-3bd)x^4}{2a^3}+\frac{3(ae-3bd)x^2}{4a^2}-\frac{d}{2a}\right)}{\left(b\,x^2+a\right)^3x^2}+\frac{\sqrt{\left(b\,x^2+a\right)^2}\,(ae-3bd)\ln(x)}{\left(b\,x^2+a\right)a^4}-\frac{\sqrt{\left(b\,x^2+a\right)^2}\,(ae-3bd)\ln(b}{2\left(b\,x^2+a\right)a^4}$
default	$-\frac{\left(2\ln\left(b\,x^2+a\right)a\,b^2e\,x^6-6\ln\left(b\,x^2+a\right)b^3d\,x^6-4\ln(x)a\,b^2e\,x^6+12\ln(x)b^3d\,x^6+4\ln\left(b\,x^2+a\right)a^2be\,x^4-12\ln\left(b\,x^2+a\right)a\,b^2d\,x^4-8\ln(x)a\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(2*ln(b*x^2+a)*a*b^2*e*x^6-6*ln(b*x^2+a)*b^3*d*x^6-4*ln(x)*a*b^2*e*x^6
+12*ln(x)*b^3*d*x^6+4*ln(b*x^2+a)*a^2*b*e*x^4-12*ln(b*x^2+a)*a*b^2*d*x^4-8*
ln(x)*a^2*b*e*x^4+24*ln(x)*a*b^2*d*x^4-2*a^2*b*e*x^4+6*a*b^2*d*x^4+2*ln(b*x
^2+a)*a^3*e*x^2-6*ln(b*x^2+a)*a^2*b*d*x^2-4*ln(x)*a^3*e*x^2+12*ln(x)*a^2*b*
d*x^2-3*a^3*e*x^2+9*a^2*b*d*x^2+2*a^3*d)*(b*x^2+a)/x^2/a^4/((b*x^2+a)^2)^(3
/2)
```

Maxima [A]
time = 0.28, size = 139, normalized size = 0.62

$$-\frac{1}{4}\,d\left(\frac{6\,b^2x^4+9\,abx^2+2\,a^2}{a^3b^2x^6+2\,a^4bx^4+a^5x^2}-\frac{6\,b\log\left(bx^2+a\right)}{a^4}+\frac{12\,b\log\left(x\right)}{a^4}\right)+\frac{1}{4}\,\left(\frac{2\,bx^2+3\,a}{a^2b^2x^4+2\,a^3bx^2+a^4}-\frac{2\,\log\left(bx^2+a\right)}{a^3}+\frac{4\,\log\left(x\right)}{a^3}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="maxima"
)
```

```
[Out] -1/4*d*((6*b^2*x^4 + 9*a*b*x^2 + 2*a^2)/(a^3*b^2*x^6 + 2*a^4*b*x^4 + a^5*x^
2) - 6*b*log(b*x^2 + a)/a^4 + 12*b*log(x)/a^4) + 1/4*((2*b*x^2 + 3*a)/(a^2*
b^2*x^4 + 2*a^3*b*x^2 + a^4) - 2*log(b*x^2 + a)/a^3 + 4*log(x)/a^3)*e
```

Fricas [A]
time = 0.37, size = 212, normalized size = 0.95

$$-\frac{6\,ab^2dx^4+9\,a^2bdx^2+2\,a^3d-\left(2\,a^2bx^4+3\,a^3x^2\right)e-2\,\left(3\,b^3dx^6+6\,ab^2dx^4+3\,a^2bdx^2-\left(ab^2x^6+2\,a^2bx^4+a^3x^2\right)e\right)\log\left(bx^2+a\right)+4\,\left(3\,b^3dx^6+6\,ab^2dx^4+3\,a^2bdx^2-\left(ab^2x^6+2\,a^2bx^4+a^3x^2\right)e\right)\log\left(x\right)}{4\,\left(a^4b^2x^6+2\,a^5bx^4+a^6x^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fricas"
)
```

```
[Out] -1/4*(6*a*b^2*d*x^4 + 9*a^2*b*d*x^2 + 2*a^3*d - (2*a^2*b*x^4 + 3*a^3*x^2)*e
 - 2*(3*b^3*d*x^6 + 6*a*b^2*d*x^4 + 3*a^2*b*d*x^2 - (a*b^2*x^6 + 2*a^2*b*x^
4 + a^3*x^2)*e)*log(b*x^2 + a) + 4*(3*b^3*d*x^6 + 6*a*b^2*d*x^4 + 3*a^2*b*d
*x^2 - (a*b^2*x^6 + 2*a^2*b*x^4 + a^3*x^2)*e)*log(x))/(a^4*b^2*x^6 + 2*a^5*
b*x^4 + a^6*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{d+ex^2}{x^3\left(\left(a+bx^2\right)^2\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/x**3/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(x**3*((a + b*x**2)**2)**(3/2)), x)
```

Giac [A]
time = 3.53, size = 188, normalized size = 0.84

$$-\frac{(3\,bd-ae)\log\left(x^2\right)}{2\,a^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{\left(3\,b^2d-abe\right)\log\left(|bx^2+a|\right)}{2\,a^4b\mathrm{sgn}\left(bx^2+a\right)}-\frac{9\,b^3dx^4-3\,ab^2x^4e+22\,ab^2dx^2-8\,a^2bx^2e+14\,a^2bd-6\,a^3e}{4\left(bx^2+a\right)^2a^4\mathrm{sgn}\left(bx^2+a\right)}+\frac{3\,bdx^2-ax^2e-ad}{2\,a^4x^2\mathrm{sgn}\left(bx^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/x^3/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/2*(3*b*d - a*e)*log(x^2)/(a^4*sgn(b*x^2 + a)) + 1/2*(3*b^2*d - a*b*e)*lo
g(abs(b*x^2 + a))/(a^4*b*sgn(b*x^2 + a)) - 1/4*(9*b^3*d*x^4 - 3*a*b^2*x^4*e
 + 22*a*b^2*d*x^2 - 8*a^2*b*x^2*e + 14*a^2*b*d - 6*a^3*e)/((b*x^2 + a)^2*a^
4*sgn(b*x^2 + a)) + 1/2*(3*b*d*x^2 - a*x^2*e - a*d)/(a^4*x^2*sgn(b*x^2 + a)
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{x^3\,{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/(x^3*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2)), x)
```

3.87 $\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{5/2} \, dx$

Optimal. Leaf size=400

$$\frac{a^5d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{a^4(5bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)\left(a+bx^2\right)}+\frac{5a^3b(2bd+ae)(fx)^{5+m}\sqrt{a^2+}}{f^5(5+m)\left(a+b\right.}$$

```
[Out] a^5*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+a^4*(a*e+5*b*d)*(f*
x)^(3+m)*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+5*a^3*b*(a*e+2*b*d)*(f*x)^
(5+m)*((b*x^2+a)^2)^(1/2)/f^5/(5+m)/(b*x^2+a)+10*a^2*b^2*(a*e+b*d)*(f*x)^(7
+m)*((b*x^2+a)^2)^(1/2)/f^7/(7+m)/(b*x^2+a)+5*a*b^3*(2*a*e+b*d)*(f*x)^(9+m)
*((b*x^2+a)^2)^(1/2)/f^9/(9+m)/(b*x^2+a)+b^4*(5*a*e+b*d)*(f*x)^(11+m)*((b*x
^2+a)^2)^(1/2)/f^11/(11+m)/(b*x^2+a)+b^5*e*(f*x)^(13+m)*((b*x^2+a)^2)^(1/2)
/f^13/(13+m)/(b*x^2+a)
```

Rubi [A]
time = 0.16, antiderivative size = 400, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1264, 459}

$$\frac{10a^2b^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+7}(ae+bd)}{f^7(m+7)\left(a+bx^2\right)}+\frac{b^5e\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+13}}{f^{13}(m+13)\left(a+bx^2\right)}+\frac{b^4\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+11}(5ae+bd)}{f^{11}(m+11)\left(a+bx^2\right)}+\frac{5ab^3\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+9}(2ae+bd)}{f^9(m+9)\left(a+bx^2\right)}+\frac{a^5d\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{a^4\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+5bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{5a^3b\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}(ae+2bd)}{f^5(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (a^5*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^4*(5*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (5*a^3*b*(2*b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a
*b*x^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (10*a^2*b^2*(b*d + a*e)*(f*x
)^(7 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (5*a
*b^3*(b*d + 2*a*e)*(f*x)^(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 +
 m)*(a + b*x^2)) + (b^4*(b*d + 5*a*e)*(f*x)^(11 + m)*Sqrt[a^2 + 2*a*b*x^2 +
 b^2*x^4])/(f^11*(11 + m)*(a + b*x^2)) + (b^5*e*(f*x)^(13 + m)*Sqrt[a^2 + 2
*a*b*x^2 + b^2*x^4])/(f^13*(13 + m)*(a + b*x^2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
```

```
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right) \left(a^2+2abx^2+b^2x^4\right)^{5/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (fx)^m \left(ab+b^2x^2\right)^5 \left(d+ex^2\right) dx}{b^4 \left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^5b^5d(fx)^m + \frac{a^4b^5(5bd+ae)(fx)^{2+m}}{f^2} + \right.}{}$$

$$= \frac{a^5d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)} + \frac{a^4(5bd+ae)(fx)^{3+m}}{f^3(3+m)}$$

Mathematica [A]
time = 0.45, size = 160, normalized size = 0.40

$$\frac{x(fx)^m\sqrt{\left(a+bx^2\right)^2}\left(\frac{a^5d}{1+m}+\frac{a^4(5bd+ae)x^2}{3+m}+\frac{5a^3b(2bd+ae)x^4}{5+m}+\frac{10a^2b^2(bd+ae)x^6}{7+m}+\frac{5ab^3(bd+2ae)x^8}{9+m}+\frac{b^4(bd+5ae)x^{10}}{11+m}+\frac{b^5ex^{12}}{13+m}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(5/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*((a^5*d)/(1 + m) + (a^4*(5*b*d + a*e)*x^2)/(
3 + m) + (5*a^3*b*(2*b*d + a*e)*x^4)/(5 + m) + (10*a^2*b^2*(b*d + a*e)*x^6)
/(7 + m) + (5*a*b^3*(b*d + 2*a*e)*x^8)/(9 + m) + (b^4*(b*d + 5*a*e)*x^10)/(
11 + m) + (b^5*e*x^12)/(13 + m)))/(a + b*x^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. 2(323) = 646.
time = 0.04, size = 1099, normalized size = 2.75 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x*(b^5*e*m^6*x^12+36*b^5*e*m^5*x^12+5*a*b^4*e*m^6*x^10+b^5*d*m^6*x^10+505*b
^5*e*m^4*x^12+190*a*b^4*e*m^5*x^10+38*b^5*d*m^5*x^10+3480*b^5*e*m^3*x^12+10
*a^2*b^3*e*m^6*x^8+5*a*b^4*d*m^6*x^8+2775*a*b^4*e*m^4*x^10+555*b^5*d*m^4*x^
10+12139*b^5*e*m^2*x^12+400*a^2*b^3*e*m^5*x^8+200*a*b^4*d*m^5*x^8+19700*a*b
^4*e*m^3*x^10+3940*b^5*d*m^3*x^10+19524*b^5*e*m*x^12+10*a^3*b^2*e*m^6*x^6+1
0*a^2*b^3*d*m^6*x^6+6130*a^2*b^3*e*m^4*x^8+3065*a*b^4*d*m^4*x^8+70195*a*b^4
```

```
*e*m^2*x^10+14039*b^5*d*m^2*x^10+10395*b^5*e*x^12+420*a^3*b^2*e*m^5*x^6+420
*a^2*b^3*d*m^5*x^6+45280*a^2*b^3*e*m^3*x^8+22640*a*b^4*d*m^3*x^8+114510*a*b
^4*e*m*x^10+22902*b^5*d*m*x^10+5*a^4*b*e*m^6*x^4+10*a^3*b^2*d*m^6*x^4+6790*
a^3*b^2*e*m^4*x^6+6790*a^2*b^3*d*m^4*x^6+166270*a^2*b^3*e*m^2*x^8+83135*a*b
^4*d*m^2*x^8+61425*a*b^4*e*x^10+12285*b^5*d*x^10+220*a^4*b*e*m^5*x^4+440*a^
3*b^2*d*m^5*x^4+52920*a^3*b^2*e*m^3*x^6+52920*a^2*b^3*d*m^3*x^6+276880*a^2*
b^3*e*m*x^8+138440*a*b^4*d*m*x^8+a^5*e*m^6*x^2+5*a^4*b*d*m^6*x^2+3765*a^4*b
*e*m^4*x^4+7530*a^3*b^2*d*m^4*x^4+203350*a^3*b^2*e*m^2*x^6+203350*a^2*b^3*d
*m^2*x^6+150150*a^2*b^3*e*x^8+75075*a*b^4*d*x^8+46*a^5*e*m^5*x^2+230*a^4*b*
d*m^5*x^2+31400*a^4*b*e*m^3*x^4+62800*a^3*b^2*d*m^3*x^4+349860*a^3*b^2*e*m*
x^6+349860*a^2*b^3*d*m*x^6+a^5*d*m^6+835*a^5*e*m^4*x^2+4175*a^4*b*d*m^4*x^2
+129895*a^4*b*e*m^2*x^4+259790*a^3*b^2*d*m^2*x^4+193050*a^3*b^2*e*x^6+19305
0*a^2*b^3*d*x^6+48*a^5*d*m^5+7540*a^5*e*m^3*x^2+37700*a^4*b*d*m^3*x^2+23718
0*a^4*b*e*m*x^4+474360*a^3*b^2*d*m*x^4+925*a^5*d*m^4+34759*a^5*e*m^2*x^2+17
3795*a^4*b*d*m^2*x^2+135135*a^4*b*e*x^4+270270*a^3*b^2*d*x^4+9120*a^5*d*m^3
+73054*a^5*e*m*x^2+365270*a^4*b*d*m*x^2+48259*a^5*d*m^2+45045*a^5*e*x^2+225
225*a^4*b*d*x^2+129072*a^5*d*m+135135*a^5*d)*(f*x)^m*((b*x^2+a)^2)^(5/2)/(1
3+m)/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^5
```

Maxima [A]
time = 0.31, size = 494, normalized size = 1.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="max
ima")
```

```
[Out] ((m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)*b^5*f^m*x^11 + 5*(m^5 +
27*m^4 + 262*m^3 + 1122*m^2 + 2041*m + 1155)*a*b^4*f^m*x^9 + 10*(m^5 + 29*m
^4 + 302*m^3 + 1366*m^2 + 2577*m + 1485)*a^2*b^3*f^m*x^7 + 10*(m^5 + 31*m^4
 + 350*m^3 + 1730*m^2 + 3489*m + 2079)*a^3*b^2*f^m*x^5 + 5*(m^5 + 33*m^4 +
406*m^3 + 2262*m^2 + 5353*m + 3465)*a^4*b*f^m*x^3 + (m^5 + 35*m^4 + 470*m^3
 + 3010*m^2 + 9129*m + 10395)*a^5*f^m*x)*d*x^m/(m^6 + 36*m^5 + 505*m^4 + 34
80*m^3 + 12139*m^2 + 19524*m + 10395) + ((m^5 + 35*m^4 + 470*m^3 + 3010*m^2
 + 9129*m + 10395)*b^5*f^m*x^13 + 5*(m^5 + 37*m^4 + 518*m^3 + 3422*m^2 + 10
617*m + 12285)*a*b^4*f^m*x^11 + 10*(m^5 + 39*m^4 + 574*m^3 + 3954*m^2 + 126
73*m + 15015)*a^2*b^3*f^m*x^9 + 10*(m^5 + 41*m^4 + 638*m^3 + 4654*m^2 + 156
81*m + 19305)*a^3*b^2*f^m*x^7 + 5*(m^5 + 43*m^4 + 710*m^3 + 5570*m^2 + 2040
9*m + 27027)*a^4*b*f^m*x^5 + (m^5 + 45*m^4 + 790*m^3 + 6750*m^2 + 28009*m +
 45045)*a^5*f^m*x^3)*e^(m*log(x) + 1)/(m^6 + 48*m^5 + 925*m^4 + 9120*m^3 +
48259*m^2 + 129072*m + 135135)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(329) = 658$.

time = 0.37, size = 866, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="fri
cas")
```

```
[Out] ((b^5*d*m^6 + 38*b^5*d*m^5 + 555*b^5*d*m^4 + 3940*b^5*d*m^3 + 14039*b^5*d*m
^2 + 22902*b^5*d*m + 12285*b^5*d)*x^11 + 5*(a*b^4*d*m^6 + 40*a*b^4*d*m^5 +
613*a*b^4*d*m^4 + 4528*a*b^4*d*m^3 + 16627*a*b^4*d*m^2 + 27688*a*b^4*d*m +
15015*a*b^4*d)*x^9 + 10*(a^2*b^3*d*m^6 + 42*a^2*b^3*d*m^5 + 679*a^2*b^3*d*m
^4 + 5292*a^2*b^3*d*m^3 + 20335*a^2*b^3*d*m^2 + 34986*a^2*b^3*d*m + 19305*a
^2*b^3*d)*x^7 + 10*(a^3*b^2*d*m^6 + 44*a^3*b^2*d*m^5 + 753*a^3*b^2*d*m^4 +
6280*a^3*b^2*d*m^3 + 25979*a^3*b^2*d*m^2 + 47436*a^3*b^2*d*m + 27027*a^3*b^
2*d)*x^5 + 5*(a^4*b*d*m^6 + 46*a^4*b*d*m^5 + 835*a^4*b*d*m^4 + 7540*a^4*b*d
*m^3 + 34759*a^4*b*d*m^2 + 73054*a^4*b*d*m + 45045*a^4*b*d)*x^3 + (a^5*d*m^
6 + 48*a^5*d*m^5 + 925*a^5*d*m^4 + 9120*a^5*d*m^3 + 48259*a^5*d*m^2 + 12907
2*a^5*d*m + 135135*a^5*d)*x + ((b^5*m^6 + 36*b^5*m^5 + 505*b^5*m^4 + 3480*b
^5*m^3 + 12139*b^5*m^2 + 19524*b^5*m + 10395*b^5)*x^13 + 5*(a*b^4*m^6 + 38*
a*b^4*m^5 + 555*a*b^4*m^4 + 3940*a*b^4*m^3 + 14039*a*b^4*m^2 + 22902*a*b^4*
m + 12285*a*b^4)*x^11 + 10*(a^2*b^3*m^6 + 40*a^2*b^3*m^5 + 613*a^2*b^3*m^4
+ 4528*a^2*b^3*m^3 + 16627*a^2*b^3*m^2 + 27688*a^2*b^3*m + 15015*a^2*b^3)*x
^9 + 10*(a^3*b^2*m^6 + 42*a^3*b^2*m^5 + 679*a^3*b^2*m^4 + 5292*a^3*b^2*m^3
+ 20335*a^3*b^2*m^2 + 34986*a^3*b^2*m + 19305*a^3*b^2)*x^7 + 5*(a^4*b*m^6 +
 44*a^4*b*m^5 + 753*a^4*b*m^4 + 6280*a^4*b*m^3 + 25979*a^4*b*m^2 + 47436*a^
4*b*m + 27027*a^4*b)*x^5 + (a^5*m^6 + 46*a^5*m^5 + 835*a^5*m^4 + 7540*a^5*m
^3 + 34759*a^5*m^2 + 73054*a^5*m + 45045*a^5)*x^3)*e)*(f*x)^m/(m^7 + 49*m^6
 + 973*m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d + ex^2\right) \left(\left(a + bx^2\right)^2\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(5/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(5/2), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2213 vs. 2(329) = 658.
time = 3.65, size = 2213, normalized size = 5.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(5/2),x, algorithm="gia
c")

[Out] ((f*x)^m*b^5*m^6*x^13*e*sgn(b*x^2 + a) + 36*(f*x)^m*b^5*m^5*x^13*e*sgn(b*x^
2 + a) + (f*x)^m*b^5*d*m^6*x^11*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*m^6*x^11*e
*sgn(b*x^2 + a) + 505*(f*x)^m*b^5*m^4*x^13*e*sgn(b*x^2 + a) + 38*(f*x)^m*b^
5*d*m^5*x^11*sgn(b*x^2 + a) + 190*(f*x)^m*a*b^4*m^5*x^11*e*sgn(b*x^2 + a) +
 3480*(f*x)^m*b^5*m^3*x^13*e*sgn(b*x^2 + a) + 5*(f*x)^m*a*b^4*d*m^6*x^9*sgn
(b*x^2 + a) + 555*(f*x)^m*b^5*d*m^4*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^
3*m^6*x^9*e*sgn(b*x^2 + a) + 2775*(f*x)^m*a*b^4*m^4*x^11*e*sgn(b*x^2 + a) +
 12139*(f*x)^m*b^5*m^2*x^13*e*sgn(b*x^2 + a) + 200*(f*x)^m*a*b^4*d*m^5*x^9*
sgn(b*x^2 + a) + 3940*(f*x)^m*b^5*d*m^3*x^11*sgn(b*x^2 + a) + 400*(f*x)^m*a
^2*b^3*m^5*x^9*e*sgn(b*x^2 + a) + 19700*(f*x)^m*a*b^4*m^3*x^11*e*sgn(b*x^2
+ a) + 19524*(f*x)^m*b^5*m*x^13*e*sgn(b*x^2 + a) + 10*(f*x)^m*a^2*b^3*d*m^6
*x^7*sgn(b*x^2 + a) + 3065*(f*x)^m*a*b^4*d*m^4*x^9*sgn(b*x^2 + a) + 14039*(
f*x)^m*b^5*d*m^2*x^11*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*m^6*x^7*e*sgn(b*x
^2 + a) + 6130*(f*x)^m*a^2*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 70195*(f*x)^m*a*b
^4*m^2*x^11*e*sgn(b*x^2 + a) + 10395*(f*x)^m*b^5*x^13*e*sgn(b*x^2 + a) + 42
0*(f*x)^m*a^2*b^3*d*m^5*x^7*sgn(b*x^2 + a) + 22640*(f*x)^m*a*b^4*d*m^3*x^9*
sgn(b*x^2 + a) + 22902*(f*x)^m*b^5*d*m*x^11*sgn(b*x^2 + a) + 420*(f*x)^m*a^
3*b^2*m^5*x^7*e*sgn(b*x^2 + a) + 45280*(f*x)^m*a^2*b^3*m^3*x^9*e*sgn(b*x^2
+ a) + 114510*(f*x)^m*a*b^4*m*x^11*e*sgn(b*x^2 + a) + 10*(f*x)^m*a^3*b^2*d*
m^6*x^5*sgn(b*x^2 + a) + 6790*(f*x)^m*a^2*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 83
135*(f*x)^m*a*b^4*d*m^2*x^9*sgn(b*x^2 + a) + 12285*(f*x)^m*b^5*d*x^11*sgn(b
x^2 + a) + 5(f*x)^m*a^4*b*m^6*x^5*e*sgn(b*x^2 + a) + 6790*(f*x)^m*a^3*b^2
*m^4*x^7*e*sgn(b*x^2 + a) + 166270*(f*x)^m*a^2*b^3*m^2*x^9*e*sgn(b*x^2 + a)
 + 61425*(f*x)^m*a*b^4*x^11*e*sgn(b*x^2 + a) + 440*(f*x)^m*a^3*b^2*d*m^5*x^
5*sgn(b*x^2 + a) + 52920*(f*x)^m*a^2*b^3*d*m^3*x^7*sgn(b*x^2 + a) + 138440*
(f*x)^m*a*b^4*d*m*x^9*sgn(b*x^2 + a) + 220*(f*x)^m*a^4*b*m^5*x^5*e*sgn(b*x^
2 + a) + 52920*(f*x)^m*a^3*b^2*m^3*x^7*e*sgn(b*x^2 + a) + 276880*(f*x)^m*a^
2*b^3*m*x^9*e*sgn(b*x^2 + a) + 5*(f*x)^m*a^4*b*d*m^6*x^3*sgn(b*x^2 + a) + 7
530*(f*x)^m*a^3*b^2*d*m^4*x^5*sgn(b*x^2 + a) + 203350*(f*x)^m*a^2*b^3*d*m^2
*x^7*sgn(b*x^2 + a) + 75075*(f*x)^m*a*b^4*d*x^9*sgn(b*x^2 + a) + (f*x)^m*a^
5*m^6*x^3*e*sgn(b*x^2 + a) + 3765*(f*x)^m*a^4*b*m^4*x^5*e*sgn(b*x^2 + a) +
203350*(f*x)^m*a^3*b^2*m^2*x^7*e*sgn(b*x^2 + a) + 150150*(f*x)^m*a^2*b^3*x^
9*e*sgn(b*x^2 + a) + 230*(f*x)^m*a^4*b*d*m^5*x^3*sgn(b*x^2 + a) + 62800*(f*
x)^m*a^3*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 349860*(f*x)^m*a^2*b^3*d*m*x^7*sgn(
b*x^2 + a) + 46*(f*x)^m*a^5*m^5*x^3*e*sgn(b*x^2 + a) + 31400*(f*x)^m*a^4*b*
m^3*x^5*e*sgn(b*x^2 + a) + 349860*(f*x)^m*a^3*b^2*m*x^7*e*sgn(b*x^2 + a) +
(f*x)^m*a^5*d*m^6*x*sgn(b*x^2 + a) + 4175*(f*x)^m*a^4*b*d*m^4*x^3*sgn(b*x^2
 + a) + 259790*(f*x)^m*a^3*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 193050*(f*x)^m*a^
2*b^3*d*x^7*sgn(b*x^2 + a) + 835*(f*x)^m*a^5*m^4*x^3*e*sgn(b*x^2 + a) + 129
895*(f*x)^m*a^4*b*m^2*x^5*e*sgn(b*x^2 + a) + 193050*(f*x)^m*a^3*b^2*x^7*e*s

```
gn(b*x^2 + a) + 48*(f*x)^m*a^5*d*m^5*x*sgn(b*x^2 + a) + 37700*(f*x)^m*a^4*b
*d*m^3*x^3*sgn(b*x^2 + a) + 474360*(f*x)^m*a^3*b^2*d*m*x^5*sgn(b*x^2 + a) +
 7540*(f*x)^m*a^5*m^3*x^3*e*sgn(b*x^2 + a) + 237180*(f*x)^m*a^4*b*m*x^5*e*s
gn(b*x^2 + a) + 925*(f*x)^m*a^5*d*m^4*x*sgn(b*x^2 + a) + 173795*(f*x)^m*a^4
*b*d*m^2*x^3*sgn(b*x^2 + a) + 270270*(f*x)^m*a^3*b^2*d*x^5*sgn(b*x^2 + a) +
 34759*(f*x)^m*a^5*m^2*x^3*e*sgn(b*x^2 + a) + 135135*(f*x)^m*a^4*b*x^5*e*sg
n(b*x^2 + a) + 9120*(f*x)^m*a^5*d*m^3*x*sgn(b*x^2 + a) + 365270*(f*x)^m*a^4
*b*d*m*x^3*sgn(b*x^2 + a) + 73054*(f*x)^m*a^5*m*x^3*e*sgn(b*x^2 + a) + 4825
9*(f*x)^m*a^5*d*m^2*x*sgn(b*x^2 + a) + 225225*(f*x)^m*a^4*b*d*x^3*sgn(b*x^2
 + a) + 45045*(f*x)^m*a^5*x^3*e*sgn(b*x^2 + a) + 129072*(f*x)^m*a^5*d*m*x*s
gn(b*x^2 + a) + 135135*(f*x)^m*a^5*d*x*sgn(b*x^2 + a))/(m^7 + 49*m^6 + 973*
m^5 + 10045*m^4 + 57379*m^3 + 177331*m^2 + 264207*m + 135135)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(5/2), x)
```

3.88 $\int (fx)^m (d+ex^2) (a^2+2abx^2+b^2x^4)^{3/2} \, dx$

Optimal. Leaf size=276

$$\frac{a^3d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)(a+bx^2)}+\frac{a^2(3bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)(a+bx^2)}+\frac{3ab(bd+ae)(fx)^{5+m}\sqrt{a^2+2}}{f^5(5+m)(a+bx}$$

```
[Out] a^3*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+a^2*(a*e+3*b*d)*(f*
x)^(3+m)*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+3*a*b*(a*e+b*d)*(f*x)^(5+m
)*((b*x^2+a)^2)^(1/2)/f^5/(5+m)/(b*x^2+a)+b^2*(3*a*e+b*d)*(f*x)^(7+m)*((b*x
^2+a)^2)^(1/2)/f^7/(7+m)/(b*x^2+a)+b^3*e*(f*x)^(9+m)*((b*x^2+a)^2)^(1/2)/f^
9/(9+m)/(b*x^2+a)
```

Rubi [A]
time = 0.10, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1264, 459}

$$\frac{b^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+7}(3ae+bd)}{f^7(m+7)(a+bx^2)}+\frac{3ab\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}(ae+bd)}{f^5(m+5)(a+bx^2)}+\frac{a^2\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+3bd)}{f^3(m+3)(a+bx^2)}+\frac{b^3e\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+9}}{f^9(m+9)(a+bx^2)}+\frac{a^3d\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+1}}{f(m+1)(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (a^3*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2
)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3
*(3 + m)*(a + b*x^2)) + (3*a*b*(b*d + a*e)*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(f^5*(5 + m)*(a + b*x^2)) + (b^2*(b*d + 3*a*e)*(f*x)^(7 + m)
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^7*(7 + m)*(a + b*x^2)) + (b^3*e*(f*x)^
(9 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^9*(9 + m)*(a + b*x^2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (fx)^m (d+ex^2) \left(a^2+2abx^2+b^2x^4\right)^{3/2} dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (fx)^m (ab+b^2x^2)^3 (d+ex^2) dx}{b^2(ab+b^2x^2)}$$
$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(a^3b^3d(fx)^m + \frac{a^2b^3(3bd+ae)(fx)^{2+m}}{f^2} + \right.}{b^2(ab+b}$$
$$= \frac{a^3d(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)(a+bx^2)} + \frac{a^2(3bd+ae)(fx)^{3+m}\sqrt{}}{f^3(3+m)(}$$

Mathematica [A]
time = 0.14, size = 112, normalized size = 0.41

$$\frac{x(fx)^m \left((a+bx^2)^2\right)^{3/2} \left(\frac{a^3d}{1+m} + \frac{a^2(3bd+ae)x^2}{3+m} + \frac{3ab(bd+ae)x^4}{5+m} + \frac{b^2(bd+3ae)x^6}{7+m} + \frac{b^3ex^8}{9+m}\right)}{(a+bx^2)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*((a + b*x^2)^2)^(3/2)*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)
/(3 + m) + (3*a*b*(b*d + a*e)*x^4)/(5 + m) + (b^2*(b*d + 3*a*e)*x^6)/(7 + m
) + (b^3*e*x^8)/(9 + m)))/(a + b*x^2)^3
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(221) = 442.
time = 0.02, size = 495, normalized size = 1.79

method	result
gosper	$\frac{x(b^3em^4x^8+16b^3em^3x^8+3ab^2em^4x^6+b^3dm^4x^6+86b^3em^2x^8+54ab^2em^3x^6+18b^3dm^3x^6+176mx^8b^3e+3a^2bem^4x^4+3ab^2dm^4}{}$
risch	$\frac{\sqrt{(bx^2+a)^2}\,(b^3em^4x^8+16b^3em^3x^8+3ab^2em^4x^6+b^3dm^4x^6+86b^3em^2x^8+54ab^2em^3x^6+18b^3dm^3x^6+176mx^8b^3e+3a^2be}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x*(b^3*e*m^4*x^8+16*b^3*e*m^3*x^8+3*a*b^2*e*m^4*x^6+b^3*d*m^4*x^6+86*b^3*e*
m^2*x^8+54*a*b^2*e*m^3*x^6+18*b^3*d*m^3*x^6+176*b^3*e*m*x^8+3*a^2*b*e*m^4*x
^4+3*a*b^2*d*m^4*x^4+312*a*b^2*e*m^2*x^6+104*b^3*d*m^2*x^6+105*b^3*e*x^8+60
*a^2*b*e*m^3*x^4+60*a*b^2*d*m^3*x^4+666*a*b^2*e*m*x^6+222*b^3*d*m*x^6+a^3*e
```

```
*m^4*x^2+3*a^2*b*d*m^4*x^2+390*a^2*b*e*m^2*x^4+390*a*b^2*d*m^2*x^4+405*a*b^
2*e*x^6+135*b^3*d*x^6+22*a^3*e*m^3*x^2+66*a^2*b*d*m^3*x^2+900*a^2*b*e*m*x^4
+900*a*b^2*d*m*x^4+a^3*d*m^4+164*a^3*e*m^2*x^2+492*a^2*b*d*m^2*x^2+567*a^2*
b*e*x^4+567*a*b^2*d*x^4+24*a^3*d*m^3+458*a^3*e*m*x^2+1374*a^2*b*d*m*x^2+206
*a^3*d*m^2+315*a^3*e*x^2+945*a^2*b*d*x^2+744*a^3*d*m+945*a^3*d)*(f*x)^m*((b
*x^2+a)^2)^(3/2)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)/(b*x^2+a)^3
```

Maxima [A]
time = 0.29, size = 246, normalized size = 0.89

$$\frac{\left(\left(m^{3}+9\,m^{2}+23\,m+15\right)b^{3}f^{m}x^{7}+3\,\left(m^{3}+11\,m^{2}+31\,m+21\right)ab^{2}f^{m}x^{5}+3\,\left(m^{3}+13\,m^{2}+47\,m+35\right)a^{2}bf^{m}x^{3}+\left(m^{3}+15\,m^{2}+71\,m+105\right)a^{3}f^{m}x\right)dx^{m}}{m^{4}+16\,m^{3}+86\,m^{2}+176\,m+105}+\frac{\left(\left(m^{3}+15\,m^{2}+71\,m+105\right)b^{3}f^{m}x^{9}+3\,\left(m^{3}+17\,m^{2}+87\,m+135\right)ab^{2}f^{m}x^{7}+3\,\left(m^{3}+19\,m^{2}+111\,m+189\right)a^{2}bf^{m}x^{5}+\left(m^{3}+21\,m^{2}+143\,m+315\right)a^{3}f^{m}x^{3}\right)e^{\left(m\log\left(x\right)+1\right)}}{m^{4}+24\,m^{3}+206\,m^{2}+744\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="max
ima")
```

```
[Out] ((m^3 + 9*m^2 + 23*m + 15)*b^3*f^m*x^7 + 3*(m^3 + 11*m^2 + 31*m + 21)*a*b^2
*f^m*x^5 + 3*(m^3 + 13*m^2 + 47*m + 35)*a^2*b*f^m*x^3 + (m^3 + 15*m^2 + 71*
m + 105)*a^3*f^m*x)*d*x^m/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105) + ((m^3 + 1
5*m^2 + 71*m + 105)*b^3*f^m*x^9 + 3*(m^3 + 17*m^2 + 87*m + 135)*a*b^2*f^m*x
^7 + 3*(m^3 + 19*m^2 + 111*m + 189)*a^2*b*f^m*x^5 + (m^3 + 21*m^2 + 143*m +
 315)*a^3*f^m*x^3)*e^(m*log(x) + 1)/(m^4 + 24*m^3 + 206*m^2 + 744*m + 945)
```

Fricas [A]
time = 0.38, size = 390, normalized size = 1.41

$$\frac{\left(\left(b^{3}dm^{4}+18\,b^{3}dm^{3}+104\,b^{3}dm^{2}+222\,b^{3}dm+135\,b^{3}d\right)x^{7}+3\,\left(ab^{2}dm^{4}+20\,ab^{2}dm^{3}+130\,ab^{2}dm^{2}+300\,ab^{2}dm+189\,ab^{2}d\right)x^{5}+3\,\left(a^{2}bdm^{4}+22\,a^{2}bdm^{3}+164\,a^{2}bdm^{2}+458\,a^{2}bdm+315\,a^{2}bd\right)x^{3}+\left(a^{3}dm^{4}+24\,a^{3}dm^{3}+206\,a^{3}dm^{2}+744\,a^{3}dm+945\,a^{3}d\right)x+\left(\left(b^{3}m^{4}+16\,b^{3}m^{3}+86\,b^{3}m^{2}+176\,b^{3}m+105\,b^{3}\right)x^{9}+3\,\left(ab^{2}m^{4}+18\,ab^{2}m^{3}+104\,ab^{2}m^{2}+222\,ab^{2}m+135\,ab^{2}\right)x^{7}+3\,\left(a^{2}bm^{4}+20\,a^{2}bm^{3}+130\,a^{2}bm^{2}+300\,a^{2}bm+189\,a^{2}b\right)x^{5}+\left(a^{3}m^{4}+22\,a^{3}m^{3}+164\,a^{3}m^{2}+458\,a^{3}m+315\,a^{3}\right)x^{3}\right)e\right)\left(fx\right)^{m}}{m^{5}+25\,m^{4}+230\,m^{3}+950\,m^{2}+1689\,m+945}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fri
cas")
```

```
[Out] ((b^3*d*m^4 + 18*b^3*d*m^3 + 104*b^3*d*m^2 + 222*b^3*d*m + 135*b^3*d)*x^7 +
 3*(a*b^2*d*m^4 + 20*a*b^2*d*m^3 + 130*a*b^2*d*m^2 + 300*a*b^2*d*m + 189*a*
b^2*d)*x^5 + 3*(a^2*b*d*m^4 + 22*a^2*b*d*m^3 + 164*a^2*b*d*m^2 + 458*a^2*b*
d*m + 315*a^2*b*d)*x^3 + (a^3*d*m^4 + 24*a^3*d*m^3 + 206*a^3*d*m^2 + 744*a^
3*d*m + 945*a^3*d)*x + ((b^3*m^4 + 16*b^3*m^3 + 86*b^3*m^2 + 176*b^3*m + 10
5*b^3)*x^9 + 3*(a*b^2*m^4 + 18*a*b^2*m^3 + 104*a*b^2*m^2 + 222*a*b^2*m + 13
5*a*b^2)*x^7 + 3*(a^2*b*m^4 + 20*a^2*b*m^3 + 130*a^2*b*m^2 + 300*a^2*b*m +
189*a^2*b)*x^5 + (a^3*m^4 + 22*a^3*m^3 + 164*a^3*m^2 + 458*a^3*m + 315*a^3)
*x^3)*e)*(f*x)^m/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{m} \left(d + e x^{2}\right) \left(\left(a + b x^{2}\right)^{2}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*((a + b*x**2)**2)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(225) = 450$.
time = 5.42, size = 1013, normalized size = 3.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b^3*m^4*x^9*e*sgn(b*x^2 + a) + 16*(f*x)^m*b^3*m^3*x^9*e*sgn(b*x^2
+ a) + (f*x)^m*b^3*d*m^4*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*m^4*x^7*e*sgn
(b*x^2 + a) + 86*(f*x)^m*b^3*m^2*x^9*e*sgn(b*x^2 + a) + 18*(f*x)^m*b^3*d*m^
3*x^7*sgn(b*x^2 + a) + 54*(f*x)^m*a*b^2*m^3*x^7*e*sgn(b*x^2 + a) + 176*(f*x
)^m*b^3*m*x^9*e*sgn(b*x^2 + a) + 3*(f*x)^m*a*b^2*d*m^4*x^5*sgn(b*x^2 + a) +
 104*(f*x)^m*b^3*d*m^2*x^7*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*m^4*x^5*e*sgn(b
*x^2 + a) + 312*(f*x)^m*a*b^2*m^2*x^7*e*sgn(b*x^2 + a) + 105*(f*x)^m*b^3*x^
9*e*sgn(b*x^2 + a) + 60*(f*x)^m*a*b^2*d*m^3*x^5*sgn(b*x^2 + a) + 222*(f*x)^
m*b^3*d*m*x^7*sgn(b*x^2 + a) + 60*(f*x)^m*a^2*b*m^3*x^5*e*sgn(b*x^2 + a) +
666*(f*x)^m*a*b^2*m*x^7*e*sgn(b*x^2 + a) + 3*(f*x)^m*a^2*b*d*m^4*x^3*sgn(b*
x^2 + a) + 390*(f*x)^m*a*b^2*d*m^2*x^5*sgn(b*x^2 + a) + 135*(f*x)^m*b^3*d*x
^7*sgn(b*x^2 + a) + (f*x)^m*a^3*m^4*x^3*e*sgn(b*x^2 + a) + 390*(f*x)^m*a^2*
b*m^2*x^5*e*sgn(b*x^2 + a) + 405*(f*x)^m*a*b^2*x^7*e*sgn(b*x^2 + a) + 66*(f
*x)^m*a^2*b*d*m^3*x^3*sgn(b*x^2 + a) + 900*(f*x)^m*a*b^2*d*m*x^5*sgn(b*x^2
+ a) + 22*(f*x)^m*a^3*m^3*x^3*e*sgn(b*x^2 + a) + 900*(f*x)^m*a^2*b*m*x^5*e*
sgn(b*x^2 + a) + (f*x)^m*a^3*d*m^4*x*sgn(b*x^2 + a) + 492*(f*x)^m*a^2*b*d*m
^2*x^3*sgn(b*x^2 + a) + 567*(f*x)^m*a*b^2*d*x^5*sgn(b*x^2 + a) + 164*(f*x)^
m*a^3*m^2*x^3*e*sgn(b*x^2 + a) + 567*(f*x)^m*a^2*b*x^5*e*sgn(b*x^2 + a) + 2
4*(f*x)^m*a^3*d*m^3*x*sgn(b*x^2 + a) + 1374*(f*x)^m*a^2*b*d*m*x^3*sgn(b*x^2
 + a) + 458*(f*x)^m*a^3*m*x^3*e*sgn(b*x^2 + a) + 206*(f*x)^m*a^3*d*m^2*x*sg
n(b*x^2 + a) + 945*(f*x)^m*a^2*b*d*x^3*sgn(b*x^2 + a) + 315*(f*x)^m*a^3*x^3
*e*sgn(b*x^2 + a) + 744*(f*x)^m*a^3*d*m*x*sgn(b*x^2 + a) + 945*(f*x)^m*a^3*
d*x*sgn(b*x^2 + a))/(m^5 + 25*m^4 + 230*m^3 + 950*m^2 + 1689*m + 945)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.89 $\int (fx)^m \left(d+ex^2\right) \sqrt{a^2+2abx^2+b^2x^4}\, dx$

Optimal. Leaf size=153

$$\frac{ad(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)}+\frac{(bd+ae)(fx)^{3+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^3(3+m)\left(a+bx^2\right)}+\frac{be(fx)^{5+m}\sqrt{a^2+2abx^2+b^2x^4}}{f^5(5+m)\left(a+bx^2\right)}$$

```
[Out] a*d*(f*x)^(1+m)*((b*x^2+a)^2)^(1/2)/f/(1+m)/(b*x^2+a)+(a*e+b*d)*(f*x)^(3+m)
*((b*x^2+a)^2)^(1/2)/f^3/(3+m)/(b*x^2+a)+b*e*(f*x)^(5+m)*((b*x^2+a)^2)^(1/2
)/f^5/(5+m)/(b*x^2+a)
```

Rubi [A]
time = 0.05, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {1264, 459}

$$\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+3}(ae+bd)}{f^3(m+3)\left(a+bx^2\right)}+\frac{ad\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+1}}{f(m+1)\left(a+bx^2\right)}+\frac{be\sqrt{a^2+2abx^2+b^2x^4}\,(fx)^{m+5}}{f^5(m+5)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f*(1 + m)*(a + b*x^2))
 + ((b*d + a*e)*(f*x)^(3 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^3*(3 + m)
*(a + b*x^2)) + (b*e*(f*x)^(5 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(f^5*(5
 + m)*(a + b*x^2))
```

Rule 459

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^n)^p*(c + d*x^
n)^q, x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] && IGt
Q[p, 0] && IGtQ[q, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a^2+2abx^2+b^2x^4}\, dx = \frac{\sqrt{a^2+2abx^2+b^2x^4} \int (fx)^m \left(ab+b^2x^2\right)\left(d+ex^2\right) dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4} \int \left(abd(fx)^m + \frac{b(bd+ae)(fx)^{2+m}}{f^2} + \frac{b^2e(fx)^{4+r}}{f^4}\right.}{ab+b^2x^2}$$

$$= \frac{ad(fx)^{1+m}\sqrt{a^2+2abx^2+b^2x^4}}{f(1+m)\left(a+bx^2\right)} + \frac{(bd+ae)(fx)^{3+m}\sqrt{a^2+2a}}{f^3(3+m)\left(a+bx\right.}$$

Mathematica [A]
time = 0.06, size = 86, normalized size = 0.56

$$\frac{x(fx)^m\sqrt{\left(a+bx^2\right)^2}\left(a(5+m)\left(d(3+m)+e(1+m)x^2\right)+b(1+m)x^2\left(d(5+m)+e(3+m)x^2\right)\right)}{(1+m)(3+m)(5+m)\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[(a + b*x^2)^2]*(a*(5 + m)*(d*(3 + m) + e*(1 + m)*x^2) + b*(
1 + m)*x^2*(d*(5 + m) + e*(3 + m)*x^2)))/((1 + m)*(3 + m)*(5 + m)*(a + b*x^
2))
```

Maple [A]
time = 0.02, size = 131, normalized size = 0.86

method	result
gosper	$\frac{x\left(be\,m^2x^4+4bem\,x^4+ae\,m^2x^2+bd\,m^2x^2+3be\,x^4+6aem\,x^2+6bdm\,x^2+ad\,m^2+5ae\,x^2+5bd\,x^2+8adm+15ad\right)(fx)^m\sqrt{\left(b\,x^2+a\right)^2}}{(5+m)(3+m)(1+m)(b\,x^2+a)}$
risch	$\frac{x\left(be\,m^2x^4+4bem\,x^4+ae\,m^2x^2+bd\,m^2x^2+3be\,x^4+6aem\,x^2+6bdm\,x^2+ad\,m^2+5ae\,x^2+5bd\,x^2+8adm+15ad\right)(fx)^m\sqrt{\left(b\,x^2+a\right)^2}}{(5+m)(3+m)(1+m)(b\,x^2+a)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x,method=_RETURNVERBOSE
)
```

```
[Out] x*(b*e*m^2*x^4+4*b*e*m*x^4+a*e*m^2*x^2+b*d*m^2*x^2+3*b*e*x^4+6*a*e*m*x^2+6*
b*d*m*x^2+a*d*m^2+5*a*e*x^2+5*b*d*x^2+8*a*d*m+15*a*d)*(f*x)^m*((b*x^2+a)^2)
^(1/2)/(5+m)/(3+m)/(1+m)/(b*x^2+a)
```

Maxima [A]
time = 0.29, size = 78, normalized size = 0.51

$$\frac{(bf^m(m+1)x^3+af^m(m+3)x)dx^m}{m^2+4\,m+3}+\frac{(bf^m(m+3)x^5+af^m(m+5)x^3)e^{(m\log(x)+1)}}{m^2+8\,m+15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] (b*f^m*(m + 1)*x^3 + a*f^m*(m + 3)*x)*d*x^m/(m^2 + 4*m + 3) + (b*f^m*(m + 3
)*x^5 + a*f^m*(m + 5)*x^3)*e^(m*log(x) + 1)/(m^2 + 8*m + 15)
```

Fricas [A]
time = 0.39, size = 98, normalized size = 0.64

$$\frac{((bdm^2 + 6\,bdm + 5\,bd)x^3 + (adm^2 + 8\,adm + 15\,ad)x + ((bm^2 + 4\,bm + 3\,b)x^5 + (am^2 + 6\,am + 5\,a)x^3)e)(fx)^m}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] ((b*d*m^2 + 6*b*d*m + 5*b*d)*x^3 + (a*d*m^2 + 8*a*d*m + 15*a*d)*x + ((b*m^2
 + 4*b*m + 3*b)*x^5 + (a*m^2 + 6*a*m + 5*a)*x^3)*e)*(f*x)^m/(m^3 + 9*m^2 +
23*m + 15)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d + ex^2\right) \sqrt{\left(a + bx^2\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt((a + b*x**2)**2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(122) = 244.
time = 2.79, size = 269, normalized size = 1.76

$$\frac{(fx)^m\,bm^2x^5e\mathrm{sgn}(bx^2+a) + 4\,(fx)^m\,bmx^5e\mathrm{sgn}(bx^2+a) + (fx)^m\,bdm^2x^3\mathrm{sgn}(bx^2+a) + (fx)^m\,am^2x^3e\mathrm{sgn}(bx^2+a) + 3\,(fx)^m\,bx^5e\mathrm{sgn}(bx^2+a) + 6\,(fx)^m\,bdmx^3\mathrm{sgn}(bx^2+a) + 6\,(fx)^m\,amx^3e\mathrm{sgn}(bx^2+a) + (fx)^m\,adm^2x\mathrm{sgn}(bx^2+a) + 5\,(fx)^m\,bdx^3\mathrm{sgn}(bx^2+a) + 5\,(fx)^m\,ax^3e\mathrm{sgn}(bx^2+a) + 8\,(fx)^m\,admx\mathrm{sgn}(bx^2+a) + 15\,(fx)^m\,adx\mathrm{sgn}(bx^2+a)}{m^3 + 9\,m^2 + 23\,m + 15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] ((f*x)^m*b*m^2*x^5*e*sgn(b*x^2 + a) + 4*(f*x)^m*b*m*x^5*e*sgn(b*x^2 + a) +
(f*x)^m*b*d*m^2*x^3*sgn(b*x^2 + a) + (f*x)^m*a*m^2*x^3*e*sgn(b*x^2 + a) + 3
*(f*x)^m*b*x^5*e*sgn(b*x^2 + a) + 6*(f*x)^m*b*d*m*x^3*sgn(b*x^2 + a) + 6*(f
*x)^m*a*m*x^3*e*sgn(b*x^2 + a) + (f*x)^m*a*d*m^2*x*sgn(b*x^2 + a) + 5*(f*x)
^m*b*d*x^3*sgn(b*x^2 + a) + 5*(f*x)^m*a*x^3*e*sgn(b*x^2 + a) + 8*(f*x)^m*a*
```

```
d*m*x*sgn(b*x^2 + a) + 15*(f*x)^m*a*d*x*sgn(b*x^2 + a))/(m^3 + 9*m^2 + 23*m
 + 15)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

3.90 $\int \frac{(fx)^m(d+ex^2)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx$

Optimal. Leaf size=134

$$\frac{e(fx)^{1+m}(a+bx^2)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)(fx)^{1+m}(a+bx^2)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{abf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] e*(f*x)^(1+m)*(b*x^2+a)/b/f/(1+m)/((b*x^2+a)^2)^(1/2)+(-a*e+b*d)*(f*x)^(1+m
)*(b*x^2+a)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-b*x^2/a)/a/b/f/(1+m)/((b*
x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1264, 470, 371}

$$\frac{(a+bx^2)(fx)^{m+1}(bd-ae)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{abf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{e(a+bx^2)(fx)^{m+1}}{bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (e*(f*x)^(1 + m)*(a + b*x^2))/(b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
 + ((b*d - a*e)*(f*x)^(1 + m)*(a + b*x^2)*Hypergeometric2F1[1, (1 + m)/2, (
3 + m)/2, -((b*x^2)/a)])/(a*b*f*(1 + m)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
```

```
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^m\left(d+ex^2\right)}{\sqrt{a^2+2abx^2+b^2x^4}}\,dx &= \frac{\left(ab+b^2x^2\right)\int \frac{(fx)^m\left(d+ex^2\right)}{ab+b^2x^2}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{e(fx)^{1+m}\left(a+bx^2\right)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}-\frac{\left(\left(-b^2d(1+m)+abe(1+m)\right)\left(ab+b^2x^2\right)\right)}{b^2(1+m)\sqrt{a^2+2abx^2+b^2x^4}}\\ &= \frac{e(fx)^{1+m}\left(a+bx^2\right)}{bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd-ae)(fx)^{1+m}\left(a+bx^2\right)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2}\right.}{abf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 78, normalized size = 0.58

$$-\frac{x(fx)^m\left(a+bx^2\right)\left(-ae+(-bd+ae)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{ab(1+m)\sqrt{\left(a+bx^2\right)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] -((x*(f*x)^m*(a + b*x^2)*(-(a*e) + (-(b*d) + a*e)*Hypergeometric2F1[1, (1 +
 m)/2, (3 + m)/2, -((b*x^2)/a)]))/(a*b*(1 + m)*Sqrt[(a + b*x^2)^2]))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\left(e\,x^2+d\right)}{\sqrt{b^2x^4+2ab\,x^2+a^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="max
ima")
```

```
[Out] integrate((x^2*e + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="fri
cas")
```

```
[Out] integral((x^2*e + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d+ex^2)}{\sqrt{(a+bx^2)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(1/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt((a + b*x**2)**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(1/2),x, algorithm="gia
c")
```

```
[Out] integrate((x^2*e + d)*(f*x)^m/sqrt(b^2*x^4 + 2*a*b*x^2 + a^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f\,x)^m\,(e\,x^2+d)}{\sqrt{a^2+2\,a\,b\,x^2+b^2\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2),x)
```

```
[Out] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(1/2), x)
```

3.91 $\int \frac{(fx)^m(d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx$

Optimal. Leaf size=154

$$\frac{(bd-ae)(fx)^{1+m}}{4abf(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(bd(3-m)+ae(1+m))(fx)^{1+m}(a+bx^2)\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)}{4a^3bf(1+m)\sqrt{a^2+2abx^2+b^2x^4}}$$

```
[Out] 1/4*(-a*e+b*d)*(f*x)^(1+m)/a/b/f/(b*x^2+a)/((b*x^2+a)^2)^(1/2)+1/4*(b*d*(3-
m)+a*e*(1+m))*(f*x)^(1+m)*(b*x^2+a)*hypergeom([2, 1/2+1/2*m],[3/2+1/2*m],-b
*x^2/a)/a^3/b/f/(1+m)/((b*x^2+a)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {1264, 468, 371}

$$\frac{(fx)^{m+1}(bd-ae)}{4abf(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}}+\frac{(a+bx^2)(fx)^{m+1}(ae(m+1)+bd(3-m))\,{}_2F_1\left(2,\frac{m+1}{2};\frac{m+3}{2};-\frac{bx^2}{a}\right)}{4a^3bf(m+1)\sqrt{a^2+2abx^2+b^2x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] ((b*d - a*e)*(f*x)^(1 + m))/(4*a*b*f*(a + b*x^2)*Sqrt[a^2 + 2*a*b*x^2 + b^2
*x^4]) + ((b*d*(3 - m) + a*e*(1 + m))*(f*x)^(1 + m)*(a + b*x^2)*Hypergeomet
ric2F1[2, (1 + m)/2, (3 + m)/2, -((b*x^2)/a)])/(4*a^3*b*f*(1 + m)*Sqrt[a^2
+ 2*a*b*x^2 + b^2*x^4])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(-(b*c - a*d))*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a
*b*e*n*(p + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*
(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e,
m, n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && Ne
Q[p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1,
 m, (-n)*(p + 1)]))
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^2)}{(a^2+2abx^2+b^2x^4)^{3/2}}\,dx = \frac{(b^2(ab+b^2x^2)) \int \frac{(fx)^m (d+ex^2)}{(ab+b^2x^2)^3}\,dx}{\sqrt{a^2+2abx^2+b^2x^4}}$$

$$= \frac{(bd-ae)(fx)^{1+m}}{4abf\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{((bd(3-m)+ae(1+m))\,(ab+b^2x}{4a\sqrt{a^2+2abx^2+b^2x}}$$

$$= \frac{(bd-ae)(fx)^{1+m}}{4abf\,(a+bx^2)\sqrt{a^2+2abx^2+b^2x^4}} + \frac{(bd(3-m)+ae(1+m))(fx)^{1+m}\,(a}{4a^3bf(1+m)\sqrt{a^2 -}}$$

Mathematica [A]
time = 0.12, size = 101, normalized size = 0.66

$$\frac{x(fx)^m (a+bx^2) \left(ae\,{}_2F_1\left(2,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)+(bd-ae)\,{}_2F_1\left(3,\frac{1+m}{2};\frac{3+m}{2};-\frac{bx^2}{a}\right)\right)}{a^3b(1+m)\sqrt{(a+bx^2)^2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a^2 + 2*a*b*x^2 + b^2*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(a + b*x^2)*(a*e*Hypergeometric2F1[2, (1 + m)/2, (3 + m)/2, -((b
*x^2)/a)] + (b*d - a*e)*Hypergeometric2F1[3, (1 + m)/2, (3 + m)/2, -((b*x^2
)/a)]))/(a^3*b*(1 + m)*Sqrt[(a + b*x^2)^2])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (e\,x^2+d)}{(b^2x^4+2ab\,x^2+a^2)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="max
ima")
[Out] integrate((x^2*e + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="fri
cas")
[Out] integral(sqrt(b^2*x^4 + 2*a*b*x^2 + a^2)*(x^2*e + d)*(f*x)^m/(b^4*x^8 + 4*a
*b^3*x^6 + 6*a^2*b^2*x^4 + 4*a^3*b*x^2 + a^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d + ex^2)}{\left((a + bx^2)^2\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(b**2*x**4+2*a*b*x**2+a**2)**(3/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)/((a + b*x**2)**2)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(b^2*x^4+2*a*b*x^2+a^2)^(3/2),x, algorithm="gia
c")
[Out] integrate((x^2*e + d)*(f*x)^m/(b^2*x^4 + 2*a*b*x^2 + a^2)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f\,x)^m\,(e\,x^2+d)}{\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a^2 + b^2*x^4 + 2*a*b*x^2)^(3/2), x)
```

3.92 $\int x(a+bx^2)(a^2+2abx^2+b^2x^4)^p\,dx$

Optimal. Leaf size=34

$$\frac{(a^2+2abx^2+b^2x^4)^{1+p}}{4b(1+p)}$$

```
[Out] 1/4*(b^2*x^4+2*a*b*x^2+a^2)^(1+p)/b/(1+p)
```

Rubi [A]
time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.069$, Rules used = {1261, 643}

$$\frac{(a^2+2abx^2+b^2x^4)^{p+1}}{4b(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2 + 2*a*b*x^2 + b^2*x^4)^(1 + p)/(4*b*(1 + p))
```

Rule 643

```
Int[((d_) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbol
] :> Simp[d*((a + b*x + c*x^2)^(p + 1)/(b*(p + 1))), x] /; FreeQ[{a, b, c,
d, e, p}, x] && EqQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx&=\frac{1}{2}\text{Subst}\left(\int(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)\\&=\frac{(a^2+2abx^2+b^2x^4)^{1+p}}{4b(1+p)}\end{aligned}$$

Mathematica [A]
time = 0.01, size = 25, normalized size = 0.74

$$\frac{\left((a+bx^2)^2\right)^{1+p}}{4b(1+p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] ((a + b*x^2)^2)^(1 + p)/(4*b*(1 + p))
```

Maple [A]
time = 0.04, size = 40, normalized size = 1.18

method	result	size
gosper	$\frac{(b\,x^2+a)^2(b^2x^4+2ab\,x^2+a^2)^p}{4b(1+p)}$	40
risch	$\frac{(b^2x^4+2ab\,x^2+a^2)\left((b\,x^2+a)^2\right)^p}{4b(1+p)}$	40
norman	$\frac{a\,x^2 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2+2p}+\frac{a^2 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4b(1+p)}+\frac{b\,x^4 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4+4p}$	103

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)^2/b/(1+p)*(b^2*x^4+2*a*b*x^2+a^2)^p
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(32) = 64$.
time = 0.30, size = 86, normalized size = 2.53

$$\frac{(bx^2+a)(bx^2+a)^{2p}a}{2\,b(2\,p+1)}+\frac{(b^2(2\,p+1)x^4+2\,abpx^2-a^2)(bx^2+a)^{2p}}{4\,(2\,p^2+3\,p+1)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*(b*x^2 + a)*(b*x^2 + a)^(2*p)*a/(b*(2*p + 1)) + 1/4*(b^2*(2*p + 1)*x^4
+ 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)/((2*p^2 + 3*p + 1)*b)
```

Fricas [A]
time = 0.38, size = 47, normalized size = 1.38

$$\frac{(b^2x^4+2\,abx^2+a^2)(b^2x^4+2\,abx^2+a^2)^p}{4\,(bp+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(b*p + b)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(27) = 54$.
time = 4.12, size = 155, normalized size = 4.56

$$\begin{cases} \frac{x^2}{2a} & \text{for } b = 0 \wedge p = -1 \\ \frac{ax^2(a^2)^p}{2} & \text{for } b = 0 \\ \frac{\log\left(x - \sqrt{-\frac{a}{b}}\right)}{2b} + \frac{\log\left(x + \sqrt{-\frac{a}{b}}\right)}{2b} & \text{for } p = -1 \\ \frac{a^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b} + \frac{2abx^2(a^2+2abx^2+b^2x^4)^p}{4bp+4b} + \frac{b^2x^4(a^2+2abx^2+b^2x^4)^p}{4bp+4b} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((x**2/(2*a), Eq(b, 0) & Eq(p, -1)), (a*x**2*(a**2)**p/2, Eq(b, 0)
), (log(x - sqrt(-a/b))/(2*b) + log(x + sqrt(-a/b))/(2*b), Eq(p, -1)), (a**
2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + 2*a*b*x**2*(a**2 + 2*a
*b*x**2 + b**2*x**4)**p/(4*b*p + 4*b) + b**2*x**4*(a**2 + 2*a*b*x**2 + b**2
*x**4)**p/(4*b*p + 4*b), True))
```

Giac [A]
time = 4.60, size = 32, normalized size = 0.94

$$\frac{(b^2x^4 + 2\,abx^2 + a^2)^{p+1}}{4\,b(p+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(b^2*x^4 + 2*a*b*x^2 + a^2)^(p + 1)/(b*(p + 1))
```

Mupad [B]
time = 0.14, size = 59, normalized size = 1.74

$$(a^2 + 2\,a\,b\,x^2 + b^2\,x^4)^p\left(\frac{a^2}{4\,b\,(p+1)} + \frac{a\,x^2}{2\,(p+1)} + \frac{b\,x^4}{4\,(p+1)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^2/(4*b*(p + 1)) + (a*x^2)/(2*(p + 1)) + (b
*x^4)/(4*(p + 1)))
```

3.93 $\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=86

$$-\frac{a(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}+\frac{(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(3+2p)}$$

```
[Out] -1/4*a*(b*x^2+a)^2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^2/(1+p)+1/2*(b*x^2+a)^3*(b^2
*x^4+2*a*b*x^2+a^2)^p/b^2/(3+2*p)
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.129$, Rules used = {1263, 784, 21, 45}

$$\frac{(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{2b^2(2p+3)}-\frac{a(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] -1/4*(a*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^2*(1 + p)) + ((a +
b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(2*b^2*(3 + 2*p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 784

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1263

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\int x^3(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx=\frac{1}{2}\text{Subst}\left(\int x(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)$$
$$=\frac{1}{2}\left(\left(b(a+bx^2)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x(a+bx)\left(ab\right.\right.$$
$$=\frac{\left(\left(b(a+bx^2)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x\left(ab+b^2x\right)^{1+2p}\right.}{2b}$$
$$=\frac{\left(\left(b(a+bx^2)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(-\frac{a(ab+b^2x)^{1+2p}}{b}+\right.\right.}{2b}$$
$$=-\frac{a(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^2(1+p)}+\frac{(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right.}{2b^2(3+2p)}$$

Mathematica [A]
time = 0.05, size = 45, normalized size = 0.52

$$\frac{\left((a+bx^2)^2\right)^{1+p}\left(-a+2b(1+p)x^2\right)}{4b^2(1+p)(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
[Out] (((a + b*x^2)^2)^(1 + p)*(-a + 2*b*(1 + p)*x^2))/(4*b^2*(1 + p)*(3 + 2*p))
```

Maple [A]
time = 0.03, size = 62, normalized size = 0.72

method	result	s
gosper	$-\frac{(b^2x^4+2ab\,x^2+a^2)^p(-2x^2pb-2b\,x^2+a)(b\,x^2+a)^2}{4b^2(2p^2+5p+3)}$	6
risch	$-\frac{(-2b^3p\,x^6-2b^3x^6-4a\,b^2p\,x^4-3a\,b^2x^4-2a^2bp\,x^2+a^3)\left((b\,x^2+a)^2\right)^p}{4(1+p)(3+2p)b^2}$	7
norman	$-\frac{a^3e^{p\ln(b^2x^4+2ab\,x^2+a^2)}}{4b^2(2p^2+5p+3)}+\frac{b\,x^6e^{p\ln(b^2x^4+2ab\,x^2+a^2)}}{6+4p}+\frac{a(4p+3)x^4e^{p\ln(b^2x^4+2ab\,x^2+a^2)}}{8p^2+20p+12}+\frac{p\,a^2x^2e^{p\ln(b^2x^4+2ab\,x^2+a^2)}}{2b(2p^2+5p+3)}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*(b^2*x^4+2*a*b*x^2+a^2)^p*(-2*b*p*x^2-2*b*x^2+a)*(b*x^2+a)^2/b^2/(2*p^
2+5*p+3)
```

Maxima [A]
time = 0.28, size = 135, normalized size = 1.57

$$\frac{(b^2(2\,p+1)x^4+2\,abpx^2-a^2)(bx^2+a)^{2\,p}a}{4\,(2\,p^2+3\,p+1)b^2}+\frac{((2\,p^2+3\,p+1)b^3x^6+(2\,p^2+p)ab^2x^4-2\,a^2bpx^2+a^3)(bx^2+a)^{2\,p}}{2\,(4\,p^3+12\,p^2+11\,p+3)b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/4*(b^2*(2*p + 1)*x^4 + 2*a*b*p*x^2 - a^2)*(b*x^2 + a)^(2*p)*a/((2*p^2 + 3
*p + 1)*b^2) + 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a
^2*b*p*x^2 + a^3)*(b*x^2 + a)^(2*p)/((4*p^3 + 12*p^2 + 11*p + 3)*b^2)
```

Fricas [A]
time = 0.37, size = 92, normalized size = 1.07

$$\frac{(2\,(b^3p+b^3)x^6+2\,a^2bpx^2+(4\,ab^2p+3\,ab^2)x^4-a^3)(b^2x^4+2\,abx^2+a^2)^p}{4\,(2\,b^2p^2+5\,b^2p+3\,b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(b^3*p + b^3)*x^6 + 2*a^2*b*p*x^2 + (4*a*b^2*p + 3*a*b^2)*x^4 - a^3)
*(b^2*x^4 + 2*a*b*x^2 + a^2)^p/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{ax^4(a^2)^p}{4} & \text{for}\: b=0 \\ \int \frac{x^3(a+bx^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx & \text{for}\: p=-\frac{3}{2} \\ -\frac{a\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^2}-\frac{a\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^2}+\frac{x^2}{2b} & \text{for}\: p=-1 \\ -\frac{a^3(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2a^2bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{4ab^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{3ab^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2b^3px^6(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2}+\frac{2b^3x^6(a^2+2abx^2+b^2x^4)^p}{8b^2p^2+20b^2p+12b^2} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((a*x**4*(a**2)**p/4, Eq(b, 0)), (Integral(x**3*(a + b*x**2)/((a +
 b*x**2)**2)**(3/2), x), Eq(p, -3/2)), (-a*log(x - sqrt(-a/b))/(2*b**2) - a
```

```
*log(x + sqrt(-a/b))/(2*b**2) + x**2/(2*b), Eq(p, -1)), (-a**3*(a**2 + 2*a*
b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*a**2*b*p*x**
2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2) +
4*a*b**2*p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2 + 20*b**2*p
 + 12*b**2) + 3*a*b**2*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**2*p**2
 + 20*b**2*p + 12*b**2) + 2*b**3*p*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/
(8*b**2*p**2 + 20*b**2*p + 12*b**2) + 2*b**3*x**6*(a**2 + 2*a*b*x**2 + b**2
*x**4)**p/(8*b**2*p**2 + 20*b**2*p + 12*b**2), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. 2(82) = 164.
time = 3.44, size = 196, normalized size = 2.28

$$\frac{2\,(b^2x^4+2\,abx^2+a^2)^pb^3px^6+2\,(b^2x^4+2\,abx^2+a^2)^pb^3x^6+4\,(b^2x^4+2\,abx^2+a^2)^pab^2px^4+3\,(b^2x^4+2\,abx^2+a^2)^pab^2x^4+2\,(b^2x^4+2\,abx^2+a^2)^pa^2bpx^2-(b^2x^4+2\,abx^2+a^2)^pa^3}{4\,(2\,b^2p^2+5\,b^2p+3\,b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^3*p*x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a
^2)^p*b^3*x^6 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^2*p*x^4 + 3*(b^2*x^4 +
2*a*b*x^2 + a^2)^p*a*b^2*x^4 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b*p*x^2
- (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3)/(2*b^2*p^2 + 5*b^2*p + 3*b^2)
```

Mupad [B]
time = 0.17, size = 108, normalized size = 1.26

$$\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)^p\,\left(\frac{b\,x^6\,\left(p+1\right)}{2\,\left(2\,p^2+5\,p+3\right)}-\frac{a^3}{4\,b^2\,\left(2\,p^2+5\,p+3\right)}+\frac{a\,x^4\,\left(4\,p+3\right)}{4\,\left(2\,p^2+5\,p+3\right)}+\frac{a^2\,p\,x^2}{2\,b\,\left(2\,p^2+5\,p+3\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*((b*x^6*(p + 1))/(2*(5*p + 2*p^2 + 3)) - a^3/
(4*b^2*(5*p + 2*p^2 + 3)) + (a*x^4*(4*p + 3))/(4*(5*p + 2*p^2 + 3)) + (a^2*
p*x^2)/(2*b*(5*p + 2*p^2 + 3)))
```

3.94 $\int x^5(a+bx^2)\left(a^2+2abx^2+b^2x^4\right)^p\,dx$

Optimal. Leaf size=128

$$\frac{a^2(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(1+p)}-\frac{a(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{b^3(3+2p)}+\frac{(a+bx^2)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(2+p)}$$

```
[Out] 1/4*a^2*(b*x^2+a)^2*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(1+p)-a*(b*x^2+a)^3*(b^2*
x^4+2*a*b*x^2+a^2)^p/b^3/(3+2*p)+1/4*(b*x^2+a)^4*(b^2*x^4+2*a*b*x^2+a^2)^p/
b^3/(2+p)
```

Rubi [A]
time = 0.08, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.129$, Rules used = {1263, 784, 21, 45}

$$\frac{(a+bx^2)^4\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+2)}-\frac{a(a+bx^2)^3\left(a^2+2abx^2+b^2x^4\right)^p}{b^3(2p+3)}+\frac{a^2(a+bx^2)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (a^2*(a + b*x^2)^2*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(1 + p)) - (a*(a +
 b*x^2)^3*(a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(b^3*(3 + 2*p)) + ((a + b*x^2)^4*(
a^2 + 2*a*b*x^2 + b^2*x^4)^p)/(4*b^3*(2 + p))
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 784

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_), x_Symbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPa
rt[p]*(b/2 + c*x)^(2*FracPart[p])), Int[(d + e*x)^m*(f + g*x)*(b/2 + c*x)^(
2*p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && EqQ[b^2 - 4*a*c, 0]
```

Rule 1263

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && EqQ[b
^2 - 4*a*c, 0] &&  !IntegerQ[p] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int x^5\left(a+bx^2\right)\left(a^2+2abx^2+b^2x^4\right)^p\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(a+bx)\left(a^2+2abx+b^2x^2\right)^p\,dx,x,x^2\right)\\ &= \frac{1}{2}\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2(a+bx)\left(a\right.\right.\\ &= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int x^2\left(ab+b^2x\right)^{1+2p}\right.}{2b}\\ &= \frac{\left(\left(b\left(a+bx^2\right)\right)^{-2p}\left(a^2+2abx^2+b^2x^4\right)^p\right)\text{Subst}\left(\int\left(\frac{a^2\left(ab+b^2x\right)^{1+2p}}{b^2}-\right.\right.}{2b}\\ &= \frac{a^2\left(a+bx^2\right)^2\left(a^2+2abx^2+b^2x^4\right)^p}{4b^3(1+p)}-\frac{a\left(a+bx^2\right)^3\left(a^2+2abx^2+b^2\right.}{b^3(3+2p)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 68, normalized size = 0.53

$$\frac{\left(\left(a+bx^2\right)^2\right)^{1+p}\left(a^2-2ab(1+p)x^2+b^2\left(3+5p+2p^2\right)x^4\right)}{4b^3(1+p)(2+p)(3+2p)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(a + b*x^2)*(a^2 + 2*a*b*x^2 + b^2*x^4)^p,x]
```

```
[Out] (((a + b*x^2)^2)^(1 + p)*(a^2 - 2*a*b*(1 + p)*x^2 + b^2*(3 + 5*p + 2*p^2)*x
^4))/(4*b^3*(1 + p)*(2 + p)*(3 + 2*p))
```

Maple [A]
time = 0.04, size = 99, normalized size = 0.77

method	result
gosper	$\frac{\left(b\,x^2+a\right)^2\left(2b^2p^2x^4+5b^2p\,x^4+3b^2x^4-2abp\,x^2-2ab\,x^2+a^2\right)\left(b^2x^4+2ab\,x^2+a^2\right)^p}{4b^3\left(2p^3+9p^2+13p+6\right)}$
risch	$\frac{\left(2b^4p^2x^8+5b^4p\,x^8+4a\,b^3p^2x^6+3b^4x^8+8a\,b^3p\,x^6+2a^2b^2p^2x^4+4a\,b^3x^6+a^2b^2p\,x^4-2a^3p\,x^2b+a^4\right)\left(\left(b\,x^2+a\right)^2\right)^p}{4(3+2p)(2+p)(1+p)b^3}$

norman	$\frac{a(1+p)x^6 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2p^2+7p+6}+\frac{a^4 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{4b^3(2p^3+9p^2+13p+6)}+\frac{b\,x^8 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{8+4p}-\frac{p\,a^3x^2 e^{p\ln\left(b^2x^4+2ab\,x^2+a^2\right)}}{2b^2(2p^3+9p^2+13p+6)}+\frac{p(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(b*x^2+a)^2*(2*b^2*p^2*x^4+5*b^2*p*x^4+3*b^2*x^4-2*a*b*p*x^2-2*a*b*x^2+
a^2)*(b^2*x^4+2*a*b*x^2+a^2)^p/b^3/(2*p^3+9*p^2+13*p+6)
```

Maxima [A]
time = 0.29, size = 196, normalized size = 1.53

$$\frac{((2p^2+3p+1)b^3x^6+(2p^2+p)ab^2x^4-2a^2bpx^2+a^3)(bx^2+a)^{2p}a}{2(4p^3+12p^2+11p+3)b^3}+\frac{((4p^3+12p^2+11p+3)b^4x^8+2(2p^3+3p^2+p)ab^3x^6-3(2p^2+p)a^2b^2x^4+6a^3bpx^2-3a^4)(bx^2+a)^{2p}}{4(4p^4+20p^3+35p^2+25p+6)b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="maxima")
```

```
[Out] 1/2*((2*p^2 + 3*p + 1)*b^3*x^6 + (2*p^2 + p)*a*b^2*x^4 - 2*a^2*b*p*x^2 + a^
3)*(b*x^2 + a)^(2*p)*a/((4*p^3 + 12*p^2 + 11*p + 3)*b^3) + 1/4*((4*p^3 + 12
*p^2 + 11*p + 3)*b^4*x^8 + 2*(2*p^3 + 3*p^2 + p)*a*b^3*x^6 - 3*(2*p^2 + p)*
a^2*b^2*x^4 + 6*a^3*b*p*x^2 - 3*a^4)*(b*x^2 + a)^(2*p)/((4*p^4 + 20*p^3 + 3
5*p^2 + 25*p + 6)*b^3)
```

Fricas [A]
time = 0.36, size = 140, normalized size = 1.09

$$\frac{((2b^4p^2+5b^4p+3b^4)x^8-2a^3bpx^2+4(ab^3p^2+2ab^3p+ab^3)x^6+(2a^2b^2p^2+a^2b^2p)x^4+a^4)(b^2x^4+2abx^2+a^2)^p}{4(2b^3p^3+9b^3p^2+13b^3p+6b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="fricas")
```

```
[Out] 1/4*((2*b^4*p^2 + 5*b^4*p + 3*b^4)*x^8 - 2*a^3*b*p*x^2 + 4*(a*b^3*p^2 + 2*a
*b^3*p + a*b^3)*x^6 + (2*a^2*b^2*p^2 + a^2*b^2*p)*x^4 + a^4)*(b^2*x^4 + 2*a
*b*x^2 + a^2)^p/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\begin{cases} \frac{ax^6(a^2)^p}{6} & \text{for}\ b=0 \\ \frac{2a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{3a^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{4abx^2}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2b^2x^4\log\left(x-\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4}+\frac{2b^2x^4\log\left(x+\sqrt{-\frac{a}{b}}\right)}{4a^2b^3+8ab^4x^2+4b^5x^4} & \text{for}\ p=-2 \\ \int\frac{x^5(a+bx^2)}{\left((a+bx^2)^2\right)^{\frac{3}{2}}}\,dx & \text{for}\ p=-\frac{3}{2} \\ \frac{a^2\log\left(x-\sqrt{-\frac{a}{b}}\right)}{2b^3}+\frac{a^2\log\left(x+\sqrt{-\frac{a}{b}}\right)}{2b^3}-\frac{ax^2}{2b^2}+\frac{x^4}{4b} & \text{for}\ p=-1 \\ \frac{a^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}-\frac{2a^3bpx^2(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{2a^2b^2p^2x^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{a^2b^2px^4(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{4ab^3p^2x^6(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{8ab^3px^6(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{4ab^3x^6(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{2b^4p^2x^8(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{5b^4px^8(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3}+\frac{3b^4x^8(a^2+2abx^2+b^2x^4)^p}{8b^3p^3+36b^3p^2+52b^3p+24b^3} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(b*x**2+a)*(b**2*x**4+2*a*b*x**2+a**2)**p,x)
```

```
[Out] Piecewise((a*x**6*(a**2)**p/6, Eq(b, 0)), (2*a**2*log(x - sqrt(-a/b))/(4*a*
*2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 2*a**2*log(x + sqrt(-a/b))/(4*a**2
*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4) + 3*a**2/(4*a**2*b**3 + 8*a*b**4*x**2
+ 4*b**5*x**4) + 4*a*b*x**2*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**
2 + 4*b**5*x**4) + 4*a*b*x**2*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x
**2 + 4*b**5*x**4) + 4*a*b*x**2/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x**4)
 + 2*b**2*x**4*log(x - sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5*x*
*4) + 2*b**2*x**4*log(x + sqrt(-a/b))/(4*a**2*b**3 + 8*a*b**4*x**2 + 4*b**5
*x**4), Eq(p, -2)), (Integral(x**5*(a + b*x**2)/((a + b*x**2)**2)**(3/2), x
), Eq(p, -3/2)), (a**2*log(x - sqrt(-a/b))/(2*b**3) + a**2*log(x + sqrt(-a/
b))/(2*b**3) - a*x**2/(2*b**2) + x**4/(4*b), Eq(p, -1)), (a**4*(a**2 + 2*a*
b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) -
 2*a**3*b*p*x**2*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*
p**2 + 52*b**3*p + 24*b**3) + 2*a**2*b**2*p**2*x**4*(a**2 + 2*a*b*x**2 + b*
*2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + a**2*b**2*
p*x**4*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*
b**3*p + 24*b**3) + 4*a*b**3*p**2*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(
8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 8*a*b**3*p*x**6*(a**2 +
 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b*
*3) + 4*a*b**3*x**6*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b*
*3*p**2 + 52*b**3*p + 24*b**3) + 2*b**4*p**2*x**8*(a**2 + 2*a*b*x**2 + b**2
*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*p + 24*b**3) + 5*b**4*p*x**
8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3 + 36*b**3*p**2 + 52*b**3*
p + 24*b**3) + 3*b**4*x**8*(a**2 + 2*a*b*x**2 + b**2*x**4)**p/(8*b**3*p**3
+ 36*b**3*p**2 + 52*b**3*p + 24*b**3), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 331 vs. 2(124) = 248.

time = 3.99, size = 331, normalized size = 2.59

$$\frac{2\,(b^2x^4+2\,abx^2+a^2)^pb^4p^2x^8+5\,(b^2x^4+2\,abx^2+a^2)^pb^4px^8+4\,(b^2x^4+2\,abx^2+a^2)^pab^3p^2x^6+3\,(b^2x^4+2\,abx^2+a^2)^pb^4x^8+8\,(b^2x^4+2\,abx^2+a^2)^pab^3px^6+2\,(b^2x^4+2\,abx^2+a^2)^pa^2b^2p^2x^4+4\,(b^2x^4+2\,abx^2+a^2)^pab^3x^6+(b^2x^4+2\,abx^2+a^2)^pa^2b^2px^4-2\,(b^2x^4+2\,abx^2+a^2)^pa^3bpx^2+(b^2x^4+2\,abx^2+a^2)^pa^4}{4\,(2\,b^3p^3+9\,b^3p^2+13\,b^3p+6\,b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(b*x^2+a)*(b^2*x^4+2*a*b*x^2+a^2)^p,x, algorithm="giac")
```

```
[Out] 1/4*(2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*b^4*p^2*x^8 + 5*(b^2*x^4 + 2*a*b*x^2 +
 a^2)^p*b^4*p*x^8 + 4*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p^2*x^6 + 3*(b^2*
x^4 + 2*a*b*x^2 + a^2)^p*b^4*x^8 + 8*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a*b^3*p*
x^6 + 2*(b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p^2*x^4 + 4*(b^2*x^4 + 2*a*b*
x^2 + a^2)^p*a*b^3*x^6 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^2*b^2*p*x^4 - 2*(b
^2*x^4 + 2*a*b*x^2 + a^2)^p*a^3*b*p*x^2 + (b^2*x^4 + 2*a*b*x^2 + a^2)^p*a^4
)/(2*b^3*p^3 + 9*b^3*p^2 + 13*b^3*p + 6*b^3)
```

Mupad [B]

time = 0.20, size = 169, normalized size = 1.32

$${\left(a^2+2\,a\,b\,x^2+b^2\,x^4\right)}^p\,\left(\frac{a^4}{4\,b^3\,\left(2\,p^3+9\,p^2+13\,p+6\right)}+\frac{a\,x^6\,{\left(p+1\right)}^2}{2\,p^3+9\,p^2+13\,p+6}+\frac{b\,x^8\,\left(2\,p^2+5\,p+3\right)}{4\,\left(2\,p^3+9\,p^2+13\,p+6\right)}-\frac{a^3\,p\,x^2}{2\,b^2\,\left(2\,p^3+9\,p^2+13\,p+6\right)}+\frac{a^2\,p\,x^4\,\left(2\,p+1\right)}{4\,b\,\left(2\,p^3+9\,p^2+13\,p+6\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(a + b*x^2)*(a^2 + b^2*x^4 + 2*a*b*x^2)^p,x)
```

```
[Out] (a^2 + b^2*x^4 + 2*a*b*x^2)^p*(a^4/(4*b^3*(13*p + 9*p^2 + 2*p^3 + 6)) + (a*
x^6*(p + 1)^2)/(13*p + 9*p^2 + 2*p^3 + 6) + (b*x^8*(5*p + 2*p^2 + 3))/(4*(1
3*p + 9*p^2 + 2*p^3 + 6)) - (a^3*p*x^2)/(2*b^2*(13*p + 9*p^2 + 2*p^3 + 6))
+ (a^2*p*x^4*(2*p + 1))/(4*b*(13*p + 9*p^2 + 2*p^3 + 6)))
```

3.95 $\int x^3(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8+\frac{1}{10}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{10}+\frac{1}{12}\left(b^3B+3\right.$$

```
[Out] 1/4*a^3*A*x^4+1/6*a^2*(3*A*b+B*a)*x^6+3/8*a*(a*b*B+A*(a*c+b^2))*x^8+1/10*(3
*a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^10+1/12*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*
b^3)*x^12+3/14*c*(A*b*c+B*a*c+B*b^2)*x^14+1/16*c^2*(A*c+3*B*b)*x^16+1/18*B*
c^3*x^18
```

Rubi [A]
time = 0.25, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {1265, 779}

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2x^6(aB+3Ab)+\frac{3}{14}cx^{14}\left(aBc+Abc+b^2B\right)+\frac{3}{8}ax^8\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{12}x^{12}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{10}x^{10}\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{16}c^2x^{16}(Ac+3bB)+\frac{1}{18}Bc^3x^{18}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

Rule 779

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx=\frac{1}{2}\text{Subst}\left(\int x(A+Bx)\left(a+bx+cx^2\right)^3\,dx,x,x^2\right)$$
$$=\frac{1}{2}\text{Subst}\left(\int\left(a^3Ax+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^3\right.\right.$$
$$=\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8+\frac{1}{10}(3a$$

Mathematica [A]
time = 0.03, size = 166, normalized size = 1.00

$$\frac{1}{4}a^3Ax^4+\frac{1}{6}a^2(3Ab+aB)x^6+\frac{3}{8}a\left(abB+A\left(b^2+ac\right)\right)x^8+\frac{1}{10}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^{10}+\frac{1}{12}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{12}+\frac{3}{14}c\left(b^2B+Abc+aBc\right)x^{14}+\frac{1}{16}c^2(3bB+Ac)x^{16}+\frac{1}{18}Bc^3x^{18}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^4)/4 + (a^2*(3*A*b + a*B)*x^6)/6 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
8)/8 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^10)/10 + ((b^3*B + 3*A*b^
2*c + 6*a*b*B*c + 3*a*A*c^2)*x^12)/12 + (3*c*(b^2*B + A*b*c + a*B*c)*x^14)/
14 + (c^2*(3*b*B + A*c)*x^16)/16 + (B*c^3*x^18)/18
```

Maple [A]
time = 0.08, size = 226, normalized size = 1.36

method	result
norman	$\frac{a^3Ax^4}{4}+\left(\frac{1}{2}a^2bA+\frac{1}{6}a^3B\right)x^6+\left(\frac{3}{8}a^2cA+\frac{3}{8}Aab^2+\frac{3}{8}Ba^2b\right)x^8+\left(\frac{3}{5}Aabc+\frac{1}{10}Ab^3+\frac{3}{10}a^2cB+\frac{3}{10}\right.$
gosper	$\frac{1}{4}a^3Ax^4+\frac{1}{2}x^6a^2bA+\frac{1}{6}x^6a^3B+\frac{3}{8}x^8a^2cA+\frac{3}{8}x^8Aab^2+\frac{3}{8}x^8Ba^2b+\frac{3}{5}x^{10}Aabc+\frac{1}{10}x^{10}Ab^3+\frac{3}{10}$
risch	$\frac{1}{4}a^3Ax^4+\frac{1}{2}x^6a^2bA+\frac{1}{6}x^6a^3B+\frac{3}{8}x^8a^2cA+\frac{3}{8}x^8Aab^2+\frac{3}{8}x^8Ba^2b+\frac{3}{5}x^{10}Aabc+\frac{1}{10}x^{10}Ab^3+\frac{3}{10}$
default	$\frac{Bc^3x^{18}}{18}+\frac{\left(c^3A+3Bbc^2\right)x^{16}}{16}+\frac{\left(3bc^2A+B\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)\right)x^{14}}{14}+\frac{\left(\left(c^2a+2b^2c+c\left(2ac+b^2\right)\right)A+B\left(4abc+b\left(2ac+b^2\right)\right)\right)x}{12}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/18*B*c^3*x^18+1/16*(A*c^3+3*B*b*c^2)*x^16+1/14*(3*b*c^2*A+B*(c^2*a+2*b^2*
c+c*(2*a*c+b^2)))*x^14+1/12*((c^2*a+2*b^2*c+c*(2*a*c+b^2))*A+B*(4*a*b*c+b*(
2*a*c+b^2)))*x^12+1/10*((4*a*b*c+b*(2*a*c+b^2))*A+B*(a*(2*a*c+b^2)+2*a*b^2+
a^2*c))*x^10+1/8*((a*(2*a*c+b^2)+2*a*b^2+a^2*c)*A+3*B*a^2*b)*x^8+1/6*(3*A*a
^2*b+B*a^3)*x^6+1/4*a^3*A*x^4
```

Maxima [A]
time = 0.27, size = 166, normalized size = 1.00

$$\frac{1}{18}Bc^3x^{18}+\frac{1}{16}\left(3Bbc^2+Ac^3\right)x^{16}+\frac{3}{14}\left(Bb^2c+(Ba+Ab)c^2\right)x^{14}+\frac{1}{12}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{12}+\frac{1}{10}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^{10}+\frac{3}{8}\left(Ba^2b+Aab^2+Aa^2c\right)x^8+\frac{1}{4}Aa^3x^4+\frac{1}{6}\left(Ba^3+3Aa^2b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A*
b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/10
*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

Fricas [A]
time = 0.35, size = 166, normalized size = 1.00

$$\frac{1}{18}Bc^3x^{18}+\frac{1}{16}\left(3Bbc^2+Ac^3\right)x^{16}+\frac{3}{14}\left(Bb^2c+(Ba+Ab)c^2\right)x^{14}+\frac{1}{12}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{12}+\frac{1}{10}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^{10}+\frac{3}{8}\left(Ba^2b+Aab^2+Aa^2c\right)x^8+\frac{1}{4}Aa^3x^4+\frac{1}{6}\left(Ba^3+3Aa^2b\right)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/18*B*c^3*x^18 + 1/16*(3*B*b*c^2 + A*c^3)*x^16 + 3/14*(B*b^2*c + (B*a + A*
b)*c^2)*x^14 + 1/12*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^12 + 1/10
*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^10 + 3/8*(B*a^2*b + A*a*b^2
+ A*a^2*c)*x^8 + 1/4*A*a^3*x^4 + 1/6*(B*a^3 + 3*A*a^2*b)*x^6
```

Sympy [A]
time = 0.02, size = 202, normalized size = 1.22

$$\frac{Aa^3x^4}{4}+\frac{Bc^3x^{18}}{18}+x^{16}\left(\frac{Ac^3}{16}+\frac{3Bbc^2}{16}\right)+x^{14}\cdot\left(\frac{3Abc^2}{14}+\frac{3Bac^2}{14}+\frac{3Bb^2c}{14}\right)+x^{12}\left(\frac{Aac^2}{4}+\frac{Ab^2c}{4}+\frac{Babc}{2}+\frac{Bb^3}{12}\right)+x^{10}\cdot\left(\frac{3Aabc}{5}+\frac{Ab^3}{10}+\frac{3Ba^2c}{10}+\frac{3Bab^2}{10}\right)+x^8\cdot\left(\frac{3Aa^2c}{8}+\frac{3Aab^2}{8}+\frac{3Ba^2b}{8}\right)+x^6\left(\frac{Aa^2b}{2}+\frac{Ba^3}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**4/4 + B*c**3*x**18/18 + x**16*(A*c**3/16 + 3*B*b*c**2/16) + x**14
*(3*A*b*c**2/14 + 3*B*a*c**2/14 + 3*B*b**2*c/14) + x**12*(A*a*c**2/4 + A*b*
*2*c/4 + B*a*b*c/2 + B*b**3/12) + x**10*(3*A*a*b*c/5 + A*b**3/10 + 3*B*a**2
*c/10 + 3*B*a*b**2/10) + x**8*(3*A*a**2*c/8 + 3*A*a*b**2/8 + 3*B*a**2*b/8)
+ x**6*(A*a**2*b/2 + B*a**3/6)
```

Giac [A]
time = 4.39, size = 193, normalized size = 1.16

$$\frac{1}{18}Bc^3x^{18}+\frac{3}{16}Bbc^2x^{16}+\frac{1}{16}Ac^3x^{16}+\frac{3}{14}Bb^2cx^{14}+\frac{3}{14}Bac^2x^{14}+\frac{3}{14}Abc^2x^{14}+\frac{1}{12}Bb^3x^{12}+\frac{1}{2}Babcx^{12}+\frac{1}{4}Ab^2cx^{12}+\frac{1}{4}Aac^2x^{12}+\frac{3}{10}Bab^2x^{10}+\frac{1}{10}Ab^3x^{10}+\frac{3}{10}Ba^2cx^{10}+\frac{3}{5}Aabcx^{10}+\frac{3}{8}Ba^2bx^8+\frac{3}{8}Aab^2x^8+\frac{3}{8}Aa^2cx^8+\frac{1}{6}Ba^3x^6+\frac{1}{2}Aa^2bx^6+\frac{1}{4}Aa^3x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/18*B*c^3*x^18 + 3/16*B*b*c^2*x^16 + 1/16*A*c^3*x^16 + 3/14*B*b^2*c*x^14 +
 3/14*B*a*c^2*x^14 + 3/14*A*b*c^2*x^14 + 1/12*B*b^3*x^12 + 1/2*B*a*b*c*x^12
 + 1/4*A*b^2*c*x^12 + 1/4*A*a*c^2*x^12 + 3/10*B*a*b^2*x^10 + 1/10*A*b^3*x^1
```

```
0 + 3/10*B*a^2*c*x^10 + 3/5*A*a*b*c*x^10 + 3/8*B*a^2*b*x^8 + 3/8*A*a*b^2*x^
8 + 3/8*A*a^2*c*x^8 + 1/6*B*a^3*x^6 + 1/2*A*a^2*b*x^6 + 1/4*A*a^3*x^4
```

Mupad [B]
time = 0.08, size = 169, normalized size = 1.02

$$x^{10}\left(\frac{3\,B\,c\,a^2}{10}+\frac{3\,B\,a\,b^2}{10}+\frac{3\,A\,c\,a\,b}{5}+\frac{A\,b^3}{10}\right)+x^{12}\left(\frac{B\,b^3}{12}+\frac{A\,b^2\,c}{4}+\frac{B\,a\,b\,c}{2}+\frac{A\,a\,c^2}{4}\right)+x^6\left(\frac{B\,a^3}{6}+\frac{A\,b\,a^2}{2}\right)+x^{16}\left(\frac{A\,c^3}{16}+\frac{3\,B\,b\,c^2}{16}\right)+x^8\left(\frac{3\,B\,a^2\,b}{8}+\frac{3\,A\,c\,a^2}{8}+\frac{3\,A\,a\,b^2}{8}\right)+x^{14}\left(\frac{3\,B\,b^2\,c}{14}+\frac{3\,A\,b\,c^2}{14}+\frac{3\,B\,a\,c^2}{14}\right)+\frac{A\,a^3\,x^4}{4}+\frac{B\,c^3\,x^{18}}{18}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^10*((A*b^3)/10 + (3*B*a*b^2)/10 + (3*B*a^2*c)/10 + (3*A*a*b*c)/5) + x^12*
((B*b^3)/12 + (A*a*c^2)/4 + (A*b^2*c)/4 + (B*a*b*c)/2) + x^6*((B*a^3)/6 + (
A*a^2*b)/2) + x^16*((A*c^3)/16 + (3*B*b*c^2)/16) + x^8*((3*A*a*b^2)/8 + (3*
A*a^2*c)/8 + (3*B*a^2*b)/8) + x^14*((3*A*b*c^2)/14 + (3*B*a*c^2)/14 + (3*B*
b^2*c)/14) + (A*a^3*x^4)/4 + (B*c^3*x^18)/18
```

3.96 $\int x^2(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a\left(abB+A\left(b^2+ac\right)\right)x^7+\frac{1}{9}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^9+\frac{1}{11}\left(b^3B+3Ab\right.$$

```
[Out] 1/3*a^3*A*x^3+1/5*a^2*(3*A*b+B*a)*x^5+3/7*a*(a*b*B+A*(a*c+b^2))*x^7+1/9*(3*
a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^9+1/11*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^
3)*x^11+3/13*c*(A*b*c+B*a*c+B*b^2)*x^13+1/15*c^2*(A*c+3*B*b)*x^15+1/17*B*c^
3*x^17
```

Rubi [A]
time = 0.10, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = $\{1275\}$

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2x^5(aB+3Ab)+\frac{3}{13}cx^{13}\left(aBc+Abc+b^2B\right)+\frac{3}{7}ax^7\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{11}x^{11}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{9}x^9\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{15}c^2x^{15}(Ac+3bB)+\frac{1}{17}Bc^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3Ax^2+a^2(3Ab+aB)x^4+3a\left(abB+A\left(b^2+ac\right)\right)x^6+\left(3aB\left(b^2\right.\right.\right.\\ &= \frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a\left(abB+A\left(b^2+ac\right)\right)x^7+\frac{1}{9}\left(3aB\left(\right.\right.\end{aligned}$$

Mathematica [A]
time = 0.04, size = 166, normalized size = 1.00

$$\frac{1}{3}a^3Ax^3+\frac{1}{5}a^2(3Ab+aB)x^5+\frac{3}{7}a\left(abB+A\left(b^2+ac\right)\right)x^7+\frac{1}{9}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^9+\frac{1}{11}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^{11}+\frac{3}{13}c\left(b^2B+Abc+aBc\right)x^{13}+\frac{1}{15}c^2(3bB+Ac)x^{15}+\frac{1}{17}Bc^3x^{17}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^3)/3 + (a^2*(3*A*b + a*B)*x^5)/5 + (3*a*(a*b*B + A*(b^2 + a*c))*x^
7)/7 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^9)/9 + ((b^3*B + 3*A*b^2*
c + 6*a*b*B*c + 3*a*A*c^2)*x^11)/11 + (3*c*(b^2*B + A*b*c + a*B*c)*x^13)/13
 + (c^2*(3*b*B + A*c)*x^15)/15 + (B*c^3*x^17)/17
```

Maple [A]
time = 0.08, size = 226, normalized size = 1.36

method	result
norman	$\frac{a^3 A x^3}{3}+\left(\frac{3}{5}a^2bA+\frac{1}{5}a^3B\right)x^5+\left(\frac{3}{7}a^2cA+\frac{3}{7}Aa\,b^2+\frac{3}{7}B\,a^2b\right)x^7+\left(\frac{2}{3}Aabc+\frac{1}{9}A\,b^3+\frac{1}{3}a^2cB+\frac{1}{3}B$
gosper	$\frac{1}{3}a^3A\,x^3+\frac{3}{5}x^5a^2bA+\frac{1}{5}x^5a^3B+\frac{3}{7}x^7a^2cA+\frac{3}{7}x^7Aa\,b^2+\frac{3}{7}x^7B\,a^2b+\frac{2}{3}x^9Aabc+\frac{1}{9}x^9A\,b^3+\frac{1}{3}x^9a$
risch	$\frac{1}{3}a^3A\,x^3+\frac{3}{5}x^5a^2bA+\frac{1}{5}x^5a^3B+\frac{3}{7}x^7a^2cA+\frac{3}{7}x^7Aa\,b^2+\frac{3}{7}x^7B\,a^2b+\frac{2}{3}x^9Aabc+\frac{1}{9}x^9A\,b^3+\frac{1}{3}x^9a$
default	$\frac{B\,c^3x^{17}}{17}+\frac{(c^3A+3Bb\,c^2)x^{15}}{15}+\frac{(3b\,c^2A+B(c^2a+2b^2c+c(2ac+b^2)))x^{13}}{13}+\frac{((c^2a+2b^2c+c(2ac+b^2))A+B(4abc+b(2ac+b^2)))x}{11}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/17*B*c^3*x^17+1/15*(A*c^3+3*B*b*c^2)*x^15+1/13*(3*b*c^2*A+B*(c^2*a+2*b^2*
c+c*(2*a*c+b^2)))*x^13+1/11*((c^2*a+2*b^2*c+c*(2*a*c+b^2))*A+B*(4*a*b*c+b*(
2*a*c+b^2)))*x^11+1/9*((4*a*b*c+b*(2*a*c+b^2))*A+B*(a*(2*a*c+b^2)+2*a*b^2+a
^2*c))*x^9+1/7*((a*(2*a*c+b^2)+2*a*b^2+a^2*c)*A+3*B*a^2*b)*x^7+1/5*(3*A*a^2
*b+B*a^3)*x^5+1/3*a^3*A*x^3
```

Maxima [A]
time = 0.28, size = 166, normalized size = 1.00

$$\frac{1}{17}Bc^3x^{17}+\frac{1}{15}\left(3\,Bbc^2+Ac^3\right)x^{15}+\frac{3}{13}\left(Bb^2c+(Ba+Ab)c^2\right)x^{13}+\frac{1}{11}\left(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c\right)x^{11}+\frac{1}{9}\left(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c\right)x^9+\frac{3}{7}\left(Ba^2b+Aab^2+Aa^2c\right)x^7+\frac{1}{3}Aa^3x^3+\frac{1}{5}\left(Ba^3+3\,Aa^2b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c + (B*a + A*
b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^11 + 1/9*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + 3/7*(B*a^2*b + A*a*b^2 +
A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

Fricas [A]
time = 0.34, size = 166, normalized size = 1.00

$$\frac{1}{17}Bc^3x^{17}+\frac{1}{15}\left(3\,Bbc^2+Ac^3\right)x^{15}+\frac{3}{13}\left(Bb^2c+(Ba+Ab)c^2\right)x^{13}+\frac{1}{11}\left(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c\right)x^{11}+\frac{1}{9}\left(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c\right)x^9+\frac{3}{7}\left(Ba^2b+Aab^2+Aa^2c\right)x^7+\frac{1}{3}Aa^3x^3+\frac{1}{5}\left(Ba^3+3\,Aa^2b\right)x^5$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/17*B*c^3*x^17 + 1/15*(3*B*b*c^2 + A*c^3)*x^15 + 3/13*(B*b^2*c + (B*a + A*
b)*c^2)*x^13 + 1/11*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^11 + 1/9*
(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^9 + 3/7*(B*a^2*b + A*a*b^2 +
A*a^2*c)*x^7 + 1/3*A*a^3*x^3 + 1/5*(B*a^3 + 3*A*a^2*b)*x^5
```

Sympy [A]
time = 0.02, size = 204, normalized size = 1.23

$$\frac{Aa^3x^3}{3}+\frac{Bc^3x^{17}}{17}+x^{15}\left(\frac{Ac^3}{15}+\frac{Bbc^2}{5}\right)+x^{13}\cdot\left(\frac{3Abc^2}{13}+\frac{3Bac^2}{13}+\frac{3Bb^2c}{13}\right)+x^{11}\cdot\left(\frac{3Aac^2}{11}+\frac{3Ab^2c}{11}+\frac{6Babc}{11}+\frac{Bb^3}{11}\right)+x^9\cdot\left(\frac{2Aabc}{3}+\frac{Ab^3}{9}+\frac{Ba^2c}{3}+\frac{Bab^2}{3}\right)+x^7\cdot\left(\frac{3Aa^2c}{7}+\frac{3Aab^2}{7}+\frac{3Ba^2b}{7}\right)+x^5\cdot\left(\frac{3Aa^2b}{5}+\frac{Ba^3}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**3/3 + B*c**3*x**17/17 + x**15*(A*c**3/15 + B*b*c**2/5) + x**13*(3
*A*b*c**2/13 + 3*B*a*c**2/13 + 3*B*b**2*c/13) + x**11*(3*A*a*c**2/11 + 3*A*
b**2*c/11 + 6*B*a*b*c/11 + B*b**3/11) + x**9*(2*A*a*b*c/3 + A*b**3/9 + B*a*
*2*c/3 + B*a*b**2/3) + x**7*(3*A*a**2*c/7 + 3*A*a*b**2/7 + 3*B*a**2*b/7) +
x**5*(3*A*a**2*b/5 + B*a**3/5)
```

Giac [A]
time = 3.67, size = 193, normalized size = 1.16

$$\frac{1}{17}Bc^3x^{17}+\frac{1}{5}Bbc^2x^{15}+\frac{1}{15}Ac^3x^{15}+\frac{3}{13}Bb^2cx^{13}+\frac{3}{13}Bac^2x^{13}+\frac{3}{13}Abc^2x^{13}+\frac{1}{11}Bb^3x^{11}+\frac{6}{11}Babcx^{11}+\frac{3}{11}Ab^2cx^{11}+\frac{3}{11}Aac^2x^{11}+\frac{1}{3}Bab^2x^9+\frac{1}{9}Ab^3x^9+\frac{1}{3}Ba^2cx^9+\frac{2}{3}Aabcx^9+\frac{3}{7}Ba^2bx^7+\frac{3}{7}Aab^2x^7+\frac{3}{7}Aa^2cx^7+\frac{1}{5}Ba^3x^5+\frac{3}{5}Aa^2bx^5+\frac{1}{3}Aa^3x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/17*B*c^3*x^17 + 1/5*B*b*c^2*x^15 + 1/15*A*c^3*x^15 + 3/13*B*b^2*c*x^13 +
3/13*B*a*c^2*x^13 + 3/13*A*b*c^2*x^13 + 1/11*B*b^3*x^11 + 6/11*B*a*b*c*x^11
 + 3/11*A*b^2*c*x^11 + 3/11*A*a*c^2*x^11 + 1/3*B*a*b^2*x^9 + 1/9*A*b^3*x^9
+ 1/3*B*a^2*c*x^9 + 2/3*A*a*b*c*x^9 + 3/7*B*a^2*b*x^7 + 3/7*A*a*b^2*x^7 + 3
/7*A*a^2*c*x^7 + 1/5*B*a^3*x^5 + 3/5*A*a^2*b*x^5 + 1/3*A*a^3*x^3
```

Mupad [B]
time = 0.10, size = 169, normalized size = 1.02

$$x^9\left(\frac{B\,c\,a^2}{3}+\frac{B\,a\,b^2}{3}+\frac{2\,A\,c\,a\,b}{3}+\frac{A\,b^3}{9}\right)+x^{11}\left(\frac{B\,b^3}{11}+\frac{3\,A\,b^2\,c}{11}+\frac{6\,B\,a\,b\,c}{11}+\frac{3\,A\,a\,c^2}{11}\right)+x^5\left(\frac{B\,a^3}{5}+\frac{3\,A\,b\,a^2}{5}\right)+x^{15}\left(\frac{A\,c^3}{15}+\frac{B\,b\,c^2}{5}\right)+x^7\left(\frac{3\,B\,a^2\,b}{7}+\frac{3\,A\,c\,a^2}{7}+\frac{3\,A\,a\,b^2}{7}\right)+x^{13}\left(\frac{3\,B\,b^2\,c}{13}+\frac{3\,A\,b\,c^2}{13}+\frac{3\,B\,a\,c^2}{13}\right)+\frac{A\,a^3\,x^3}{3}+\frac{B\,c^3\,x^{17}}{17}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^9*((A*b^3)/9 + (B*a*b^2)/3 + (B*a^2*c)/3 + (2*A*a*b*c)/3) + x^11*((B*b^3)
/11 + (3*A*a*c^2)/11 + (3*A*b^2*c)/11 + (6*B*a*b*c)/11) + x^5*((B*a^3)/5 +
(3*A*a^2*b)/5) + x^15*((A*c^3)/15 + (B*b*c^2)/5) + x^7*((3*A*a*b^2)/7 + (3*
A*a^2*c)/7 + (3*B*a^2*b)/7) + x^13*((3*A*b*c^2)/13 + (3*B*a*c^2)/13 + (3*B*
b^2*c)/13) + (A*a^3*x^3)/3 + (B*c^3*x^17)/17
```

3.97 $\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx$

Optimal. Leaf size=166

$$\frac{1}{2}a^3Ax^2+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{1}{2}a\left(abB+A\left(b^2+ac\right)\right)x^6+\frac{1}{8}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^8+\frac{1}{10}\left(b^3B+3A\right.$$

```
[Out] 1/2*a^3*A*x^2+1/4*a^2*(3*A*b+B*a)*x^4+1/2*a*(a*b*B+A*(a*c+b^2))*x^6+1/8*(3*
a*B*(a*c+b^2)+A*(6*a*b*c+b^3))*x^8+1/10*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^
3)*x^10+1/4*c*(A*b*c+B*a*c+B*b^2)*x^12+1/14*c^2*(A*c+3*B*b)*x^14+1/16*B*c^3
*x^16
```

Rubi [A]
time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1261, 645}

$$\frac{1}{2}a^3Ax^2+\frac{1}{4}a^2x^4(aB+3Ab)+\frac{1}{4}cx^{12}\left(aBc+Abc+b^2B\right)+\frac{1}{2}ax^6\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{10}x^{10}\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{8}x^8\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{14}c^2x^{14}(Ac+3bB)+\frac{1}{16}Bc^3x^{16}$$

Antiderivative was successfully verified.

```
[In] Int[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*A*x^2)/2 + (a^2*(3*A*b + a*B)*x^4)/4 + (a*(a*b*B + A*(b^2 + a*c))*x^6)
/2 + ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^8)/8 + ((b^3*B + 3*A*b^2*c
+ 6*a*b*B*c + 3*a*A*c^2)*x^10)/10 + (c*(b^2*B + A*b*c + a*B*c)*x^12)/4 + (c
^2*(3*b*B + A*c)*x^14)/14 + (B*c^3*x^16)/16
```

Rule 645

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_Symbo
l] :> Int[ExpandIntegrand[(d + e*x)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a
, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || Eq
Q[a, 0])
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x(A+Bx^2)\left(a+bx^2+cx^4\right)^3\,dx = \frac{1}{2}\text{Subst}\left(\int (A+Bx)\left(a+bx+cx^2\right)^3\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(a^3A+a^2(3Ab+aB)x+3a\left(abB+A\left(b^2+ac\right)\right)x^2+(3a\right.\right.$$
$$= \frac{1}{2}a^3Ax^2+\frac{1}{4}a^2(3Ab+aB)x^4+\frac{1}{2}a\left(abB+A\left(b^2+ac\right)\right)x^6+\frac{1}{8}\left(3aB\left(b\right.\right.$$

Mathematica [A]
time = 0.04, size = 154, normalized size = 0.93

$$\frac{1}{560}x^2\left(280a^3A+140a^2(3Ab+aB)x^2+280a\left(abB+A\left(b^2+ac\right)\right)x^4+70\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^6+56\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^8+140c\left(b^2B+Abc+aBc\right)x^{10}+40c^2(3bB+Ac)x^{12}+35Bc^3x^{14}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x^2*(280*a^3*A + 140*a^2*(3*A*b + a*B)*x^2 + 280*a*(a*b*B + A*(b^2 + a*c))
*x^4 + 70*(3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6 + 56*(b^3*B + 3*A*b^2
*c + 6*a*b*B*c + 3*a*A*c^2)*x^8 + 140*c*(b^2*B + A*b*c + a*B*c)*x^10 + 40*c
^2*(3*b*B + A*c)*x^12 + 35*B*c^3*x^14))/560
```

Maple [A]
time = 0.09, size = 226, normalized size = 1.36

method	result
norman	$\frac{a^3Ax^2}{2}+\left(\frac{3}{4}a^2bA+\frac{1}{4}a^3B\right)x^4+\left(\frac{1}{2}a^2cA+\frac{1}{2}Aab^2+\frac{1}{2}Ba^2b\right)x^6+\left(\frac{3}{4}Aabc+\frac{1}{8}Ab^3+\frac{3}{8}a^2cB+\frac{3}{8}Ba\right.$
gosper	$\frac{1}{2}a^3Ax^2+\frac{3}{4}x^4a^2bA+\frac{1}{4}x^4a^3B+\frac{1}{2}x^6a^2cA+\frac{1}{2}x^6Aab^2+\frac{1}{2}x^6Ba^2b+\frac{3}{4}x^8Aabc+\frac{1}{8}x^8Ab^3+\frac{3}{8}x^8a^2c$
risch	$\frac{1}{2}a^3Ax^2+\frac{3}{4}x^4a^2bA+\frac{1}{4}x^4a^3B+\frac{1}{2}x^6a^2cA+\frac{1}{2}x^6Aab^2+\frac{1}{2}x^6Ba^2b+\frac{3}{4}x^8Aabc+\frac{1}{8}x^8Ab^3+\frac{3}{8}x^8a^2c$
default	$\frac{Bc^3x^{16}}{16}+\frac{(c^3A+3Bbc^2)x^{14}}{14}+\frac{(3bc^2A+B(c^2a+2b^2c+c(2ac+b^2)))x^{12}}{12}+\frac{((c^2a+2b^2c+c(2ac+b^2))A+B(4abc+b(2ac+b^2)))x^{10}}{10}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*B*c^3*x^16+1/14*(A*c^3+3*B*b*c^2)*x^14+1/12*(3*b*c^2*A+B*(c^2*a+2*b^2*
c+c*(2*a*c+b^2)))*x^12+1/10*((c^2*a+2*b^2*c+c*(2*a*c+b^2))*A+B*(4*a*b*c+b*(
2*a*c+b^2)))*x^10+1/8*((4*a*b*c+b*(2*a*c+b^2))*A+B*(a*(2*a*c+b^2)+2*a*b^2+a
^2*c))*x^8+1/6*((a*(2*a*c+b^2)+2*a*b^2+a^2*c)*A+3*B*a^2*b)*x^6+1/4*(3*A*a^2
*b+B*a^3)*x^4+1/2*a^3*A*x^2
```

Maxima [A]
time = 0.28, size = 166, normalized size = 1.00

$$\frac{1}{16}Bc^3x^{16}+\frac{1}{14}\left(3Bbc^2+Ac^3\right)x^{14}+\frac{1}{4}\left(Bb^2c+(Ba+Ab)c^2\right)x^{12}+\frac{1}{10}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^{10}+\frac{1}{8}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^8+\frac{1}{2}\left(Ba^2b+Aab^2+Aa^2c\right)x^6+\frac{1}{2}Aa^3x^2+\frac{1}{4}\left(Ba^3+3Aa^2b\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c + (B*a + A*b
)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^10 + 1/8*(
3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2 + A
*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Fricas [A]
time = 0.33, size = 166, normalized size = 1.00

$$\frac{1}{16}\,Bc^3x^{16}+\frac{1}{14}\,(3\,Bbc^2+Ac^3)x^{14}+\frac{1}{4}\,(Bb^2c+(Ba+Ab)c^2)x^{12}+\frac{1}{10}\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^{10}+\frac{1}{8}\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^8+\frac{1}{2}\,(Ba^2b+Aab^2+Aa^2c)x^6+\frac{1}{2}\,Aa^3x^2+\frac{1}{4}\,(Ba^3+3\,Aa^2b)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*B*c^3*x^16 + 1/14*(3*B*b*c^2 + A*c^3)*x^14 + 1/4*(B*b^2*c + (B*a + A*b
)*c^2)*x^12 + 1/10*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^10 + 1/8*(
3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^8 + 1/2*(B*a^2*b + A*a*b^2 + A
*a^2*c)*x^6 + 1/2*A*a^3*x^2 + 1/4*(B*a^3 + 3*A*a^2*b)*x^4
```

Sympy [A]
time = 0.02, size = 199, normalized size = 1.20

$$\frac{Aa^3x^2}{2}+\frac{Bc^3x^{16}}{16}+x^{14}\left(\frac{Ac^3}{14}+\frac{3Bbc^2}{14}\right)+x^{12}\left(\frac{Abc^2}{4}+\frac{Bac^2}{4}+\frac{Bb^2c}{4}\right)+x^{10}\cdot\left(\frac{3Aac^2}{10}+\frac{3Ab^2c}{10}+\frac{3Babc}{5}+\frac{Bb^3}{10}\right)+x^8\cdot\left(\frac{3Aabc}{4}+\frac{Ab^3}{8}+\frac{3Ba^2c}{8}+\frac{3Bab^2}{8}\right)+x^6\left(\frac{Aa^2c}{2}+\frac{Aab^2}{2}+\frac{Ba^2b}{2}\right)+x^4\cdot\left(\frac{3Aa^2b}{4}+\frac{Ba^3}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x**2/2 + B*c**3*x**16/16 + x**14*(A*c**3/14 + 3*B*b*c**2/14) + x**12
*(A*b*c**2/4 + B*a*c**2/4 + B*b**2*c/4) + x**10*(3*A*a*c**2/10 + 3*A*b**2*c
/10 + 3*B*a*b*c/5 + B*b**3/10) + x**8*(3*A*a*b*c/4 + A*b**3/8 + 3*B*a**2*c/
8 + 3*B*a*b**2/8) + x**6*(A*a**2*c/2 + A*a*b**2/2 + B*a**2*b/2) + x**4*(3*A
*a**2*b/4 + B*a**3/4)
```

Giac [A]
time = 3.05, size = 193, normalized size = 1.16

$$\frac{1}{16}\,Bc^3x^{16}+\frac{3}{14}\,Bbc^2x^{14}+\frac{1}{14}\,Ac^3x^{14}+\frac{1}{4}\,Bb^2cx^{12}+\frac{1}{4}\,Bac^2x^{12}+\frac{1}{4}\,Abc^2x^{12}+\frac{1}{10}\,Bb^3x^{10}+\frac{3}{5}\,Babcx^{10}+\frac{3}{10}\,Ab^2cx^{10}+\frac{3}{10}\,Aac^2x^{10}+\frac{3}{8}\,Bab^2x^8+\frac{1}{8}\,Ab^3x^8+\frac{3}{8}\,Ba^2cx^8+\frac{3}{4}\,Aabcx^8+\frac{1}{2}\,Ba^2bx^6+\frac{1}{2}\,Aab^2x^6+\frac{1}{2}\,Aa^2cx^6+\frac{1}{4}\,Ba^3x^4+\frac{3}{4}\,Aa^2bx^4+\frac{1}{2}\,Aa^3x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/16*B*c^3*x^16 + 3/14*B*b*c^2*x^14 + 1/14*A*c^3*x^14 + 1/4*B*b^2*c*x^12 +
1/4*B*a*c^2*x^12 + 1/4*A*b*c^2*x^12 + 1/10*B*b^3*x^10 + 3/5*B*a*b*c*x^10 +
3/10*A*b^2*c*x^10 + 3/10*A*a*c^2*x^10 + 3/8*B*a*b^2*x^8 + 1/8*A*b^3*x^8 + 3
```

```
/8*B*a^2*c*x^8 + 3/4*A*a*b*c*x^8 + 1/2*B*a^2*b*x^6 + 1/2*A*a*b^2*x^6 + 1/2*
A*a^2*c*x^6 + 1/4*B*a^3*x^4 + 3/4*A*a^2*b*x^4 + 1/2*A*a^3*x^2
```

Mupad [B]
time = 0.05, size = 169, normalized size = 1.02

$$x^8\left(\frac{3\,B\,c\,a^2}{8}+\frac{3\,B\,a\,b^2}{8}+\frac{3\,A\,c\,a\,b}{4}+\frac{A\,b^3}{8}\right)+x^{10}\left(\frac{B\,b^3}{10}+\frac{3\,A\,b^2\,c}{10}+\frac{3\,B\,a\,b\,c}{5}+\frac{3\,A\,a\,c^2}{10}\right)+x^4\left(\frac{B\,a^3}{4}+\frac{3\,A\,b\,a^2}{4}\right)+x^{14}\left(\frac{A\,c^3}{14}+\frac{3\,B\,b\,c^2}{14}\right)+x^6\left(\frac{B\,a^2\,b}{2}+\frac{A\,c\,a^2}{2}+\frac{A\,a\,b^2}{2}\right)+x^{12}\left(\frac{B\,b^2\,c}{4}+\frac{A\,b\,c^2}{4}+\frac{B\,a\,c^2}{4}\right)+\frac{A\,a^3\,x^2}{2}+\frac{B\,c^3\,x^{16}}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^8*((A*b^3)/8 + (3*B*a*b^2)/8 + (3*B*a^2*c)/8 + (3*A*a*b*c)/4) + x^10*((B*
b^3)/10 + (3*A*a*c^2)/10 + (3*A*b^2*c)/10 + (3*B*a*b*c)/5) + x^4*((B*a^3)/4
 + (3*A*a^2*b)/4) + x^14*((A*c^3)/14 + (3*B*b*c^2)/14) + x^6*((A*a*b^2)/2 +
 (A*a^2*c)/2 + (B*a^2*b)/2) + x^12*((A*b*c^2)/4 + (B*a*c^2)/4 + (B*b^2*c)/4
) + (A*a^3*x^2)/2 + (B*c^3*x^16)/16
```

3.98 $\int (A+Bx^2)\left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=161

$$a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5+\frac{1}{7}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^7+\frac{1}{9}\left(b^3B+3Ab^2c\right.$$

```
[Out] a^3*A*x+1/3*a^2*(3*A*b+B*a)*x^3+3/5*a*(a*b*B+A*(a*c+b^2))*x^5+1/7*(3*a*B*(a
*c+b^2)+A*(6*a*b*c+b^3))*x^7+1/9*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^9+
3/11*c*(A*b*c+B*a*c+B*b^2)*x^11+1/13*c^2*(A*c+3*B*b)*x^13+1/15*B*c^3*x^15
```

Rubi [A]
time = 0.07, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1167}

$$a^3Ax+\frac{1}{3}a^2x^3(aB+3Ab)+\frac{3}{11}cx^{11}\left(aBc+Abc+b^2B\right)+\frac{3}{5}ax^5\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{9}x^9\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{7}x^7\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{13}c^2x^{13}(Ac+3bB)+\frac{1}{15}Bc^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(A+Bx^2\right)\left(a+bx^2+cx^4\right)^3\,dx &= \int \left(a^3A+a^2(3Ab+aB)x^2+3a\left(abB+A\left(b^2+ac\right)\right)x^4+\left(3aB\left(b^2+\right.\right.\right.\\ &= a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5+\frac{1}{7}\left(3aB\left(b^2+\right.\right.\end{aligned}$$

Mathematica [A]
time = 0.03, size = 161, normalized size = 1.00

$$a^3Ax+\frac{1}{3}a^2(3Ab+aB)x^3+\frac{3}{5}a\left(abB+A\left(b^2+ac\right)\right)x^5+\frac{1}{7}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^7+\frac{1}{9}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^9+\frac{3}{11}c\left(b^2B+Abc+aBc\right)x^{11}+\frac{1}{13}c^2(3bB+Ac)x^{13}+\frac{1}{15}Bc^3x^{15}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] a^3*A*x + (a^2*(3*A*b + a*B)*x^3)/3 + (3*a*(a*b*B + A*(b^2 + a*c))*x^5)/5 +
 ((3*a*B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^7)/7 + ((b^3*B + 3*A*b^2*c + 6*
a*b*B*c + 3*a*A*c^2)*x^9)/9 + (3*c*(b^2*B + A*b*c + a*B*c)*x^11)/11 + (c^2*
(3*b*B + A*c)*x^13)/13 + (B*c^3*x^15)/15
```

Maple [A]
time = 0.09, size = 223, normalized size = 1.39

method	result
norman	$a^3Ax + \left(a^2bA + \frac{1}{3}a^3B\right)x^3 + \left(\frac{3}{5}a^2cA + \frac{3}{5}Aa\,b^2 + \frac{3}{5}B\,a^2b\right)x^5 + \left(\frac{6}{7}Aabc + \frac{1}{7}A\,b^3 + \frac{3}{7}a^2cB + \frac{3}{7}Ba\,b^2\right.$
gosper	$a^3Ax + x^3a^2bA + \frac{1}{3}x^3a^3B + \frac{3}{5}x^5a^2cA + \frac{3}{5}x^5Aa\,b^2 + \frac{3}{5}x^5B\,a^2b + \frac{6}{7}x^7Aabc + \frac{1}{7}x^7A\,b^3 + \frac{3}{7}x^7a^2cB +$
risch	$a^3Ax + x^3a^2bA + \frac{1}{3}x^3a^3B + \frac{3}{5}x^5a^2cA + \frac{3}{5}x^5Aa\,b^2 + \frac{3}{5}x^5B\,a^2b + \frac{6}{7}x^7Aabc + \frac{1}{7}x^7A\,b^3 + \frac{3}{7}x^7a^2cB +$
default	$\frac{B\,c^3x^{15}}{15} + \frac{(c^3A+3Bb\,c^2)x^{13}}{13} + \frac{(3b\,c^2A+B(c^2a+2b^2c+c(2ac+b^2)))x^{11}}{11} + \frac{((c^2a+2b^2c+c(2ac+b^2))A+B(4abc+b(2ac+b^2)))x^9}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*B*c^3*x^15+1/13*(A*c^3+3*B*b*c^2)*x^13+1/11*(3*b*c^2*A+B*(c^2*a+2*b^2*
c+c*(2*a*c+b^2)))*x^11+1/9*((c^2*a+2*b^2*c+c*(2*a*c+b^2))*A+B*(4*a*b*c+b*(2
*a*c+b^2)))*x^9+1/7*((4*a*b*c+b*(2*a*c+b^2))*A+B*(a*(2*a*c+b^2)+2*a*b^2+a^2
*c))*x^7+1/5*((a*(2*a*c+b^2)+2*a*b^2+a^2*c)*A+3*B*a^2*b)*x^5+1/3*(3*A*a^2*b
+B*a^3)*x^3+a^3*A*x
```

Maxima [A]
time = 0.28, size = 163, normalized size = 1.01

$$\frac{1}{15}Bc^3x^{15} + \frac{1}{13}\left(3Bbc^2 + Ac^3\right)x^{13} + \frac{3}{11}\left(Bb^2c + (Ba + Ab)c^2\right)x^{11} + \frac{1}{9}\left(Bb^3 + 3Aac^2 + 3\left(2Bab + Ab^2\right)c\right)x^9 + \frac{1}{7}\left(3Bab^2 + Ab^3 + 3\left(Ba^2 + 2Aab\right)c\right)x^7 + \frac{3}{5}\left(Ba^2b + Aab^2 + Aa^2c\right)x^5 + Aa^3x + \frac{1}{3}\left(Ba^3 + 3Aa^2b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c + (B*a + A*
b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Fricas [A]
time = 0.35, size = 163, normalized size = 1.01

$$\frac{1}{15}Bc^3x^{15} + \frac{1}{13}\left(3Bbc^2 + Ac^3\right)x^{13} + \frac{3}{11}\left(Bb^2c + (Ba + Ab)c^2\right)x^{11} + \frac{1}{9}\left(Bb^3 + 3Aac^2 + 3\left(2Bab + Ab^2\right)c\right)x^9 + \frac{1}{7}\left(3Bab^2 + Ab^3 + 3\left(Ba^2 + 2Aab\right)c\right)x^7 + \frac{3}{5}\left(Ba^2b + Aab^2 + Aa^2c\right)x^5 + Aa^3x + \frac{1}{3}\left(Ba^3 + 3Aa^2b\right)x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/15*B*c^3*x^15 + 1/13*(3*B*b*c^2 + A*c^3)*x^13 + 3/11*(B*b^2*c + (B*a + A*
b)*c^2)*x^11 + 1/9*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^9 + 1/7*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^7 + 3/5*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^5 + A*a^3*x + 1/3*(B*a^3 + 3*A*a^2*b)*x^3
```

Sympy [A]
time = 0.02, size = 199, normalized size = 1.24

$$Aa^3x + \frac{Bc^3x^{15}}{15} + x^{13}\left(\frac{Ac^3}{13} + \frac{3Bbc^2}{13}\right) + x^{11}\cdot\left(\frac{3Abc^2}{11} + \frac{3Bac^2}{11} + \frac{3Bb^2c}{11}\right) + x^9\left(\frac{Aac^2}{3} + \frac{Ab^2c}{3} + \frac{2Babc}{3} + \frac{Bb^3}{9}\right) + x^7\cdot\left(\frac{6Aabc}{7} + \frac{Ab^3}{7} + \frac{3Ba^2c}{7} + \frac{3Bab^2}{7}\right) + x^5\cdot\left(\frac{3Aa^2c}{5} + \frac{3Aab^2}{5} + \frac{3Ba^2b}{5}\right) + x^3\left(Aa^2b + \frac{Ba^3}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] A*a**3*x + B*c**3*x**15/15 + x**13*(A*c**3/13 + 3*B*b*c**2/13) + x**11*(3*A
*b*c**2/11 + 3*B*a*c**2/11 + 3*B*b**2*c/11) + x**9*(A*a*c**2/3 + A*b**2*c/3
 + 2*B*a*b*c/3 + B*b**3/9) + x**7*(6*A*a*b*c/7 + A*b**3/7 + 3*B*a**2*c/7 +
3*B*a*b**2/7) + x**5*(3*A*a**2*c/5 + 3*A*a*b**2/5 + 3*B*a**2*b/5) + x**3*(A
*a**2*b + B*a**3/3)
```

Giac [A]
time = 4.13, size = 189, normalized size = 1.17

$$\frac{1}{15}Bc^3x^{15} + \frac{3}{13}Bbc^2x^{13} + \frac{1}{13}Ac^3x^{13} + \frac{3}{11}Bb^2cx^{11} + \frac{3}{11}Bac^2x^{11} + \frac{3}{11}Abc^2x^{11} + \frac{1}{9}Bb^3x^9 + \frac{2}{3}Babcx^9 + \frac{1}{3}Ab^2cx^9 + \frac{1}{3}Aac^2x^9 + \frac{3}{7}Bab^2x^7 + \frac{1}{7}Ab^3x^7 + \frac{3}{7}Ba^2cx^7 + \frac{6}{7}Aabcx^7 + \frac{3}{5}Ba^2bx^5 + \frac{3}{5}Aab^2x^5 + \frac{3}{5}Aa^2cx^5 + \frac{1}{3}Ba^3x^3 + Aa^2bx^3 + Aa^3x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/15*B*c^3*x^15 + 3/13*B*b*c^2*x^13 + 1/13*A*c^3*x^13 + 3/11*B*b^2*c*x^11 +
 3/11*B*a*c^2*x^11 + 3/11*A*b*c^2*x^11 + 1/9*B*b^3*x^9 + 2/3*B*a*b*c*x^9 +
1/3*A*b^2*c*x^9 + 1/3*A*a*c^2*x^9 + 3/7*B*a*b^2*x^7 + 1/7*A*b^3*x^7 + 3/7*B
*a^2*c*x^7 + 6/7*A*a*b*c*x^7 + 3/5*B*a^2*b*x^5 + 3/5*A*a*b^2*x^5 + 3/5*A*a^
2*c*x^5 + 1/3*B*a^3*x^3 + A*a^2*b*x^3 + A*a^3*x
```

Mupad [B]
time = 0.05, size = 165, normalized size = 1.02

$$x^7\left(\frac{3\,B\,c\,a^2}{7} + \frac{3\,B\,a\,b^2}{7} + \frac{6\,A\,c\,a\,b}{7} + \frac{A\,b^3}{7}\right) + x^9\left(\frac{B\,b^3}{9} + \frac{A\,b^2\,c}{3} + \frac{2\,B\,a\,b\,c}{3} + \frac{A\,a\,c^2}{3}\right) + x^3\left(\frac{B\,a^3}{3} + A\,b\,a^2\right) + x^{13}\left(\frac{A\,c^3}{13} + \frac{3\,B\,b\,c^2}{13}\right) + x^5\left(\frac{3\,B\,a^2\,b}{5} + \frac{3\,A\,c\,a^2}{5} + \frac{3\,A\,a\,b^2}{5}\right) + x^{11}\left(\frac{3\,B\,b^2\,c}{11} + \frac{3\,A\,b\,c^2}{11} + \frac{3\,B\,a\,c^2}{11}\right) + \frac{B\,c^3\,x^{15}}{15} + A\,a^3\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)*(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] x^7*((A*b^3)/7 + (3*B*a*b^2)/7 + (3*B*a^2*c)/7 + (6*A*a*b*c)/7) + x^9*((B*b
^3)/9 + (A*a*c^2)/3 + (A*b^2*c)/3 + (2*B*a*b*c)/3) + x^3*((B*a^3)/3 + A*a^2
*b) + x^13*((A*c^3)/13 + (3*B*b*c^2)/13) + x^5*((3*A*a*b^2)/5 + (3*A*a^2*c)
/5 + (3*B*a^2*b)/5) + x^11*((3*A*b*c^2)/11 + (3*B*a*c^2)/11 + (3*B*b^2*c)/1
1) + (B*c^3*x^15)/15 + A*a^3*x
```

3.99 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x}\,dx$

Optimal. Leaf size=162

$$\frac{1}{2}a^2(3Ab+aB)x^2+\frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4+\frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^6+\frac{1}{8}\left(b^3B+3Ab^2c+6abBc\right.$$

```
[Out] 1/2*a^2*(3*A*b+B*a)*x^2+3/4*a*(a*b*B+A*(a*c+b^2))*x^4+1/6*(3*a*B*(a*c+b^2)+
A*(6*a*b*c+b^3))*x^6+1/8*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^8+3/10*c*(
A*b*c+B*a*c+B*b^2)*x^10+1/12*c^2*(A*c+3*B*b)*x^12+1/14*B*c^3*x^14+a^3*A*ln(
x)
```

Rubi [A]
time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1265, 779}

$$a^3A\log(x)+\frac{1}{2}a^2x^2(aB+3Ab)+\frac{3}{10}cx^{10}\left(aBc+Abc+b^2B\right)+\frac{3}{4}ax^4\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{8}x^8\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{6}x^6\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{12}c^2x^{12}(Ac+3bB)+\frac{1}{14}Bc^3x^{14}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

Rule 779

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(A+Bx^2)\left(a+bx^2+cx^4\right)^3}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\left(a+bx+cx^2\right)^3}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(a^2(3Ab+aB)+\frac{a^3A}{x}+3a\big(abB+A\big(b^2+ac\big)\big)\,x+\big(3aB$$
$$= \frac{1}{2}a^2(3Ab+aB)x^2+\frac{3}{4}a\big(abB+A\big(b^2+ac\big)\big)\,x^4+\frac{1}{6}\big(3aB\big(b^2+ac\big)+A$$

Mathematica [A]
time = 0.04, size = 162, normalized size = 1.00

$$\frac{1}{2}a^2(3Ab+aB)x^2+\frac{3}{4}a\left(abB+A\left(b^2+ac\right)\right)x^4+\frac{1}{6}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^6+\frac{1}{8}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^8+\frac{3}{10}c\left(b^2B+Abc+aBc\right)x^{10}+\frac{1}{12}c^2(3bB+Ac)x^{12}+\frac{1}{14}Bc^3x^{14}+a^3A\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x]
```

```
[Out] (a^2*(3*A*b + a*B)*x^2)/2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^4)/4 + ((3*a*B*(
b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^6)/6 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c +
3*a*A*c^2)*x^8)/8 + (3*c*(b^2*B + A*b*c + a*B*c)*x^10)/10 + (c^2*(3*b*B + A
*c)*x^12)/12 + (B*c^3*x^14)/14 + a^3*A*Log[x]
```

Maple [A]
time = 0.02, size = 191, normalized size = 1.18

method	result
norman	$\left(\frac{1}{12}c^3A+\frac{1}{4}Bbc^2\right)x^{12}+\left(\frac{3}{2}a^2bA+\frac{1}{2}a^3B\right)x^2+\left(\frac{3}{4}a^2cA+\frac{3}{4}Aab^2+\frac{3}{4}Ba^2b\right)x^4+\left(\frac{3}{10}bc^2A+\frac{3}{10}c^2\right.$
default	$\frac{Bc^3x^{14}}{14}+\frac{Ac^3x^{12}}{12}+\frac{Bbc^2x^{12}}{4}+\frac{3Abc^2x^{10}}{10}+\frac{3Bac^2x^{10}}{10}+\frac{3Bb^2cx^{10}}{10}+\frac{3Aac^2x^8}{8}+\frac{3Ab^2cx^8}{8}+\frac{3Babcx^8}{4}+\frac{Bb^3x^8}{8}$
risch	$\frac{Bc^3x^{14}}{14}+\frac{Ac^3x^{12}}{12}+\frac{Bbc^2x^{12}}{4}+\frac{3Abc^2x^{10}}{10}+\frac{3Bac^2x^{10}}{10}+\frac{3Bb^2cx^{10}}{10}+\frac{3Aac^2x^8}{8}+\frac{3Ab^2cx^8}{8}+\frac{3Babcx^8}{4}+\frac{Bb^3x^8}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/14*B*c^3*x^14+1/12*A*c^3*x^12+1/4*B*b*c^2*x^12+3/10*A*b*c^2*x^10+3/10*B*a
*c^2*x^10+3/10*B*b^2*c*x^10+3/8*A*a*c^2*x^8+3/8*A*b^2*c*x^8+3/4*B*a*b*c*x^8
+1/8*B*b^3*x^8+A*a*b*c*x^6+1/6*A*b^3*x^6+1/2*B*a^2*c*x^6+1/2*B*a*b^2*x^6+3/
4*x^4*a^2*c*A+3/4*A*a*b^2*x^4+3/4*B*a^2*b*x^4+3/2*a^2*A*b*x^2+1/2*a^3*B*x^2
+a^3*A*ln(x)
```

Maxima [A]
time = 0.29, size = 167, normalized size = 1.03

$$\frac{1}{14}Bc^3x^{14}+\frac{1}{12}\left(3Bbc^2+Ac^3\right)x^{12}+\frac{3}{10}\left(Bb^2c+(Ba+Ab)c^2\right)x^{10}+\frac{1}{8}\left(Bb^3+3Aac^2+3\left(2Bab+Ab^2\right)c\right)x^8+\frac{1}{6}\left(3Bab^2+Ab^3+3\left(Ba^2+2Aab\right)c\right)x^6+\frac{3}{4}\left(Ba^2b+Aab^2+Aa^2c\right)x^4+\frac{1}{2}Aa^3\log\left(x^2\right)+\frac{1}{2}\left(Ba^3+3Aa^2b\right)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="maxima")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + 1/2*A*a^3*log(x^2) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Fricas [A]
time = 0.35, size = 164, normalized size = 1.01

$$\frac{1}{14}\,Bc^3x^{14} + \frac{1}{12}\,(3\,Bbc^2 + Ac^3)x^{12} + \frac{3}{10}\,(Bb^2c + (Ba + Ab)c^2)x^{10} + \frac{1}{8}\,(Bb^3 + 3\,Aac^2 + 3\,(2\,Bab + Ab^2)c)x^8 + \frac{1}{6}\,(3\,Bab^2 + Ab^3 + 3\,(Ba^2 + 2\,Aab)c)x^6 + \frac{3}{4}\,(Ba^2b + Aab^2 + Aa^2c)x^4 + Aa^3\log{(x)} + \frac{1}{2}\,(Ba^3 + 3\,Aa^2b)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="fricas")
```

```
[Out] 1/14*B*c^3*x^14 + 1/12*(3*B*b*c^2 + A*c^3)*x^12 + 3/10*(B*b^2*c + (B*a + A*
b)*c^2)*x^10 + 1/8*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 1/6*(3
*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 3/4*(B*a^2*b + A*a*b^2 + A*
a^2*c)*x^4 + A*a^3*log(x) + 1/2*(B*a^3 + 3*A*a^2*b)*x^2
```

Sympy [A]
time = 0.13, size = 199, normalized size = 1.23

$$Aa^3\log{(x)} + \frac{Bc^3x^{14}}{14} + x^{12}\left(\frac{Ac^3}{12} + \frac{Bbc^2}{4}\right) + x^{10}\cdot\left(\frac{3Abc^2}{10} + \frac{3Bac^2}{10} + \frac{3Bb^2c}{10}\right) + x^8\cdot\left(\frac{3Aac^2}{8} + \frac{3Ab^2c}{8} + \frac{3Babc}{4} + \frac{Bb^3}{8}\right) + x^6\left(Aabc + \frac{Ab^3}{6} + \frac{Ba^2c}{2} + \frac{Bab^2}{2}\right) + x^4\cdot\left(\frac{3Aa^2c}{4} + \frac{3Aab^2}{4} + \frac{3Ba^2b}{4}\right) + x^2\cdot\left(\frac{3Aa^2b}{2} + \frac{Ba^3}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x,x)
```

```
[Out] A*a**3*log(x) + B*c**3*x**14/14 + x**12*(A*c**3/12 + B*b*c**2/4) + x**10*(3
*A*b*c**2/10 + 3*B*a*c**2/10 + 3*B*b**2*c/10) + x**8*(3*A*a*c**2/8 + 3*A*b*
*2*c/8 + 3*B*a*b*c/4 + B*b**3/8) + x**6*(A*a*b*c + A*b**3/6 + B*a**2*c/2 +
B*a*b**2/2) + x**4*(3*A*a**2*c/4 + 3*A*a*b**2/4 + 3*B*a**2*b/4) + x**2*(3*A
*a**2*b/2 + B*a**3/2)
```

Giac [A]
time = 5.28, size = 193, normalized size = 1.19

$$\frac{1}{14}\,Bc^3x^{14} + \frac{1}{4}\,Bbc^2x^{12} + \frac{1}{12}\,Ac^3x^{12} + \frac{3}{10}\,Bb^2cx^{10} + \frac{3}{10}\,Bac^2x^{10} + \frac{3}{10}\,Abc^2x^{10} + \frac{1}{8}\,Bb^3x^8 + \frac{3}{4}\,Babcx^8 + \frac{3}{8}\,Ab^2cx^8 + \frac{3}{8}\,Aac^2x^8 + \frac{1}{2}\,Bab^2x^6 + \frac{1}{6}\,Ab^3x^6 + \frac{1}{2}\,Ba^2cx^6 + Aabcx^6 + \frac{3}{4}\,Ba^2bx^4 + \frac{3}{4}\,Aab^2x^4 + \frac{3}{4}\,Aa^2cx^4 + \frac{1}{2}\,Ba^3x^2 + \frac{3}{2}\,Aa^2bx^2 + \frac{1}{2}\,Aa^3\log{(x^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x,x, algorithm="giac")
```

```
[Out] 1/14*B*c^3*x^14 + 1/4*B*b*c^2*x^12 + 1/12*A*c^3*x^12 + 3/10*B*b^2*c*x^10 +
3/10*B*a*c^2*x^10 + 3/10*A*b*c^2*x^10 + 1/8*B*b^3*x^8 + 3/4*B*a*b*c*x^8 + 3
/8*A*b^2*c*x^8 + 3/8*A*a*c^2*x^8 + 1/2*B*a*b^2*x^6 + 1/6*A*b^3*x^6 + 1/2*B*
```

```
a^2*c*x^6 + A*a*b*c*x^6 + 3/4*B*a^2*b*x^4 + 3/4*A*a*b^2*x^4 + 3/4*A*a^2*c*x
^4 + 1/2*B*a^3*x^2 + 3/2*A*a^2*b*x^2 + 1/2*A*a^3*log(x^2)
```

Mupad [B]
time = 0.10, size = 166, normalized size = 1.02

$$x^6\left(\frac{B\,c\,a^2}{2}+\frac{B\,a\,b^2}{2}+A\,c\,a\,b+\frac{A\,b^3}{6}\right)+x^8\left(\frac{B\,b^3}{8}+\frac{3\,A\,b^2\,c}{8}+\frac{3\,B\,a\,b\,c}{4}+\frac{3\,A\,a\,c^2}{8}\right)+x^2\left(\frac{B\,a^3}{2}+\frac{3\,A\,b\,a^2}{2}\right)+x^{12}\left(\frac{A\,c^3}{12}+\frac{B\,b\,c^2}{4}\right)+x^4\left(\frac{3\,B\,a^2\,b}{4}+\frac{3\,A\,c\,a^2}{4}+\frac{3\,A\,a\,b^2}{4}\right)+x^{10}\left(\frac{3\,B\,b^2\,c}{10}+\frac{3\,A\,b\,c^2}{10}+\frac{3\,B\,a\,c^2}{10}\right)+\frac{B\,c^3\,x^{14}}{14}+A\,a^3\,\ln\left(x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x,x)
```

```
[Out] x^6*((A*b^3)/6 + (B*a*b^2)/2 + (B*a^2*c)/2 + A*a*b*c) + x^8*((B*b^3)/8 + (3
*A*a*c^2)/8 + (3*A*b^2*c)/8 + (3*B*a*b*c)/4) + x^2*((B*a^3)/2 + (3*A*a^2*b)
/2) + x^12*((A*c^3)/12 + (B*b*c^2)/4) + x^4*((3*A*a*b^2)/4 + (3*A*a^2*c)/4
+ (3*B*a^2*b)/4) + x^10*((3*A*b*c^2)/10 + (3*B*a*c^2)/10 + (3*B*b^2*c)/10)
+ (B*c^3*x^14)/14 + A*a^3*log(x)
```

3.100 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx$

Optimal. Leaf size=156

$$-\frac{a^3A}{x}+a^2(3Ab+aB)x+a\left(abB+A\left(b^2+ac\right)\right)x^3+\frac{1}{5}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^5+\frac{1}{7}\left(b^3B+3Ab^2c+6a\right.$$

```
[Out] -a^3*A/x+a^2*(3*A*b+B*a)*x+a*(a*b*B+A*(a*c+b^2))*x^3+1/5*(3*a*B*(a*c+b^2)+A
*(6*a*b*c+b^3))*x^5+1/7*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^7+1/3*c*(A*
b*c+B*a*c+B*b^2)*x^9+1/11*c^2*(A*c+3*B*b)*x^11+1/13*B*c^3*x^13
```

Rubi [A]
time = 0.07, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1275}

$$-\frac{a^3A}{x}+a^2x(aB+3Ab)+\frac{1}{3}cx^9\left(aBc+Abc+b^2B\right)+ax^3\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{7}x^7\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{5}x^5\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{11}c^2x^{11}(Ac+3bB)+\frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^2}\,dx = \int \left(a^2(3Ab+aB)+\frac{a^3A}{x^2}+3a\left(abB+A\left(b^2+ac\right)\right)x^2+\left(3aB\left(b^2+ac\right)\right.\right.$$
$$= -\frac{a^3A}{x}+a^2(3Ab+aB)x+a\left(abB+A\left(b^2+ac\right)\right)x^3+\frac{1}{5}\left(3aB\left(b^2+ac\right)\right.$$

Mathematica [A]
time = 0.06, size = 156, normalized size = 1.00

$$-\frac{a^3A}{x}+a^2(3Ab+aB)x+a\left(abB+A\left(b^2+ac\right)\right)x^3+\frac{1}{5}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^5+\frac{1}{7}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right)x^7+\frac{1}{3}c\left(b^2B+Abc+aBc\right)x^9+\frac{1}{11}c^2(3bB+Ac)x^{11}+\frac{1}{13}Bc^3x^{13}$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x]
```

```
[Out] -((a^3*A)/x) + a^2*(3*A*b + a*B)*x + a*(a*b*B + A*(b^2 + a*c))*x^3 + ((3*a*
B*(b^2 + a*c) + A*(b^3 + 6*a*b*c))*x^5)/5 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c
 + 3*a*A*c^2)*x^7)/7 + (c*(b^2*B + A*b*c + a*B*c)*x^9)/3 + (c^2*(3*b*B + A*
c)*x^11)/11 + (B*c^3*x^13)/13
```

Maple [A]
time = 0.02, size = 186, normalized size = 1.19

method	result
norman	$\frac{-a^3A+(3a^2bA+a^3B)x^2+(a^2cA+Aa\,b^2+B\,a^2b)x^4+(\frac{6}{5}Aabc+\frac{1}{5}A\,b^3+\frac{3}{5}a^2cB+\frac{3}{5}Ba\,b^2)x^6+(\frac{3}{7}c^2aA+\frac{3}{7}A\,b^2c+\frac{6}{7}abBc+\frac{1}{7}b^3B)x^8+(}{x}$
default	$\frac{B\,c^3x^{13}}{13}+\frac{A\,c^3x^{11}}{11}+\frac{3Bb\,c^2x^{11}}{11}+\frac{Ab\,c^2x^9}{3}+\frac{Ba\,c^2x^9}{3}+\frac{B\,b^2c\,x^9}{3}+\frac{3Aa\,c^2x^7}{7}+\frac{3A\,b^2c\,x^7}{7}+\frac{6Babc\,x^7}{7}+\frac{B\,b^3x^7}{7}+$
risch	$\frac{B\,c^3x^{13}}{13}+\frac{A\,c^3x^{11}}{11}+\frac{3Bb\,c^2x^{11}}{11}+\frac{Ab\,c^2x^9}{3}+\frac{Ba\,c^2x^9}{3}+\frac{B\,b^2c\,x^9}{3}+\frac{3Aa\,c^2x^7}{7}+\frac{3A\,b^2c\,x^7}{7}+\frac{6Babc\,x^7}{7}+\frac{B\,b^3x^7}{7}+$
gosper	$-\frac{-1155B\,c^3x^{14}-1365A\,c^3x^{12}-4095Bb\,c^2x^{12}-5005Ab\,c^2x^{10}-5005Ba\,c^2x^{10}-5005B\,b^2c\,x^{10}-6435Aa\,c^2x^8-6435A\,b^2c\,x^8-12870}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/13*B*c^3*x^13+1/11*A*c^3*x^11+3/11*B*b*c^2*x^11+1/3*A*b*c^2*x^9+1/3*B*a*c
^2*x^9+1/3*B*b^2*c*x^9+3/7*A*a*c^2*x^7+3/7*A*b^2*c*x^7+6/7*B*a*b*c*x^7+1/7*
B*b^3*x^7+6/5*A*a*b*c*x^5+1/5*A*b^3*x^5+3/5*a^2*c*B*x^5+3/5*B*a*b^2*x^5+x^3
*a^2*c*A+A*a*b^2*x^3+B*a^2*b*x^3+3*a^2*b*A*x+a^3*B*x-a^3*A/x
```

Maxima [A]
time = 0.28, size = 162, normalized size = 1.04

$$\frac{1}{13}\,Bc^3x^{13}+\frac{1}{11}\,(3\,Bbc^2+Ac^3)x^{11}+\frac{1}{3}\,(Bb^2c+(Ba+Ab)c^2)x^9+\frac{1}{7}\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^7+\frac{1}{5}\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^5+(Ba^2b+Aab^2+Aa^2c)x^3-\frac{Aa^3}{x}+(Ba^3+3\,Aa^2b)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="maxima")
```

```
[Out] 1/13*B*c^3*x^13 + 1/11*(3*B*b*c^2 + A*c^3)*x^11 + 1/3*(B*b^2*c + (B*a + A*b
)*c^2)*x^9 + 1/7*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^7 + 1/5*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^5 + (B*a^2*b + A*a*b^2 + A*a^2*c)
*x^3 - A*a^3/x + (B*a^3 + 3*A*a^2*b)*x
```

Fricas [A]
time = 0.34, size = 168, normalized size = 1.08

$$\frac{1155\,Bc^3x^{14}+1365\,(3\,Bbc^2+Ac^3)x^{12}+5005\,(Bb^2c+(Ba+Ab)c^2)x^{10}+2145\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^8+3003\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^6+15015\,(Ba^2b+Aab^2+Aa^2c)x^4-15015\,Aa^3+15015\,(Ba^3+3\,Aa^2b)x^2}{15015\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="fricas")
```

```
[Out] 1/15015*(1155*B*c^3*x^14 + 1365*(3*B*b*c^2 + A*c^3)*x^12 + 5005*(B*b^2*c +
(B*a + A*b)*c^2)*x^10 + 2145*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^
8 + 3003*(3*B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 15015*(B*a^2*b +
 A*a*b^2 + A*a^2*c)*x^4 - 15015*A*a^3 + 15015*(B*a^3 + 3*A*a^2*b)*x^2)/x
```

Sympy [A]
time = 0.12, size = 185, normalized size = 1.19

$$-\frac{Aa^3}{x} + \frac{Bc^3x^{13}}{13} + x^{11}\left(\frac{Ac^3}{11} + \frac{3Bbc^2}{11}\right) + x^9\left(\frac{Abc^2}{3} + \frac{Bac^2}{3} + \frac{Bb^2c}{3}\right) + x^7\cdot\left(\frac{3Aac^2}{7} + \frac{3Ab^2c}{7} + \frac{6Babc}{7} + \frac{Bb^3}{7}\right) + x^5\cdot\left(\frac{6Aabc}{5} + \frac{Ab^3}{5} + \frac{3Ba^2c}{5} + \frac{3Bab^2}{5}\right) + x^3\left(Aa^2c + Aab^2 + Ba^2b\right) + x\left(3Aa^2b + Ba^3\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**2,x)
```

```
[Out] -A*a**3/x + B*c**3*x**13/13 + x**11*(A*c**3/11 + 3*B*b*c**2/11) + x**9*(A*b
*c**2/3 + B*a*c**2/3 + B*b**2*c/3) + x**7*(3*A*a*c**2/7 + 3*A*b**2*c/7 + 6*
B*a*b*c/7 + B*b**3/7) + x**5*(6*A*a*b*c/5 + A*b**3/5 + 3*B*a**2*c/5 + 3*B*a
*b**2/5) + x**3*(A*a**2*c + A*a*b**2 + B*a**2*b) + x*(3*A*a**2*b + B*a**3)
```

Giac [A]
time = 4.84, size = 185, normalized size = 1.19

$$\frac{1}{13}Bc^3x^{13} + \frac{3}{11}Bbc^2x^{11} + \frac{1}{11}Ac^3x^{11} + \frac{1}{3}Bb^2cx^9 + \frac{1}{3}Bac^2x^9 + \frac{1}{3}Abc^2x^9 + \frac{1}{7}Bb^3x^7 + \frac{6}{7}Babcx^7 + \frac{3}{7}Ab^2cx^7 + \frac{3}{7}Aac^2x^7 + \frac{3}{5}Bab^2x^5 + \frac{1}{5}Ab^3x^5 + \frac{3}{5}Ba^2cx^5 + \frac{6}{5}Aabcx^5 + Ba^2bx^3 + Aab^2x^3 + Aa^2cx^3 + Ba^3x + 3\,Aa^2bx - \frac{Aa^3}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^2,x, algorithm="giac")
```

```
[Out] 1/13*B*c^3*x^13 + 3/11*B*b*c^2*x^11 + 1/11*A*c^3*x^11 + 1/3*B*b^2*c*x^9 + 1
/3*B*a*c^2*x^9 + 1/3*A*b*c^2*x^9 + 1/7*B*b^3*x^7 + 6/7*B*a*b*c*x^7 + 3/7*A*
b^2*c*x^7 + 3/7*A*a*c^2*x^7 + 3/5*B*a*b^2*x^5 + 1/5*A*b^3*x^5 + 3/5*B*a^2*c
*x^5 + 6/5*A*a*b*c*x^5 + B*a^2*b*x^3 + A*a*b^2*x^3 + A*a^2*c*x^3 + B*a^3*x
+ 3*A*a^2*b*x - A*a^3/x
```

Mupad [B]
time = 0.05, size = 163, normalized size = 1.04

$$x^5\left(\frac{3\,B\,c\,a^2}{5} + \frac{3\,B\,a\,b^2}{5} + \frac{6\,A\,c\,a\,b}{5} + \frac{A\,b^3}{5}\right) + x^7\left(\frac{B\,b^3}{7} + \frac{3\,A\,b^2\,c}{7} + \frac{6\,B\,a\,b\,c}{7} + \frac{3\,A\,a\,c^2}{7}\right) + x\left(B\,a^3 + 3\,A\,b\,a^2\right) + x^{11}\left(\frac{A\,c^3}{11} + \frac{3\,B\,b\,c^2}{11}\right) + x^3\left(B\,a^2\,b + A\,c\,a^2 + A\,a\,b^2\right) + x^9\left(\frac{B\,b^2\,c}{3} + \frac{A\,b\,c^2}{3} + \frac{B\,a\,c^2}{3}\right) - \frac{A\,a^3}{x} + \frac{B\,c^3\,x^{13}}{13}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^2,x)
```

```
[Out] x^5*((A*b^3)/5 + (3*B*a*b^2)/5 + (3*B*a^2*c)/5 + (6*A*a*b*c)/5) + x^7*((B*b
^3)/7 + (3*A*a*c^2)/7 + (3*A*b^2*c)/7 + (6*B*a*b*c)/7) + x*(B*a^3 + 3*A*a^2
*b) + x^11*((A*c^3)/11 + (3*B*b*c^2)/11) + x^3*(A*a*b^2 + A*a^2*c + B*a^2*b
) + x^9*((A*b*c^2)/3 + (B*a*c^2)/3 + (B*b^2*c)/3) - (A*a^3)/x + (B*c^3*x^13
)/13
```

3.101 $\int \frac{(A+Bx^2)(a+bx^2+cx^4)^3}{x^3} \, dx$

Optimal. Leaf size=162

$$-\frac{a^3A}{2x^2}+\frac{3}{2}a\left(abB+A\left(b^2+ac\right)\right)x^2+\frac{1}{4}\left(3aB\left(b^2+ac\right)+A\left(b^3+6abc\right)\right)x^4+\frac{1}{6}\left(b^3B+3Ab^2c+6abBc+3aAc^2\right.$$

```
[Out] -1/2*a^3*A/x^2+3/2*a*(a*b*B+A*(a*c+b^2))*x^2+1/4*(3*a*B*(a*c+b^2)+A*(6*a*b*
c+b^3))*x^4+1/6*(3*A*a*c^2+3*A*b^2*c+6*B*a*b*c+B*b^3)*x^6+3/8*c*(A*b*c+B*a*
c+B*b^2)*x^8+1/10*c^2*(A*c+3*B*b)*x^10+1/12*B*c^3*x^12+a^2*(3*A*b+B*a)*ln(x
)
```

Rubi [A]

time = 0.14, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1265, 779}

$$-\frac{a^3A}{2x^2}+a^2\log(x)(aB+3Ab)+\frac{3}{8}cx^8\left(aBc+Abc+b^2B\right)+\frac{3}{2}ax^2\left(A\left(ac+b^2\right)+abB\right)+\frac{1}{6}x^6\left(3aAc^2+6abBc+3Ab^2c+b^3B\right)+\frac{1}{4}x^4\left(A\left(6abc+b^3\right)+3aB\left(ac+b^2\right)\right)+\frac{1}{10}c^2x^{10}(Ac+3bB)+\frac{1}{12}Bc^3x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*A)/x^2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

Rule 779

```
Int[((e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_
)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(f + g*x)*(a + b*x + c
*x^2)^p, x], x] /; FreeQ[{a, b, c, e, f, g, m}, x] && IntegerQ[p] && (GtQ[p
, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(A+Bx^2)\,(a+bx^2+cx^4)^3}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(A+Bx)\,(a+bx+cx^2)^3}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(3a\big(abB+A(b^2+ac)\big)+\frac{a^3A}{x^2}+\frac{a^2(3Ab+aB)}{x}+\big(3aB(b$$
$$= -\frac{a^3A}{2x^2}+\frac{3}{2}a\big(abB+A(b^2+ac)\big)\,x^2+\frac{1}{4}\big(3aB(b^2+ac)+A(b^3+6abc)\big)$$

Mathematica [A]
time = 0.05, size = 162, normalized size = 1.00

$$-\frac{a^3A}{2x^2}+\frac{3}{2}a\big(abB+A(b^2+ac)\big)\,x^2+\frac{1}{4}\big(3aB(b^2+ac)+A(b^3+6abc)\big)\,x^4+\frac{1}{6}\big(b^3B+3Ab^2c+6abBc+3aAc^2\big)\,x^6+\frac{3}{8}c\big(b^2B+Abc+aBc\big)\,x^8+\frac{1}{10}c^2(3bB+Ac)x^{10}+\frac{1}{12}Bc^3x^{12}+a^2(3Ab+aB)\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x]
```

```
[Out] -1/2*(a^3*A)/x^2 + (3*a*(a*b*B + A*(b^2 + a*c))*x^2)/2 + ((3*a*B*(b^2 + a*c
) + A*(b^3 + 6*a*b*c))*x^4)/4 + ((b^3*B + 3*A*b^2*c + 6*a*b*B*c + 3*a*A*c^2
)*x^6)/6 + (3*c*(b^2*B + A*b*c + a*B*c)*x^8)/8 + (c^2*(3*b*B + A*c)*x^10)/1
0 + (B*c^3*x^12)/12 + a^2*(3*A*b + a*B)*Log[x]
```

Maple [A]
time = 0.02, size = 188, normalized size = 1.16

method	result
norman	$\frac{(\frac{1}{10}c^3A+\frac{3}{10}Bb\,c^2)x^{12}+(\frac{3}{2}a^2cA+\frac{3}{2}Aa\,b^2+\frac{3}{2}B\,a^2b)x^4+(\frac{3}{8}b\,c^2A+\frac{3}{8}c^2aB+\frac{3}{8}B\,b^2c)x^{10}+(\frac{1}{2}c^2aA+\frac{1}{2}A\,b^2c+abBc+\frac{1}{6}b^3B)x^8+(\frac{3}{2}Aabc+}{x^2}$
default	$\frac{B\,c^3x^{12}}{12}+\frac{A\,c^3x^{10}}{10}+\frac{3Bb\,c^2x^{10}}{10}+\frac{3Ab\,c^2x^8}{8}+\frac{3Ba\,c^2x^8}{8}+\frac{3B\,b^2c\,x^8}{8}+\frac{x^6c^2aA}{2}+\frac{A\,b^2c\,x^6}{2}+Babc\,x^6+\frac{B\,b^3x^6}{6}+$
risch	$\frac{B\,c^3x^{12}}{12}+\frac{A\,c^3x^{10}}{10}+\frac{3Bb\,c^2x^{10}}{10}+\frac{3Ab\,c^2x^8}{8}+\frac{3Ba\,c^2x^8}{8}+\frac{3B\,b^2c\,x^8}{8}+\frac{x^6c^2aA}{2}+\frac{A\,b^2c\,x^6}{2}+Babc\,x^6+\frac{B\,b^3x^6}{6}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*B*c^3*x^12+1/10*A*c^3*x^10+3/10*B*b*c^2*x^10+3/8*A*b*c^2*x^8+3/8*B*a*c
^2*x^8+3/8*B*b^2*c*x^8+1/2*x^6*c^2*a*A+1/2*A*b^2*c*x^6+B*a*b*c*x^6+1/6*B*b^
3*x^6+3/2*A*a*b*c*x^4+1/4*A*b^3*x^4+3/4*a^2*c*B*x^4+3/4*B*a*b^2*x^4+3/2*A*a
^2*c*x^2+3/2*A*a*b^2*x^2+3/2*B*a^2*b*x^2-1/2*a^3*A/x^2+a^2*(3*A*b+B*a)*ln(x
)
```

Maxima [A]
time = 0.29, size = 167, normalized size = 1.03

$$\frac{1}{12}\,Bc^3x^{12}+\frac{1}{10}\,(3\,Bbc^2+Ac^3)x^{10}+\frac{3}{8}\,(Bb^2c+(Ba+Ab)c^2)x^8+\frac{1}{6}\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^6+\frac{1}{4}\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^4+\frac{3}{2}\,(Ba^2b+Aab^2+Aa^2c)x^2-\frac{Aa^3}{2\,x^2}+\frac{1}{2}\,(Ba^3+3\,Aa^2b)\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="maxima")
```

```
[Out] 1/12*B*c^3*x^12 + 1/10*(3*B*b*c^2 + A*c^3)*x^10 + 3/8*(B*b^2*c + (B*a + A*b
)*c^2)*x^8 + 1/6*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^6 + 1/4*(3*B
*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^4 + 3/2*(B*a^2*b + A*a*b^2 + A*a^
2*c)*x^2 - 1/2*A*a^3/x^2 + 1/2*(B*a^3 + 3*A*a^2*b)*log(x^2)
```

Fricas [**A**]
time = 0.35, size = 170, normalized size = 1.05

$$\frac{10\,Bc^3x^{14}+12\,(3\,Bbc^2+Ac^3)x^{12}+45\,(Bb^2c+(Ba+Ab)c^2)x^{10}+20\,(Bb^3+3\,Aac^2+3\,(2\,Bab+Ab^2)c)x^8+30\,(3\,Bab^2+Ab^3+3\,(Ba^2+2\,Aab)c)x^6+180\,(Ba^2b+Aab^2+Aa^2c)x^4-60\,Aa^3+120\,(Ba^3+3\,Aa^2b)x^2\log{(x)}}{120\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="fricas")
```

```
[Out] 1/120*(10*B*c^3*x^14 + 12*(3*B*b*c^2 + A*c^3)*x^12 + 45*(B*b^2*c + (B*a + A
*b)*c^2)*x^10 + 20*(B*b^3 + 3*A*a*c^2 + 3*(2*B*a*b + A*b^2)*c)*x^8 + 30*(3*
B*a*b^2 + A*b^3 + 3*(B*a^2 + 2*A*a*b)*c)*x^6 + 180*(B*a^2*b + A*a*b^2 + A*a
^2*c)*x^4 - 60*A*a^3 + 120*(B*a^3 + 3*A*a^2*b)*x^2*log(x))/x^2
```

Sympy [**A**]
time = 0.18, size = 197, normalized size = 1.22

$$-\frac{Aa^3}{2x^2}+\frac{Bc^3x^{12}}{12}+a^2\cdot(3Ab+Ba)\log{(x)}+x^{10}\left(\frac{Ac^3}{10}+\frac{3Bbc^2}{10}\right)+x^8\cdot\left(\frac{3Abc^2}{8}+\frac{3Bac^2}{8}+\frac{3Bb^2c}{8}\right)+x^6\left(\frac{Aac^2}{2}+\frac{Ab^2c}{2}+Babc+\frac{Bb^3}{6}\right)+x^4\cdot\left(\frac{3Aabc}{2}+\frac{Ab^3}{4}+\frac{3Ba^2c}{4}+\frac{3Bab^2}{4}\right)+x^2\cdot\left(\frac{3Aa^2c}{2}+\frac{3Aab^2}{2}+\frac{3Ba^2b}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)*(c*x**4+b*x**2+a)**3/x**3,x)
```

```
[Out] -A*a**3/(2*x**2) + B*c**3*x**12/12 + a**2*(3*A*b + B*a)*log(x) + x**10*(A*c
**3/10 + 3*B*b*c**2/10) + x**8*(3*A*b*c**2/8 + 3*B*a*c**2/8 + 3*B*b**2*c/8)
 + x**6*(A*a*c**2/2 + A*b**2*c/2 + B*a*b*c + B*b**3/6) + x**4*(3*A*a*b*c/2
+ A*b**3/4 + 3*B*a**2*c/4 + 3*B*a*b**2/4) + x**2*(3*A*a**2*c/2 + 3*A*a*b**2
/2 + 3*B*a**2*b/2)
```

Giac [**A**]
time = 4.21, size = 212, normalized size = 1.31

$$\frac{1}{12}\,Bc^3x^{12}+\frac{3}{10}\,Bbc^2x^{10}+\frac{1}{10}\,Ac^3x^{10}+\frac{3}{8}\,Bb^2cx^8+\frac{3}{8}\,Bac^2x^8+\frac{3}{8}\,Abc^2x^8+\frac{1}{6}\,Bb^3x^6+Babcx^6+\frac{1}{2}\,Ab^2cx^6+\frac{1}{2}\,Aac^2x^6+\frac{3}{4}\,Bab^2x^4+\frac{1}{4}\,Ab^3x^4+\frac{3}{4}\,Ba^2cx^4+\frac{3}{2}\,Aabcx^4+\frac{3}{2}\,Ba^2bx^2+\frac{3}{2}\,Aab^2x^2+\frac{3}{2}\,Aa^2cx^2+\frac{1}{2}\,(Ba^3+3\,Aa^2b)\log{(x^2)}-\frac{Ba^3x^2+3\,Aa^2bx^2+Aa^3}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)*(c*x^4+b*x^2+a)^3/x^3,x, algorithm="giac")
```

```
[Out] 1/12*B*c^3*x^12 + 3/10*B*b*c^2*x^10 + 1/10*A*c^3*x^10 + 3/8*B*b^2*c*x^8 + 3
/8*B*a*c^2*x^8 + 3/8*A*b*c^2*x^8 + 1/6*B*b^3*x^6 + B*a*b*c*x^6 + 1/2*A*b^2*
c*x^6 + 1/2*A*a*c^2*x^6 + 3/4*B*a*b^2*x^4 + 1/4*A*b^3*x^4 + 3/4*B*a^2*c*x^4
```

```
 + 3/2*A*a*b*c*x^4 + 3/2*B*a^2*b*x^2 + 3/2*A*a*b^2*x^2 + 3/2*A*a^2*c*x^2 +
1/2*(B*a^3 + 3*A*a^2*b)*log(x^2) - 1/2*(B*a^3*x^2 + 3*A*a^2*b*x^2 + A*a^3)/
x^2
```

Mupad [B]

time = 0.06, size = 166, normalized size = 1.02

$$x^4\left(\frac{3\,B\,c\,a^2}{4}+\frac{3\,B\,a\,b^2}{4}+\frac{3\,A\,c\,a\,b}{2}+\frac{A\,b^3}{4}\right)+x^6\left(\frac{B\,b^3}{6}+\frac{A\,b^2\,c}{2}+B\,a\,b\,c+\frac{A\,a\,c^2}{2}\right)+x^{10}\left(\frac{A\,c^3}{10}+\frac{3\,B\,b\,c^2}{10}\right)+\ln\left(x\right)\,\left(B\,a^3+3\,A\,b\,a^2\right)+x^2\left(\frac{3\,B\,a^2\,b}{2}+\frac{3\,A\,c\,a^2}{2}+\frac{3\,A\,a\,b^2}{2}\right)+x^8\left(\frac{3\,B\,b^2\,c}{8}+\frac{3\,A\,b\,c^2}{8}+\frac{3\,B\,a\,c^2}{8}\right)-\frac{A\,a^3}{2\,x^2}+\frac{B\,c^3\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B*x^2)*(a + b*x^2 + c*x^4)^3)/x^3,x)
```

```
[Out] x^4*((A*b^3)/4 + (3*B*a*b^2)/4 + (3*B*a^2*c)/4 + (3*A*a*b*c)/2) + x^6*((B*b
^3)/6 + (A*a*c^2)/2 + (A*b^2*c)/2 + B*a*b*c) + x^10*((A*c^3)/10 + (3*B*b*c^
2)/10) + log(x)*(B*a^3 + 3*A*a^2*b) + x^2*((3*A*a*b^2)/2 + (3*A*a^2*c)/2 +
(3*B*a^2*b)/2) + x^8*((3*A*b*c^2)/8 + (3*B*a*c^2)/8 + (3*B*b^2*c)/8) - (A*a
^3)/(2*x^2) + (B*c^3*x^12)/12
```

3.102 $\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=133

$$-\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{(b^3B-Ab^2c-3abBc+2aAc^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b^2B-Abc-aBc)\log(a+\cdots}{4c^3}$$

```
[Out] -1/2*(-A*c+B*b)*x^2/c^2+1/4*B*x^4/c+1/4*(-A*b*c-B*a*c+B*b^2)*ln(c*x^4+b*x^2
+a)/c^3+1/2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)*arctanh((2*c*x^2+b)/(-4*a*c
+b^2)^(1/2))/c^3/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 814, 648, 632, 212, 642}

$$\frac{(-aBc-Abc+b^2B)\log(a+bx^2+cx^4)}{4c^3}+\frac{(2aAc^2-3abBc-Ab^2c+b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}-\frac{x^2(bB-Ac)}{2c^2}+\frac{Bx^4}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*((b*B - A*c)*x^2)/c^2 + (B*x^4)/(4*c) + ((b^3*B - A*b^2*c - 3*a*b*B*c
+ 2*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*Sqrt[b^2 - 4*
a*c]) + ((b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(A+Bx^2)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{bB-Ac}{c^2}+\frac{Bx}{c}+\frac{a(bB-Ac)+(b^2B-Abc-aBc)\,x}{c^2\,(a+bx+cx^2)}\right)\,dx,x,x^2\right)\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{\text{Subst}\left(\int \frac{a(bB-Ac)+(b^2B-Abc-aBc)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c^2}\\
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{(b^2B-Abc-aBc)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^3}-\frac{(b^3B-A
\end{aligned}$$

$$\begin{aligned}
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{(b^2B-Abc-aBc)\log\left(a+bx^2+cx^4\right)}{4c^3}+\frac{(b^3B-Ab^2c-3a
\end{aligned}$$

$$\begin{aligned}
&= -\frac{(bB-Ac)x^2}{2c^2}+\frac{Bx^4}{4c}+\frac{(b^3B-Ab^2c-3abBc+2aAc^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}}+\frac{(b
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 126, normalized size = 0.95

$$\frac{2c(-bB+Ac)x^2+Bc^2x^4+\frac{2\left(-b^3B+Ab^2c+3abBc-2aAc^2\right)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(b^2B-Abc-aBc)\log\left(a+bx^2+cx^4\right)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(-(b*B) + A*c)*x^2 + B*c^2*x^4 + (2*(-(b^3*B) + A*b^2*c + 3*a*b*B*c -
2*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c] + (
b^2*B - A*b*c - a*B*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A]
time = 0.09, size = 136, normalized size = 1.02

method	result	siz
default	$\frac{\frac{1}{2}Bc\,x^4+Ac\,x^2-Bb\,x^2}{2c^2}+\frac{\frac{\left(-bcA-acB+b^2B\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(-acA+abB-\frac{\left(-bcA-acB+b^2B\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2c^2}$	13
risch	Expression too large to display	21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c^2*(1/2*B*c*x^4+A*c*x^2-B*b*x^2)+1/2/c^2*(1/2*(-A*b*c-B*a*c+B*b^2)/c*l
n(c*x^4+b*x^2+a)+2*(-a*c*A+a*b*B-1/2*(-A*b*c-B*a*c+B*b^2)*b/c)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.39, size = 421, normalized size = 3.17

$$\left[\frac{(Bb^2c^2-4\,Bac^3)x^4-2\,(Bb^3c+4\,Aac^3-(4\,Bab+Ab^2)c^2)x^2+(Bb^3+2\,Aac^2-(3\,Bab+Ab^2)c)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(Bb^4+4\,(Ba^2+Aab)c^2-(5\,Bab^2+Ab^3)c)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^3-4\,ac^4)},\ \frac{(Bb^2c^2-4\,Bac^3)x^4-2\,(Bb^3c+4\,Aac^3-(4\,Bab+Ab^2)c^2)x^2+2\,(Bb^3+2\,Aac^2-(3\,Bab+Ab^2)c)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(Bb^4+4\,(Ba^2+Aab)c^2-(5\,Bab^2+Ab^3)c)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c^3-4\,ac^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*b^2*c^2 - 4*B*a*c^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b
^2)*c^2)*x^2 + (B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(b^2 - 4*a*c)*
log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))
/(c*x^4 + b*x^2 + a)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*b^2 + A*b^3
)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4), 1/4*((B*b^2*c^2 - 4*B*a*c
^3)*x^4 - 2*(B*b^3*c + 4*A*a*c^3 - (4*B*a*b + A*b^2)*c^2)*x^2 + 2*(B*b^3 +
2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*s
qrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*b^4 + 4*(B*a^2 + A*a*b)*c^2 - (5*B*a*
b^2 + A*b^3)*c)*log(c*x^4 + b*x^2 + a))/(b^2*c^3 - 4*a*c^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.58, size = 126, normalized size = 0.95

$$\frac{Bcx^4 - 2\,Bbx^2 + 2\,Acx^2}{4\,c^2} + \frac{(Bb^2 - Bac - Abc)\log\left(cx^4 + bx^2 + a\right)}{4\,c^3} - \frac{(Bb^3 - 3\,Babc - Ab^2c + 2\,Aac^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(B*c*x^4 - 2*B*b*x^2 + 2*A*c*x^2)/c^2 + 1/4*(B*b^2 - B*a*c - A*b*c)*log
(c*x^4 + b*x^2 + a)/c^3 - 1/2*(B*b^3 - 3*B*a*b*c - A*b^2*c + 2*A*a*c^2)*arc
tan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c^3)
```

Mupad [**B**]
time = 0.46, size = 1343, normalized size = 10.10

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x^2*(A/(2*c) - (B*b)/(2*c^2)) + (B*x^4)/(4*c) - (log(a + b*x^2 + c*x^4)*(2*
B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4
 - 4*b^2*c^3)) + (atan((2*c^4*(4*a*c - b^2)*(x^2*(((((6*A*b^2*c^4 - 6*B*b^3
*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*
```

```
A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A
*a*c^2 - A*b^2*c - 3*B*a*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (b*(B*b^3 + 2*
A*a*c^2 - A*b^2*c - 3*B*a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b
*c^2 - 10*B*a*b^2*c))/(2*c*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a -
 (b*((((6*A*b^2*c^4 - 6*B*b^3*c^3 - 4*A*a*c^5 + 10*B*a*b*c^4)/c^4 - (4*b*c^
2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c
^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*
b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) - (B^2*b^5 + A^2*b^3*c^2 - 2*A*B*b^4*c -
 A*B*a^2*c^3 - A^2*a*b*c^3 - 3*B^2*a*b^3*c + 2*B^2*a^2*b*c^2 + 4*A*B*a*b^2*
c^2)/c^4 + (b*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(2*c^4*(4*a*c -
b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + ((((8*B*a^2*c^4 + 8*A*a*b*c^4 - 8*B*a*
b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 -
10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a
*b*c))/(8*c^3*(4*a*c - b^2)^(1/2)) - (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*
a*b*c)*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(c
*(4*a*c - b^2)^(1/2)*(16*a*c^4 - 4*b^2*c^3)))/a - (b*((((8*B*a^2*c^4 + 8*A*
a*b*c^4 - 8*B*a*b^2*c^3)/c^4 - (8*a*c^2*(2*B*b^4 + 8*B*a^2*c^2 - 2*A*b^3*c
+ 8*A*a*b*c^2 - 10*B*a*b^2*c))/(16*a*c^4 - 4*b^2*c^3))*(2*B*b^4 + 8*B*a^2*c
^2 - 2*A*b^3*c + 8*A*a*b*c^2 - 10*B*a*b^2*c))/(2*(16*a*c^4 - 4*b^2*c^3)) -
(B^2*a*b^4 + B^2*a^3*c^2 + A^2*a*b^2*c^2 - 2*B^2*a^2*b^2*c - 2*A*B*a*b^3*c
+ 2*A*B*a^2*b*c^2)/c^4 + (a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c)^2)/(c
^4*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))))/(B^2*b^6 + 4*A^2*a^2*c^4 +
A^2*b^4*c^2 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2
*c^3 + 10*A*B*a*b^3*c^2 - 12*A*B*a^2*b*c^3))*(B*b^3 + 2*A*a*c^2 - A*b^2*c -
 3*B*a*b*c))/(2*c^3*(4*a*c - b^2)^(1/2))
```

3.103 $\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=97

$$\frac{Bx^2}{2c}-\frac{(b^2B-Abc-2aBc)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

```
[Out] 1/2*B*x^2/c-1/4*(-A*c+B*b)*ln(c*x^4+b*x^2+a)/c^2-1/2*(-A*b*c-2*B*a*c+B*b^2)
*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 787, 648, 632, 212, 642}

$$-\frac{(-2aBc-Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}+\frac{Bx^2}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x^2)/(2*c) - ((b^2*B - A*b*c - 2*a*B*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 -
 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]) - ((b*B - A*c)*Log[a + b*x^2 + c*x^4])/
(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3(A+Bx^2)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{a+bx+cx^2}\,dx,x,x^2\right)\\
&= \frac{Bx^2}{2c}+\frac{\text{Subst}\left(\int \frac{-aB+(-bB+Ac)x}{a+bx+cx^2}\,dx,x,x^2\right)}{2c}\\
&= \frac{Bx^2}{2c}-\frac{(bB-Ac)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4c^2}+\frac{(b^2B-Abc-2aBc)\,\text{Subst}\left(\int \frac{1}{a+bx+
\end{aligned}$$

$$\begin{aligned}
&= \frac{Bx^2}{2c}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}-\frac{(b^2B-Abc-2aBc)\,\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,
\end{aligned}$$

$$\begin{aligned}
&= \frac{Bx^2}{2c}-\frac{(b^2B-Abc-2aBc)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}}-\frac{(bB-Ac)\log\left(a+bx^2+cx^4\right.}{4c^2}
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 93, normalized size = 0.96

$$\frac{2Bcx^2+\frac{2(b^2B-Abc-2aBc)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+(-bB+Ac)\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*B*c*x^2 + (2*(b^2*B - A*b*c - 2*a*B*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 +
4*a*c]])/Sqrt[-b^2 + 4*a*c] + (-(b*B) + A*c)*Log[a + b*x^2 + c*x^4])/(4*c^2
)
```

Maple [A]
time = 0.06, size = 98, normalized size = 1.01

method	result
default	$\frac{B\,x^{2}}{2c}+\frac{\frac{(Ac-bB)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(-aB-\frac{(Ac-bB)b}{2c}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2c}$
risch	$\frac{B\,x^{2}}{2c}+\frac{\ln\left(\left(\left(4Aab\,c^{2}-A\,b^{3}c+8a^{2}B\,c^{2}-6a\,b^{2}Bc+b^{4}B-\sqrt{-\left(4ac-b^{2}\right)\left(bcA+2acB-b^{2}B\right)^{2}}\,b\right)x^{2}-2\sqrt{-\left(4ac-\right.}\right.\right.}{4ac-b^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c+1/2/c*(1/2*(A*c-B*b)/c*ln(c*x^4+b*x^2+a)+2*(-a*B-1/2*(A*c-B*b)*
b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 312, normalized size = 3.22

$$\left[\frac{2\,(Bb^{2}c-4\,Bac^{2})x^{2}-(Bb^{2}-(2\,Ba+Ab)c)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-(Bb^{3}+4\,Aac^{2}-(4\,Bab+Ab^{2})c)\log\left(cx^{4}+bx^{2}+a\right)}{4\,(b^{2}c^{2}-4\,ac^{3})},\frac{2\,(Bb^{2}c-4\,Bac^{2})x^{2}-2\,(Bb^{2}-(2\,Ba+Ab)c)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-(Bb^{3}+4\,Aac^{2}-(4\,Bab+Ab^{2})c)\log\left(cx^{4}+bx^{2}+a\right)}{4\,(b^{2}c^{2}-4\,ac^{3})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 - (B*b^2 - (2*B*a + A*b)*c)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
```

```
a*c))/(c*x^4 + b*x^2 + a)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*log(
c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3), 1/4*(2*(B*b^2*c - 4*B*a*c^2)*x^2 -
 2*(B*b^2 - (2*B*a + A*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(
-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (B*b^3 + 4*A*a*c^2 - (4*B*a*b + A*b^2)*c)*lo
g(c*x^4 + b*x^2 + a))/(b^2*c^2 - 4*a*c^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(90) = 180$.

time = 71.09, size = 434, normalized size = 4.47

$$\frac{Bx^2}{2c} + \left(-\frac{-Ac+Bb}{4c^2} - \frac{\sqrt{-4ac+b^2}\,(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{2Aac - Bab - 8ac^2\left(-\frac{-Ac+Bb}{4c^2} - \frac{\sqrt{-4ac+b^2}\,(Abc+2Bac-Bb^2)}{4c^2\,(4ac-b^2)}\right) + 2b^2c\left(-\frac{-Ac+Bb}{4c^2} - \frac{\sqrt{-4ac+b^2}\,(Abc+2Bac-Bb^2)}{4c^2\,(4ac-b^2)}\right)}{Abc+2Bac-Bb^2}\right) + \left(-\frac{-Ac+Bb}{4c^2} + \frac{\sqrt{-4ac+b^2}\,(Abc+2Bac-Bb^2)}{4c^2\cdot(4ac-b^2)}\right)\log\left(x^2 + \frac{2Aac - Bab - 8ac^2\left(-\frac{-Ac+Bb}{4c^2} + \frac{\sqrt{-4ac+b^2}\,(Abc+2Bac-Bb^2)}{4c^2\,(4ac-b^2)}\right) + 2b^2c\left(-\frac{-Ac+Bb}{4c^2} + \frac{\sqrt{-4ac+b^2}\,(Abc+2Bac-Bb^2)}{4c^2\,(4ac-b^2)}\right)}{Abc+2Bac-Bb^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] B*x**2/(2*c) + (-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) - sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
c - B*b**2)) + (-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a
*c - B*b**2)/(4*c**2*(4*a*c - b**2)))*log(x**2 + (2*A*a*c - B*a*b - 8*a*c**
2*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c + b**2)*(A*b*c + 2*B*a*c - B*b**2)/
(4*c**2*(4*a*c - b**2))) + 2*b**2*c*(-(-A*c + B*b)/(4*c**2) + sqrt(-4*a*c +
 b**2)*(A*b*c + 2*B*a*c - B*b**2)/(4*c**2*(4*a*c - b**2))))/(A*b*c + 2*B*a*
c - B*b**2))
```

Giac [A]

time = 3.40, size = 91, normalized size = 0.94

$$\frac{Bx^2}{2\,c} - \frac{(Bb - Ac)\log\left(cx^4 + bx^2 + a\right)}{4\,c^2} + \frac{(Bb^2 - 2\,Bac - Abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\;c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c - 1/4*(B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^2 + 1/2*(B*b^2 - 2*B
*a*c - A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*
c^2)
```

Mupad [B]

time = 0.65, size = 979, normalized size = 10.09

$$\frac{Bx^2}{2c} + \frac{\ln\left(cx^4+bx^2+a\right)\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{2\left(16ac^3-4b^2c^2\right)} - \frac{\mathrm{atan}\left(\frac{2c^2\left(4ac-b^2\right)\left(\frac{\frac{\left(\frac{8Aac^3-8Babc^2}{c^2}-\frac{8ac^2\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{16ac^3-4b^2c^2}\right)\left(-Bb^2+Acb+2Bac\right)}{8c^2\sqrt{4ac-b^2}}-\frac{a\left(-Bb^2+Acb+2Bac\right)\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{\sqrt{4ac-b^2}\left(16ac^3-4b^2c^2\right)}}{a}+x^2\left(\frac{\frac{\left(\frac{-6Bb^2c^2+6Abc^3+4Bac^3}{c^2}-\frac{4bc^2\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{16ac^3-4b^2c^2}\right)\left(-Bb^2+Acb+2Bac\right)}{8c^2\sqrt{4ac-b^2}}-\frac{b\left(-Bb^2+Acb+2Bac\right)\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{2\sqrt{4ac-b^2}\left(16ac^3-4b^2c^2\right)}}{a}+\frac{b\left(\frac{\left(\frac{-6Bb^2c^2+6Abc^3+4Bac^3}{c^2}-\frac{4bc^2\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{16ac^3-4b^2c^2}\right)\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{2\left(16ac^3-4b^2c^2\right)}-\frac{A^2bc^2-2ABb^2c+aABc^2+B^2b^3-aB^2bc}{c^2}+\frac{b\left(-Bb^2+Acb+2Bac\right)^2}{2c^2\left(4ac-b^2\right)}\right)}{2a\sqrt{4ac-b^2}}\right)+\frac{b\left(\frac{\left(\frac{8Aac^3-8Babc^2}{c^2}-\frac{8ac^2\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{16ac^3-4b^2c^2}\right)\left(2Bb^3-2Ab^2c-8Babc+8Aac^2\right)}{2\left(16ac^3-4b^2c^2\right)}-\frac{aA^2c^2-2aABbc+aB^2b^2}{c^2}+\frac{a\left(-Bb^2+Acb+2Bac\right)^2}{c^2\left(4ac-b^2\right)}\right)}{2a\sqrt{4ac-b^2}}\right)}{A^2b^2c^2+4ABabc^2-2ABb^3c+4B^2a^2c^2-4B^2ab^2c+B^2b^4}\right)\left(-Bb^2+Acb+2Bac\right)}{2c^2\sqrt{4ac-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (B*x^2)/(2*c) + (log(a + b*x^2 + c*x^4)*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c -
8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (atan((2*c^2*(4*a*c - b^2)*(((((8*
A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*
B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(8*c^2*(4*a*c
- b^2)^(1/2)) - (a*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2
*c - 8*B*a*b*c))/((4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^2)))/a + x^2*((((
(6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^3 + 8*A*a*c^2 -
 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(A*b*c - B*b^2 + 2*B*a*c))
/(8*c^2*(4*a*c - b^2)^(1/2)) - (b*(A*b*c - B*b^2 + 2*B*a*c)*(2*B*b^3 + 8*A*
a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(4*a*c - b^2)^(1/2)*(16*a*c^3 - 4*b^2*c^
2)))/a + (b*((((6*A*b*c^3 - 6*B*b^2*c^2 + 4*B*a*c^3)/c^2 - (4*b*c^2*(2*B*b^
3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 +
8*A*a*c^2 - 2*A*b^2*c - 8*B*a*b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (B^2*b^3 +
 A^2*b*c^2 + A*B*a*c^2 - 2*A*B*b^2*c - B^2*a*b*c)/c^2 + (b*(A*b*c - B*b^2 +
 2*B*a*c)^2)/(2*c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2)^(1/2))) + (b*((((8*
A*a*c^3 - 8*B*a*b*c^2)/c^2 - (8*a*c^2*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*
B*a*b*c))/(16*a*c^3 - 4*b^2*c^2))*(2*B*b^3 + 8*A*a*c^2 - 2*A*b^2*c - 8*B*a*
b*c))/(2*(16*a*c^3 - 4*b^2*c^2)) - (A^2*a*c^2 + B^2*a*b^2 - 2*A*B*a*b*c)/c^
2 + (a*(A*b*c - B*b^2 + 2*B*a*c)^2)/(c^2*(4*a*c - b^2))))/(2*a*(4*a*c - b^2
)^(1/2))))/(B^2*b^4 + A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b
^2*c + 4*A*B*a*b*c^2))*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)^(1/2
))
```

3.104 $\int \frac{x(A+Bx^2)}{a+bx^2+cx^4} dx$

Optimal. Leaf size=71

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

```
[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+1/2*(-2*A*c+B*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1261, 648, 632, 212, 642}

$$\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c*Sqrt[b^2 - 4*
a*c]) + (B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{a+bx+cx^2}\,dx, x, x^2\right)\\ &= \frac{B\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c}+\frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{4c}\\ &= \frac{B\log\left(a+bx^2+cx^4\right)}{4c}-\frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{2c}\\ &= \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}}+\frac{B\log\left(a+bx^2+cx^4\right)}{4c}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 1.00

$$\frac{-\frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}+B\log\left(a+bx^2+cx^4\right)}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-2*(b*B - 2*A*c)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*
a*c] + B*Log[a + b*x^2 + c*x^4])/(4*c)
```

Maple [A]
time = 0.06, size = 65, normalized size = 0.92

method	result

default	$\frac{B\ln(c\,x^4+b\,x^2+a)}{4c}+\frac{\left(A-\frac{Bb}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}$
risch	$\frac{\ln\left(\left(\left(-8c^2aA+2A\,b^2c+4abBc-b^3B-\sqrt{-\left(4ac-b^2\right)\left(2Ac-bB\right)^2}\,b\right)x^2-2\sqrt{-\left(4ac-b^2\right)\left(2Ac-bB\right)^2}\,a\right)a}{4ac-b^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B*ln(c*x^4+b*x^2+a)/c+(A-1/2*B*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b
)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 219, normalized size = 3.08

$$\left[-\frac{(Bb-2\,Ac)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-(Bb^2-4\,Bac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)},\ \frac{2\,(Bb-2\,Ac)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(Bb^2-4\,Bac)\log\left(cx^4+bx^2+a\right)}{4\,(b^2c-4\,ac^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((B*b - 2*A*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (B*b^2 - 4*B
*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2), 1/4*(2*(B*b - 2*A*c)*sqrt(
-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + (B*
b^2 - 4*B*a*c)*log(c*x^4 + b*x^2 + a))/(b^2*c - 4*a*c^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(61) = 122$.
time = 5.04, size = 287, normalized size = 4.04

$$\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}-\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)}{-2Ac+Bb}\right)+\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c\,(4ac-b^2)}\right)\log\left(x^2+\frac{-Ab+2Ba-8ac\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)+2b^2\left(\frac{B}{4c}+\frac{(-2Ac+Bb)\sqrt{-4ac+b^2}}{4c(4ac-b^2)}\right)}{-2Ac+Bb}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] (B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**
2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*
c*(4*a*c - b**2))) + 2*b**2*(B/(4*c) - (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(
4*c*(4*a*c - b**2))))/(-2*A*c + B*b)) + (B/(4*c) + (-2*A*c + B*b)*sqrt(-4*a
*c + b**2)/(4*c*(4*a*c - b**2)))*log(x**2 + (-A*b + 2*B*a - 8*a*c*(B/(4*c)
+ (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))) + 2*b**2*(B/(4*c
) + (-2*A*c + B*b)*sqrt(-4*a*c + b**2)/(4*c*(4*a*c - b**2))))/(-2*A*c + B*b
))
```

Giac [A]
time = 5.40, size = 67, normalized size = 0.94

$$\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c}-\frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*B*log(c*x^4 + b*x^2 + a)/c - 1/2*(B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqr
t(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*c)
```

Mupad [B]
time = 0.50, size = 606, normalized size = 8.54

$$-\frac{\ln\left(c\,x^4+b\,x^2+a\right)\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)}-\frac{\mathrm{atan}\left(\frac{2\,\left(4\,a\,c-b^2\right)\,\left(x^2\,\left(\frac{\frac{\left(2\,A\,c-B\,b\right)\,\left(6\,B\,b\,c-4\,A\,c^2+\frac{4\,b\,c^2\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{16\,a\,c^2-4\,b^2\,c}\right)}{8\,c\,\sqrt{4\,a\,c-b^2}}+\frac{b\,c\,\left(2\,B\,b^2-8\,B\,a\,c\right)\,\left(2\,A\,c-B\,b\right)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)\,\sqrt{4\,a\,c-b^2}}}{a}+\frac{b\,\left(B^2\,b-A\,B\,c-\frac{b\,{\left(2\,A\,c-B\,b\right)}^2}{2\,\left(4\,a\,c-b^2\right)}+\frac{\left(2\,B\,b^2-8\,B\,a\,c\right)\,\left(6\,B\,b\,c-4\,A\,c^2+\frac{4\,b\,c^2\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{16\,a\,c^2-4\,b^2\,c}\right)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)}\right)}{2\,a\,\sqrt{4\,a\,c-b^2}}\right)+\frac{\frac{\left(8\,B\,a\,c+\frac{8\,a\,c^2\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{16\,a\,c^2-4\,b^2\,c}\right)\,\left(2\,A\,c-B\,b\right)}{8\,c\,\sqrt{4\,a\,c-b^2}}+\frac{a\,c\,\left(2\,B\,b^2-8\,B\,a\,c\right)\,\left(2\,A\,c-B\,b\right)}{\left(16\,a\,c^2-4\,b^2\,c\right)\,\sqrt{4\,a\,c-b^2}}}{a}+\frac{b\,\left(B^2\,a+\frac{\left(2\,B\,b^2-8\,B\,a\,c\right)\,\left(8\,B\,a\,c+\frac{8\,a\,c^2\,\left(2\,B\,b^2-8\,B\,a\,c\right)}{16\,a\,c^2-4\,b^2\,c}\right)}{2\,\left(16\,a\,c^2-4\,b^2\,c\right)}-\frac{a\,{\left(2\,A\,c-B\,b\right)}^2}{4\,a\,c-b^2}\right)}{2\,a\,\sqrt{4\,a\,c-b^2}}\right)}{4\,A^2\,c^2-4\,A\,B\,b\,c+B^2\,b^2}\right)\,\left(2\,A\,c-B\,b\right)}{2\,c\,\sqrt{4\,a\,c-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] - (log(a + b*x^2 + c*x^4)*(2*B*b^2 - 8*B*a*c))/(2*(16*a*c^2 - 4*b^2*c)) - (
atan((2*(4*a*c - b^2)*(x^2*((((2*A*c - B*b)*(6*B*b*c - 4*A*c^2 + (4*b*c^2*(
2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))/(8*c*(4*a*c - b^2)^(1/2)) + (b*c
*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*b))/(2*(16*a*c^2 - 4*b^2*c)*(4*a*c - b^2)^(
1/2)))/a + (b*(B^2*b - A*B*c - (b*(2*A*c - B*b)^2)/(2*(4*a*c - b^2)) + ((2*
B*b^2 - 8*B*a*c)*(6*B*b*c - 4*A*c^2 + (4*b*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c
^2 - 4*b^2*c)))/(2*(16*a*c^2 - 4*b^2*c))))/(2*a*(4*a*c - b^2)^(1/2))) + (((
8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c))*(2*A*c - B*b)
)/(8*c*(4*a*c - b^2)^(1/2)) + (a*c*(2*B*b^2 - 8*B*a*c)*(2*A*c - B*b))/((16*
```

```
a*c^2 - 4*b^2*c)*(4*a*c - b^2)^(1/2)))/a + (b*(B^2*a + ((2*B*b^2 - 8*B*a*c)
*(8*B*a*c + (8*a*c^2*(2*B*b^2 - 8*B*a*c))/(16*a*c^2 - 4*b^2*c)))/(2*(16*a*c
^2 - 4*b^2*c)) - (a*(2*A*c - B*b)^2)/(4*a*c - b^2)))/(2*a*(4*a*c - b^2)^(1/
2))))/(4*A^2*c^2 + B^2*b^2 - 4*A*B*b*c))*(2*A*c - B*b))/(2*c*(4*a*c - b^2)^
(1/2))
```

3.105 $\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=78

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}$$

```
[Out] A*ln(x)/a-1/4*A*ln(c*x^4+b*x^2+a)/a+1/2*(A*b-2*B*a)*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 814, 648, 632, 212, 642}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}+\frac{A\log(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*Sqrt[b^2 - 4*
a*c]) + (A*Log[x])/a - (A*Log[a + b*x^2 + c*x^4])/(4*a)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax}+\frac{-Ab+aB-Acx}{a\left(a+bx+cx^2\right)}\right)dx,x,x^2\right)\\
&= \frac{A\log(x)}{a}+\frac{\text{Subst}\left(\int \frac{-Ab+aB-Acx}{a+bx+cx^2}\,dx,x,x^2\right)}{2a}\\
&= \frac{A\log(x)}{a}-\frac{A\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4a}+\frac{(-Ab+2aB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx,x\right.}{4a}\\
&= \frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}-\frac{(-Ab+2aB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx,x,b+2c\right.}{2a}\\
&= \frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}}+\frac{A\log(x)}{a}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 128, normalized size = 1.64

$$\frac{4A\sqrt{b^2-4ac}\,\log(x)-\left(-2aB+A\left(b+\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)+\left(-2aB+A\left(b-\sqrt{b^2-4ac}\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{4a\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*A*Sqrt[b^2 - 4*a*c]*Log[x] - (-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*Log[b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2] + (-2*a*B + A*(b - Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(4*a*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.04, size = 76, normalized size = 0.97

method	result
default	$-\frac{\frac{A\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}+\frac{2\left(\frac{Ab}{2}-aB\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a}+\frac{A\ln\left(x\right)}{a}$
risch	$\frac{A\ln\left(x\right)}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c-a\,b^{2}\right)\_Z^{2}+\left(4acA-A\,b^{2}\right)\_Z+A^{2}c-bBA+B^{2}a\right)}\_R\ln\left(\left(\left(10ac-3b^{2}\right)\_R^{2}+\left(5Ac-bB\right)\_R+2B^{2}\right)x^{2}-\right.\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a*(1/2*A*ln(c*x^4+b*x^2+a)+2*(1/2*A*b-a*B)/(4*a*c-b^2)^(1/2)*arctan((2
*c*x^2+b)/(4*a*c-b^2)^(1/2)))+A*ln(x)/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.37, size = 249, normalized size = 3.19

$$\left[-\frac{(2\,Ba-Ab)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+(Ab^2-4\,Aac)\log\left(cx^4+bx^2+a\right)-4\,(Ab^2-4\,Aac)\log\left(x\right)}{4\,(ab^2-4\,a^2c)},-\frac{2\,(2\,Ba-Ab)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+(Ab^2-4\,Aac)\log\left(cx^4+bx^2+a\right)-4\,(Ab^2-4\,Aac)\log\left(x\right)}{4\,(ab^2-4\,a^2c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((2*B*a - A*b)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + (A*b^2 - 4*A
```

```
*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c
), -1/4*(2*(2*B*a - A*b)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2
 + 4*a*c)/(b^2 - 4*a*c)) + (A*b^2 - 4*A*a*c)*log(c*x^4 + b*x^2 + a) - 4*(A*
b^2 - 4*A*a*c)*log(x))/(a*b^2 - 4*a^2*c)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.88, size = 78, normalized size = 1.00

$$-\frac{A\log\left(cx^4+bx^2+a\right)}{4\,a}+\frac{A\log\left(x^2\right)}{2\,a}+\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*A*log(c*x^4 + b*x^2 + a)/a + 1/2*A*log(x^2)/a + 1/2*(2*B*a - A*b)*arct
an((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(sqrt(-b^2 + 4*a*c)*a)
```

Mupad [**B**]
time = 4.48, size = 2424, normalized size = 31.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (A*log(x))/a - (log((A*B^2*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^
2)))^(1/2))*(B^2*a*c^2 + ((A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/
2))*(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 +
 (b*c^2*(A + a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x
^2 - 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))/(4*a)
 + B^3*c^2*x^2)*(A*B^2*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))
^(1/2))*(B^2*a*c^2 + ((A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*
(4*A*b^2*c^2 + 2*c^2*x^2*(5*A*b*c - 4*B*b^2 + 10*B*a*c) - 4*B*a*b*c^2 + (b*
c^2*(A - a*(-(A*b - 2*B*a)^2/(a^2*(4*a*c - b^2)))^(1/2))*(a*b + 3*b^2*x^2 -
 10*a*c*x^2))/a))/(4*a) - 4*A*B*b*c^2 - B*c^2*x^2*(5*A*c + B*b)))/(4*a) + B
```

```
^3*c^2*x^2))*(2*A*b^2 - 8*A*a*c))/(2*(4*a*b^2 - 16*a^2*c)) - (atan((2*(4*a*
c - b^2)^(3/2)*(3*A*b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(A*B^2*c^2 + ((2*A
*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2
*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a*b^2 - 16*a^2*c)) +
 B^2*a*c^2 - 4*A*B*b*c^2))/(2*(4*a*b^2 - 16*a^2*c)) - ((A*b - 2*B*a)*(((A*b
 - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4
*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A*a*
c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(4*a*(4*a*
c - b^2)^(1/2)) - (b^2*c^2*(2*A*b^2 - 8*A*a*c)*(A*b - 2*B*a)^2)/(8*a*(4*a*b
^2 - 16*a^2*c)*(4*a*c - b^2))))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c^2 - 4*A*B*a
*b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)) - (16*a^3*x^2*(((3*A*
b^3 - B*a*b^2 + B*a^2*c - 8*A*a*b*c)*(((2*A*b^2 - 8*A*a*c)*(B^2*b*c^2 + 5*A
*B*c^3 - ((2*A*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^
3))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(
4*a*b^2 - 16*a^2*c))))/(2*(4*a*b^2 - 16*a^2*c)) - B^3*c^2 + ((A*b - 2*B*a)*
(((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^
2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2)
^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8*
a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(4*a*(4*a*c - b^2)^(1/2)) + (
(2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^2)/(32*a^2*(4*a
*b^2 - 16*a^2*c)*(4*a*c - b^2))))/(8*a^3*c^2*(6*A^2*b^2 - B^2*a^2 - 25*A^2*
a*c + A*B*a*b)) + ((((12*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a)^3)/(64*a^3*(4*
a*c - b^2)^(3/2)) - ((2*A*b^2 - 8*A*a*c)*(((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a
*c)*(12*b^3*c^2 - 40*a*b*c^3))/(2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*
A*b*c^3 + 20*B*a*c^3))/(4*a*(4*a*c - b^2)^(1/2)) + ((2*A*b^2 - 8*A*a*c)*(12
*b^3*c^2 - 40*a*b*c^3)*(A*b - 2*B*a))/(8*a*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^
2)^(1/2))))/(2*(4*a*b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*(B^2*b*c^2 + 5*A*B*c^
3 - ((2*A*b^2 - 8*A*a*c)*(((2*A*b^2 - 8*A*a*c)*(12*b^3*c^2 - 40*a*b*c^3))/(
2*(4*a*b^2 - 16*a^2*c)) - 8*B*b^2*c^2 + 10*A*b*c^3 + 20*B*a*c^3))/(2*(4*a*b
^2 - 16*a^2*c))))/(4*a*(4*a*c - b^2)^(1/2)))*(3*A*b^4 + 10*A*a^2*c^2 - B*a*
b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(8*a^3*c^2*(4*a*c - b^2)^(1/2)*(6*A^2*b^
2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)))*(4*a*c - b^2)^(3/2))/(A^2*b^2*c^2 + 4
*B^2*a^2*c^2 - 4*A*B*a*b*c^2) + (2*(4*a*c - b^2)*(((2*A*b^2 - 8*A*a*c)*(((A
*b - 2*B*a)*(4*A*b^2*c^2 - 4*B*a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/
(4*a*b^2 - 16*a^2*c)))/(4*a*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*b^2 - 8*A*
a*c)*(A*b - 2*B*a))/(2*(4*a*b^2 - 16*a^2*c)*(4*a*c - b^2)^(1/2))))/(2*(4*a*
b^2 - 16*a^2*c)) + ((A*b - 2*B*a)*(((2*A*b^2 - 8*A*a*c)*(4*A*b^2*c^2 - 4*B*
a*b*c^2 + (2*a*b^2*c^2*(2*A*b^2 - 8*A*a*c))/(4*a*b^2 - 16*a^2*c)))/(2*(4*a*
b^2 - 16*a^2*c)) + B^2*a*c^2 - 4*A*B*b*c^2))/(4*a*(4*a*c - b^2)^(1/2)) - (b
^2*c^2*(A*b - 2*B*a)^3)/(16*a^2*(4*a*c - b^2)^(3/2)))*(3*A*b^4 + 10*A*a^2*c
^2 - B*a*b^3 - 14*A*a*b^2*c + 3*B*a^2*b*c))/(c^2*(A^2*b^2*c^2 + 4*B^2*a^2*c
^2 - 4*A*B*a*b*c^2)*(6*A^2*b^2 - B^2*a^2 - 25*A^2*a*c + A*B*a*b)))*(A*b - 2
*B*a))/(2*a*(4*a*c - b^2)^(1/2))
```

3.106 $\int \frac{A+Bx^2}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=112

$$-\frac{A}{2ax^2}-\frac{(Ab^2-abB-2aAc)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}$$

```
[Out] -1/2*A/a/x^2-(A*b-B*a)*ln(x)/a^2+1/4*(A*b-B*a)*ln(c*x^4+b*x^2+a)/a^2-1/2*(-
2*A*a*c+A*b^2-B*a*b)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^
2)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 814, 648, 632, 212, 642}

$$-\frac{(-2aAc-abB+Ab^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{\log(x)(Ab-aB)}{a^2}-\frac{A}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*A/(a*x^2) - ((A*b^2 - a*b*B - 2*a*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2
- 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]) - ((A*b - a*B)*Log[x])/a^2 + ((A*b - a
*B)*Log[a + b*x^2 + c*x^4])/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{A}{ax^2}+\frac{-Ab+aB}{a^2x}+\frac{-abB+A(b^2-ac)+(Ab-aB)cx}{a^2\left(a+bx+cx^2\right)}\right)\,dx, x, x^2\right)\\
&= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{\text{Subst}\left(\int \frac{-abB+A(b^2-ac)+(Ab-aB)cx}{a+bx+cx^2}\,dx, x, x^2\right)}{2a^2}\\
&= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a^2}+\frac{(-abB+\ldots}{}\\
&= -\frac{A}{2ax^2}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^2}-\frac{(-abB+A(b^2-\ldots}{}\\
&= -\frac{A}{2ax^2}+\frac{(abB-A(b^2-2ac))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}}-\frac{(Ab-aB)\log(x)}{a^2}+\frac{(A\ldots}{}
\end{aligned}
$$

Mathematica [A]

time = 0.15, size = 186, normalized size = 1.66

$$
\frac{-\frac{2aA}{x^2}+4(-Ab+aB)\log(x)+\frac{\left(-aB\left(b+\sqrt{b^2-4ac}\right)+A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}+\frac{\left(aB\left(b-\sqrt{b^2-4ac}\right)+A\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\sqrt{b^2-4ac}}}{4a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^2 + 4*(-(A*b) + a*B)*Log[x] + ((-(a*B*(b + Sqrt[b^2 - 4*a*c]))
+ A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/Sqrt[b^2 - 4*a*c] + ((a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c + b
*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/Sqrt[b^2 - 4*a*c
])/(4*a^2)
```

Maple [A]
time = 0.05, size = 126, normalized size = 1.12

method	result
default	$-\frac{\frac{(-bcA+acB)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(acA-A\,b^2+abB-\frac{(-bcA+acB)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2a^2}-\frac{A}{2a\,x^2}+\frac{(-Ab+aB)\ln(x)}{a^2}$
risch	$-\frac{A}{2a\,x^2}-\frac{\ln(x)Ab}{a^2}+\frac{\ln(x)B}{a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c-a^2b^2\right)\_Z^2+\left(-4Aabc+A\,b^3+4a^2cB-Ba\,b^2\right)\_Z+A^2c^2-ABbc+B^2ac\right)}\_R\ln(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*(1/2*(-A*b*c+B*a*c)/c*ln(c*x^4+b*x^2+a)+2*(a*c*A-A*b^2+a*b*B-1/2*(
-A*b*c+B*a*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
-1/2*A/a/x^2+1/a^2*(-A*b+B*a)*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.48, size = 385, normalized size = 3.44

$$\left[\frac{(Bab-Ab^2+2\,Aac)\sqrt{b^2-4\,ac}\,x^2\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-2\,Aab^2+8\,Aa^2c-(Bab^2-Ab^3-4\,(Ba^2-Aab)c)x^2\log\left(cx^4+bx^2+a\right)+4\,(Bab^2-Ab^3-4\,(Ba^2-Aab)c)x^2\log\left(x\right)}{4\,(a^2b^2-4\,a^3c)x^2},\ \frac{2\,(Bab-Ab^2+2\,Aac)\sqrt{-b^2+4\,ac}\,x^2\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-2\,Aab^2+8\,Aa^2c-(Bab^2-Ab^3-4\,(Ba^2-Aab)c)x^2\log\left(cx^4+bx^2+a\right)+4\,(Bab^2-Ab^3-4\,(Ba^2-Aab)c)x^2\log\left(x\right)}{4\,(a^2b^2-4\,a^3c)x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((B*a*b - A*b^2 + 2*A*a*c)*sqrt(b^2 - 4*a*c)*x^2*log((2*c^2*x^4 + 2*b*
c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a))
 - 2*A*a*b^2 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(
c*x^4 + b*x^2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/
((a^2*b^2 - 4*a^3*c)*x^2), 1/4*(2*(B*a*b - A*b^2 + 2*A*a*c)*sqrt(-b^2 + 4*a
*c)*x^2*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*A*a*b^2
 + 8*A*a^2*c - (B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(c*x^4 + b*x^
2 + a) + 4*(B*a*b^2 - A*b^3 - 4*(B*a^2 - A*a*b)*c)*x^2*log(x))/((a^2*b^2 -
4*a^3*c)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.46, size = 124, normalized size = 1.11

$$-\frac{(Ba-Ab)\log\left(cx^4+bx^2+a\right)}{4\,a^2}+\frac{(Ba-Ab)\log\left(x^2\right)}{2\,a^2}-\frac{(Bab-Ab^2+2\,Aac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\sqrt{-b^2+4\,ac}\;a^2}-\frac{Bax^2-Abx^2+Aa}{2\,a^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(B*a - A*b)*log(c*x^4 + b*x^2 + a)/a^2 + 1/2*(B*a - A*b)*log(x^2)/a^2
- 1/2*(B*a*b - A*b^2 + 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(s
qrt(-b^2 + 4*a*c)*a^2) - 1/2*(B*a*x^2 - A*b*x^2 + A*a)/(a^2*x^2)
```

Mupad [**B**]
time = 4.85, size = 2500, normalized size = 22.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - A/(2*a*x^2) - (log(x)*(A*b - B*a))/a^2 - (log(((A^3*c^5*x^2)/a^3 - (((((4
*b*c^2*(A*a*c - A*b^2 + B*a*b))/a - (2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b)
```

)/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*(B*a - A*b + a^2*(-(2*A*a*c - A
*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/a^2)*(B*a - A*b + a^2*(-(2*A*a
*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A*c^3*(A*a*c
- 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*b - 5*B*a))/a^2)*(B*a - A*b + a
^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))/(4*a^2) + (A^
2*c^4*(A*b - B*a))/a^3)*((((((2*c^3*x^2*(A*b^2 + 10*A*a*c - 5*B*a*b))/a - (
4*b*c^2*(A*a*c - A*b^2 + B*a*b))/a + (b*c^2*(a*b + 3*b^2*x^2 - 10*a*c*x^2)*
(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(1/2)))
/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c - b^2)))^(
1/2)))/(4*a^2) + (A*c^3*(A*a*c - 4*A*b^2 + 4*B*a*b))/a^2 - (A*c^4*x^2*(6*A*
b - 5*B*a))/a^2)*(A*b - B*a + a^2*(-(2*A*a*c - A*b^2 + B*a*b)^2/(a^4*(4*a*c
 - b^2)))^(1/2)))/(4*a^2) + (A^3*c^5*x^2)/a^3 + (A^2*c^4*(A*b - B*a))/a^3))
*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2))
- (atan((16*a^6*x^2*((((((5*A*B*a^2*c^4 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*
c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c
^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2
b^2)))(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2
b^2)))(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2
*b^2)) - (A^3*c^5)/a^3 + (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c
^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c
 - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(
4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*
b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c -
 b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a^2*(4*a
*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A*b^2 + B*a*
b)^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(32*a^7*(4*a*c - b^2)*(
16*a^3*c - 4*a^2*b^2)))*(3*A*b^4 + A*a^2*c^2 - 3*B*a*b^3 - 9*A*a*b^2*c + 8*
B*a^2*b*c))/(8*a^3*c^2*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*
b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)) + (((((5*A*B*a^2*c^4
 - 6*A^2*a*b*c^4)/a^3 - (((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*c^3)
/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c -
8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*
c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/(4*a
^2*(4*a*c - b^2)^(1/2)) - (((((20*A*a^3*c^4 - 10*B*a^3*b*c^3 + 2*A*a^2*b^2*
c^3)/a^3 + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*
c - 8*A*a*b*c))/(2*a^3*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c - A*b^2 + B*a*b))/
(4*a^2*(4*a*c - b^2)^(1/2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*A*a*c - A
*b^2 + B*a*b)*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(8*a^5*(4*a*c
- b^2)^(1/2)*(16*a^3*c - 4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*
A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)) + ((40*a^4*b*c^3 - 12*a^3*b^3*c^2)*(2*
A*a*c - A*b^2 + B*a*b)^3)/(64*a^9*(4*a*c - b^2)^(3/2)))*(6*A*b^5 - 20*B*a^3
*c^2 - 6*B*a*b^4 - 30*A*a*b^3*c + 26*A*a^2*b*c^2 + 28*B*a^2*b^2*c))/(16*a^3
c^2(4*a*c - b^2)^(1/2)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^
2*b^2 + 12*A*B*a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)))*(4*a*c - b^2)^(3/
2))/(4*A^2*a^2*c^4 + A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*

```
B*a*b^3*c^2 + 4*A*B*a^2*b*c^3) + (a^3*(4*a*c - b^2)*((((((4*A*a^3*b*c^3 - 4
*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 +
 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*a*c - A*b^2 + B*a*b))
/(4*a^2*(4*a*c - b^2)^(1/2)) + (b^2*c^2*(2*A*a*c - A*b^2 + B*a*b)*(2*A*b^3
- 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*a*(4*a*c - b^2)^(1/2)*(16*a^3*c -
4*a^2*b^2)))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c -
4*a^2*b^2)) + (((A^2*a^2*c^4 - 4*A^2*a*b^2*c^3 + 4*A*B*a^2*b*c^3)/a^3 + (((
4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*a*b^2*c^2*(2*A*
b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*b^2))*(2*A*b^3
- 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(2*(16*a^3*c - 4*a^2*b^2)))*(2*A*a*c
- A*b^2 + B*a*b))/(4*a^2*(4*a*c - b^2)^(1/2)) - (b^2*c^2*(2*A*a*c - A*b^2 +
 B*a*b)^3)/(16*a^5*(4*a*c - b^2)^(3/2)))*(6*A*b^5 - 20*B*a^3*c^2 - 6*B*a*b^
4 - 30*A*a*b^3*c + 26*A*a^2*b*c^2 + 28*B*a^2*b^2*c))/(c^2*(4*A^2*a^2*c^4 +
A^2*b^4*c^2 + B^2*a^2*b^2*c^2 - 4*A^2*a*b^2*c^3 - 2*A*B*a*b^3*c^2 + 4*A*B*a
^2*b*c^3)*(25*B^2*a^3*c - 6*A^2*b^4 + A^2*a^2*c^2 - 6*B^2*a^2*b^2 + 12*A*B*
a*b^3 + 24*A^2*a*b^2*c - 49*A*B*a^2*b*c)) - (2*a^3*(4*a*c - b^2)^(3/2)*((A^
3*b*c^4 - A^2*B*a*c^4)/a^3 - (((A^2*a^2*c^4 - 4*A^2*a*b^2*c^3 + 4*A*B*a^2*b
*c^3)/a^3 + (((4*A*a^3*b*c^3 - 4*A*a^2*b^3*c^2 + 4*B*a^3*b^2*c^2)/a^3 + (2*
a*b^2*c^2*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*b*c))/(16*a^3*c - 4*a^2*
b^2))*(2*A*b^3 - 2*B*a*b^2 + 8*B*a^2*c - 8*A*a*...
```

3.107 $\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=261

$$-\frac{(bB-Ac)x}{c^2}+\frac{Bx^3}{3c}+\frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(b^2B-\right.}{}$$

```
[Out] -(-A*c+B*b)*x/c^2+1/3*B*x^3/c+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2))*(b^2*B-A*b*c-a*B*c+(-2*A*a*c^2+A*b^2*c+3*B*a*b*c-B*b^3)/(-4*a
*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^
(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*B-A*b*c-a*B*c+(2*A*a*c^2-A
*b^2*c-3*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A]
time = 0.89, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1293, 1180, 211}

$$\frac{\left(-\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{2aAc^2-3abBc-Ab^2c+b^3B}{\sqrt{b^2-4ac}}-aBc-Abc+b^2B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(bB-Ac)}{c^2}+\frac{Bx^3}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b*B - A*c)*x)/c^2) + (B*x^3)/(3*c) + ((b^2*B - A*b*c - a*B*c - (b^3*B -
 A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a
*c]]) + ((b^2*B - A*b*c - a*B*c + (b^3*B - A*b^2*c - 3*a*b*B*c + 2*a*A*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx^3}{3c} - \frac{\int \frac{x^2(3aB+3(bB-Ac)x^2)}{a+bx^2+cx^4}\,dx}{3c}$$

$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\int \frac{3a(bB-Ac)+3(b^2B-Abc-aBc)x^2}{a+bx^2+cx^4}\,dx}{3c^2}$$

$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}}}{2c^2}$$

$$= -\frac{(bB-Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(b^2B-Abc-aBc-\frac{b^3B-Ab^2c-3abBc+2aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b-\sqrt{}}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.27, size = 327, normalized size = 1.25

$$\frac{(-bB+Ac)x}{c^2} + \frac{Bx^3}{3c} + \frac{\left(-b^3B+Ab^2c+3abBc-2aAc^2+b^2B\sqrt{b^2-4ac}-Abc\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\left(b^3B-Ab^2c-3abBc+2aAc^2+b^2B\sqrt{b^2-4ac}-Abc\sqrt{b^2-4ac}-aBc\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((-(b*B) + A*c)*x)/c^2 + (B*x^3)/(3*c) + ((-(b^3*B) + A*b^2*c + 3*a*b*B*c -
 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 - 4*a*c] - a*B*c*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sq
rt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B - A*
b^2*c - 3*a*b*B*c + 2*a*A*c^2 + b^2*B*Sqrt[b^2 - 4*a*c] - A*b*c*Sqrt[b^2 -
```

```
4*a*c] - a*B*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*
c]])
```

Maple [A]
time = 0.09, size = 289, normalized size = 1.11

method	result
risch	$\frac{B x^3}{3c}+\frac{Ax}{c}-\frac{bBx}{c^2}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2 b+a\right)}\frac{\left(\left(-bcA-acB+b^2B\right)\_R^2-acA+abB\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c^2}$
default	$\frac{\frac{1}{3}Bcx^3+Acx-bBx}{c^2}+\frac{-\frac{\left(-bcA\sqrt{-4ac+b^2}-2c^2aA+Ab^2c-acB\sqrt{-4ac+b^2}+b^2B\sqrt{-4ac+b^2}+3abBc-b^3B\right)\sqrt{2}\ a}{2c\sqrt{-4ac+b^2}\ \sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c^2*(1/3*B*c*x^3+A*c*x-b*B*x)+4/c*(-1/8*(-b*c*A*(-4*a*c+b^2)^(1/2)-2*c^2*
a*A+A*b^2*c-a*c*B*(-4*a*c+b^2)^(1/2)+b^2*B*(-4*a*c+b^2)^(1/2)+3*a*b*B*c-b^3
*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-b*c*A*(-4*a*c+b^2)^(1/
2)+2*c^2*a*A-A*b^2*c-a*c*B*(-4*a*c+b^2)^(1/2)+b^2*B*(-4*a*c+b^2)^(1/2)-3*a*
b*B*c+b^3*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*
arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/3*(B*c*x^3 - 3*(B*b - A*c)*x)/c^2 - integrate(-(B*a*b - A*a*c + (B*b^2 -
(B*a + A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/c^2
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5140 vs. $2(225) = 450$.
time = 2.94, size = 5140, normalized size = 19.69

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] 1/6*(2*B*c*x^3 + 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c
^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*
c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^
3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2
+ 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*
A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2
*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b
^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3
*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 -
3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x
 + sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A
^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)
*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 +
 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3
)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a
*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3
+ A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2
*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2
*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))*sqrt(-(B^2*b^5 - (4
*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*
B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6
- 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*
b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 +
 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 +
 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^
2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) - 3*sqrt(1/2)*c^2*sqrt(-(B^2*b^5
 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*B*a*b^2 + A^2*b^3)*c^2
- (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*sqrt((B^4*b^8 + A^4*a^2
*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3
*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*
b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*
b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c
)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-2*(B^4*a^2*b^4 - A*B^3*
a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^2)*c^3 + (B^4*a^4 + 3*A*B^3*
a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2 - (3*B^4*a^3*b^2 - A*B^3*a^2
*b^3 - 3*A^2*B^2*a*b^4)*c)*x - sqrt(1/2)*(B^3*b^7 - 4*A^3*a^2*c^5 + (4*A*B^
2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*B^3*a^3*b + 29*A*B^2*a^2*b^2
 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*b^3 + 19*A*B^2*a*b^4 + 3*A^2
*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c - (B*b^4*c^5 + 4*(2*B*a^2 + A*a
*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2
*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B
^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^
2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*
```

```
b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a
*c^11)))*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b + 8*A*
B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c + (b^2*c^5 - 4*a*c^6)*
sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c
^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b
^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A^3*B*
b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(3*B^4
*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))) + 3*
sqrt(1/2)*c^2*sqrt(-(B^2*b^5 - (4*A*B*a^2 + 3*A^2*a*b)*c^3 + (5*B^2*a^2*b +
 8*A*B*a*b^2 + A^2*b^3)*c^2 - (5*B^2*a*b^3 + 2*A*B*b^4)*c - (b^2*c^5 - 4*a*
c^6)*sqrt((B^4*b^8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b
^2)*c^5 + (B^4*a^4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 +
A^4*b^4)*c^4 - 2*(3*B^4*a^3*b^2 + 14*A*B^3*a^2*b^3 + 12*A^2*B^2*a*b^4 + 2*A
^3*B*b^5)*c^3 + (11*B^4*a^2*b^4 + 20*A*B^3*a*b^5 + 6*A^2*B^2*b^6)*c^2 - 2*(
3*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*
log(-2*(B^4*a^2*b^4 - A*B^3*a*b^5 - A^4*a^2*c^4 + (5*A^3*B*a^2*b + A^4*a*b^
2)*c^3 + (B^4*a^4 + 3*A*B^3*a^3*b - 6*A^2*B^2*a^2*b^2 - 3*A^3*B*a*b^3)*c^2
- (3*B^4*a^3*b^2 - A*B^3*a^2*b^3 - 3*A^2*B^2*a*b^4)*c)*x + sqrt(1/2)*(B^3*b
^7 - 4*A^3*a^2*c^5 + (4*A*B^2*a^3 + 20*A^2*B*a^2*b + 5*A^3*a*b^2)*c^4 - (4*
B^3*a^3*b + 29*A*B^2*a^2*b^2 + 17*A^2*B*a*b^3 + A^3*b^4)*c^3 + (13*B^3*a^2*
b^3 + 19*A*B^2*a*b^4 + 3*A^2*B*b^5)*c^2 - (7*B^3*a*b^5 + 3*A*B^2*b^6)*c + (
B*b^4*c^5 + 4*(2*B*a^2 + A*a*b)*c^7 - (6*B*a*b^2 + A*b^3)*c^6)*sqrt((B^4*b^
8 + A^4*a^2*c^6 - 2*(A^2*B^2*a^3 + 4*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (B^4*a^
4 + 8*A*B^3*a^3*b + 24*A^2*B^2*a^2*b^2 + 12*A^3...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4391 vs. $2(225) = 450$.
time = 4.80, size = 4391, normalized size = 16.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
```

c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*c^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 32*a^3*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^4 + 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(c) - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 2*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*B*abs(c) - (2*b^5*c^5 - 12*a*b^3*c^6 + 16*a^2*b*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^5 + 4*(b^2 - 4*a*c)*a*b*c^6)*A + (2*b^6*c^4 - 14*a*b^4*c^5 + 24*a^2*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -

```
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*
a*b^2*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c^3 + sqrt(b^2*c^6 - 4*a*c^7))/c
^4))/((a*b^4*c^4 - 8*a^2*b^2*c^5 - 2*a*b^3*c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 +
 a*b^2*c^6 - 4*a^2*c^7)*c^2) + 1/8*((2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b
^2 - 4*a*c)*a*b*c^4)*A*c^2 - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 3
2*a^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 +
 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 +...
```

Mupad [B]
time = 1.59, size = 2500, normalized size = 9.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] x*(A/c - (B*b)/c^2) - atan(((((16*A*a^2*c^5 - 4*A*a*b^2*c^4 + 4*B*a*b^3*c^3
 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^2*b^7 + A^2*b^5
*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2
+ A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/
2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 -
A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3
- 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(
-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2
*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*
B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*
(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*
b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c -
b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^
4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6 + 2*A^2*a^2*c^4 + A^2*b^4*c^2
```

- 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*b^4*c - 4*A^2*a
*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2*b^7 + A^2*b^5*
c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3*c^2 +
A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c^4 - A
^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2*c^3 -
3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-
(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2
*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((16*A*a^2*c^5 - 4*A*a*b^2*c^4
+ 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 + (2*x*(4*b^3*c^5 - 16*a*b*c^6)*(-(B^
2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B
^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A
^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A
*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2
- 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*(-(B^2*b^7 + A^
2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^2*a^2*b^3
*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c - b^2)^3
)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^2*a^2*b*c
^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*B*a^2*b^2
*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 - 2*A*B*b^
3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1/2))/(8*(
16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*x*(B^2*b^6 + 2*A^2*a^2*c^4
+ A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2*c^2 - 6*B^2*a*
b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^3))/c^3)*(-(B^2
*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c + 25*B^
2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-(4*a*c
- b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 + 12*A^
2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 - 36*A*
B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4*c^2 -
2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)^3)^(1
/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((((16*A*a^2*c^5 -
4*A*a*b^2*c^4 + 4*B*a*b^3*c^3 - 16*B*a^2*b*c^4)/c^3 - (2*x*(4*b^3*c^5 - 16
*a*b*c^6)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*b^6*c + 25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a
*b^3*c^3 + 12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a
^3*b*c^3 - 36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16
*A*B*a*b^4*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*
a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2))/c^3)*
(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^6*c +
25*B^2*a^2*b^3*c^2 + A^2*b^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*c^2*(-
(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^3*c^4 - 9*B^2*a*b^5*c - 7*A^2*a*b^3*c^3 +
12*A^2*a^2*b*c^4 - A^2*a*c^3*(-(4*a*c - b^2)^3)^(1/2) - 20*B^2*a^3*b*c^3 -

```
36*A*B*a^2*b^2*c^3 - 3*B^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a*b^4
*c^2 - 2*A*B*b^3*c*(-(4*a*c - b^2)^3)^(1/2) + 4*A*B*a*b*c^2*(-(4*a*c - b^2)
^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*x*(B^2*b^6
+ 2*A^2*a^2*c^4 + A^2*b^4*c^2 - 2*B^2*a^3*c^3 - 2*A*B*b^5*c + 9*B^2*a^2*b^2
*c^2 - 6*B^2*a*b^4*c - 4*A^2*a*b^2*c^3 + 10*A*B*a*b^3*c^2 - 10*A*B*a^2*b*c^
3))/c^3)*(-(B^2*b^7 + A^2*b^5*c^2 + B^2*b^4*(-(...
```

3.108 $\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=208

$$\frac{Bx}{c}-\frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(bB-Ac+\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] B*x/c-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*B-A*c+(
A*b*c+2*B*a*c-B*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*B-
A*c+(-A*b*c-2*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)/(b+(-4*a*c+b
^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.35, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1293, 1180, 211}

$$-\frac{\left(-\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{-2aBc-Abc+b^2B}{\sqrt{b^2-4ac}}-Ac+bB\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Bx}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((b*B - A*c - (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b
- Sqrt[b^2 - 4*a*c]]) - ((b*B - A*c + (b^2*B - A*b*c - 2*a*B*c)/Sqrt[b^2 -
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c
^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2(A+Bx^2)}{a+bx^2+cx^4}\,dx = \frac{Bx}{c} - \frac{\int \frac{aB+(bB-Ac)x^2}{a+bx^2+cx^4}\,dx}{c}$$

$$= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2c} - \frac{\left(bB-Ac+\frac{b^2B-Abc-\cdots}{\sqrt{b^2-\cdots}}\right.\cdots}{\cdots}$$

$$= \frac{Bx}{c} - \frac{\left(bB-Ac-\frac{b^2B-Abc-2aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(bB-Ac+\frac{b^2B\cdots}{\sqrt{\cdots}}\right.\cdots}{\cdots}$$

Mathematica [A]
time = 0.11, size = 251, normalized size = 1.21

$$\frac{Bx}{c} - \frac{\left(-b^2B+Abc+2aBc+bB\sqrt{b^2-4ac}-Ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\left(b^2B-Abc-2aBc+bB\sqrt{b^2-4ac}-Ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (B*x)/c - ((-(b^2*B) + A*b*c + 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((b^2*B - A*b*
c - 2*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - A*c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*
c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Maple [A]
time = 0.06, size = 212, normalized size = 1.02

method	result
risch	$\frac{Bx}{c}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left((Ac-bB)\_R^2-aB\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{2c}$
default	$\frac{Bx}{c}-\frac{\left(Ac\sqrt{-4ac+b^2}-bcA-bB\sqrt{-4ac+b^2}-2acB+b^2B\right)\sqrt{2}\ \mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{2c\sqrt{-4ac+b^2}\ \sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/c-1/2*(A*c*(-4*a*c+b^2)^(1/2)-b*c*A-b*B*(-4*a*c+b^2)^(1/2)-2*a*c*B+b^2*
B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/2*(A*c*(-4*a*c+b^2)^(1/2)+b
*c*A-b*B*(-4*a*c+b^2)^(1/2)+2*a*c*B-b^2*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] B*x/c + integrate(-((B*b - A*c)*x^2 + B*a)/(c*x^4 + b*x^2 + a), x)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2632 vs. 2(172) = 344.
time = 0.75, size = 2632, normalized size = 12.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*
B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A
^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 +
2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2
 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b
```

^2)*c)*x + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x - sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c - (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c + (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x + sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) - sqrt(1/2)*c*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(2*(B^4*a*b^2 - A*B^3*b^3 - 3*A^3*B*b*c^2 + A^4*c^3 - (B^4*a^2 + A*B^3*a*b - 3*A^2*B^2*b^2)*c)*x - sqrt(1/2)*(B^3*b^4 - 4*A^2*B*a*c^3 + (4*B^3*a^2 + 8*A*B^2*a*b + A^2*B*b^2)*c^2 - (5*B^3*a*b^2 + 2*A*B^2*b^3)*c + (B*b^3*c^3 + 8*A*a*c^5 - 2*(2*B*a*b + A*b^2)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2 + 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))*sqrt(-(B^2*b^3 + (4*A*B*a + A^2*b)*c^2 - (3*B^2*a*b + 2*A*B*b^2)*c - (b^2*c^3 - 4*a*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(A^2*B^2*a + 2

```
*A^3*B*b)*c^3 + (B^4*a^2 + 4*A*B^3*a*b + 6*A^2*B^2*b^2)*c^2 - 2*(B^4*a*b^2
+ 2*A*B^3*b^3)*c)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))) + 2*B*x)/c
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3179 vs. 2(172) = 344.
time = 8.69, size = 3179, normalized size = 15.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] B*x/c - 1/8*((2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b
^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2
*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*B*c^2 + 2*(sqrt(2)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*
a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^4 - 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^5 + 32*a^3*c^5 - 2*(b^2 - 4*a*c)*a*b^2*c^3 + 8*(b^2 - 4*a*c)*a^2
*c^4)*B*abs(c) - (2*b^4*c^5 - 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
```

```
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b^2*c^5 - 2*(b^2 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^
5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b
^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^
3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 -
 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c
)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*B)*arctan(2*sqrt(1/2)*x/sqrt((b*c + sq
rt(b^2*c^2 - 4*a*c^3))/c^2))/((a*b^4*c^3 - 8*a^2*b^2*c^4 - 2*a*b^3*c^4 + 16
*a^3*c^5 + 8*a^2*b*c^5 + a*b^2*c^5 - 4*a^2*c^6)*c^2) - 1/8*((2*b^4*c^3 - 16
*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c
^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c
)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*A*c^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a
^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(
b^2 - 4*a*c)*a*b*c^3)*B*c^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*
b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*
b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 +
2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*B*abs(c) - (2*b^4*c^5
- 8*a*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 2*(b^2
 - 4*a*c)*b^2*c^5)*A + (2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^...
```

Mupad [B]

time = 1.26, size = 2500, normalized size = 12.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4),x)

[Out] (B*x)/c - atan(((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i - (((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*B*a^2*c^3 - 4*B*a*b^2*c^2)/c - (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-

```
(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1
/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (
2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*b^3*c - 4*
B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*
c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*c - 16*A*B
*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2)^3)^(1/2)
 + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2
)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (((16*B*a^2*c^3 - 4*B*a
*b^2*c^2)/c + (2*x*(4*b^3*c^3 - 16*a*b*c^4)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*
c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4
*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*
B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2))/c)*(-(B^2*b^5
 + A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)
^3)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c +
B^2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c -
b^2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))
^(1/2) + (2*x*(B^2*b^4 - 2*A^2*a*c^3 + A^2*b^2*c^2 + 2*B^2*a^2*c^2 - 2*A*B*
b^3*c - 4*B^2*a*b^2*c + 6*A*B*a*b*c^2))/c)*(-(B^2*b^5 + A^2*b^3*c^2 - A^2*c
^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*b^4*
c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B
*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(A^3*a*c^2
 - B^3*a^2*b + A*B^2*a*b^2 + A*B^2*a^2*c - 2*A^2*B*a*b*c))/c))*(-(B^2*b^5 +
 A^2*b^3*c^2 - A^2*c^2*(-(4*a*c - b^2)^3)^(1/2) - B^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) - 2*A*B*b^4*c - 16*A*B*a^2*c^3 - 4*A^2*a*b*c^3 - 7*B^2*a*b^3*c + B^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) + 12*B^2*a^2*b*c^2 + 2*A*B*b*c*(-(4*a*c - b^
2)^3)^(1/2) + 12*A*B*a*b^2*c^2)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*2i - atan(((((16*B*a^2*c^3 - 4*B*a*b^2*c^2...
```

3.109 $\int \frac{A+Bx^2}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=172

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(B+(2*A*c-B*b)/(
-4*a*c+b^2)^(1/2))*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(B+(-2*A*c+B*b)/(-4*a*c+b^2
)^(1/2))*2^(1/2)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {1180, 211}

$$\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{bB-2Ac}{\sqrt{b^2-4ac}}+B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((B - (b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((B +
(b*B - 2*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b
^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rubi steps

$$\int \frac{A+Bx^2}{a+bx^2+cx^4}\,dx=\frac{1}{2}\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx+\frac{1}{2}\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\int \frac{}{\frac{b}{2}+}$$

$$=\frac{\left(B-\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(B+\frac{bB-2Ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{}{\sqrt{}}\right.}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2}}}$$

Mathematica [A]
time = 0.06, size = 173, normalized size = 1.01

$$\frac{\frac{\left(-bB+2Ac+B\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(bB-2Ac+B\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}}}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (((-(b*B) + 2*A*c + B*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
- Sqrt[b^2 - 4*a*c]]])/Sqrt[b - Sqrt[b^2 - 4*a*c]] + ((b*B - 2*A*c + B*Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/Sqr
t[b + Sqrt[b^2 - 4*a*c]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [A]
time = 0.04, size = 164, normalized size = 0.95

method	result
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(B\_R^2+A\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{2}$
default	$4c\left(-\frac{\left(2Ac+B\sqrt{-4ac+b^2}-bB\right)\sqrt{2}\ \mathrm{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-2Ac+B\sqrt{-4ac+b^2}-\right.}{8c\sqrt{-4ac+}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4*c*(-1/8*(2*A*c+B*(-4*a*c+b^2)^(1/2)-b*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-
b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))
*c)^(1/2))+1/8*(-2*A*c+B*(-4*a*c+b^2)^(1/2)+b*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/
2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/
2))*c)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(c*x^4 + b*x^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1569 vs. $2(138) = 276$.
time = 0.52, size = 1569, normalized size = 9.12

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*
sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2
*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x + sqr
t(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3
 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c
 + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*
c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*sqrt(-(B^2*a*b -
 (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c
+ A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a
^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*
c^2 - (4*A*B^2*a^2 + A^3*b^2)*c + (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2
 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c + (a*b^2*c - 4*a^2*c^2)*sqrt(
(B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c -
```

```
4*a^2*c^2))) + 1/2*sqrt(1/2)*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*
c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^
3*c^3)))/(a*b^2*c - 4*a^2*c^2))*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A
^4*c^2)*x + sqrt(1/2)*(A*B^2*a*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*
c - (4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A
*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4
*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2))) - 1/2*sqrt(1/2)*s
qrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4*a^2*c^2)*sqrt((B^4*a^2 -
 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3)))/(a*b^2*c - 4*a^2*c^2)
)*log(-2*(B^4*a^2 - A*B^3*a*b + A^3*B*b*c - A^4*c^2)*x - sqrt(1/2)*(A*B^2*a
*b^2 + 4*A^3*a*c^2 - (4*A*B^2*a^2 + A^3*b^2)*c - (4*(2*B*a^3 - A*a^2*b)*c^2
 - (2*B*a^2*b^2 - A*a*b^3)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2
*b^2*c^2 - 4*a^3*c^3)))*sqrt(-(B^2*a*b - (4*A*B*a - A^2*b)*c - (a*b^2*c - 4
*a^2*c^2)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^2*c^2 - 4*a^3*c^3
)))/(a*b^2*c - 4*a^2*c^2)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1400 vs. $2(138) = 276$.
time = 5.87, size = 1400, normalized size = 8.14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*
c - 2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^2*c^2 + 16*a*b^2*c^2 + 2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*c^3 - 32*a^2*c^3 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c - 8*(b^2 - 4*a*c)*a*c^2 - 2*(b^2 - 4*a*c)
```

```
*b*c^2)*A - 2*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arctan(2*sqrt(1/2)*x/sqrt((b + sqrt(b^2
- 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2
+ a*b^2*c^2 - 4*a^2*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 2*b^4*c + 16*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a^2*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 16*a*b^2*c^2 - 2*b^3*
c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^3 + 32*a^2*c^3 + 8*a*b*
c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 - 4*sqr
t(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c + sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b^2*c + 8*(b
^2 - 4*a*c)*a*c^2 + 2*(b^2 - 4*a*c)*b*c^2)*A + 2*(2*a*b^2*c^2 - 8*a^2*c^3 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2 + 4*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c + 2*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*a*c)*a*c^2)*B)*arcta
n(2*sqrt(1/2)*x/sqrt((b - sqrt(b^2 - 4*a*c))/c))/((a*b^4 - 8*a^2*b^2*c - 2*
a*b^3*c + 16*a^3*c^2 + 8*a^2*b*c^2 + a*b^2*c^2 - 4*a^2*c^3)*abs(c))
```

Mupad [B]
time = 1.00, size = 2500, normalized size = 14.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] - atan((((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*
(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c -
 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3
*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*
c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2
*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)
 - 4*A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4
*A*B*b*c^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^
2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b
*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i +
((-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c
 - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a
*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*
```

(8*b^3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^
2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 -
 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))
^(1/2) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c
^2))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4
*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A
*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((-(B^2*
a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^
3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/
(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^
3)*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a
*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B
*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*A*b^2*c^2 +
 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(
B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b
^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2
c)/(8(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - ((-(B^2*a*b^3 + B^2
a(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2) +
16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*
c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^3*c^2 - 32*a*b
c^3)(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(
4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*
A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 16*A*a*c^3
) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*(-(B^2*a*b^3 +
 B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c*(-(4*a*c - b^2)^3)^(1/2
) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*
a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) + 2*A^2*B*c^2 + 2*B^3*a*c - 2*A*
B^2*b*c))*(-(B^2*a*b^3 + B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c - A^2*c
*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c
- 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*2i - ata
n((((-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*
a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*
B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2
- 32*a*b*c^3)*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A
^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*
b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2) - 4*
A*b^2*c^2 + 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*
b*c^2))*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(
-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c -
4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i + ((-(B
^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^
2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*
c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(4*A*b^2*c^2 + x*(8*b^
3*c^2 - 32*a*b*c^3)*(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3
*c + A^2*c*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^

```
2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2
) - 16*A*a*c^3) + x*(4*A^2*c^3 - 4*B^2*a*c^2 + 2*B^2*b^2*c - 4*A*B*b*c^2))*
(-(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c
- b^2)^3)^(1/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*
b^2*c)/(8*(16*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*1i)/(((-(B^2*a*b^3
 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A^2*b^3*c + A^2*c*(-(4*a*c - b^2)^3)^(1
/2) + 16*A*B*a^2*c^2 - 4*A^2*a*b*c^2 - 4*B^2*a^2*b*c - 4*A*B*a*b^2*c)/(8*(1
6*a^3*c^3 - 8*a^2*b^2*c^2 + a*b^4*c)))^(1/2)*(x*(8*b^3*c^2 - 32*a*b*c^3)*(-
(B^2*a*b^3 - B^2*a*(-(4*a*c - b^2)^3)^(1/2) + A...
```

3.110 $\int \frac{A+Bx^2}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=189

$$-\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] -A/a/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(
A+(A*b-2*B*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A+(-A*b+2
*B*a)/(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1295, 1180, 211}

$$-\frac{\sqrt{c}\left(\frac{Ab-2aB}{\sqrt{b^2-4ac}}+A\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{A}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(A/(a*x)) - (Sqrt[c]*(A + (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b - Sqrt[b^2 - 4*
a*c]]) - (Sqrt[c]*(A - (A*b - 2*a*B)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqr
t[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)}\,dx &= -\frac{A}{ax}-\frac{\int \frac{Ab-aB+Acx^2}{a+bx^2+cx^4}\,dx}{a}\\ &= -\frac{A}{ax}-\frac{\left(c\left(A-\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2a}-\frac{\left(c\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right.\right.}{}\\ &= -\frac{A}{ax}-\frac{\sqrt{c}\left(A+\frac{Ab-2aB}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A-\frac{Ab-}{\sqrt{b^2-}}\right.}{\sqrt{}}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 206, normalized size = 1.09

$$-\frac{\frac{2A}{x}+\frac{\sqrt{2}\sqrt{c}\left(-2aB+A\left(b+\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2aB+A\left(-b+\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((2*A)/x + (Sqrt[2]*Sqrt[c]*(-2*a*B + A*(b + Sqrt[b^2 - 4*a*c]))*ArcTa
n[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a*B + A*(-b + Sqrt[b^2 - 4*a
*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 -
4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/a
```

Maple [A]
time = 0.05, size = 173, normalized size = 0.92

method	result
default	$\frac{4c\left(-\frac{\left(-A\sqrt{-4ac+b^2}-Ab+2aB\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(-A\sqrt{-4ac+b^2}+Ab-2aB\right)\sqrt{}}{8\sqrt{-4ac+b^2}\,\sqrt{}}\right.}{a}$
risch	$-\frac{A}{ax}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(16a^5c^2-8a^4b^2c+a^3b^4\right)\_Z^4+\left(12A^2a^2b\,c^2-7A^2a\,b^3c+A^2b^5-16AB\,a^3c^2+12AB\,a^2b^2c-2ABa\,b^4-4B^2a^3bc+B^2a^2b^3\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a*c*(-1/8*(-A*(-4*a*c+b^2)^(1/2)-A*b+2*a*B)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))+1/8*(-A*(-4*a*c+b^2)^(1/2)+A*b-2*a*B)/(-4*a*c+b^2)^(1/2)*2^(1/2
)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)))-A/a/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -integrate((A*c*x^2 - B*a + A*b)/(c*x^4 + b*x^2 + a), x)/a - A/(a*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2914 vs. $2(155) = 310$.
time = 0.89, size = 2914, normalized size = 15.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 -
3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^
2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*
B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A
```

```
^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2
*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2
*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*
a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^
5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^
2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*
A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*
a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4
*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^
2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c))
)/(a^3*b^2 - 4*a^4*c))) - sqrt(1/2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^
2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A
*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*
(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2
 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*
B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*
A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (
4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*a*b^3)*c - (B*a^4*b^3
 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 -
 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2
 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt
(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c + (a^3*b^2
 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b
^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) + sqrt(1/2)*a*x*sqrt(-(B^2*a^2
*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c
)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b
^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2
 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (A^3*B*a*b - A^4*b^2)*
c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3*B*b^3)*c)*x + sqrt(1
/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A^3*b^5 + 4*(A^2*B*a^3
 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A^2*B*a^2*b^2 - 5*A^3*
a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2*B*a^5*b - 3*A*a^4*b^
2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A
^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*
b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*
A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*
a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*
a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))) - sqrt(1/
2)*a*x*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c
 - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 -
4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b + A^
4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(2*(A^4*a*c^3 + (
A^3*B*a*b - A^4*b^2)*c^2 - (B^4*a^3 - 3*A*B^3*a^2*b + 3*A^2*B^2*a*b^2 - A^3
*B*b^3)*c)*x - sqrt(1/2)*(B^3*a^3*b^2 - 3*A*B^2*a^2*b^3 + 3*A^2*B*a*b^4 - A
```

```
^3*b^5 + 4*(A^2*B*a^3 - A^3*a^2*b)*c^2 - (4*B^3*a^4 - 12*A*B^2*a^3*b + 13*A
^2*B*a^2*b^2 - 5*A^3*a*b^3)*c + (B*a^4*b^3 - A*a^3*b^4 - 8*A*a^5*c^2 - 2*(2
*B*a^5*b - 3*A*a^4*b^2)*c)*sqrt((B^4*a^4 - 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A^2*B^2*a^3 - 2*A^3*B*a^2*b
+ A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))*sqrt(-(B^2*a^2*b - 2*A*B*a*b^2 + A^2*
b^3 + (4*A*B*a^2 - 3*A^2*a*b)*c - (a^3*b^2 - 4*a^4*c)*sqrt((B^4*a^4 - 4*A*B
^3*a^3*b + 6*A^2*B^2*a^2*b^2 - 4*A^3*B*a*b^3 + A^4*b^4 + A^4*a^2*c^2 - 2*(A
^2*B^2*a^3 - 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
 4*a^4*c))) - 2*A)/(a*x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2805 vs. $2(155) = 310$.
time = 4.28, size = 2805, normalized size = 14.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -A/(a*x) - 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A*a^2 + 2*(sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*
c)*a^2*b*c^2)*A*abs(a) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4
 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*sqrt(2)*sqrt(b*c + sqr
```

t(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b^2*c^2
 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 32*a^4*c^3 + 2*(b^2
- 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2)*B*abs(a) + (2*a^2*b^4*c^2 - 8
*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2
*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2
*(b^2 - 4*a*c)*a^2*b^2*c^2)*A - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^3*b*c^2)*B)*
arctan(2*sqrt(1/2)*x/sqrt((a*b + sqrt(a^2*b^2 - 4*a^3*c))/(a*c)))/((a^3*b^4
 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a
^4*c^3)*abs(a)*abs(c)) + 1/8*((2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*A
a^2 - 2(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*
c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^3*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c +
 8*(b^2 - 4*a*c)*a^2*b*c^2)*A*abs(a) + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - 2*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*a^2*b^4*c + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 -
16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*a^4
c^3 - 2(b^2 - 4*a*c)*a^2*b^2*c + 8*(b^2 - 4*a*c)*a^3*c^2)*B*abs(a) + (2*a
^2*b^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*A - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^
3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*

```
a^3*b*c^2)*B)*arctan(2*sqrt(1/2)*x/sqrt((a*b - sqrt(a^2*b^2 - 4*a^3*c))/(a*
c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3
*b^2*c^2 - 4*a^4*c^3)*abs(a)*abs(c))
```

Mupad [B]
time = 1.35, size = 2500, normalized size = 13.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - atan(((x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b
*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a
^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c
- A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A
*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^
2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2
*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1
/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2
)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3
)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)
 - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*B*a^5*b^2*c^2))*(-(A
^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c
- b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-
(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*
a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^
2*c)))^(1/2)*1i + (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4
*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/
2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^
2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b
*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 +
 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a^6*b*c^3 - 8*a^5*
b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c
 - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*
A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c
^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c^2 - 4*B*a^5*b^2*
c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2
*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c -
A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B
*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2
- 8*a^4*b^2*c)))^(1/2)*1i)/((x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^2*a^3*b
^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b
^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*
```

```
c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12
*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*
(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(x*(32*a^6*b*c^3 - 8*a^5*b^3*c
^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*(
-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*a*b^3*c - A^
2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c^2 - 2*A*B*a
*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 16*a^5*c^2 -
8*a^4*b^2*c)))^(1/2) - 16*B*a^6*c^3 + 16*A*a^5*b*c^3 - 4*A*a^4*b^3*c^2 + 4*
B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^(1/2)
 + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7*A^2*
a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^2*b*c
^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^4 + 1
6*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - (x*(4*A^2*a^4*c^4 - 4*B^2*a^5*c^3 - 2*A^
2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) + (-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*
a*c - b^2)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A
*B*a^3*c^2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b
*c + 12*A^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2
*c)/(8*(a^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*(16*B*a^6*c^3 + x*(32*a
^6*b*c^3 - 8*a^5*b^3*c^2)*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2
)^3)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^
2 - 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A
^2*a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a
^3*b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) - 16*A*a^5*b*c^3 + 4*A*a^4*b^3*c
^2 - 4*B*a^5*b^2*c^2))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3
)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 -
 7*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*
a^2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*
b^4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2) + 2*A^3*a^3*c^4 + 2*A*B^2*a^4*c^3 -
 2*A^2*B*a^3*b*c^3))*(-(A^2*b^5 + B^2*a^2*b^3 + A^2*b^2*(-(4*a*c - b^2)^3)^
(1/2) + B^2*a^2*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^4 - 16*A*B*a^3*c^2 - 7
*A^2*a*b^3*c - A^2*a*c*(-(4*a*c - b^2)^3)^(1/2) - 4*B^2*a^3*b*c + 12*A^2*a^
2*b*c^2 - 2*A*B*a*b*(-(4*a*c - b^2)^3)^(1/2) + 12*A*B*a^2*b^2*c)/(8*(a^3*b^
4 + 16*a^5*c^2 - 8*a^4*b^2*c)))^(1/2)*2i - atan(((x*(4*A^2*a^4*c^4 - 4*B^2*
a^5*c^3 - 2*A^2*a^3*b^2*c^3 + 4*A*B*a^4*b*c^3) ...
```

3.111 $\int \frac{A+Bx^2}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=271

$$-\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}-\frac{\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/3*A/a/x^3+(A*b-B*a)/a^2/x-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2))*c^(1/2)*(a*B*(b+(-4*a*c+b^2)^(1/2))-A*(b^2-2*a*c+b*(-4*a*c+b^2
)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*a
rctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(b-(-4*a
*c+b^2)^(1/2))-A*(b^2-2*a*c-b*(-4*a*c+b^2)^(1/2)))/a^2*2^(1/2)/(-4*a*c+b^2)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1295, 1180, 211}

$$-\frac{\sqrt{c}\left(aB\left(\sqrt{b^2-4ac}+b\right)-A\left(b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(-b\sqrt{b^2-4ac}-2ac+b^2\right)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}+\frac{Ab-aB}{a^2x}-\frac{A}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*A/(a*x^3) + (A*b - a*B)/(a^2*x) - (Sqrt[c]*(a*B*(b + Sqrt[b^2 - 4*a*c]
) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[
b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) - A*(b^2 - 2*a*c - b*Sqrt
[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)}\,dx &= -\frac{A}{3ax^3}-\frac{\int \frac{3(Ab-aB)+3Acx^2}{x^2(a+bx^2+cx^4)}\,dx}{3a}\\
&= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{\int \frac{3\left(Ab^2-abB-aAc\right)+3(Ab-aB)cx^2}{a+bx^2+cx^4}\,dx}{3a^2}\\
&= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}+\frac{\left(c\left(aB\left(b-\sqrt{b^2-4ac}\right)-A\left(b^2-2ac-b\sqrt{b^2-4ac}\right)\right)\right)}{2a^2\sqrt{b^2-4ac}}\\
&= -\frac{A}{3ax^3}+\frac{Ab-aB}{a^2x}-\frac{\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4}}}
\end{aligned}
$$

Mathematica [A]
time = 0.22, size = 267, normalized size = 0.99

$$
\frac{-\frac{2aA}{x^3}+\frac{6Ab-6aB}{x}-\frac{3\sqrt{2}\sqrt{c}\left(aB\left(b+\sqrt{b^2-4ac}\right)-A\left(b^2-2ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(aB\left(b-\sqrt{b^2-4ac}\right)+A\left(-b^2+2ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}}}{6a^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*a*A)/x^3 + (6*A*b - 6*a*B)/x - (3*Sqrt[2]*Sqrt[c]*(a*B*(b + Sqrt[b^2 -
 4*a*c]) - A*(b^2 - 2*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*
c]]) + (3*Sqrt[2]*Sqrt[c]*(a*B*(b - Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 2*a*c +
b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]
]])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(6*a^2)
```

Maple [A]
time = 0.06, size = 232, normalized size = 0.86

method	result
default	$\frac{4c\left(-\frac{\left(Ab\sqrt{-4ac+b^2}-2acA+Ab^2-aB\sqrt{-4ac+b^2}-abB\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(Ab\sqrt{-\right.}{}\right.}{}}{a^2}$
risch	$\frac{\frac{(Ab-aB)x^2}{a^2}-\frac{A}{3a}}{x^3}+\frac{\left(\sum_{\_R=\operatorname{RootOf}\left(\left(16a^7c^2-8b^2c\,a^6+b^4a^5\right)\_Z^4+\left(-20A^2a^3b\,c^3+25A^2a^2b^3c^2-9A^2a\,b^5c+A^2b^7+16AB\,a^4c^3-36AB\,a^3b^2c^2-\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/a^2*c*(-1/8*(A*b*(-4*a*c+b^2)^(1/2)-2*a*c*A+A*b^2-a*B*(-4*a*c+b^2)^(1/2)-
a*b*B)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh
(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(A*b*(-4*a*c+b^2)^(1/2)
+2*a*c*A-A*b^2-a*B*(-4*a*c+b^2)^(1/2)+a*b*B)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b
+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)
^(1/2)))-1/3*A/a/x^3-1/a^2*(-A*b+B*a)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(-((B*a - A*b)*c*x^2 + B*a*b - A*b^2 + A*a*c)/(c*x^4 + b*x^2 + a),
 x)/a^2 - 1/3*(3*(B*a - A*b)*x^2 + A*a)/(a^2*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5442 vs. $2(227) = 454$.
time = 3.24, size = 5442, normalized size = 20.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*(3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*
B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c +
(a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6
 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b +
 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^
3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A
^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))
/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4
- (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3
 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x + s
qrt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a
^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b
 - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b
^3 - 18*A*B^2*a^3*b^4 + 21*A^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*
a^5*b^5 + 4*(2*B*a^8 - 3*A*a^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt
((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b
^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B
^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2
*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*
B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*
A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*
a^2*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*
a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^
2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b +
 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b
^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)
*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))) - 3*sqrt(1/2)*a^2*x^3*sqr
t(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (
3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (a^5*b^2 - 4*a^6*c)*sqrt((B^
4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 +
 A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a
^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4
)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^
2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*
(A^4*a^2*c^5 + 3*(A^3*B*a^2*b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b +
 6*A^2*B^2*a^2*b^2 - A^3*B*a*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^
2*b^3 + 3*A^2*B^2*a*b^4 - A^3*B*b^5)*c^2)*x - sqrt(1/2)*(B^3*a^3*b^5 - 3*A*
B^2*a^2*b^6 + 3*A^2*B*a*b^7 - A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A
^2*B*a^4*b + 17*A^3*a^3*b^2)*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2
*B*a^3*b^3 - 20*A^3*a^2*b^4)*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A
^2*B*a^2*b^5 - 8*A^3*a*b^6)*c - (B*a^6*b^4 - A*a^5*b^5 + 4*(2*B*a^8 - 3*A*a
^7*b)*c^2 - (6*B*a^7*b^2 - 7*A*a^6*b^3)*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*
b^5 + 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^
2*a^5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*
A^2*B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 -
```

```
 6*A*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/
(a^10*b^2 - 4*a^11*c)))*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^4 + A^2*b^5 - (4*A*B
*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2 + 5*A^2*a*b^3)*c + (
a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5 + 6*A^2*B^2*a^2*b^6
- 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^5 - 4*A^3*B*a^4*b +
3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*B^2*a^4*b^2 - 28*A^3
*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A*B^3*a^4*b^3 + 12*A^
2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^10*b^2 - 4*a^11*c)))/
(a^5*b^2 - 4*a^6*c))) + 3*sqrt(1/2)*a^2*x^3*sqrt(-(B^2*a^2*b^3 - 2*A*B*a*b^
4 + A^2*b^5 - (4*A*B*a^3 - 5*A^2*a^2*b)*c^2 - (3*B^2*a^3*b - 8*A*B*a^2*b^2
+ 5*A^2*a*b^3)*c - (a^5*b^2 - 4*a^6*c)*sqrt((B^4*a^4*b^4 - 4*A*B^3*a^3*b^5
+ 6*A^2*B^2*a^2*b^6 - 4*A^3*B*a*b^7 + A^4*b^8 + A^4*a^4*c^4 - 2*(A^2*B^2*a^
5 - 4*A^3*B*a^4*b + 3*A^4*a^3*b^2)*c^3 + (B^4*a^6 - 8*A*B^3*a^5*b + 24*A^2*
B^2*a^4*b^2 - 28*A^3*B*a^3*b^3 + 11*A^4*a^2*b^4)*c^2 - 2*(B^4*a^5*b^2 - 6*A
*B^3*a^4*b^3 + 12*A^2*B^2*a^3*b^4 - 10*A^3*B*a^2*b^5 + 3*A^4*a*b^6)*c)/(a^1
0*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(2*(A^4*a^2*c^5 + 3*(A^3*B*a^2*
b - A^4*a*b^2)*c^4 - (B^4*a^4 - 5*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 - A^3*B*a
*b^3 - A^4*b^4)*c^3 + (B^4*a^3*b^2 - 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 - A^
3*B*b^5)*c^2)*x + sqrt(1/2)*(B^3*a^3*b^5 - 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7
- A^3*b^8 - 4*A^3*a^4*c^4 + (4*A*B^2*a^5 - 20*A^2*B*a^4*b + 17*A^3*a^3*b^2)
*c^3 + (4*B^3*a^5*b - 25*A*B^2*a^4*b^2 + 41*A^2*B*a^3*b^3 - 20*A^3*a^2*b^4)
*c^2 - (5*B^3*a^4*b^3 - 18*A*B^2*a^3*b^4 + 21*A...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2870 vs. 2(227) = 454.
time = 5.14, size = 2870, normalized size = 10.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*
c - 2*b^6*c + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(
```

b^2 - 4*a*c)*c)*b^4*c^2 + 18*a*b^4*c^2 + 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 48*a^2*b^2*c^3 - 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3*c^4 + 24*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 10*(b^2 - 4*a*c)*a*b^2*c^2 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3 + 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 + 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 - 12*a^2*b^2*c^3 + 16*a^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 4*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b + sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)) + 1/4*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 18*a*b^4*c^2 - 2*b^5*c^2 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 48*a^2*b^2*c^3 + 14*a*b^3*c^3 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 - 24*a^2*b*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c + 10*(b^2 - 4*a*c)*a*b^2*c^2 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 -

```
 4*a*c)*a^2*c^3 - 6*(b^2 - 4*a*c)*a*b*c^3)*A - (sqrt(2)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c -
2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*a*b^5*c + 16*sqrt(2)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2
- 16*a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a^2*b*c^3 + 32*a^3*b*c^3 + 12*a^2*b^2*c^3 - 16*a^3*c^4 + sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*a^2*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c + 8*(b^2 - 4*a*c)*a^2*b*c^2 +
 2*(b^2 - 4*a*c)*a*b^2*c^2 - 4*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(2*sqrt(1/2)
*x/sqrt((a^2*b - sqrt(a^4*b^2 - 4*a^5*c))/(a^2*c)))/((a^3*b^4 - 8*a^4*b^2*c
 - 2*a^3*b^3*c + 16*a^5*c^2 + 8*a^4*b*c^2 + a^3*b^2*c^2 - 4*a^4*c^3)*abs(c)
) - 1/3*(3*B*a*x^2 - 3*A*b*x^2 + A*a)/(a^2*x^3)
```

Mupad [B]
time = 2.19, size = 2500, normalized size = 9.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] - (A/(3*a) - (x^2*(A*b - B*a))/a^2)/x^3 - atan((((-(A^2*b^7 + B^2*a^2*b^5 +
 A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*
a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 1
6*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2
*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^
2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) +
 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 1
6*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10
*b^3*c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A
*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*
a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*
a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^
3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*
A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*
a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B
*a^10*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*
A^2*a^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2
*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*
```

b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i - ((-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 - x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) + x*(4*A^2*a^8*c^5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*1i)/(((-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2)*(16*A*a^10*c^4 + x*(32*a^11*b*c^3 - 8*a^10*b^3*c^2)*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25*A^2*a^2*b^3*c^2 + A^2*a^2*c^2*(-(4*a*c - b^2)^3)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^4*c^3 - 9*A^2*a*b^5*c - 20*A^2*a^3*b*c^3 - 7*B^2*a^3*b^3*c + 12*B^2*a^4*b*c^2 - B^2*a^3*c*(-(4*a*c - b^2)^3)^(1/2) - 36*A*B*a^3*b^2*c^2 - 3*A^2*a*b^2*c*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^3*(-(4*a*c - b^2)^3)^(1/2) + 16*A*B*a^2*b^4*c + 4*A*B*a^2*b*c*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4 + 16*a^7*c^2 - 8*a^6*b^2*c)))^(1/2) + 16*B*a^10*b*c^

```
3 + 4*A*a^8*b^4*c^2 - 20*A*a^9*b^2*c^3 - 4*B*a^9*b^3*c^2) - x*(4*A^2*a^8*c^
5 - 4*B^2*a^9*c^4 + 2*A^2*a^6*b^4*c^3 - 8*A^2*a^7*b^2*c^4 + 2*B^2*a^8*b^2*c
^3 - 4*A*B*a^7*b^3*c^3 + 12*A*B*a^8*b*c^4))*(-(A^2*b^7 + B^2*a^2*b^5 + A^2*
b^4*(-(4*a*c - b^2)^3)^(1/2) - 2*A*B*a*b^6 + 25...
```

3.112 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=212

$$\frac{(2b^2B-Abc-6aBc)\,x^2}{2c^2\,(b^2-4ac)}-\frac{x^4(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{(2b^4B-Ab^3c-12ab^2Bc+6aAbc^2}{2c^3\,(b^2-}$$

```
[Out] 1/2*(-A*b*c-6*B*a*c+2*B*b^2)*x^2/c^2/(-4*a*c+b^2)-1/2*x^4*(a*(-2*A*c+B*b)+(
-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-1/2*(6*A*a*b*c^2-
A*b^3*c+12*B*a^2*c^2-12*B*a*b^2*c+2*B*b^4)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)
^(1/2))/c^3/(-4*a*c+b^2)^(3/2)-1/4*(-A*c+2*B*b)*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A]
time = 0.25, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1265, 832, 787, 648, 632, 212, 642}

$$-\frac{(12a^2Bc^2+6aAbc^2-12ab^2Bc-Ab^3c+2b^4B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{3/2}}+\frac{x^2(-6aBc-Abc+2b^2B)}{2c^2\,(b^2-4ac)}-\frac{x^4(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac))}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{(2bB-Ac)\log(a+bx^2+cx^4)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*b^2*B - A*b*c - 6*a*B*c)*x^2)/(2*c^2*(b^2 - 4*a*c)) - (x^4*(a*(b*B - 2*
A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(2*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) - ((2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2)*ArcT
anh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(3/2)) - ((2*b*B
 - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= -\frac{x^4(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{x(2a(bB-2Ac)+(2b^2B-Abc-6a\cdots}{a+bx+cx^2}\cdots\right.}{2c\,(b^2-4ac)}$$

$$= \frac{(2b^2B-Abc-6aBc)\,x^2}{2c^2\,(b^2-4ac)} - \frac{x^4(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\cdots\right.}{\cdots}$$

$$= \frac{(2b^2B-Abc-6aBc)\,x^2}{2c^2\,(b^2-4ac)} - \frac{x^4(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(2bB-\cdots}{\cdots}$$

$$= \frac{(2b^2B-Abc-6aBc)\,x^2}{2c^2\,(b^2-4ac)} - \frac{x^4(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(2bB-\cdots}{\cdots}$$

$$= \frac{(2b^2B-Abc-6aBc)\,x^2}{2c^2\,(b^2-4ac)} - \frac{x^4(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} - \frac{(2b^4B-\cdots}{\cdots}$$

Mathematica [A]

time = 0.19, size = 208, normalized size = 0.98

$$\frac{2Bcx^2 - \frac{2(b^3(bB-Ac)x^2+a^2c(-3bB+2c(A+Bx^2))+ab(b^2B+3Ac^2x^2-bc(A+4Bx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} - \frac{2(2b^4B-Ab^3c-12ab^2Bc+6aAbc^2+12a^2Bc^2)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + (-2bB+Ac)\log(a+bx^2+cx^4)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (2*B*c*x^2 - (2*(b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a
*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 +
c*x^4)) - (2*(2*b^4*B - A*b^3*c - 12*a*b^2*B*c + 6*a*A*b*c^2 + 12*a^2*B*c^2
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/2) + (-2*b*B
+ A*c)*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Maple [A]

time = 0.10, size = 282, normalized size = 1.33

method	result
default	$\frac{Bx^2}{2c^2} + \frac{\frac{(3Aab\,c^2-A\,b^3c+2a^2B\,c^2-4a\,b^2Bc+b^4B)x^2}{c(4ac-b^2)}+\frac{a(2c^2aA-A\,b^2c-3abBc+b^3B)}{c(4ac-b^2)}}{c\,x^4+b\,x^2+a} + \frac{\frac{(4c^2aA-A\,b^2c-8abBc+2b^3B)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2(\cdots}{\cdots}}{2c^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c^2+1/2/c^2*(((3*A*a*b*c^2-A*b^3*c+2*B*a^2*c^2-4*B*a*b^2*c+B*b^4)
/c/(4*a*c-b^2)*x^2+a*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/c/(4*a*c-b^2))/(c*
x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*A*a*c^2-A*b^2*c-8*B*a*b*c+2*B*b^3)/c*ln(
c*x^4+b*x^2+a)+2*(-A*a*b*c-6*a^2*c*B+2*B*a*b^2-1/2*(4*A*a*c^2-A*b^2*c-8*B*a
*b*c+2*B*b^3)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 650 vs. $2(200) = 400$.
time = 0.44, size = 1323, normalized size = 6.24

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + (2*B*a*b^4 + (2*B*b^4*c + 6*(
2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2*b
)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*x^
2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^
2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2
*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*
a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3 - (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(
4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6 - 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A
```

```
*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log
(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2),
 -1/4*(2*B*a*b^5 - 16*A*a^3*c^3 - 2*(B*b^4*c^2 - 8*B*a*b^2*c^3 + 16*B*a^2*c
^4)*x^6 - 2*(B*b^5*c - 8*B*a*b^3*c^2 + 16*B*a^2*b*c^3)*x^4 + 12*(2*B*a^3*b
+ A*a^2*b^2)*c^2 + 2*(B*b^6 - 12*(2*B*a^3 + A*a^2*b)*c^3 + (26*B*a^2*b^2 +
7*A*a*b^3)*c^2 - (9*B*a*b^4 + A*b^5)*c)*x^2 + 2*(2*B*a*b^4 + (2*B*b^4*c + 6
*(2*B*a^2 + A*a*b)*c^3 - (12*B*a*b^2 + A*b^3)*c^2)*x^4 + 6*(2*B*a^3 + A*a^2
*b)*c^2 + (2*B*b^5 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (12*B*a*b^3 + A*b^4)*c)*
x^2 - (12*B*a^2*b^2 + A*a*b^3)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*
sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(7*B*a^2*b^3 + A*a*b^4)*c + (2*B*a*b^
5 - 16*A*a^3*c^3 + (2*B*b^5*c - 16*A*a^2*c^4 + 8*(4*B*a^2*b + A*a*b^2)*c^3
- (16*B*a*b^3 + A*b^4)*c^2)*x^4 + 8*(4*B*a^3*b + A*a^2*b^2)*c^2 + (2*B*b^6
- 16*A*a^2*b*c^3 + 8*(4*B*a^2*b^2 + A*a*b^3)*c^2 - (16*B*a*b^4 + A*b^5)*c)*
x^2 - (16*B*a^2*b^3 + A*a*b^4)*c)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^3 - 8*a^
2*b^2*c^4 + 16*a^3*c^5 + (b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^4 + (b^5*c^
3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.88, size = 239, normalized size = 1.13

$$\frac{Bx^2}{2\,c^2}+\frac{\left(2\,Bb^4-12\,Bab^2c-Ab^3c+12\,Ba^2c^2+6\,Aabc^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(b^2c^3-4\,ac^4\right)\sqrt{-b^2+4\,ac}}+\frac{2\,Bb^3x^4-8\,Babcx^4-Ab^2cx^4+4\,Aac^2x^4+Ab^3x^2-4\,Ba^2cx^2-2\,Aabcx^2-2\,Ba^2b+Aab^2}{4\,\left(cx^4+bx^2+a\right)\left(b^2c^2-4\,ac^3\right)}-\frac{\left(2\,Bb-Ac\right)\log\left(cx^4+bx^2+a\right)}{4\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*B*x^2/c^2 + 1/2*(2*B*b^4 - 12*B*a*b^2*c - A*b^3*c + 12*B*a^2*c^2 + 6*A*
a*b*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2*c^3 - 4*a*c^4)*sqrt
(-b^2 + 4*a*c)) + 1/4*(2*B*b^3*x^4 - 8*B*a*b*c*x^4 - A*b^2*c*x^4 + 4*A*a*c^
2*x^4 + A*b^3*x^2 - 4*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + A*a*b^2)/((
c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)) - 1/4*(2*B*b - A*c)*log(c*x^4 + b*x
^2 + a)/c^3
```

Mupad [B]
time = 0.83, size = 2282, normalized size = 10.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((a*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c*(4*a*c - b^2)) + (x^2*(
B*b^4 + 2*B*a^2*c^2 - A*b^3*c + 3*A*a*b*c^2 - 4*B*a*b^2*c))/(2*c*(4*a*c - b
^2)))/(a*c^2 + c^3*x^4 + b*c^2*x^2) + (B*x^2)/(2*c^2) + (log(a + b*x^2 + c*
x^4)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 -
 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*c^6 -
 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)) - (atan(((8*a*c^5*(4*a*c - b^
2)^3 - 2*b^2*c^4*(4*a*c - b^2)^3)*(x^2*(((((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12
*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c
^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A
*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4
*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5
)))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c))/(8*c^3
*(4*a*c - b^2)^(3/2)) + ((8*b^3*c^6 - 32*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 -
 A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c
 - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192
*B*a^2*b^3*c^2))/(16*c^3*(4*a*c - b^2)^(3/2)*(4*a*c^5 - b^2*c^4)*(256*a^3*c
^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*(
(4*B^2*b^5 + A^2*b^3*c^2 - 4*A*B*b^4*c - 6*A*B*a^2*c^3 - 5*A^2*a*b*c^3 - 20
*B^2*a*b^3*c + 12*B^2*a^2*b*c^2 + 20*A*B*a*b^2*c^2)/(4*a*c^5 - b^2*c^4) + (
((24*B*a^2*c^5 - 6*A*b^3*c^4 + 12*B*b^4*c^3 + 28*A*a*b*c^5 - 56*B*a*b^2*c^4
)/(4*a*c^5 - b^2*c^4) + ((8*b^3*c^6 - 32*a*b*c^7)*(4*B*b^7 + 128*A*a^3*c^4
- 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^
2*c^3 + 192*B*a^2*b^3*c^2))/(2*(4*a*c^5 - b^2*c^4)*(256*a^3*c^6 - 4*b^6*c^3
 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5)))*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c -
 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B
*a^2*b^3*c^2))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5
)) - (((b^3*c^6)/2 - 2*a*b*c^7)*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b
*c^2 - 12*B*a*b^2*c)^2)/(c^6*(4*a*c - b^2)^3*(4*a*c^5 - b^2*c^4))))/(2*a*(4
*a*c - b^2)^(3/2))) + ((((8*A*a*c^4 - 16*B*a*b*c^3)/c^4 - (8*a*c^2*(4*B*b^7
 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*
c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(256*a^3*c^6 - 4*b^6*c^3 + 48*
a*b^4*c^4 - 192*a^2*b^2*c^5))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c
^2 - 12*B*a*b^2*c))/(8*c^3*(4*a*c - b^2)^(3/2)) - (a*(2*B*b^4 + 12*B*a^2*c^
2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^
6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 +
192*B*a^2*b^3*c^2))/(c*(4*a*c - b^2)^(3/2)*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*
b^4*c^4 - 192*a^2*b^2*c^5)))/(a*(4*a*c - b^2)) + (b*((((8*A*a*c^4 - 16*B*a*
b*c^3)/c^4 - (8*a*c^2*(4*B*b^7 + 128*A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c +
 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 96*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/
(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^4*c^4 - 192*a^2*b^2*c^5))*(4*B*b^7 + 128*
A*a^3*c^4 - 2*A*b^6*c - 48*B*a*b^5*c + 24*A*a*b^4*c^2 - 256*B*a^3*b*c^3 - 9
```

```
6*A*a^2*b^2*c^3 + 192*B*a^2*b^3*c^2))/(2*(256*a^3*c^6 - 4*b^6*c^3 + 48*a*b^
4*c^4 - 192*a^2*b^2*c^5)) - (A^2*a*c^2 + 4*B^2*a*b^2 - 4*A*B*a*b*c)/c^4 + (
a*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c + 6*A*a*b*c^2 - 12*B*a*b^2*c)^2)/(c^4*(
4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2))))/(4*B^2*b^8 + A^2*b^6*c^2 + 14
4*B^2*a^4*c^4 - 4*A*B*b^7*c + 36*A^2*a^2*b^2*c^4 + 192*B^2*a^2*b^4*c^2 - 28
8*B^2*a^3*b^2*c^3 - 48*B^2*a*b^6*c - 12*A^2*a*b^4*c^3 - 168*A*B*a^2*b^3*c^3
 + 48*A*B*a*b^5*c^2 + 144*A*B*a^3*b*c^4))*(2*B*b^4 + 12*B*a^2*c^2 - A*b^3*c
 + 6*A*a*b*c^2 - 12*B*a*b^2*c))/(2*c^3*(4*a*c - b^2)^(3/2))
```

3.113 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=147

$$-\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{(b^3B-6abBc+4aAc^2)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}}+\frac{B\log(a+}{4}$$

```
[Out] -1/2*x^2*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)+1/2*(4*A*a*c^2-6*B*a*b*c+B*b^3)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(
1/2))/c^2/(-4*a*c+b^2)^(3/2)+1/4*B*ln(c*x^4+b*x^2+a)/c^2
```

Rubi [A]
time = 0.12, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 832, 648, 632, 212, 642}

$$-\frac{x^2(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac))}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{(4aAc^2-6abBc+b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\,(b^2-4ac)^{3/2}}+\frac{B\log(a+bx^2+cx^4)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(x^2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(c*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)) + ((b^3*B - 6*a*b*B*c + 4*a*A*c^2)*ArcTanh[(b + 2*c
*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*(b^2 - 4*a*c)^(3/2)) + (B*Log[a + b*x^2 +
c*x^4])/(4*c^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)$$

$$= -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{\text{Subst}\left(\int \frac{a(bB-2Ac)+B(b^2-4ac)x}{a+bx+cx^2}\,dx,\right.}{2c\,(b^2-4ac)}$$

$$= -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{B\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c^2} -$$

$$= -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{B\log\left(a+bx^2+cx^4\right)}{4c^2} + \frac{(b^3B-}{}$$

$$= -\frac{x^2(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)} + \frac{(b^3B-6abBc+4aAc^2)\tanh^{-1}\left(\right.}{2c^2\,(b^2-4ac)^{3/2}}$$

Mathematica [A]
time = 0.13, size = 160, normalized size = 1.09

$$\frac{-\frac{2(2a^2Bc+b^2(-bB+Ac)x^2+a(-b^2B-2Ac^2x^2+bc(A+3Bx^2)))}{(b^2-4ac)(a+bx^2+cx^4)} + \frac{2(b^3B-6abBc+4aAc^2)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}} + B\log\left(a+bx^2+cx^4\right)}{4c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((-2*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*
(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (2*(b^3*B - 6*a*b*B*
c + 4*a*A*c^2)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(3/
2) + B*Log[a + b*x^2 + c*x^4])/(4*c^2)
```

Maple [A]
time = 0.08, size = 211, normalized size = 1.44

method	result
default	$\frac{-\frac{\left(2c^2aA-A\,b^2c-3abBc+b^3B\right)x^2}{c^2\left(4ac-b^2\right)}+\frac{a\left(bcA+2acB-b^2B\right)}{c^2\left(4ac-b^2\right)}}{2c\,x^4+2b\,x^2+2a} + \frac{\frac{\left(4acB-b^2B\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(2acA-abB-\frac{\left(4acB-b^2B\right)b}{2c}\right)\arctan\left(\frac{2c}{\sqrt{4a}}\right.}{\sqrt{4ac-b^2}}}{2(4ac-b^2)c}$
risch	$\frac{-\frac{\left(2c^2aA-A\,b^2c-3abBc+b^3B\right)x^2}{2c^2\left(4ac-b^2\right)}+\frac{a\left(bcA+2acB-b^2B\right)}{2c^2\left(4ac-b^2\right)}}{c\,x^4+b\,x^2+a} + \frac{4\ln\left(\left(\left(-16A\,a^2c^3+4Aa\,b^2c^2+24B\,a^2b\,c^2-10Ba\,b^3c+B\,b^5-\sqrt{-\left(4ac\right.}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/c^2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^2+a*(A*b*c+2*
B*a*c-B*b^2)/c^2/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/2/(4*a*c-b^2)/c*(1/2*(4*B*a
*c-B*b^2)/c*ln(c*x^4+b*x^2+a)+2*(2*a*c*A-a*b*B-1/2*(4*B*a*c-B*b^2)*b/c)/(4*
a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(137) = 274.
time = 0.40, size = 849, normalized size = 5.78

$$\left[\frac{2\,Bab^4 + 8\,(2\,Ba^3 + Aa^2b)c^2 + 2\,(Bb^5 - 8\,Aa^2c^3 + 6\,(2\,Ba^2b + Aab^2)c^2 - (7\,Bab^3 + Ab^4)c)x^2 - (Bab^3 - 6\,Ba^2bc + 4\,Aa^2c^2 + (Bb^3c - 6\,Babc^2 + 4\,Aac^3)x^4 + (Bb^4 - 6\,Bab^2c + 4\,Aabc^2)x^2)\sqrt{b^2 - 4\,ac}\,\log\left(\frac{2\,c^2x^4 + 2\,bcx^2 + b^2 - 2\,ac - (2\,cx^2 + b)\sqrt{b^2 - 4\,ac}}{cx^4 + bx^2 + a}\right) - 2\,(6\,Ba^2b^2 + Aab^3)c + (Bab^4 - 8\,Ba^2b^2c + 16\,Ba^3c^2 + (Bb^4c - 8\,Bab^2c^2 + 16\,Ba^2c^3)x^4 + (Bb^5 - 8\,Bab^3c + 16\,Ba^2bc^2)x^2)\log\left(cx^4 + bx^2 + a\right)}{4\,(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + (b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5)x^4 + (b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4)x^2)}, \frac{2\,Bab^4 + 8\,(2\,Ba^3 + Aa^2b)c^2 + 2\,(Bb^5 - 8\,Aa^2c^3 + 6\,(2\,Ba^2b + Aab^2)c^2 - (7\,Bab^3 + Ab^4)c)x^2 + 2\,(Bab^3 - 6\,Ba^2bc + 4\,Aa^2c^2 + (Bb^3c - 6\,Babc^2 + 4\,Aac^3)x^4 + (Bb^4 - 6\,Bab^2c + 4\,Aabc^2)x^2)\sqrt{-b^2 + 4\,ac}\,\arctan\left(-\frac{(2\,cx^2 + b)\sqrt{-b^2 + 4\,ac}}{b^2 - 4\,ac}\right) - 2\,(6\,Ba^2b^2 + Aab^3)c + (Bab^4 - 8\,Ba^2b^2c + 16\,Ba^3c^2 + (Bb^4c - 8\,Bab^2c^2 + 16\,Ba^2c^3)x^4 + (Bb^5 - 8\,Bab^3c + 16\,Ba^2bc^2)x^2)\log\left(cx^4 + bx^2 + a\right)}{4\,(ab^4c^2 - 8\,a^2b^2c^3 + 16\,a^3c^4 + (b^4c^3 - 8\,ab^2c^4 + 16\,a^2c^5)x^4 + (b^5c^2 - 8\,ab^3c^3 + 16\,a^2bc^4)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*c^3 + 6*(2
*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 - (B*a*b^3 - 6*B*a^2*b
*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B*b^4 - 6*B*a
*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b
^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(6*B
*a^2*b^2 + A*a*b^3)*c + (B*a*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c
- 8*B*a*b^2*c^2 + 16*B*a^2*c^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2
)*x^2)*log(c*x^4 + b*x^2 + a))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b
^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*
c^4)*x^2), 1/4*(2*B*a*b^4 + 8*(2*B*a^3 + A*a^2*b)*c^2 + 2*(B*b^5 - 8*A*a^2*
c^3 + 6*(2*B*a^2*b + A*a*b^2)*c^2 - (7*B*a*b^3 + A*b^4)*c)*x^2 + 2*(B*a*b^3
 - 6*B*a^2*b*c + 4*A*a^2*c^2 + (B*b^3*c - 6*B*a*b*c^2 + 4*A*a*c^3)*x^4 + (B
*b^4 - 6*B*a*b^2*c + 4*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2
+ b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 2*(6*B*a^2*b^2 + A*a*b^3)*c + (B*a
*b^4 - 8*B*a^2*b^2*c + 16*B*a^3*c^2 + (B*b^4*c - 8*B*a*b^2*c^2 + 16*B*a^2*c
^3)*x^4 + (B*b^5 - 8*B*a*b^3*c + 16*B*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a
))/(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4 + (b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)*x^4 + (b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.77, size = 194, normalized size = 1.32

$$-\frac{\left(Bb^3-6\,Babc+4\,Aac^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(b^2c^2-4\,ac^3)\sqrt{-b^2+4\,ac}}+\frac{B\log\left(cx^4+bx^2+a\right)}{4\,c^2}-\frac{Bb^2cx^4-4\,Bac^2x^4-Bb^3x^2+2\,Babcx^2+2\,Ab^2cx^2-4\,Aac^2x^2-Bab^2+2\,Aabc}{4\,(cx^4+bx^2+a)(b^2c^2-4\,ac^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^3 - 6*B*a*b*c + 4*A*a*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((b^2*c^2 - 4*a*c^3)*sqrt(-b^2 + 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/
c^2 - 1/4*(B*b^2*c*x^4 - 4*B*a*c^2*x^4 - B*b^3*x^2 + 2*B*a*b*c*x^2 + 2*A*b^
2*c*x^2 - 4*A*a*c^2*x^2 - B*a*b^2 + 2*A*a*b*c)/((c*x^4 + b*x^2 + a)*(b^2*c^
2 - 4*a*c^3))
```

Mupad [**B**]
time = 1.22, size = 1527, normalized size = 10.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(B*b^3 + 2*A*a*c^2 - A*b^2*c - 3*B*a*b*c))/(2*c^2*(4*a*c - b^2)) -
(a*(A*b*c - B*b^2 + 2*B*a*c))/(2*c^2*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) -
(log(a + b*x^2 + c*x^4)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*
b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) -
(atan(((8*a*c^3*(4*a*c - b^2)^3 - 2*b^2*c^2*(4*a*c - b^2)^3)*((((8*B*a + (8
*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(256*a^
3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(B*b^3 + 4*A*a*c^2 - 6
*B*a*b*c))/(8*c^2*(4*a*c - b^2)^(3/2)) + (a*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c)
*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/((4*a*c - b^2
)^(3/2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))/(a*(4*
a*c - b^2)) - x^2*(((((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(4*a*c^3 - b
^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c
```

```
 + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*
a*b^4*c^3 - 192*a^2*b^2*c^4)))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c))/(8*c^2*(4*a
*c - b^2)^(3/2)) + ((8*b^3*c^4 - 32*a*b*c^5)*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c
)*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(16*c^2*(4*a
*c - b^2)^(3/2)*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3
 - 192*a^2*b^2*c^4)))/(a*(4*a*c - b^2)) + (b*((B^2*b^3 + 2*A*B*a*c^2 - 5*B^
2*a*b*c)/(4*a*c^3 - b^2*c^2) + (((6*B*b^3*c^2 + 8*A*a*c^4 - 28*B*a*b*c^3)/(
4*a*c^3 - b^2*c^2) + ((8*b^3*c^4 - 32*a*b*c^5)*(2*B*b^6 - 128*B*a^3*c^3 - 2
4*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(4*a*c^3 - b^2*c^2)*(256*a^3*c^5 - 4*b^
6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)))*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a
*b^4*c + 96*B*a^2*b^2*c^2))/(2*(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 19
2*a^2*b^2*c^4)) - (((b^3*c^4)/2 - 2*a*b*c^5)*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c
)^2)/(c^4*(4*a*c - b^2)^3*(4*a*c^3 - b^2*c^2))))/(2*a*(4*a*c - b^2)^(3/2)))
 + (b*(((8*B*a + (8*a*c^2*(2*B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^
2*b^2*c^2))/(256*a^3*c^5 - 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4))*(2*
B*b^6 - 128*B*a^3*c^3 - 24*B*a*b^4*c + 96*B*a^2*b^2*c^2))/(2*(256*a^3*c^5 -
 4*b^6*c^2 + 48*a*b^4*c^3 - 192*a^2*b^2*c^4)) + (B^2*a)/c^2 - (a*(B*b^3 + 4
*A*a*c^2 - 6*B*a*b*c)^2)/(c^2*(4*a*c - b^2)^3)))/(2*a*(4*a*c - b^2)^(3/2)))
)/(B^2*b^6 + 16*A^2*a^2*c^4 + 36*B^2*a^2*b^2*c^2 - 12*B^2*a*b^4*c + 8*A*B*a
*b^3*c^2 - 48*A*B*a^2*b*c^3))*(B*b^3 + 4*A*a*c^2 - 6*B*a*b*c))/(2*c^2*(4*a*
c - b^2)^(3/2))
```

3.114 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=107

$$-\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(-a*(-2*A*c+B*b)-(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^
2+a)-(A*b-2*B*a)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.07, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1265, 791, 632, 212}

$$-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB-2Ac)}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(c*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(b^2 - 4*a*c)^(3/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
```

```
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)\\ &= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{(Ab-2aB)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2\,(b^2-4ac)}\\ &= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{(Ab-2aB)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b\ldots\right)}{b^2-4ac}\\ &= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{2c\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 111, normalized size = 1.04

$$\frac{abB+b(bB-Ac)x^2-2ac(A+Bx^2)}{2c\,(-b^2+4ac)\,(a+bx^2+cx^4)}-\frac{(Ab-2aB)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(2*c*(-b^2 + 4*a*c)*(a + b*
x^2 + c*x^4)) - ((A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-
b^2 + 4*a*c)^(3/2)
```

Maple [A]
time = 0.05, size = 126, normalized size = 1.18

method	result
default	$\frac{-\frac{\left(bcA+2acB-b^{2}B\right)x^{2}}{c\left(4ac-b^{2}\right)}-\frac{a(2Ac-bB)}{\left(4ac-b^{2}\right)c}}{2c\,x^{4}+2b\,x^{2}+2a}-\frac{(Ab-2aB)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{(4ac-b^{2})^{\frac{3}{2}}}$
risch	$\frac{-\frac{\left(bcA+2acB-b^{2}B\right)x^{2}}{2c\left(4ac-b^{2}\right)}-\frac{a(2Ac-bB)}{2\left(4ac-b^{2}\right)c}}{c\,x^{4}+b\,x^{2}+a}+\frac{\ln\left(\left(-(-4ac+b^{2})^{\frac{3}{2}}+4abc-b^{3}\right)x^{2}+8a^{2}c-2a\,b^{2}\right)Ab}{2(-4ac+b^{2})^{\frac{3}{2}}}-\frac{\ln\left(\left(-(-4ac+b^{2})^{\frac{3}{2}}+4abc-b^{3}\right)x^{2}\right.}{(-4ac+b^{2})^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^2-a*(2*A*c-B*b)/(4*a*c-b^2)/c)/
(c*x^4+b*x^2+a)-(A*b-2*B*a)/(4*a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2
)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 257 vs. 2(101) = 202.
time = 0.36, size = 538, normalized size = 5.03

$$\left[-\frac{Bab^{3}+8\,Aa^{2}c^{2}+(Bb^{4}+4\,(2\,Ba^{2}+Aab)c^{2}-(6\,Bab^{2}+Ab^{3})c)x^{2}-((2\,Ba-Ab)c^{2}x^{4}+(2\,Bab-Ab^{2})cx^{2}+(2\,Ba^{2}-Aab)c)\sqrt{b^{2}-4\,ac}\,\log\left(\frac{2\,c^{2}x^{4}+2\,bcx^{2}+b^{2}-2\,ac+(2\,cx^{2}+b)\sqrt{b^{2}-4\,ac}}{cx^{4}+bx^{2}+a}\right)-2\,(2\,Ba^{2}b+Aab^{2})c}{2\,(ab^{4}c-8\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3}+(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4})x^{4}+(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3})x^{2})},\ -\frac{Bab^{3}+8\,Aa^{2}c^{2}+(Bb^{4}+4\,(2\,Ba^{2}+Aab)c^{2}-(6\,Bab^{2}+Ab^{3})c)x^{2}-2\,((2\,Ba-Ab)c^{2}x^{4}+(2\,Bab-Ab^{2})cx^{2}+(2\,Ba^{2}-Aab)c)\sqrt{-b^{2}+4\,ac}\,\arctan\left(-\frac{(2\,cx^{2}+b)\sqrt{-b^{2}+4\,ac}}{b^{2}-4\,ac}\right)-2\,(2\,Ba^{2}b+Aab^{2})c}{2\,(ab^{4}c-8\,a^{2}b^{2}c^{2}+16\,a^{3}c^{3}+(b^{4}c^{2}-8\,ab^{2}c^{3}+16\,a^{2}c^{4})x^{4}+(b^{5}c-8\,ab^{3}c^{2}+16\,a^{2}bc^{3})x^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/2*(B*a*b^3 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^
2 + A*b^3)*c)*x^2 - ((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B
*a^2 - A*a*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c
 + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 2*(2*B*a^2*b + A
*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (b^4*c^2 - 8*a*b^2*c^3 +
 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^2), -1/2*(B*a*b^3
 + 8*A*a^2*c^2 + (B*b^4 + 4*(2*B*a^2 + A*a*b)*c^2 - (6*B*a*b^2 + A*b^3)*c)*
```

```
x^2 - 2*((2*B*a - A*b)*c^2*x^4 + (2*B*a*b - A*b^2)*c*x^2 + (2*B*a^2 - A*a*b
)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)) - 2*(2*B*a^2*b + A*a*b^2)*c)/(a*b^4*c - 8*a^2*b^2*c^2 + 16*a^3*c^3 + (
b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^4 + (b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c
^3)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 394 vs. $2(99) = 198$.
time = 4.28, size = 394, normalized size = 3.68

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)\log\left(x^2+\frac{-Ab^2+2Bab-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)-b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)}{-2Abc+4Bac}\right)}{2}+\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)\log\left(x^2+\frac{-Ab^2+2Bab+16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)+b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-Ab+2Ba)}{-2Abc+4Bac}\right)}{2}+\frac{-2Aac+Bab+x^2(-Abc-2Bac+Bb^2)}{8a^2c^2-2ab^2c+x^4\cdot(8ac^3-2b^2c^2)+x^2\cdot(8abc^2-2b^3c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a)*log(x**2 + (-A*b**2 + 2*B*a*b -
16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + 8*a*b**2*c*sqrt(-1
/(4*a*c - b**2)**3)*(-A*b + 2*B*a) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b
+ 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*
a)*log(x**2 + (-A*b**2 + 2*B*a*b + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-A*b + 2*B*a) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-A*b + 2*B*a))/(-2*A*b*c + 4*B*a*c))/2 + (-2*
A*a*c + B*a*b + x**2*(-A*b*c - 2*B*a*c + B*b**2))/(8*a**2*c**2 - 2*a*b**2*c
 + x**4*(8*a*c**3 - 2*b**2*c**2) + x**2*(8*a*b*c**2 - 2*b**3*c))
```

Giac [A]
time = 5.10, size = 120, normalized size = 1.12

$$-\frac{(2\,Ba-Ab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}-\frac{Bb^2x^2-2\,Bacx^2-Abcx^2+Bab-2\,Aac}{2\,(cx^4+bx^2+a)(b^2c-4\,ac^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -(2*B*a - A*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt
(-b^2 + 4*a*c)) - 1/2*(B*b^2*x^2 - 2*B*a*c*x^2 - A*b*c*x^2 + B*a*b - 2*A*a*
c)/((c*x^4 + b*x^2 + a)*(b^2*c - 4*a*c^2))
```

Mupad [B]
time = 0.32, size = 283, normalized size = 2.64

$$-\frac{\frac{x^2\,\left(-B\,b^2+A\,c\,b+2\,B\,a\,c\right)}{2\,c\,\left(4\,a\,c-b^2\right)}+\frac{a\,\left(2\,A\,c-B\,b\right)}{2\,c\,\left(4\,a\,c-b^2\right)}}{c\,x^4+b\,x^2+a}-\frac{\mathrm{atan}\left(\frac{{\left(4\,a\,c-b^2\right)}^4\,\left(x^2\,\left(\frac{\left(A\,b-2\,B\,a\right)\,\left(A\,b\,c^2-2\,B\,a\,c^2\right)}{a\,{\left(4\,a\,c-b^2\right)}^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,{\left(A\,b-2\,B\,a\right)}^2\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,{\left(4\,a\,c-b^2\right)}^{13/2}}\right)-\frac{2\,c^2\,{\left(A\,b-2\,B\,a\right)}^2\,\left(b^3-4\,a\,b\,c\right)}{{\left(4\,a\,c-b^2\right)}^{11/2}}\right)}{2\,A^2\,b^2\,c^2-8\,A\,B\,a\,b\,c^2+8\,B^2\,a^2\,c^2}\right)\,\left(A\,b-2\,B\,a\right)}{{\left(4\,a\,c-b^2\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] - ((x^2*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (a*(2*A*c - B*b))/
(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - (atan(((4*a*c - b^2)^4*(x^2*(((A
*b - 2*B*a)*(A*b*c^2 - 2*B*a*c^2))/(a*(4*a*c - b^2)^(7/2)) + ((2*b^3*c^2 -
8*a*b*c^3)*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(2*a*(4*a*c - b^2)^(13/2))) - (
2*c^2*(A*b - 2*B*a)^2*(b^3 - 4*a*b*c))/(4*a*c - b^2)^(11/2)))/(2*A^2*b^2*c^
2 + 8*B^2*a^2*c^2 - 8*A*B*a*b*c^2))*(A*b - 2*B*a))/(4*a*c - b^2)^(3/2)
```

3.115 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2} \, dx$

Optimal. Leaf size=94

$$-\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}$$

```
[Out] 1/2*(-A*b+2*a*B+(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)-(-2*A*c+B*b)
*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(3/2)
```

Rubi [A]
time = 0.06, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {1261, 652, 632, 212}

$$-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4))
- ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(3
/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
```

```
NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(a+bx+cx^2)^2}\,dx, x, x^2\right)\\
&= -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{(bB-2Ac)\text{Subst}\left(\int \frac{1}{a+bx+cx^2}\,dx, x, x^2\right)}{2(b^2-4ac)}\\
&= -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(bB-2Ac)\text{Subst}\left(\int \frac{1}{b^2-4ac-x^2}\,dx, x, b+2cx^2\right)}{b^2-4ac}\\
&= -\frac{Ab-2aB-(bB-2Ac)x^2}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.05, size = 101, normalized size = 1.07

$$
\frac{\frac{B(2a+bx^2)-A(b+2cx^2)}{a+bx^2+cx^4} + \frac{2(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{2(b^2-4ac)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((B*(2*a + b*x^2) - A*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + (2*(b*B - 2*A*c)
*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(2*(b^2 - 4*
a*c))
```

Maple [A]
time = 0.04, size = 95, normalized size = 1.01

method	result

default	$\frac{(2Ac-bB)x^2+Ab-2aB}{2(4ac-b^2)(c\,x^4+b\,x^2+a)}+\frac{(2Ac-bB)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}$
risch	$\frac{\frac{(2Ac-bB)x^2}{8ac-2b^2}+\frac{Ab-2aB}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8a^2c-2a\,b^2\right)Ac}{(-4ac+b^2)^{\frac{3}{2}}}-\frac{\ln\left(\left(\left(-4ac+b^2\right)^{\frac{3}{2}}+4abc-b^3\right)x^2+8a^2c-2a\,b^2\right)bB}{2(-4ac+b^2)^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*((2*A*c-B*b)*x^2+A*b-2*a*B)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+(2*A*c-B*b)/(4*
a*c-b^2)^(3/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(88) = 176$.
time = 0.38, size = 474, normalized size = 5.04

$$\left[\frac{2\,Bab^2-Ab^3+(Bb^3+8\,Aac^2-2\,(2\,Bab+Ab^2)c)x^2+((Bbc-2\,Ac^2)x^4+Bab-2\,Aac+(Bb^2-2\,Abc)x^2)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-4\,(2\,Ba^2-Aab)c}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)},\frac{2\,Bab^2-Ab^3+(Bb^3+8\,Aac^2-2\,(2\,Bab+Ab^2)c)x^2-2\,((Bbc-2\,Ac^2)x^4+Bab-2\,Aac+(Bb^2-2\,Abc)x^2)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-4\,(2\,Ba^2-Aab)c}{2\,(ab^4-8\,a^2b^2c+16\,a^3c^2+(b^4c-8\,ab^2c^2+16\,a^2c^3)x^4+(b^5-8\,ab^3c+16\,a^2bc^2)x^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B*a*b + A*b^2)*c)*x^2 +
 ((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2 - 2*A*b*c)*x^2)*sqrt(b^2
 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2
 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2 + (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c
 + 16*a^2*b*c^2)*x^2), 1/2*(2*B*a*b^2 - A*b^3 + (B*b^3 + 8*A*a*c^2 - 2*(2*B
*a*b + A*b^2)*c)*x^2 - 2*((B*b*c - 2*A*c^2)*x^4 + B*a*b - 2*A*a*c + (B*b^2
- 2*A*b*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)
/(b^2 - 4*a*c)) - 4*(2*B*a^2 - A*a*b)*c)/(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
```

```
+ (b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*x^4 + (b^5 - 8*a*b^3*c + 16*a^2*b*c^2)
*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(83) = 166$.
time = 2.35, size = 374, normalized size = 3.98

$$\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-2Ac+Bb)\log\left(x^2+\frac{-2Abc+Bb^2-16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-2Ac+Bb)+8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-2Ac+Bb)-b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-2Ac+Bb)}{-4Ac^2+2Bbc}\right)}{2}-\frac{\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-2Ac+Bb)\log\left(x^2+\frac{-2Abc+Bb^2+16a^2c^2\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-2Ac+Bb)-8ab^2c\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-2Ac+Bb)+b^4\sqrt{-\frac{1}{(4ac-b^2)^3}}\,(-2Ac+Bb)}{-4Ac^2+2Bbc}\right)}{2}+\frac{Ab-2Ba+x^2\cdot(2Ac-Bb)}{8a^2c-2ab^2+x^4\cdot(8ac^2-2b^2c)+x^2\cdot(8abc-2b^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b)*log(x**2 + (-2*A*b*c + B*b**2 - 1
6*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + 8*a*b**2*c*sqrt(-1/
(4*a*c - b**2)**3)*(-2*A*c + B*b) - b**4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c
 + B*b))/(-4*A*c**2 + 2*B*b*c))/2 - sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*
b)*log(x**2 + (-2*A*b*c + B*b**2 + 16*a**2*c**2*sqrt(-1/(4*a*c - b**2)**3)*
(-2*A*c + B*b) - 8*a*b**2*c*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b) + b**
4*sqrt(-1/(4*a*c - b**2)**3)*(-2*A*c + B*b))/(-4*A*c**2 + 2*B*b*c))/2 + (A*
b - 2*B*a + x**2*(2*A*c - B*b))/(8*a**2*c - 2*a*b**2 + x**4*(8*a*c**2 - 2*b
**2*c) + x**2*(8*a*b*c - 2*b**3))
```

Giac [A]
time = 4.65, size = 102, normalized size = 1.09

$$\frac{(Bb-2\,Ac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^2-4\,ac)\sqrt{-b^2+4\,ac}}+\frac{Bbx^2-2\,Acx^2+2\,Ba-Ab}{2\,(cx^4+bx^2+a)(b^2-4\,ac)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] (B*b - 2*A*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^2 - 4*a*c)*sqrt(
-b^2 + 4*a*c)) + 1/2*(B*b*x^2 - 2*A*c*x^2 + 2*B*a - A*b)/((c*x^4 + b*x^2 +
a)*(b^2 - 4*a*c))
```

Mupad [B]
time = 0.30, size = 264, normalized size = 2.81

$$\frac{\frac{A\,b-2\,B\,a}{2\,(4\,a\,c-b^2)}+\frac{x^2\,(2\,A\,c-B\,b)}{2\,(4\,a\,c-b^2)}}{c\,x^4+b\,x^2+a}+\frac{\mathrm{atan}\left(\frac{\left(x^2\,\left(\frac{(2\,A\,c-B\,b)\,\left(2\,A\,c^3-B\,b\,c^2\right)}{a\,\left(4\,a\,c-b^2\right)^{7/2}}+\frac{\left(2\,b^3\,c^2-8\,a\,b\,c^3\right)\,(2\,A\,c-B\,b)^2\,\left(b^3-4\,a\,b\,c\right)}{2\,a\,\left(4\,a\,c-b^2\right)^{13/2}}\right)-\frac{2\,c^2\,(2\,A\,c-B\,b)^2\,\left(b^3-4\,a\,b\,c\right)}{\left(4\,a\,c-b^2\right)^{11/2}}\right)\,\left(4\,a\,c-b^2\right)^4}{8\,A^2\,c^4-8\,A\,B\,b\,c^3+2\,B^2\,b^2\,c^2}\right)\,(2\,A\,c-B\,b)}{(4\,a\,c-b^2)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((A*b - 2*B*a)/(2*(4*a*c - b^2)) + (x^2*(2*A*c - B*b))/(2*(4*a*c - b^2)))/(
a + b*x^2 + c*x^4) + (atan(((x^2*(((2*A*c - B*b)*(2*A*c^3 - B*b*c^2))/(a*(4
*a*c - b^2)^(7/2)) + ((2*b^3*c^2 - 8*a*b*c^3)*(2*A*c - B*b)^2*(b^3 - 4*a*b*
c))/(2*a*(4*a*c - b^2)^(13/2))) - (2*c^2*(2*A*c - B*b)^2*(b^3 - 4*a*b*c))/(
4*a*c - b^2)^(11/2))*(4*a*c - b^2)^4)/(8*A^2*c^4 + 2*B^2*b^2*c^2 - 8*A*B*b*
c^3))*(2*A*c - B*b))/(4*a*c - b^2)^(3/2)
```

3.116 $\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=150

$$-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\left(4a^2Bc+A(b^3-6abc)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}+\frac{A\log(x)}{a^2}-\frac{A\log}{}$$

```
[Out] 1/2*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)+1/2*(4*a^2*B*c+A*(-6*a*b*c+b^3))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
a^2/(-4*a*c+b^2)^(3/2)+A*ln(x)/a^2-1/4*A*ln(c*x^4+b*x^2+a)/a^2
```

Rubi [A]
time = 0.22, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1265, 836, 814, 648, 632, 212, 642}

$$\frac{\left(4a^2Bc+A(b^3-6abc)\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\left(b^2-4ac\right)^{3/2}}-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^2}+\frac{A\log(x)}{a^2}-\frac{-A(b^2-2ac)-\left(cx^2(Ab-2aB)\right)+abB}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(a*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c))*ArcTanh[(b + 2*c*x^2)/Sq
rt[b^2 - 4*a*c]])/(2*a^2*(b^2 - 4*a*c)^(3/2)) + (A*Log[x])/a^2 - (A*Log[a +
 b*x^2 + c*x^4])/(4*a^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-A(b^2-4ac)-(Ab-2aB)cx}{x(a+bx+cx^2)}\,dx, x, x\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{A(-b^2+4ac)}{ax} + \frac{2a^2Bc+A(b^3-5ab}{a(a+bx}\right.\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{\text{Subst}\left(\int \frac{2a^2Bc+A(b^3-5abc)+}{a+bx+cx}\right.}{2a^2\left(b^2-\right.}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x,\right.}{4a^2}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{A\log(x)}{a^2} - \frac{A\log\left(a+bx^2+cx^4\right)}{4a^2} + \frac{(}{}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(4a^2Bc+A(b^3-6abc)\right)\tanh^{-1}\left(\frac{b}{\sqrt{b}}\right.}{2a^2\left(b^2-4ac\right)^{3/2}}$$

Mathematica [A]
time = 0.23, size = 243, normalized size = 1.62

$$\frac{-\frac{2a\left(aB(b+2cx^2)-A(b^2-2ac+bcx^2)\right)}{(b^2-4ac)(a+bx^2+cx^4)} + 4A\log(x) - \frac{\left(4a^2Bc+A\left(b^3-6abc+b^2\sqrt{b^2-4ac}-4ac\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}} + \frac{\left(4a^2Bc+A\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{3/2}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + 4*A*Log[x] - ((4*a^2*B*c + A*(b^3 - 6*a*b*c + b^2*Sqrt[b
^2 - 4*a*c] - 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
])/(b^2 - 4*a*c)^(3/2) + ((4*a^2*B*c + A*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*
a*c] + 4*a*c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2
 - 4*a*c)^(3/2))/(4*a^2)
```

Maple [A]
time = 0.07, size = 212, normalized size = 1.41

method	result

default	$-\frac{\frac{\frac{ac(Ab-2aB)x^2}{4ac-b^2}-\frac{a\left(2acA-A\,b^2+abB\right)}{4ac-b^2}}{c\,x^4+b\,x^2+a}+\frac{\frac{\left(4c^2aA-A\,b^2c\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(5Aabc-A\,b^3-2a^2cB-\frac{\left(4c^2aA-A\,b^2c\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-}}\right.}{\sqrt{4ac-b^2}}}{4ac-b^2}}{2a^2}$
risch	$\frac{-\frac{c(Ab-2aB)x^2}{2a\left(4ac-b^2\right)}+\frac{2acA-A\,b^2+abB}{2\left(4ac-b^2\right)a}}{c\,x^4+b\,x^2+a}+\frac{A\ln(x)}{a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(64a^5c^3-48a^4b^2c^2+12a^3b^4c-a^2b^6\right)\_Z^2+\left(64a^3c^3A-48a^2b^2c^2A+12a\,b^4cA\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^2*((a*c*(A*b-2*B*a)/(4*a*c-b^2)*x^2-a*(2*A*a*c-A*b^2+B*a*b)/(4*a*c-b
^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(4*A*a*c^2-A*b^2*c)/c*ln(c*x^4+b*x^
2+a)+2*(5*A*a*b*c-A*b^3-2*a^2*c*B-1/2*(4*A*a*c^2-A*b^2*c)*b/c)/(4*a*c-b^2)^
(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))+A*ln(x)/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(140) = 280$.
time = 0.73, size = 1014, normalized size = 6.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^
2 - (2*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A
*a*b)*c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3
*A*a^2*b)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (
2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(2*B*a^3*b - 3*A*a
^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*
```

```
c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(c
*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^3*c^2 + (A*b^4*c -
8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c + 16*A*a^2*b*c^2)*
x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + (a^2*b^4*c - 8*a^3*b^2*c
^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2), -1/4*(2
*B*a^2*b^3 - 2*A*a*b^4 - 16*A*a^3*c^2 - 2*(4*(2*B*a^3 - A*a^2*b)*c^2 - (2*B
*a^2*b^2 - A*a*b^3)*c)*x^2 - 2*(A*a*b^3 + (A*b^3*c + 2*(2*B*a^2 - 3*A*a*b)*
c^2)*x^4 + (A*b^4 + 2*(2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + 2*(2*B*a^3 - 3*A*a^2
*b)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - 4*(2*B*a^3*b - 3*A*a^2*b^2)*c + (A*a*b^4 - 8*A*a^2*b^2*c + 16*A*a^
3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8*A*a*b^3*c
 + 16*A*a^2*b*c^2)*x^2)*log(c*x^4 + b*x^2 + a) - 4*(A*a*b^4 - 8*A*a^2*b^2*c
 + 16*A*a^3*c^2 + (A*b^4*c - 8*A*a*b^2*c^2 + 16*A*a^2*c^3)*x^4 + (A*b^5 - 8
*A*a*b^3*c + 16*A*a^2*b*c^2)*x^2)*log(x))/(a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c
^2 + (a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)*x^4 + (a^2*b^5 - 8*a^3*b^3*c
+ 16*a^4*b*c^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.49, size = 201, normalized size = 1.34

$$-\frac{\left(Ab^{3}+4\,Ba^{2}c-6\,Aabc\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(a^{2}b^{2}-4\,a^{3}c\right)\sqrt{-b^{2}+4\,ac}}-\frac{A\log\left(cx^{4}+bx^{2}+a\right)}{4\,a^{2}}+\frac{A\log\left(x^{2}\right)}{2\,a^{2}}+\frac{Ab^{2}cx^{4}-4\,Aac^{2}x^{4}+Ab^{3}x^{2}-4\,Ba^{2}cx^{2}-2\,Aabcx^{2}-2\,Ba^{2}b+3\,Aab^{2}-8\,Aa^{2}c}{4\,\left(cx^{4}+bx^{2}+a\right)\left(a^{2}b^{2}-4\,a^{3}c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c
))/((a^2*b^2 - 4*a^3*c)*sqrt(-b^2 + 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/
a^2 + 1/2*A*log(x^2)/a^2 + 1/4*(A*b^2*c*x^4 - 4*A*a*c^2*x^4 + A*b^3*x^2 - 4
*B*a^2*c*x^2 - 2*A*a*b*c*x^2 - 2*B*a^2*b + 3*A*a*b^2 - 8*A*a^2*c)/((c*x^4 +
 b*x^2 + a)*(a^2*b^2 - 4*a^3*c))
```

Mupad [**B**]
time = 7.88, size = 2500, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] ((2*A*a*c - A*b^2 + B*a*b)/(2*a*(4*a*c - b^2)) - (c*x^2*(A*b - 2*B*a))/(2*a
*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) + (A*log(x))/a^2 - (log((((A + a^2*(-(
A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(((A + a^2*(
-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*((4*b*c^2*
(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*(A + a^2*(-(A*b
^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^
2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B*a^2*c - 10*A*a*
b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A*b^3 + 2*B*a^2*c
 - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2*B*a)*(10*A*a*c
- 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x^2*(A*b - 2*B*a
)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4*a*c - b^2)^2))
*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2
))*(((A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1
/2))*((4*b*c^2*(A*b^3 + 2*B*a^2*c - 5*A*a*b*c))/(a*(4*a*c - b^2)) - (b*c^2*
(A - a^2*(-(A*b^3 + 4*B*a^2*c - 6*A*a*b*c)^2/(a^4*(4*a*c - b^2)^3))^(1/2))*
(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^2 + (2*c^3*x^2*(A*b^3 + 8*B*a*b^2 - 20*B*
a^2*c - 10*A*a*b*c))/(a*(4*a*c - b^2))))/(4*a^2) + (c^3*(A*b - 2*B*a)*(4*A*
b^3 + 2*B*a^2*c - 17*A*a*b*c))/(a^2*(4*a*c - b^2)^2) - (2*c^4*x^2*(A*b - 2*
B*a)*(10*A*a*c - 3*A*b^2 + B*a*b))/(a^2*(4*a*c - b^2)^2)))/(4*a^2) + (c^5*x
^2*(A*b - 2*B*a)^3)/(a^3*(4*a*c - b^2)^3) - (A*c^4*(A*b - 2*B*a)^2)/(a^3*(4
*a*c - b^2)^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2
))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)) - (atan((
x^2*((((A^3*b^3*c^5 - 8*B^3*a^3*c^5 + 12*A*B^2*a^2*b*c^5 - 6*A^2*B*a*b^2*c^
5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) + (((44*A^2*a^2*b
^3*c^5 - 4*B^2*a^3*b^3*c^4 + 160*A*B*a^4*c^6 - 6*A^2*a*b^5*c^4 - 80*A^2*a^3
*b*c^6 + 16*B^2*a^4*b*c^5 + 14*A*B*a^2*b^4*c^4 - 96*A*B*a^3*b^2*c^5)/(a^3*b
^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2) - (((640*B*a^6*c^6 + 320*A
*a^5*b*c^6 - 2*A*a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*
a^3*b^6*c^3 + 168*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3
- 12*a^4*b^4*c + 48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c
 + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1
056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*
c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c
^2)))*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^
2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(2*A*b^6 - 128*A*a^
3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2))/(2*(4*a^2*b^6 - 256*a^5*c^3 - 48*
a^3*b^4*c + 192*a^4*b^2*c^2)) + (((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*
a^2*b^7*c^3 + 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168
*B*a^4*b^4*c^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c +
48*a^5*b^2*c^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c
^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 -
```

```
 2688*a^6*b^3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^
2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*
B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 + 4*B*a^2*c - 6
*A*a*b*c)*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560
*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6
*b^3*c^5))/(8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c
+ 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2
)))*(A*b^3 + 4*B*a^2*c - 6*A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3
+ 4*B*a^2*c - 6*A*a*b*c)^2*(2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a
^2*b^2*c^2)*(2560*a^7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b
^5*c^4 - 2688*a^6*b^3*c^5))/(32*a^4*(4*a*c - b^2)^3*(a^3*b^6 - 64*a^6*c^3 -
 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^6 - 256*a^5*c^3 - 48*a^3*b^4*c + 1
92*a^4*b^2*c^2)))*(3*A*b^5 - 2*B*a^3*c^2 - 21*A*a*b^3*c + 33*A*a^2*b*c^2 +
2*B*a^2*b^2*c))/(8*a^3*c^2*(4*a*c - b^2)^3*(400*A^2*a^3*c^3 - 6*A^2*b^6 + 4
*B^2*a^4*c^2 - 291*A^2*a^2*b^2*c^2 + 72*A^2*a*b^4*c + 2*A*B*a^2*b^3*c - 12*
A*B*a^3*b*c^2)) + (((((((640*B*a^6*c^6 + 320*A*a^5*b*c^6 - 2*A*a^2*b^7*c^3
+ 36*A*a^3*b^5*c^4 - 192*A*a^4*b^3*c^5 - 16*B*a^3*b^6*c^3 + 168*B*a^4*b^4*c
^4 - 576*B*a^5*b^2*c^5)/(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c
^2) - ((2*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^
7*b*c^6 + 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^
3*c^5))/(2*(a^3*b^6 - 64*a^6*c^3 - 12*a^4*b^4*c + 48*a^5*b^2*c^2)*(4*a^2*b^
6 - 256*a^5*c^3 - 48*a^3*b^4*c + 192*a^4*b^2*c^2)))*(A*b^3 + 4*B*a^2*c - 6*
A*a*b*c))/(4*a^2*(4*a*c - b^2)^(3/2)) - ((A*b^3 + 4*B*a^2*c - 6*A*a*b*c)*(2
*A*b^6 - 128*A*a^3*c^3 - 24*A*a*b^4*c + 96*A*a^2*b^2*c^2)*(2560*a^7*b*c^6 +
 12*a^3*b^9*c^2 - 184*a^4*b^7*c^3 + 1056*a^5*b^5*c^4 - 2688*a^6*b^3*c^5))/(
8*a^2*(4*a*c - b^2)^(3/2)*(a^3*b^6 - 64*a^6*c^3...
```

3.117 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=223

$$-\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2}-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)}+\frac{(abB(b^2-6ac)-2A(b^4-6ab^2c+6a^2c^2))\,\text{ta}}{2a^3\left(b^2-4ac\right)^{3/2}}$$

```
[Out] 1/2*(6*A*a*c-2*A*b^2+B*a*b)/a^2/(-4*a*c+b^2)/x^2+1/2*(-a*b*B+A*(-2*a*c+b^2)
+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4+b*x^2+a)+1/2*(a*b*B*(-6*a*c+b
^2)-2*A*(6*a^2*c^2-6*a*b^2*c+b^4))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/
a^3/(-4*a*c+b^2)^(3/2)-(2*A*b-B*a)*ln(x)/a^3+1/4*(2*A*b-B*a)*ln(c*x^4+b*x^2
+a)/a^3
```

Rubi [A]
time = 0.28, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1265, 836, 814, 648, 632, 212, 642}

$$\frac{(2Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^3}-\frac{\log(x)(2Ab-aB)}{a^3}-\frac{-6aAc-abB+2Ab^2}{2a^2x^2\left(b^2-4ac\right)}+\frac{(abB(b^2-6ac)-2A(6a^2c^2-6ab^2c+b^4))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{3/2}}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(2*A*b^2 - a*b*B - 6*a*A*c)/(a^2*(b^2 - 4*a*c)*x^2) - (a*b*B - A*(b^2
- 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^2*(a + b*x^2 + c*x^4))
 + ((a*b*B*(b^2 - 6*a*c) - 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2))*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(3/2)) - ((2*A*b - a*B)*L
og[x])/a^3 + ((2*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-2Ab^2+abB+6aAc-2(Ab-2aB)cx}{x^2(a+bx+cx^2)}\,d\right.}{2a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{-2Ab^2+abB+6aAc}{ax^2} + \frac{(-2Ab+aB)}{a^2}\right.\right.}{}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\,\text{lo}}{a^3}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\,\text{lo}}{a^3}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} - \frac{(2Ab-aB)\,\text{lo}}{a^3}$$

$$= -\frac{2Ab^2-abB-6aAc}{2a^2\left(b^2-4ac\right)x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)} + \frac{(abB(b^2-6ac}{}$$

Mathematica [A]
time = 0.38, size = 379, normalized size = 1.70

$$\frac{-\frac{2aA}{x^2}-\frac{2a\left(aB\left(-b^2+2ac-bcx^2\right)+A\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+4(-2Ab+aB)\log(x)+\frac{\left(aB\left(-b^3+6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)+2A\left(b^4-6ab^2c+6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}+\frac{\left(aB\left(b^3-6abc-b^2\sqrt{b^2-4ac}+4ac\sqrt{b^2-4ac}\right)+2A\left(-b^4+6ab^2c-6a^2c^2+b^3\sqrt{b^2-4ac}-4abc\sqrt{b^2-4ac}\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{\left(b^2-4ac\right)^{3/2}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-2*a*A)/x^2 - (2*a*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + 4*(-2*A*b + a
*B)*Log[x] + ((a*B*(-b^3 + 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2
 - 4*a*c]) + 2*A*(b^4 - 6*a*b^2*c + 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a
*b*c*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c
)^(3/2) + ((a*B*(b^3 - 6*a*b*c - b^2*Sqrt[b^2 - 4*a*c] + 4*a*c*Sqrt[b^2 - 4
*a*c]) + 2*A*(-b^4 + 6*a*b^2*c - 6*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 4*a*b*
c*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(
3/2))/(4*a^3)
```

Maple [A]
time = 0.08, size = 300, normalized size = 1.35

method	result
default	$-\frac{\frac{\frac{ac\left(2acA-A\,b^{2}+abB\right)x^{2}}{4ac-b^{2}}+\frac{a\left(3Aabc-A\,b^{3}-2a^{2}cB+Ba\,b^{2}\right)}{4ac-b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{\frac{\left(-8Aab\,c^{2}+2A\,b^{3}c+4a^{2}B\,c^{2}-a\,b^{2}Bc\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(6A\,a^{2}c^{2}-10Aa\,b^{2}c \cdots\right.}{\cdots}}{\cdots}}{2a^{3}}\cdots$
risch	$\frac{-\frac{c\left(6acA-2A\,b^{2}+abB\right)x^{4}}{2a^{2}\left(4ac-b^{2}\right)}-\frac{\left(7Aabc-2A\,b^{3}-2a^{2}cB+Ba\,b^{2}\right)x^{2}}{2\left(4ac-b^{2}\right)a^{2}}-\frac{A}{2a}}{x^{2}(c\,x^{4}+b\,x^{2}+a)}-\frac{2\ln(x)Ab}{a^{3}}+\frac{\ln(x)B}{a^{2}}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(64a^{6}c^{3}-48a^{5}b^{2}c^{2}+12a^{4}b \cdots\right.\right.}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*((a*c*(2*A*a*c-A*b^2+B*a*b)/(4*a*c-b^2)*x^2+a*(3*A*a*b*c-A*b^3-2*B
*a^2*c+B*a*b^2)/(4*a*c-b^2))/(c*x^4+b*x^2+a)+1/(4*a*c-b^2)*(1/2*(-8*A*a*b*c
^2+2*A*b^3*c+4*B*a^2*c^2-B*a*b^2*c)/c*ln(c*x^4+b*x^2+a)+2*(6*A*a^2*c^2-10*A
*a*b^2*c+2*A*b^4+5*a^2*b*B*c-B*a*b^3-1/2*(-8*A*a*b*c^2+2*A*b^3*c+4*B*a^2*c^
2-B*a*b^2*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
-1/2/a^2*A/x^2+(-2*A*b+B*a)/a^3*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 806 vs. $2(207) = 414$.
time = 1.34, size = 1635, normalized size = 7.33

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3*c^3 + 2*(2
*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2*(B*a^2*b^4
```

```
 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*A*a^2*b^3)*
c)*x^2 + ((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b^3 - 2*A*b^4)
*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 - 2*A*a*b^3)*c
)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2*A*a^2*b^2)*c
)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^
2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + ((16*(B*a^3 - 2*A*a^2*b)*c
^3 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5
- 2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x
^4 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2
*A*a^2*b^3)*c)*x^2)*log(c*x^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3
 - 8*(B*a^2*b^2 - 2*A*a*b^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 -
2*A*b^6 + 16*(B*a^3*b - 2*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4
 + (B*a^2*b^4 - 2*A*a*b^5 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A
*a^2*b^3)*c)*x^2)*log(x))/((a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (
a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6
*c^2)*x^2), -1/4*(2*A*a^2*b^4 - 16*A*a^3*b^2*c + 32*A*a^4*c^2 + 2*(24*A*a^3
*c^3 + 2*(2*B*a^3*b - 7*A*a^2*b^2)*c^2 - (B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 - 2
*(B*a^2*b^4 - 2*A*a*b^5 + 4*(2*B*a^4 - 7*A*a^3*b)*c^2 - 3*(2*B*a^3*b^2 - 5*
A*a^2*b^3)*c)*x^2 + 2*((12*A*a^2*c^3 + 6*(B*a^2*b - 2*A*a*b^2)*c^2 - (B*a*b
^3 - 2*A*b^4)*c)*x^6 - (B*a*b^4 - 2*A*b^5 - 12*A*a^2*b*c^2 - 6*(B*a^2*b^2 -
 2*A*a*b^3)*c)*x^4 - (B*a^2*b^3 - 2*A*a*b^4 - 12*A*a^3*c^2 - 6*(B*a^3*b - 2
*A*a^2*b^2)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) + ((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b
^3)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2
*A*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5
 + 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(c*x
^4 + b*x^2 + a) - 4*((16*(B*a^3 - 2*A*a^2*b)*c^3 - 8*(B*a^2*b^2 - 2*A*a*b^3
)*c^2 + (B*a*b^4 - 2*A*b^5)*c)*x^6 + (B*a*b^5 - 2*A*b^6 + 16*(B*a^3*b - 2*A
*a^2*b^2)*c^2 - 8*(B*a^2*b^3 - 2*A*a*b^4)*c)*x^4 + (B*a^2*b^4 - 2*A*a*b^5 +
 16*(B*a^4 - 2*A*a^3*b)*c^2 - 8*(B*a^3*b^2 - 2*A*a^2*b^3)*c)*x^2)*log(x))/(
(a^3*b^4*c - 8*a^4*b^2*c^2 + 16*a^5*c^3)*x^6 + (a^3*b^5 - 8*a^4*b^3*c + 16*
a^5*b*c^2)*x^4 + (a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.44, size = 250, normalized size = 1.12

$$-\frac{\left(Bab^3-2\,Ab^4-6\,Ba^2bc+12\,Aab^2c-12\,Aa^2c^2\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(a^3b^2-4\,a^4c\right)\sqrt{-b^2+4\,ac}}+\frac{Babcx^4-2\,Ab^2cx^4+6\,Aac^2x^4+Bab^2x^2-2\,Ab^3x^2-2\,Ba^2cx^2+7\,Aabcx^2-Aab^2+4\,Aa^2c}{2\,\left(cx^6+bx^4+ax^2\right)\left(a^2b^2-4\,a^3c\right)}-\frac{\left(Ba-2\,Ab\right)\log\left(cx^4+bx^2+a\right)}{4\,a^3}+\frac{\left(Ba-2\,Ab\right)\log\left(x^2\right)}{2\,a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^3 - 2*A*b^4 - 6*B*a^2*b*c + 12*A*a*b^2*c - 12*A*a^2*c^2)*arctan
((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^2 - 4*a^4*c)*sqrt(-b^2 + 4*a*c))
 + 1/2*(B*a*b*c*x^4 - 2*A*b^2*c*x^4 + 6*A*a*c^2*x^4 + B*a*b^2*x^2 - 2*A*b^3
*x^2 - 2*B*a^2*c*x^2 + 7*A*a*b*c*x^2 - A*a*b^2 + 4*A*a^2*c)/((c*x^6 + b*x^4
 + a*x^2)*(a^2*b^2 - 4*a^3*c)) - 1/4*(B*a - 2*A*b)*log(c*x^4 + b*x^2 + a)/a
^3 + 1/2*(B*a - 2*A*b)*log(x^2)/a^3
```

Mupad [B]
time = 9.09, size = 2500, normalized size = 11.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] (log(((c^4*(2*A*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)^
2) - ((((B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2
*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((b*c^2*(B*a - 2*A*b + a^
3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*
(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (4*b*c^2*(2*
A*b^4 + 6*A*a^2*c^2 - B*a*b^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*a*c -
b^2)) + (2*c^3*x^2*(2*A*b^4 - 60*A*a^2*c^2 - B*a*b^3 + 4*A*a*b^2*c + 10*B*a
^2*b*c))/(a^2*(4*a*c - b^2))))/(4*a^3) + (c^3*(36*A^2*a^3*c^3 - 16*A^2*b^6
- 4*B^2*a^2*b^4 + 16*A*B*a*b^5 - 216*A^2*a^2*b^2*c^2 + 116*A^2*a*b^4*c + 17
*B^2*a^3*b^2*c - 92*A*B*a^2*b^3*c + 108*A*B*a^3*b*c^2))/(a^4*(4*a*c - b^2)^
2) - (2*c^4*x^2*(12*A^2*b^5 + 3*B^2*a^2*b^3 - 12*A*B*a*b^4 - 60*A*B*a^3*c^2
 - 82*A^2*a*b^3*c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B*a^2*b^2*c))
/(a^4*(4*a*c - b^2)^2))*(B*a - 2*A*b + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*
b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3)
+ (c^5*x^2*(6*A*a*c - 2*A*b^2 + B*a*b)^3)/(a^6*(4*a*c - b^2)^3))*((c^4*(2*A
*b - B*a)*(6*A*a*c - 2*A*b^2 + B*a*b)^2)/(a^6*(4*a*c - b^2)^2) - ((((2*A*b
- B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*
c)^2/(a^6*(4*a*c - b^2)^3))^(1/2))*((4*b*c^2*(2*A*b^4 + 6*A*a^2*c^2 - B*a*b
^3 - 10*A*a*b^2*c + 5*B*a^2*b*c))/(a^2*(4*a*c - b^2)) - (b*c^2*(2*A*b - B*a
 + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/
(a^6*(4*a*c - b^2)^3))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^3 + (2*c^3*
x^2*(2*A*b^4 - 60*A*a^2*c^2 - B*a*b^3 + 4*A*a*b^2*c + 10*B*a^2*b*c))/(a^2*(
4*a*c - b^2))))/(4*a^3) - (c^3*(36*A^2*a^3*c^3 - 16*A^2*b^6 - 4*B^2*a^2*b^4
 + 16*A*B*a*b^5 - 216*A^2*a^2*b^2*c^2 + 116*A^2*a*b^4*c + 17*B^2*a^3*b^2*c
- 92*A*B*a^2*b^3*c + 108*A*B*a^3*b*c^2))/(a^4*(4*a*c - b^2)^2) + (2*c^4*x^2
*(12*A^2*b^5 + 3*B^2*a^2*b^3 - 12*A*B*a*b^4 - 60*A*B*a^3*c^2 - 82*A^2*a*b^3
*c - 10*B^2*a^3*b*c + 138*A^2*a^2*b*c^2 + 61*A*B*a^2*b^2*c))/(a^4*(4*a*c -
```

b^2)^2))*(2*A*b - B*a + a^3*(-(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)^2/(a^6*(4*a*c - b^2)^3))^(1/2)))/(4*a^3) + (c^5*x^2*(6*A*a*c - 2*A*b^2 + B*a*b)^3)/(a^6*(4*a*c - b^2)^3)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) - (log(x)*(2*A*b - B*a))/a^3 - (A/(2*a) - (x^2*(2*A*b^3 - B*a*b^2 + 2*B*a^2*c - 7*A*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(6*A*a*c - 2*A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)))/(a*x^2 + b*x^4 + c*x^6) + (atan((x^2*((((216*A^3*a^3*c^8 - 8*A^3*b^6*c^5 - 216*A^3*a^2*b^2*c^7 + B^3*a^3*b^3*c^5 + 72*A^3*a*b^4*c^6 + 12*A^2*B*a*b^5*c^5 + 108*A^2*B*a^3*b*c^7 - 6*A*B^2*a^2*b^4*c^5 + 18*A*B^2*a^3*b^2*c^6 - 72*A^2*B*a^2*b^3*c^6)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((24*A^2*a^2*b^7*c^4 - 260*A^2*a^3*b^5*c^5 + 932*A^2*a^4*b^3*c^6 + 6*B^2*a^4*b^5*c^4 - 44*B^2*a^5*b^3*c^5 + 480*A*B*a^6*c^7 - 1104*A^2*a^5*b*c^7 + 80*B^2*a^6*b*c^6 - 24*A*B*a^3*b^6*c^4 + 218*A*B*a^4*b^4*c^5 - 608*A*B*a^5*b^2*c^6)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) + (((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)) - (((((1920*A*a^8*c^7 - 320*B*a^8*b*c^6 - 4*A*a^4*b^8*c^3 + 24*A*a^5*b^6*c^4 + 120*A*a^6*b^4*c^5 - 1088*A*a^7*b^2*c^6 + 2*B*a^5*b^7*c^3 - 36*B*a^6*b^5*c^4 + 192*B*a^7*b^3*c^5)/(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*A*a*b^5*c - 256*A*a^3*b*c^3 + 24*B*a^2*b^4*c + 192*A*a^2*b^3*c^2 - 96*B*a^3*b^2*c^2))/(2*(a^6*b^6 - 64*a^9*c^3 - 12*a^7*b^4*c + 48*a^8*b^2*c^2)*(4*a^3*b^6 - 256*a^6*c^3 - 48*a^4*b^4*c + 192*a^5*b^2*c^2)))*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c))/(4*a^3*(4*a*c - b^2)^(3/2)) - ((2560*a^10*b*c^6 + 12*a^6*b^9*c^2 - 184*a^7*b^7*c^3 + 1056*a^8*b^5*c^4 - 2688*a^9*b^3*c^5)*(2*A*b^4 + 12*A*a^2*c^2 - B*a*b^3 - 12*A*a*b^2*c + 6*B*a^2*b*c)*(4*A*b^7 + 128*B*a^4*c^3 - 2*B*a*b^6 - 48*...

3.118 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=425

$$\frac{(3b^2B-Abc-10aBc)\,x}{2c^2\,(b^2-4ac)}-\frac{(bB-2Ac)x^3}{2c\,(b^2-4ac)}-\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{\Big(3b^3B-Ab^2c-13abBc+6a}{}$$

```
[Out] 1/2*(-A*b*c-10*B*a*c+3*B*b^2)*x/c^2/(-4*a*c+b^2)-1/2*(-2*A*c+B*b)*x^3/c/(-4
*a*c+b^2)-1/2*x^5*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)
-1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*B-A*b^2*
c-13*a*b*B*c+6*a*A*c^2+(-8*A*a*b*c^2+A*b^3*c-20*B*a^2*c^2+19*B*a*b^2*c-3*B*
b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2)
)^(1/2)-1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^3*B
-A*b^2*c-13*a*b*B*c+6*a*A*c^2+(8*A*a*b*c^2-A*b^3*c+20*B*a^2*c^2-19*B*a*b^2*
c+3*B*b^4)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A]
time = 2.62, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1289, 1293, 1180, 211}

$$-\frac{\left(-\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{5/2}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\left(\frac{20a^2Bc^2+8aAbc^2-19ab^2Bc-Ab^3c+3b^4B}{\sqrt{b^2-4ac}}+6aAc^2-13abBc-Ab^2c+3b^3B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{5/2}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(-10aBc-Abc+3b^2B)}{2c^2\,(b^2-4ac)}-\frac{x^3(bB-2Ac)}{2c\,(b^2-4ac)}-\frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((3*b^2*B - A*b*c - 10*a*B*c)*x)/(2*c^2*(b^2 - 4*a*c)) - ((b*B - 2*A*c)*x^3
)/(2*c*(b^2 - 4*a*c)) - (x^5*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/(2*(b^2 - 4
*a*c)*(a + b*x^2 + c*x^4)) - ((3*b^3*B - A*b^2*c - 13*a*b*B*c + 6*a*A*c^2 -
 (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2 + 20*a^2*B*c^2)/Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2
]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - ((3*b^3*B - A*b^2*c
- 13*a*b*B*c + 6*a*A*c^2 + (3*b^4*B - A*b^3*c - 19*a*b^2*B*c + 8*a*A*b*c^2
+ 20*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt
[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c
]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{x^4(5(Ab-2aB)-3(bB-2Ac)x^2)}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{(bB-2Ac)x^3}{2c(b^2-4ac)} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{x^2(-9a(bB-2Ac)-3(3b^2B-Abc-}{a+bx^2+cx^4}}{6c(b^2-4ac)}$$

$$= \frac{(3b^2B-Abc-10aBc)\,x}{2c^2(b^2-4ac)} - \frac{(bB-2Ac)x^3}{2c(b^2-4ac)} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \int$$

$$= \frac{(3b^2B-Abc-10aBc)\,x}{2c^2(b^2-4ac)} - \frac{(bB-2Ac)x^3}{2c(b^2-4ac)} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - $$

$$= \frac{(3b^2B-Abc-10aBc)\,x}{2c^2(b^2-4ac)} - \frac{(bB-2Ac)x^3}{2c(b^2-4ac)} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - $$

Mathematica [**A**]
time = 0.77, size = 455, normalized size = 1.07

$$\frac{4B\sqrt{c}\,x+\frac{2\sqrt{c}\,x(-2a^2Bc+b^2(bB-Ac)x^2+a(b^2B+2Ac^2x^2-bc(A+3Bx^2)))}{(b^2-4ac)(a+bx^2+cx^4)}-\frac{\sqrt{2}\left(-3b^4B+b^2c\left(19aB-A\sqrt{b^2-4ac}\right)+2ac^2\left(-10aB+3A\sqrt{b^2-4ac}\right)+b^3\left(Ac+3B\sqrt{b^2-4ac}\right)-abc\left(8Ac+13B\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2}\left(3b^4B-b^2c\left(19aB+A\sqrt{b^2-4ac}\right)+2ac^2\left(10aB+3A\sqrt{b^2-4ac}\right)+abc\left(8Ac-13B\sqrt{b^2-4ac}\right)+b^3\left(-Ac+3B\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (4*B*Sqrt[c]*x + (2*Sqrt[c]*x*(-2*a^2*B*c + b^2*(b*B - A*c)*x^2 + a*(b^2*B
+ 2*A*c^2*x^2 - b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) -
(Sqrt[2]*(-3*b^4*B + b^2*c*(19*a*B - A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(-10*a*
B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - a*b*c*(8*A
*c + 13*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2
- 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2]*(3
*b^4*B - b^2*c*(19*a*B + A*Sqrt[b^2 - 4*a*c]) + 2*a*c^2*(10*a*B + 3*A*Sqrt[
b^2 - 4*a*c]) + a*b*c*(8*A*c - 13*B*Sqrt[b^2 - 4*a*c]) + b^3*(-(A*c) + 3*B*
Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]]
)/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(5/2))
```

Maple [**A**]
time = 0.07, size = 447, normalized size = 1.05

method	result

risch	$\frac{Bx}{c^2}+\frac{-\frac{(2c^2aA-A\,b^2c-3abBc+b^3B)x^3}{2(4ac-b^2)}+\frac{a(bcA+2acB-b^2B)x}{8ac-2b^2}}{c^2(c\,x^4+b\,x^2+a)}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(\frac{(6c^2aA-A\,b^2c-13abBc+3b^3B)\_R^2}{4ac-b^2}\cdots\right.}{2c\_R^3\cdots}}{4c^2}$
default	$\frac{Bx}{c^2}+\frac{-\frac{(2c^2aA-A\,b^2c-3abBc+b^3B)x^3}{2(4ac-b^2)}+\frac{a(bcA+2acB-b^2B)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-\frac{\left(6c^2aA\sqrt{-4ac+b^2}-A\,b^2c\sqrt{-4ac+b^2}-8Aab\,c^2+A\,b^3c\cdots\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] B/c^2*x+1/c^2*((-1/2*(2*A*a*c^2-A*b^2*c-3*B*a*b*c+B*b^3)/(4*a*c-b^2)*x^3+1/
2*a*(A*b*c+2*B*a*c-B*b^2)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-
1/8*(6*c^2*a*A*(-4*a*c+b^2)^(1/2)-A*b^2*c*(-4*a*c+b^2)^(1/2)-8*A*a*b*c^2+A*
b^3*c-13*a*b*B*c*(-4*a*c+b^2)^(1/2)+3*b^3*B*(-4*a*c+b^2)^(1/2)-20*a^2*B*c^2
+19*a*b^2*B*c-3*b^4*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(6*c^
2*a*A*(-4*a*c+b^2)^(1/2)-A*b^2*c*(-4*a*c+b^2)^(1/2)+8*A*a*b*c^2-A*b^3*c-13*
a*b*B*c*(-4*a*c+b^2)^(1/2)+3*b^3*B*(-4*a*c+b^2)^(1/2)+20*a^2*B*c^2-19*a*b^2
*B*c+3*b^4*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)
*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b^3 + 2*A*a*c^2 - (3*B*a*b + A*b^2)*c)*x^3 + (B*a*b^2 - (2*B*a^2 +
A*a*b)*c)*x)/(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 -
4*a*b*c^3)*x^2) + B*x/c^2 - 1/2*integrate((3*B*a*b^2 + (3*B*b^3 + 6*A*a*c^2
 - (13*B*a*b + A*b^2)*c)*x^2 - (10*B*a^2 + A*a*b)*c)/(c*x^4 + b*x^2 + a), x
)/(b^2*c^2 - 4*a*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7252 vs. $2(379) = 758$.
time = 8.30, size = 7252, normalized size = 17.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(4*(B*b^2*c - 4*B*a*c^2)*x^5 + 2*(3*B*b^3 + 2*A*a*c^2 - (11*B*a*b + A*b
^2)*c)*x^3 - sqrt(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (
b^3*c^2 - 4*a*b*c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4
 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 +
80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 1
2*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^
6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2
200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6
*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^
3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*
a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^
3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*
B^4*a^2*b^6 - 135*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*
a^2*b^2)*c^4 - (2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 64
7*A^3*B*a^2*b^3 - 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3
 - 186*A^2*B^2*a^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3
*a^2*b^5 - 5*A^2*B^2*a*b^6)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2
*B*a^4 + A^3*a^3*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*
b^2 + 11*A^3*a^2*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A
^2*B*a^2*b^4 + 17*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 +
 153*A^2*B*a*b^6 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2
*B*b^8)*c^2 - 27*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^
10 + 128*(8*B*a^4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8
+ 24*(14*B*a^2*b^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*
b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5
 + (625*B^4*a^4 + 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3
 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^
4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6
)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a
^2*b^2*c^12 - 64*a^3*c^13)))*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*
c^4 - 15*(28*B^2*a^3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3
 + 80*A*B*a*b^4 + A^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5
- 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2
*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4
+ 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)
*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B
^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64
*a^3*c^13)))/(b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))) + sqr
t(1/2)*(a*b^2*c^2 - 4*a^2*c^3 + (b^2*c^3 - 4*a*c^4)*x^4 + (b^3*c^2 - 4*a*b*
c^3)*x^2)*sqrt(-(9*B^2*b^7 + 60*(4*A*B*a^3 + A^2*a^2*b)*c^4 - 15*(28*B^2*a^
```

```
3*b + 20*A*B*a^2*b^2 + A^2*a*b^3)*c^3 + (385*B^2*a^2*b^3 + 80*A*B*a*b^4 + A
^2*b^5)*c^2 - 3*(35*B^2*a*b^5 + 2*A*B*b^6)*c + (b^6*c^5 - 12*a*b^4*c^6 + 48
*a^2*b^2*c^7 - 64*a^3*c^8)*sqrt((81*B^4*b^8 + 81*A^4*a^2*c^6 - 18*(25*A^2*B
^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4 + 2200*A*B^3*a^3*b
+ 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(425*B^4*a^3*b^
2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)*c^3 + 27*(113*B^4*
a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B^4*a*b^6 + 2*A*B^3*
b^7)*c)/(b^6*c^10 - 12*a*b^4*c^11 + 48*a^2*b^2*c^12 - 64*a^3*c^13)))/(b^6*c
^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8))*log((189*B^4*a^2*b^6 - 13
5*A*B^3*a*b^7 + 324*A^4*a^3*c^5 - 81*(28*A^3*B*a^3*b + A^4*a^2*b^2)*c^4 - (
2500*B^4*a^5 + 2500*A*B^3*a^4*b - 5016*A^2*B^2*a^3*b^2 - 647*A^3*B*a^2*b^3
- 5*A^4*a*b^4)*c^3 + 9*(625*B^4*a^4*b^2 - 303*A*B^3*a^3*b^3 - 186*A^2*B^2*a
^2*b^4 - 5*A^3*B*a*b^5)*c^2 - 27*(73*B^4*a^3*b^4 - 49*A*B^3*a^2*b^5 - 5*A^2
*B^2*a*b^6)*c)*x - 1/2*sqrt(1/2)*(27*B^3*b^10 + 144*(10*A^2*B*a^4 + A^3*a^3
*b)*c^6 - 8*(500*B^3*a^5 + 930*A*B^2*a^4*b + 252*A^2*B*a^3*b^2 + 11*A^3*a^2
*b^3)*c^5 + (11360*B^3*a^4*b^2 + 7608*A*B^2*a^3*b^3 + 882*A^2*B*a^2*b^4 + 1
7*A^3*a*b^5)*c^4 - (8818*B^3*a^3*b^4 + 2841*A*B^2*a^2*b^5 + 153*A^2*B*a*b^6
 + A^3*b^7)*c^3 + 9*(329*B^3*a^2*b^6 + 51*A*B^2*a*b^7 + A^2*B*b^8)*c^2 - 27
*(17*B^3*a*b^8 + A*B^2*b^9)*c - (3*B*b^9*c^5 - 768*A*a^4*c^10 + 128*(8*B*a^
4*b + 5*A*a^3*b^2)*c^9 - 192*(5*B*a^3*b^3 + A*a^2*b^4)*c^8 + 24*(14*B*a^2*b
^5 + A*a*b^6)*c^7 - (52*B*a*b^7 + A*b^8)*c^6)*sqrt((81*B^4*b^8 + 81*A^4*a^2
*c^6 - 18*(25*A^2*B^2*a^3 + 44*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (625*B^4*a^4
+ 2200*A*B^3*a^3*b + 2904*A^2*B^2*a^2*b^2 + 196*A^3*B*a*b^3 + A^4*b^4)*c^4
- 6*(425*B^4*a^3*b^2 + 798*A*B^3*a^2*b^3 + 132*A^2*B^2*a*b^4 + 2*A^3*B*b^5)
*c^3 + 27*(113*B^4*a^2*b^4 + 52*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(17*B
^4*a*b^6 + 2*A*B^3*b^7)*c)/(b^6*c^10 - 12*a*b^4...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5681 vs. 2(379) = 758.
time = 6.51, size = 5681, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] B*x/c^2 + 1/2*(B*b^3*x^3 - 3*B*a*b*c*x^3 - A*b^2*c*x^3 + 2*A*a*c^2*x^3 + B*
a*b^2*x - 2*B*a^2*c*x - A*a*b*c*x)/((c*x^4 + b*x^2 + a)*(b^2*c^2 - 4*a*c^3)
) + 1/16*((2*b^4*c^3 - 20*a*b^2*c^4 + 48*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 12*(b^2 - 4*a*c)*a*c^4)*(b^2*c^2 - 4
*a*c^3)^2*A - (6*b^5*c^2 - 50*a*b^3*c^3 + 104*a^2*b*c^4 - 3*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 + 25*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^3*c^2 + 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b*c^3 - 6*(b^2 - 4*a*c)*b^3*c^2 + 26*(b^2 - 4*a*c)*a*b*c^3
)*(b^2*c^2 - 4*a*c^3)^2*B + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^
5*c^4 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*a*b^5*c^5 + 16*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a^2*b^2*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 - 16*a^
2*b^3*c^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 + 32*a^3*b*
c^7 - 2*(b^2 - 4*a*c)*a*b^3*c^5 + 8*(b^2 - 4*a*c)*a^2*b*c^6)*A*abs(-b^2*c^2
 + 4*a*c^3) - 2*(3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 34*s
qrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 6*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 6*a*b^6*c^4 + 128*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*b^2*c^5 + 44*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*b^3*c^5 + 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 68*a^2*b^4
*c^5 - 160*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^6 - 80*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 22*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^2*b^2*c^6 + 256*a^3*b^2*c^6 + 40*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a^3*c^7 - 320*a^4*c^7 - 6*(b^2 - 4*a*c)*a*b^4*c^4 + 44*(b^2 - 4*a*
c)*a^2*b^2*c^5 - 80*(b^2 - 4*a*c)*a^3*c^6)*B*abs(-b^2*c^2 + 4*a*c^3) - (2*b
^8*c^7 - 32*a*b^6*c^8 + 160*a^2*b^4*c^9 - 256*a^3*b^2*c^10 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 16*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
- sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 2*(b^2 - 4*a*c)*b^6*c^7 + 24*(b^2 - 4*
a*c)*a*b^4*c^8 - 64*(b^2 - 4*a*c)*a^2*b^2*c^9)*A + (6*b^9*c^6 - 86*a*b^7*c^
7 + 440*a^2*b^5*c^8 - 928*a^3*b^3*c^9 + 640*a^4*b*c^10 - 3*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 43*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 6*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 - 220*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - 62*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 464*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 31*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 320*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 - 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 6*(b^2 - 4*a*c)*b^7*c^6 + 62*(b^2 - 4*
a*c)*a*b^5*c^7 - 192*(b^2 - 4*a*c)*a^2*b^3*c^8 + 160*(b^2 - 4*a*c)*a^3*b*c^
9)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c^2 - 4*a*b*c^3 + sqrt((b^3*c^2 - 4*a*
b*c^3)^2 - 4*(a*b^2*c^2 - 4*a^2*c^3)*(b^2*c^3 - 4*a*c^4)))/(b^2*c^3 - 4*a*c
^4)))/((a*b^6*c^5 - 12*a^2*b^4*c^6 - 2*a*b^5*c^6 + 48*a^3*b^2*c^7 + 16*a^2*
b^3*c^7 + a*b^4*c^7 - 64*a^4*c^8 - 32*a^3*b*c^8 - 8*a^2*b^2*c^8 + 16*a^3*c^
9)*abs(-b^2*c^2 + 4*a*c^3)*abs(c)) - 1/16*((2*b...
```

Mupad [B]
time = 4.36, size = 2500, normalized size = 5.88

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (B*x)/c^2 - atan((((((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 +
48*B*a*b^8*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4
+ 4224*B*a^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a
*b^4*c^4 - 48*a^2*b^2*c^5)) - (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2
*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*
b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c
^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c -
 b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^
7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*
(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 806
4*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*
a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*
c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*
c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840
```

*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 102
4*a^3*b*c^8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*(
(9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^1
2*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2
077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800
*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-
(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^
9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B
^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*
b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) -
152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2
*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 +
240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10))
)^(1/2) - (x*(9*B^2*b^8 - 72*A^2*a^3*c^5 + A^2*b^6*c^2 + 200*B^2*a^4*c^4 -
6*A*B*b^7*c + 74*A^2*a^2*b^2*c^4 + 481*B^2*a^2*b^4*c^2 - 718*B^2*a^3*b^2*c^
3 - 114*B^2*a*b^6*c - 16*A^2*a*b^4*c^3 - 374*A*B*a^2*b^3*c^3 + 86*A*B*a*b^5
*c^2 + 472*A*B*a^3*b*c^4))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^
2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c -
288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B
^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*
a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*
c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3
+ 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^
6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c
^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*
A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 44*A*B*a*b*c^2*(-(4
*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^12*c^5 - 24*a*b^10*c^6 + 240*a
^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c^9 - 6144*a^5*b^2*c^10)))^(1/
2)*1i - (((10240*B*a^5*c^7 - 16*A*a*b^7*c^4 + 1024*A*a^4*b*c^7 + 48*B*a*b^8
*c^3 + 192*A*a^2*b^5*c^5 - 768*A*a^3*b^3*c^6 - 736*B*a^2*b^6*c^4 + 4224*B*a
^3*b^4*c^5 - 10752*B*a^4*b^2*c^6)/(8*(64*a^3*c^6 - b^6*c^3 + 12*a*b^4*c^4 -
48*a^2*b^2*c^5)) + (x*((9*B^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2
- 9*B^2*b^13 + 6*A*B*b^12*c - 288*A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 -
3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240
*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(
1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^
2*a*b^11*c + 27*A^2*a*b^9*c^3 + 3840*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c -
b^2)^9)^(1/2) - 26880*B^2*a^6*b*c^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*
b^6*c^4 + 22400*A*B*a^4*b^4*c^5 - 30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-
(4*a*c - b^2)^9)^(1/2) - 152*A*B*a*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9
)^(1/2) + 44*A*B*a*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(4096*a^6*c^11 + b^1
2*c^5 - 24*a*b^10*c^6 + 240*a^2*b^8*c^7 - 1280*a^3*b^6*c^8 + 3840*a^4*b^4*c
^9 - 6144*a^5*b^2*c^10)))^(1/2)*(16*b^7*c^5 - 192*a*b^5*c^6 - 1024*a^3*b*c^
8 + 768*a^2*b^3*c^7))/(2*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))*((9*B^2*b^4
*(-(4*a*c - b^2)^9)^(1/2) - A^2*b^11*c^2 - 9*B^2*b^13 + 6*A*B*b^12*c - 288*

```
A^2*a^2*b^7*c^4 + 1504*A^2*a^3*b^5*c^5 - 3840*A^2*a^4*b^3*c^6 - 2077*B^2*a^
2*b^9*c^2 + 10656*B^2*a^3*b^7*c^3 - 30240*B^2*a^4*b^5*c^4 + 44800*B^2*a^5*b
^3*c^5 + A^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + 25*B^2*a^2*c^2*(-(4*a*c - b
^2)^9)^(1/2) + 15360*A*B*a^6*c^7 + 213*B^2*a*b^11*c + 27*A^2*a*b^9*c^3 + 38
40*A^2*a^5*b*c^7 - 9*A^2*a*c^3*(-(4*a*c - b^2)^9)^(1/2) - 26880*B^2*a^6*b*c
^6 + 1548*A*B*a^2*b^8*c^3 - 8064*A*B*a^3*b^6*c^4 + 22400*A*B*a^4*b^4*c^5 -
30720*A*B*a^5*b^2*c^6 - 51*B^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 152*A*B*a
*b^10*c^2 - 6*A*B*b^3*c*(-(4*a*c - b^2)^9)^(1/2...
```

3.119 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=336

$$-\frac{(bB-2Ac)x}{2c\,(b^2-4ac)}-\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab^2c-8abBc+4aAc^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\quad}{\sqrt{\quad}}\right.}{2\sqrt{2}\,c^{3/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/2*(-2*A*c+B*b)*x/c/(-4*a*c+b^2)-1/2*x^3*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4
*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*(b^2*B+A*b*c-6*a*B*c+(-4*A*a*c^2-A*b^2*c+8*B*a*b*c-B*b^3)/(-4*a*c
+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*B+A*b*c-6*a*B*c
+(4*A*a*c^2+A*b^2*c-8*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^
2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.22, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1289, 1293, 1180, 211}

$$\frac{\left(-\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,c^{3/2}\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aAc^2-8abBc+Ab^2c+b^3B}{\sqrt{b^2-4ac}}-6aBc+Abc+b^2B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,c^{3/2}\,(b^2-4ac)\,\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^3(-2aB-(x^2(bB-2Ac))+Ab)}{2\,(b^2-4ac)\,(a+bx^2+cx^4)}-\frac{x(bB-2Ac)}{2c\,(b^2-4ac)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*((b*B - 2*A*c)*x)/(c*(b^2 - 4*a*c)) - (x^3*(A*b - 2*a*B - (b*B - 2*A*c
)*x^2))/(2*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + ((b^2*B + A*b*c - 6*a*B*c -
 (b^3*B + A*b^2*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c
)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^2*B + A*b*c - 6*a*B*c + (b^3*B + A*b^2
*c - 8*a*b*B*c + 4*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\int \frac{x^2(3(Ab-2aB)+(-bB+2Ac)x^2)}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{(bB-2Ac)x}{2c(b^2-4ac)} - \frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} - \frac{\int \frac{-a(bB-2Ac)+(-b^2B-Abc+6aBc)x^2}{a+bx^2+cx^4}}{2c(b^2-4ac)}$$

$$= -\frac{(bB-2Ac)x}{2c(b^2-4ac)} - \frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab}{\sqrt{}}\right.}{4}$$

$$= -\frac{(bB-2Ac)x}{2c(b^2-4ac)} - \frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)} + \frac{\left(b^2B+Abc-6aBc-\frac{b^3B+Ab}{\sqrt{}}\right.}{2\sqrt{2}\,c^{3/2}\,(b}$$

Mathematica [A]
time = 0.57, size = 362, normalized size = 1.08

$$\frac{\frac{2\sqrt{c}\left(-abBx+b(-bB+Ac)x^3+2acx(A+Bx^2)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-b^3B+bc\left(8aB+A\sqrt{b^2-4ac}\right)+b^2\left(-Ac+B\sqrt{b^2-4ac}\right)-2ac\left(2Ac+3B\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^3B+2ac\left(2Ac-3B\sqrt{b^2-4ac}\right)+b^2\left(Ac+B\sqrt{b^2-4ac}\right)+b\left(-8aBc+Ac\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*Sqrt[c]*(-(a*b*B*x) + b*(-(b*B) + A*c)*x^3 + 2*a*c*x*(A + B*x^2)))/((b^
2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*(-(b^3*B) + b*c*(8*a*B + A*Sqrt[
b^2 - 4*a*c]) + b^2*(-(A*c) + B*Sqrt[b^2 - 4*a*c]) - 2*a*c*(2*A*c + 3*B*Sqr
t[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(
(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^3*B + 2*a*c*
(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^2*(A*c + B*Sqrt[b^2 - 4*a*c]) + b*(-8*a
*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2
 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*c^(3/2))
```

Maple [A]
time = 0.07, size = 359, normalized size = 1.07

method	result
risch	$\frac{-\frac{\left(bcA+2acB-b^2B\right)x^3}{2c\left(4ac-b^2\right)}-\frac{a(2Ac-bB)x}{2\left(4ac-b^2\right)c}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(-\frac{\left(bcA-6acB+b^2B\right)\_R^2}{4ac-b^2}+\frac{a(2Ac-bB)}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}}{4c}$
default	$\frac{-\frac{\left(bcA+2acB-b^2B\right)x^3}{2c\left(4ac-b^2\right)}-\frac{a(2Ac-bB)x}{2\left(4ac-b^2\right)c}}{cx^4+bx^2+a}+\frac{-\frac{\left(-bcA\sqrt{-4ac+b^2}+4c^2aA+Ab^2c+6acB\sqrt{-4ac+b^2}-b^2B\sqrt{-4ac+b^2}-8a\ldots\right.}{4c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4\ldots\right.}}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/2*(A*b*c+2*B*a*c-B*b^2)/c/(4*a*c-b^2)*x^3-1/2*a*(2*A*c-B*b)/(4*a*c-b^2)
/c*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*(-1/8*(-b*c*A*(-4*a*c+b^2)^(1/2)+4*c^2*
a*A+A*b^2*c+6*a*c*B*(-4*a*c+b^2)^(1/2)-b^2*B*(-4*a*c+b^2)^(1/2)-8*a*b*B*c+b
^3*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan
h(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-b*c*A*(-4*a*c+b^2)^(
1/2)-4*c^2*a*A-A*b^2*c+6*a*c*B*(-4*a*c+b^2)^(1/2)-b^2*B*(-4*a*c+b^2)^(1/2)+
8*a*b*B*c-b^3*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1
/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((B*b^2 - (2*B*a + A*b)*c)*x^3 + (B*a*b - 2*A*a*c)*x)/((b^2*c^2 - 4*a*
c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2) + 1/2*integrate((
B*a*b - 2*A*a*c + (B*b^2 - (6*B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b
^2*c - 4*a*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4658 vs. 2(292) = 584.
time = 2.50, size = 4658, normalized size = 13.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(B*b^2 - (2*B*a + A*b)*c)*x^3 + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4
+ a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B
*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2
*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c
^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a
^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(
b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4
*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^
4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b
 + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 +
 9*A^2*B^2*b^4)*c)*x + 1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2
*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b
+ 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A
*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 -
64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6
*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b
)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2
- 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
```

```
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))) - sqrt(1/2)
*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 + (b^3*c - 4*a*b*c^2)*x^2)*
sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^
2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c + (b^6*c^3 - 12*a*b^4*c^4 +
 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*
A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4
*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*
c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6))*log(-(5*B^4*
a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b - 3*A^4*b^2)*c^3 + 3*(108
*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3*A^3*B*b^3)*c^2 - (81*B^4*
a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x - 1/2*sqrt(1/2)*(B^3*b^7 - 1
7*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3 - 3*A^2*B*a^2*b + A^3*a*b
^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12*A^2*B*a*b^3 + A^3*b^4)*c^
3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^5)*c^2 - (B*b^8*c^3 + 256*
(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*a^2*b^3)*c^6 + 48*(4*B*a^2
*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*sqrt((B^4*b^4 + A^4*c^4 -
2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A^2*B^2*
b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7 + 48*a^
2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c +
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))) + sqrt(1/2)*((b^2*c^2 - 4*a*c^3)*x^4 + a*b^2*c - 4*a^2*c^2 +
 (b^3*c - 4*a*b*c^2)*x^2)*sqrt(-(B^2*b^5 - 12*(4*A*B*a^2 - A^2*a*b)*c^3 + (
60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*B^2*a*b^3 - 2*A*B*b^4)*c -
 (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*sqrt((B^4*b^4 + A^4
*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 - 12*A*B^3*a*b + 2*A
^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*c^6 - 12*a*b^4*c^7
+ 48*a^2*b^2*c^8 - 64*a^3*c^9)))/(b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 -
 64*a^3*c^6))*log(-(5*B^4*a*b^4 - 3*A*B^3*b^5 - 4*A^4*a*c^4 + (20*A^3*B*a*b
 - 3*A^4*b^2)*c^3 + 3*(108*B^4*a^3 - 108*A*B^3*a^2*b + 28*A^2*B^2*a*b^2 - 3
*A^3*B*b^3)*c^2 - (81*B^4*a^2*b^2 - 65*A*B^3*a*b^3 + 9*A^2*B^2*b^4)*c)*x +
1/2*sqrt(1/2)*(B^3*b^7 - 17*B^3*a*b^5*c - 32*A^3*a^2*c^5 + 16*(18*A*B^2*a^3
 - 3*A^2*B*a^2*b + A^3*a*b^2)*c^4 - 2*(72*B^3*a^3*b + 72*A*B^2*a^2*b^2 - 12
*A^2*B*a*b^3 + A^3*b^4)*c^3 + (88*B^3*a^2*b^3 + 18*A*B^2*a*b^4 - 3*A^2*B*b^
5)*c^2 + (B*b^8*c^3 + 256*(3*B*a^4 - A*a^3*b)*c^7 - 64*(10*B*a^3*b^2 - 3*A*
a^2*b^3)*c^6 + 48*(4*B*a^2*b^4 - A*a*b^5)*c^5 - 4*(6*B*a*b^6 - A*b^7)*c^4)*
sqrt((B^4*b^4 + A^4*c^4 - 2*(9*A^2*B^2*a - 2*A^3*B*b)*c^3 + 3*(27*B^4*a^2 -
 12*A*B^3*a*b + 2*A^2*B^2*b^2)*c^2 - 2*(9*B^4*a*b^2 - 2*A*B^3*b^3)*c)/(b^6*
c^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)))*sqrt(-(B^2*b^5 - 12*(4*
A*B*a^2 - A^2*a*b)*c^3 + (60*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c^2 - (15*
B^2*a*b^3 - 2*A*B*b^4)*c - (b^6*c^3 - 12*a*b^4*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4538 vs. 2(292) = 584.
time = 7.12, size = 4538, normalized size = 13.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^2*x^3 - 2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - 2*A*a*c*x)/((c*x^4 +
b*x^2 + a)*(b^2*c - 4*a*c^2)) + 1/16*((2*b^3*c^3 - 8*a*b*c^4 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3)*(b^2*c - 4*a*c^2)
^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^
3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b^2*c - 4*a*c^2)^2*B
 - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^4 - 2*a*b^4*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*c^5 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 16*a^2*b^2*c^5 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*c^6 - 32*a^3*c^6 + 2*(b^2 - 4*a*c)*a*b^2*c^4 -
 8*(b^2 - 4*a*c)*a^2*c^5)*A*abs(b^2*c - 4*a*c^2) + 2*(sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c
^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
```

```
)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^
2*b*c^4)*B*abs(b^2*c - 4*a*c^2) - (2*b^7*c^5 - 8*a*b^5*c^6 - 32*a^2*b^3*c^7
 + 128*a^3*b*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^7*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^
4 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^
5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 64*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 32*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 16*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 2*(b^2
- 4*a*c)*b^5*c^5 + 32*(b^2 - 4*a*c)*a^2*b*c^7)*A - (2*b^8*c^4 - 32*a*b^6*c^
5 + 160*a^2*b^4*c^6 - 256*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^8*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^7*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b^4*c^4 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b^6*c^4 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b^2*c^5 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*b^3*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c^5 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b^2*c^6 - 2*(b^2 - 4*a*c)*b^6*c^4 + 24*(b^2 - 4*a*c)*a*b^4*c^5 - 64*(
b^2 - 4*a*c)*a^2*b^2*c^6)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3*c - 4*a*b*c^2 +
 sqrt((b^3*c - 4*a*b*c^2)^2 - 4*(a*b^2*c - 4*a^2*c^2)*(b^2*c^2 - 4*a*c^3)))
/(b^2*c^2 - 4*a*c^3)))/((a*b^6*c^3 - 12*a^2*b^4*c^4 - 2*a*b^5*c^4 + 48*a^3*
b^2*c^5 + 16*a^2*b^3*c^5 + a*b^4*c^5 - 64*a^4*c^6 - 32*a^3*b*c^6 - 8*a^2*b^
2*c^6 + 16*a^3*c^7)*abs(b^2*c - 4*a*c^2)*abs(c)) - 1/16*((2*b^3*c^3 - 8*a*b
*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c + 4*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 2*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^3 - 2*(b^2 - 4*a*c)*b*c^3
)*(b^2*c - 4*a*c^2)^2*A + (2*b^4*c^2 - 20*a*b^2*c^3 + 48*a^2*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 10*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^2*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 12*(b^2 - 4*a*c)*a*c^3)*(b
^2*c - 4*a*c^2)^2*B + 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3
- 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2...
```

Mupad [B]
time = 5.18, size = 2500, normalized size = 7.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)

[Out] - ((x^3*(A*b*c - B*b^2 + 2*B*a*c))/(2*c*(4*a*c - b^2)) + (x*(2*A*a*c - B*a*b))/(2*c*(4*a*c - b^2)))/(a + b*x^2 + c*x^4) - atan(((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2) - (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 72*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2)*1i - (((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) + (x*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)

```
^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4
+ 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)
))^(1/2)*(16*b^7*c^3 - 192*a*b^5*c^4 - 1024*a^3*b*c^6 + 768*a^2*b^3*c^5))/(
2*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*
(-(4*a*c - b^2)^9)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c
- 96*A^2*a^2*b^5*c^4 + 512*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2
*a^3*b^5*c^3 + 3840*B^2*a^4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9
*B^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5
+ 192*A*B*a^2*b^6*c^3 - 128*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*
b*c*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c
^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7
- 6144*a^5*b^2*c^8)))^(1/2) + (x*(B^2*b^6 + 8*A^2*a^2*c^4 + A^2*b^4*c^2 - 7
2*B^2*a^3*c^3 + 2*A*B*b^5*c + 74*B^2*a^2*b^2*c^2 - 16*B^2*a*b^4*c + 2*A^2*a
*b^2*c^3 - 14*A*B*a*b^3*c^2 - 8*A*B*a^2*b*c^3))/(2*(b^4*c + 16*a^2*c^3 - 8*
a*b^2*c^2)))*(-(B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2)^9)^(1/2) +
 B^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*b^10*c - 96*A^2*a^2*b^5*c^4 + 512
*A^2*a^3*b^3*c^5 + 288*B^2*a^2*b^7*c^2 - 1504*B^2*a^3*b^5*c^3 + 3840*B^2*a^
4*b^3*c^4 + 3072*A*B*a^5*c^6 - 27*B^2*a*b^9*c - 9*B^2*a*c*(-(4*a*c - b^2)^9
)^(1/2) - 768*A^2*a^4*b*c^6 - 3840*B^2*a^5*b*c^5 + 192*A*B*a^2*b^6*c^3 - 12
8*A*B*a^3*b^4*c^4 - 1536*A*B*a^4*b^2*c^5 + 2*A*B*b*c*(-(4*a*c - b^2)^9)^(1/
2) - 36*A*B*a*b^8*c^2)/(32*(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a
^2*b^8*c^5 - 1280*a^3*b^6*c^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)))^(1/2
)*1i)/((((2048*A*a^4*c^6 - 32*A*a*b^6*c^3 + 16*B*a*b^7*c^2 - 1024*B*a^4*b*c
^5 + 384*A*a^2*b^4*c^4 - 1536*A*a^3*b^2*c^5 - 192*B*a^2*b^5*c^3 + 768*B*a^3
*b^3*c^4)/(8*(b^6*c - 64*a^3*c^4 - 12*a*b^4*c^2 + 48*a^2*b^2*c^3)) - (x*(-(
B^2*b^11 + A^2*b^9*c^2 + A^2*c^2*(-(4*a*c - b^2...
```

3.120 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=276

$$-\frac{x(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(bB-2Ac\right.}{2\sqrt{}}$$

```
[Out] -1/2*x*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)+1/4*arctan
(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*B-2*A*c+(4*A*b*c-4*B*a*
c-B*b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*B
-2*A*c+(-4*A*b*c+4*B*a*c+B*b^2)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)*2^(1/2)/c^
(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1289, 1180, 211}

$$\frac{\left(-\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{4aBc-4Abc+b^2B}{\sqrt{b^2-4ac}}-2Ac+bB\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(-2aB-(x^2(bB-2Ac))+Ab)}{2(b^2-4ac)(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] -1/2*(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + ((b*B - 2*A*c - (b^2*B - 4*A*b*c + 4*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2
- 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b*B - 2*A*c + (b^2*B - 4*A*b*c +
4*a*B*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 -
4*a*c]]])/(2*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^2}\,dx = -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\int \frac{Ab-2aB+(bB-2Ac)x^2}{a+bx^2+cx^4}\,dx}{2(b^2-4ac)}$$

$$= -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4a}}}{4(b^2-4ac)}$$

$$= -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{2(b^2-4ac)(a+bx^2+cx^4)}+\frac{\left(bB-2Ac-\frac{b^2B-4Abc+4aBc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{b-}}\right.}{2\sqrt{2}\sqrt{c}\,(b^2-4ac)\sqrt{b-\sqrt{b^2-4a}}}$$

Mathematica [A]
time = 0.43, size = 298, normalized size = 1.08

$$\frac{1}{4}\left(\frac{2x(B(2a+bx^2)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{\sqrt{2}\left(-b^2B+4Abc-4aBc+bB\sqrt{b^2-4ac}-2Ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\left(b^2B-4Abc+4aBc+bB\sqrt{b^2-4ac}-2Ac\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{c}\,(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)) + (Sqrt[2]*(-(b^2*B) + 4*A*b*c - 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*
c*Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]
])/(Sqrt[c]*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*(b^
2*B - 4*A*b*c + 4*a*B*c + b*B*Sqrt[b^2 - 4*a*c] - 2*A*c*Sqrt[b^2 - 4*a*c])*
```

```
ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[c]*(b^2 - 4*
a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/4
```

Maple [A]
time = 0.07, size = 290, normalized size = 1.05

method	result
risch	$\frac{\frac{(2Ac-bB)x^3}{8ac-2b^2}+\frac{(Ab-2aB)x}{8ac-2b^2}}{cx^4+bx^2+a}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+\_Z^2b+a\right)}\frac{\left(\frac{(2Ac-bB)\_R^2}{4ac-b^2}-\frac{Ab-2aB}{4ac-b^2}\right)\ln\left(x-\_R\right)}{2c\_R^3+\_Rb}\right)}{4}$
default	$\frac{\frac{(2Ac-bB)x^3}{8ac-2b^2}+\frac{(Ab-2aB)x}{8ac-2b^2}}{cx^4+bx^2+a}+\frac{2c\left(-\frac{\left(2Ac\sqrt{-4ac+b^2}-4bcA-bB\sqrt{-4ac+b^2}+4acB+b^2B\right)\sqrt{2}\,\mathrm{arctanh}\left(\frac{cx\sqrt{\ }}{\sqrt{\left(-b+\sqrt{-}\right.}}\right.}{8c\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] (1/2*(2*A*c-B*b)/(4*a*c-b^2)*x^3+1/2*(A*b-2*B*a)/(4*a*c-b^2)*x)/(c*x^4+b*x^
2+a)+2/(4*a*c-b^2)*c*(-1/8*(2*A*c*(-4*a*c+b^2)^(1/2)-4*b*c*A-b*B*(-4*a*c+b^
2)^(1/2)+4*a*c*B+b^2*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(2*A
*c*(-4*a*c+b^2)^(1/2)+4*b*c*A-b*B*(-4*a*c+b^2)^(1/2)-4*a*c*B-b^2*B)/c/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((B*b - 2*A*c)*x^3 + (2*B*a - A*b)*x)/((b^2*c - 4*a*c^2)*x^4 + a*b^2 -
4*a^2*c + (b^3 - 4*a*b*c)*x^2) - 1/2*integrate(-((B*b - 2*A*c)*x^2 - 2*B*a
+ A*b)/(c*x^4 + b*x^2 + a), x)/(b^2 - 4*a*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3467 vs. $2(234) = 468$.
time = 1.22, size = 3467, normalized size = 12.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/4*(2*(B*b - 2*A*c)*x^3 - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a*b^4 - A^3*b^5)*c + (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c + (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) - sqrt(1/2)*((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*a^3 - 1

```
2*A*B^3*a^2*b + A^3*B*b^3)*c)*x + 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B^2*a*b^
5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b + 2*A^2
*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*A^2*B*a
*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^5*b + A
*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a*b^8)*c
)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 +
48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A^2*a*b)
*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2*b^4*c^
2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(
a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))/(a*b^6*c - 12
*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))) + sqrt(1/2)*((b^2*c - 4*a*c^2
)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(-(B^2*a*b^3 - 4*(4*A*B*
a^2 - 3*A^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c
 - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B^4*a^2 - 2*A^2*B^2*
a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)
))/(a*b^6*c - 12*a^2*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4))*log(-(3*B^4*a^
2*b^2 - A*B^3*a*b^3 - 4*A^4*a*c^3 + 3*(4*A^3*B*a*b - A^4*b^2)*c^2 + (4*B^4*
a^3 - 12*A*B^3*a^2*b + A^3*B*b^3)*c)*x - 1/2*sqrt(1/2)*(2*B^3*a^2*b^4 - A*B
^2*a*b^5 - 16*(2*A^2*B*a^3 - A^3*a^2*b)*c^3 + 8*(4*B^3*a^4 - 2*A*B^2*a^3*b
+ 2*A^2*B*a^2*b^2 - A^3*a*b^3)*c^2 - (16*B^3*a^3*b^2 - 8*A*B^2*a^2*b^3 + 2*
A^2*B*a*b^4 - A^3*b^5)*c - (192*B*a^4*b^3*c^3 + 256*A*a^5*c^5 - 128*(2*B*a^
5*b + A*a^4*b^2)*c^4 - 8*(6*B*a^3*b^5 - A*a^2*b^6)*c^2 + (4*B*a^2*b^7 - A*a
*b^8)*c)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^6*c^2 - 12*a^3*b^4
*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5)))*sqrt(-(B^2*a*b^3 - 4*(4*A*B*a^2 - 3*A
^2*a*b)*c^2 + (12*B^2*a^2*b - 12*A*B*a*b^2 + A^2*b^3)*c - (a*b^6*c - 12*a^2
*b^4*c^2 + 48*a^3*b^2*c^3 - 64*a^4*c^4)*sqrt((B...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3776 vs. $2(234) = 468$.
time = 6.53, size = 3776, normalized size = 13.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*b*x^3 - 2*A*c*x^3 + 2*B*a*x - A*b*x)/((c*x^4 + b*x^2 + a)*(b^2 - 4*a
*c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*c^
3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)^2*
B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3
 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4
*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*
a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^
4 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a*c)
 - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^3 + 8*(b^2 - 4*a*c)*a*b^2*c^4)*A
+ (2*b^7*c^2 - 8*a*b^5*c^3 - 32*a^2*b^3*c^4 + 128*a^3*b*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 32*(b^2 - 4*a*c)*a^2*
b*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^3 - 4*a*b*c + sqrt((b^3 - 4*a*b*c)^2
 - 4*(a*b^2 - 4*a^2*c)*(b^2*c - 4*a*c^2)))/(b^2*c - 4*a*c^2)))/((a*b^6*c -
12*a^2*b^4*c^2 - 2*a*b^5*c^2 + 48*a^3*b^2*c^3 + 16*a^2*b^3*c^3 + a*b^4*c^3
- 64*a^4*c^4 - 32*a^3*b*c^4 - 8*a^2*b^2*c^4 + 16*a^3*c^5)*abs(b^2 - 4*a*c)*
abs(c)) + 1/16*(2*(2*b^2*c^3 - 8*a*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
 - sqrt(b^2 - 4*a*c)*c)*b^2*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*c^3 - 2*(b^2 - 4*a*c)*c^3)*(b^2 - 4*a*c)^2*A - (2*b^3*c^2 - 8*a*b*c^3 - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(b^2 - 4*a*c)
^2*B + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*b^4*c^2 + 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*
c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*a*b*c^4 + 32*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2
- 4*a*c)*a*b*c^3)*A*abs(b^2 - 4*a*c) - 4*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a
*c)*c)*a*b^4*c - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*
sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*a*b^4*c^2 + 16*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 -
16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 32*a^3
*c^4 - 2*(b^2 - 4*a*c)*a*b^2*c^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*B*abs(b^2 - 4*a
*c) - 4*(2*b^6*c^3 - 16*a*b^4*c^4 + 32*a^2*b^2*...
```

Mupad [B]
time = 4.41, size = 2500, normalized size = 9.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] ((x*(A*b - 2*B*a))/(2*(4*a*c - b^2)) + (x^3*(2*A*c - B*b))/(2*(4*a*c - b^2)
))/(a + b*x^2 + c*x^4) - atan(((((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b
^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*
b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*
b^4*c)) - (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^
2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 9
6*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*
c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*
B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4
*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*
c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c
 + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^
4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a
```

^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i - (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 + 768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2) + (x*(B^2*b^4*c - 8*A^2*a*c^4 + 10*A^2*b^2*c^3 + 8*B^2*a^2*c^3 - 6*A*B*b^3*c^2 + 2*B^2*a*b^2*c^2 - 8*A*B*a*b*c^3))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*b^2*c^6 + a*b^12*c)))^(1/2)*1i)/((8*A^3*a*c^4 + 6*A^3*b^2*c^3 + A*B^2*b^4*c - 3*B^3*a*b^3*c + 8*A*B^2*a^2*c^3 - 5*A^2*B*b^3*c^2 - 4*B^3*a^2*b*c^2 + 18*A*B^2*a*b^2*c^2 - 28*A^2*B*a*b*c^3)/(4*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) + (((16*A*b^7*c^2 + 2048*B*a^4*c^5 - 192*A*a*b^5*c^3 - 1024*A*a^3*b*c^5 - 32*B*a*b^6*c^2 + 768*A*a^2*b^3*c^4 + 384*B*a^2*b^4*c^3 - 1536*B*a^3*b^2*c^4)/(8*(b^6 - 64*a^3*c^3 + 48*a^2*b^2*c^2 - 12*a*b^4*c)) - (x*(-(B^2*a*b^9 - B^2*a*(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144*a^6*

```
b^2*c^6 + a*b^12*c)))^(1/2)*(16*b^7*c^2 - 192*a*b^5*c^3 - 1024*a^3*b*c^5 +
768*a^2*b^3*c^4))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))*(-(B^2*a*b^9 - B^2*a*
(-(4*a*c - b^2)^9)^(1/2) + A^2*b^9*c + A^2*c*(-(4*a*c - b^2)^9)^(1/2) - 96*
A^2*a^2*b^5*c^3 + 512*A^2*a^3*b^3*c^4 - 96*B^2*a^3*b^5*c^2 + 512*B^2*a^4*b^
3*c^3 + 1024*A*B*a^5*c^5 - 768*A^2*a^4*b*c^5 - 768*B^2*a^5*b*c^4 + 128*A*B*
a^2*b^6*c^2 - 384*A*B*a^3*b^4*c^3 - 12*A*B*a*b^8*c)/(32*(4096*a^7*c^7 - 24*
a^2*b^10*c^2 + 240*a^3*b^8*c^3 - 1280*a^4*b^6*c^4 + 3840*a^5*b^4*c^5 - 6144
*a^6*b^2*c^6 + a*b^12*c)))^(1/2) - (x*(B^2*b^4*...
```

3.121 $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=293

$$\frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}+\frac{\sqrt{c}\left(Ab-2aB+\frac{4abB+A(b^2-12ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*x*(A*b^2-a*b*B-2*a*A*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(A*b-2
*a*B+(4*a*b*B+A*(-12*a*c+b^2))/(-4*a*c+b^2)^(1/2))/a/(-4*a*c+b^2)*2^(1/2)/(
b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*c^(1/2)*(A*b-2*a*B+(12*A*a*c-A*b^2-4*B*a*b)/(-4*a*c+b^2)^(1/2))
/a/(-4*a*c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1192, 1180, 211}

$$\frac{\sqrt{c}\left(\frac{A(b^2-12ac)+4abB}{\sqrt{b^2-4ac}}-2aB+Ab\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}\,a\,(b^2-4ac)\,\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{-12aAc+4abB+Ab^2}{\sqrt{b^2-4ac}}-2aB+Ab\right)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}\,a\,(b^2-4ac)\,\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(cx^2(Ab-2aB)-2aAc-abB+Ab^2)}{2a\,(b^2-4ac)\,(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(2*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)) + (Sqrt[c]*(A*b - 2*a*B + (4*a*b*B + A*(b^2 - 12*a*c))/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2
*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(A*b - 2*a
*B - (A*b^2 + 4*a*b*B - 12*a*A*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a*(b^2 - 4*a*c)*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{A+Bx^2}{\left(a+bx^2+cx^4\right)^2}\,dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-Ab^2-abB+6aAc-(Ab-2aB)cx^2}{a+bx^2+cx^4}\,dx}{2a\left(b^2-4ac\right)}$$

$$= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\left(c\left(Ab-2aB-\frac{Ab^2+4abB-12aAc}{\sqrt{b^2-4ac}}\right)\right)\int \cdots}{4a\left(b^2-4ac\right)}$$

$$= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{2a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\left(2aB\left(2b-\sqrt{b^2-4ac}\right)+A\left(b^2-\cdots\right.\right.}{2\sqrt{2}\,a\left(b^2-\cdots\right.}$$

Mathematica [A]

time = 0.50, size = 304, normalized size = 1.04

$$\frac{\frac{2x\left(-aB\left(b+2cx^2\right)+A\left(b^2-2ac+bcx^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(-2aB\left(-2b+\sqrt{b^2-4ac}\right)+A\left(b^2-12ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-2aB\left(2b+\sqrt{b^2-4ac}\right)+A\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((2*x*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(-2*a*B*(-2*b + Sqrt[b^2 - 4*a*c]) +
A*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (
```

```
Sqrt[2]*Sqrt[c]*(-2*a*B*(2*b + Sqrt[b^2 - 4*a*c]) + A*(-b^2 + 12*a*c + b*Sq
rt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/
((b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(4*a)
```

Maple [A]
time = 0.09, size = 464, normalized size = 1.58

method	result
risch	$\frac{-\frac{c(Ab-2aB)x^3}{2a(4ac-b^2)}+\frac{(2acA-Ab^2+abB)x}{2(4ac-b^2)a}}{cx^4+bx^2+a}+\frac{\sum_{\_R=\mathrm{RootOf}(c\_Z^4+\_Z^2b+a)}\frac{\left(-\frac{c(Ab-2aB)\_R^2}{4ac-b^2}+\frac{6acA-Ab^2-abB}{4ac-b^2}\right)\ln(x-\_R)}{2c\_R^3+\_Rb}}{4a}$
default	$16c^2\left(-\frac{-\frac{\left(A\sqrt{-4ac+b^2}-Ab+2aB\right)\sqrt{-4ac+b^2}\,x}{16ac\left(x^2+\frac{\sqrt{-4ac+b^2}}{2c}+\frac{b}{2c}\right)}-\frac{\left(-28Aabc+3Ab^3+12A\sqrt{-4ac+b^2}\,ac-3A\sqrt{-4ac+b^2}\ldots\right.}{16a(4ac+3\ldots}}{4c(4ac-b^2)\sqrt{-4ac+b^2}\ldots}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 16*c^2*(-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*(-1/16*(A*(-4*a*c+b^2)^(1/2)-
A*b+2*a*B)*(-4*a*c+b^2)^(1/2)/a/c*x/(x^2+1/2/c*(-4*a*c+b^2)^(1/2)+1/2*b/c)-
1/16*(-28*A*a*b*c+3*A*b^3+12*A*(-4*a*c+b^2)^(1/2)*a*c-3*A*(-4*a*c+b^2)^(1/2
)*b^2+8*a^2*c*B+6*B*a*b^2)*(2*b+(-4*a*c+b^2)^(1/2))/a/(4*a*c+3*b^2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)))-1/4/c/(4*a*c-b^2)/(-4*a*c+b^2)^(1/2)*(-1/16*(-A*(-4*a*c+b^2)^(
1/2)-A*b+2*a*B)*(-4*a*c+b^2)^(1/2)/a/c*x/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1
/2))-1/16*(12*A*(-4*a*c+b^2)^(1/2)*a*c-3*A*(-4*a*c+b^2)^(1/2)*b^2+28*A*a*b*
c-3*A*b^3-8*a^2*c*B-6*B*a*b^2)*(-2*b+(-4*a*c+b^2)^(1/2))/a/(4*a*c+3*b^2)*2^
(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^
2)^(1/2))*c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*B*a - A*b)*c*x^3 + (B*a*b - A*b^2 + 2*A*a*c)*x)/((a*b^2*c - 4*a^2*
c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2) + 1/2*integrate(-((
2*B*a - A*b)*c*x^2 - B*a*b - A*b^2 + 6*A*a*c)/(c*x^4 + b*x^2 + a), x)/(a*b^
2 - 4*a^2*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4885 vs. $2(254) = 508$.
time = 3.26, size = 4885, normalized size = 16.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*B*a - A*b)*c*x^3 - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^
2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A
^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2
- 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*s
qrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4
+ 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6
 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c +
48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^
4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*
a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4
 + A^3*B*b^5)*c)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B
*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A
^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 9
5*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 +
 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8
- 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*
A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a
*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b
^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))*sqrt(-(B^2*
a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B
^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48*a^
5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 +
 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3)))/(
a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))) + sqrt(1/2)*((a*b^2
*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^3 - 4*a^2*b*c)*x^2)*sqrt(-(B
^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3 - 5*A^2*a^2*b)*c^2 + 3*(
4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^3*b^6 - 12*a^4*b^4*c + 48
*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^
2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^
2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*a^9*c^3))
```

```
)/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3))*log((324*A^4*a^2*
c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B^4*a^4 - 20*A*B^3*a^3*b - 84
*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 - 3*(B^4*a^3*b^2 + 3*A*B
^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x - 1/2*sqrt(1/2)*(B^3*a^3*b^5
 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 + 864*A^3*a^4*c^4 - 48*(2*A*B^
2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2*(8*B^3*a^5*b + 48*A*B^2*a^4
*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2 - (8*B^3*a^4*b^3 + 30*A*B^2*
a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c - (B*a^4*b^8 + A*a^3*b^9 + 144
*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 + 64*(2*B*a^7*b^2 - 7*A*a^6*b^
3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt((B^4*a^4 + 4*A*B^3*a^3*b + 6
*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*(A^2*B^2*a
^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2
 - 64*a^9*c^3)))*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*a^3
 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c + (a^
3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*B^3
*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 - 18*
(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 48*a
^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^
6*c^3))) - sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*x^4 + a^2*b^2 - 4*a^3*c + (a*b^
3 - 4*a^2*b*c)*x^2)*sqrt(-(B^2*a^2*b^3 + 2*A*B*a*b^4 + A^2*b^5 - 12*(4*A*B*
a^3 - 5*A^2*a^2*b)*c^2 + 3*(4*B^2*a^3*b - 4*A*B*a^2*b^2 - 5*A^2*a*b^3)*c -
(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64*a^6*c^3)*sqrt((B^4*a^4 + 4*A*
B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81*A^4*a^2*c^2 -
18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^6 - 12*a^7*b^4*c + 4
8*a^8*b^2*c^2 - 64*a^9*c^3)))/(a^3*b^6 - 12*a^4*b^4*c + 48*a^5*b^2*c^2 - 64
*a^6*c^3))*log((324*A^4*a^2*c^4 - 81*(4*A^3*B*a^2*b + A^4*a*b^2)*c^3 - (4*B
^4*a^4 - 20*A*B^3*a^3*b - 84*A^2*B^2*a^2*b^2 - 65*A^3*B*a*b^3 - 5*A^4*b^4)*
c^2 - 3*(B^4*a^3*b^2 + 3*A*B^3*a^2*b^3 + 3*A^2*B^2*a*b^4 + A^3*B*b^5)*c)*x
+ 1/2*sqrt(1/2)*(B^3*a^3*b^5 + 3*A*B^2*a^2*b^6 + 3*A^2*B*a*b^7 + A^3*b^8 +
864*A^3*a^4*c^4 - 48*(2*A*B^2*a^5 + 7*A^2*B*a^4*b + 14*A^3*a^3*b^2)*c^3 + 2
*(8*B^3*a^5*b + 48*A*B^2*a^4*b^2 + 108*A^2*B*a^3*b^3 + 95*A^3*a^2*b^4)*c^2
- (8*B^3*a^4*b^3 + 30*A*B^2*a^3*b^4 + 45*A^2*B*a^2*b^5 + 23*A^3*a*b^6)*c +
(B*a^4*b^8 + A*a^3*b^9 + 144*A*a^5*b^5*c^2 - 256*(B*a^8 - 2*A*a^7*b)*c^4 +
64*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^3 - 4*(2*B*a^5*b^6 + 5*A*a^4*b^7)*c)*sqrt(
(B^4*a^4 + 4*A*B^3*a^3*b + 6*A^2*B^2*a^2*b^2 + 4*A^3*B*a*b^3 + A^4*b^4 + 81
*A^4*a^2*c^2 - 18*(A^2*B^2*a^3 + 2*A^3*B*a^2*b ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4426 vs. $2(254) = 508$.
time = 6.80, size = 4426, normalized size = 15.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*B*a*c*x^3 - A*b*c*x^3 + B*a*b*x - A*b^2*x + 2*A*a*c*x)/((c*x^4 + b*
x^2 + a)*(a*b^2 - 4*a^2*c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*A - 2
*(2*a*b^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b
^2 - 4*a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*a*b^6*c + 64*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 + 28*a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 4
8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 10*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 128*a^3*b^2*c^3 + 24*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 192*a^4*c^4 + 2*(b^2 - 4*a*c)*a*b^4*c - 20*
(b^2 - 4*a*c)*a^2*b^2*c^2 + 48*(b^2 - 4*a*c)*a^3*c^3)*A*abs(a*b^2 - 4*a^2*c
) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5 - 8*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^2*b^4*c - 2*a^2*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 16*a^3*b^3*c^2 - 4*sqrt(2)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 32*a^4*b*c^3 + 2*(b^2 - 4*a*c)*a^2*
b^3*c - 8*(b^2 - 4*a*c)*a^3*b*c^2)*B*abs(a*b^2 - 4*a^2*c) + (2*a^2*b^7*c^2
- 40*a^3*b^5*c^3 + 224*a^4*b^3*c^4 - 384*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
```

```
 + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 + 32*(b^2 -
 4*a*c)*a^3*b^3*c^3 - 96*(b^2 - 4*a*c)*a^4*b*c^4)*A + 4*(2*a^3*b^6*c^2 - 16
*a^4*b^4*c^3 + 32*a^5*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^3*b^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^4*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^3*b^5*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a^3*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^4*b^2*c^3 - 2*(b^2 - 4*a*c)*a^3*b^4*c^2 + 8*(b^2 - 4*a*c)*a^4*b^2*c^3)*B)
*arctan(2*sqrt(1/2)*x/sqrt((a*b^3 - 4*a^2*b*c + sqrt((a*b^3 - 4*a^2*b*c)^2
- 4*(a^2*b^2 - 4*a^3*c)*(a*b^2*c - 4*a^2*c^2)))/(a*b^2*c - 4*a^2*c^2)))/((a
^3*b^6 - 12*a^4*b^4*c - 2*a^3*b^5*c + 48*a^5*b^2*c^2 + 16*a^4*b^3*c^2 + a^3
*b^4*c^2 - 64*a^6*c^3 - 32*a^5*b*c^3 - 8*a^4*b^2*c^3 + 16*a^5*c^4)*abs(a*b^
2 - 4*a^2*c)*abs(c)) + 1/16*((2*b^3*c^2 - 8*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
- 4*a*c)*c)*b*c^2 - 2*(b^2 - 4*a*c)*b*c^2)*(a*b^2 - 4*a^2*c)^2*A - 2*(2*a*b
^2*c^2 - 8*a^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2
*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c - sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*c^2 - 2*(b^2 - 4*
a*c)*a*c^2)*(a*b^2 - 4*a^2*c)^2*B + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)
*c)*a*b^6 - 14*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c - 2*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*a*b^6*c + 64*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 + 20*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^3*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 28*
a^2*b^4*c^2 - 96*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 48*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - ...
```

Mupad [B]
time = 4.84, size = 2500, normalized size = 8.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] atan(((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6
*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a
^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 4
8*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)
^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*
```

c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 51
2*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c
 - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*
c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2
)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c
 + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5
)))^(1/2)*(1024*a^5*b*c^5 - 16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*
c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 +
 A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*
B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^
4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*
b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a
^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4
 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4
096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7
*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2) + (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8
*B^2*a^3*c^4 + 10*B^2*a^2*b^2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40
*A*B*a^2*b*c^4))/(2*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^
2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(
1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2
*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5
 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5
 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*
a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a
^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3
 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*1i - (((6144*A*a^5*c^6 + 16
*A*a*b^8*c^2 - 1024*B*a^5*b*c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 -
5632*A*a^4*b^2*c^5 + 16*B*a^2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c
^4)/(8*(a^2*b^6 - 64*a^5*c^3 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) + (x*(-(A^2*
b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c -
b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 +
 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B
*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a
^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 -
1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*
c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^
6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)))^(1/2)*(1024*a^5*b*c^5 -
16*a^2*b^7*c^2 + 192*a^3*b^5*c^3 - 768*a^4*b^3*c^4))/(2*(a^2*b^4 + 16*a^4*c
^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c - b^2)^9)^
(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^2*a^2*b^7*c
^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^5*c^2 + 512
*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c*(-(4*a*c -
 b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*B*a^3*b^6*c
^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4*a*c - b^2)
^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*a^4*b^10*c

```
+ 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a^8*b^2*c^5)
))^(1/2) - (x*(72*A^2*a^2*c^5 + A^2*b^4*c^3 - 8*B^2*a^3*c^4 + 10*B^2*a^2*b^
2*c^3 - 14*A^2*a*b^2*c^4 + 2*A*B*a*b^3*c^3 - 40*A*B*a^2*b*c^4))/(2*(a^2*b^4
 + 16*a^4*c^2 - 8*a^3*b^2*c)))*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2*(-(4*a*c
 - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10 + 288*A^
2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 + 3840*A^2*a^4*b^3*c^4 - 96*B^2*a^4*b^
5*c^2 + 512*B^2*a^5*b^3*c^3 + 3072*A*B*a^6*c^5 - 27*A^2*a*b^9*c - 9*A^2*a*c
*(-(4*a*c - b^2)^9)^(1/2) - 3840*A^2*a^5*b*c^5 - 768*B^2*a^6*b*c^4 + 192*A*
B*a^3*b^6*c^2 - 128*A*B*a^4*b^4*c^3 - 1536*A*B*a^5*b^2*c^4 + 2*A*B*a*b*(-(4
*a*c - b^2)^9)^(1/2) - 36*A*B*a^2*b^8*c)/(32*(a^3*b^12 + 4096*a^9*c^6 - 24*
a^4*b^10*c + 240*a^5*b^8*c^2 - 1280*a^6*b^6*c^3 + 3840*a^7*b^4*c^4 - 6144*a
^8*b^2*c^5)))^(1/2)*1i)/((((6144*A*a^5*c^6 + 16*A*a*b^8*c^2 - 1024*B*a^5*b*
c^5 - 288*A*a^2*b^6*c^3 + 1920*A*a^3*b^4*c^4 - 5632*A*a^4*b^2*c^5 + 16*B*a^
2*b^7*c^2 - 192*B*a^3*b^5*c^3 + 768*B*a^4*b^3*c^4)/(8*(a^2*b^6 - 64*a^5*c^3
 - 12*a^3*b^4*c + 48*a^4*b^2*c^2)) - (x*(-(A^2*b^11 + B^2*a^2*b^9 + A^2*b^2
*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*(-(4*a*c - b^2)^9)^(1/2) + 2*A*B*a*b^10
 + 288*A^2*a^2*b^7*c^2 - 1504*A^2*a^3*b^5*c^3 +...
```

3.122 $\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=389

$$-\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x}-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)}+\frac{\sqrt{c}\left(aB\left(b^2-12ac+b\sqrt{b^2-4ac}\right)-A\left(3\right.\right.}{2\ldots}$$

```
[Out] 1/2*(10*A*a*c-3*A*b^2+B*a*b)/a^2/(-4*a*c+b^2)/x+1/2*(-a*b*B+A*(-2*a*c+b^2)+
(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x/(c*x^4+b*x^2+a)+1/4*arctan(x*2^(1/2)*c^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(b^2-12*a*c+b*(-4*a*c+b^2)
^(1/2))-A*(3*b^3-16*a*b*c+3*b^2*(-4*a*c+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2
)))/a^2/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/4*arctan(
x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(3*A*b^2-a*b*B-10*a
*A*c+(a*B*(-12*a*c+b^2)-A*(-16*a*b*c+3*b^3))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*
c+b^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.83, antiderivative size = 389, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1291, 1295, 1180, 211}

$$\frac{\sqrt{c}\left(aB\left(b\sqrt{b^2-4ac}-12ac+b^2\right)-A\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(\frac{aB(b^2-12ac)-A(3b^3-16abc)}{\sqrt{b^2-4ac}}-10aAc-abB+3Ab^2\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^2\left(b^2-4ac\right)\sqrt{\sqrt{b^2-4ac}+b}}-\frac{-10aAc-abB+3Ab^2}{2a^2x\left(b^2-4ac\right)}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/2*(3*A*b^2 - a*b*B - 10*a*A*c)/(a^2*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2 -
 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x*(a + b*x^2 + c*x^4)) +
(Sqrt[c]*(a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) - A*(3*b^3 - 16*a*b*c +
3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)^(3/2)*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(3*A*b^2 - a*b*B - 10*a*A*c + (a*B*(b^2
- 12*a*c) - A*(3*b^3 - 16*a*b*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c
]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^2*(b^2 - 4*a*c)*Sqrt[b + Sq
rt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-3Ab^2+abB+10aAc-3(Ab-2aB)cx^2}{x^2(a+bx^2+cx^4)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\int \frac{aB(b^2-6ac)}{}}{}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\Big(c\Big(aB\Big(b^2}{}$$

$$= -\frac{3Ab^2-abB-10aAc}{2a^2\left(b^2-4ac\right)x} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x\left(a+bx^2+cx^4\right)} + \frac{\sqrt{c}\,\Big(aB\Big(b}{}$$

Mathematica [A]
time = 0.69, size = 382, normalized size = 0.98

$$\frac{-\frac{4A}{x}+\frac{2x\left(aB\left(b^2-2ac+bcx^2\right)-A\left(b^3-3abc+b^2cx^2-2ac^2x^2\right)\right)}{\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}+\frac{\sqrt{2}\sqrt{c}\left(aB\left(b^2-12ac+b\sqrt{b^2-4ac}\right)+A\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(aB\left(-b^2+12ac+b\sqrt{b^2-4ac}\right)+A\left(3b^3-16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\left(b^2-4ac\right)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{4a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*A)/x + (2*x*(a*B*(b^2 - 2*a*c + b*c*x^2) - A*(b^3 - 3*a*b*c + b^2*c*x^
2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(
a*B*(b^2 - 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(-3*b^3 + 16*a*b*c - 3*b^2*Sqr
t[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt
[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]])
 + (Sqrt[2]*Sqrt[c]*(a*B*(-b^2 + 12*a*c + b*Sqrt[b^2 - 4*a*c]) + A*(3*b^3 -
 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(3/2)*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]))/(4*a^2)
```

Maple [A]
time = 0.10, size = 379, normalized size = 0.97

method	result
default	$-\frac{\frac{c\left(2acA-A\,b^2+abB\right)x^3}{8ac-2b^2}+\frac{\left(3Aabc-A\,b^3-2a^2cB+Ba\,b^2\right)x}{8ac-2b^2}}{c\,x^4+b\,x^2+a}+\frac{2c\left(-\frac{\left(10A\sqrt{-4ac+b^2}\,ac-3A\sqrt{-4ac+b^2}\,b^2+16Aabc-3A\,b^3+abB\right.}{8\sqrt{-4ac+b^2}}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*((1/2*c*(2*A*a*c-A*b^2+B*a*b)/(4*a*c-b^2)*x^3+1/2*(3*A*a*b*c-A*b^3-2
*B*a^2*c+B*a*b^2)/(4*a*c-b^2)*x)/(c*x^4+b*x^2+a)+2/(4*a*c-b^2)*c*(-1/8*(10*
A*(-4*a*c+b^2)^(1/2)*a*c-3*A*(-4*a*c+b^2)^(1/2)*b^2+16*A*a*b*c-3*A*b^3+a*b*
B*(-4*a*c+b^2)^(1/2)-12*a^2*c*B+B*a*b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^
(1/2))+1/8*(10*A*(-4*a*c+b^2)^(1/2)*a*c-3*A*(-4*a*c+b^2)^(1/2)*b^2-16*A*a*b
*c+3*A*b^3+a*b*B*(-4*a*c+b^2)^(1/2)+12*a^2*c*B-B*a*b^2)/(-4*a*c+b^2)^(1/2)*
2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))))-1/a^2*A/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 2*A*a*b^2 + 8*A*a^2*c + (B*a*
b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2)/((a^2*b^2*c - 4*a^3*c^2)*x^5 +
 (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x) + 1/2*integrate((B*a*b^
2 - 3*A*b^3 + (10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^2 - (6*B*a^2 - 13*A*a*b)
*c)/(c*x^4 + b*x^2 + a), x)/(a^2*b^2 - 4*a^3*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7583 vs. 2(338) = 676.
time = 9.28, size = 7583, normalized size = 19.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(2*(10*A*a*c^2 + (B*a*b - 3*A*b^2)*c)*x^4 - 4*A*a*b^2 + 16*A*a^2*c + 2*
(B*a*b^2 - 3*A*b^3 - (2*B*a^2 - 11*A*a*b)*c)*x^2 - sqrt(1/2)*((a^2*b^2*c -
4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(
B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 +
5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 -
 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^
2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 -
 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^
3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2
*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^
2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*
b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b
^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 6
25*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b +
 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^
4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A
^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*
B*b^7)*c^2)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b
^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 -
 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^
6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 +
 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^
```

7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*
a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^10 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b -
17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3 - 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 4
9*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 - 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12
*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^
4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(2
7*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 10
17*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^3
*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 48
*a^12*b^2*c^2 - 64*a^13*c^3)))*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b^7
 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 + 77
*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c + (
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 - 1
2*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A
^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3*(
27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 + 1
017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*a^
3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c + 4
8*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 -
64*a^8*c^3))) + sqrt(1/2)*((a^2*b^2*c - 4*a^3*c^2)*x^5 + (a^2*b^3 - 4*a^3*b
*c)*x^3 + (a^3*b^2 - 4*a^4*c)*x)*sqrt(-(B^2*a^2*b^5 - 6*A*B*a*b^6 + 9*A^2*b
^7 + 60*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 5*(12*B^2*a^4*b - 60*A*B*a^3*b^2 +
77*A^2*a^2*b^3)*c^2 - 5*(3*B^2*a^3*b^3 - 16*A*B*a^2*b^4 + 21*A^2*a*b^5)*c +
 (a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3)*sqrt((B^4*a^4*b^4 -
 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625
*A^4*a^4*c^4 - 50*(9*A^2*B^2*a^5 - 44*A^3*B*a^4*b + 51*A^4*a^3*b^2)*c^3 + 3
*(27*B^4*a^6 - 264*A*B^3*a^5*b + 968*A^2*B^2*a^4*b^2 - 1596*A^3*B*a^3*b^3 +
 1017*A^4*a^2*b^4)*c^2 - 2*(9*B^4*a^5*b^2 - 98*A*B^3*a^4*b^3 + 396*A^2*B^2*
a^3*b^4 - 702*A^3*B*a^2*b^5 + 459*A^4*a*b^6)*c)/(a^10*b^6 - 12*a^11*b^4*c +
 48*a^12*b^2*c^2 - 64*a^13*c^3)))/(a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2
- 64*a^8*c^3))*log((2500*A^4*a^3*c^6 + 625*(4*A^3*B*a^3*b - 9*A^4*a^2*b^2)*
c^5 - 3*(108*B^4*a^5 - 756*A*B^3*a^4*b + 1672*A^2*B^2*a^3*b^2 - 909*A^3*B*a
^2*b^3 - 657*A^4*a*b^4)*c^4 + (81*B^4*a^4*b^2 - 647*A*B^3*a^3*b^3 + 1674*A^
2*B^2*a^2*b^4 - 1323*A^3*B*a*b^5 - 189*A^4*b^6)*c^3 - 5*(B^4*a^3*b^4 - 9*A*
B^3*a^2*b^5 + 27*A^2*B^2*a*b^6 - 27*A^3*B*b^7)*c^2)*x - 1/2*sqrt(1/2)*(B^3*
a^3*b^8 - 9*A*B^2*a^2*b^9 + 27*A^2*B*a*b^10 - 27*A^3*b^11 - 400*(6*A^2*B*a^
6 - 13*A^3*a^5*b)*c^5 + 8*(108*B^3*a^7 - 762*A*B^2*a^6*b + 1956*A^2*B*a^5*b
^2 - 1801*A^3*a^4*b^3)*c^4 - (672*B^3*a^6*b^2 - 4968*A*B^2*a^5*b^3 + 12414*
A^2*B*a^4*b^4 - 10549*A^3*a^3*b^5)*c^3 + 5*(38*B^3*a^5*b^4 - 297*A*B^2*a^4*
b^5 + 771*A^2*B*a^3*b^6 - 666*A^3*a^2*b^7)*c^2 - (23*B^3*a^4*b^6 - 192*A*B^
2*a^3*b^7 + 531*A^2*B*a^2*b^8 - 486*A^3*a*b^9)*c - (B*a^6*b^9 - 3*A*a^5*b^1
0 + 1280*A*a^10*c^5 + 128*(4*B*a^10*b - 17*A*a^9*b^2)*c^4 - 448*(B*a^9*b^3
- 3*A*a^8*b^4)*c^3 + 8*(18*B*a^8*b^5 - 49*A*a^7*b^6)*c^2 - 5*(4*B*a^7*b^7 -
 11*A*a^6*b^8)*c)*sqrt((B^4*a^4*b^4 - 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6
 - 108*A^3*B*a*b^7 + 81*A^4*b^8 + 625*A^4*a^4*c...

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5408 vs. $2(338) = 676$.
time = 6.33, size = 5408, normalized size = 13.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*(B*a*b*c*x^4 - 3*A*b^2*c*x^4 + 10*A*a*c^2*x^4 + B*a*b^2*x^2 - 3*A*b^3*x
^2 - 2*B*a^2*c*x^2 + 11*A*a*b*c*x^2 - 2*A*a*b^2 + 8*A*a^2*c)/((c*x^5 + b*x^
3 + a*x)*(a^2*b^2 - 4*a^3*c)) - 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^
4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 20*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 6*(b^2 - 4*a*c)*b^2*c^2 + 20*(b^2 -
 4*a*c)*a*c^3)*(a^2*b^2 - 4*a^3*c)^2*A - (2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a^2*b
^2 - 4*a^3*c)^2*B + 2*(3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 -
37*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c - 6*sqrt(2)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c - 6*a^2*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^4*b^3*c^2 + 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^4*c^2 + 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 74*a^3
*b^5*c^2 - 208*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^3 - 104*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 304*a^4*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^4*b*c^4 + 416*a^5*b*c^4 + 6*(b^2 - 4*a*c)*a^2*b^5*c - 5
0*(b^2 - 4*a*c)*a^3*b^3*c^2 + 104*(b^2 - 4*a*c)*a^4*b*c^3)*A*abs(a^2*b^2 -
4*a^3*c) - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6 - 14*sqrt(2)*
```

```
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^3*b^5*c - 2*a^3*b^6*c + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^2*c^2 + 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^2 +
 sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 28*a^4*b^4*c^2 - 96*
sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^3 - 48*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^5*b*c^3 - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
4*b^2*c^3 - 128*a^5*b^2*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
5*c^4 + 192*a^6*c^4 + 2*(b^2 - 4*a*c)*a^3*b^4*c - 20*(b^2 - 4*a*c)*a^4*b^2*
c^2 + 48*(b^2 - 4*a*c)*a^5*c^3)*B*abs(a^2*b^2 - 4*a^3*c) + (6*a^4*b^8*c^2 -
 80*a^5*b^6*c^3 + 352*a^6*b^4*c^4 - 512*a^7*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 - 6*(b^2 - 4*a*c)*a^4*b^6*c^2 +
 56*(b^2 - 4*a*c)*a^5*b^4*c^3 - 128*(b^2 - 4*a*c)*a^6*b^2*c^4)*A - (2*a^5*b
^7*c^2 - 40*a^6*b^5*c^3 + 224*a^7*b^3*c^4 - 384*a^8*b*c^5 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^7 + 20*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c - 112*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^3*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b*c^3 + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 48*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 2*(b^2 - 4*a*c)*a^5*b^5*c^2 + 32
*(b^2 - 4*a*c)*a^6*b^3*c^3 - 96*(b^2 - 4*a*c)*a^7*b*c^4)*B)*arctan(2*sqrt(1
/2)*x/sqrt((a^2*b^3 - 4*a^3*b*c + sqrt((a^2*b^3 - 4*a^3*b*c)^2 - 4*(a^3*b^2
 - 4*a^4*c)*(a^2*b^2*c - 4*a^3*c^2)))/(a^2*b^2*c - 4*a^3*c^2)))/((a^5*b^6 -
 12*a^6*b^4*c - 2*a^5*b^5*c + 48*a^7*b^2*c^2 + 16*a^6*b^3*c^2 + a^5*b^4*c^2
 - 64*a^8*c^3 - 32*a^7*b*c^3 - 8*a^6*b^2*c^3 + 16*a^7*c^4)*abs(a^2*b^2 - 4*
a^3*c)*abs(c)) + 1/16*((6*b^4*c^2 - 44*a*b^2*c^3 + 80*a^2*c^4 - 3*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4 + 22*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c - 40*...
```

Mupad [B]
time = 5.38, size = 2500, normalized size = 6.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^2),x)

[Out] - (A/a - (x^2*(3*A*b^3 - B*a*b^2 + 2*B*a^2*c - 11*A*a*b*c))/(2*a^2*(4*a*c - b^2)) + (c*x^4*(10*A*a*c - 3*A*b^2 + B*a*b))/(2*a^2*(4*a*c - b^2)))/(a*x + b*x^3 + c*x^5) - atan((((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c^5)))^(1/2)*(1048576*a^16*b*c^8 + 256*a^10*b^13*c^2 - 6144*a^11*b^11*c^3 + 61440*a^12*b^9*c^4 - 327680*a^13*b^7*c^5 + 983040*a^14*b^5*c^6 - 1572864*a^15*b^3*c^7) - 393216*B*a^15*c^8 + 851968*A*a^14*b*c^8 + 192*A*a^8*b^13*c^2 - 4672*A*a^9*b^11*c^3 + 47360*A*a^10*b^9*c^4 - 256000*A*a^11*b^7*c^5 + 778240*A*a^12*b^5*c^6 - 1261568*A*a^13*b^3*c^7 - 64*B*a^9*b^12*c^2 + 1664*B*a^10*b^10*c^3 - 17920*B*a^11*b^8*c^4 + 102400*B*a^12*b^6*c^5 - 327680*B*a^13*b^4*c^6 + 557056*B*a^14*b^2*c^7) + x*(204800*A^2*a^12*c^9 - 73728*B^2*a^13*c^8 + 144*A^2*a^6*b^12*c^3 - 3264*A^2*a^7*b^10*c^4 + 30112*A^2*a^8*b^8*c^5 - 143360*A^2*a^9*b^6*c^6 + 365568*A^2*a^10*b^4*c^7 - 458752*A^2*a^11*b^2*c^8 + 16*B^2*a^8*b^10*c^3 - 416*B^2*a^9*b^8*c^4 + 4608*B^2*a^10*b^6*c^5 - 25600*B^2*a^11*b^4*c^6 + 69632*B^2*a^12*b^2*c^7 - 96*A*B*a^7*b^11*c^3 + 2336*A*B*a^8*b^9*c^4 - 22528*A*B*a^9*b^7*c^5 + 107520*A*B*a^10*b^5*c^6 - 253952*A*B*a^11*b^3*c^7 + 237568*A*B*a^12*b*c^8))*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1504*B^2*a^5*b^5*

```
c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*b^11*c + 26880*A
^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^2*a^3*c*(-(4*a*c
 - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*c^3 - 22400*A*B*
a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a*c - b^2)^9)^(1/
2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^10*c + 44*A*B*a^2
*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^6 - 24*a^6*b^10*
c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 - 6144*a^10*b^2*c
^5)))^(1/2)*1i + ((-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(4*a*c - b^2)^
9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3*b^7*c^3 + 30
240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(-(4*a*c - b^2
)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^4*b^7*c^2 - 1
504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6 - 213*A^2*a*
b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7*b*c^5 - 9*B^
2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064*A*B*a^4*b^6*
c^3 - 22400*A*B*a^5*b^4*c^4 + 30720*A*B*a^6*b^2*c^5 - 51*A^2*a*b^2*c*(-(4*a
*c - b^2)^9)^(1/2) - 6*A*B*a*b^3*(-(4*a*c - b^2)^9)^(1/2) + 152*A*B*a^2*b^1
0*c + 44*A*B*a^2*b*c*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^5*b^12 + 4096*a^11*c^
6 - 24*a^6*b^10*c + 240*a^7*b^8*c^2 - 1280*a^8*b^6*c^3 + 3840*a^9*b^4*c^4 -
 6144*a^10*b^2*c^5)))^(1/2)*(x*(-(9*A^2*b^13 + B^2*a^2*b^11 + 9*A^2*b^4*(-(
4*a*c - b^2)^9)^(1/2) - 6*A*B*a*b^12 + 2077*A^2*a^2*b^9*c^2 - 10656*A^2*a^3
*b^7*c^3 + 30240*A^2*a^4*b^5*c^4 - 44800*A^2*a^5*b^3*c^5 + 25*A^2*a^2*c^2*(
-(4*a*c - b^2)^9)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^9)^(1/2) + 288*B^2*a^
4*b^7*c^2 - 1504*B^2*a^5*b^5*c^3 + 3840*B^2*a^6*b^3*c^4 - 15360*A*B*a^7*c^6
 - 213*A^2*a*b^11*c + 26880*A^2*a^6*b*c^6 - 27*B^2*a^3*b^9*c - 3840*B^2*a^7
*b*c^5 - 9*B^2*a^3*c*(-(4*a*c - b^2)^9)^(1/2) - 1548*A*B*a^3*b^8*c^2 + 8064
*A*B*a^4*b^6*c^3 - 22400*A*B*a^5*b^4*c^4 + 3072...
```

3.123 $\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx$

Optimal. Leaf size=522

$$-\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3}-\frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3\left(b^2-4ac\right)x}-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)}-\frac{\sqrt{c}\,(\ldots}{\ldots}$$

```
[Out] 1/6*(14*A*a*c-5*A*b^2+3*B*a*b)/a^2/(-4*a*c+b^2)/x^3+1/2*(-a*B*(-10*a*c+3*b^
2)+A*(-19*a*b*c+5*b^3))/a^3/(-4*a*c+b^2)/x+1/2*(-a*b*B+A*(-2*a*c+b^2)+(A*b-
2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^3/(c*x^4+b*x^2+a)-1/4*arctan(x*2^(1/2)*c^(1/
2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(3*b^3-16*a*b*c+3*b^2*(-4*a*c
+b^2)^(1/2)-10*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29*a*b^2*c+28*a^2*c^2+5*(-4
*a*c+b^2)^(1/2)*b^3-19*(-4*a*c+b^2)^(1/2)*a*b*c))/a^3/(-4*a*c+b^2)^(3/2)*2^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/4*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(3*b^3-16*a*b*c-3*b^2*(-4*a*c+b^2)^(1/2)+10
*a*c*(-4*a*c+b^2)^(1/2))-A*(5*b^4-29*a*b^2*c+28*a^2*c^2-5*(-4*a*c+b^2)^(1/2
)*b^3+19*(-4*a*c+b^2)^(1/2)*a*b*c))/a^3/(-4*a*c+b^2)^(3/2)*2^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.93, antiderivative size = 522, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1291, 1295, 1180, 211}

$$-\frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3x\left(b^2-4ac\right)}-\frac{-14aAc-3abB+5Ab^2}{6a^2x^3\left(b^2-4ac\right)}-\frac{\sqrt{c}\left(aB\left(3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c-19abc\sqrt{b^2-4ac}+5b^3\sqrt{b^2-4ac}+5b^4\right)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(aB\left(-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}-16abc+3b^3\right)-A\left(28a^2c^2-29ab^2c+19abc\sqrt{b^2-4ac}-5b^3\sqrt{b^2-4ac}+5b^4\right)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{2\sqrt{2}a^3\left(b^2-4ac\right)^{3/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{2ax^3\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] -1/6*(5*A*b^2 - 3*a*b*B - 14*a*A*c)/(a^2*(b^2 - 4*a*c)*x^3) - (a*B*(3*b^2 -
 10*a*c) - A*(5*b^3 - 19*a*b*c))/(2*a^3*(b^2 - 4*a*c)*x) - (a*b*B - A*(b^2
- 2*a*c) - (A*b - 2*a*B)*c*x^2)/(2*a*(b^2 - 4*a*c)*x^3*(a + b*x^2 + c*x^4))
 - (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[
b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c
] - 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b
^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[c]*(a*B*(3*b^3 - 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sq
rt[b^2 - 4*a*c]) - A*(5*b^4 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*
a*c] + 19*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqr
t[b^2 - 4*a*c]]])/(2*Sqrt[2]*a^3*(b^2 - 4*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*
a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\left(a+bx^2+cx^4\right)^2}\,dx = -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} - \frac{\int \frac{-5Ab^2+3abB+14aAc-5(Ab-2aB)cx^2}{x^4(a+bx^2+cx^4)}\,dx}{2a\left(b^2-4ac\right)}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{2a\left(b^2-4ac\right)x^3\left(a+bx^2+cx^4\right)} + \frac{\int \frac{-3(5Ab^3-}{}}{}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A(b^2-}{2a\left(b^2-4a\right.}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A(b^2-}{2a\left(b^2-4a\right.}$$

$$= -\frac{5Ab^2-3abB-14aAc}{6a^2\left(b^2-4ac\right)x^3} - \frac{aB(3b^2-10ac)-A(5b^3-19abc)}{2a^3\left(b^2-4ac\right)x} - \frac{abB-A(b^2-}{2a\left(b^2-4a\right.}$$

Mathematica [A]
time = 0.80, size = 487, normalized size = 0.93

$$\frac{-\frac{4aA}{x^3}+\frac{24Ab-12aB}{x}+\frac{6x\left(aB\left(-b^3+3abc-b^2cx^2+2ac^2x^2\right)+A\left(b^4-4ab^2c+2a^2c^2+b^3cx^2-3abc^2x^2\right)\right)}{(b^2-4ac)(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(aB\left(-3b^3+16abc-3b^2\sqrt{b^2-4ac}+10ac\sqrt{b^2-4ac}\right)+A\left(5b^4-29ab^2c+28a^2c^2+5b^3\sqrt{b^2-4ac}-19abc\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\sqrt{c}\left(aB\left(-3b^3+16abc+3b^2\sqrt{b^2-4ac}-10ac\sqrt{b^2-4ac}\right)+A\left(5b^4-29ab^2c+28a^2c^2-5b^3\sqrt{b^2-4ac}+19abc\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{3/2}\sqrt{b+\sqrt{b^2-4ac}}}}{12a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x]
```

```
[Out] ((-4*a*A)/x^3 + (24*A*b - 12*a*B)/x + (6*x*(a*B*(-b^3 + 3*a*b*c - b^2*c*x^2
 + 2*a*c^2*x^2) + A*(b^4 - 4*a*b^2*c + 2*a^2*c^2 + b^3*c*x^2 - 3*a*b*c^2*x^
2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)) + (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c - 3*b^2*Sqrt[b^2 - 4*a*c] + 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 + 5*b^3*Sqrt[b^2 - 4*a*c] - 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (3*Sqrt[2]*Sqrt[c]*(a*B*(-3*b^3
+ 16*a*b*c + 3*b^2*Sqrt[b^2 - 4*a*c] - 10*a*c*Sqrt[b^2 - 4*a*c]) + A*(5*b^4
 - 29*a*b^2*c + 28*a^2*c^2 - 5*b^3*Sqrt[b^2 - 4*a*c] + 19*a*b*c*Sqrt[b^2 -
4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4
*a*c)^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(12*a^3)
```

Maple [A]
time = 0.09, size = 484, normalized size = 0.93

method	result

default	$-\frac{-\frac{c\left(3Aabc-A\,b^{3}-2a^{2}cB+Ba\,b^{2}\right)x^{3}}{2\left(4ac-b^{2}\right)}+\frac{\left(2A\,a^{2}c^{2}-4Aa\,b^{2}c+A\,b^{4}+3a^{2}bBc-Ba\,b^{3}\right)x}{8ac-2b^{2}}}{c\,x^{4}+b\,x^{2}+a}+\frac{2c\left(-\frac{\left(-19Aabc\sqrt{-4ac+b^{2}}+5A\,b^{3}\sqrt{-4ac+\cdots}\right.}{\cdots}\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*((-1/2*c*(3*A*a*b*c-A*b^3-2*B*a^2*c+B*a*b^2)/(4*a*c-b^2)*x^3+1/2*(2*
A*a^2*c^2-4*A*a*b^2*c+A*b^4+3*B*a^2*b*c-B*a*b^3)/(4*a*c-b^2)*x)/(c*x^4+b*x^
2+a)+2/(4*a*c-b^2)*c*(-1/8*(-19*A*a*b*c*(-4*a*c+b^2)^(1/2)+5*A*b^3*(-4*a*c+
b^2)^(1/2)+28*A*a^2*c^2-29*A*a*b^2*c+5*A*b^4+10*a^2*c*B*(-4*a*c+b^2)^(1/2)-
3*B*a*b^2*(-4*a*c+b^2)^(1/2)+16*a^2*b*B*c-3*B*a*b^3)/(-4*a*c+b^2)^(1/2)*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2
)^(1/2))*c)^(1/2))+1/8*(-19*A*a*b*c*(-4*a*c+b^2)^(1/2)+5*A*b^3*(-4*a*c+b^2)
^(1/2)-28*A*a^2*c^2+29*A*a*b^2*c-5*A*b^4+10*a^2*c*B*(-4*a*c+b^2)^(1/2)-3*B*
a*b^2*(-4*a*c+b^2)^(1/2)-16*a^2*b*B*c+3*B*a*b^3)/(-4*a*c+b^2)^(1/2)*2^(1/2)
/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2))))-1/3/a^2*A/x^3-(-2*A*b+B*a)/a^3/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 2*A*a^2*
b^2 + 8*A*a^3*c - (9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b - 62*A
*a*b^2)*c)*x^4 - 2*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c)*x^
2)/((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*
a^5*c)*x^3) - 1/2*integrate((3*B*a*b^3 - 5*A*b^4 - 14*A*a^2*c^2 - ((10*B*a^
2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^2 - (13*B*a^2*b - 24*A*a*b^2
)*c)/(c*x^4 + b*x^2 + a), x)/(a^3*b^2 - 4*a^4*c)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10190 vs. $2(460) = 920$.
time = 21.18, size = 10190, normalized size = 19.52

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")

[Out] 1/12*(6*((10*B*a^2 - 19*A*a*b)*c^2 - (3*B*a*b^2 - 5*A*b^3)*c)*x^6 - 4*A*a^2*b^2 + 16*A*a^3*c - 2*(9*B*a*b^3 - 15*A*b^4 - 14*A*a^2*c^2 - (33*B*a^2*b - 62*A*a*b^2)*c)*x^4 - 4*(3*B*a^2*b^2 - 5*A*a*b^3 - 4*(3*B*a^3 - 5*A*a^2*b)*c)*x^2 - 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3))*log((9604*A^4*a^4*c^8 + 7203*(4*A^3*B*a^4*b - 7*A^4*a^3*b^2)*c^7 - (2500*B^4*a^6 - 22500*A*B^3*a^5*b + 43524*A^2*B^2*a^4*b^2 + 4343*A^3*B*a^3*b^3 - 43410*A^4*a^2*b^4)*c^6 + (5625*B^4*a^5*b^2 - 31137*A*B^3*a^4*b^3 + 52821*A^2*B^2*a^3*b^4 - 20190*A^3*B*a^2*b^5 - 12325*A^4*a*b^6)*c^5 - 3*(657*B^4*a^4*b^4 - 3351*A*B^3*a^3*b^5 + 5560*A^2*B^2*a^2*b^6 - 2775*A^3*B*a*b^7 - 375*A^4*b^8)*c^4 + 7*(27*B^4*a^3*b^6 - 135*A*B^3*a^2*b^7 + 225*A^2*B^2*a*b^8 - 125*A^3*B*b^9)*c^3)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b^11 - 135*A*B^2*a^2*b^12 + 225*A^2*B*a*b^13 - 125*A^3*b^14 + 10976*A^3*a^7*c^7 - 112*(50*A*B^2*a^8 - 463*A^2*B*a^7*b + 709*A^3*a^6*b^2)*c^6 - 2*(2600*B^3*a^8*b - 31256*A*B^2*a^7*b^2 + 96044*A^2*B*a^6*b^3 - 86495*A^3*a^5*b^4)*c^5 + (14408*B^3*a^7*b^3 - 101006*A*B^2*a^6*b^4 + 224705*A^2*B*a^5*b^5 - 160932*A^3*a^4*b^6)*c^4 - 7*(1507*B^3*a^6*b^5 - 8820*A*B^2*a^5*b^6 + 16991*A^2*B*a^4*b^7 - 10797*A^3*a^3*b^8)*c^3 + (3330*B^3*a^5*b^7 - 17889*A*B^2*a^4*b^8 + 31929*A^2*B*a^3*b^9 - 18940*A^3*a^2*b^10)*c^2 - (486*B^3*a^4*b^9 - 2493*A*B^2*a^3*b^10 + 4260*A^2*B*a^2*b^11 - 2425*A^3*a*b^12)*c - (3*B*a^8*b^10 - 5*A*a^7*b^11 - 256*(5*B*a^13 - 13*A*a^12*b)*c^5 + 64*(34*B*a^12*b^2 - 73*A*a^11*b^3)*c^4 - 112*(12*B*a^11*b^4 - 23*A*a^10*b^5)*c^3 + 28*(14*B*a^10*b^6 - 25*A*a^9*b^7)*c^2 - (55*B*a^9*b^8 - 94*A*a^8*b^9)*c)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b^4 - 77424*A^3*B*a^4*b^5 +

```
41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7872*A*B^3*a^5*b^5 + 22508*A
^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4*a^2*b^8)*c^2 - 2*(459*B^4*
a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*b^8 - 9550*A^3*B*a^2*b^9 +
4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c + 48*a^16*b^2*c^2 - 64*a^17*c
^3)))*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8 + 25*A^2*b^9 - 140*(4*A*B*a^5 - 9
*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 +
 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3
*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c + (a^7*b^6 - 12*a^8*b^4*c + 48*a^9*
b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^8 - 540*A*B^3*a^3*b^9 + 1350*A^2*
B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4*b^12 + 2401*A^4*a^6*c^6 - 98*(25
*A^2*B^2*a^7 - 186*A^3*B*a^6*b + 246*A^4*a^5*b^2)*c^5 + (625*B^4*a^8 - 9300
*A*B^3*a^7*b + 51894*A^2*B^2*a^6*b^2 - 109544*A^3*B*a^5*b^3 + 76686*A^4*a^4
*b^4)*c^4 - 2*(1275*B^4*a^7*b^2 - 14086*A*B^3*a^6*b^3 + 51336*A^2*B^2*a^5*b
^4 - 77424*A^3*B*a^4*b^5 + 41815*A^4*a^3*b^6)*c^3 + 3*(1017*B^4*a^6*b^4 - 7
872*A*B^3*a^5*b^5 + 22508*A^2*B^2*a^4*b^6 - 28260*A^3*B*a^3*b^7 + 13175*A^4
*a^2*b^8)*c^2 - 2*(459*B^4*a^5*b^6 - 3186*A*B^3*a^4*b^7 + 8280*A^2*B^2*a^3*
b^8 - 9550*A^3*B*a^2*b^9 + 4125*A^4*a*b^10)*c)/(a^14*b^6 - 12*a^15*b^4*c +
48*a^16*b^2*c^2 - 64*a^17*c^3)))/(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 -
 64*a^10*c^3))) + 3*sqrt(1/2)*((a^3*b^2*c - 4*a^4*c^2)*x^7 + (a^3*b^3 - 4*a
^4*b*c)*x^5 + (a^4*b^2 - 4*a^5*c)*x^3)*sqrt(-(9*B^2*a^2*b^7 - 30*A*B*a*b^8
+ 25*A^2*b^9 - 140*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 - 105*(4*B^2*a^5*b - 20*A*
B*a^4*b^2 + 23*A^2*a^3*b^3)*c^3 + 7*(55*B^2*a^4*b^3 - 210*A*B*a^3*b^4 + 198
*A^2*a^2*b^5)*c^2 - 7*(15*B^2*a^3*b^5 - 52*A*B*a^2*b^6 + 45*A^2*a*b^7)*c +
(a^7*b^6 - 12*a^8*b^4*c + 48*a^9*b^2*c^2 - 64*a^10*c^3)*sqrt((81*B^4*a^4*b^
8 - 540*A*B^3*a^3*b^9 + 1350*A^2*B^2*a^2*b^10 - 1500*A^3*B*a*b^11 + 625*A^4
*b^12 + 2401*A^4*a^6*c^6 - 98*(25*A^2*B^2*a^7 -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6327 vs. $2(460) = 920$.
time = 8.81, size = 6327, normalized size = 12.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^2*c*x^3 - A*b^3*c*x^3 - 2*B*a^2*c^2*x^3 + 3*A*a*b*c^2*x^3 + B*a
*b^3*x - A*b^4*x - 3*B*a^2*b*c*x + 4*A*a*b^2*c*x - 2*A*a^2*c^2*x)/((a^3*b^2
 - 4*a^4*c)*(c*x^4 + b*x^2 + a)) + 1/16*((10*b^5*c^2 - 78*a*b^3*c^3 + 152*a
^2*b*c^4 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
+ 39*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 10
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 76*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 38*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 5*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 19*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 10*(b^2 - 4*a*c)*b^3*
c^2 + 38*(b^2 - 4*a*c)*a*b*c^3)*(a^3*b^2 - 4*a^4*c)^2*A - (6*a*b^4*c^2 - 44
*a^2*b^2*c^3 + 80*a^3*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^4 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^3*c - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^3*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*
b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c
^2 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 -
 6*(b^2 - 4*a*c)*a*b^2*c^2 + 20*(b^2 - 4*a*c)*a^2*c^3)*(a^3*b^2 - 4*a^4*c)^
2*B + 2*(5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^8 - 64*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c - 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^3*b^7*c - 10*a^3*b^8*c + 286*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^5*b^4*c^2 + 88*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 +
 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^2 + 128*a^4*b^6*c^2 -
496*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^3 - 220*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3 - 44*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^4*c^3 - 572*a^5*b^4*c^3 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^7*c^4 + 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 +
 110*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^4 + 992*a^6*b^2*c^4
- 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^5 - 448*a^7*c^5 + 10*(b^
2 - 4*a*c)*a^3*b^6*c - 88*(b^2 - 4*a*c)*a^4*b^4*c^2 + 220*(b^2 - 4*a*c)*a^5
*b^2*c^3 - 112*(b^2 - 4*a*c)*a^6*c^4)*A*abs(a^3*b^2 - 4*a^4*c) - 2*(3*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7 - 37*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^5*b^5*c - 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*
c - 6*a^4*b^7*c + 152*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^2 +
 50*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^2 + 3*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^2 + 74*a^5*b^5*c^2 - 208*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^3 - 104*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^6*b^2*c^3 - 25*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^3
- 304*a^6*b^3*c^3 + 52*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^4 +
416*a^7*b*c^4 + 6*(b^2 - 4*a*c)*a^4*b^5*c - 50*(b^2 - 4*a*c)*a^5*b^3*c^2 +
104*(b^2 - 4*a*c)*a^6*b*c^3)*B*abs(a^3*b^2 - 4*a^4*c) + (10*a^6*b^9*c^2 - 1
38*a^7*b^7*c^3 + 680*a^8*b^5*c^4 - 1376*a^9*b^3*c^5 + 896*a^10*b*c^6 - 5*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^9 + 69*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^7*c + 10*sqrt(2)*
```

```
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^8*c - 340*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^5*c^2 - 98*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^6*c^2 - 5*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^7*c^2 + 688*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 288*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^4*c^3 + 49*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^5*c^3 - 448*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^10*b*c^4 - 224*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b^2*c^4 - 144*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^8*b^3*c^4 + 112*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^9*b*c^5 - 10*(b^2 -
4*a*c)*a^6*b^7*c^2 + 98*(b^2 - 4*a*c)*a^7*b^5*c^3 - 288*(b^2 - 4*a*c)*a^8*b
^3*c^4 + 224*(b^2 - 4*a*c)*a^9*b*c^5)*A - (6*a^7*b^8*c^2 - 80*a^8*b^6*c^3 +
 352*a^9*b^4*c^4 - 512*a^10*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a^7*b^8 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^8*b^6*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^7*b^7*c - 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^9*b^4*c^2 - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^8*b^5*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^7*b^6*c^2 + 256*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^10*b^2*c^3 + 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^9*b^3*c^3 + 28*sqrt(2)*sqrt(...
```

Mupad [B]
time = 5.70, size = 2500, normalized size = 4.79

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^2),x)
```

```
[Out] - atan((((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1
/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 11692
8*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^
2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2)
 + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 4
4800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*
a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 268
80*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*
a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*
a^6*b^4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(
1/2) + 51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c -
b^2)^9)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(
1/2) + 186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^1
3*c^6 - 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4
```

*c^4 - 6144*a^12*b^2*c^5)))^(1/2)*(917504*A*a^19*c^9 + x*(-(25*A^2*b^15 + 9
*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*
A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*
A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9
)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 1
0656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B
^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c
- 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*
a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^
4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A*B*a^
7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) + 51*B^2*a^3*b^2*c*(-(
4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9)^(1/2) + 724*A*B*a^2
*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) + 186*A*B*a^3*b*c^2*(-
(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6 - 24*a^8*b^10*c + 240
*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 - 6144*a^12*b^2*c^5)))
^(1/2)*(1048576*a^21*b*c^8 + 256*a^15*b^13*c^2 - 6144*a^16*b^11*c^3 + 61440
*a^17*b^9*c^4 - 327680*a^18*b^7*c^5 + 983040*a^19*b^5*c^6 - 1572864*a^20*b^
3*c^7) + 851968*B*a^19*b*c^8 - 320*A*a^12*b^14*c^2 + 7936*A*a^13*b^12*c^3 -
82816*A*a^14*b^10*c^4 + 468480*A*a^15*b^8*c^5 - 1536000*A*a^16*b^6*c^6 + 2
867200*A*a^17*b^4*c^7 - 2719744*A*a^18*b^2*c^8 + 192*B*a^13*b^13*c^2 - 4672
*B*a^14*b^11*c^3 + 47360*B*a^15*b^9*c^4 - 256000*B*a^16*b^7*c^5 + 778240*B*
a^17*b^5*c^6 - 1261568*B*a^18*b^3*c^7) - x*(401408*A^2*a^16*c^10 - 204800*B
^2*a^17*c^9 - 400*A^2*a^9*b^14*c^3 + 9440*A^2*a^10*b^12*c^4 - 92816*A^2*a^1
1*b^10*c^5 + 488096*A^2*a^12*b^8*c^6 - 1458688*A^2*a^13*b^6*c^7 + 2401280*A
^2*a^14*b^4*c^8 - 1871872*A^2*a^15*b^2*c^9 - 144*B^2*a^11*b^12*c^3 + 3264*B
^2*a^12*b^10*c^4 - 30112*B^2*a^13*b^8*c^5 + 143360*B^2*a^14*b^6*c^6 - 36556
8*B^2*a^15*b^4*c^7 + 458752*B^2*a^16*b^2*c^8 + 480*A*B*a^10*b^13*c^3 - 1110
4*A*B*a^11*b^11*c^4 + 105824*A*B*a^12*b^9*c^5 - 530432*A*B*a^13*b^7*c^6 + 1
469440*A*B*a^14*b^5*c^7 - 2121728*A*B*a^15*b^3*c^8 + 1236992*A*B*a^16*b*c^9
))*(-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*b^6*(-(4*a*c - b^2)^9)^(1/2) -
30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 35767*A^2*a^3*b^9*c^3 + 116928*A^2*
a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 215040*A^2*a^6*b^3*c^6 + 49*A^2*a^3*
c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4*(-(4*a*c - b^2)^9)^(1/2) + 207
7*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 + 30240*B^2*a^6*b^5*c^4 - 44800*B
^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b^2)^9)^(1/2) + 35840*A*B*a^8*c^
7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 - 213*B^2*a^3*b^11*c + 26880*B^2
*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c - b^2)^9)^(1/2) - 7278*A*B*a^3*b^
10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*a^5*b^6*c^4 + 201600*A*B*a^6*b^
4*c^5 - 161280*A*B*a^7*b^2*c^6 + 165*A^2*a*b^4*c*(-(4*a*c - b^2)^9)^(1/2) +
51*B^2*a^3*b^2*c*(-(4*a*c - b^2)^9)^(1/2) + 30*A*B*a*b^5*(-(4*a*c - b^2)^9
)^(1/2) + 724*A*B*a^2*b^12*c - 184*A*B*a^2*b^3*c*(-(4*a*c - b^2)^9)^(1/2) +
186*A*B*a^3*b*c^2*(-(4*a*c - b^2)^9)^(1/2))/(32*(a^7*b^12 + 4096*a^13*c^6
- 24*a^8*b^10*c + 240*a^9*b^8*c^2 - 1280*a^10*b^6*c^3 + 3840*a^11*b^4*c^4 -
6144*a^12*b^2*c^5)))^(1/2)*1i - ((-(25*A^2*b^15 + 9*B^2*a^2*b^13 - 25*A^2*
b^6*(-(4*a*c - b^2)^9)^(1/2) - 30*A*B*a*b^14 + 6366*A^2*a^2*b^11*c^2 - 3576

```
7*A^2*a^3*b^9*c^3 + 116928*A^2*a^4*b^7*c^4 - 219744*A^2*a^5*b^5*c^5 + 21504
0*A^2*a^6*b^3*c^6 + 49*A^2*a^3*c^3*(-(4*a*c - b^2)^9)^(1/2) - 9*B^2*a^2*b^4
*(-(4*a*c - b^2)^9)^(1/2) + 2077*B^2*a^4*b^9*c^2 - 10656*B^2*a^5*b^7*c^3 +
30240*B^2*a^6*b^5*c^4 - 44800*B^2*a^7*b^3*c^5 - 25*B^2*a^4*c^2*(-(4*a*c - b
^2)^9)^(1/2) + 35840*A*B*a^8*c^7 - 615*A^2*a*b^13*c - 80640*A^2*a^7*b*c^7 -
 213*B^2*a^3*b^11*c + 26880*B^2*a^8*b*c^6 - 246*A^2*a^2*b^2*c^2*(-(4*a*c -
b^2)^9)^(1/2) - 7278*A*B*a^3*b^10*c^2 + 39132*A*B*a^4*b^8*c^3 - 119616*A*B*
a^5*b^6*c^4 + 201600*A*B*a^6*b^4*c^5 - 161280*A...
```

3.124 $\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=365

$$\frac{(3b^4B - Ab^3c - 21ab^2Bc + 7aAbc^2 + 30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2} - \frac{x^8(a(bB-2Ac) + (b^2B - Abc - 2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^4(a(3b^3\ldots}{}$$

```
[Out] 1/2*(7*A*a*b*c^2-A*b^3*c+30*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)*x^2/c^3/(-4*a*c
+b^2)^2-1/4*x^8*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/
(c*x^4+b*x^2+a)^2-1/4*x^4*(a*(16*A*a*c^2-A*b^2*c-18*B*a*b*c+3*B*b^3)+(10*A*
a*b*c^2-A*b^3*c+20*B*a^2*c^2-20*B*a*b^2*c+3*B*b^4)*x^2)/c^2/(-4*a*c+b^2)^2/
(c*x^4+b*x^2+a)-1/2*(-30*A*a^2*b*c^3+10*A*a*b^3*c^2-A*b^5*c-60*B*a^3*c^3+90
*B*a^2*b^2*c^2-30*B*a*b^4*c+3*B*b^6)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/c^4/(-4*a*c+b^2)^(5/2)-1/4*(-A*c+3*B*b)*ln(c*x^4+b*x^2+a)/c^4
```

Rubi [A]
time = 0.97, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1265, 832, 787, 648, 632, 212, 642}

$$-\frac{x^4(x^2(20a^2Bc^2+10aAbc^2-20ab^2Bc-Ab^3c+3b^4B)+a(16aAc^2-18abBc-Ab^2c+3b^3B))}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{x^2(30a^2Bc^2+7aAbc^2-21ab^2Bc-Ab^3c+3b^4B)}{2c^3\,(b^2-4ac)^2} - \frac{(-60a^3Bc^3-30a^2Abc^3+90a^2b^2Bc^2+10aAb^3c^2-30ab^4Bc-Ab^5c+3b^6B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^4\,(b^2-4ac)^{5/2}} - \frac{x^8(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac))}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{(3bB-Ac)\log(a+bx^2+cx^4)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Int[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((3*b^4*B - A*b^3*c - 21*a*b^2*B*c + 7*a*A*b*c^2 + 30*a^2*B*c^2)*x^2)/(2*c^
3*(b^2 - 4*a*c)^2) - (x^8*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)
)/(4*c*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^4*(a*(3*b^3*B - A*b^2*c -
18*a*b*B*c + 16*a*A*c^2) + (3*b^4*B - A*b^3*c - 20*a*b^2*B*c + 10*a*A*b*c^2
 + 20*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((3*b^
6*B - A*b^5*c - 30*a*b^4*B*c + 10*a*A*b^3*c^2 + 90*a^2*b^2*B*c^2 - 30*a^2*A
*b*c^3 - 60*a^3*B*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^4*(b^
2 - 4*a*c)^(5/2)) - ((3*b*B - A*c)*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 787

```
Int[(((d_.) + (e_.)*(x_))*((f_) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) + (c_.)*
(x_)^2), x_Symbol] :> Simp[e*g*(x/c), x] + Dist[1/c, Int[(c*d*f - a*e*g + (
c*e*f + c*d*g - b*e*g)*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f, g}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^{11}(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^5(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^8(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\text{Subst}\Big(\int \frac{x^3(4a(bB-2Ac)+(3b^2B-Abc-}{(a+bx+cx^2)^2}}{4c\,(b^2-4ac)}$$

$$= -\frac{x^8(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^4(a(3b^3B-Ab^2c-18abBc+1}{4}$$

$$= \frac{(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2} - \frac{x^8(a(bB-2Ac)+(b^2B-A}{4c\,(b^2-4ac)\,(a+bx^2}$$

$$= \frac{(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2} - \frac{x^8(a(bB-2Ac)+(b^2B-A}{4c\,(b^2-4ac)\,(a+bx^2}$$

$$= \frac{(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2} - \frac{x^8(a(bB-2Ac)+(b^2B-A}{4c\,(b^2-4ac)\,(a+bx^2}$$

$$= \frac{(3b^4B-Ab^3c-21ab^2Bc+7aAbc^2+30a^2Bc^2)\,x^2}{2c^3\,(b^2-4ac)^2} - \frac{x^8(a(bB-2Ac)+(b^2B-A}{4c\,(b^2-4ac)\,(a+bx^2}$$

Mathematica [**A**]
time = 0.45, size = 435, normalized size = 1.19

$$\frac{2Bc^2x^2 + \frac{b^7B-b^6c(A+6Bx^2)+4a^3c^4(8A+9Bx^2)-3a^2b^2c^3(13A+34Bx^2)+ab^4c^2(11A+48Bx^2)+ab^3c^2(61aB-30Acx^2)+2b^5c(-7aB+2Acx^2)+2a^2bc^3(-39aB+25Acx^2)}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{b^5(-bB+Ac)x^2+a^3c^2(-5bB+2c(A+Bx^2))+ab^3(-b^2B-5Ac^2x^2+bc(A+6Bx^2))+a^2bc(5b^2B+5Ac^2x^2-bc(4A+9Bx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{2c(-3b^6B+Ab^5c+30ab^4Bc-10aAb^3c^2-90a^2b^2Bc^2+30a^2Abc^3+60a^3Bc^3)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}} + c(-3bB+Ac)\log(a+bx^2+cx^4)}{4c^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (2*B*c^2*x^2 + (b^7*B - b^6*c*(A + 6*B*x^2) + 4*a^3*c^4*(8*A + 9*B*x^2) - 3
*a^2*b^2*c^3*(13*A + 34*B*x^2) + a*b^4*c^2*(11*A + 48*B*x^2) + a*b^3*c^2*(6
1*a*B - 30*A*c*x^2) + 2*b^5*c*(-7*a*B + 2*A*c*x^2) + 2*a^2*b*c^3*(-39*a*B +
 25*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (b^5*(-(b*B) + A*c)*x
^2 + a^3*c^2*(-5*b*B + 2*c*(A + B*x^2)) + a*b^3*(-(b^2*B) - 5*A*c^2*x^2 + b
*c*(A + 6*B*x^2)) + a^2*b*c*(5*b^2*B + 5*A*c^2*x^2 - b*c*(4*A + 9*B*x^2)))/
((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(-3*b^6*B + A*b^5*c + 30*a*b^4
*B*c - 10*a*A*b^3*c^2 - 90*a^2*b^2*B*c^2 + 30*a^2*A*b*c^3 + 60*a^3*B*c^3)*A
rcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2) + c*(-3*b*B +
 A*c)*Log[a + b*x^2 + c*x^4])/(4*c^5)
```

Maple [**A**]
time = 0.14, size = 624, normalized size = 1.71

method	result
default	$\frac{B x^2}{2c^3} + \frac{\frac{\left(25A\,a^2 b\,c^3-15Aa\,b^3c^2+2A\,b^5c+18B\,a^3c^3-51B\,a^2b^2c^2+24Ba\,b^4c-3B\,b^6\right)x^6}{16a^2c^2-8a\,b^2c+b^4}+\frac{\left(32A\,a^3c^4+11A\,a^2b^2c^3-19Aa\,b^4c^2+3A\,b^6c-42B\,a^3b\,c^3\right.}{2c\left(16a^2c^2-8a\,b^2c+b^4\right.}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*x^2/c^3+1/2/c^3*(((25*A*a^2*b*c^3-15*A*a*b^3*c^2+2*A*b^5*c+18*B*a^3*c
^3-51*B*a^2*b^2*c^2+24*B*a*b^4*c-3*B*b^6)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/
2*(32*A*a^3*c^4+11*A*a^2*b^2*c^3-19*A*a*b^4*c^2+3*A*b^6*c-42*B*a^3*b*c^3-41
*B*a^2*b^3*c^2+34*B*a*b^5*c-5*B*b^7)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+a*(31
*A*a^2*b*c^3-22*A*a*b^3*c^2+3*A*b^5*c+14*B*a^3*c^3-71*B*a^2*b^2*c^2+38*B*a*
b^4*c-5*B*b^6)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(24*A*a^2*c^3-21*A*
a*b^2*c^2+3*A*b^4*c-58*B*a^2*b*c^2+36*B*a*b^3*c-5*B*b^5)/c/(16*a^2*c^2-8*a*
b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*A*a^2*c
^3-8*A*a*b^2*c^2+A*b^4*c-48*B*a^2*b*c^2+24*B*a*b^3*c-3*B*b^5)/c*ln(c*x^4+b*
x^2+a)+2*(-7*A*a^2*b*c^2+A*a*b^3*c-30*a^3*B*c^2+21*B*a^2*b^2*c-3*B*a*b^4-1/
2*(16*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-48*B*a^2*b*c^2+24*B*a*b^3*c-3*B*b^5)*
b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1586 vs. 2(351) = 702.
time = 0.62, size = 3196, normalized size = 8.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10
 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^
3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 -
(254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4 + (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(
13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3
*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 + 87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*
A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2
)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 3
1*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 119*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*
A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*A*a*b^7)*c)*x^2 - (3*B*a^2*b^6 + (3*B*b^
6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B
*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 +
10*(9*B*a^2*b^3 + A*a*b^4)*c^3 - (30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 -
 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a
^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4
*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b +
 A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^
6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2*b^5)*c)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4
+ 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2
 + a)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (3*
B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*b^
3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^3*
b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^3
- (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4*b
 + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 + 5*
A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c^3
 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48*(
4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*a^
2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2 +
 a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c^6
 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^5*
c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a^2
*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*c^
5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2), 1/4*(2*(B*b^6*c^3 - 12*B*a*b^4*c^4
 + 48*B*a^2*b^2*c^5 - 64*B*a^3*c^6)*x^10 - 5*B*a^2*b^7 - 96*A*a^5*c^4 + 4*(
B*b^7*c^2 - 12*B*a*b^5*c^3 + 48*B*a^2*b^3*c^4 - 64*B*a^3*b*c^5)*x^8 - 2*(2*
B*b^8*c + 100*(2*B*a^4 + A*a^3*b)*c^5 - (254*B*a^3*b^2 + 85*A*a^2*b^3)*c^4
+ (123*B*a^2*b^4 + 23*A*a*b^5)*c^3 - 2*(13*B*a*b^6 + A*b^7)*c^2)*x^6 - (5*B
*b^9 + 128*A*a^4*c^5 + 4*(22*B*a^4*b + 3*A*a^3*b^2)*c^4 - (314*B*a^3*b^3 +
87*A*a^2*b^4)*c^3 + (225*B*a^2*b^5 + 31*A*a*b^6)*c^2 - (58*B*a*b^7 + 3*A*b^
8)*c)*x^4 + 4*(58*B*a^5*b + 27*A*a^4*b^2)*c^3 - (202*B*a^4*b^3 + 33*A*a^3*b
^4)*c^2 - 2*(5*B*a*b^8 + 4*(30*B*a^5 + 31*A*a^4*b)*c^4 - (346*B*a^4*b^2 + 1
19*A*a^3*b^3)*c^3 + (235*B*a^3*b^4 + 34*A*a^2*b^5)*c^2 - (59*B*a^2*b^6 + 3*
```

```
A*a*b^7)*c)*x^2 - 2*(3*B*a^2*b^6 + (3*B*b^6*c^2 - 30*(2*B*a^3 + A*a^2*b)*c^
5 + 10*(9*B*a^2*b^2 + A*a*b^3)*c^4 - (30*B*a*b^4 + A*b^5)*c^3)*x^8 + 2*(3*B
*b^7*c - 30*(2*B*a^3*b + A*a^2*b^2)*c^4 + 10*(9*B*a^2*b^3 + A*a*b^4)*c^3 -
(30*B*a*b^5 + A*b^6)*c^2)*x^6 + (3*B*b^8 - 60*(2*B*a^4 + A*a^3*b)*c^4 + 10*
(12*B*a^3*b^2 - A*a^2*b^3)*c^3 + 2*(15*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (24*B*a
*b^6 + A*b^7)*c)*x^4 - 30*(2*B*a^5 + A*a^4*b)*c^3 + 10*(9*B*a^4*b^2 + A*a^3
*b^3)*c^2 + 2*(3*B*a*b^7 - 30*(2*B*a^4*b + A*a^3*b^2)*c^3 + 10*(9*B*a^3*b^3
 + A*a^2*b^4)*c^2 - (30*B*a^2*b^5 + A*a*b^6)*c)*x^2 - (30*B*a^3*b^4 + A*a^2
*b^5)*c)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 -
 4*a*c)) + (56*B*a^3*b^5 + 3*A*a^2*b^6)*c - (3*B*a^2*b^7 + 64*A*a^5*c^4 + (
3*B*b^7*c^2 + 64*A*a^3*c^6 - 48*(4*B*a^3*b + A*a^2*b^2)*c^5 + 12*(12*B*a^2*
b^3 + A*a*b^4)*c^4 - (36*B*a*b^5 + A*b^6)*c^3)*x^8 + 2*(3*B*b^8*c + 64*A*a^
3*b*c^5 - 48*(4*B*a^3*b^2 + A*a^2*b^3)*c^4 + 12*(12*B*a^2*b^4 + A*a*b^5)*c^
3 - (36*B*a*b^6 + A*b^7)*c^2)*x^6 + (3*B*b^9 + 128*A*a^4*c^5 - 32*(12*B*a^4
*b + A*a^3*b^2)*c^4 + 24*(4*B*a^3*b^3 - A*a^2*b^4)*c^3 + 2*(36*B*a^2*b^5 +
5*A*a*b^6)*c^2 - (30*B*a*b^7 + A*b^8)*c)*x^4 - 48*(4*B*a^5*b + A*a^4*b^2)*c
^3 + 12*(12*B*a^4*b^3 + A*a^3*b^4)*c^2 + 2*(3*B*a*b^8 + 64*A*a^4*b*c^4 - 48
*(4*B*a^4*b^2 + A*a^3*b^3)*c^3 + 12*(12*B*a^3*b^4 + A*a^2*b^5)*c^2 - (36*B*
a^2*b^6 + A*a*b^7)*c)*x^2 - (36*B*a^3*b^5 + A*a^2*b^6)*c)*log(c*x^4 + b*x^2
 + a))/(a^2*b^6*c^4 - 12*a^3*b^4*c^5 + 48*a^4*b^2*c^6 - 64*a^5*c^7 + (b^6*c
^6 - 12*a*b^4*c^7 + 48*a^2*b^2*c^8 - 64*a^3*c^9)*x^8 + 2*(b^7*c^5 - 12*a*b^
5*c^6 + 48*a^2*b^3*c^7 - 64*a^3*b*c^8)*x^6 + (b^8*c^4 - 10*a*b^6*c^5 + 24*a
^2*b^4*c^6 + 32*a^3*b^2*c^7 - 128*a^4*c^8)*x^4 + 2*(a*b^7*c^4 - 12*a^2*b^5*
c^5 + 48*a^3*b^3*c^6 - 64*a^4*b*c^7)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**11*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.35, size = 598, normalized size = 1.64

$$\frac{\left(3\,Bb^6-30\,Bab^4c-Ab^5c+90\,Ba^2b^2c^2+10\,Aab^3c^2-60\,Ba^3c^3-30\,Aa^2bc^3\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)\sqrt{-b^2+4\,ac}}+\frac{Bx^2}{2\,c^3}+\frac{9\,Bb^5c^2x^8-72\,Bab^3c^3x^8-3\,Ab^4c^3x^8+144\,Ba^2bc^4x^8+24\,Aab^2c^4x^8-48\,Aa^2c^5x^8+6\,Bb^6cx^6-48\,Bab^4c^2x^6+2\,Ab^5c^2x^6+84\,Ba^2b^2c^3x^6-12\,Aab^3c^3x^6+72\,Ba^3c^4x^6+4\,Aa^2bc^4x^6-Bb^7x^4+14\,Bab^5cx^4+3\,Ab^6cx^4-82\,Ba^2b^3c^2x^4-20\,Aab^4c^2x^4+204\,Ba^3bc^3x^4+22\,Aa^2b^2c^3x^4-32\,Aa^3c^4x^4-2\,Bab^6x^2+8\,Ba^2b^4cx^2+6\,Aab^5cx^2+4\,Ba^3b^2c^2x^2-40\,Aa^2b^3c^2x^2+56\,Ba^4c^3x^2+28\,Aa^3bc^3x^2-Ba^2b^5+3\,Aa^2b^4c+28\,Ba^4bc^2-18\,Aa^3b^2c^2}{8\,\left(b^4c^4-8\,ab^2c^5+16\,a^2c^6\right)\left(cx^4+bx^2+a\right)^2}-\frac{\left(3\,Bb-Ac\right)\log\left(cx^4+bx^2+a\right)}{4\,c^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^11*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/2*(3*B*b^6 - 30*B*a*b^4*c - A*b^5*c + 90*B*a^2*b^2*c^2 + 10*A*a*b^3*c^2 -
 60*B*a^3*c^3 - 30*A*a^2*b*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((
b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*sqrt(-b^2 + 4*a*c)) + 1/2*B*x^2/c^3 + 1
```

```
/8*(9*B*b^5*c^2*x^8 - 72*B*a*b^3*c^3*x^8 - 3*A*b^4*c^3*x^8 + 144*B*a^2*b*c^
4*x^8 + 24*A*a*b^2*c^4*x^8 - 48*A*a^2*c^5*x^8 + 6*B*b^6*c*x^6 - 48*B*a*b^4*
c^2*x^6 + 2*A*b^5*c^2*x^6 + 84*B*a^2*b^2*c^3*x^6 - 12*A*a*b^3*c^3*x^6 + 72*
B*a^3*c^4*x^6 + 4*A*a^2*b*c^4*x^6 - B*b^7*x^4 + 14*B*a*b^5*c*x^4 + 3*A*b^6*
c*x^4 - 82*B*a^2*b^3*c^2*x^4 - 20*A*a*b^4*c^2*x^4 + 204*B*a^3*b*c^3*x^4 + 2
2*A*a^2*b^2*c^3*x^4 - 32*A*a^3*c^4*x^4 - 2*B*a*b^6*x^2 + 8*B*a^2*b^4*c*x^2
+ 6*A*a*b^5*c*x^2 + 4*B*a^3*b^2*c^2*x^2 - 40*A*a^2*b^3*c^2*x^2 + 56*B*a^4*c
^3*x^2 + 28*A*a^3*b*c^3*x^2 - B*a^2*b^5 + 3*A*a^2*b^4*c + 28*B*a^4*b*c^2 -
18*A*a^3*b^2*c^2)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*(c*x^4 + b*x^2 + a)
^2) - 1/4*(3*B*b - A*c)*log(c*x^4 + b*x^2 + a)/c^4
```

Mupad [B]
time = 4.66, size = 2500, normalized size = 6.85

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^11*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^6*(18*B*a^3*c^3 - 3*B*b^6 + 2*A*b^5*c + 24*B*a*b^4*c - 15*A*a*b^3*c^2 +
 25*A*a^2*b*c^3 - 51*B*a^2*b^2*c^2))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
a*(24*A*a^3*c^3 - 5*B*a*b^5 + 3*A*a*b^4*c + 36*B*a^2*b^3*c - 58*B*a^3*b*c^2
 - 21*A*a^2*b^2*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(14*B*a^4
*c^3 - 5*B*a*b^6 + 3*A*a*b^5*c + 31*A*a^3*b*c^3 + 38*B*a^2*b^4*c - 22*A*a^2
*b^3*c^2 - 71*B*a^3*b^2*c^2))/(2*c*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x^4*(
5*B*b^7 - 32*A*a^3*c^4 - 3*A*b^6*c - 34*B*a*b^5*c + 19*A*a*b^4*c^2 + 42*B*a
^3*b*c^3 - 11*A*a^2*b^2*c^3 + 41*B*a^2*b^3*c^2))/(4*c*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)))/(a^2*c^3 + c^5*x^8 + x^4*(2*a*c^4 + b^2*c^3) + 2*b*c^4*x^6 + 2*
a*b*c^3*x^2) + (B*x^2)/(2*c^3) + (log(((a*(A*c - 3*B*b)^2)/c^6 - (((8*a*(A*
c - 3*B*b))/c^2 - (2*(2*a + b*x^2)*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B
*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*
b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/c^2 + (2*x^2*(60*B*a^3*c^3 - 9*B*
b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 - 186*B*a^
2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(A*c - 3*B*b + c^4*(-(60*B*a^3*c^3 - 3*B
*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*
b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3*B*b)*(30*
B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23*A*a^2*b*c
^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2))*((a*(A*c - 3*B*b)^2)/c^6 + (
((2*(2*a + b*x^2)*(3*B*b - A*c + c^4*(-(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c +
30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)^2/(c^8*(
4*a*c - b^2)^5))^(1/2)))/c^2 + (8*a*(A*c - 3*B*b))/c^2 + (2*x^2*(60*B*a^3*c
^3 - 9*B*b^6 + 3*A*b^5*c + 78*B*a*b^4*c - 26*A*a*b^3*c^2 + 62*A*a^2*b*c^3 -
 186*B*a^2*b^2*c^2))/(c^2*(4*a*c - b^2)^2))*(3*B*b - A*c + c^4*(-(60*B*a^3*
c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 -
90*B*a^2*b^2*c^2)^2/(c^8*(4*a*c - b^2)^5))^(1/2)))/(4*c^4) + (x^2*(A*c - 3*
```

```
B*b)*(30*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 27*B*a*b^4*c - 9*A*a*b^3*c^2 + 23*
A*a^2*b*c^3 - 69*B*a^2*b^2*c^2))/(c^6*(4*a*c - b^2)^2)))*(6*B*b^11 + 2048*A
*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 -
 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^
7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10
*c^4 + 80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8
)) - (atan(((32*a^2*c^8*(4*a*c - b^2)^5 + 2*b^4*c^6*(4*a*c - b^2)^5 - 16*a*
b^2*c^7*(4*a*c - b^2)^5)*(x^2*(((((6*A*b^5*c^5 + 120*B*a^3*c^7 - 18*B*b^6*c
^4 - 52*A*a*b^3*c^6 + 124*A*a^2*b*c^7 + 156*B*a*b^4*c^5 - 372*B*a^2*b^2*c^6
)/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7) - ((8*b^5*c^8 - 64*a*b^3*c^9 + 128*a
^2*b*c^10)*(6*B*b^11 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a
*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560
*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^
4))/(2*(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80
*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))*(60*B
*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*a^2*b*c
^3 - 90*B*a^2*b^2*c^2))/(8*c^4*(4*a*c - b^2)^(5/2)) - ((8*b^5*c^8 - 64*a*b^
3*c^9 + 128*a^2*b*c^10)*(60*B*a^3*c^3 - 3*B*b^6 + A*b^5*c + 30*B*a*b^4*c -
10*A*a*b^3*c^2 + 30*A*a^2*b*c^3 - 90*B*a^2*b^2*c^2)*(6*B*b^11 + 2048*A*a^5*
c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*
A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2
 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(16*c^4*(4*a*c - b^2)^(5/2)*(1
6*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^
5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))/(a*(4*a*c - b^
2)^2) + (b*((((6*A*b^5*c^5 + 120*B*a^3*c^7 - 18*B*b^6*c^4 - 52*A*a*b^3*c^6
+ 124*A*a^2*b*c^7 + 156*B*a*b^4*c^5 - 372*B*a^2*b^2*c^6)/(16*a^2*c^8 + b^4*
c^6 - 8*a*b^2*c^7) - ((8*b^5*c^8 - 64*a*b^3*c^9 + 128*a^2*b*c^10)*(6*B*b^11
 + 2048*A*a^5*c^6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^
5*b*c^5 - 320*A*a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960
*B*a^2*b^7*c^2 - 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(16*a^2*c^8 +
 b^4*c^6 - 8*a*b^2*c^7)*(4096*a^5*c^9 - 4*b^10*c^4 + 80*a*b^8*c^5 - 640*a^2
*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)))*(6*B*b^11 + 2048*A*a^5*c^
6 - 2*A*b^10*c - 120*B*a*b^9*c + 40*A*a*b^8*c^2 - 6144*B*a^5*b*c^5 - 320*A*
a^2*b^6*c^3 + 1280*A*a^3*b^4*c^4 - 2560*A*a^4*b^2*c^5 + 960*B*a^2*b^7*c^2 -
 3840*B*a^3*b^5*c^3 + 7680*B*a^4*b^3*c^4))/(2*(4096*a^5*c^9 - 4*b^10*c^4 +
80*a*b^8*c^5 - 640*a^2*b^6*c^6 + 2560*a^3*b^4*c^7 - 5120*a^4*b^2*c^8)) - (9
*B^2*b^7 + A^2*b^5*c^2 - 6*A*B*b^6*c + 207*B^2*a^2*b^3*c^2 + 30*A*B*a^3*c^4
 - 81*B^2*a*b^5*c - 9*A^2*a*b^3*c^3 + 23*A^2*a^2*b*c^4 - 90*B^2*a^3*b*c^3 -
 138*A*B*a^2*b^2*c^3 + 54*A*B*a*b^4*c^2)/(16*a^2*c^8 + b^4*c^6 - 8*a*b^2*c^
7) + (((b^5*c^8)/2 - 4*a*b^3*c^9 + 8*a^2*b*c^10)*(60*B*a^3*c^3 - 3*B*b^6 +
A*b^5*c + 30*B*a*b^4*c - 10*A*a*b^3*c^2 + 30*A*...
```

3.125 $\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=254

$$-\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{x^2(2a(b^3B-7abBc+6aAc^2)+(2b^4B-15ab^2Bc+6aAbc^2}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

```
[Out] -1/4*x^6*(a*(-2*A*c+B*b)+(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+
b*x^2+a)^2-1/4*x^2*(2*a*(6*A*a*c^2-7*B*a*b*c+B*b^3)+(6*A*a*b*c^2+16*B*a^2*c
^2-15*B*a*b^2*c+2*B*b^4)*x^2)/c^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/2*(-12*A
*a^2*c^3+30*B*a^2*b*c^2-10*B*a*b^3*c+B*b^5)*arctanh((2*c*x^2+b)/(-4*a*c+b^2
)^(1/2))/c^3/(-4*a*c+b^2)^(5/2)+1/4*B*ln(c*x^4+b*x^2+a)/c^3
```

Rubi [A]
time = 0.27, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {1265, 832, 648, 632, 212, 642}

$$\frac{(-12a^2Ac^3+30a^2bBc^2-10ab^3Bc+b^5B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\,(b^2-4ac)^{5/2}}-\frac{x^2(x^2(16a^2Bc^2+6aAbc^2-15ab^2Bc+2b^4B)+2a(6aAc^2-7abBc+b^3B))}{4c^2\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{x^6(x^2(-2aBc-Abc+b^2B)+a(bB-2Ac))}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{B\log(a+bx^2+cx^4)}{4c^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^6*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2))/(c*(b^2 - 4*a*
c)*(a + b*x^2 + c*x^4)^2) - (x^2*(2*a*(b^3*B - 7*a*b*B*c + 6*a*A*c^2) + (2*
b^4*B - 15*a*b^2*B*c + 6*a*A*b*c^2 + 16*a^2*B*c^2)*x^2))/(4*c^2*(b^2 - 4*a*
c)^2*(a + b*x^2 + c*x^4)) + ((b^5*B - 10*a*b^3*B*c + 30*a^2*b*B*c^2 - 12*a^
2*A*c^3)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*c^3*(b^2 - 4*a*c)^(5/
2)) + (B*Log[a + b*x^2 + c*x^4])/(4*c^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^9(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\text{Subst}\left(\int \frac{x^2(3a(bB-2Ac)+2B(b^2-4ac)x}{(a+bx+cx^2)^2}\right.}{4c\,(b^2-4ac)}$$

$$= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+}{4c^2\,(b^2-}$$

$$= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+}{4c^2\,(b^2-}$$

$$= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+}{4c^2\,(b^2-}$$

$$= -\frac{x^6(a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2)}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^2(2a(b^3B-7abBc+6aAc^2)+}{4c^2\,(b^2-}$$

Mathematica [A]
time = 0.32, size = 354, normalized size = 1.39

$$\frac{\frac{-b^6B+b^5c(A+4Bx^2)-2ab^3c^2(4A+15Bx^2)+2a^2bc^3(11A+25Bx^2)+4a^2c^3(8aB-5Acx^2)+b^4c(11aB-2Acx^2)+ab^2c^2(-39aB+16Acx^2)}{(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{2a^3Bc^2+b^4(bB-Ac)x^2+ab^2(b^2B+4Ac^2x^2-bc(A+5Bx^2))+a^2c(-4b^2B-2Ac^2x^2+bc(3A+5Bx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{2c(b^5B-10ab^3Bc+30a^2bBc^2-12a^2Ac^3)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}} + Bc\log\left(a+bx^2+cx^4\right)}{4c^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^6*B) + b^5*c*(A + 4*B*x^2) - 2*a*b^3*c^2*(4*A + 15*B*x^2) + 2*a^2*b*c
^3*(11*A + 25*B*x^2) + 4*a^2*c^3*(8*a*B - 5*A*c*x^2) + b^4*c*(11*a*B - 2*A*
c*x^2) + a*b^2*c^2*(-39*a*B + 16*A*c*x^2))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*
x^4)) + (2*a^3*B*c^2 + b^4*(b*B - A*c)*x^2 + a*b^2*(b^2*B + 4*A*c^2*x^2 - b
*c*(A + 5*B*x^2)) + a^2*c*(-4*b^2*B - 2*A*c^2*x^2 + b*c*(3*A + 5*B*x^2)))/(
(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (2*c*(b^5*B - 10*a*b^3*B*c + 30*a^2*
b*B*c^2 - 12*a^2*A*c^3)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4
*a*c)^(5/2) + B*c*Log[a + b*x^2 + c*x^4])/(4*c^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 494 vs. 2(242) = 484.
time = 0.11, size = 495, normalized size = 1.95

method	result

default	$\frac{-\frac{\left(10A\,a^2c^3-8Aa\,b^2c^2+A\,b^4c-25B\,a^2b\,c^2+15Ba\,b^3c-2B\,b^5\right)x^6}{c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{\left(2A\,a^2b\,c^3+8Aa\,b^3c^2-A\,b^5c+32B\,a^3c^3+11B\,a^2b^2c^2-19Ba\,b^4c+3B\,b^6\right)x^4}{2c^3\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a(\cdots}{}}{2(c\,x^4+b\,x^2+a)^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-1/c^2*(10*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c-25*B*a^2*b*c^2+15*B*a*b^3*c
-2*B*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(2*A*a^2*b*c^3+8*A*a*b^3*c^2-A
*b^5*c+32*B*a^3*c^3+11*B*a^2*b^2*c^2-19*B*a*b^4*c+3*B*b^6)/c^3/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^4-a*(6*A*a^2*c^3-10*A*a*b^2*c^2+A*b^4*c-31*B*a^2*b*c^2+22*
B*a*b^3*c-3*B*b^5)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(10*A*a*b*c^2
-A*b^3*c+24*B*a^2*c^2-21*B*a*b^2*c+3*B*b^4)/c^3/(16*a^2*c^2-8*a*b^2*c+b^4))
/(c*x^4+b*x^2+a)^2+1/2/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(16*B*a^2*c^2-8*
B*a*b^2*c+B*b^4)/c*ln(c*x^4+b*x^2+a)+2*(6*A*a^2*c^2-7*a^2*b*B*c+B*a*b^3-1/2
*(16*B*a^2*c^2-8*B*a*b^2*c+B*b^4)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)
/(4*a*c-b^2)^(1/2)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. 2(242) = 484.
time = 0.48, size = 2167, normalized size = 8.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^2*
b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^6
```

+ (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c^3
 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12*B
*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7 +
 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A*a
^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 - ((B*b^5*c^2 - 10*B*a*b^3*c^
3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 30*B*
a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c^3 -
 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a^3*c
^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c + 30
*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*
b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a
)) - (33*B*a^3*b^4 + A*a^2*b^5)*c + (B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*
b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2 - 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 6
4*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^
3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3
- 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 - 12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*
B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 4
8*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*
a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x
^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^
7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2
), 1/4*(3*B*a^2*b^6 + 2*(2*B*b^7*c + 40*A*a^3*c^5 - 2*(50*B*a^3*b + 21*A*a^
2*b^2)*c^4 + (85*B*a^2*b^3 + 12*A*a*b^4)*c^3 - (23*B*a*b^5 + A*b^6)*c^2)*x^
6 + (3*B*b^8 - 8*(16*B*a^4 + A*a^3*b)*c^4 - 6*(2*B*a^3*b^2 + 5*A*a^2*b^3)*c
^3 + 3*(29*B*a^2*b^4 + 4*A*a*b^5)*c^2 - (31*B*a*b^6 + A*b^7)*c)*x^4 - 8*(12
*B*a^5 + 5*A*a^4*b)*c^3 + 2*(54*B*a^4*b^2 + 7*A*a^3*b^3)*c^2 + 2*(3*B*a*b^7
 + 24*A*a^4*c^4 - 2*(62*B*a^4*b + 23*A*a^3*b^2)*c^3 + 7*(17*B*a^3*b^3 + 2*A
*a^2*b^4)*c^2 - (34*B*a^2*b^5 + A*a*b^6)*c)*x^2 + 2*((B*b^5*c^2 - 10*B*a*b^
3*c^3 + 30*B*a^2*b*c^4 - 12*A*a^2*c^5)*x^8 + B*a^2*b^5 - 10*B*a^3*b^3*c + 3
0*B*a^4*b*c^2 - 12*A*a^4*c^3 + 2*(B*b^6*c - 10*B*a*b^4*c^2 + 30*B*a^2*b^2*c
^3 - 12*A*a^2*b*c^4)*x^6 + (B*b^7 - 8*B*a*b^5*c + 10*B*a^2*b^3*c^2 - 24*A*a
^3*c^4 + 12*(5*B*a^3*b - A*a^2*b^2)*c^3)*x^4 + 2*(B*a*b^6 - 10*B*a^2*b^4*c
+ 30*B*a^3*b^2*c^2 - 12*A*a^3*b*c^3)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x
^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (33*B*a^3*b^4 + A*a^2*b^5)*c +
(B*a^2*b^6 - 12*B*a^3*b^4*c + 48*B*a^4*b^2*c^2 - 64*B*a^5*c^3 + (B*b^6*c^2
- 12*B*a*b^4*c^3 + 48*B*a^2*b^2*c^4 - 64*B*a^3*c^5)*x^8 + 2*(B*b^7*c - 12*B
*a*b^5*c^2 + 48*B*a^2*b^3*c^3 - 64*B*a^3*b*c^4)*x^6 + (B*b^8 - 10*B*a*b^6*c
 + 24*B*a^2*b^4*c^2 + 32*B*a^3*b^2*c^3 - 128*B*a^4*c^4)*x^4 + 2*(B*a*b^7 -
12*B*a^2*b^5*c + 48*B*a^3*b^3*c^2 - 64*B*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a))/(a^2*b^6*c^3 - 12*a^3*b^4*c^4 + 48*a^4*b^2*c^5 - 64*a^5*c^6 + (b^6*c^
5 - 12*a*b^4*c^6 + 48*a^2*b^2*c^7 - 64*a^3*c^8)*x^8 + 2*(b^7*c^4 - 12*a*b^5
*c^5 + 48*a^2*b^3*c^6 - 64*a^3*b*c^7)*x^6 + (b^8*c^3 - 10*a*b^6*c^4 + 24*a^
2*b^4*c^5 + 32*a^3*b^2*c^6 - 128*a^4*c^7)*x^4 + 2*(a*b^7*c^3 - 12*a^2*b^5*c
^4 + 48*a^3*b^3*c^5 - 64*a^4*b*c^6)*x^2)]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 7.58, size = 466, normalized size = 1.83

$$-\frac{\left(Bb^{5} - 10\,Bab^{3}c + 30\,Ba^{2}bc^{2} - 12\,Aa^{2}c^{3}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(b^{4}c^{3} - 8\,ab^{2}c^{4} + 16\,a^{2}c^{5}\right)\sqrt{-b^{2}+4\,ac}} + \frac{B\log\left(cx^{4}+bx^{2}+a\right)}{4\,c^{3}} - \frac{3\,Bb^{4}c^{2}x^{8} - 24\,Bab^{2}c^{3}x^{8} + 48\,Ba^{2}c^{4}x^{8} - 2\,Bb^{5}cx^{6} + 12\,Bab^{3}c^{2}x^{6} + 4\,Ab^{4}c^{2}x^{6} - 4\,Ba^{2}bc^{3}x^{6} - 32\,Aab^{2}c^{3}x^{6} + 40\,Aa^{2}c^{4}x^{6} - 3\,Bb^{6}x^{4} + 20\,Bab^{4}cx^{4} + 2\,Ab^{5}cx^{4} - 22\,Ba^{2}b^{2}c^{2}x^{4} - 16\,Aab^{3}c^{2}x^{4} + 32\,Ba^{3}c^{3}x^{4} - 4\,Aa^{2}bc^{3}x^{4} - 6\,Bab^{5}x^{2} + 40\,Ba^{2}b^{3}cx^{2} + 4\,Aab^{4}cx^{2} - 28\,Ba^{3}bc^{2}x^{2} - 40\,Aa^{2}b^{2}c^{2}x^{2} + 24\,Aa^{3}c^{3}x^{2} - 3\,Ba^{2}b^{4} + 18\,Ba^{3}b^{2}c + 2\,Aa^{2}b^{3}c - 20\,Aa^{3}bc^{2}}{8\,\left(b^{4}c^{3} - 8\,ab^{2}c^{4} + 16\,a^{2}c^{5}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*b^5 - 10*B*a*b^3*c + 30*B*a^2*b*c^2 - 12*A*a^2*c^3)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*sqrt(-b^2 +
 4*a*c)) + 1/4*B*log(c*x^4 + b*x^2 + a)/c^3 - 1/8*(3*B*b^4*c^2*x^8 - 24*B*a
*b^2*c^3*x^8 + 48*B*a^2*c^4*x^8 - 2*B*b^5*c*x^6 + 12*B*a*b^3*c^2*x^6 + 4*A*
b^4*c^2*x^6 - 4*B*a^2*b*c^3*x^6 - 32*A*a*b^2*c^3*x^6 + 40*A*a^2*c^4*x^6 - 3
*B*b^6*x^4 + 20*B*a*b^4*c*x^4 + 2*A*b^5*c*x^4 - 22*B*a^2*b^2*c^2*x^4 - 16*A
*a*b^3*c^2*x^4 + 32*B*a^3*c^3*x^4 - 4*A*a^2*b*c^3*x^4 - 6*B*a*b^5*x^2 + 40*
B*a^2*b^3*c*x^2 + 4*A*a*b^4*c*x^2 - 28*B*a^3*b*c^2*x^2 - 40*A*a^2*b^2*c^2*x
^2 + 24*A*a^3*c^3*x^2 - 3*B*a^2*b^4 + 18*B*a^3*b^2*c + 2*A*a^2*b^3*c - 20*A
*a^3*b*c^2)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B]
time = 5.15, size = 2500, normalized size = 9.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^9*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^4*(3*B*b^6 + 32*B*a^3*c^3 - A*b^5*c - 19*B*a*b^4*c + 8*A*a*b^3*c^2 + 2*
A*a^2*b*c^3 + 11*B*a^2*b^2*c^2))/(4*c^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (
x^6*(2*B*b^5 - 10*A*a^2*c^3 - A*b^4*c - 15*B*a*b^3*c + 8*A*a*b^2*c^2 + 25*B
*a^2*b*c^2))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*(24*B*a^3*c^2 + 3*
B*a*b^4 - A*a*b^3*c + 10*A*a^2*b*c^2 - 21*B*a^2*b^2*c))/(4*c^3*(b^4 + 16*a^
2*c^2 - 8*a*b^2*c)) - (x^2*(6*A*a^3*c^3 - 3*B*a*b^5 + A*a*b^4*c + 22*B*a^2*
b^3*c - 31*B*a^3*b*c^2 - 10*A*a^2*b^2*c^2))/(2*c^3*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - (log
```

(((B^2*a)/c^4 - ((B + c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*((8*B*a)/c - (2*(B + c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*(2*a + b*x^2))/c + (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B*a^2*b*c^2))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6*A*a^2*c^3 - 9*B*a*b^3*c + 23*B*a^2*b*c^2))/(c^4*(4*a*c - b^2)^2))*((B^2*a)/c^4 - ((B - c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*((8*B*a)/c - (2*(B - c^3*(-(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2/(c^6*(4*a*c - b^2)^5))^(1/2))*(2*a + b*x^2))/c + (2*x^2*(3*B*b^5 - 12*A*a^2*c^3 - 26*B*a*b^3*c + 62*B*a^2*b*c^2))/(c*(4*a*c - b^2)^2)))/(4*c^3) + (B*x^2*(B*b^5 - 6*A*a^2*c^3 - 9*B*a*b^3*c + 23*B*a^2*b*c^2))/(c^4*(4*a*c - b^2)^2)))*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) + (atan(((32*a^2*c^6*(4*a*c - b^2)^5 + 2*b^4*c^4*(4*a*c - b^2)^5 - 16*a*b^2*c^5*(4*a*c - b^2)^5)*(x^2*(((((24*A*a^2*c^6 - 6*B*b^5*c^3 + 52*B*a*b^3*c^4 - 124*B*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) - ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2))/(8*c^3*(4*a*c - b^2)^(5/2)) - ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(16*c^3*(4*a*c - b^2)^(5/2)*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))/(a*(4*a*c - b^2)^2) - (b*((((24*A*a^2*c^6 - 6*B*b^5*c^3 + 52*B*a*b^3*c^4 - 124*B*a^2*b*c^5)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) - ((8*b^5*c^6 - 64*a*b^3*c^7 + 128*a^2*b*c^8)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)))*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (B^2*b^5 - 6*A*B*a^2*c^3 - 9*B^2*a*b^3*c + 23*B^2*a^2*b*c^2)/(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5) + (((b^5*c^6)/2 - 4*a*b^3*c^7 + 8*a^2*b*c^8)*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2)/(c^6*(4*a*c - b^2)^5*(16*a^2*c^6 + b^4*c^4 - 8*a*b^2*c^5)))))/(2*a*(4*a*c - b^2)^(5/2))) - ((((8*B*a)/c + (8*a*c^2*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2))/(8*c^3*(4*a*c - b^2)^(5/2)) + (a*(B*b^5 - 12*A*a^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2

```
*b^6*c^2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(c*(4*a*c - b^2)^(5/2)
*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4
*c^6 - 5120*a^4*b^2*c^7)))/(a*(4*a*c - b^2)^2) + (b*((B^2*a)/c^4 + (((8*B*a
)/c + (8*a*c^2*(2*B*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^
2 - 1280*B*a^3*b^4*c^3 + 2560*B*a^4*b^2*c^4))/(4096*a^5*c^8 - 4*b^10*c^3 +
80*a*b^8*c^4 - 640*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7))*(2*B
*b^10 - 2048*B*a^5*c^5 - 40*B*a*b^8*c + 320*B*a^2*b^6*c^2 - 1280*B*a^3*b^4*
c^3 + 2560*B*a^4*b^2*c^4))/(2*(4096*a^5*c^8 - 4*b^10*c^3 + 80*a*b^8*c^4 - 6
40*a^2*b^6*c^5 + 2560*a^3*b^4*c^6 - 5120*a^4*b^2*c^7)) - (a*(B*b^5 - 12*A*a
^2*c^3 - 10*B*a*b^3*c + 30*B*a^2*b*c^2)^2)/(c^4*(4*a*c - b^2)^5)))/(2*a*(4*
a*c - b^2)^(5/2))))/(B^2*b^10 + 144*A^2*a^4*c^6 + 160*B^2*a^2*b^6*c^2 - 600
*B^2*a^3*b^4*c^3 + 900*B^2*a^4*b^2*c^4 - 20*B^2*a*b^8*c - 24*A*B*a^2*b^5*c^
3 + 240*A*B*a^3*b^3*c^4 - 720*A*B*a^4*b*c^5))*(B*b^5 - 12*A*a^2*c^3 - 10*B*
a*b^3*c + 30*B*a^2*b*c^2))/(2*c^3*(4*a*c - b^2)...
```

3.126 $\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=146

$$-\frac{x^6(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3(Ab-2aB)x^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] -1/4*x^6*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/4*(A
*b-2*B*a)*x^2*(b*x^2+2*a)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*a*(A*b-2*B*a)*ar
ctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.10, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 818, 736, 632, 212}

$$\frac{3a(Ab-2aB)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{3x^2(2a+bx^2)(Ab-2aB)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^6(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^6*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)^2) + (3*(A*b - 2*a*B)*x^2*(2*a + b*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2
 + c*x^4)) + (3*a*(A*b - 2*a*B)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 736

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m - 1)*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b*x
```

```
+ c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*(2*p + 3)*((c*d^2 -
b*d*e + a*e^2)/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m - 2)*(a + b*x + c
*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] &
& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
0] && LtQ[p, -1]
```

Rule 818

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(b*f - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[m*((b*(
e*f + d*g) - 2*(c*d*f + a*e*g))/((p + 1)*(b^2 - 4*a*c))), Int[(d + e*x)^(m
- 1)*(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] &
& NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m +
2*p + 3], 0] && LtQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^7(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^6(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{(3(Ab-2aB))\text{Subst}\left(\int \frac{x^2}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4(b^2-4ac)}$$

$$= -\frac{x^6(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3(Ab-2aB)x^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{(3a(Ab-2aB}{}$$

$$= -\frac{x^6(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3(Ab-2aB)x^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{(3a(Ab-2aB}{}$$

$$= -\frac{x^6(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{3(Ab-2aB)x^2(2a+bx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{3a(Ab-2aB)}{(b}$$

Mathematica [A]
time = 0.18, size = 261, normalized size = 1.79

$$\frac{1}{4}\left(\frac{b^5B-8ab^3Bc-b^4c(A+2Bx^2)-4a^2c^3(4A+5Bx^2)+ab^2c^2(5A+16Bx^2)+2abc^2(11aB-3Acx^2)}{c^3(b^2-4ac)^2(a+bx^2+cx^4)} + \frac{b^3(bB-Ac)x^2+a^2c(-3bB+2c(A+Bx^2))+ab(b^2B+3Ac^2x^2-bc(A+4Bx^2))}{c^3(-b^2+4ac)(a+bx^2+cx^4)^2} - \frac{12a(Ab-2aB)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((b^5*B - 8*a*b^3*B*c - b^4*c*(A + 2*B*x^2) - 4*a^2*c^3*(4*A + 5*B*x^2) + a
*b^2*c^2*(5*A + 16*B*x^2) + 2*a*b*c^2*(11*a*B - 3*A*c*x^2))/(c^3*(b^2 - 4*a
*c)^2*(a + b*x^2 + c*x^4)) + (b^3*(b*B - A*c)*x^2 + a^2*c*(-3*b*B + 2*c*(A
+ B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*B*x^2)))/(c^3*(-b^2 + 4*a
*c)*(a + b*x^2 + c*x^4)^2) - (12*a*(A*b - 2*a*B)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. $2(138) = 276$.
time = 0.07, size = 343, normalized size = 2.35

method	result
default	$\frac{-\frac{\left(3Aab\,c^{2}+10a^{2}B\,c^{2}-8a\,b^{2}Bc+b^{4}B\right)x^{6}}{c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(16A\,a^{2}c^{3}+Aa\,b^{2}c^{2}+A\,b^{4}c-2B\,a^{2}b\,c^{2}-8Ba\,b^{3}c+B\,b^{5}\right)x^{4}}{2c^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(5Aab\,c^{2}+A\,b^{3}c+6a^{2}B\,c^{2}-10a\,b^{2}Bc+b\right.}{\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$
risch	$\frac{-\frac{\left(3Aab\,c^{2}+10a^{2}B\,c^{2}-8a\,b^{2}Bc+b^{4}B\right)x^{6}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{\left(16A\,a^{2}c^{3}+Aa\,b^{2}c^{2}+A\,b^{4}c-2B\,a^{2}b\,c^{2}-8Ba\,b^{3}c+B\,b^{5}\right)x^{4}}{4c^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(5Aab\,c^{2}+A\,b^{3}c+6a^{2}B\,c^{2}-10a\,b^{2}Bc+b\right.}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)c^{2}}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-(3*A*a*b*c^2+10*B*a^2*c^2-8*B*a*b^2*c+B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+
b^4)*x^6-1/2*(16*A*a^2*c^3+A*a*b^2*c^2+A*b^4*c-2*B*a^2*b*c^2-8*B*a*b^3*c+B*
b^5)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-a*(5*A*a*b*c^2+A*b^3*c+6*B*a^2*c^2-
10*B*a*b^2*c+B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)/c^2*x^2-1/2*a^2/c^2*(8*A*a*c
^2+A*b^2*c-10*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-
3*a*(A*b-2*B*a)/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^
2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(140) = 280$.
time = 0.40, size = 1378, normalized size = 9.44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^4*c^2 - 4*(10*B*a^3
 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6 + (B*b^7 - 64*A*a^3
*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3 - A*a*b^4)*c^2 - (
12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*c^2 + 2*(B*a*b^6 -
4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)*c^2 - (14*B*a^2*b^
4 - A*a*b^5)*c)*x^2 + 6*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*(2*B*a^2*b - A*a*b^2
)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B*a^3 - A*a^2*b)*c^3
+ (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b)*c^2)*sqrt(b^2 - 4*
a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*
a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 1
2*a^3*b^4*c^3 + 48*a^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*
a^2*b^2*c^6 - 64*a^3*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5
- 64*a^3*b*c^6)*x^6 + (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2
*c^5 - 128*a^4*c^6)*x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 -
64*a^4*b*c^5)*x^2), -1/4*(B*a^2*b^5 - 32*A*a^4*c^3 + 2*(B*b^6*c - 12*B*a*b^
4*c^2 - 4*(10*B*a^3 + 3*A*a^2*b)*c^4 + 3*(14*B*a^2*b^2 + A*a*b^3)*c^3)*x^6
+ (B*b^7 - 64*A*a^3*c^4 + 4*(2*B*a^3*b + 3*A*a^2*b^2)*c^3 + 3*(10*B*a^2*b^3
 - A*a*b^4)*c^2 - (12*B*a*b^5 - A*b^6)*c)*x^4 + 4*(10*B*a^4*b + A*a^3*b^2)*
c^2 + 2*(B*a*b^6 - 4*(6*B*a^4 + 5*A*a^3*b)*c^3 + (46*B*a^3*b^2 + A*a^2*b^3)
*c^2 - (14*B*a^2*b^4 - A*a*b^5)*c)*x^2 + 12*((2*B*a^2 - A*a*b)*c^4*x^8 + 2*
(2*B*a^2*b - A*a*b^2)*c^3*x^6 + 2*(2*B*a^3*b - A*a^2*b^2)*c^2*x^2 + (2*(2*B
*a^3 - A*a^2*b)*c^3 + (2*B*a^2*b^2 - A*a*b^3)*c^2)*x^4 + (2*B*a^4 - A*a^3*b
)*c^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)) - (14*B*a^3*b^3 - A*a^2*b^4)*c)/(a^2*b^6*c^2 - 12*a^3*b^4*c^3 + 48*a
^4*b^2*c^4 - 64*a^5*c^5 + (b^6*c^4 - 12*a*b^4*c^5 + 48*a^2*b^2*c^6 - 64*a^3
*c^7)*x^8 + 2*(b^7*c^3 - 12*a*b^5*c^4 + 48*a^2*b^3*c^5 - 64*a^3*b*c^6)*x^6
+ (b^8*c^2 - 10*a*b^6*c^3 + 24*a^2*b^4*c^4 + 32*a^3*b^2*c^5 - 128*a^4*c^6)*
x^4 + 2*(a*b^7*c^2 - 12*a^2*b^5*c^3 + 48*a^3*b^3*c^4 - 64*a^4*b*c^5)*x^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(140) = 280.
time = 7.42, size = 318, normalized size = 2.18

$$\frac{3\,(2\,Ba^2-Aab)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{2\,Bb^4cx^6-16\,Bab^2c^2x^6+20\,Ba^2c^3x^6+6\,Aabc^3x^6+Bb^5x^4-8\,Bab^3cx^4+Ab^4cx^4-2\,Ba^2bc^2x^4+Aab^2c^2x^4+16\,Aa^2c^3x^4+2\,Bab^4x^2-20\,Ba^2b^2cx^2+2\,Aab^3cx^2+12\,Ba^3c^2x^2+10\,Aa^2bc^2x^2+Ba^2b^3-10\,Ba^3bc+Aa^2b^2c+8\,Aa^3c^2}{4\,(b^4c^2-8\,ab^2c^3+16\,a^2c^4)(cx^4+bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3*(2*B*a^2 - A*a*b)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b^
2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(2*B*b^4*c*x^6 - 16*B*a*b^2*c^2
*x^6 + 20*B*a^2*c^3*x^6 + 6*A*a*b*c^3*x^6 + B*b^5*x^4 - 8*B*a*b^3*c*x^4 + A
*b^4*c*x^4 - 2*B*a^2*b*c^2*x^4 + A*a*b^2*c^2*x^4 + 16*A*a^2*c^3*x^4 + 2*B*a
*b^4*x^2 - 20*B*a^2*b^2*c*x^2 + 2*A*a*b^3*c*x^2 + 12*B*a^3*c^2*x^2 + 10*A*a
^2*b*c^2*x^2 + B*a^2*b^3 - 10*B*a^3*b*c + A*a^2*b^2*c + 8*A*a^3*c^2)/((b^4*
c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B]
time = 0.69, size = 593, normalized size = 4.06

$$\frac{3\,a\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3\,(A\,b-2\,B\,a)\left(6\,B\,a^2\,c^2-3\,A\,a\,b\,c^2\right)}{\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{9\,a\,b\,(A\,b-2\,B\,a)^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)-\frac{18\,a^2\,b\,c^2\,(A\,b-2\,B\,a)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\left(4\,a\,c-b^2\right)^5\right)}{18\,A^2\,a^2\,b^2\,c^2-72\,A\,B\,a^3\,b\,c^2+72\,B^2\,a^4\,c^2}\right)(A\,b-2\,B\,a)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{\frac{x^4\left(-2\,B\,a^2\,b\,c^2+16\,A\,a^2\,c^3-8\,B\,a\,b^3\,c+A\,a\,b^2\,c^2+B\,b^5+A\,b^4\,c\right)}{4\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a^2\left(B\,b^3+A\,b^2\,c-10\,B\,a\,b\,c+8\,A\,a\,c^2\right)}{4\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^6\left(10\,B\,a^2\,c^2-8\,B\,a\,b^2\,c+3\,A\,a\,b\,c^2+B\,b^4\right)}{2\,c\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,x^2\left(6\,B\,a^2\,c^2-10\,B\,a\,b^2\,c+5\,A\,a\,b\,c^2+B\,b^4+A\,b^3\,c\right)}{2\,c^2\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*a*atan(((x^2*((3*(A*b - 2*B*a)*(6*B*a^2*c^2 - 3*A*a*b*c^2))/((4*a*c - b^
2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (9*a*b*(A*b - 2*B*a)^2*(2*b^5*c^
2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c))) - (18*a^2*b*c^2*(A*b - 2*B*a)^2)/(4*a*c - b^2)^(15/2))*(b^4
*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))
/(72*B^2*a^4*c^2 + 18*A^2*a^2*b^2*c^2 - 72*A*B*a^3*b*c^2))*(A*b - 2*B*a))/(
4*a*c - b^2)^(5/2) - ((x^4*(B*b^5 + 16*A*a^2*c^3 + A*b^4*c - 8*B*a*b^3*c +
A*a*b^2*c^2 - 2*B*a^2*b*c^2))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2
*(B*b^3 + 8*A*a*c^2 + A*b^2*c - 10*B*a*b*c))/(4*c^2*(b^4 + 16*a^2*c^2 - 8*a
*b^2*c)) + (x^6*(B*b^4 + 10*B*a^2*c^2 + 3*A*a*b*c^2 - 8*B*a*b^2*c))/(2*c*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a*x^2*(B*b^4 + 6*B*a^2*c^2 + A*b^3*c + 5*A
*a*b*c^2 - 10*B*a*b^2*c))/(2*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a
*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.127 $\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=185

$$-\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{a(b^2B-6Abc+8aBc)+(b^3B-4Ab^2c+2abBc+4aAc^2)x^2}{4c(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{(3abB-\cdots}{\cdots}$$

```
[Out] -1/4*x^4*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+1/4*(-
a*(-6*A*b*c+8*B*a*c+B*b^2)-(4*A*a*c^2-4*A*b^2*c+2*B*a*b*c+B*b^3)*x^2)/c/(-4
*a*c+b^2)^2/(c*x^4+b*x^2+a)+(3*a*b*B-A*(2*a*c+b^2))*arctanh((2*c*x^2+b)/(-4
*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.17, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 834, 791, 632, 212}

$$\frac{(3abB-A(2ac+b^2))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{x^4(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{a(8aBc-6Abc+b^2B)+x^2(4aAc^2+2abBc-4Ab^2c+b^3B)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)^2) - (a*(b^2*B - 6*A*b*c + 8*a*B*c) + (b^3*B - 4*A*b^2*c + 2*a*b*B*c +
 4*a*A*c^2)*x^2)/(4*c*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*a*b*B - A*
(b^2 + 2*a*c))*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
```

```
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 834

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^m*(a + b*x + c*x^2)^(p + 1)*(
(f*b - 2*a*g + (2*c*f - b*g)*x)/((p + 1)*(b^2 - 4*a*c))), x] + Dist[1/((p +
 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 1)*(a + b*x + c*x^2)^(p + 1)*Simp[g*
(2*a*e*m + b*d*(2*p + 3)) - f*(b*e*m + 2*c*d*(2*p + 3)) - e*(2*c*f - b*g)*(
m + 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 0] && (I
ntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^5(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{x(-2(Ab-2aB)-(bB-2Ac)x)}{(a+bx+cx^2)^2}\,dx, x, x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{a(b^2B-6Abc+8aBc)+(b^3B-4Ab^2c+2a}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

$$= -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{a(b^2B-6Abc+8aBc)+(b^3B-4Ab^2c+2a}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

$$= -\frac{x^4(Ab-2aB-(bB-2Ac)x^2)}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{a(b^2B-6Abc+8aBc)+(b^3B-4Ab^2c+2a}{4c\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right.}$$

Mathematica [A]

time = 0.17, size = 233, normalized size = 1.26

$$\frac{1}{4}\left(\frac{-b^4B+Ab^3c+2abc^2(A-3Bx^2)+4ac^2(-4aB+Acx^2)+b^2c(5aB+2Acx^2)}{c^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{2a^2Bc+b^2(-bB+Ac)x^2+a(-b^2B-2Ac^2x^2+bc(A+3Bx^2))}{c^2\left(-b^2+4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{4(-3abB+A(b^2+2ac))\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\left(-b^2+4ac\right)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-(b^4*B) + A*b^3*c + 2*a*b*c^2*(A - 3*B*x^2) + 4*a*c^2*(-4*a*B + A*c*x^2)
 + b^2*c*(5*a*B + 2*A*c*x^2))/(c^2*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (
2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A*c^2*x^2 + b*c*(A + 3
*B*x^2)))/(c^2*(-b^2 + 4*a*c)*(a + b*x^2 + c*x^4)^2) + (4*(-3*a*b*B + A*(b^
2 + 2*a*c))*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))
/4
```

Maple [A]
time = 0.07, size = 303, normalized size = 1.64

method	result
default	$\frac{\frac{c\left(2acA+A\,b^{2}-3abB\right)x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}+\frac{\left(6Aab\,c^{2}+3A\,b^{3}c-16a^{2}B\,c^{2}-a\,b^{2}Bc-b^{4}B\right)x^{4}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(2c^{2}aA-5A\,b^{2}c+5abBc+b^{3}B\right)x^{2}}{c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{a^{2}\left(6bcA-8acB-b^{2}B\right)}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{2\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{(2ac}{}$
risch	$\frac{\frac{c\left(2acA+A\,b^{2}-3abB\right)x^{6}}{32a^{2}c^{2}-16a\,b^{2}c+2b^{4}}+\frac{\left(6Aab\,c^{2}+3A\,b^{3}c-16a^{2}B\,c^{2}-a\,b^{2}Bc-b^{4}B\right)x^{4}}{4c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{a\left(2c^{2}aA-5A\,b^{2}c+5abBc+b^{3}B\right)x^{2}}{2c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{a^{2}\left(6bcA-8acB-b^{2}B\right)}{4c\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}-\frac{\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(c*(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*(6*A*a*b*
c^2+3*A*b^3*c-16*B*a^2*c^2-B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-8*a*b^2*c+b^4)*x^
4-a/c*(2*A*a*c^2-5*A*b^2*c+5*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+
1/2*a^2*(6*A*b*c-8*B*a*c-B*b^2)/c/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+
a)^2+(2*A*a*c+A*b^2-3*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*a
rctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. $2(177) = 354$.
time = 0.40, size = 1369, normalized size = 7.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c^3 + (3*B*a*b^
3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 + 6*(2*B*a^2*b^2
 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3*A*a^3*b)*c^2
+ 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^2 + (B*a^2*b^3
 - 5*A*a*b^4)*c)*x^2 - 2*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)*x^8 + 2*(2*A*
a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a^2*c^3 - 2*(3*
B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*A*a^2*b*c^2 -
(3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*sqrt(b^2 - 4*a*
c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*
c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12
*a^3*b^4*c^2 + 48*a^4*b^2*c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a
^2*b^2*c^5 - 64*a^3*c^6)*x^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 -
 64*a^3*b*c^5)*x^6 + (b^8*c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^
4 - 128*a^4*c^5)*x^4 + 2*(a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^
4*b*c^4)*x^2), -1/4*(B*a^2*b^4 + 2*(8*A*a^2*c^4 - 2*(6*B*a^2*b - A*a*b^2)*c
^3 + (3*B*a*b^3 - A*b^4)*c^2)*x^6 + (B*b^6 - 8*(8*B*a^3 - 3*A*a^2*b)*c^3 +
6*(2*B*a^2*b^2 + A*a*b^3)*c^2 - 3*(B*a*b^4 + A*b^5)*c)*x^4 - 8*(4*B*a^4 - 3
*A*a^3*b)*c^2 + 2*(B*a*b^5 - 8*A*a^3*c^3 - 2*(10*B*a^3*b - 11*A*a^2*b^2)*c^
2 + (B*a^2*b^3 - 5*A*a*b^4)*c)*x^2 + 4*((2*A*a*c^4 - (3*B*a*b - A*b^2)*c^3)
*x^8 + 2*(2*A*a*b*c^3 - (3*B*a*b^2 - A*b^3)*c^2)*x^6 + 2*A*a^3*c^2 + (4*A*a
^2*c^3 - 2*(3*B*a^2*b - 2*A*a*b^2)*c^2 - (3*B*a*b^3 - A*b^4)*c)*x^4 + 2*(2*
A*a^2*b*c^2 - (3*B*a^2*b^2 - A*a*b^3)*c)*x^2 - (3*B*a^3*b - A*a^2*b^2)*c)*s
qrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) +
 2*(2*B*a^3*b^2 - 3*A*a^2*b^3)*c)/(a^2*b^6*c - 12*a^3*b^4*c^2 + 48*a^4*b^2*
c^3 - 64*a^5*c^4 + (b^6*c^3 - 12*a*b^4*c^4 + 48*a^2*b^2*c^5 - 64*a^3*c^6)*x
^8 + 2*(b^7*c^2 - 12*a*b^5*c^3 + 48*a^2*b^3*c^4 - 64*a^3*b*c^5)*x^6 + (b^8*
c - 10*a*b^6*c^2 + 24*a^2*b^4*c^3 + 32*a^3*b^2*c^4 - 128*a^4*c^5)*x^4 + 2*(
a*b^7*c - 12*a^2*b^5*c^2 + 48*a^3*b^3*c^3 - 64*a^4*b*c^4)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.39, size = 268, normalized size = 1.45

$$-\frac{(3\,Bab-Ab^2-2\,Aac)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{6\,Babc^2x^6-2\,Ab^2c^2x^6-4\,Aac^3x^6+Bb^4x^4+Bab^2cx^4-3\,Ab^3cx^4+16\,Ba^2c^2x^4-6\,Aabc^2x^4+2\,Bab^3x^2+10\,Ba^2bcx^2-10\,Aab^2cx^2+4\,Aa^2c^2x^2+Ba^2b^2+8\,Ba^3c-6\,Aa^2bc}{4\,(b^4c-8\,ab^2c^2+16\,a^2c^3)(cx^4+bx^2+a)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -(3*B*a*b - A*b^2 - 2*A*a*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
 - 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*a*b*c^2*x^6 - 2*A
*b^2*c^2*x^6 - 4*A*a*c^3*x^6 + B*b^4*x^4 + B*a*b^2*c*x^4 - 3*A*b^3*c*x^4 +
16*B*a^2*c^2*x^4 - 6*A*a*b*c^2*x^4 + 2*B*a*b^3*x^2 + 10*B*a^2*b*c*x^2 - 10*
A*a*b^2*c*x^2 + 4*A*a^2*c^2*x^2 + B*a^2*b^2 + 8*B*a^3*c - 6*A*a^2*b*c)/((b^
4*c - 8*a*b^2*c^2 + 16*a^2*c^3)*(c*x^4 + b*x^2 + a)^2)
```

Mupad [B]
time = 0.68, size = 625, normalized size = 3.38

$$\frac{\mathrm{atan}\left(\frac{\left(x^2\left(\frac{\left(A\,b^2\,c^2-3\,B\,a\,b\,c^2+2\,A\,a\,c^3\right)\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)}{a\,{\left(4\,a\,c-b^2\right)}^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\,{\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)}^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\,{\left(4\,a\,c-b^2\right)}^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{2\,b\,c^2\,{\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)}^2}{{\left(4\,a\,c-b^2\right)}^{15/2}}\right)\left(b^4\,{\left(4\,a\,c-b^2\right)}^5+16\,a^2\,c^2\,{\left(4\,a\,c-b^2\right)}^5-8\,a\,b^2\,c\,{\left(4\,a\,c-b^2\right)}^5\right)}{8\,A^2\,a^2\,c^4+8\,A^2\,a\,b^2\,c^3+2\,A^2\,b^4\,c^2-24\,A\,B\,a^2\,b\,c^3-12\,A\,B\,a\,b^3\,c^2+18\,B^2\,a^2\,b^2\,c^2}\right)\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{\frac{x^4\,\left(16\,B\,a^2\,c^2+B\,a\,b^2\,c-6\,A\,a\,b\,c^2+B\,b^4-3\,A\,b^3\,c\right)}{4\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{c\,x^6\,\left(A\,b^2-3\,B\,a\,b+2\,A\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{a\,\left(8\,B\,c\,a^2+B\,a\,b^2-6\,A\,c\,a\,b\right)}{4\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(5\,B\,a^2\,b\,c+2\,A\,a^2\,c^2+B\,a\,b^3-5\,A\,a\,b^2\,c\right)}{2\,c\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan(((x^2*(((A*b^2*c^2 + 2*A*a*c^3 - 3*B*a*b*c^2)*(A*b^2 + 2*A*a*c - 3*B*
a*b))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(A*b^2 +
2*A*a*c - 3*B*a*b)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c
 - b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(A*b^2 + 2*A*a*c
 - 3*B*a*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a
*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(8*A^2*a^2*c^4 + 2*A^2*b^4*c^2 +
18*B^2*a^2*b^2*c^2 + 8*A^2*a*b^2*c^3 - 12*A*B*a*b^3*c^2 - 24*A*B*a^2*b*c^3)
)*(A*b^2 + 2*A*a*c - 3*B*a*b))/(4*a*c - b^2)^(5/2) - ((x^4*(B*b^4 + 16*B*a^
2*c^2 - 3*A*b^3*c - 6*A*a*b*c^2 + B*a*b^2*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) - (c*x^6*(A*b^2 + 2*A*a*c - 3*B*a*b))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^
2*c)) + (a*(B*a*b^2 + 8*B*a^2*c - 6*A*a*b*c))/(4*c*(b^4 + 16*a^2*c^2 - 8*a*
b^2*c)) + (x^2*(2*A*a^2*c^2 + B*a*b^3 - 5*A*a*b^2*c + 5*B*a^2*b*c))/(2*c*(b
^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x
^2 + 2*b*c*x^6)
```

3.128 $\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3} dx$

Optimal. Leaf size=170

$$-\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)x^2}{4c(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{(b^2B-3Abc+2aBc)(b+2cx^2)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{(b^2B-3Abc+2aBc)\tanh^{-1}}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/4*(-a*(-2*A*c+B*b)-(-A*b*c-2*B*a*c+B*b^2)*x^2)/c/(-4*a*c+b^2)/(c*x^4+b*x^
2+a)^2+1/4*(-3*A*b*c+2*B*a*c+B*b^2)*(2*c*x^2+b)/c/(-4*a*c+b^2)^2/(c*x^4+b*x
^2+a)-(-3*A*b*c+2*B*a*c+B*b^2)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*
a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.12, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 791, 628, 632, 212}

$$-\frac{(2aBc-3Abc+b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}+\frac{(b+2cx^2)(2aBc-3Abc+b^2B)}{4c(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{x^2(-2aBc-Abc+b^2B)+a(bB-2Ac)}{4c(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(a*(b*B - 2*A*c) + (b^2*B - A*b*c - 2*a*B*c)*x^2)/(c*(b^2 - 4*a*c)*(a
+ b*x^2 + c*x^4)^2) + ((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(4*c*(b^2
 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - ((b^2*B - 3*A*b*c + 2*a*B*c)*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{(a+bx+cx^2)^3}\,dx, x, x^2\right)$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{(b^2B-3Abc+2aBc)\,\text{Subst}\left(\int \frac{1}{(a+bx+c\cdots}\right.}{4c\,(b^2-4ac)}$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \cdots$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \cdots$$

$$= -\frac{a(bB-2Ac)+(b^2B-Abc-2aBc)\,x^2}{4c\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{4c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} - \cdots$$

Mathematica [A]
time = 0.14, size = 172, normalized size = 1.01

$$\frac{1}{4}\left(\frac{(b^2B-3Abc+2aBc)\,(b+2cx^2)}{c\,(b^2-4ac)^2\,(a+bx^2+cx^4)} + \frac{abB+b(bB-Ac)x^2-2ac(A+Bx^2)}{c\,(-b^2+4ac)\,(a+bx^2+cx^4)^2} + \frac{4(b^2B-3Abc+2aBc)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{(-b^2+4ac)^{5/2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (((b^2*B - 3*A*b*c + 2*a*B*c)*(b + 2*c*x^2))/(c*(b^2 - 4*a*c)^2*(a + b*x^2
 + c*x^4)) + (a*b*B + b*(b*B - A*c)*x^2 - 2*a*c*(A + B*x^2))/(c*(-b^2 + 4*a*
c)*(a + b*x^2 + c*x^4)^2) + (4*(b^2*B - 3*A*b*c + 2*a*B*c)*ArcTan[(b + 2*c*
x^2)/Sqrt[-b^2 + 4*a*c]])/(-b^2 + 4*a*c)^(5/2))/4
```

Maple [A]
time = 0.07, size = 273, normalized size = 1.61

method	result
default	$\frac{-\frac{c\left(3bcA-2acB-b^2B\right)x^6}{16a^2c^2-8a\,b^2c+b^4}-\frac{3b\left(3bcA-2acB-b^2B\right)x^4}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(5Aabc+A\,b^3+2a^2cB-5Ba\,b^2\right)x^2}{16a^2c^2-8a\,b^2c+b^4}-\frac{a\left(8acA+A\,b^2-6abB\right)}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}}{2\left(c\,x^4+b\,x^2+a\right)^2}-\frac{\left(3bcA-2acB-b^2B\right)a}{(16a^2c^2-8a\,b^2c-}$
risch	$\frac{-\frac{c\left(3bcA-2acB-b^2B\right)x^6}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{3b\left(3bcA-2acB-b^2B\right)x^4}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{\left(5Aabc+A\,b^3+2a^2cB-5Ba\,b^2\right)x^2}{2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(8acA+A\,b^2-6abB\right)}{4\left(16a^2c^2-8a\,b^2c+b^4\right)}}{\left(c\,x^4+b\,x^2+a\right)^2}-\frac{3\ln\left(\left(-\left(-4ac+b^2\right)^{\frac{5}{2}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(-c*(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6-3/2*b*(3*A*b
*c-2*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4-(5*A*a*b*c+A*b^3+2*B*a^2*c
-5*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a*(8*A*a*c+A*b^2-6*B*a*b)/(1
6*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2-(3*A*b*c-2*B*a*c-B*b^2)/(16*a^2
*c^2-8*a*b^2*c+b^4)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. 2(162) = 324.
time = 0.39, size = 1226, normalized size = 7.21

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^2)*x
^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3*A*a
*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*B*a^
3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 2*((B*b^2*c^2 + (2*B
*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^2*b^
2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 + 2*(
B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqrt(b^
2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^
2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - 4*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 -
 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c +
 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 6
4*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 12
8*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x
^2), 1/4*(2*(B*b^4*c - 4*(2*B*a^2 - 3*A*a*b)*c^3 - (2*B*a*b^2 + 3*A*b^3)*c^
2)*x^6 + 6*B*a^2*b^3 - A*a*b^4 + 32*A*a^3*c^2 + 3*(B*b^5 - 4*(2*B*a^2*b - 3
*A*a*b^2)*c^2 - (2*B*a*b^3 + 3*A*b^4)*c)*x^4 + 2*(5*B*a*b^4 - A*b^5 + 4*(2*
B*a^3 + 5*A*a^2*b)*c^2 - (22*B*a^2*b^2 + A*a*b^3)*c)*x^2 - 4*((B*b^2*c^2 +
(2*B*a - 3*A*b)*c^3)*x^8 + 2*(B*b^3*c + (2*B*a*b - 3*A*b^2)*c^2)*x^6 + B*a^
2*b^2 + (B*b^4 + 2*(2*B*a^2 - 3*A*a*b)*c^2 + (4*B*a*b^2 - 3*A*b^3)*c)*x^4 +
 2*(B*a*b^3 + (2*B*a^2*b - 3*A*a*b^2)*c)*x^2 + (2*B*a^3 - 3*A*a^2*b)*c)*sqr
t(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - 4
*(6*B*a^3*b + A*a^2*b^2)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*
a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^
7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c
 + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b
^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 789 vs. $2(165) = 330$.
time = 204.27, size = 789, normalized size = 4.64

$$-\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(-3Abc+2Bac+Bb^2)\log\left(x^2+\frac{-3Ab^2c+2Babc+Bb^3-64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-3Abc+2Bac+Bb^2)+48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(-3Abc+2Bac+Bb^2)-12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-3Abc+2Bac+Bb^2)+b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}(-3Abc+2Bac+Bb^2)}{-6Abc^2+4Bac^2+2Bb^2c}\right)}{2}+\frac{\sqrt{-\frac{1}{(4ac-b^2)^5}}\,(-3Abc+2Bac+Bb^2)\log\left(x^2+\frac{-3Ab^2c+2Babc+Bb^3+64a^3c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-3Abc+2Bac+Bb^2)-48a^2b^2c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(-3Abc+2Bac+Bb^2)+12ab^4c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-3Abc+2Bac+Bb^2)-b^6\sqrt{-\frac{1}{(4ac-b^2)^5}}(-3Abc+2Bac+Bb^2)}{-6Abc^2+4Bac^2+2Bb^2c}\right)}{2}+\frac{-8Aa^2c-Aab^2+6Ba^2b+x^6(-6Abc^2+4Bac^2+2Bb^2c)+x^4(-9Ab^2c+6Babc+3Bb^3)+x^2(-10Aabc-2Ab^3-4Ba^2c+10Bab^2)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6\cdot(128a^2bc^3-64ab^3c^2+8b^5c)+x^4\cdot(128a^3c^3-24ab^4c+4b^6)+x^2\cdot(128a^3bc^2-64a^2b^3c+8ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] -sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*
b**2*c + 2*B*a*b*c + B*b**3 - 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-3*A
*b*c + 2*B*a*c + B*b**2) + 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-3
*A*b*c + 2*B*a*c + B*b**2) - 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b
*c + 2*B*a*c + B*b**2) + b**6*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*
c + B*b**2))/(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + sqrt(-1/(4*a*c -
```

```
b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2)*log(x**2 + (-3*A*b**2*c + 2*B*a*b*c
 + B*b**3 + 64*a**3*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B
*b**2) - 48*a**2*b**2*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c +
 B*b**2) + 12*a*b**4*c*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b
**2) - b**6*sqrt(-1/(4*a*c - b**2)**5)*(-3*A*b*c + 2*B*a*c + B*b**2))/(-6*A
*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c))/2 + (-8*A*a**2*c - A*a*b**2 + 6*B*a**2*
b + x**6*(-6*A*b*c**2 + 4*B*a*c**2 + 2*B*b**2*c) + x**4*(-9*A*b**2*c + 6*B*
a*b*c + 3*B*b**3) + x**2*(-10*A*a*b*c - 2*A*b**3 - 4*B*a**2*c + 10*B*a*b**2
))/(64*a**4*c**2 - 32*a**3*b**2*c + 4*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a
*b**2*c**3 + 4*b**4*c**2) + x**6*(128*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5
*c) + x**4*(128*a**3*c**3 - 24*a*b**4*c + 4*b**6) + x**2*(128*a**3*b*c**2 -
 64*a**2*b**3*c + 8*a*b**5))
```

Giac [A]
time = 7.24, size = 228, normalized size = 1.34

$$\frac{(Bb^2+2\,Bac-3\,Abc)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}+\frac{2\,Bb^2cx^6+4\,Bac^2x^6-6\,Abc^2x^6+3\,Bb^3x^4+6\,Babcx^4-9\,Ab^2cx^4+10\,Bab^2x^2-2\,Ab^3x^2-4\,Ba^2cx^2-10\,Aabcx^2+6\,Ba^2b-Aab^2-8\,Aa^2c}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] (B*b^2 + 2*B*a*c - 3*A*b*c)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4
- 8*a*b^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) + 1/4*(2*B*b^2*c*x^6 + 4*B*a*
c^2*x^6 - 6*A*b*c^2*x^6 + 3*B*b^3*x^4 + 6*B*a*b*c*x^4 - 9*A*b^2*c*x^4 + 10*
B*a*b^2*x^2 - 2*A*b^3*x^2 - 4*B*a^2*c*x^2 - 10*A*a*b*c*x^2 + 6*B*a^2*b - A*
a*b^2 - 8*A*a^2*c)/((c*x^4 + b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

Mupad [B]
time = 0.66, size = 587, normalized size = 3.45

$$\frac{\mathrm{atan}\left(\frac{\left(x^2\left(\frac{\left(B\,b^2\,c^2-3\,A\,b\,c^3+2\,B\,a\,c^3\right)\left(B\,b^2-3\,A\,c\,b+2\,B\,a\,c\right)}{a\,{\left(4\,a\,c-b^2\right)}^{9/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{b\,{\left(B\,b^2-3\,A\,c\,b+2\,B\,a\,c\right)}^2\,\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\,{\left(4\,a\,c-b^2\right)}^{15/2}\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{2\,b\,c^2\,{\left(B\,b^2-3\,A\,c\,b+2\,B\,a\,c\right)}^2}{{\left(4\,a\,c-b^2\right)}^{15/2}}\right)\left(b^4\,{\left(4\,a\,c-b^2\right)}^5+16\,a^2\,c^2\,{\left(4\,a\,c-b^2\right)}^5-8\,a\,b^2\,c\,{\left(4\,a\,c-b^2\right)}^5\right)}{18\,A^2\,b^2\,c^4-24\,A\,B\,a\,b\,c^4-12\,A\,B\,b^3\,c^3+8\,B^2\,a^2\,c^4+8\,B^2\,a\,b^2\,c^3+2\,B^2\,b^4\,c^2}\right)\left(B\,b^2-3\,A\,c\,b+2\,B\,a\,c\right)}{{\left(4\,a\,c-b^2\right)}^{5/2}}-\frac{\frac{-6\,B\,a^2\,b+8\,A\,c\,a^2+A\,a\,b^2}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{x^2\,\left(2\,B\,c\,a^2-5\,B\,a\,b^2+5\,A\,c\,a\,b+A\,b^3\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{3\,b\,x^4\,\left(B\,b^2-3\,A\,c\,b+2\,B\,a\,c\right)}{4\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}-\frac{c\,x^6\,\left(B\,b^2-3\,A\,c\,b+2\,B\,a\,c\right)}{2\,\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}}{x^4\,\left(b^2+2\,a\,c\right)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (atan(((x^2*(((B*b^2*c^2 - 3*A*b*c^3 + 2*B*a*c^3)*(B*b^2 - 3*A*b*c + 2*B*a*
c))/(a*(4*a*c - b^2)^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (b*(B*b^2 - 3*
A*b*c + 2*B*a*c)^2*(2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c -
 b^2)^(15/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c))) + (2*b*c^2*(B*b^2 - 3*A*b*c +
 2*B*a*c)^2)/(4*a*c - b^2)^(15/2))*(b^4*(4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c
 - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(18*A^2*b^2*c^4 + 8*B^2*a^2*c^4 + 2
*B^2*b^4*c^2 - 12*A*B*b^3*c^3 + 8*B^2*a*b^2*c^3 - 24*A*B*a*b*c^4))*(B*b^2 -
 3*A*b*c + 2*B*a*c))/(4*a*c - b^2)^(5/2) - ((A*a*b^2 + 8*A*a^2*c - 6*B*a^2*
b)/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^3 - 5*B*a*b^2 + 2*B*a^2*c
 + 5*A*a*b*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (3*b*x^4*(B*b^2 - 3*A*b
```

```
*c + 2*B*a*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (c*x^6*(B*b^2 - 3*A*b*c
 + 2*B*a*c))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 +
 c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.129 $\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=139

$$-\frac{Ab-2aB-(bB-2Ac)x^2}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{3(bB-2Ac)(b+2cx^2)}{4(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}$$

```
[Out] 1/4*(-A*b+2*a*B+(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-3/4*(-2*A*
c+B*b)*(2*c*x^2+b)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+3*c*(-2*A*c+B*b)*arctanh(
(2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^(5/2)
```

Rubi [A]
time = 0.09, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1261, 652, 628, 632, 212}

$$\frac{3c(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{(b^2-4ac)^{5/2}}-\frac{3(b+2cx^2)(bB-2Ac)}{4(b^2-4ac)^2(a+bx^2+cx^4)}-\frac{-2aB-(x^2(bB-2Ac))+Ab}{4(b^2-4ac)(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(A*b - 2*a*B - (b*B - 2*A*c)*x^2)/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2
) - (3*(b*B - 2*A*c)*(b + 2*c*x^2))/(4*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4))
 + (3*c*(b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(b^2 - 4*a*
c)^(5/2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c))), x] - Dist[2*c*((2*p +
 3)/((p + 1)*(b^2 - 4*a*c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2] && Int
egerQ[4*p]
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{(a+bx+cx^2)^3}\,dx,x,x^2\right)$$

$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{(3(bB-2Ac))\text{Subst}\left(\int \frac{1}{(a+bx+cx^2)^2}\,dx,x,x^2\right)}{4\left(b^2-4ac\right)}$$

$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} - \frac{(3c(bB-2Ac))\text{S}\cdots}{2\cdots}$$

$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{(3c(bB-2Ac))\text{S}\cdots}{\cdots}$$

$$= -\frac{Ab-2aB-(bB-2Ac)x^2}{4\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{3(bB-2Ac)\left(b+2cx^2\right)}{4\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)} + \frac{3c(bB-2Ac)\tan\cdots}{(b^2-\cdots}$$

Mathematica [A]
time = 0.09, size = 142, normalized size = 1.02

$$\frac{-\frac{3(bB-2Ac)(b+2cx^2)}{a+bx^2+cx^4} + \frac{(b^2-4ac)\left(B\left(2a+bx^2\right)-A\left(b+2cx^2\right)\right)}{\left(a+bx^2+cx^4\right)^2} - \frac{12c(bB-2Ac)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{\sqrt{-b^2+4ac}}}{4\left(b^2-4ac\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-3*(b*B - 2*A*c)*(b + 2*c*x^2))/(a + b*x^2 + c*x^4) + ((b^2 - 4*a*c)*(B*(
2*a + b*x^2) - A*(b + 2*c*x^2)))/(a + b*x^2 + c*x^4)^2 - (12*c*(b*B - 2*A*c
)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])/Sqrt[-b^2 + 4*a*c])/(4*(b^2 - 4
*a*c)^2)
```

Maple [A]
time = 0.08, size = 147, normalized size = 1.06

method	result
default	$\frac{(2Ac-bB)x^2+Ab-2aB}{4(4ac-b^2)(cx^4+bx^2+a)^2}+\frac{3(2Ac-bB)\left(\frac{2cx^2+b}{(4ac-b^2)(cx^4+bx^2+a)}+\frac{4c\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{(4ac-b^2)^{\frac{3}{2}}}\right)}{4(4ac-b^2)}$
risch	$\frac{\frac{3c^2(2Ac-bB)x^6}{2(16a^2c^2-8a\,b^2c+b^4)}+\frac{9bc(2Ac-bB)x^4}{4(16a^2c^2-8a\,b^2c+b^4)}+\frac{(5ac+b^2)(2Ac-bB)x^2}{32a^2c^2-16a\,b^2c+2b^4}+\frac{10Aabc-A\,b^3-8a^2cB-Ba\,b^2}{64a^2c^2-32a\,b^2c+4b^4}}{(c\,x^4+b\,x^2+a)^2}-3c^2\ln\left(\left(\left(-4ac+b^2\right)^{\frac{5}{2}}-16a^2b\,c\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*((2*A*c-B*b)*x^2+A*b-2*a*B)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^2+3/4*(2*A*c-B*
b)/(4*a*c-b^2)*((2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)+4*c/(4*a*c-b^2)^(3/
2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(131) = 262$.
time = 0.39, size = 1109, normalized size = 7.98

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 + A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 + 6*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2), -1/4*(6*(B*b^3*c^2 + 8*A*a*c^4 - 2*(2*B*a*b + A*b^2)*c^3)*x^6 + B*a*b^4 + A*b^5 + 9*(B*b^4*c + 8*A*a*b*c^3 - 2*(2*B*a*b^2 + A*b^3)*c^2)*x^4 - 8*(4*B*a^3 - 5*A*a^2*b)*c^2 + 2*(B*b^5 + 40*A*a^2*c^3 - 2*(10*B*a^2*b + A*a*b^2)*c^2 + (B*a*b^3 - 2*A*b^4)*c)*x^2 - 12*((B*b*c^3 - 2*A*c^4)*x^8 + 2*(B*b^2*c^2 - 2*A*b*c^3)*x^6 + B*a^2*b*c - 2*A*a^2*c^2 + (B*b^3*c - 4*A*a*c^3 + 2*(B*a*b - A*b^2)*c^2)*x^4 + 2*(B*a*b^2*c - 2*A*a*b*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) + 2*(2*B*a^2*b^2 - 7*A*a*b^3)*c)/((b^6*c^2 - 12*a*b^4*c^3 + 48*a^2*b^2*c^4 - 64*a^3*c^5)*x^8 + a^2*b^6 - 12*a^3*b^4*c + 48*a^4*b^2*c^2 - 64*a^5*c^3 + 2*(b^7*c - 12*a*b^5*c^2 + 48*a^2*b^3*c^3 - 64*a^3*b*c^4)*x^6 + (b^8 - 10*a*b^6*c + 24*a^2*b^4*c^2 + 32*a^3*b^2*c^3 - 128*a^4*c^4)*x^4 + 2*(a*b^7 - 12*a^2*b^5*c + 48*a^3*b^3*c^2 - 64*a^4*b*c^3)*x^2)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(134) = 268$.

time = 163.74, size = 661, normalized size = 4.76

$$\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c-192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)+144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)-36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)+3b^6c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)}{-12Ac^3+6Bbc^2}\right)}{2}-\frac{3c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)\log\left(x^2+\frac{-6Abc^2+3Bb^2c+192a^3c^4\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)-144a^2b^2c^3\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)+36ab^4c^2\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)-3b^6c\sqrt{-\frac{1}{(4ac-b^2)^5}}(-2Ac+Bb)}{-12Ac^3+6Bbc^2}\right)}{2}+\frac{10Aabc-Ab^3-8Ba^2c-Bab^2+x^6\cdot(12Ac^3-6Bbc^2)+x^4\cdot(18Abc^2-9Bb^2c)+x^2\cdot(20Aac^2+4Ab^2c-10Babc-2Bb^3)}{64a^4c^2-32a^3b^2c+4a^2b^4+x^8\cdot(64a^2c^4-32ab^2c^3+4b^4c^2)+x^6\cdot(128a^2bc^3-64ab^3c^2+8b^5c)+x^4\cdot(128a^3c^3-24ab^4c+4b^6)+x^2\cdot(128a^3bc^2-64a^2b^3c+8ab^5)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)

[Out] 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c - 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 - 3*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b)*log(x**2 + (-6*A*b*c**2 + 3*B*b**2*c + 192*a**3*c**4*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 144*a**2*b**2*c**3*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) + 36*a*b**4*c**2*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b) - 3*b**6*c*sqrt(-1/(4*a*c - b**2)**5)*(-2*A*c + B*b))/(-12*A*c**3 + 6*B*b*c**2))/2 + (10*A*a*b*c - A*b**3 - 8*B*a**2*c - B*a*b**2 + x**6*(1

```
2*A*c**3 - 6*B*b*c**2) + x**4*(18*A*b*c**2 - 9*B*b**2*c) + x**2*(20*A*a*c**
2 + 4*A*b**2*c - 10*B*a*b*c - 2*B*b**3))/(64*a**4*c**2 - 32*a**3*b**2*c + 4
*a**2*b**4 + x**8*(64*a**2*c**4 - 32*a*b**2*c**3 + 4*b**4*c**2) + x**6*(128
*a**2*b*c**3 - 64*a*b**3*c**2 + 8*b**5*c) + x**4*(128*a**3*c**3 - 24*a*b**4
*c + 4*b**6) + x**2*(128*a**3*b*c**2 - 64*a**2*b**3*c + 8*a*b**5))
```

Giac [A]
time = 5.90, size = 208, normalized size = 1.50

$$-\frac{3\,(Bbc-2\,Ac^2)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{(b^4-8\,ab^2c+16\,a^2c^2)\sqrt{-b^2+4\,ac}}-\frac{6\,Bbc^2x^6-12\,Ac^3x^6+9\,Bb^2cx^4-18\,Abc^2x^4+2\,Bb^3x^2+10\,Babcx^2-4\,Ab^2cx^2-20\,Aac^2x^2+Bab^2+Ab^3+8\,Ba^2c-10\,Aabc}{4\,(cx^4+bx^2+a)^2(b^4-8\,ab^2c+16\,a^2c^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -3*(B*b*c - 2*A*c^2)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((b^4 - 8*a*b
^2*c + 16*a^2*c^2)*sqrt(-b^2 + 4*a*c)) - 1/4*(6*B*b*c^2*x^6 - 12*A*c^3*x^6
+ 9*B*b^2*c*x^4 - 18*A*b*c^2*x^4 + 2*B*b^3*x^2 + 10*B*a*b*c*x^2 - 4*A*b^2*c
*x^2 - 20*A*a*c^2*x^2 + B*a*b^2 + A*b^3 + 8*B*a^2*c - 10*A*a*b*c)/((c*x^4 +
 b*x^2 + a)^2*(b^4 - 8*a*b^2*c + 16*a^2*c^2))
```

Mupad [B]
time = 0.59, size = 517, normalized size = 3.72

$$\frac{3\,c\,\mathrm{atan}\left(\frac{\left(x^2\left(\frac{3\,c\,(2\,A\,c-B\,b)\left(6\,A\,c^4-3\,B\,b\,c^3\right)}{a\left(4\,a\,c-b^2\right)^{9/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}+\frac{9\,b\,c^2\,(2\,A\,c-B\,b)^2\left(32\,a^2\,b\,c^4-16\,a\,b^3\,c^3+2\,b^5\,c^2\right)}{2\,a\left(4\,a\,c-b^2\right)^{15/2}\left(16\,a^2\,c^2-8\,a\,b^2\,c+b^4\right)}\right)+\frac{18\,b\,c^4\,(2\,A\,c-B\,b)^2}{\left(4\,a\,c-b^2\right)^{15/2}}\right)\left(b^4\left(4\,a\,c-b^2\right)^5+16\,a^2\,c^2\left(4\,a\,c-b^2\right)^5-8\,a\,b^2\,c\left(4\,a\,c-b^2\right)^5\right)}{72\,A^2\,c^6-72\,A\,B\,b\,c^5+18\,B^2\,b^2\,c^4}\right)(2\,A\,c-B\,b)}{\left(4\,a\,c-b^2\right)^{5/2}}-\frac{\frac{8\,B\,c\,a^2+B\,a\,b^2-10\,A\,c\,a\,b+A\,b^3}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{9\,x^4\,(2\,A\,b\,c^2-B\,b^2\,c)}{4\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}+\frac{x^2\,(B\,b^3-2\,A\,b^2\,c+5\,B\,a\,b\,c-10\,A\,a\,c^2)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}-\frac{3\,c^2\,x^6\,(2\,A\,c-B\,b)}{2\,(16\,a^2\,c^2-8\,a\,b^2\,c+b^4)}}{x^4\,(b^2+2\,a\,c)+a^2+c^2\,x^8+2\,a\,b\,x^2+2\,b\,c\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] (3*c*atan(((x^2*((3*c*(2*A*c - B*b)*(6*A*c^4 - 3*B*b*c^3))/(a*(4*a*c - b^2)
^(9/2)*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (9*b*c^2*(2*A*c - B*b)^2*(2*b^5*c^
2 - 16*a*b^3*c^3 + 32*a^2*b*c^4))/(2*a*(4*a*c - b^2)^(15/2)*(b^4 + 16*a^2*c
^2 - 8*a*b^2*c))) + (18*b*c^4*(2*A*c - B*b)^2)/(4*a*c - b^2)^(15/2))*(b^4*(
4*a*c - b^2)^5 + 16*a^2*c^2*(4*a*c - b^2)^5 - 8*a*b^2*c*(4*a*c - b^2)^5))/(
72*A^2*c^6 + 18*B^2*b^2*c^4 - 72*A*B*b*c^5))*(2*A*c - B*b))/(4*a*c - b^2)^(
5/2) - ((A*b^3 + B*a*b^2 + 8*B*a^2*c - 10*A*a*b*c)/(4*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)) - (9*x^4*(2*A*b*c^2 - B*b^2*c))/(4*(b^4 + 16*a^2*c^2 - 8*a*b^2*c
)) + (x^2*(B*b^3 - 10*A*a*c^2 - 2*A*b^2*c + 5*B*a*b*c))/(2*(b^4 + 16*a^2*c^
2 - 8*a*b^2*c)) - (3*c^2*x^6*(2*A*c - B*b))/(2*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6)
```

3.130 $\int \frac{A+Bx^2}{x(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=252

$$-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{6a^2bBc+A(2b^4-15ab^2c+16a^2c^2)+2c(6a^2Bc+A(b^3-7abc))\,x^2}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/4*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)^2+1/4*(6*a^2*b*B*c+A*(16*a^2*c^2-15*a*b^2*c+2*b^4)+2*c*(6*a^2*B*c+A*(-7*a
*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)-1/2*(12*a^3*B*c^2-A*(30*
a^2*b*c^2-10*a*b^3*c+b^5))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^3/(-4*
a*c+b^2)^(5/2)+A*ln(x)/a^3-1/4*A*ln(c*x^4+b*x^2+a)/a^3
```

Rubi [A]
time = 0.36, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1265, 836, 814, 648, 632, 212, 642}

$$-\frac{A\log\left(a+bx^2+cx^4\right)}{4a^3}+\frac{A\log(x)}{a^3}+\frac{2cx^2(6a^2Bc+A(b^3-7abc))+A(16a^2c^2-15ab^2c+2b^4)+6a^2bBc}{4a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}-\frac{(12a^3Bc^2-A(30a^2bc^2-10ab^3c+b^5))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\left(b^2-4ac\right)^{5/2}}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] -1/4*(a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(a*(b^2 - 4*a*c)*(a +
b*x^2 + c*x^4)^2) + (6*a^2*b*B*c + A*(2*b^4 - 15*a*b^2*c + 16*a^2*c^2) + 2*
c*(6*a^2*B*c + A*(b^3 - 7*a*b*c))*x^2)/(4*a^2*(b^2 - 4*a*c)^2*(a + b*x^2 +
c*x^4)) - ((12*a^3*B*c^2 - A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2))*ArcTanh[(b
+ 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^3*(b^2 - 4*a*c)^(5/2)) + (A*Log[x])/a^3
 - (A*Log[a + b*x^2 + c*x^4])/(4*a^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-2A(b^2-4ac)-3(Ab-2aB)cx}{x(a+bx+cx^2)^2}\,dx, x,\right.}{4a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A(2b^4-15ab^2c+16a^2c^2)+}{4a^2\left(b^2-4ac\right)^2(a+}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A(2b^4-15ab^2c+16a^2c^2)+}{4a^2\left(b^2-4ac\right)^2(a+}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A(2b^4-15ab^2c+16a^2c^2)+}{4a^2\left(b^2-4ac\right)^2(a+}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A(2b^4-15ab^2c+16a^2c^2)+}{4a^2\left(b^2-4ac\right)^2(a+}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A(2b^4-15ab^2c+16a^2c^2)+}{4a^2\left(b^2-4ac\right)^2(a+}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2} + \frac{6a^2bBc+A(2b^4-15ab^2c+16a^2c^2)+}{4a^2\left(b^2-4ac\right)^2(a+}$$

Mathematica [A]
time = 0.43, size = 396, normalized size = 1.57

$$\frac{\frac{a^2(-aB(b+2cx^2)+A(b^2-2ac+bcx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{a(2Ab^3(b+cx^2)-aAbc(15b+14cx^2)+2a^2c(3bB+8Ac+6Bcx^2))}{(b^2-4ac)^2(a+bx^2+cx^4)}+4A\log(x)-\frac{\left(-12a^3Bc^2+A\left(b^5-10ab^3c+30a^2bc^2+b^4\sqrt{b^2-4ac}-8ab^2c\sqrt{b^2-4ac}+16a^2c^2\sqrt{b^2-4ac}\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}-\frac{\left(12a^3Bc^2+A\left(-b^5+10ab^3c-30a^2bc^2+b^4\sqrt{b^2-4ac}-8ab^2c\sqrt{b^2-4ac}+16a^2c^2\sqrt{b^2-4ac}\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{(b^2-4ac)^{5/2}}}{4a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((a^2*(-(a*B*(b + 2*c*x^2)) + A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (a*(2*A*b^3*(b + c*x^2) - a*A*b*c*(15*b + 14*c*x^2)
+ 2*a^2*c*(3*b*B + 8*A*c + 6*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4
)) + 4*A*Log[x] - ((-12*a^3*B*c^2 + A*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^
4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4
*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2) - ((12*a^
3*B*c^2 + A*(-b^5 + 10*a*b^3*c - 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a
*b^2*c*Sqrt[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c]))*Log[b + Sqrt[b^2
- 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^3)
```

Maple [A]
time = 0.10, size = 442, normalized size = 1.75

method	result
default	$-\frac{\frac{c^{2}a\left(7Aabc-A\,b^{3}-6a^{2}cB\right)x^{6}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{ca\left(16A\,a^{2}c^{2}-29Aa\,b^{2}c+4A\,b^{4}+18a^{2}bBc\right)x^{4}}{2\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{a\left(A\,a^{2}b\,c^{2}+6Aa\,b^{3}c-A\,b^{5}-10a^{3}B\,c^{2}-2B\,a^{2}b^{2}c\right)x^{2}}{16a^{2}c^{2}-8a\,b^{2}c+b^{4}}-\frac{a^{2}\left(24A\,a\right.}{}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$
risch	$\frac{-\frac{c^{2}\left(7Aabc-A\,b^{3}-6a^{2}cB\right)x^{6}}{2a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{c\left(16A\,a^{2}c^{2}-29Aa\,b^{2}c+4A\,b^{4}+18a^{2}bBc\right)x^{4}}{4\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)a^{2}}-\frac{\left(A\,a^{2}b\,c^{2}+6Aa\,b^{3}c-A\,b^{5}-10a^{3}B\,c^{2}-2B\,a^{2}b^{2}c\right)x^{2}}{2a^{2}\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{24A\,a^{2}c^{2}-21}{4a}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*((c^2*a*(7*A*a*b*c-A*b^3-6*B*a^2*c)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^6
-1/2*c*a*(16*A*a^2*c^2-29*A*a*b^2*c+4*A*b^4+18*B*a^2*b*c)/(16*a^2*c^2-8*a*b
^2*c+b^4)*x^4+a*(A*a^2*b*c^2+6*A*a*b^3*c-A*b^5-10*B*a^3*c^2-2*B*a^2*b^2*c)/
(16*a^2*c^2-8*a*b^2*c+b^4)*x^2-1/2*a^2*(24*A*a^2*c^2-21*A*a*b^2*c+3*A*b^4+1
0*B*a^2*b*c-B*a*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4))/(c*x^4+b*x^2+a)^2+1/(16*a^
2*c^2-8*a*b^2*c+b^4)*(1/2*(16*A*a^2*c^3-8*A*a*b^2*c^2+A*b^4*c)/c*ln(c*x^4+b
*x^2+a)+2*(23*A*a^2*b*c^2-9*A*a*b^3*c+A*b^5-6*a^3*B*c^2-1/2*(16*A*a^2*c^3-8
*A*a*b^2*c^2+A*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)
^(1/2))))+A*ln(x)/a^3
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. $2(240) = 480$.
time = 2.08, size = 2494, normalized size = 9.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")

[Out] [-1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4
 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c -
64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2
*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a
^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a
^2*b^5)*c)*x^2 - ((A*b^5*c^2 - 10*A*a*b^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4
)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a
^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 - 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3
*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*
c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*s
qrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*s
qrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c +
(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2
- 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A
*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c
 + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 -
12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2
+ a) - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A
*b^6*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7
*c - 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*
A*a*b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A
*a*b^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(
a^5*b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^
4*b^4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^
2 + 48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b
^4*c^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48
*a^6*b^3*c^2 - 64*a^7*b*c^3)*x^2), -1/4*(B*a^3*b^5 - 3*A*a^2*b^6 + 96*A*a^5
*c^3 - 2*(A*a*b^5*c^2 - 4*(6*B*a^4 - 7*A*a^3*b)*c^4 + (6*B*a^3*b^2 - 11*A*a
^2*b^3)*c^3)*x^6 - (4*A*a*b^6*c - 64*A*a^4*c^4 - 12*(6*B*a^4*b - 11*A*a^3*b
^2)*c^3 + 9*(2*B*a^3*b^3 - 5*A*a^2*b^4)*c^2)*x^4 + 4*(10*B*a^5*b - 27*A*a^4
*b^2)*c^2 - 2*(A*a*b^7 - 4*(10*B*a^5 - A*a^4*b)*c^3 + (2*B*a^4*b^2 + 23*A*a
^3*b^3)*c^2 + 2*(B*a^3*b^4 - 5*A*a^2*b^5)*c)*x^2 - 2*((A*b^5*c^2 - 10*A*a*b
^3*c^3 - 6*(2*B*a^3 - 5*A*a^2*b)*c^4)*x^8 + A*a^2*b^5 - 10*A*a^3*b^3*c + 2*
(A*b^6*c - 10*A*a*b^4*c^2 - 6*(2*B*a^3*b - 5*A*a^2*b^2)*c^3)*x^6 + (A*b^7 -
 8*A*a*b^5*c - 12*(2*B*a^4 - 5*A*a^3*b)*c^3 - 2*(6*B*a^3*b^2 - 5*A*a^2*b^3)
*c^2)*x^4 - 6*(2*B*a^5 - 5*A*a^4*b)*c^2 + 2*(A*a*b^6 - 10*A*a^2*b^4*c - 6*(
2*B*a^4*b - 5*A*a^3*b^2)*c^2)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)
*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - (14*B*a^4*b^3 - 33*A*a^3*b^4)*c + (A*a
^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6*c^2 - 12
*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c - 12*A*a*b
^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*b^6*c + 2
4*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b^7 - 12*A
*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(c*x^4 + b*x^2 + a)
```

```
 - 4*(A*a^2*b^6 - 12*A*a^3*b^4*c + 48*A*a^4*b^2*c^2 - 64*A*a^5*c^3 + (A*b^6
*c^2 - 12*A*a*b^4*c^3 + 48*A*a^2*b^2*c^4 - 64*A*a^3*c^5)*x^8 + 2*(A*b^7*c -
 12*A*a*b^5*c^2 + 48*A*a^2*b^3*c^3 - 64*A*a^3*b*c^4)*x^6 + (A*b^8 - 10*A*a*
b^6*c + 24*A*a^2*b^4*c^2 + 32*A*a^3*b^2*c^3 - 128*A*a^4*c^4)*x^4 + 2*(A*a*b
^7 - 12*A*a^2*b^5*c + 48*A*a^3*b^3*c^2 - 64*A*a^4*b*c^3)*x^2)*log(x))/(a^5*
b^6 - 12*a^6*b^4*c + 48*a^7*b^2*c^2 - 64*a^8*c^3 + (a^3*b^6*c^2 - 12*a^4*b^
4*c^3 + 48*a^5*b^2*c^4 - 64*a^6*c^5)*x^8 + 2*(a^3*b^7*c - 12*a^4*b^5*c^2 +
48*a^5*b^3*c^3 - 64*a^6*b*c^4)*x^6 + (a^3*b^8 - 10*a^4*b^6*c + 24*a^5*b^4*c
^2 + 32*a^6*b^2*c^3 - 128*a^7*c^4)*x^4 + 2*(a^4*b^7 - 12*a^5*b^5*c + 48*a^6
*b^3*c^2 - 64*a^7*b*c^3)*x^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.67, size = 421, normalized size = 1.67

$$-\frac{\left(Ab^{5}-10\,Aab^{3}c-12\,Ba^{3}c^{2}+30\,Aa^{2}bc^{2}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(a^{3}b^{4}-8\,a^{4}b^{2}c+16\,a^{5}c^{2}\right)\sqrt{-b^{2}+4\,ac}}-\frac{A\log\left(cx^{4}+bx^{2}+a\right)}{4\,a^{3}}+\frac{A\log\left(x^{2}\right)}{2\,a^{3}}+\frac{3\,Ab^{4}c^{2}x^{8}-24\,Aab^{2}c^{3}x^{8}+48\,Aa^{2}c^{4}x^{8}+6\,Ab^{5}cx^{6}-44\,Aab^{3}c^{2}x^{6}+24\,Ba^{3}c^{3}x^{6}+68\,Aa^{2}bc^{3}x^{6}+3\,Ab^{6}x^{4}-10\,Aab^{4}cx^{4}+36\,Ba^{3}bc^{2}x^{4}-58\,Aa^{2}b^{2}c^{2}x^{4}+128\,Aa^{3}c^{3}x^{4}+10\,Aab^{5}x^{2}+8\,Ba^{3}b^{2}cx^{2}-72\,Aa^{2}b^{3}cx^{2}+40\,Ba^{4}c^{2}x^{2}+92\,Aa^{3}bc^{2}x^{2}-2\,Ba^{3}b^{3}+9\,Aa^{2}b^{4}+20\,Ba^{4}bc-66\,Aa^{3}b^{2}c+96\,Aa^{4}c^{2}}{8\,\left(a^{3}b^{4}-8\,a^{4}b^{2}c+16\,a^{5}c^{2}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(A*b^5 - 10*A*a*b^3*c - 12*B*a^3*c^2 + 30*A*a^2*b*c^2)*arctan((2*c*x^2
 + b)/sqrt(-b^2 + 4*a*c))/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*sqrt(-b^2 +
 4*a*c)) - 1/4*A*log(c*x^4 + b*x^2 + a)/a^3 + 1/2*A*log(x^2)/a^3 + 1/8*(3*A
*b^4*c^2*x^8 - 24*A*a*b^2*c^3*x^8 + 48*A*a^2*c^4*x^8 + 6*A*b^5*c*x^6 - 44*A
*a*b^3*c^2*x^6 + 24*B*a^3*c^3*x^6 + 68*A*a^2*b*c^3*x^6 + 3*A*b^6*x^4 - 10*A
*a*b^4*c*x^4 + 36*B*a^3*b*c^2*x^4 - 58*A*a^2*b^2*c^2*x^4 + 128*A*a^3*c^3*x^
4 + 10*A*a*b^5*x^2 + 8*B*a^3*b^2*c*x^2 - 72*A*a^2*b^3*c*x^2 + 40*B*a^4*c^2*
x^2 + 92*A*a^3*b*c^2*x^2 - 2*B*a^3*b^3 + 9*A*a^2*b^4 + 20*B*a^4*b*c - 66*A*
a^3*b^2*c + 96*A*a^4*c^2)/((a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(c*x^4 + b*
x^2 + a)^2)
```

Mupad [**B**]
time = 11.57, size = 2500, normalized size = 9.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^3),x)

[Out] ((3*A*b^4 + 24*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 10*B*a^2*b*c)/(4*a*(b^4
 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^2*(A*b^5 + 10*B*a^3*c^2 - 6*A*a*b^3*c - A*
a^2*b*c^2 + 2*B*a^2*b^2*c))/(2*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(
16*A*a^2*c^3 + 4*A*b^4*c - 29*A*a*b^2*c^2 + 18*B*a^2*b*c^2))/(4*a^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^6*(A*b^3 + 6*B*a^2*c - 7*A*a*b*c))/(2*a^2
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*
b*x^2 + 2*b*c*x^6) + (A*log(x))/a^3 - (log(((c^5*x^2*(A*b^3 + 6*B*a^2*c - 7
*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10
*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4*A^2*b
^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A^2*a*
b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(a^4*(
4*a*c - b^2)^4) - ((A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a
^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^3*c^2
 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^2) +
(4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4*a*c
- b^2)^2) + (b*c^2*(A + a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 30*A*a
^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a
^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4*c^3
- 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3*b^2*c
^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^3 + 6
*B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4))*((c^5*x^2*(A*b^3 + 6*B*a^2*
c - 7*A*a*b*c)^3)/(a^6*(4*a*c - b^2)^6) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2
 - 10*A*a*b^3*c + 30*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((c^3*(4*
A^2*b^8 - 36*B^2*a^5*c^3 + 302*A^2*a^2*b^4*c^2 - 497*A^2*a^3*b^2*c^3 - 61*A
^2*a*b^6*c - 204*A*B*a^3*b^3*c^2 + 24*A*B*a^2*b^5*c + 468*A*B*a^4*b*c^3))/(
a^4*(4*a*c - b^2)^4) - ((A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3
0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*((2*c^3*x^2*(A*b^5 + 60*B*a^
3*c^2 - 2*A*a*b^3*c + 10*A*a^2*b*c^2 - 24*B*a^2*b^2*c))/(a^2*(4*a*c - b^2)^
2) + (4*b*c^2*(A*b^5 - 6*B*a^3*c^2 - 9*A*a*b^3*c + 23*A*a^2*b*c^2))/(a^2*(4
*a*c - b^2)^2) + (b*c^2*(A - a^3*(-(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*c + 3
0*A*a^2*b*c^2)^2/(a^6*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^
2))/a^3))/(4*a^3) + (c^4*x^2*(6*A^2*b^7 + 409*A^2*a^2*b^3*c^2 + 480*A*B*a^4
*c^3 - 89*A^2*a*b^5*c - 560*A^2*a^3*b*c^3 + 36*B^2*a^4*b*c^2 - 324*A*B*a^3*
b^2*c^2 + 42*A*B*a^2*b^4*c))/(a^4*(4*a*c - b^2)^4)))/(4*a^3) + (A*c^4*(A*b^
3 + 6*B*a^2*c - 7*A*a*b*c)^2)/(a^6*(4*a*c - b^2)^4)))*(2*A*b^10 - 2048*A*a^
5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*
b^2*c^4))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 -
2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)) - (atan((x^2*((((((((30720*B*a^11*c^9
 + 5120*A*a^10*b*c^9 + 2*A*a^4*b^13*c^3 - 36*A*a^5*b^11*c^4 + 276*A*a^6*b^9
*c^5 - 1216*A*a^7*b^7*c^6 + 3456*A*a^8*b^5*c^7 - 6144*A*a^9*b^3*c^8 - 48*B*
a^6*b^10*c^4 + 888*B*a^7*b^8*c^5 - 6528*B*a^8*b^6*c^6 + 23808*B*a^9*b^4*c^7
 - 43008*B*a^10*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^
8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5) - ((2
```

```
*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^
4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*
b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*b^9*c^5 - 97280*a^10*b^7*c^6 + 227
328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/(2*(4*a^3*b^10 - 4096*a^8*c^5 - 80
*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^
12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3
840*a^10*b^4*c^4 - 6144*a^11*b^2*c^5)))*(A*b^5 - 12*B*a^3*c^2 - 10*A*a*b^3*
c + 30*A*a^2*b*c^2))/(4*a^3*(4*a*c - b^2)^(5/2)) - ((A*b^5 - 12*B*a^3*c^2 -
 10*A*a*b^3*c + 30*A*a^2*b*c^2)*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c +
 320*A*a^2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4)*(163840*a^13*
b*c^9 - 12*a^6*b^15*c^2 + 328*a^7*b^13*c^3 - 3840*a^8*b^11*c^4 + 24960*a^9*
b^9*c^5 - 97280*a^10*b^7*c^6 + 227328*a^11*b^5*c^7 - 294912*a^12*b^3*c^8))/
(8*a^3*(4*a*c - b^2)^(5/2)*(4*a^3*b^10 - 4096*a^8*c^5 - 80*a^4*b^8*c + 640*
a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2*c^4)*(a^6*b^12 + 4096*a^12*c^
6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4
- 6144*a^11*b^2*c^5)))*(2*A*b^10 - 2048*A*a^5*c^5 - 40*A*a*b^8*c + 320*A*a^
2*b^6*c^2 - 1280*A*a^3*b^4*c^3 + 2560*A*a^4*b^2*c^4))/(2*(4*a^3*b^10 - 4096
*a^8*c^5 - 80*a^4*b^8*c + 640*a^5*b^6*c^2 - 2560*a^6*b^4*c^3 + 5120*a^7*b^2
*c^4)) - (((6*A^2*a^2*b^11*c^4 - 137*A^2*a^3*b^9*c^5 + 1217*A^2*a^4*b^7*c^6
 - 5256*A^2*a^5*b^5*c^7 + 11024*A^2*a^6*b^3*c^8 + 36*B^2*a^6*b^5*c^6 - 288*
B^2*a^7*b^3*c^7 + 7680*A*B*a^8*c^9 - 8960*A^2*a^7*b*c^9 + 576*B^2*a^8*b*c^8
 + 42*A*B*a^4*b^8*c^5 - 660*A*B*a^5*b^6*c^6 + 3744*A*B*a^6*b^4*c^7 - 9024*A
*B*a^7*b^2*c^8)/(a^6*b^12 + 4096*a^12*c^6 - 24*a^7*b^10*c + 240*a^8*b^8*c^2
 - 1280*a^9*b^6*c^3 + 3840*a^10*b^4*c^4 - 6144*...
```

3.131 $\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3} \, dx$

Optimal. Leaf size=363

$$\frac{abB(b^2-7ac)-3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2}-\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}-\frac{abB(b^2-10ac)-A(3b^4$$

```
[Out] 1/2*(a*b*B*(-7*a*c+b^2)-3*A*(10*a^2*c^2-7*a*b^2*c+b^4))/a^3/(-4*a*c+b^2)^2/
x^2+1/4*(-a*b*B+A*(-2*a*c+b^2)+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/x^2/(c*x^4
+b*x^2+a)^2+1/4*(-a*b*B*(-10*a*c+b^2)+A*(20*a^2*c^2-20*a*b^2*c+3*b^4)-c*(a*
B*(-16*a*c+b^2)-3*A*(-6*a*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/x^2/(c*x^4+b*x^
2+a)+1/2*(a*b*B*(30*a^2*c^2-10*a*b^2*c+b^4)-3*A*(-20*a^3*c^3+30*a^2*b^2*c^2
-10*a*b^4*c+b^6))*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/a^4/(-4*a*c+b^2)^
(5/2)-(3*A*b-B*a)*ln(x)/a^4+1/4*(3*A*b-B*a)*ln(c*x^4+b*x^2+a)/a^4
```

Rubi [A]

time = 0.49, antiderivative size = 363, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1265, 836, 814, 648, 632, 212, 642}

$$\frac{(3Ab-aB)\log\left(a+bx^2+cx^4\right)}{4a^4}-\frac{\log(x)(3Ab-aB)}{a^4}-\frac{-A(20a^2c^2-20ab^2c+3b^4)+cx^2(aB(b^2-16ac)-3A(b^3-6abc))+abB(b^2-10ac)}{4a^2x^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{abB(b^2-7ac)-3A(10a^2c^2-7ab^2c+b^4)}{2a^3x^2\left(b^2-4ac\right)^2}+\frac{(abB(30a^2c^2-10ab^2c+b^4)-3A(-20a^3c^3+30a^2b^2c^2-10ab^4c+b^6))\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^4\left(b^2-4ac\right)^{5/2}}-\frac{-A(b^2-2ac)-(cx^2(Ab-2aB))+abB}{4ax^2\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] (a*b*B*(b^2 - 7*a*c) - 3*A*(b^4 - 7*a*b^2*c + 10*a^2*c^2))/(2*a^3*(b^2 - 4*
a*c)^2*x^2) - (a*b*B - A*(b^2 - 2*a*c) - (A*b - 2*a*B)*c*x^2)/(4*a*(b^2 - 4
*a*c)*x^2*(a + b*x^2 + c*x^4)^2) - (a*b*B*(b^2 - 10*a*c) - A*(3*b^4 - 20*a*
b^2*c + 20*a^2*c^2) + c*(a*B*(b^2 - 16*a*c) - 3*A*(b^3 - 6*a*b*c))*x^2)/(4*
a^2*(b^2 - 4*a*c)^2*x^2*(a + b*x^2 + c*x^4)) + ((a*b*B*(b^4 - 10*a*b^2*c +
30*a^2*c^2) - 3*A*(b^6 - 10*a*b^4*c + 30*a^2*b^2*c^2 - 20*a^3*c^3))*ArcTanh
[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^4*(b^2 - 4*a*c)^(5/2)) - ((3*A*b -
a*B)*Log[x])/a^4 + ((3*A*b - a*B)*Log[a + b*x^2 + c*x^4])/(4*a^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\left(a+bx^2+cx^4\right)^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\left(a+bx+cx^2\right)^3}\,dx, x, x^2\right)$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{\text{Subst}\left(\int \frac{-3Ab^2+abB+10aAc-4(Ab-2aB)cx}{x^2(a+bx+cx^2)^2}\right.}{4a\left(b^2-4ac\right)}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB(b^2-10ac)-A(3b^4-20ab^2c+2}{4a^2\left(b^2-4a\right.}$$

$$= -\frac{abB-A(b^2-2ac)-(Ab-2aB)cx^2}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2} - \frac{abB(b^2-10ac)-A(3b^4-20ab^2c+2}{4a^2\left(b^2-4a\right.}$$

$$= \frac{abB(b^2-7ac)-3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)c}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}$$

$$= \frac{abB(b^2-7ac)-3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)c}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}$$

$$= \frac{abB(b^2-7ac)-3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)c}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}$$

$$= \frac{abB(b^2-7ac)-3A(b^4-7ab^2c+10a^2c^2)}{2a^3\left(b^2-4ac\right)^2x^2} - \frac{abB-A(b^2-2ac)-(Ab-2aB)c}{4a\left(b^2-4ac\right)x^2\left(a+bx^2+cx^4\right)^2}$$

Mathematica [A]
time = 0.94, size = 642, normalized size = 1.77

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x]
```

```
[Out] ((-2*a*A)/x^2 - (a^2*(a*B*(-b^2 + 2*a*c - b*c*x^2) + A*(b^3 - 3*a*b*c + b^2
*c*x^2 - 2*a*c^2*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (a*(a*B*(2*
b^4 - 15*a*b^2*c + 16*a^2*c^2 + 2*b^3*c*x^2 - 14*a*b*c^2*x^2) - A*(4*b^5 -
29*a*b^3*c + 46*a^2*b*c^2 + 4*b^4*c*x^2 - 26*a*b^2*c^2*x^2 + 28*a^2*c^3*x^2
)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + 4*(-3*A*b + a*B)*Log[x] + ((-(a
*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 + b^4*Sqrt[b^2 - 4*a*c] - 8*a*b^2*c*Sqr
t[b^2 - 4*a*c] + 16*a^2*c^2*Sqrt[b^2 - 4*a*c])) + 3*A*(b^6 - 10*a*b^4*c + 3
0*a^2*b^2*c^2 - 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*a*c] - 8*a*b^3*c*Sqrt[b^2 - 4
*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^
2])/(b^2 - 4*a*c)^(5/2) + ((a*B*(b^5 - 10*a*b^3*c + 30*a^2*b*c^2 - b^4*Sqrt
```

```
[b^2 - 4*a*c] + 8*a*b^2*c*Sqrt[b^2 - 4*a*c] - 16*a^2*c^2*Sqrt[b^2 - 4*a*c])
 + 3*A*(-b^6 + 10*a*b^4*c - 30*a^2*b^2*c^2 + 20*a^3*c^3 + b^5*Sqrt[b^2 - 4*
a*c] - 8*a*b^3*c*Sqrt[b^2 - 4*a*c] + 16*a^2*b*c^2*Sqrt[b^2 - 4*a*c]))*Log[b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(b^2 - 4*a*c)^(5/2))/(4*a^4)
```

Maple [A]
time = 0.11, size = 616, normalized size = 1.70

method	result
default	$-\frac{\frac{c^{2} a \left(14 A \,a^{2} c^{2}-13 A a \,b^{2} c +2 A \,b^{4}+7 a^{2} b B c -B a \,b^{3}\right) x^{6}}{16 a^{2} c^{2}-8 a \,b^{2} c +b^{4}}+\frac{c a \left(74 A \,a^{2} b \,c^{2}-55 A a \,b^{3} c +8 A \,b^{5}-16 a^{3} B \,c^{2}+29 B \,a^{2} b^{2} c -4 B a \,b^{4}\right) x^{4}}{32 a^{2} c^{2}-16 a \,b^{2} c +2 b^{4}}+\frac{a \left(18 A \,a^{3} c^{3}+7 A \,a \cdots\right.}{\cdots}}{\left(c \,x^{4}+b \,x^{2}+a \right)^{2}} \cdots$
risch	$\frac{-\frac{c^{2} \left(30 A \,a^{2} c^{2}-21 A a \,b^{2} c +3 A \,b^{4}+7 a^{2} b B c -B a \,b^{3}\right) x^{8}}{2 a^{3} \left(16 a^{2} c^{2}-8 a \,b^{2} c +b^{4}\right)}-\frac{c \left(138 A \,a^{2} b \,c^{2}-87 A a \,b^{3} c +12 A \,b^{5}-16 a^{3} B \,c^{2}+29 B \,a^{2} b^{2} c -4 B a \,b^{4}\right) x^{6}}{4 \left(16 a^{2} c^{2}-8 a \,b^{2} c +b^{4}\right) a^{3}}-\frac{\left(50 A \,a^{3} c^{3}+7 A \,a^{2} \cdots\right.}{\cdots}}{x^{2} \left(c \,x^{4}+b \,x^{2}+a \right) \cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^4*((c^2*a*(14*A*a^2*c^2-13*A*a*b^2*c+2*A*b^4+7*B*a^2*b*c-B*a*b^3)/(1
6*a^2*c^2-8*a*b^2*c+b^4)*x^6+1/2*c*a*(74*A*a^2*b*c^2-55*A*a*b^3*c+8*A*b^5-1
6*B*a^3*c^2+29*B*a^2*b^2*c-4*B*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^4+a*(18*
A*a^3*c^3+7*A*a^2*b^2*c^2-12*A*a*b^4*c+2*A*b^6+B*a^3*b*c^2+6*B*a^2*b^3*c-B*
a*b^5)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^2+1/2*a^2*(58*A*a^2*b*c^2-36*A*a*b^3*c+
5*A*b^5-24*B*a^3*c^2+21*B*a^2*b^2*c-3*B*a*b^4)/(16*a^2*c^2-8*a*b^2*c+b^4))/
(c*x^4+b*x^2+a)^2+1/(16*a^2*c^2-8*a*b^2*c+b^4)*(1/2*(-48*A*a^2*b*c^3+24*A*a
*b^3*c^2-3*A*b^5*c+16*B*a^3*c^3-8*B*a^2*b^2*c^2+B*a*b^4*c)/c*ln(c*x^4+b*x^2
+a)+2*(30*A*a^3*c^3-69*A*a^2*b^2*c^2+27*A*a*b^4*c-3*A*b^6+23*B*a^3*b*c^2-9*
B*a^2*b^3*c+B*a*b^5-1/2*(-48*A*a^2*b*c^3+24*A*a*b^3*c^2-3*A*b^5*c+16*B*a^3*
c^3-8*B*a^2*b^2*c^2+B*a*b^4*c)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4
*a*c-b^2)^(1/2))))-1/2/a^3*A/x^2+(-3*A*b+B*a)/a^4*ln(x)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1966 vs. $2(346) = 692$.
time = 4.30, size = 3956, normalized size = 10.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*A*a^6*c^3 - 2*
(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^3*b^3 - 3*A*a^
2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 - 69*A*a^4*b)*c
^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2
 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 + 200*A*a^5*
c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A*a^3*b^4)*c^2
- 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^2*b^7 - 8*(12*
B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^2 - (33*B*a^4*
b^4 - 104*A*a^3*b^5)*c)*x^2 - ((60*A*a^3*c^5 + 30*(B*a^3*b - 3*A*a^2*b^2)*c
^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c^2)*x^10 + 2*(60
*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2*b^4 - 3*A*a*b^5
)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 120*A*a^4*c^4 + 6
0*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4)*c^2 - 8*(B*a^2
*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*a^4*b*c^3 + 30*(
B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)*c)*x^4 + (B*a^3
*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b^2)*c^2 - 10*(B*
a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2
 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((
64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b
^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*
a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6
)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*
a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5
)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(
B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b
^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b
)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2 - 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*
x^2)*log(c*x^4 + b*x^2 + a) + 4*((64*(B*a^4 - 3*A*a^3*b)*c^5 - 48*(B*a^3*b^
2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B*a*b^6 - 3*A*b^7)
*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3*b^3 - 3*A*a^2*b^
4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b^8)*c)*x^8 - (B*a
*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b^2 - 3*A*a^3*b^3)
*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6
 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c^3 + 48*(B*a^4*b^
3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (B*a^3*b^6 - 3
*A*a^2*b^7 - 64*(B*a^6 - 3*A*a^5*b)*c^3 + 48*(B*a^5*b^2 - 3*A*a^4*b^3)*c^2
```

```
- 12*(B*a^4*b^4 - 3*A*a^3*b^5)*c)*x^2)*log(x))/((a^4*b^6*c^2 - 12*a^5*b^4*c
^3 + 48*a^6*b^2*c^4 - 64*a^7*c^5)*x^10 + 2*(a^4*b^7*c - 12*a^5*b^5*c^2 + 48
*a^6*b^3*c^3 - 64*a^7*b*c^4)*x^8 + (a^4*b^8 - 10*a^5*b^6*c + 24*a^6*b^4*c^2
 + 32*a^7*b^2*c^3 - 128*a^8*c^4)*x^6 + 2*(a^5*b^7 - 12*a^6*b^5*c + 48*a^7*b
^3*c^2 - 64*a^8*b*c^3)*x^4 + (a^6*b^6 - 12*a^7*b^4*c + 48*a^8*b^2*c^2 - 64*
a^9*c^3)*x^2), -1/4*(2*A*a^3*b^6 - 24*A*a^4*b^4*c + 96*A*a^5*b^2*c^2 - 128*
A*a^6*c^3 - 2*(120*A*a^4*c^5 + 2*(14*B*a^4*b - 57*A*a^3*b^2)*c^4 - 11*(B*a^
3*b^3 - 3*A*a^2*b^4)*c^3 + (B*a^2*b^5 - 3*A*a*b^6)*c^2)*x^8 + (8*(8*B*a^5 -
 69*A*a^4*b)*c^4 - 6*(22*B*a^4*b^2 - 81*A*a^3*b^3)*c^3 + 45*(B*a^3*b^4 - 3*
A*a^2*b^5)*c^2 - 4*(B*a^2*b^6 - 3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^
8 + 200*A*a^5*c^4 + 2*(2*B*a^5*b - 11*A*a^4*b^2)*c^3 + (23*B*a^4*b^3 - 79*A
*a^3*b^4)*c^2 - 10*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4 - (3*B*a^3*b^6 - 9*A*a^
2*b^7 - 8*(12*B*a^6 - 61*A*a^5*b)*c^3 + 2*(54*B*a^5*b^2 - 197*A*a^4*b^3)*c^
2 - (33*B*a^4*b^4 - 104*A*a^3*b^5)*c)*x^2 - 2*((60*A*a^3*c^5 + 30*(B*a^3*b
 - 3*A*a^2*b^2)*c^4 - 10*(B*a^2*b^3 - 3*A*a*b^4)*c^3 + (B*a*b^5 - 3*A*b^6)*c
^2)*x^10 + 2*(60*A*a^3*b*c^4 + 30*(B*a^3*b^2 - 3*A*a^2*b^3)*c^3 - 10*(B*a^2
*b^4 - 3*A*a*b^5)*c^2 + (B*a*b^6 - 3*A*b^7)*c)*x^8 + (B*a*b^7 - 3*A*b^8 + 1
20*A*a^4*c^4 + 60*(B*a^4*b - 2*A*a^3*b^2)*c^3 + 10*(B*a^3*b^3 - 3*A*a^2*b^4
)*c^2 - 8*(B*a^2*b^5 - 3*A*a*b^6)*c)*x^6 + 2*(B*a^2*b^6 - 3*A*a*b^7 + 60*A*
a^4*b*c^3 + 30*(B*a^4*b^2 - 3*A*a^3*b^3)*c^2 - 10*(B*a^3*b^4 - 3*A*a^2*b^5)
*c)*x^4 + (B*a^3*b^5 - 3*A*a^2*b^6 + 60*A*a^5*c^3 + 30*(B*a^5*b - 3*A*a^4*b
^2)*c^2 - 10*(B*a^4*b^3 - 3*A*a^3*b^4)*c)*x^2)*sqrt(-b^2 + 4*a*c)*arctan(-(
2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((64*(B*a^4 - 3*A*a^3*b)*c
^5 - 48*(B*a^3*b^2 - 3*A*a^2*b^3)*c^4 + 12*(B*a^2*b^4 - 3*A*a*b^5)*c^3 - (B
*a*b^6 - 3*A*b^7)*c^2)*x^10 + 2*(64*(B*a^4*b - 3*A*a^3*b^2)*c^4 - 48*(B*a^3
*b^3 - 3*A*a^2*b^4)*c^3 + 12*(B*a^2*b^5 - 3*A*a*b^6)*c^2 - (B*a*b^7 - 3*A*b
^8)*c)*x^8 - (B*a*b^8 - 3*A*b^9 - 128*(B*a^5 - 3*A*a^4*b)*c^4 + 32*(B*a^4*b
^2 - 3*A*a^3*b^3)*c^3 + 24*(B*a^3*b^4 - 3*A*a^2*b^5)*c^2 - 10*(B*a^2*b^6 -
3*A*a*b^7)*c)*x^6 - 2*(B*a^2*b^7 - 3*A*a*b^8 - 64*(B*a^5*b - 3*A*a^4*b^2)*c
^3 + 48*(B*a^4*b^3 - 3*A*a^3*b^4)*c^2 - 12*(B*a^3*b^5 - 3*A*a^2*b^6)*c)*x^4
 - (B*a^3*b^6 - 3*A*a^2*b^7 - 64*(B*a^6 - 3*A*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.30, size = 648, normalized size = 1.79

$$-\frac{\left(Bab^{5}-3\,Ab^{6}-10\,Ba^{2}b^{3}c+30\,Aab^{4}c+30\,Ba^{3}bc^{2}-90\,Aa^{2}b^{2}c^{2}+60\,Aa^{3}c^{3}\right)\arctan\left(\frac{2\,cx^{2}+b}{\sqrt{-b^{2}+4\,ac}}\right)}{2\,\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\sqrt{-b^{2}+4\,ac}}+\frac{3\,Bab^{4}c^{2}x^{8}-9\,Ab^{5}c^{2}x^{8}-24\,Ba^{2}b^{2}c^{3}x^{8}+72\,Aab^{3}c^{3}x^{8}+48\,Ba^{3}c^{4}x^{8}-144\,Aa^{2}bc^{4}x^{8}+6\,Bab^{5}cx^{6}-18\,Ab^{6}cx^{6}-44\,Ba^{2}b^{3}c^{2}x^{6}+136\,Aab^{4}c^{2}x^{6}+68\,Ba^{3}bc^{3}x^{6}-236\,Aa^{2}b^{2}c^{3}x^{6}-56\,Aa^{3}c^{4}x^{6}+3\,Bab^{6}x^{4}-9\,Ab^{7}x^{4}-10\,Ba^{2}b^{4}cx^{4}+38\,Aab^{5}cx^{4}-58\,Ba^{3}b^{2}c^{2}x^{4}+110\,Aa^{2}b^{3}c^{2}x^{4}+128\,Ba^{4}c^{3}x^{4}-436\,Aa^{3}bc^{3}x^{4}+10\,Ba^{2}b^{5}x^{2}-26\,Aab^{6}x^{2}-72\,Ba^{3}b^{3}cx^{2}+192\,Aa^{2}b^{4}cx^{2}+92\,Ba^{4}bc^{2}x^{2}-316\,Aa^{3}b^{2}c^{2}x^{2}-72\,Aa^{4}c^{3}x^{2}+9\,Ba^{3}b^{4}-19\,Aa^{2}b^{5}-66\,Ba^{4}b^{2}c+144\,Aa^{3}b^{3}c+96\,Ba^{5}c^{2}-260\,Aa^{4}bc^{2}}{8\,\left(a^{4}b^{4}-8\,a^{5}b^{2}c+16\,a^{6}c^{2}\right)\left(cx^{4}+bx^{2}+a\right)^{2}}-\frac{\left(Ba-3\,Ab\right)\log\left(cx^{4}+bx^{2}+a\right)}{4\,a^{4}}+\frac{\left(Ba-3\,Ab\right)\log\left(x^{2}\right)}{2\,a^{4}}-\frac{Bax^{2}-3\,Abx^{2}+Aa}{2\,a^{4}x^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] -1/2*(B*a*b^5 - 3*A*b^6 - 10*B*a^2*b^3*c + 30*A*a*b^4*c + 30*B*a^3*b*c^2 -
90*A*a^2*b^2*c^2 + 60*A*a^3*c^3)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/(
(a^4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2)*sqrt(-b^2 + 4*a*c)) + 1/8*(3*B*a*b^4*c
^2*x^8 - 9*A*b^5*c^2*x^8 - 24*B*a^2*b^2*c^3*x^8 + 72*A*a*b^3*c^3*x^8 + 48*B
*a^3*c^4*x^8 - 144*A*a^2*b*c^4*x^8 + 6*B*a*b^5*c*x^6 - 18*A*b^6*c*x^6 - 44*
B*a^2*b^3*c^2*x^6 + 136*A*a*b^4*c^2*x^6 + 68*B*a^3*b*c^3*x^6 - 236*A*a^2*b^
2*c^3*x^6 - 56*A*a^3*c^4*x^6 + 3*B*a*b^6*x^4 - 9*A*b^7*x^4 - 10*B*a^2*b^4*c
*x^4 + 38*A*a*b^5*c*x^4 - 58*B*a^3*b^2*c^2*x^4 + 110*A*a^2*b^3*c^2*x^4 + 12
8*B*a^4*c^3*x^4 - 436*A*a^3*b*c^3*x^4 + 10*B*a^2*b^5*x^2 - 26*A*a*b^6*x^2 -
 72*B*a^3*b^3*c*x^2 + 192*A*a^2*b^4*c*x^2 + 92*B*a^4*b*c^2*x^2 - 316*A*a^3*
b^2*c^2*x^2 - 72*A*a^4*c^3*x^2 + 9*B*a^3*b^4 - 19*A*a^2*b^5 - 66*B*a^4*b^2*
c + 144*A*a^3*b^3*c + 96*B*a^5*c^2 - 260*A*a^4*b*c^2)/((a^4*b^4 - 8*a^5*b^2
*c + 16*a^6*c^2)*(c*x^4 + b*x^2 + a)^2) - 1/4*(B*a - 3*A*b)*log(c*x^4 + b*x
^2 + a)/a^4 + 1/2*(B*a - 3*A*b)*log(x^2)/a^4 - 1/2*(B*a*x^2 - 3*A*b*x^2 + A
*a)/(a^4*x^2)
```

Mupad [B]
time = 15.91, size = 2500, normalized size = 6.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^3),x)
```

```
[Out] (log(((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b
*c)^3)/(a^9*(4*a*c - b^2)^6) - ((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^
6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2
*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((B*a - 3*A*b + a^4*(-(60*A*a^3*c^3
- 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A
*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3*c^3 - 3*A
*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 - 69*A*a^2*b
^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*c^3 - 3*A*
b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2*c^2))/(a^
3*(4*a*c - b^2)^2) + (b*c^2*(B*a - 3*A*b + a^4*(-(60*A*a^3*c^3 - 3*A*b^6 +
B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a^2*b^2*c^2
)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))/a^4))/(4*
a^4) + (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 24*A*B*a*b^9 -
 3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b^2*c^4 - 302*B
^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*B^2*a^3*b^6*c +
 1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*b^7*c + 2340*A*B
*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) - (c^4*x^2*(54*A^2*b^9 + 6*B^2*a^2*b^7 -
 36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c^3 + 409*B^2*a^4*b
```

^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a^4*b*c^4 - 89*B^2*a
^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 + 4980*A*B*a^4*b^2*c^3
+ 534*A*B*a^2*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (c^4*(3*A*b - B*a)*
(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c)^2)/(a^9*(4*
a*c - b^2)^4))*((c^5*x^2*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c +
 7*B*a^2*b*c)^3)/(a^9*(4*a*c - b^2)^6) - ((3*A*b - B*a + a^4*(-(60*A*a^3*c^
3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90
*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(((3*A*b - B*a + a^4*(-(60*
A*a^3*c^3 - 3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*
c^2 - 90*A*a^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*((4*b*c^2*(30*A*a^3
*c^3 - 3*A*b^6 + B*a*b^5 + 27*A*a*b^4*c - 9*B*a^2*b^3*c + 23*B*a^3*b*c^2 -
69*A*a^2*b^2*c^2))/(a^3*(4*a*c - b^2)^2) + (2*c^3*x^2*(B*a*b^5 - 300*A*a^3*
c^3 - 3*A*b^6 + 6*A*a*b^4*c - 2*B*a^2*b^3*c + 10*B*a^3*b*c^2 + 90*A*a^2*b^2
c^2))/(a^3(4*a*c - b^2)^2) - (b*c^2*(3*A*b - B*a + a^4*(-(60*A*a^3*c^3 -
3*A*b^6 + B*a*b^5 + 30*A*a*b^4*c - 10*B*a^2*b^3*c + 30*B*a^3*b*c^2 - 90*A*a
^2*b^2*c^2)^2/(a^8*(4*a*c - b^2)^5))^(1/2))*(a*b + 3*b^2*x^2 - 10*a*c*x^2))
/a^4))/(4*a^4) - (c^3*(900*A^2*a^5*c^5 - 36*A^2*b^10 - 4*B^2*a^2*b^8 + 24*A
*B*a*b^9 - 3078*A^2*a^2*b^6*c^2 + 7533*A^2*a^3*b^4*c^3 - 7020*A^2*a^4*b^2*c
^4 - 302*B^2*a^4*b^4*c^2 + 497*B^2*a^5*b^2*c^3 + 549*A^2*a*b^8*c + 61*B^2*a
^3*b^6*c + 1932*A*B*a^3*b^5*c^2 - 4002*A*B*a^4*b^3*c^3 - 366*A*B*a^2*b^7*c
+ 2340*A*B*a^5*b*c^4))/(a^6*(4*a*c - b^2)^4) + (c^4*x^2*(54*A^2*b^9 + 6*B^2
*a^2*b^7 - 36*A*B*a*b^8 + 4311*A^2*a^2*b^5*c^2 - 9900*A^2*a^3*b^3*c^3 + 409
*B^2*a^4*b^3*c^2 - 2400*A*B*a^5*c^4 - 801*A^2*a*b^7*c + 8100*A^2*a^4*b*c^4
- 89*B^2*a^3*b^5*c - 560*B^2*a^5*b*c^3 - 2664*A*B*a^3*b^4*c^2 + 4980*A*B*a^
4*b^2*c^3 + 534*A*B*a^2*b^6*c))/(a^6*(4*a*c - b^2)^4)))/(4*a^4) + (c^4*(3*A
*b - B*a)*(3*A*b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c)^2
)/(a^9*(4*a*c - b^2)^4)))*(6*A*b^11 + 2048*B*a^6*c^5 - 2*B*a*b^10 - 120*A*a
*b^9*c - 6144*A*a^5*b*c^5 + 40*B*a^2*b^8*c + 960*A*a^2*b^7*c^2 - 3840*A*a^3
*b^5*c^3 + 7680*A*a^4*b^3*c^4 - 320*B*a^3*b^6*c^2 + 1280*B*a^4*b^4*c^3 - 25
60*B*a^5*b^2*c^4))/(2*(4*a^4*b^10 - 4096*a^9*c^5 - 80*a^5*b^8*c + 640*a^6*b
^6*c^2 - 2560*a^7*b^4*c^3 + 5120*a^8*b^2*c^4)) - (log(x)*(3*A*b - B*a))/a^4
 - (A/(2*a) + (x^2*(9*A*b^5 - 24*B*a^3*c^2 - 3*B*a*b^4 - 68*A*a*b^3*c + 122
*A*a^2*b*c^2 + 21*B*a^2*b^2*c))/(4*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x
^6*(16*B*a^3*c^3 - 12*A*b^5*c + 4*B*a*b^4*c + 87*A*a*b^3*c^2 - 138*A*a^2*b*
c^3 - 29*B*a^2*b^2*c^2))/(4*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^4*(3*A
*b^6 + 50*A*a^3*c^3 - B*a*b^5 - 18*A*a*b^4*c + 6*B*a^2*b^3*c + B*a^3*b*c^2
+ 7*A*a^2*b^2*c^2))/(2*a^3*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c^2*x^8*(3*A*
b^4 + 30*A*a^2*c^2 - B*a*b^3 - 21*A*a*b^2*c + 7*B*a^2*b*c))/(2*a^3*(b^4 + 1
6*a^2*c^2 - 8*a*b^2*c)))/(x^6*(2*a*c + b^2) + a^2*x^2 + c^2*x^10 + 2*a*b*x^
4 + 2*b*c*x^8) - (atan((x^2*((((((((153600*A*a^13*c^10 - 5120*B*a^13*b*c^9
+ 6*A*a^6*b^14*c^3 - 108*A*a^7*b^12*c^4 + 588*A*a^8*b^10*c^5 + 792*A*a^9*b^
8*c^6 - 22272*A*a^10*b^6*c^7 + 100608*A*a^11*b^4*c^8 - 199680*A*a^12*b^2*c^
9 - 2*B*a^7*b^13*c^3 + 36*B*a^8*b^11*c^4 - 276*B*a^9*b^9*c^5 + 1216*B*a^10*
b^7*c^6 - 3456*B*a^11*b^5*c^7 + 6144*B*a^12*b^3*c^8)/(a^9*b^12 + 4096*a^15*
c^6 - 24*a^10*b^10*c + 240*a^11*b^8*c^2 - 1280*a^12*b^6*c^3 + 3840*a^13*b^4

```
*c^4 - 6144*a^14*b^2*c^5) - ((163840*a^16*b*c^9 - 12*a^9*b^15*c^2 + 328*a^1
0*b^13*c^3 - 3840*a^11*b^11*c^4 + 24960*a^12*b^9*c^5 - 97280*a^13*b^7*c^6 +
 227328*a^14*b^5*c^7 - 294912*a^15*b^3*c^8)*(6*A*b^11 + 2048*B*a^6*c^5 - 2*
B*a*b^10 - 120*A*a*b^9*c - 6144*A*a^5*b*c^5 + 4...
```

3.132 $\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=554

$$-\frac{(3b^3B+Ab^2c-24abBc+20aAc^2)\,x}{8c^2\,(b^2-4ac)^2}+\frac{(b^2B+12Abc-28aBc)\,x^3}{8c\,(b^2-4ac)^2}-\frac{x^7(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{x^5(}{}$$

```
[Out] -1/8*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)*x/c^2/(-4*a*c+b^2)^2+1/8*(12*A
*b*c-28*B*a*c+B*b^2)*x^3/c/(-4*a*c+b^2)^2-1/4*x^7*(A*b-2*a*B-(-2*A*c+B*b)*x
^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x^5*(7*A*b^2-12*a*b*B-4*a*A*c+(12*A*
b*c-28*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2^(1/
2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(3*b^4*B+A*b^3*c-27*a*b^2*B*c-16*a
*A*b*c^2+84*a^2*B*c^2+(40*A*a^2*c^3+18*A*a*b^2*c^2-A*b^4*c-132*B*a^2*b*c^2+
33*B*a*b^3*c-3*B*b^5)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b
-(-4*a*c+b^2)^(1/2))^(1/2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*(3*b^4*B+A*b^3*c-27*a*b^2*B*c-16*a*A*b*c^2+84*a^2*B*c^2+(-40*A*
a^2*c^3-18*A*a*b^2*c^2+A*b^4*c+132*B*a^2*b*c^2-33*B*a*b^3*c+3*B*b^5)/(-4*a*
c+b^2)^(1/2))/c^(5/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 7.64, antiderivative size = 554, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1289, 1293, 1180, 211}

$$\frac{\left(-\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{5/2}\,(b^2-4ac)^2\,\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Ac^3+132a^2bBc^2-18aAb^2c^2-33ab^3Bc+Ab^4c+3b^5B}{\sqrt{b^2-4ac}}+84a^2Bc^2-16aAbc^2-27ab^2Bc+Ab^3c+3b^4B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,c^{5/2}\,(b^2-4ac)^2\,\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x^3(-28aBc+12Abc+b^2B)}{8c\,(b^2-4ac)^2}-\frac{x^7(-2aB-(x^2(bB-2Ac))+Ab)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{x^5(x^2(-28aBc+12Abc+b^2B)-4aAc-12abB+7Ab^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{x(20aAc^2-24abBc+Ab^2c+3b^3B)}{8c^2\,(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*((3*b^3*B + A*b^2*c - 24*a*b*B*c + 20*a*A*c^2)*x)/(c^2*(b^2 - 4*a*c)^2
) + ((b^2*B + 12*A*b*c - 28*a*B*c)*x^3)/(8*c*(b^2 - 4*a*c)^2) - (x^7*(A*b -
 2*a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^5
*(7*A*b^2 - 12*a*b*B - 4*a*A*c + (b^2*B + 12*A*b*c - 28*a*B*c)*x^2))/(8*(b^
2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16
*a*A*b*c^2 + 84*a^2*B*c^2 - (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*
c^2 + 132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sq
rt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*S
qrt[b - Sqrt[b^2 - 4*a*c]]) + ((3*b^4*B + A*b^3*c - 27*a*b^2*B*c - 16*a*A*b
*c^2 + 84*a^2*B*c^2 + (3*b^5*B + A*b^4*c - 33*a*b^3*B*c - 18*a*A*b^2*c^2 +
132*a^2*b*B*c^2 - 40*a^2*A*c^3)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(5/2)*(b^2 - 4*a*c)^2*Sqrt[b
+ Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^8(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x^7(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} + \frac{\int \frac{x^6(7(Ab-2aB)+(-bB+2Ac)x^2)}{(a+bx^2+cx^4)^2}\,dx}{4\,(b^2-4ac)}$$

$$= -\frac{x^7(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^5(7Ab^2-12abB-4aAc+(b^2B+12Abc-}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

$$= \frac{(b^2B+12Abc-28aBc)\,x^3}{8c\,(b^2-4ac)^2} - \frac{x^7(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2} - \frac{x^5(7Ab^2-12ab}{8\,(}$$

$$= -\frac{(3b^3B+Ab^2c-24abBc+20aAc^2)\,x}{8c^2\,(b^2-4ac)^2} + \frac{(b^2B+12Abc-28aBc)\,x^3}{8c\,(b^2-4ac)^2} - \frac{x^7(Ab-}{4\,(b^2-}$$

$$= -\frac{(3b^3B+Ab^2c-24abBc+20aAc^2)\,x}{8c^2\,(b^2-4ac)^2} + \frac{(b^2B+12Abc-28aBc)\,x^3}{8c\,(b^2-4ac)^2} - \frac{x^7(Ab-}{4\,(b^2-}$$

$$= -\frac{(3b^3B+Ab^2c-24abBc+20aAc^2)\,x}{8c^2\,(b^2-4ac)^2} + \frac{(b^2B+12Abc-28aBc)\,x^3}{8c\,(b^2-4ac)^2} - \frac{x^7(Ab-}{4\,(b^2-}$$

Mathematica [A]
time = 1.51, size = 644, normalized size = 1.16

Antiderivative was successfully verified.

```
[In] Integrate[(x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(2*b^5*B - b^4*c*(2*A + 5*B*x^2) - 4*a^2*c^3*(9*A + 11*B*x^2) + a*b^2
*c^2*(11*A + 37*B*x^2) + 16*a*b*c^2*(3*a*B - A*c*x^2) + b^3*c*(-17*a*B + A*
c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) - (4*x*(b^3*(b*B - A*c)*x^2
+ a^2*c*(-3*b*B + 2*c*(A + B*x^2)) + a*b*(b^2*B + 3*A*c^2*x^2 - b*c*(A + 4*
B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sqrt[2]*Sqrt[c]*(-3*b^5*
B + b^3*c*(33*a*B + A*Sqrt[b^2 - 4*a*c]) - 4*a*b*c^2*(33*a*B + 4*A*Sqrt[b^2
 - 4*a*c]) + 9*a*b^2*c*(2*A*c - 3*B*Sqrt[b^2 - 4*a*c]) + b^4*(-(A*c) + 3*B*
Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(10*A*c + 21*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(3*b^5*B + 4*a*b*c^2*(33*a*B - 4*
A*Sqrt[b^2 - 4*a*c]) + b^4*(A*c + 3*B*Sqrt[b^2 - 4*a*c]) - 9*a*b^2*c*(2*A*c
 + 3*B*Sqrt[b^2 - 4*a*c]) + 4*a^2*c^2*(-10*A*c + 21*B*Sqrt[b^2 - 4*a*c]) +
b^3*(-33*a*B*c + A*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
```

```
+ Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(
16*c^3)
```

Maple [A]
time = 0.08, size = 710, normalized size = 1.28

method	result
risch	$\frac{-\frac{\left(16Aab\,c^2-A\,b^3c+44a^2B\,c^2-37a\,b^2Bc+5b^4B\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(36A\,a^2c^3+5Aa\,b^2c^2+A\,b^4c-4B\,a^2b\,c^2-20Ba\,b^3c+3B\,b^5\right)x^5}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(28Aab\,c^2+2A\,b^3c+28a^2B\,c\right.}{8c^2\left(16a^2c^2-8c\right.}}{\left(c\,x^4+b\,x^2+a\right)^2}$
default	$\frac{-\frac{\left(16Aab\,c^2-A\,b^3c+44a^2B\,c^2-37a\,b^2Bc+5b^4B\right)x^7}{8\left(16a^2c^2-8a\,b^2c+b^4\right)c}-\frac{\left(36A\,a^2c^3+5Aa\,b^2c^2+A\,b^4c-4B\,a^2b\,c^2-20Ba\,b^3c+3B\,b^5\right)x^5}{8c^2\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(28Aab\,c^2+2A\,b^3c+28a^2B\,c\right.}{8c^2\left(16a^2c^2-8c\right.}}{\left(c\,x^4+b\,x^2+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (-1/8*(16*A*a*b*c^2-A*b^3*c+44*B*a^2*c^2-37*B*a*b^2*c+5*B*b^4)/(16*a^2*c^2-
8*a*b^2*c+b^4)/c*x^7-1/8*(36*A*a^2*c^3+5*A*a*b^2*c^2+A*b^4*c-4*B*a^2*b*c^2-
20*B*a*b^3*c+3*B*b^5)/c^2/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*a/c^2*(28*A*a*
b*c^2+2*A*b^3*c+28*B*a^2*c^2-49*B*a*b^2*c+6*B*b^4)/(16*a^2*c^2-8*a*b^2*c+b^
4)*x^3-1/8*a^2*(20*A*a*c^2+A*b^2*c-24*B*a*b*c+3*B*b^3)/c^2/(16*a^2*c^2-8*a*
b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/2/c/(16*a^2*c^2-8*a*b^2*c+b^4)*(-1/8*(-16
*A*a*b*c^2*(-4*a*c+b^2)^(1/2)+A*b^3*c*(-4*a*c+b^2)^(1/2)+40*A*a^2*c^3+18*A*
a*b^2*c^2-A*b^4*c+84*a^2*B*c^2*(-4*a*c+b^2)^(1/2)-27*a*b^2*B*c*(-4*a*c+b^2)
^(1/2)+3*b^4*B*(-4*a*c+b^2)^(1/2)-132*B*a^2*b*c^2+33*B*a*b^3*c-3*B*b^5)/c/(
-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(
1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-16*A*a*b*c^2*(-4*a*c+b^2)^(1/
2)+A*b^3*c*(-4*a*c+b^2)^(1/2)-40*A*a^2*c^3-18*A*a*b^2*c^2+A*b^4*c+84*a^2*B*
c^2*(-4*a*c+b^2)^(1/2)-27*a*b^2*B*c*(-4*a*c+b^2)^(1/2)+3*b^4*B*(-4*a*c+b^2)
^(1/2)+132*B*a^2*b*c^2-33*B*a*b^3*c+3*B*b^5)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*
c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2)*x
^7 + (3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b^3 -
A*b^4)*c)*x^5 + (6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^2 - 2*A
*a*b^3)*c)*x^3 + (3*B*a^2*b^3 + 20*A*a^3*c^2 - (24*B*a^3*b - A*a^2*b^2)*c)*
x)/((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3
+ 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*
a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*
x^2) - 1/8*integrate(-(3*B*a*b^3 + 20*A*a^2*c^2 + (3*B*b^4 + 4*(21*B*a^2 -
4*A*a*b)*c^2 - (27*B*a*b^2 - A*b^3)*c)*x^2 - (24*B*a^2*b - A*a*b^2)*c)/(c*x
^4 + b*x^2 + a), x)/(b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9636 vs. $2(507) = 1014$.
time = 19.29, size = 9636, normalized size = 17.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] -1/16*(2*(5*B*b^4*c + 4*(11*B*a^2 + 4*A*a*b)*c^3 - (37*B*a*b^2 + A*b^3)*c^2
)*x^7 + 2*(3*B*b^5 + 36*A*a^2*c^3 - (4*B*a^2*b - 5*A*a*b^2)*c^2 - (20*B*a*b
^3 - A*b^4)*c)*x^5 + 2*(6*B*a*b^4 + 28*(B*a^3 + A*a^2*b)*c^2 - (49*B*a^2*b^
2 - 2*A*a*b^3)*c)*x^3 - sqrt(1/2)*((b^4*c^4 - 8*a*b^2*c^5 + 16*a^2*c^6)*x^8
 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2*(b^5*c^3 - 8*a*b^3*c^4 + 16
*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a^3*c^5)*x^4 + 2*(a*b^5*c^2 -
 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^
2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(
216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168
*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20
*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^
5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*
B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*
B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*
A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4
- 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)
/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^
4*b^2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 -
640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))*log(-(1701*B^4*a^2*b^8
 - 945*A*B^3*a*b^9 - 10000*A^4*a^4*c^6 + 15000*(6*A^3*B*a^4*b - A^4*a^3*b^2
)*c^5 + 3*(1037232*B^4*a^6 - 1037232*A*B^3*a^5*b + 287712*A^2*B^2*a^4*b^2 -
 32952*A^3*B*a^3*b^3 + 497*A^4*a^2*b^4)*c^4 - (1555848*B^4*a^5*b^2 - 129837
6*A*B^3*a^4*b^3 + 238464*A^2*B^2*a^3*b^4 - 11277*A^3*B*a^2*b^5 + 35*A^4*a*b
^6)*c^3 + 9*(37701*B^4*a^4*b^4 - 26973*A*B^3*a^3*b^5 + 3066*A^2*B^2*a^2*b^6
 - 35*A^3*B*a*b^7)*c^2 - 27*(1341*B^4*a^3*b^6 - 819*A*B^3*a^2*b^7 + 35*A^2*
```

```
B^2*a*b^8)*c)*x + 1/2*sqrt(1/2)*(27*B^3*b^13 + 32000*A^3*a^5*c^8 - 640*(882
*A*B^2*a^6 - 156*A^2*B*a^5*b + 37*A^3*a^4*b^2)*c^7 + 64*(10584*B^3*a^6*b +
5562*A*B^2*a^5*b^2 - 1083*A^2*B*a^4*b^3 + 89*A^3*a^3*b^4)*c^6 - 8*(93096*B^
3*a^5*b^3 + 3816*A*B^2*a^4*b^4 - 1746*A^2*B*a^3*b^5 + 49*A^3*a^2*b^6)*c^5 +
 (337392*B^3*a^4*b^5 - 24120*A*B^2*a^3*b^6 - 84*A^2*B*a^2*b^7 - 17*A^3*a*b^
8)*c^4 - (81324*B^3*a^3*b^7 - 6993*A*B^2*a^2*b^8 + 195*A^2*B*a*b^9 - A^3*b^
10)*c^3 + 9*(1239*B^3*a^2*b^9 - 79*A*B^2*a*b^10 + A^2*B*b^11)*c^2 - 27*(31*
B^3*a*b^11 - A*B^2*b^12)*c - (3*B*b^14*c^5 - 4096*(42*B*a^7 - 13*A*a^6*b)*c
^12 + 6144*(40*B*a^6*b^2 - 11*A*a^5*b^3)*c^11 - 768*(194*B*a^5*b^4 - 45*A*a
^4*b^5)*c^10 + 1280*(39*B*a^4*b^6 - 7*A*a^3*b^7)*c^9 - 240*(42*B*a^3*b^8 -
5*A*a^2*b^9)*c^8 + 24*(52*B*a^2*b^10 - 3*A*a*b^11)*c^7 - (90*B*a*b^12 - A*b
^13)*c^6)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^
3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^
2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 444
6*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^
4 - 116*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*
c)/(b^10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*
a^4*b^2*c^14 - 1024*a^5*c^15)))*sqrt(-(9*B^2*b^9 - 1680*(4*A*B*a^4 - A^2*a^
3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b^2 + A^2*a^2*b^3)*c^4 - 35*(216*
B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c^3 + (1701*B^2*a^2*b^5 - 168*A*B
*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2*A*B*b^8)*c + (b^10*c^5 - 20*a*b
^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^
10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50*(441*A^2*B^2*a^3 - 108*A^3*B*a^
2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95256*A*B^3*a^3*b + 17496*A^2*B^2*
a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 - 6*(14553*B^4*a^3*b^2 - 4446*A*B^
3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5)*c^3 + 27*(657*B^4*a^2*b^4 - 11
6*A*B^3*a*b^5 + 2*A^2*B^2*b^6)*c^2 - 54*(33*B^4*a*b^6 - 2*A*B^3*b^7)*c)/(b^
10*c^10 - 20*a*b^8*c^11 + 160*a^2*b^6*c^12 - 640*a^3*b^4*c^13 + 1280*a^4*b^
2*c^14 - 1024*a^5*c^15)))/(b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*
a^3*b^4*c^8 + 1280*a^4*b^2*c^9 - 1024*a^5*c^10))) + sqrt(1/2)*((b^4*c^4 - 8
*a*b^2*c^5 + 16*a^2*c^6)*x^8 + a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4 + 2
*(b^5*c^3 - 8*a*b^3*c^4 + 16*a^2*b*c^5)*x^6 + (b^6*c^2 - 6*a*b^4*c^3 + 32*a
^3*c^5)*x^4 + 2*(a*b^5*c^2 - 8*a^2*b^3*c^3 + 16*a^3*b*c^4)*x^2)*sqrt(-(9*B^
2*b^9 - 1680*(4*A*B*a^4 - A^2*a^3*b)*c^5 + 280*(54*B^2*a^4*b - 12*A*B*a^3*b
^2 + A^2*a^2*b^3)*c^4 - 35*(216*B^2*a^3*b^3 - 36*A*B*a^2*b^4 + A^2*a*b^5)*c
^3 + (1701*B^2*a^2*b^5 - 168*A*B*a*b^6 + A^2*b^7)*c^2 - 3*(63*B^2*a*b^7 - 2
*A*B*b^8)*c + (b^10*c^5 - 20*a*b^8*c^6 + 160*a^2*b^6*c^7 - 640*a^3*b^4*c^8
+ 1280*a^4*b^2*c^9 - 1024*a^5*c^10)*sqrt((81*B^4*b^8 + 625*A^4*a^2*c^6 - 50
*(441*A^2*B^2*a^3 - 108*A^3*B*a^2*b + A^4*a*b^2)*c^5 + (194481*B^4*a^4 - 95
256*A*B^3*a^3*b + 17496*A^2*B^2*a^2*b^2 - 516*A^3*B*a*b^3 + A^4*b^4)*c^4 -
6*(14553*B^4*a^3*b^2 - 4446*A*B^3*a^2*b^3 + 324*A^2*B^2*a*b^4 - 2*A^3*B*b^5
)*c^3 + 27*(657*B^4*a^2*b^4 - 116*A*B^3*a*b^5 +...
```

Sympy [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3987 vs. 2(507) = 1014.
time = 7.75, size = 3987, normalized size = 7.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*b^5*c^2 - 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^3 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 24*a*b^4*c^3 - 2*b^5*c^3 + 320*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(2)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^
4 + 288*a^2*b^2*c^4 + 112*a*b^3*c^4 - 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^5 - 640*a^3*c^5 - 416*a^2*b*c^5 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 208*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 + 104*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*b^3*c^3 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a*b*c^4 + 2*(b^2 - 4*a*c)*b^4*c^2 + 32*(b^2 - 4*a*c)*a*b^2*c^3 + 2*(
b^2 - 4*a*c)*b^3*c^3 - 160*(b^2 - 4*a*c)*a^2*c^4 - 104*(b^2 - 4*a*c)*a*b*c^
4)*A + 3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 - 16*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^6*c - 2*b^7*c + 80*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 +
 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + sqrt(2)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*a*b^5*c^2 - 2*b^6*c^2 - 128*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b^2*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 -
 160*a^2*b^3*c^3 + 28*a*b^4*c^3 + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^4 + 256*a^3*b*c^4 - 192*a^2*b^2*c^4 + 448*a^3*c^5 + sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 14*sqrt(2)*sqrt(b^2 - 4*
```

```
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 96*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 - 112*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^3 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b^2*c^3 + 56*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 + 2*(b^2 - 4*a*c)*b^5*c - 24*(b^2 - 4*a*c)*a*b^3*c^2 + 2
*(b^2 - 4*a*c)*b^4*c^2 + 64*(b^2 - 4*a*c)*a^2*b*c^3 - 20*(b^2 - 4*a*c)*a*b^
2*c^3 + 112*(b^2 - 4*a*c)*a^2*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5*c^2 -
8*a*b^3*c^3 + 16*a^2*b*c^4 + sqrt((b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)^2
- 4*(a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*(b^4*c^3 - 8*a*b^2*c^4 + 16*a^
2*c^5)))/(b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)))/((b^8*c^2 - 16*a*b^6*c^3 -
2*b^7*c^3 + 96*a^2*b^4*c^4 + 24*a*b^5*c^4 + b^6*c^4 - 256*a^3*b^2*c^5 - 96*
a^2*b^3*c^5 - 12*a*b^4*c^5 + 256*a^4*c^6 + 128*a^3*b*c^6 + 48*a^2*b^2*c^6 -
 64*a^3*c^7)*abs(c)) + 1/32*((sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c
 + 12*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*sqrt(2)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 2*b^6*c^2 - 144*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*
b^3*c^3 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 24*a*b^4*c^3 +
2*b^5*c^3 + 320*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 160*sqrt(
2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a*b^2*c^4 - 288*a^2*b^2*c^4 - 112*a*b^3*c^4 - 80*sqrt(2)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 640*a^3*c^5 + 416*a^2*b*c^5 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 56*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 208*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 104*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 - 32*(b^2 -
 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 + 160*(b^2 - 4*a*c)*a^2*c^4 + 1
04*(b^2 - 4*a*c)*a*b*c^4)*A + 3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^
7 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 2*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^6*c + 2*b^7*c + 80*sqrt(2)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c^2 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^
2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 32*a*b^5*c^2 + 2*b^6*
c^2 - 128*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 64*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 12*...
```

Mupad [B]
time = 5.05, size = 2500, normalized size = 4.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^8*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] - ((x^5*(3*B*b^5 + 36*A*a^2*c^3 + A*b^4*c - 20*B*a*b^3*c + 5*A*a*b^2*c^2 -
4*B*a^2*b*c^2))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^7*(5*B*b^4 + 44
*B*a^2*c^2 - A*b^3*c + 16*A*a*b*c^2 - 37*B*a*b^2*c))/(8*c*(b^4 + 16*a^2*c^2
 - 8*a*b^2*c)) + (x^3*(28*B*a^3*c^2 + 6*B*a*b^4 + 2*A*a*b^3*c + 28*A*a^2*b*
c^2 - 49*B*a^2*b^2*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (a^2*x*(3*B
*b^3 + 20*A*a*c^2 + A*b^2*c - 24*B*a*b*c))/(8*c^2*(b^4 + 16*a^2*c^2 - 8*a*b
^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) - atan(
((((256*A*a*b^12*c^4 - 5242880*A*a^7*c^10 + 768*B*a*b^13*c^3 + 6291456*B*a^
7*b*c^9 - 61440*A*a^3*b^8*c^6 + 655360*A*a^4*b^6*c^7 - 2949120*A*a^5*b^4*c^
8 + 6291456*A*a^6*b^2*c^9 - 21504*B*a^2*b^11*c^4 + 245760*B*a^3*b^9*c^5 - 1
474560*B*a^4*b^7*c^6 + 4915200*B*a^5*b^5*c^7 - 8650752*B*a^6*b^3*c^8)/(512*
(4096*a^6*c^9 + b^12*c^3 - 24*a*b^10*c^4 + 240*a^2*b^8*c^5 - 1280*a^3*b^6*c
^6 + 3840*a^4*b^4*c^7 - 6144*a^5*b^2*c^8)) - (x*(-(9*B^2*b^19 + A^2*b^17*c^
2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*
c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^
7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^
2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*
c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8
*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15
*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15
482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377
280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 -
1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c
 - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1
/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 +
b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4
*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^
12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14)))^(1/2)*(256*b^11*c^5 - 5
120*a*b^9*c^6 - 262144*a^5*b*c^10 + 40960*a^2*b^7*c^7 - 163840*a^3*b^5*c^8
+ 327680*a^4*b^3*c^9))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b
^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*
b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b
^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^
13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^
6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c
^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) +
6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a
^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9
 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5
```

- 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*a^2*b^16*c^7 - 7680*a^3*b^14*c^8 + 53760*a^4*b^12*c^9 - 258048*a^5*b^10*c^10 + 860160*a^6*b^8*c^11 - 1966080*a^7*b^6*c^12 + 2949120*a^8*b^4*c^13 - 2621440*a^9*b^2*c^14))))^(1/2) - (x*(9*B^2*b^10 + 800*A^2*a^4*c^6 + A^2*b^8*c^2 - 14112*B^2*a^5*c^5 + 6*A*B*b^9*c + 314*A^2*a^2*b^4*c^4 + 208*A^2*a^3*b^2*c^5 + 1881*B^2*a^2*b^6*c^2 - 9090*B^2*a^3*b^4*c^3 + 21312*B^2*a^4*b^2*c^4 - 198*B^2*a*b^8*c - 36*A^2*a*b^6*c^3 + 1422*A*B*a^2*b^5*c^3 - 4464*A*B*a^3*b^3*c^4 - 174*A*B*a*b^7*c^2 + 96*A*B*a^4*b*c^5))/(32*(256*a^4*c^7 + b^8*c^3 - 16*a*b^6*c^4 + 96*a^2*b^4*c^5 - 256*a^3*b^2*c^6)))*(-(9*B^2*b^19 + A^2*b^17*c^2 + 9*B^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^18*c + 1140*A^2*a^2*b^13*c^4 - 10160*A^2*a^3*b^11*c^5 + 34880*A^2*a^4*b^9*c^6 + 43776*A^2*a^5*b^7*c^7 - 680960*A^2*a^6*b^5*c^8 + 1863680*A^2*a^7*b^3*c^9 + 6921*B^2*a^2*b^15*c^2 - 77580*B^2*a^3*b^13*c^3 + 570960*B^2*a^4*b^11*c^4 - 2851776*B^2*a^5*b^9*c^5 + 9628416*B^2*a^6*b^7*c^6 - 21095424*B^2*a^7*b^5*c^7 + 27095040*B^2*a^8*b^3*c^8 + A^2*b^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 441*B^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + 6881280*A*B*a^9*c^10 - 369*B^2*a*b^17*c - 55*A^2*a*b^15*c^3 - 1720320*A^2*a^8*b*c^10 - 25*A^2*a*c^3*(-(4*a*c - b^2)^15)^(1/2) - 15482880*B^2*a^9*b*c^9 + 5580*A*B*a^2*b^14*c^3 - 59280*A*B*a^3*b^12*c^4 + 377280*A*B*a^4*b^10*c^5 - 1430784*A*B*a^5*b^8*c^6 + 2860032*A*B*a^6*b^6*c^7 - 1290240*A*B*a^7*b^4*c^8 - 5160960*A*B*a^8*b^2*c^9 - 99*B^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a*b^16*c^2 + 6*A*B*b^3*c*(-(4*a*c - b^2)^15)^(1/2) - 108*A*B*a*b*c^2*(-(4*a*c - b^2)^15)^(1/2))/(512*(1048576*a^10*c^15 + b^20*c^5 - 40*a*b^18*c^6 + 720*...

3.133 $\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=461

$$-\frac{(b^2B-12Abc+20aBc)\,x}{8c\,(b^2-4ac)^2}-\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{x^3(5Ab^2-12abB+4aAc-(b^2B-12Abc}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}$$

```
[Out] -1/8*(-12*A*b*c+20*B*a*c+B*b^2)*x/c/(-4*a*c+b^2)^2-1/4*x^5*(A*b-2*a*B-(-2*A
*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x^3*(5*A*b^2-12*a*b*B+4*a*A
*c-(-12*A*b*c+20*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^3*B+3*A*b^2*c-16*a*b*
B*c+12*a*A*c^2+(-36*A*a*b*c^2-3*A*b^3*c+40*B*a^2*c^2+18*B*a*b^2*c-B*b^4)/(-
4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^3*B+3*A*b
^2*c-16*a*b*B*c+12*a*A*c^2+(36*A*a*b*c^2+3*A*b^3*c-40*B*a^2*c^2-18*B*a*b^2*
c+B*b^4)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A]

time = 3.23, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1289, 1293, 1180, 211}

$$\frac{\left(-\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,c^{3/2}\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\left(\frac{-40a^2Bc^2+36aAbc^2-18ab^2Bc+3Ab^3c+b^4B}{\sqrt{b^2-4ac}}+12aAc^2-16abBc+3Ab^2c+b^3B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,c^{3/2}\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x^5(-2aB-(x^2(bB-2Ac))+Ab)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}-\frac{x^3(-x^2(20aBc-12Abc+b^2B)+4aAc-12abB+5Ab^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{x(20aBc-12Abc+b^2B)}{8c\,(b^2-4ac)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/8*((b^2*B - 12*A*b*c + 20*a*B*c)*x)/(c*(b^2 - 4*a*c)^2) - (x^5*(A*b - 2*
a*B - (b*B - 2*A*c)*x^2))/(4*(b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) - (x^3*(5
*A*b^2 - 12*a*b*B + 4*a*A*c - (b^2*B - 12*A*b*c + 20*a*B*c)*x^2))/(8*(b^2 -
 4*a*c)^2*(a + b*x^2 + c*x^4)) + ((b^3*B + 3*A*b^2*c - 16*a*b*B*c + 12*a*A*
c^2 - (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + ((b^3*B + 3*
A*b^2*c - 16*a*b*B*c + 12*a*A*c^2 + (b^4*B + 3*A*b^3*c - 18*a*b^2*B*c + 36*
a*A*b*c^2 - 40*a^2*B*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqr
t[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*c^(3/2)*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt
[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^6(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{\int \frac{x^4(5(Ab-2aB)+(bB-2Ac)x^2)}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)}$$

$$= -\frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x^3(5Ab^2-12abB+4aAc-(b^2B-12Abc+\cdots}{8(b^2-4ac)^2(a+bx^2+cx^4)}$$

$$= -\frac{(b^2B-12Abc+20aBc)x}{8c(b^2-4ac)^2} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x^3(5Ab^2-12ab\cdots}{8\cdots}$$

$$= -\frac{(b^2B-12Abc+20aBc)x}{8c(b^2-4ac)^2} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x^3(5Ab^2-12ab\cdots}{8\cdots}$$

$$= -\frac{(b^2B-12Abc+20aBc)x}{8c(b^2-4ac)^2} - \frac{x^5(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x^3(5Ab^2-12ab\cdots}{8\cdots}$$

Mathematica [A]
time = 1.30, size = 543, normalized size = 1.18

$$\frac{\frac{2x(-2b^4B+4abc^2(A-4Bx^2)+b^3c(2A+Bx^2)+12ac^2(-3aB+Acx^2)+b^2c(11aB+3Acx^2))}{(b^2-4ac)^2(a+bx^2+cx^4)} - \frac{4x(2a^2Bc+b^2(-bB+Ac)x^2+a(-b^2B-2Ac^2x^2+bc(A+3Bx^2)))}{(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{\sqrt{2}\sqrt{c}\left(-b^4B+3b^2c\left(6aB+A\sqrt{b^2-4ac}\right)+4ac^2\left(10aB+3A\sqrt{b^2-4ac}\right)+b^3\left(-3Ac+B\sqrt{b^2-4ac}\right)-4abc\left(9Ac+4B\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}} + \frac{\sqrt{2}\sqrt{c}\left(b^4B+3b^2c\left(-6aB+A\sqrt{b^2-4ac}\right)+4ac^2\left(-10aB+3A\sqrt{b^2-4ac}\right)+4abc\left(9Ac-4B\sqrt{b^2-4ac}\right)+b^3\left(3Ac+B\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((2*x*(-2*b^4*B + 4*a*b*c^2*(A - 4*B*x^2) + b^3*c*(2*A + B*x^2) + 12*a*c^2*
(-3*a*B + A*c*x^2) + b^2*c*(11*a*B + 3*A*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x
^2 + c*x^4)) - (4*x*(2*a^2*B*c + b^2*(-(b*B) + A*c)*x^2 + a*(-(b^2*B) - 2*A
*c^2*x^2 + b*c*(A + 3*B*x^2))))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^4)^2) + (Sq
rt[2]*Sqrt[c]*(-(b^4*B) + 3*b^2*c*(6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(
10*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + b^3*(-3*A*c + B*Sqrt[b^2 - 4*a*c]) - 4*a*
b*c*(9*A*c + 4*B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sq
rt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqr
t[2]*Sqrt[c]*(b^4*B + 3*b^2*c*(-6*a*B + A*Sqrt[b^2 - 4*a*c]) + 4*a*c^2*(-10
*a*B + 3*A*Sqrt[b^2 - 4*a*c]) + 4*a*b*c*(9*A*c - 4*B*Sqrt[b^2 - 4*a*c]) + b
^3*(3*A*c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[
b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c^2)
```

Maple [A]
time = 0.08, size = 591, normalized size = 1.28

method	result
risch	$\frac{\frac{\left(12c^2aA+3A\,b^2c-16abBc+b^3B\right)x^7}{128a^2c^2-64a\,b^2c+8b^4}+\frac{\left(16Aab\,c^2+5A\,b^3c-36a^2B\,c^2-5a\,b^2Bc-b^4B\right)x^5}{8c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(4c^2aA-19A\,b^2c+28abBc+2b^3B\right)x^3}{8c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{a^2\left(12bcA-20ac\right.}{8c\left(16a^2c^2-8a\,\right.}}{\left(c\,x^4+b\,x^2+a\right)^2}$
default	$\frac{\frac{\left(12c^2aA+3A\,b^2c-16abBc+b^3B\right)x^7}{128a^2c^2-64a\,b^2c+8b^4}+\frac{\left(16Aab\,c^2+5A\,b^3c-36a^2B\,c^2-5a\,b^2Bc-b^4B\right)x^5}{8c\left(16a^2c^2-8a\,b^2c+b^4\right)}-\frac{a\left(4c^2aA-19A\,b^2c+28abBc+2b^3B\right)x^3}{8c\left(16a^2c^2-8a\,b^2c+b^4\right)}+\frac{a^2\left(12bcA-20ac\right.}{8c\left(16a^2c^2-8a\,\right.}}{\left(c\,x^4+b\,x^2+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(12*A*a*c^2+3*A*b^2*c-16*B*a*b*c+B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7
+1/8*(16*A*a*b*c^2+5*A*b^3*c-36*B*a^2*c^2-5*B*a*b^2*c-B*b^4)/c/(16*a^2*c^2-
8*a*b^2*c+b^4)*x^5-1/8*a/c*(4*A*a*c^2-19*A*b^2*c+28*B*a*b*c+2*B*b^3)/(16*a^
2*c^2-8*a*b^2*c+b^4)*x^3+1/8*a^2*(12*A*b*c-20*B*a*c-B*b^2)/c/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/2/(16*a^2*c^2-8*a*b^2*c+b^4)*(-1/8*(12*
c^2*a*A*(-4*a*c+b^2)^(1/2)+3*A*b^2*c*(-4*a*c+b^2)^(1/2)-36*A*a*b*c^2-3*A*b^
3*c-16*a*b*B*c*(-4*a*c+b^2)^(1/2)+b^3*B*(-4*a*c+b^2)^(1/2)+40*a^2*B*c^2+18*
a*b^2*B*c-b^4*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(
1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(12*c^2*a*A
*(-4*a*c+b^2)^(1/2)+3*A*b^2*c*(-4*a*c+b^2)^(1/2)+36*A*a*b*c^2+3*A*b^3*c-16*
a*b*B*c*(-4*a*c+b^2)^(1/2)+b^3*B*(-4*a*c+b^2)^(1/2)-40*a^2*B*c^2-18*a*b^2*B
*c+b^4*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arc
tan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - (B*b^4 + 4*(9*
B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - (2*B*a*b^3 + 4*A*a^2*c^
2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - (B*a^2*b^2 + 4*(5*B*a^3 - 3*A*a^2*b)
*c)*x)/((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^
2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*
a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^
2) + 1/8*integrate((B*a*b^2 + (B*b^3 + 12*A*a*c^2 - (16*B*a*b - 3*A*b^2)*c)
```

```
*x^2 + 4*(5*B*a^2 - 3*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(b^4*c - 8*a*b^2*c^
2 + 16*a^2*c^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7060 vs. $2(414) = 828$.
time = 6.16, size = 7060, normalized size = 15.31

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(B*b^3*c + 12*A*a*c^3 - (16*B*a*b - 3*A*b^2)*c^2)*x^7 - 2*(B*b^4 +
4*(9*B*a^2 - 4*A*a*b)*c^2 + 5*(B*a*b^2 - A*b^3)*c)*x^5 - 2*(2*B*a*b^3 + 4*A
*a^2*c^2 + (28*B*a^2*b - 19*A*a*b^2)*c)*x^3 - sqrt(1/2)*((b^4*c^3 - 8*a*b^2
*c^4 + 16*a^2*c^5)*x^8 + a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^
2 - 8*a*b^3*c^3 + 16*a^2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^
4 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4
*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*
b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5
 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*
c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*
A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)
*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2
*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3
 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 10
24*a^5*c^8))*log(-(35*B^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14
*A^3*B*a^2*b - 5*A^4*a*b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936
*A^2*B^2*a^2*b^2 + 1080*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^
2 - 3864*A*B^3*a^2*b^3 + 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B
^4*a^2*b^4 - 315*A*B^3*a*b^5 + 45*A^2*B^2*b^6)*c)*x + 1/2*sqrt(1/2)*(B^3*b^
10 - 2304*(5*A^2*B*a^4 - 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4
*b + 198*A^2*B*a^3*b^2 - 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*
A*B^2*a^3*b^3 + 324*A^2*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4
 - 1332*A*B^2*a^2*b^5 + 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^
6 - 492*A*B^2*a*b^7 + 63*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c -
(B*b^13*c^3 - 24576*A*a^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536
*(44*B*a^5*b^3 - 5*A*a^4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 -
160*(56*B*a^3*b^7 - 15*A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5
- 18*(4*B*a*b^11 - A*b^12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2
*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 -
2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^
8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))*sqrt(-(B^2*b^7 -
 240*(4*A*B*a^3 - 3*A^2*a^2*b)*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3
*A^2*a*b^3)*c^3 + (280*B^2*a^2*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^
```

```
2*a*b^5 - 6*A*B*b^6)*c + (b^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a
^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 -
18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B
^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 +
160*a^2*b^6*c^8 - 640*a^3*b^4*c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b
^10*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c
^7 - 1024*a^5*c^8))) + sqrt(1/2)*((b^4*c^3 - 8*a*b^2*c^4 + 16*a^2*c^5)*x^8
+ a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3 + 2*(b^5*c^2 - 8*a*b^3*c^3 + 16*a^
2*b*c^4)*x^6 + (b^6*c - 6*a*b^4*c^2 + 32*a^3*c^4)*x^4 + 2*(a*b^5*c - 8*a^2*
b^3*c^2 + 16*a^3*b*c^3)*x^2)*sqrt(-(B^2*b^7 - 240*(4*A*B*a^3 - 3*A^2*a^2*b)
*c^4 + 120*(14*B^2*a^3*b - 16*A*B*a^2*b^2 + 3*A^2*a*b^3)*c^3 + (280*B^2*a^2
*b^3 - 60*A*B*a*b^4 + 9*A^2*b^5)*c^2 - (35*B^2*a*b^5 - 6*A*B*b^6)*c + (b^10
*c^3 - 20*a*b^8*c^4 + 160*a^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7
- 1024*a^5*c^8)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*
c^3 + (625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2
- 6*A*B^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b^6*c^8 - 640*a^3*b^4*
c^9 + 1280*a^4*b^2*c^10 - 1024*a^5*c^11)))/(b^10*c^3 - 20*a*b^8*c^4 + 160*a
^2*b^6*c^5 - 640*a^3*b^4*c^6 + 1280*a^4*b^2*c^7 - 1024*a^5*c^8))*log(-(35*B
^4*a*b^6 - 15*A*B^3*b^7 - 1296*A^4*a^2*c^5 + 648*(14*A^3*B*a^2*b - 5*A^4*a*
b^2)*c^4 + (10000*B^4*a^4 - 30000*A*B^3*a^3*b + 9936*A^2*B^2*a^2*b^2 + 1080
*A^3*B*a*b^3 - 405*A^4*b^4)*c^3 + 3*(5000*B^4*a^3*b^2 - 3864*A*B^3*a^2*b^3
+ 1080*A^2*B^2*a*b^4 - 135*A^3*B*b^5)*c^2 - 3*(497*B^4*a^2*b^4 - 315*A*B^3*
a*b^5 + 45*A^2*B^2*b^6)*c)*x - 1/2*sqrt(1/2)*(B^3*b^10 - 2304*(5*A^2*B*a^4
- 3*A^3*a^3*b)*c^6 + 64*(500*B^3*a^5 - 420*A*B^2*a^4*b + 198*A^2*B*a^3*b^2
- 81*A^3*a^2*b^3)*c^5 - 16*(1480*B^3*a^4*b^2 - 1284*A*B^2*a^3*b^3 + 324*A^2
*B*a^2*b^4 - 81*A^3*a*b^5)*c^4 + 4*(1424*B^3*a^3*b^4 - 1332*A*B^2*a^2*b^5 +
 234*A^2*B*a*b^6 - 27*A^3*b^7)*c^3 - (392*B^3*a^2*b^6 - 492*A*B^2*a*b^7 + 6
3*A^2*B*b^8)*c^2 - (17*B^3*a*b^8 + 6*A*B^2*b^9)*c - (B*b^13*c^3 - 24576*A*a
^6*c^10 + 4096*(13*B*a^6*b + 3*A*a^5*b^2)*c^9 - 1536*(44*B*a^5*b^3 - 5*A*a^
4*b^4)*c^8 + 3840*(9*B*a^4*b^5 - 2*A*a^3*b^6)*c^7 - 160*(56*B*a^3*b^7 - 15*
A*a^2*b^8)*c^6 + 48*(25*B*a^2*b^9 - 7*A*a*b^10)*c^5 - 18*(4*B*a*b^11 - A*b^
12)*c^4)*sqrt((B^4*b^4 + 81*A^4*c^4 - 18*(25*A^2*B^2*a - 6*A^3*B*b)*c^3 + (
625*B^4*a^2 - 300*A*B^3*a*b + 54*A^2*B^2*b^2)*c^2 - 2*(25*B^4*a*b^2 - 6*A*B
^3*b^3)*c)/(b^10*c^6 - 20*a*b^8*c^7 + 160*a^2*b...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7578 vs. $2(414) = 828$.

time = 8.37, size = 7578, normalized size = 16.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*(3*(2*b^4*c^3 - 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^
3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(
b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*(b^4*c - 8*a*b^2*c^2 + 16*a^2
*c^3)^2*A + (2*b^5*c^2 - 40*a*b^3*c^3 + 128*a^2*b*c^4 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*b^3*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a*c)*a*b*c^3)*(b^4
*c - 8*a*b^2*c^2 + 16*a^2*c^3)^2*B - 24*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^7*c^3 - 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 -
2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 2*a*b^7*c^4 + 48*sqrt
(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b
^5*c^5 + 24*a^2*b^5*c^5 - 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*
c^6 - 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 - 8*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 - 96*a^3*b^3*c^6 + 16*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 + 128*a^4*b*c^7 + 2*(b^2 - 4*a*c)*a*
b^5*c^4 - 16*(b^2 - 4*a*c)*a^2*b^3*c^5 + 32*(b^2 - 4*a*c)*a^3*b*c^6)*A*abs(
b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3) + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^8*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^3 - 2*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 2*a*b^8*c^3 - 192*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^2*b^5*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6
*c^4 - 16*a^2*b^6*c^4 + 896*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2
*c^5 + 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^5 + 12*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + 384*a^3*b^4*c^5 - 1280*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^6 - 640*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^4*b*c^6 - 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b
^2*c^6 - 1792*a^4*b^2*c^6 + 320*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4
*c^7 + 2560*a^5*c^7 + 2*(b^2 - 4*a*c)*a*b^6*c^3 + 24*(b^2 - 4*a*c)*a^2*b^4*
c^4 - 288*(b^2 - 4*a*c)*a^3*b^2*c^5 + 640*(b^2 - 4*a*c)*a^4*c^6)*B*abs(b^4*
```

```
c - 8*a*b^2*c^2 + 16*a^2*c^3) - 3*(2*b^12*c^5 - 8*a*b^10*c^6 - 192*a^2*b^8*
c^7 + 1792*a^3*b^6*c^8 - 5632*a^4*b^4*c^9 + 6144*a^5*b^2*c^10 - sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 + 2*sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 96*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 - sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^10*c^5 - 896*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c^6 - 192*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c^6 + 2816*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^7 + 1024*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^7 + 96*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^7 - 3072*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^8 - 1536*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^8 - 512*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^8 + 768*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^9 - 2*(b^2 - 4*a*c)*b
^10*c^5 + 192*(b^2 - 4*a*c)*a^2*b^6*c^7 - 1024*(b^2 - 4*a*c)*a^3*b^4*c^8 +
1536*(b^2 - 4*a*c)*a^4*b^2*c^9)*A - (2*b^13*c^4 - 68*a*b^11*c^5 + 688*a^2*b
^9*c^6 - 2688*a^3*b^7*c^7 + 2048*a^4*b^5*c^8 + 11264*a^5*b^3*c^9 - 20480*a^
6*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^13*c
^2 + 34*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^11*c^
3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^12*c^3 -
344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^9*c^4 -
 60*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^10*c^4 -
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^11*c^4 + 1344*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^7*c^5 + 448*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8*c^5 + 30*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9*c^5 - 1024*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -...
```

Mupad [B]
time = 3.95, size = 2500, normalized size = 5.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6*b*c^8 - 256*
B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 1966080*A*a^4*b
^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*B*a^4*b^6*c^5
 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c + 4096*a^6*c
^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840*a^4*b^4*c^5
- 6144*a^5*b^2*c^6)) - (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c
 - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040
```

*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A
^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2
*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*
a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*
c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*
a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^
10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*
b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 18
0*A*B*a*b^14*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*
a^2*b^16*c^5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8
 + 860160*a^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 26214
40*a^9*b^2*c^12)))^(1/2)*(256*b^11*c^3 - 5120*a*b^9*c^4 - 262144*a^5*b*c^8
+ 40960*a^2*b^7*c^5 - 163840*a^3*b^5*c^6 + 327680*a^4*b^3*c^7))/(32*(b^8*c
+ 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*a^3*b^2*c^4)))*(-(B^2*b
^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a
*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*
b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^
3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*
c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*
c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)
^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8
+ 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 +
992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 +
 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^
10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^5 - 7680*a^3*b^14*c^6 +
 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a^6*b^8*c^9 - 1966080*a^
7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*c^12)))^(1/2) - (x*(B^2
*b^8 - 288*A^2*a^3*c^5 + 9*A^2*b^6*c^2 + 800*B^2*a^4*c^4 + 6*A*B*b^7*c + 57
6*A^2*a^2*b^2*c^4 + 314*B^2*a^2*b^4*c^2 + 208*B^2*a^3*b^2*c^3 - 36*B^2*a*b^
6*c + 126*A^2*a*b^4*c^3 - 816*A*B*a^2*b^3*c^3 - 66*A*B*a*b^5*c^2 - 672*A*B*
a^3*b*c^4))/(32*(b^8*c + 256*a^4*c^5 - 16*a*b^6*c^2 + 96*a^2*b^4*c^3 - 256*
a^3*b^2*c^4)))*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2*c^2*(-(4*a*c - b^2)^15)
^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*b^16*c - 5040*A^2*a^2*b^
11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7*c^6 - 9216*A^2*a^5*b^5*
c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^2 - 10160*B^2*a^3*b^11*c
^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5 - 680960*B^2*a^6*b^5*c^6
 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 55*B^2*a*b^15*c - 25*B^2*
a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3 - 737280*A^2*a^7*b*c^9 -
 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 24000*A*B*a^3*b^10*c^4 - 24
1920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 1781760*A*B*a^6*b^4*c^7 + 7
37280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a*b^1
4*c^2)/(512*(1048576*a^10*c^13 + b^20*c^3 - 40*a*b^18*c^4 + 720*a^2*b^16*c^
5 - 7680*a^3*b^14*c^6 + 53760*a^4*b^12*c^7 - 258048*a^5*b^10*c^8 + 860160*a
^6*b^8*c^9 - 1966080*a^7*b^6*c^10 + 2949120*a^8*b^4*c^11 - 2621440*a^9*b^2*
c^12)))^(1/2)*1i - (((5242880*B*a^7*c^8 + 3072*A*a*b^11*c^3 - 3145728*A*a^6

```
*b*c^8 - 256*B*a*b^12*c^2 - 61440*A*a^2*b^9*c^4 + 491520*A*a^3*b^7*c^5 - 19
66080*A*a^4*b^5*c^6 + 3932160*A*a^5*b^3*c^7 + 61440*B*a^3*b^8*c^4 - 655360*
B*a^4*b^6*c^5 + 2949120*B*a^5*b^4*c^6 - 6291456*B*a^6*b^2*c^7)/(512*(b^12*c
 + 4096*a^6*c^7 - 24*a*b^10*c^2 + 240*a^2*b^8*c^3 - 1280*a^3*b^6*c^4 + 3840
*a^4*b^4*c^5 - 6144*a^5*b^2*c^6)) + (x*(-(B^2*b^17 + 9*A^2*b^15*c^2 + 9*A^2
*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*
b^16*c - 5040*A^2*a^2*b^11*c^4 + 37440*A^2*a^3*b^9*c^5 - 103680*A^2*a^4*b^7
*c^6 - 9216*A^2*a^5*b^5*c^7 + 552960*A^2*a^6*b^3*c^8 + 1140*B^2*a^2*b^13*c^
2 - 10160*B^2*a^3*b^11*c^3 + 34880*B^2*a^4*b^9*c^4 + 43776*B^2*a^5*b^7*c^5
- 680960*B^2*a^6*b^5*c^6 + 1863680*B^2*a^7*b^3*c^7 + 983040*A*B*a^8*c^9 - 5
5*B^2*a*b^15*c - 25*B^2*a*c*(-(4*a*c - b^2)^15)^(1/2) + 180*A^2*a*b^13*c^3
- 737280*A^2*a^7*b*c^9 - 1720320*B^2*a^8*b*c^8 + 240*A*B*a^2*b^12*c^3 + 240
00*A*B*a^3*b^10*c^4 - 241920*A*B*a^4*b^8*c^5 + 992256*A*B*a^5*b^6*c^6 - 178
1760*A*B*a^6*b^4*c^7 + 737280*A*B*a^7*b^2*c^8 + 6*A*B*b*c*(-(4*a*c - b^2)^1
5)^(1/2) - 180*A*B*a*b^14*c^2)/(512*(1048576*a^...
```

3.134 $\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=380

$$-\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}+\frac{3x(4abB-A(b^2+4ac)+(b^2B-4Abc+4aBc)\,x^2)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}+\frac{3\Big(b^2B-4Abc+}{}$$

```
[Out] -1/4*x^3*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2+3/8*x*
(4*a*b*B-A*(4*a*c+b^2)+(-4*A*b*c+4*B*a*c+B*b^2)*x^2)/(-4*a*c+b^2)^2/(c*x^4+
b*x^2+a)+3/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*B
-4*A*b*c+4*a*B*c+(8*A*a*c^2+6*A*b^2*c-12*B*a*b*c-B*b^3)/(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^2*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+3/16*arctan(x*
2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*B-4*A*b*c+4*a*B*c+(-8*A*
a*c^2-6*A*b^2*c+12*B*a*b*c+B*b^3)/(-4*a*c+b^2)^(1/2))/(-4*a*c+b^2)^2*2^(1/2
)/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.94, antiderivative size = 380, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1289, 1180, 211}

$$\frac{3\left(-\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\sqrt{c}\,(b^2-4ac)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\left(\frac{-8aAc^2+12abBc-6Ab^2c+b^3B}{\sqrt{b^2-4ac}}+4aBc-4Abc+b^2B\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\sqrt{c}\,(b^2-4ac)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{3x(x^2(4aBc-4Abc+b^2B)-A(4ac+b^2)+4abB)}{8\,(b^2-4ac)^2\,(a+bx^2+cx^4)}-\frac{x^3(-2aB-(x^2(bB-2Ac))+Ab)}{4\,(b^2-4ac)\,(a+bx^2+cx^4)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x^3*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*
x^4)^2) + (3*x*(4*a*b*B - A*(b^2 + 4*a*c) + (b^2*B - 4*A*b*c + 4*a*B*c)*x^2
))/(8*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (3*(b^2*B - 4*A*b*c + 4*a*B*c
- (b^3*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4
*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*(b^2*B - 4*A*b*c + 4*a*B*c + (b^3
*B - 6*A*b^2*c + 12*a*b*B*c - 8*a*A*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*Sqrt[c]*(b^2 - 4*a*c)^
2*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^4(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx &= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{\int \frac{x^2(3(Ab-2aB)+3(bB-2Ac)x^2)}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)}\\
&= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x(4abB-A(b^2+4ac)+(b^2B-4Abc+4aB}{8(b^2-4ac)^2(a+bx^2+cx^4)}\\
&= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x(4abB-A(b^2+4ac)+(b^2B-4Abc+4aB}{8(b^2-4ac)^2(a+bx^2+cx^4)}\\
&= -\frac{x^3(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{3x(4abB-A(b^2+4ac)+(b^2B-4Abc+4aB}{8(b^2-4ac)^2(a+bx^2+cx^4)}
\end{aligned}
$$

Mathematica [A]
time = 1.08, size = 447, normalized size = 1.18

$$
\frac{\frac{-4abBx+4b(-bB+Ac)x^3+8acx(A+Bx^2)}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(2b^3B+4ac^2(A+3Bx^2)+4bc(aB-3Acx^2)+b^2(-7Ac+3Bcx^2))}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{3\sqrt{2}\sqrt{c}\left(-b^3B-4bc\left(3aB+A\sqrt{b^2-4ac}\right)+4ac\left(2Ac+B\sqrt{b^2-4ac}\right)+b^2\left(6Ac+B\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{3\sqrt{2}\sqrt{c}\left(b^3B+4bc\left(3aB-A\sqrt{b^2-4ac}\right)+b^2\left(-6Ac+B\sqrt{b^2-4ac}\right)+4ac\left(-2Ac+B\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16c}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

[Out] ((-4*a*b*B*x + 4*b*(-(b*B) + A*c)*x^3 + 8*a*c*x*(A + B*x^2))/((b^2 - 4*a*c)
*(a + b*x^2 + c*x^4)^2) + (2*x*(2*b^3*B + 4*a*c^2*(A + 3*B*x^2) + 4*b*c*(a*
B - 3*A*c*x^2) + b^2*(-7*A*c + 3*B*c*x^2)))/((b^2 - 4*a*c)^2*(a + b*x^2 + c
*x^4)) + (3*Sqrt[2]*Sqrt[c]*(-(b^3*B) - 4*b*c*(3*a*B + A*Sqrt[b^2 - 4*a*c])
 + 4*a*c*(2*A*c + B*Sqrt[b^2 - 4*a*c]) + b^2*(6*A*c + B*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5
/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (3*Sqrt[2]*Sqrt[c]*(b^3*B + 4*b*c*(3*a*B
 - A*Sqrt[b^2 - 4*a*c]) + b^2*(-6*A*c + B*Sqrt[b^2 - 4*a*c]) + 4*a*c*(-2*A*
c + B*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*
a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/(16*c)

Maple [A]
time = 0.08, size = 495, normalized size = 1.30

method	result
risch	$\frac{-\frac{3c\left(4bcA-4acB-b^{2}B\right)x^{7}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(4c^{2}aA-19A\,b^{2}c+16abBc+5b^{3}B\right)x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}-\frac{\left(16Aabc+5A\,b^{3}+4a^{2}cB-19Ba\,b^{2}\right)x^{3}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3a\left(4acA+A\,b^{2}-4abB\right)x}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3\left(-\cdots\right.}{\cdots}$
default	$\frac{-\frac{3c\left(4bcA-4acB-b^{2}B\right)x^{7}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}+\frac{\left(4c^{2}aA-19A\,b^{2}c+16abBc+5b^{3}B\right)x^{5}}{128a^{2}c^{2}-64a\,b^{2}c+8b^{4}}-\frac{\left(16Aabc+5A\,b^{3}+4a^{2}cB-19Ba\,b^{2}\right)x^{3}}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}-\frac{3a\left(4acA+A\,b^{2}-4abB\right)x}{8\left(16a^{2}c^{2}-8a\,b^{2}c+b^{4}\right)}}{\left(c\,x^{4}+b\,x^{2}+a\right)^{2}}+\frac{3c\left(-\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)

[Out] (-3/8*c*(4*A*b*c-4*B*a*c-B*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*(4*A*a*c
^2-19*A*b^2*c+16*B*a*b*c+5*B*b^3)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5-1/8*(16*A*
a*b*c+5*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^3-3/8*a*(4
*A*a*c+A*b^2-4*B*a*b)/(16*a^2*c^2-8*a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+3/2/(
16*a^2*c^2-8*a*b^2*c+b^4)*c*(-1/8*(-4*b*c*A*(-4*a*c+b^2)^(1/2)+8*c^2*a*A+6*
A*b^2*c+4*a*c*B*(-4*a*c+b^2)^(1/2)+b^2*B*(-4*a*c+b^2)^(1/2)-12*a*b*B*c-b^3*
B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c
*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-4*b*c*A*(-4*a*c+b^2)^(1
/2)-8*c^2*a*A-6*A*b^2*c+4*a*c*B*(-4*a*c+b^2)^(1/2)+b^2*B*(-4*a*c+b^2)^(1/2)
+12*a*b*B*c+b^3*B)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^
(1/2)*arctan(c*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + (5*B*b^3 + 4*A*a*c^2 + (16*B*a*b
 - 19*A*b^2)*c)*x^5 + (19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x^3 +
3*(4*B*a^2*b - A*a*b^2 - 4*A*a^2*c)*x)/((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4
)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c
+ 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c
+ 16*a^3*b*c^2)*x^2) - 3/8*integrate((4*B*a*b - A*b^2 - 4*A*a*c - (B*b^2 +
4*(B*a - A*b)*c)*x^2)/(c*x^4 + b*x^2 + a), x)/(b^4 - 8*a*b^2*c + 16*a^2*c^2
)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5650 vs. $2(336) = 672$.
time = 4.31, size = 5650, normalized size = 14.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(6*(B*b^2*c + 4*(B*a - A*b)*c^2)*x^7 + 2*(5*B*b^3 + 4*A*a*c^2 + (16*B*
a*b - 19*A*b^2)*c)*x^5 + 2*(19*B*a*b^2 - 5*A*b^3 - 4*(B*a^2 + 4*A*a*b)*c)*x
^3 - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 + 2*(b^5*c - 8*a
*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2 + (b^6 -
6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3*b*c^2)*x^2)*s
qrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*
B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c +
(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b
^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c
^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6
- 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^
4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*b^4 - A*B^3*a*
b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (16*B^4*a^4 - 8
0*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3*b^2 - 40*A*B^
3*a^2*b^3 + A^3*B*b^5)*c)*x + 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 - A*B^2*a*b^8 -
 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3*b^2)*c^4 - 64
*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(24*B^3*a^4*b^3
+ 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^2*a^2*b^6 + 4*
A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 - 2048*(2*B*a^7*
b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5 + 1280*(2*B*a
```

```
^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 + 8*(14*B*a^3*b
^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^4*a^2 - 2*A^2*
B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a
^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*b^5 - 16*(4*A*
B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2
 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2
 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqr
t((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*
a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*
c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 -
 1024*a^6*c^6))) + 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a^2*c^4)*x^8 +
2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*b^2*c + 16*a^4
*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*b^3*c + 16*a^3
*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^
2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 +
 A^2*b^5)*c + (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^
4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^
2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 128
0*a^6*b^2*c^6 - 1024*a^7*c^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^
3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6))*log(-27*(5*B^4*a^2*
b^4 - A*B^3*a*b^5 - 16*A^4*a^2*c^4 + 40*(2*A^3*B*a^2*b - A^4*a*b^2)*c^3 + (
16*B^4*a^4 - 80*A*B^3*a^3*b + 40*A^3*B*a*b^3 - 5*A^4*b^4)*c^2 + (40*B^4*a^3
*b^2 - 40*A*B^3*a^2*b^3 + A^3*B*b^5)*c)*x - 27/2*sqrt(1/2)*(4*B^3*a^2*b^7 -
 A*B^2*a*b^8 - 256*A^3*a^4*c^5 + 128*(2*A*B^2*a^5 + 2*A^2*B*a^4*b + A^3*a^3
*b^2)*c^4 - 64*(4*B^3*a^5*b + 2*A*B^2*a^4*b^2 + 3*A^2*B*a^3*b^3)*c^3 + 8*(2
4*B^3*a^4*b^3 + 6*A^2*B*a^2*b^5 - A^3*a*b^6)*c^2 - (48*B^3*a^3*b^5 - 8*A*B^
2*a^2*b^6 + 4*A^2*B*a*b^7 - A^3*b^8)*c - (4096*(2*B*a^8 - 3*A*a^7*b)*c^7 -
2048*(2*B*a^7*b^2 - 7*A*a^6*b^3)*c^6 - 1280*(2*B*a^6*b^4 + 5*A*a^5*b^5)*c^5
 + 1280*(2*B*a^5*b^6 + A*a^4*b^7)*c^4 - 80*(10*B*a^4*b^8 + A*a^3*b^9)*c^3 +
 8*(14*B*a^3*b^10 - A*a^2*b^11)*c^2 - (6*B*a^2*b^12 - A*a*b^13)*c)*sqrt((B^
4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b
^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))*sqrt(-(B^2*a*
b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 +
A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 - 20*A*B*a*b^4 + A^2*b^5)*c + (a*b^10*c -
20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 102
4*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*
b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c
^7)))/(a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 - 640*a^4*b^4*c^4 + 1280
*a^5*b^2*c^5 - 1024*a^6*c^6))) - 3*sqrt(1/2)*((b^4*c^2 - 8*a*b^2*c^3 + 16*a
^2*c^4)*x^8 + 2*(b^5*c - 8*a*b^3*c^2 + 16*a^2*b*c^3)*x^6 + a^2*b^4 - 8*a^3*
b^2*c + 16*a^4*c^2 + (b^6 - 6*a*b^4*c + 32*a^3*c^3)*x^4 + 2*(a*b^5 - 8*a^2*
b^3*c + 16*a^3*b*c^2)*x^2)*sqrt(-(B^2*a*b^5 - 16*(4*A*B*a^3 - 5*A^2*a^2*b)*
c^3 + 40*(2*B^2*a^3*b - 4*A*B*a^2*b^2 + A^2*a*b^3)*c^2 + (40*B^2*a^2*b^3 -
20*A*B*a*b^4 + A^2*b^5)*c - (a*b^10*c - 20*a^2*b^8*c^2 + 160*a^3*b^6*c^3 -
640*a^4*b^4*c^4 + 1280*a^5*b^2*c^5 - 1024*a^6*c^6)*sqrt((B^4*a^2 - 2*A^2*B^
```

```
2*a*c + A^4*c^2)/(a^2*b^10*c^2 - 20*a^3*b^8*c^3 + 160*a^4*b^6*c^4 - 640*a^5
*b^4*c^5 + 1280*a^6*b^2*c^6 - 1024*a^7*c^7)))/(...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3164 vs. $2(336) = 672$.
time = 6.50, size = 3164, normalized size = 8.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 3/32*((sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5
*c - 2*b^6*c - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 8*a*b^4*c^2 + 2*b^5*c^2 + 64
*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt(2)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 128*a^3*c^4 - 96*a^2*b*c^4 - sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b*c^3 + 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2 - 4*a*c)*b^3
*c^2 - 32*(b^2 - 4*a*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*A - 2*(2*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^2*b^3*c - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 4
*a*b^5*c + 32*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 16*sqrt(2
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a*b^3*c^2 + 32*a^2*b^3*c^2 + 6*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 64*a^3*b*c^3 - 16*a^2*b^2*c^3 - 32*a^3
*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4 +
8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 16*sqrt(
```

```
2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 4*sqrt(2)*sqrt(b^2
 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 4*(b^2 - 4*a*c)*a*b^3*c
 - 16*(b^2 - 4*a*c)*a^2*b*c^2 - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)
*a^2*c^3)*B)*arctan(2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 + sq
rt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2
)*(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^2*c^3)))
/((a*b^8 - 16*a^2*b^6*c - 2*a*b^7*c + 96*a^3*b^4*c^2 + 24*a^2*b^5*c^2 + a*b
^6*c^2 - 256*a^4*b^2*c^3 - 96*a^3*b^3*c^3 - 12*a^2*b^4*c^3 + 256*a^5*c^4 +
128*a^4*b*c^4 + 48*a^3*b^2*c^4 - 64*a^4*c^5)*abs(c)) + 3/32*((sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*b^6 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^4*c - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c + 2*b^6*c - 16*sq
rt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqrt(b*c - sqrt
(b^2 - 4*a*c)*c)*b^4*c^2 - 8*a*b^4*c^2 + 2*b^5*c^2 + 64*sqrt(2)*sqrt(b*c -
sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 32*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^2*b*c^3 - 32*a^2*b^2*c^3 + 16*a*b^3*c^3 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*c^4 + 128*a^3*c^4 - 96*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*b^4*c + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a^2*b*c^2 + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^4*c - 2*(b^2 - 4*a*c)*b^3*c^2 + 32*(b^2 - 4*a
*c)*a^2*c^3 - 24*(b^2 - 4*a*c)*a*b*c^3)*A - 2*(2*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a*b^5 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c
- 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*a*b^5*c + 32*sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 16*sqrt(2)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*a^2*b^2*c^2 + 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3
*c^2 - 32*a^2*b^3*c^2 + 6*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*b*c^3 + 64*a^3*b*c^3 - 16*a^2*b^2*c^3 - 32*a^3*c^4 - 3*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4 + 8*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c + 6*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^2 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c -
 sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 4*(b^2 - 4*a*c)*a*b^3*c + 16*(b^2 - 4*a*c)*
a^2*b*c^2 - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*B)*arctan(
2*sqrt(1/2)*x/sqrt((b^5 - 8*a*b^3*c + 16*a^2*b*c^2 - sqrt((b^5 - 8*a*b^3*c
+ 16*a^2*b*c^2)^2 - 4*(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2)*(b^4*c - 8*a*b^2*c
^2 + 16*a^2*c^3)))/(b^4*c - 8*a*b^2*c^2 + 16*a^...
```

Mupad [B]

time = 3.49, size = 2500, normalized size = 6.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] atan(((((3*(1048576*A*a^6*c^8 - 256*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B
*a*b^11*c^2 - 1048576*B*a^6*b*c^7 - 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*
c^6 - 1048576*A*a^5*b^2*c^7 - 20480*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 -
655360*B*a^4*b^5*c^5 + 1310720*B*a^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 +
240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 -
24*a*b^10*c)) - (x*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2
*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2
*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6
*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*
c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20
*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b
*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 +
 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 2
0*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c
^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*
a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*
c^10 + a*b^20*c)))^(1/2)*(256*b^11*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7
+ 40960*a^2*b^7*c^4 - 163840*a^3*b^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 +
256*a^4*c^4 + 96*a^2*b^4*c^2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*
b^15 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)
^15)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^
7*c^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2
 + 4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61
440*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7
*b*c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 6
4*A*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360
*A*B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*
a^11*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*
a^5*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c
^8 + 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2) - (x*(
9*B^2*b^6*c + 288*A^2*a^2*c^5 + 234*A^2*b^4*c^3 - 288*B^2*a^3*c^4 + 576*B^2
*a^2*b^2*c^3 - 90*A*B*b^5*c^2 + 144*A^2*a*b^2*c^4 + 126*B^2*a*b^4*c^2 - 720
*A*B*a*b^3*c^3 - 288*A*B*a^2*b*c^4))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^
2 - 256*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2
)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^
11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^
6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 4160*B^2*a^4*b^9*c^3 - 1
1520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440*B^2*a^7*b^3*c^6 + 65536
```

```
*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*c^8 + 20*B^2*a^2*b^13*c
- 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A*B*a^3*b^10*c^3 - 11520*
A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*B*a^6*b^4*c^6 + 81920*A*
B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^11*c^11 - 40*a^2*b^18*c^2
 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5*b^12*c^5 - 258048*a^6*b
^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8 + 2949120*a^9*b^4*c^9 -
2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*1i - (((3*(1048576*A*a^6*c^8 - 25
6*A*b^12*c^2 + 4096*A*a*b^10*c^3 + 1024*B*a*b^11*c^2 - 1048576*B*a^6*b*c^7
- 20480*A*a^2*b^8*c^4 + 327680*A*a^4*b^4*c^6 - 1048576*A*a^5*b^2*c^7 - 2048
0*B*a^2*b^9*c^3 + 163840*B*a^3*b^7*c^4 - 655360*B*a^4*b^5*c^5 + 1310720*B*a
^5*b^3*c^6))/(512*(b^12 + 4096*a^6*c^6 + 240*a^2*b^8*c^2 - 1280*a^3*b^6*c^3
 + 3840*a^4*b^4*c^4 - 6144*a^5*b^2*c^5 - 24*a*b^10*c)) + (x*(-(9*(B^2*a*b^1
5 + B^2*a*(-(4*a*c - b^2)^15)^(1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15
)^(1/2) - 560*A^2*a^2*b^11*c^3 + 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c
^5 - 1024*A^2*a^5*b^5*c^6 + 61440*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 +
4160*B^2*a^4*b^9*c^3 - 11520*B^2*a^5*b^7*c^4 - 1024*B^2*a^6*b^5*c^5 + 61440
*B^2*a^7*b^3*c^6 + 65536*A*B*a^8*c^8 + 20*A^2*a*b^13*c^2 - 81920*A^2*a^7*b*
c^8 + 20*B^2*a^2*b^13*c - 81920*B^2*a^8*b*c^7 + 240*A*B*a^2*b^12*c^2 - 64*A
*B*a^3*b^10*c^3 - 11520*A*B*a^4*b^8*c^4 + 66560*A*B*a^5*b^6*c^5 - 143360*A*
B*a^6*b^4*c^6 + 81920*A*B*a^7*b^2*c^7 - 20*A*B*a*b^14*c))/(512*(1048576*a^1
1*c^11 - 40*a^2*b^18*c^2 + 720*a^3*b^16*c^3 - 7680*a^4*b^14*c^4 + 53760*a^5
*b^12*c^5 - 258048*a^6*b^10*c^6 + 860160*a^7*b^8*c^7 - 1966080*a^8*b^6*c^8
+ 2949120*a^9*b^4*c^9 - 2621440*a^10*b^2*c^10 + a*b^20*c)))^(1/2)*(256*b^11
*c^2 - 5120*a*b^9*c^3 - 262144*a^5*b*c^7 + 40960*a^2*b^7*c^4 - 163840*a^3*b
^5*c^5 + 327680*a^4*b^3*c^6))/(32*(b^8 + 256*a^4*c^4 + 96*a^2*b^4*c^2 - 256
*a^3*b^2*c^3 - 16*a*b^6*c)))*(-(9*(B^2*a*b^15 + B^2*a*(-(4*a*c - b^2)^15)^(
1/2) + A^2*b^15*c - A^2*c*(-(4*a*c - b^2)^15)^(1/2) - 560*A^2*a^2*b^11*c^3
+ 4160*A^2*a^3*b^9*c^4 - 11520*A^2*a^4*b^7*c^5 - 1024*A^2*a^5*b^5*c^6 + 614
40*A^2*a^6*b^3*c^7 - 560*B^2*a^3*b^11*c^2 + 416...
```

3.135 $\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=438

$$-\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{x(aB(7b^2-4ac)-A(b^3+8abc)+c(12abB-A(b^2+20ac))x^2)}{8a(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{\sqrt{c}\,(6a$$

```
[Out] -1/4*x*(A*b-2*a*B-(-2*A*c+B*b)*x^2)/(-4*a*c+b^2)/(c*x^4+b*x^2+a)^2-1/8*x*(a
*B*(-4*a*c+7*b^2)-A*(8*a*b*c+b^3)+c*(12*a*b*B-A*(20*a*c+b^2))*x^2)/a/(-4*a*
c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2
))^(1/2))*c^(1/2)*(6*a*B*(3*b^2+4*a*c-2*b*(-4*a*c+b^2)^(1/2))+A*(b^3-52*a*b
*c+b^2*(-4*a*c+b^2)^(1/2)+20*a*c*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(5/2)*
2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a
*c+b^2)^(1/2))^(1/2))*c^(1/2)*(6*a*B*(3*b^2+4*a*c+2*b*(-4*a*c+b^2)^(1/2))+A
*(b^3-52*a*b*c-b^2*(-4*a*c+b^2)^(1/2)-20*a*c*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c
+b^2)^(5/2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 0.69, antiderivative size = 438, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1289, 1192, 1180, 211}

$$\frac{\sqrt{c}\left(A\left(b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(-2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a\,(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(A\left(-b^2\sqrt{b^2-4ac}-20ac\sqrt{b^2-4ac}-52abc+b^3\right)+6aB\left(2b\sqrt{b^2-4ac}+4ac+3b^2\right)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a\,(b^2-4ac)^{5/2}\sqrt{\sqrt{b^2-4ac}+b}}-\frac{x(-2aB-(x^2(bB-2Ac))+Ab)}{4(b^2-4ac)(a+bx^2+cx^4)^2}-\frac{x(-A(8abc+b^3)+cx^2(12abB-A(20ac+b^2))+aB(7b^2-4ac))}{8a(b^2-4ac)^2(a+bx^2+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] -1/4*(x*(A*b - 2*a*B - (b*B - 2*A*c)*x^2))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) - (x*(a*B*(7*b^2 - 4*a*c) - A*(b^3 + 8*a*b*c) + c*(12*a*b*B - A*(b^2
+ 20*a*c))*x^2))/(8*a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(6*a*
B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c + b^2*Sqrt[b^
2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b -
 Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
4*a*c]]) - (Sqrt[c]*(6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3
 - 52*a*b*c - b^2*Sqrt[b^2 - 4*a*c] - 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a*(b^2 - 4*a*c)^(
5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2(A+Bx^2)}{(a+bx^2+cx^4)^3}\,dx = -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{\int \frac{Ab-2aB+5(bB-2Ac)x^2}{(a+bx^2+cx^4)^2}\,dx}{4(b^2-4ac)}$$

$$= -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x(aB(7b^2-4ac)-A(b^3+8abc)+c(12abB-}{8a(b^2-4ac)^2(a+bx^2+cx^4}$$

$$= -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x(aB(7b^2-4ac)-A(b^3+8abc)+c(12abB-}{8a(b^2-4ac)^2(a+bx^2+cx^4}$$

$$= -\frac{x(Ab-2aB-(bB-2Ac)x^2)}{4(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{x(aB(7b^2-4ac)-A(b^3+8abc)+c(12abB-}{8a(b^2-4ac)^2(a+bx^2+cx^4}$$

Mathematica [A]

time = 1.02, size = 436, normalized size = 1.00

$$\frac{1}{16}\left(\frac{4x(B(2a+bx^2)-A(b+2cx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(aB(-7b^2+4ac-12bcx^2)+A(b^3+8abc+b^2cx^2+20ac^2x^2))}{a(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(6aB\left(3b^2+4ac-2b\sqrt{b^2-4ac}\right)+A\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(-6aB\left(3b^2+4ac+2b\sqrt{b^2-4ac}\right)+A\left(-b^3+52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{a(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((4*x*(B*(2*a + b*x^2) - A*(b + 2*c*x^2)))/((b^2 - 4*a*c)*(a + b*x^2 + c*x^
4)^2) + (2*x*(a*B*(-7*b^2 + 4*a*c - 12*b*c*x^2) + A*(b^3 + 8*a*b*c + b^2*c*
x^2 + 20*a*c^2*x^2)))/(a*(b^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sq
rt[c]*(6*a*B*(3*b^2 + 4*a*c - 2*b*Sqrt[b^2 - 4*a*c]) + A*(b^3 - 52*a*b*c +
b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^(5/2)*Sqrt[b - Sqrt[b^2 -
 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(-6*a*B*(3*b^2 + 4*a*c + 2*b*Sqrt[b^2 - 4*a*c]
) + A*(-b^3 + 52*a*b*c + b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]))
*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(a*(b^2 - 4*a*c)^
(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]))/16
```

Maple [A]

time = 0.08, size = 524, normalized size = 1.20

method	result
risch	$\frac{\frac{c^2\left(20acA+Ab^2-12abB\right)x^7}{8a\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{c\left(28Aabc+2Ab^3+4a^2cB-19Bab^2\right)x^5}{8a\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{\left(36Aa^2c^2+5Aab^2c+Ab^4-16a^2bBc-5Bab^3\right)x^3}{8a\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{\left(16Aabc-Ab^3-12a^2cB\right.}{128a^2c^2-64ab^2c}}{\left(cx^4+bx^2+a\right)^2}$
default	$\frac{\frac{c^2\left(20acA+Ab^2-12abB\right)x^7}{8a\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{c\left(28Aabc+2Ab^3+4a^2cB-19Bab^2\right)x^5}{8a\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{\left(36Aa^2c^2+5Aab^2c+Ab^4-16a^2bBc-5Bab^3\right)x^3}{8a\left(16a^2c^2-8ab^2c+b^4\right)}+\frac{\left(16Aabc-Ab^3-12a^2cB\right.}{128a^2c^2-64ab^2c}}{\left(cx^4+bx^2+a\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*c^2*(20*A*a*c+A*b^2-12*B*a*b)/a/(16*a^2*c^2-8*a*b^2*c+b^4)*x^7+1/8*c/a
*(28*A*a*b*c+2*A*b^3+4*B*a^2*c-19*B*a*b^2)/(16*a^2*c^2-8*a*b^2*c+b^4)*x^5+1
/8*(36*A*a^2*c^2+5*A*a*b^2*c+A*b^4-16*B*a^2*b*c-5*B*a*b^3)/a/(16*a^2*c^2-8*
a*b^2*c+b^4)*x^3+1/8*(16*A*a*b*c-A*b^3-12*B*a^2*c-3*B*a*b^2)/(16*a^2*c^2-8*
a*b^2*c+b^4)*x)/(c*x^4+b*x^2+a)^2+1/2/a/(16*a^2*c^2-8*a*b^2*c+b^4)*c*(-1/8*
(20*A*(-4*a*c+b^2)^(1/2)*a*c+A*(-4*a*c+b^2)^(1/2)*b^2-52*A*a*b*c+A*b^3-12*a
```

```
*b*B*(-4*a*c+b^2)^(1/2)+24*a^2*c*B+18*B*a*b^2)/(-4*a*c+b^2)^(1/2)*2^(1/2)/(
(-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2
))*c)^(1/2))+1/8*(20*A*(-4*a*c+b^2)^(1/2)*a*c+A*(-4*a*c+b^2)^(1/2)*b^2+52*A
*a*b*c-A*b^3-12*a*b*B*(-4*a*c+b^2)^(1/2)-24*a^2*c*B-18*B*a*b^2)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + (4*(B*a^2 + 7*A*a*b)*c^2 -
 (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - (5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 + (16*B*
a^2*b - 5*A*a*b^2)*c)*x^3 - (3*B*a^2*b^2 + A*a*b^3 + 4*(3*B*a^3 - 4*A*a^2*b
)*c)*x)/((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2
*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 -
 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x
^2) + 1/8*integrate((3*B*a*b^2 + A*b^3 + (20*A*a*c^2 - (12*B*a*b - A*b^2)*c
)*x^2 + 4*(3*B*a^2 - 4*A*a*b)*c)/(c*x^4 + b*x^2 + a), x)/(a*b^4 - 8*a^2*b^2
*c + 16*a^3*c^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7270 vs. $2(382) = 764$.
time = 8.31, size = 7270, normalized size = 16.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(20*A*a*c^3 - (12*B*a*b - A*b^2)*c^2)*x^7 + 2*(4*(B*a^2 + 7*A*a*b)*
c^2 - (19*B*a*b^2 - 2*A*b^3)*c)*x^5 - 2*(5*B*a*b^3 - A*b^4 - 36*A*a^2*c^2 +
 (16*B*a^2*b - 5*A*a*b^2)*c)*x^3 + sqrt(1/2)*((a*b^4*c^2 - 8*a^2*b^2*c^3 +
16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b
^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^
2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6
 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*
a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a
*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 12
80*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*
B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^
3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^
```

2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a
^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*
c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*
(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3
 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2 - 216*A*B^3*a^3*b^3 - 648*A^2*B^
2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^2 - 15*(27*B^4*a^3*b^4 + 27*A*B^
3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)*x + 1/2*sqrt(1/2)*(27*B^3*a^3*b
^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^3*b^11 + 6400*(3*A^2*B*a^6 - 4*A
^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*a^6*b + 66*A^2*B*a^5*b^2 - 341*A
^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b
^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a^4*b^5 + 102*A^2*B*a^3*b^6 + 47*
A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8
 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*b^14 + 40960*A*a^10*c^7 - 4096*(
9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*a^9*b^3 + A*a^8*b^4)*c^5 - 6400*
(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a^7*b^7 - 17*A*a^6*b^8)*c^3 - 240
*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a^5*b^11 + 19*A*a^4*b^12)*c)*sqrt
((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*
b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(
a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2
*c^4 - 1024*a^11*c^5)))*sqrt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*
(4*A*B*a^4 - 7*A^2*a^3*b)*c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a
^2*b^3)*c^2 + 5*(72*B^2*a^3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^
10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 -
1024*a^8*c^5)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*
A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b
 + A^4*a*b^2)*c)/(a^6*b^10 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c
^3 + 1280*a^10*b^2*c^4 - 1024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))) - sqrt(1/2
)*((a*b^4*c^2 - 8*a^2*b^2*c^3 + 16*a^3*c^4)*x^8 + a^3*b^4 - 8*a^4*b^2*c + 1
6*a^5*c^2 + 2*(a*b^5*c - 8*a^2*b^3*c^2 + 16*a^3*b*c^3)*x^6 + (a*b^6 - 6*a^2
*b^4*c + 32*a^4*c^3)*x^4 + 2*(a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)*x^2)*sq
rt(-(9*B^2*a^2*b^5 + 6*A*B*a*b^6 + A^2*b^7 - 240*(4*A*B*a^4 - 7*A^2*a^3*b)*
c^3 + 40*(18*B^2*a^4*b - 48*A*B*a^3*b^2 + 7*A^2*a^2*b^3)*c^2 + 5*(72*B^2*a^
3*b^3 - 12*A*B*a^2*b^4 - 7*A^2*a*b^5)*c + (a^3*b^10 - 20*a^4*b^8*c + 160*a^
5*b^6*c^2 - 640*a^6*b^4*c^3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5)*sqrt((81*B^4
*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 62
5*A^4*a^2*c^2 - 50*(9*A^2*B^2*a^3 + 6*A^3*B*a^2*b + A^4*a*b^2)*c)/(a^6*b^10
 - 20*a^7*b^8*c + 160*a^8*b^6*c^2 - 640*a^9*b^4*c^3 + 1280*a^10*b^2*c^4 - 1
024*a^11*c^5)))/(a^3*b^10 - 20*a^4*b^8*c + 160*a^5*b^6*c^2 - 640*a^6*b^4*c^
3 + 1280*a^7*b^2*c^4 - 1024*a^8*c^5))*log((10000*A^4*a^3*c^5 - 15000*(2*A^3
*B*a^3*b - A^4*a^2*b^2)*c^4 - 3*(432*B^4*a^5 - 3024*A*B^3*a^4*b - 3312*A^2*
B^2*a^3*b^2 + 3864*A^3*B*a^2*b^3 + 497*A^4*a*b^4)*c^3 - 5*(648*B^4*a^4*b^2
 - 216*A*B^3*a^3*b^3 - 648*A^2*B^2*a^2*b^4 - 189*A^3*B*a*b^5 - 7*A^4*b^6)*c^
2 - 15*(27*B^4*a^3*b^4 + 27*A*B^3*a^2*b^5 + 9*A^2*B^2*a*b^6 + A^3*B*b^7)*c)
*x - 1/2*sqrt(1/2)*(27*B^3*a^3*b^8 + 27*A*B^2*a^2*b^9 + 9*A^2*B*a*b^10 + A^

```
3*b^11 + 6400*(3*A^2*B*a^6 - 4*A^3*a^5*b)*c^5 - 64*(108*B^3*a^7 - 72*A*B^2*
a^6*b + 66*A^2*B*a^5*b^2 - 341*A^3*a^4*b^3)*c^4 + 16*(216*B^3*a^6*b^2 - 324
*A*B^2*a^5*b^3 - 288*A^2*B*a^4*b^4 - 427*A^3*a^3*b^5)*c^3 + 20*(108*A*B^2*a
^4*b^5 + 102*A^2*B*a^3*b^6 + 47*A^3*a^2*b^7)*c^2 - (216*B^3*a^4*b^6 + 396*A
*B^2*a^3*b^7 + 267*A^2*B*a^2*b^8 + 53*A^3*a*b^9)*c - (3*B*a^4*b^13 + A*a^3*
b^14 + 40960*A*a^10*c^7 - 4096*(9*B*a^10*b + 8*A*a^9*b^2)*c^6 + 1536*(28*B*
a^9*b^3 + A*a^8*b^4)*c^5 - 6400*(3*B*a^8*b^5 - A*a^7*b^6)*c^4 + 160*(24*B*a
^7*b^7 - 17*A*a^6*b^8)*c^3 - 240*(B*a^6*b^9 - 2*A*a^5*b^10)*c^2 - 2*(12*B*a
^5*b^11 + 19*A*a^4*b^12)*c)*sqrt((81*B^4*a^4 + 108*A*B^3*a^3*b + 54*A^2*B^2
*a^2*b^2 + 12*A^3*B*a*b^3 + A^4*b^4 + 625*A^4*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7267 vs. 2(382) = 764.
time = 6.99, size = 7267, normalized size = 16.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] 1/64*((2*b^4*c^2 + 32*a*b^2*c^3 - 160*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^3*c + 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^2*c^2 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c
^3 - 2*(b^2 - 4*a*c)*b^2*c^2 - 40*(b^2 - 4*a*c)*a*c^3)*(a*b^4 - 8*a^2*b^2*c
 + 16*a^3*c^2)^2*A - 12*(2*a*b^3*c^2 - 8*a^2*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*a*b*c^2)*(a*b^4 - 8*a^2*b^2*c +
16*a^3*c^2)^2*B + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^9 - 28*sqr
t(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a*b^8*c - 2*a*b^9*c + 240*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
```

*c)*c)*a^3*b^5*c^2 + 48*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 56*a^2*b^7*c^2 - 832 *sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 288*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a* c)*c)*a^2*b^5*c^3 - 480*a^3*b^5*c^3 + 1024*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4* a*c)*c)*a^5*b*c^4 + 512*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + 1664*a^4*b^3*c ^4 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5 - 2048*a^5*b*c^5 + 2*(b^2 - 4*a*c)*a*b^7*c - 48*(b^2 - 4*a*c)*a^2*b^5*c^2 + 288*(b^2 - 4*a* c)*a^3*b^3*c^3 - 512*(b^2 - 4*a*c)*a^4*b*c^4)*A*abs(a*b^4 - 8*a^2*b^2*c + 1 6*a^3*c^2) + 6*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^8 - 8*sqrt(2) *sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 2*a^2*b^8*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 + 16 *a^3*b^6*c^2 + 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^3 + 32 *sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*c^4 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 16*s qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^4 - 256*a^5*b^2*c^4 + 64*s qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 + 512*a^6*c^5 + 2*(b^2 - 4*a *c)*a^2*b^6*c - 8*(b^2 - 4*a*c)*a^3*b^4*c^2 - 32*(b^2 - 4*a*c)*a^4*b^2*c^3 + 128*(b^2 - 4*a*c)*a^5*c^4)*B*abs(a*b^4 - 8*a^2*b^2*c + 16*a^3*c^2) + (2*a ^2*b^12*c^2 - 136*a^3*b^10*c^3 + 1856*a^4*b^8*c^4 - 10496*a^5*b^6*c^5 + 271 36*a^6*b^4*c^6 - 26624*a^7*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s qrt(b^2 - 4*a*c)*c)*a^2*b^12 + 68*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt (b^2 - 4*a*c)*c)*a^3*b^10*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b ^2 - 4*a*c)*c)*a^2*b^11*c - 928*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b ^2 - 4*a*c)*c)*a^4*b^8*c^2 - 128*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^9*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^10*c^2 + 5248*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^6*c^3 + 1344*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr t(b^2 - 4*a*c)*c)*a^4*b^7*c^3 + 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr t(b^2 - 4*a*c)*c)*a^3*b^8*c^3 - 13568*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*c^4 - 5120*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^4 - 672*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^4 + 13312*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^5 + 6656*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr t(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^5 + 2560*sqrt(2)*sqrt(b^2 - 4*a*c)*s qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^5 - 3328*sqrt(2)*sqrt(b^2 - 4*a*c) *sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^6 - 2*(b^2 - 4*a*c)*a^2*b^10*c^2 + 128*(b^2 - 4*a*c)*a^3*b^8*c^3 - 1344*(b^2 - 4*a*c)*a^4*b^6*c^4 + 5120*(b ^2 - 4*a*c)*a^5*b^4*c^5 - 6656*(b^2 - 4*a*c)*a^6*b^2*c^6)*A + 6*(6*a^3*b^11 *c^2 - 88*a^4*b^9*c^3 + 448*a^5*b^7*c^4 - 768*a^6*b^5*c^5 - 512*a^7*b^3*c^6 + 2048*a^8*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^11 + 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c

```
)*a^4*b^9*c + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^3*b^10*c - 224*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^5*b^7*c^2 - 64*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^4*b^8*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^
3*b^9*c^2 + 384*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a
^6*b^5*c^3 + 192*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt...
```

Mupad [B]
time = 3.92, size = 2500, normalized size = 5.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(A*b^4 + 36*A*a^2*c^2 - 5*B*a*b^3 + 5*A*a*b^2*c - 16*B*a^2*b*c))/(8*a
*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) - (x*(A*b^3 + 3*B*a*b^2 + 12*B*a^2*c - 16*
A*a*b*c))/(8*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(4*B*a^2*c^2 + 2*A*b^3*
c + 28*A*a*b*c^2 - 19*B*a*b^2*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c
*x^7*(20*A*a*c^2 + A*b^2*c - 12*B*a*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*
c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + atan((((
(256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^
11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^
6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 614
40*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*
b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 +
3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) - (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A
^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*
A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4
*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7
*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b
^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8
 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8
*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 +
 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 -
 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^
2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*
a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 25
8048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^1
1*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*(262144*a^7*b*c^7 - 256*a^2*b^11*
c^2 + 5120*a^3*b^9*c^3 - 40960*a^4*b^7*c^4 + 163840*a^5*b^5*c^5 - 327680*a^
6*b^3*c^6))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 25
6*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)
^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*
b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b
```

^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2) + (x*(A^2*b^6*c^3 - 800*A^2*a^3*c^6 + 288*B^2*a^4*c^5 + 1472*A^2*a^2*b^2*c^5 + 234*B^2*a^2*b^4*c^3 + 144*B^2*a^3*b^2*c^4 - 34*A^2*a*b^4*c^4 - 1104*A*B*a^2*b^3*c^4 + 6*A*B*a*b^5*c^3 - 288*A*B*a^3*b*c^5))/(32*(a^2*b^8 + 256*a^6*c^4 - 16*a^3*b^6*c + 96*a^4*b^4*c^2 - 256*a^5*b^2*c^3)))*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B^2*a^5*b^9*c^3 - 103680*B^2*a^6*b^7*c^4 - 9216*B^2*a^7*b^5*c^5 + 552960*B^2*a^8*b^3*c^6 + 983040*A*B*a^9*c^8 - 55*A^2*a*b^15*c - 25*A^2*a*c*(-(4*a*c - b^2)^15)^(1/2) - 1720320*A^2*a^8*b*c^8 + 180*B^2*a^3*b^13*c - 737280*B^2*a^9*b*c^7 + 240*A*B*a^3*b^12*c^2 + 24000*A*B*a^4*b^10*c^3 - 241920*A*B*a^5*b^8*c^4 + 992256*A*B*a^6*b^6*c^5 - 1781760*A*B*a^7*b^4*c^6 + 737280*A*B*a^8*b^2*c^7 + 6*A*B*a*b*(-(4*a*c - b^2)^15)^(1/2) - 180*A*B*a^2*b^14*c)/(512*(a^3*b^20 + 1048576*a^13*c^10 - 40*a^4*b^18*c + 720*a^5*b^16*c^2 - 7680*a^6*b^14*c^3 + 53760*a^7*b^12*c^4 - 258048*a^8*b^10*c^5 + 860160*a^9*b^8*c^6 - 1966080*a^10*b^6*c^7 + 2949120*a^11*b^4*c^8 - 2621440*a^12*b^2*c^9)))^(1/2)*1i - (((256*A*a*b^13*c^2 - 3145728*B*a^8*c^8 + 4194304*A*a^7*b*c^8 - 9216*A*a^2*b^11*c^3 + 122880*A*a^3*b^9*c^4 - 819200*A*a^4*b^7*c^5 + 2949120*A*a^5*b^5*c^6 - 5505024*A*a^6*b^3*c^7 + 768*B*a^2*b^12*c^2 - 12288*B*a^3*b^10*c^3 + 61440*B*a^4*b^8*c^4 - 983040*B*a^6*b^4*c^6 + 3145728*B*a^7*b^2*c^7)/(512*(a^2*b^12 + 4096*a^8*c^6 - 24*a^3*b^10*c + 240*a^4*b^8*c^2 - 1280*a^5*b^6*c^3 + 3840*a^6*b^4*c^4 - 6144*a^7*b^2*c^5)) + (x*(-(A^2*b^17 + 9*B^2*a^2*b^15 + A^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 9*B^2*a^2*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^16 + 1140*A^2*a^2*b^13*c^2 - 10160*A^2*a^3*b^11*c^3 + 34880*A^2*a^4*b^9*c^4 + 43776*A^2*a^5*b^7*c^5 - 680960*A^2*a^6*b^5*c^6 + 1863680*A^2*a^7*b^3*c^7 - 5040*B^2*a^4*b^11*c^2 + 37440*B...

3.136 $\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx$

Optimal. Leaf size=460

$$\frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}+\frac{x(abB(b^2+8ac)+A(3b^4-25ab^2c+28a^2c^2)+c(aB(b^2+20ac)}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}$$

```
[Out] 1/4*x*(A*b^2-a*b*B-2*a*A*c+(A*b-2*B*a)*c*x^2)/a/(-4*a*c+b^2)/(c*x^4+b*x^2+a
)^2+1/8*x*(a*b*B*(8*a*c+b^2)+A*(28*a^2*c^2-25*a*b^2*c+3*b^4)+c*(a*B*(20*a*c
+b^2)+3*A*(-8*a*b*c+b^3))*x^2)/a^2/(-4*a*c+b^2)^2/(c*x^4+b*x^2+a)+1/16*arct
an(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(20*a*c+b^2
)+3*A*(-8*a*b*c+b^3)+(a*b*B*(-52*a*c+b^2)+3*A*(56*a^2*c^2-10*a*b^2*c+b^4))/
(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)
+1/16*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a*B*(
20*a*c+b^2)+3*A*(-8*a*b*c+b^3)+(-a*b*B*(-52*a*c+b^2)-3*A*(56*a^2*c^2-10*a*b
^2*c+b^4))/(-4*a*c+b^2)^(1/2))/a^2/(-4*a*c+b^2)^2*2^(1/2)/(b+(-4*a*c+b^2)^(
1/2))^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1192, 1180, 211}

$$\frac{\sqrt{c}\left(\frac{3A(56a^2c^2-10ab^2c+b^4)+abB(b^2-52ac)}{\sqrt{b^2-4ac}}+3A(b^3-8abc)+aB(20ac+b^2)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{c}\left(-\frac{3A(56a^2c^2-10ab^2c+b^4)+abB(b^2-52ac)}{\sqrt{b^2-4ac}}+3A(b^3-8abc)+aB(20ac+b^2)\right)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)}{8\sqrt{2}\,a^2\left(b^2-4ac\right)^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{x(A(28a^2c^2-25ab^2c+3b^4)+cx^2(3A(b^3-8abc)+aB(20ac+b^2))+abB(8ac+b^2))}{8a^2\left(b^2-4ac\right)^2\left(a+bx^2+cx^4\right)}+\frac{x(cx^2(Ab-2aB)-2aAc-abB+Ab^2)}{4a\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (x*(A*b^2 - a*b*B - 2*a*A*c + (A*b - 2*a*B)*c*x^2))/(4*a*(b^2 - 4*a*c)*(a +
 b*x^2 + c*x^4)^2) + (x*(a*b*B*(b^2 + 8*a*c) + A*(3*b^4 - 25*a*b^2*c + 28*a
^2*c^2) + c*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c))*x^2))/(8*a^2*(b^2 -
4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[c]*(a*B*(b^2 + 20*a*c) + 3*A*(b^3 - 8
*a*b*c) + (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*a*b^2*c + 56*a^2*c^2))/Sqrt
[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(8*
Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[c]*(a*B*(b
^2 + 20*a*c) + 3*A*(b^3 - 8*a*b*c) - (a*b*B*(b^2 - 52*a*c) + 3*A*(b^4 - 10*
a*b^2*c + 56*a^2*c^2))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
 + Sqrt[b^2 - 4*a*c]]])/(8*Sqrt[2]*a^2*(b^2 - 4*a*c)^2*Sqrt[b + Sqrt[b^2 -
4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rubi steps

$$\int \frac{A+Bx^2}{(a+bx^2+cx^4)^3}\,dx = \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} - \frac{\int \frac{-3Ab^2-abB+14aAc-5(Ab-2aB)cx^2}{(a+bx^2+cx^4)^2}\,dx}{4a(b^2-4ac)}$$

$$= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(abB(b^2+8ac)+A(3b^4-25ab^2c+2\cdots}{8a^2(b^2-4\cdots}$$

$$= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(abB(b^2+8ac)+A(3b^4-25ab^2c+2\cdots}{8a^2(b^2-4\cdots}$$

$$= \frac{x(Ab^2-abB-2aAc+(Ab-2aB)cx^2)}{4a(b^2-4ac)(a+bx^2+cx^4)^2} + \frac{x(abB(b^2+8ac)+A(3b^4-25ab^2c+2\cdots}{8a^2(b^2-4\cdots}$$

Mathematica [A]
time = 1.39, size = 516, normalized size = 1.12

$$\frac{-\frac{4ax(aB(b+2cx^2)-A(b^2-2ac+bcx^2))}{(b^2-4ac)(a+bx^2+cx^4)^2}+\frac{2x(aB(b^3+8abc+b^2cx^2+20ac^2x^2)+A(3b^4-25ab^2c+28a^2c^2+3b^3cx^2-24abc^2x^2))}{(b^2-4ac)^2(a+bx^2+cx^4)}+\frac{\sqrt{2}\sqrt{c}\left(aB\left(b^3-52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)+3A\left(b^4-10ab^2c+56a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(aB\left(-b^3+52abc+b^2\sqrt{b^2-4ac}+20ac\sqrt{b^2-4ac}\right)+3A\left(-b^4+10ab^2c-56a^2c^2+b^3\sqrt{b^2-4ac}-8abc\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{(b^2-4ac)^{5/2}\sqrt{b+\sqrt{b^2-4ac}}}}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] ((-4*a*x*(a*B*(b + 2*c*x^2) - A*(b^2 - 2*a*c + b*c*x^2)))/((b^2 - 4*a*c)*(a
 + b*x^2 + c*x^4)^2) + (2*x*(a*B*(b^3 + 8*a*b*c + b^2*c*x^2 + 20*a*c^2*x^2)
 + A*(3*b^4 - 25*a*b^2*c + 28*a^2*c^2 + 3*b^3*c*x^2 - 24*a*b*c^2*x^2)))/((b
^2 - 4*a*c)^2*(a + b*x^2 + c*x^4)) + (Sqrt[2]*Sqrt[c]*(a*B*(b^3 - 52*a*b*c
+ b^2*Sqrt[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(b^4 - 10*a*b^2*c
 + 56*a^2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt
[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(a*B*(-b^3 + 52*a*b*c + b^2*Sqr
t[b^2 - 4*a*c] + 20*a*c*Sqrt[b^2 - 4*a*c]) + 3*A*(-b^4 + 10*a*b^2*c - 56*a^
2*c^2 + b^3*Sqrt[b^2 - 4*a*c] - 8*a*b*c*Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/((b^2 - 4*a*c)^(5/2)*Sqrt[b + Sqr
t[b^2 - 4*a*c]]))/(16*a^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. $2(416) = 832$.
time = 0.22, size = 1552, normalized size = 3.37

method	result
risch	$\frac{-\frac{c^{2}\left(24 A a b c-3 A b^{3}-20 a^{2} c B-B a b^{2}\right) x^{7}}{8 a^{2}\left(16 a^{2} c^{2}-8 a b^{2} c+b^{4}\right)}+\frac{c\left(28 A a^{2} c^{2}-49 A a b^{2} c+6 A b^{4}+28 a^{2} b B c+2 B a b^{3}\right) x^{5}}{8 a^{2}\left(16 a^{2} c^{2}-8 a b^{2} c+b^{4}\right)}-\frac{(4 A a^{2} b c^{2}+20 A a b^{3} c-3 A b^{5}-36 a^{3} B c^{2}-5 B a^{2}}{8 a^{2}\left(16 a^{2} c^{2}-8 a b^{2} c+b^{4}\right)}}{\left(c x^{4}+b x^{2}+a\right)^{2}}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 64*c^3*(-1/64/(-4*a*c+b^2)^(5/2)/c/(4*a*c-b^2)^2*((-1/32/c^2/a^2*(20*a*c*(-
4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)+4*a*b*c-b^3)*(2880*A*(-4*a*c+b^2)^(
1/2)*a^3*c^3-1968*A*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2+444*A*(-4*a*c+b^2)^(1/2)
*a*b^4*c-33*A*(-4*a*c+b^2)^(1/2)*b^6-4416*A*a^3*b*c^3+2736*A*a^2*b^3*c^2-54
0*A*a*b^5*c+33*A*b^7+3200*B*a^4*c^3-1248*B*a^3*b^2*c^2+24*B*a^2*b^4*c+22*B*
a*b^6)/(100*a*c+11*b^2)*x^3+1/16/c^2/a*(-6*b*(-4*a*c+b^2)^(1/2)+28*a*c-7*b^
2)*(4928*A*(-4*a*c+b^2)^(1/2)*a^3*c^3-3504*A*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2
+828*A*(-4*a*c+b^2)^(1/2)*a*b^4*c-65*A*(-4*a*c+b^2)^(1/2)*b^6-6464*A*a^3*b*
c^3+4272*A*a^2*b^3*c^2-924*A*a*b^5*c+65*A*b^7+6272*B*a^4*c^3-3552*B*a^3*b^2
*c^2+600*B*a^2*b^4*c-26*B*a*b^6)/(196*a*c-13*b^2)*x)/(x^2+1/2/c*(-4*a*c+b^2
)^(1/2)+1/2*b/c)^2-1/32*(20*a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^2)^(1/2)+5
2*a*b*c-b^3)*(26880*A*(-4*a*c+b^2)^(1/2)*a^4*c^4-26880*A*(-4*a*c+b^2)^(1/2)
*a^3*b^2*c^3+10080*A*(-4*a*c+b^2)^(1/2)*a^2*b^4*c^2-1680*A*(-4*a*c+b^2)^(1/
2)*a*b^6*c+105*A*(-4*a*c+b^2)^(1/2)*b^8-85248*A*a^4*b*c^4+61440*A*a^3*b^3*c
```

```
^3-16416*A*a^2*b^5*c^2+2016*A*a*b^7*c-105*A*b^9+12800*B*a^5*c^4+13312*B*a^4
*b^2*c^3-10176*B*a^3*b^4*c^2+1792*B*a^2*b^6*c-70*B*a*b^8)/a^2/(400*a^2*c^2+
616*a*b^2*c-35*b^4)/c*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2
^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/64/(-4*a*c+b^2)^(5/2)/c/(4*a*c-
b^2)^2*((-1/32/c^2/a^2*(-20*a*c*(-4*a*c+b^2)^(1/2)-b^2*(-4*a*c+b^2)^(1/2)+4
*a*b*c-b^3)*(-2880*A*(-4*a*c+b^2)^(1/2)*a^3*c^3+1968*A*(-4*a*c+b^2)^(1/2)*a
^2*b^2*c^2-444*A*(-4*a*c+b^2)^(1/2)*a*b^4*c+33*A*(-4*a*c+b^2)^(1/2)*b^6-441
6*A*a^3*b*c^3+2736*A*a^2*b^3*c^2-540*A*a*b^5*c+33*A*b^7+3200*B*a^4*c^3-1248
*B*a^3*b^2*c^2+24*B*a^2*b^4*c+22*B*a*b^6)/(100*a*c+11*b^2)*x^3+1/16/c^2/a*(
6*b*(-4*a*c+b^2)^(1/2)+28*a*c-7*b^2)*(-4928*A*(-4*a*c+b^2)^(1/2)*a^3*c^3+35
04*A*(-4*a*c+b^2)^(1/2)*a^2*b^2*c^2-828*A*(-4*a*c+b^2)^(1/2)*a*b^4*c+65*A*(
-4*a*c+b^2)^(1/2)*b^6-6464*A*a^3*b*c^3+4272*A*a^2*b^3*c^2-924*A*a*b^5*c+65*
A*b^7+6272*B*a^4*c^3-3552*B*a^3*b^2*c^2+600*B*a^2*b^4*c-26*B*a*b^6)/(196*a*
c-13*b^2)*x)/(x^2+1/2*b/c-1/2/c*(-4*a*c+b^2)^(1/2))^2+1/32*(b^3-52*a*b*c+b^
2*(-4*a*c+b^2)^(1/2)+20*a*c*(-4*a*c+b^2)^(1/2))*(26880*A*(-4*a*c+b^2)^(1/2)
*a^4*c^4-26880*A*(-4*a*c+b^2)^(1/2)*a^3*b^2*c^3+10080*A*(-4*a*c+b^2)^(1/2)*
a^2*b^4*c^2-1680*A*(-4*a*c+b^2)^(1/2)*a*b^6*c+105*A*(-4*a*c+b^2)^(1/2)*b^8+
85248*A*a^4*b*c^4-61440*A*a^3*b^3*c^3+16416*A*a^2*b^5*c^2-2016*A*a*b^7*c+10
5*A*b^9-12800*B*a^5*c^4-13312*B*a^4*b^2*c^3+10176*B*a^3*b^4*c^2-1792*B*a^2*
b^6*c+70*B*a*b^8)/a^2/(400*a^2*c^2+616*a*b^2*c-35*b^4)/c*2^(1/2)/((-b+(-4*a
*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/
2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + (28*A*a^2*
c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + (B*a*b
^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)*c)*x^3
 - (B*a^2*b^3 - 5*A*a*b^4 - 44*A*a^3*c^2 - (16*B*a^3*b - 37*A*a^2*b^2)*c)*x
)/((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b^4 - 8*a^5*b^2*c +
 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3)*x^6 + (a^2*b^6 -
 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c + 16*a^5*b*c^2)*x
^2) - 1/8*integrate(-(B*a*b^3 + 3*A*b^4 + 84*A*a^2*c^2 + (4*(5*B*a^2 - 6*A*
a*b)*c^2 + (B*a*b^2 + 3*A*b^3)*c)*x^2 - (16*B*a^2*b + 27*A*a*b^2)*c)/(c*x^4
 + b*x^2 + a), x)/(a^2*b^4 - 8*a^3*b^2*c + 16*a^4*c^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 9909 vs. $2(417) = 834$.
time = 22.37, size = 9909, normalized size = 21.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(2*(4*(5*B*a^2 - 6*A*a*b)*c^3 + (B*a*b^2 + 3*A*b^3)*c^2)*x^7 + 2*(28*A
*a^2*c^3 + 7*(4*B*a^2*b - 7*A*a*b^2)*c^2 + 2*(B*a*b^3 + 3*A*b^4)*c)*x^5 + 2
*(B*a*b^4 + 3*A*b^5 + 4*(9*B*a^3 - A*a^2*b)*c^2 + 5*(B*a^2*b^2 - 4*A*a*b^3)
*c)*x^3 - sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^4*b
^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*c^3
)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^3*c
 + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 1680*(
4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*a^3
*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 - 7*
(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b^8*
c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5)*s
qrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b^7
+ 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*b +
 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b + 17496*A^2*B^2*a^4*
b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(25*B^4*a^5*b^2 + 25
8*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^5 + 891*A^4*a*b^6)
*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a^13*b^4*c^3 + 1280
*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2
 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5))*log((3111696*A^4*a^
4*c^7 - 1555848*(2*A^3*B*a^4*b + A^4*a^3*b^2)*c^6 - (10000*B^4*a^6 - 90000*
A*B^3*a^5*b - 863136*A^2*B^2*a^4*b^2 - 1298376*A^3*B*a^3*b^3 - 339309*A^4*a
^2*b^4)*c^5 - 3*(5000*B^4*a^5*b^2 + 32952*A*B^3*a^4*b^3 + 79488*A^2*B^2*a^3
*b^4 + 80919*A^3*B*a^2*b^5 + 12069*A^4*a*b^6)*c^4 + 21*(71*B^4*a^4*b^4 + 53
7*A*B^3*a^3*b^5 + 1314*A^2*B^2*a^2*b^6 + 1053*A^3*B*a*b^7 + 81*A^4*b^8)*c^3
 - 35*(B^4*a^3*b^6 + 9*A*B^3*a^2*b^7 + 27*A^2*B^2*a*b^8 + 27*A^3*B*b^9)*c^2
)*x + 1/2*sqrt(1/2)*(B^3*a^3*b^11 + 9*A*B^2*a^2*b^12 + 27*A^2*B*a*b^13 + 27
*A^3*b^14 - 2370816*A^3*a^7*c^7 + 2688*(50*A*B^2*a^8 + 384*A^2*B*a^7*b + 11
43*A^3*a^6*b^2)*c^6 - 64*(400*B^3*a^8*b + 4062*A*B^2*a^7*b^2 + 17541*A^2*B*
a^6*b^3 + 26865*A^3*a^5*b^4)*c^5 + 8*(2728*B^3*a^7*b^3 + 20520*A*B^2*a^6*b^
4 + 62694*A^2*B*a^5*b^5 + 67797*A^3*a^4*b^6)*c^4 - 7*(976*B^3*a^6*b^5 + 674
4*A*B^2*a^5*b^6 + 16884*A^2*B*a^4*b^7 + 14985*A^3*a^3*b^8)*c^3 + (940*B^3*a
^5*b^7 + 6591*A*B^2*a^4*b^8 + 15489*A^2*B*a^3*b^9 + 12528*A^3*a^2*b^10)*c^2
 - (53*B^3*a^4*b^9 + 414*A*B^2*a^3*b^10 + 1053*A^2*B*a^2*b^11 + 864*A^3*a*b
^12)*c - (B*a^6*b^14 + 3*A*a^5*b^15 + 4096*(10*B*a^13 - 33*A*a^12*b)*c^7 -
2048*(16*B*a^12*b^2 - 99*A*a^11*b^3)*c^6 + 768*(2*B*a^11*b^4 - 169*A*a^10*b
^5)*c^5 + 1280*(5*B*a^10*b^6 + 36*A*a^9*b^7)*c^4 - 80*(34*B*a^9*b^8 + 123*A
*a^8*b^9)*c^3 + 24*(20*B*a^8*b^10 + 53*A*a^7*b^11)*c^2 - (38*B*a^7*b^12 + 9
3*A*a^6*b^13)*c)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6
+ 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 +
 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B^3*a^5*b +
17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)*c^2 - 2*(2
```

```
5*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A^3*B*a^2*b^
5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*c^2 - 640*a
^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))*sqrt(-(B^2*a^2*b^7 + 6*A
*B*a*b^8 + 9*A^2*b^9 - 1680*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*
b - 4*A*B*a^4*b^2 - 9*A^2*a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^
4 + 243*A^2*a^2*b^5)*c^2 - 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7
)*c + (a^5*b^10 - 20*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a
^9*b^2*c^4 - 1024*a^10*c^5)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B
^2*a^2*b^6 + 108*A^3*B*a*b^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^
2*B^2*a^5 + 108*A^3*B*a^4*b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A*B
^3*a^5*b + 17496*A^2*B^2*a^4*b^2 + 26676*A^3*B*a^3*b^3 + 17739*A^4*a^2*b^4)
*c^2 - 2*(25*B^4*a^5*b^2 + 258*A*B^3*a^4*b^3 + 972*A^2*B^2*a^3*b^4 + 1566*A
^3*B*a^2*b^5 + 891*A^4*a*b^6)*c)/(a^10*b^10 - 20*a^11*b^8*c + 160*a^12*b^6*
c^2 - 640*a^13*b^4*c^3 + 1280*a^14*b^2*c^4 - 1024*a^15*c^5)))/(a^5*b^10 - 2
0*a^6*b^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a
^10*c^5))) + sqrt(1/2)*((a^2*b^4*c^2 - 8*a^3*b^2*c^3 + 16*a^4*c^4)*x^8 + a^
4*b^4 - 8*a^5*b^2*c + 16*a^6*c^2 + 2*(a^2*b^5*c - 8*a^3*b^3*c^2 + 16*a^4*b*
c^3)*x^6 + (a^2*b^6 - 6*a^3*b^4*c + 32*a^5*c^3)*x^4 + 2*(a^3*b^5 - 8*a^4*b^
3*c + 16*a^5*b*c^2)*x^2)*sqrt(-(B^2*a^2*b^7 + 6*A*B*a*b^8 + 9*A^2*b^9 - 168
0*(4*A*B*a^5 - 9*A^2*a^4*b)*c^4 + 840*(2*B^2*a^5*b - 4*A*B*a^4*b^2 - 9*A^2*
a^3*b^3)*c^3 + 7*(40*B^2*a^4*b^3 + 180*A*B*a^3*b^4 + 243*A^2*a^2*b^5)*c^2 -
 7*(5*B^2*a^3*b^5 + 24*A*B*a^2*b^6 + 27*A^2*a*b^7)*c + (a^5*b^10 - 20*a^6*b
^8*c + 160*a^7*b^6*c^2 - 640*a^8*b^4*c^3 + 1280*a^9*b^2*c^4 - 1024*a^10*c^5
)*sqrt((B^4*a^4*b^4 + 12*A*B^3*a^3*b^5 + 54*A^2*B^2*a^2*b^6 + 108*A^3*B*a*b
^7 + 81*A^4*b^8 + 194481*A^4*a^4*c^4 - 882*(25*A^2*B^2*a^5 + 108*A^3*B*a^4*
b + 99*A^4*a^3*b^2)*c^3 + (625*B^4*a^6 + 5400*A...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**3,x)
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 4613 vs. 2(417) = 834.
time = 6.60, size = 4613, normalized size = 10.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

[Out] 1/32*(3*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^8 - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7*c - 2*b^8*c + 116*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 34*a*b^6*c^2 + 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 736*a^3*b^2*c^4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 - 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 176*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 44*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^6*c - 26*(b^2 - 4*a*c)*a*b^4*c^2 - 2*(b^2 - 4*a*c)*b^5*c^2 + 128*(b^2 - 4*a*c)*a^2*b^2*c^3 + 22*(b^2 - 4*a*c)*a*b^3*c^3 - 224*(b^2 - 4*a*c)*a^3*c^4 - 88*(b^2 - 4*a*c)*a^2*b*c^4)*A + (sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^7 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 144*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 48*a^2*b^5*c^2 + 2*a*b^6*c^2 - 256*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^3 - 128*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 20*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 288*a^3*b^3*c^3 - 44*a^2*b^4*c^3 + 64*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 512*a^4*b*c^4 + 64*a^3*b^2*c^4 + 320*a^4*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 + 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 36*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 160*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 - 80*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 2*(b^2 - 4*a*c)*a*b^5*c - 40*(b^2 - 4*a*c)*a^2*b^3*c^2 - 2*(b^2 - 4*a*c)*a*b^4*c^2 + 128*(b^2 - 4*a*c)*a^3*b*c^3 + 36*(b^2 - 4*a*c)*a^2*b^2*c^3 + 80*(b^2 - 4*a*c)*a^3*c^4)*B)*arctan(2*sqrt(1/2)*x/sqrt((a^2*b^5 - 8*a^3*

```
b^3*c + 16*a^4*b*c^2 + sqrt((a^2*b^5 - 8*a^3*b^3*c + 16*a^4*b*c^2)^2 - 4*(a
^3*b^4 - 8*a^4*b^2*c + 16*a^5*c^2)*(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)
))/(a^2*b^4*c - 8*a^3*b^2*c^2 + 16*a^4*c^3)))/((a^3*b^8 - 16*a^4*b^6*c - 2*
a^3*b^7*c + 96*a^5*b^4*c^2 + 24*a^4*b^5*c^2 + a^3*b^6*c^2 - 256*a^6*b^2*c^3
 - 96*a^5*b^3*c^3 - 12*a^4*b^4*c^3 + 256*a^7*c^4 + 128*a^6*b*c^4 + 48*a^5*b
^2*c^4 - 64*a^6*c^5)*abs(c)) + 1/32*(3*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*b^8 - 17*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*sqrt(2)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c + 2*b^8*c + 116*sqrt(2)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*a^2*b^4*c^2 + 26*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
a*b^5*c^2 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 34*a*b^6*c^2
+ 2*b^7*c^2 - 368*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 - 128
*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 13*sqrt(2)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 232*a^2*b^4*c^3 - 30*a*b^5*c^3 + 448*sqr
t(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^4 + 224*sqrt(2)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^4 + 64*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^2*c^4 - 736*a^3*b^2*c^4 + 176*a^2*b^3*c^4 - 112*sqrt(2)*sqrt(b*c - sqrt(b
^2 - 4*a*c)*c)*a^3*c^5 + 896*a^4*c^5 - 352*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 88*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 22*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2...
```

Mupad [B]
time = 4.61, size = 2500, normalized size = 5.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^3,x)
```

```
[Out] ((x^3*(3*A*b^5 + 36*B*a^3*c^2 + B*a*b^4 - 20*A*a*b^3*c - 4*A*a^2*b*c^2 + 5*
B*a^2*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (x^5*(28*A*a^2*c^3 +
 6*A*b^4*c + 2*B*a*b^3*c - 49*A*a*b^2*c^2 + 28*B*a^2*b*c^2))/(8*a^2*(b^4 +
16*a^2*c^2 - 8*a*b^2*c)) + (x*(5*A*b^4 + 44*A*a^2*c^2 - B*a*b^3 - 37*A*a*b^
2*c + 16*B*a^2*b*c))/(8*a*(b^4 + 16*a^2*c^2 - 8*a*b^2*c)) + (c*x^7*(20*B*a^
2*c^2 + 3*A*b^3*c - 24*A*a*b*c^2 + B*a*b^2*c))/(8*a^2*(b^4 + 16*a^2*c^2 - 8
*a*b^2*c)))/(x^4*(2*a*c + b^2) + a^2 + c^2*x^8 + 2*a*b*x^2 + 2*b*c*x^6) + a
tan(((((4194304*B*a^9*b*c^8 - 22020096*A*a^9*c^9 + 768*A*a^2*b^14*c^2 - 222
72*A*a^3*b^12*c^3 + 282624*A*a^4*b^10*c^4 - 2027520*A*a^5*b^8*c^5 + 8847360
*A*a^6*b^6*c^6 - 23396352*A*a^7*b^4*c^7 + 34603008*A*a^8*b^2*c^8 + 256*B*a^
3*b^13*c^2 - 9216*B*a^4*b^11*c^3 + 122880*B*a^5*b^9*c^4 - 819200*B*a^6*b^7*
c^5 + 2949120*B*a^7*b^5*c^6 - 5505024*B*a^8*b^3*c^7)/(512*(a^4*b^12 + 4096*
a^10*c^6 - 24*a^5*b^10*c + 240*a^6*b^8*c^2 - 1280*a^7*b^6*c^3 + 3840*a^8*b^
4*c^4 - 6144*a^9*b^2*c^5)) - (x*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-
```

```
(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*
a^3*b^13*c^3 + 570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*
A^2*a^6*b^7*c^6 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441
*A^2*a^2*c^2*(-(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1
/2) + 1140*B^2*a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^
4 + 43776*B^2*a^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^
7 + 6881280*A*B*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B
^2*a^3*b^15*c - 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(
1/2) + 5580*A*B*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10
*c^4 - 1430784*A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*
b^4*c^7 - 5160960*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2
) + 6*A*B*a*b^3*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^
2*b*c*(-(4*a*c - b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^
6*b^18*c + 720*a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 2580
48*a^10*b^10*c^5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^1
3*b^4*c^8 - 2621440*a^14*b^2*c^9)))^(1/2)*(262144*a^9*b*c^7 - 256*a^4*b^11*
c^2 + 5120*a^5*b^9*c^3 - 40960*a^6*b^7*c^4 + 163840*a^7*b^5*c^5 - 327680*a^
8*b^3*c^6))/(32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 25
6*a^7*b^2*c^3)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^1
5)^(1/2) + 6*A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 +
570960*A^2*a^4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6
 - 21095424*A^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-
(4*a*c - b^2)^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*
a^4*b^13*c^2 - 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a
^7*b^7*c^5 - 680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B
*a^10*c^9 - 369*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c -
 1720320*B^2*a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B
*a^3*b^14*c^2 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*
A*B*a^6*b^8*c^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 51609
60*A*B*a^9*b^2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3
*(-(4*a*c - b^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c
- b^2)^15)^(1/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^6*b^18*c + 720*
a^7*b^16*c^2 - 7680*a^8*b^14*c^3 + 53760*a^9*b^12*c^4 - 258048*a^10*b^10*c^
5 + 860160*a^11*b^8*c^6 - 1966080*a^12*b^6*c^7 + 2949120*a^13*b^4*c^8 - 262
1440*a^14*b^2*c^9)))^(1/2) + (x*(14112*A^2*a^4*c^7 + 9*A^2*b^8*c^3 - 800*B^
2*a^5*c^6 + 1530*A^2*a^2*b^4*c^5 - 6192*A^2*a^3*b^2*c^6 + B^2*a^2*b^6*c^3 -
 34*B^2*a^3*b^4*c^4 + 1472*B^2*a^4*b^2*c^5 - 180*A^2*a*b^6*c^4 - 162*A*B*a^
2*b^5*c^4 + 1104*A*B*a^3*b^3*c^5 + 6*A*B*a*b^7*c^3 - 6816*A*B*a^4*b*c^6))/(
32*(a^4*b^8 + 256*a^8*c^4 - 16*a^5*b^6*c + 96*a^6*b^4*c^2 - 256*a^7*b^2*c^3
)))*(-(9*A^2*b^19 + B^2*a^2*b^17 + 9*A^2*b^4*(-(4*a*c - b^2)^15)^(1/2) + 6*
A*B*a*b^18 + 6921*A^2*a^2*b^15*c^2 - 77580*A^2*a^3*b^13*c^3 + 570960*A^2*a^
4*b^11*c^4 - 2851776*A^2*a^5*b^9*c^5 + 9628416*A^2*a^6*b^7*c^6 - 21095424*A
^2*a^7*b^5*c^7 + 27095040*A^2*a^8*b^3*c^8 + 441*A^2*a^2*c^2*(-(4*a*c - b^2)
^15)^(1/2) + B^2*a^2*b^2*(-(4*a*c - b^2)^15)^(1/2) + 1140*B^2*a^4*b^13*c^2
- 10160*B^2*a^5*b^11*c^3 + 34880*B^2*a^6*b^9*c^4 + 43776*B^2*a^7*b^7*c^5 -
```

```
680960*B^2*a^8*b^5*c^6 + 1863680*B^2*a^9*b^3*c^7 + 6881280*A*B*a^10*c^9 - 3
69*A^2*a*b^17*c - 15482880*A^2*a^9*b*c^9 - 55*B^2*a^3*b^15*c - 1720320*B^2*
a^10*b*c^8 - 25*B^2*a^3*c*(-(4*a*c - b^2)^15)^(1/2) + 5580*A*B*a^3*b^14*c^2
 - 59280*A*B*a^4*b^12*c^3 + 377280*A*B*a^5*b^10*c^4 - 1430784*A*B*a^6*b^8*c
^5 + 2860032*A*B*a^7*b^6*c^6 - 1290240*A*B*a^8*b^4*c^7 - 5160960*A*B*a^9*b^
2*c^8 - 99*A^2*a*b^2*c*(-(4*a*c - b^2)^15)^(1/2) + 6*A*B*a*b^3*(-(4*a*c - b
^2)^15)^(1/2) - 288*A*B*a^2*b^16*c - 108*A*B*a^2*b*c*(-(4*a*c - b^2)^15)^(1
/2))/(512*(a^5*b^20 + 1048576*a^15*c^10 - 40*a^...
```

3.137 $\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)+3/2*ln(-x^2+4)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {1261, 646, 31}

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{-7+4x}{4-5x+x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{-4+x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\log\left(1-x^2\right) + \frac{3}{2}\log\left(4-x^2\right)$$

Mathematica [A]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{1}{2}\log\left(1-x^2\right) + \frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(-7 + 4*x^2))/(4 - 5*x^2 + x^4),x]
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Maple [A]
time = 0.02, size = 18, normalized size = 0.72

method	result	size
default	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
risch	$\frac{\ln(x^2-1)}{2}+\frac{3\ln(x^2-4)}{2}$	18
norman	$\frac{3\ln(x-2)}{2}+\frac{\ln(-1+x)}{2}+\frac{\ln(1+x)}{2}+\frac{3\ln(x+2)}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(4*x^2-7)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
[Out] 1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

Maxima [A]
time = 0.28, size = 17, normalized size = 0.68

$$\frac{1}{2}\log\left(x^2-1\right) + \frac{3}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="maxima")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Fricas [A]
time = 0.35, size = 17, normalized size = 0.68

$$\frac{1}{2}\log\left(x^{2}-1\right)+\frac{3}{2}\log\left(x^{2}-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Sympy [A]
time = 0.05, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^{2}-4\right)}{2}+\frac{\log\left(x^{2}-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x**2-7)/(x**4-5*x**2+4),x)
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

Giac [A]
time = 4.19, size = 19, normalized size = 0.76

$$\frac{1}{2}\log\left(\left|x^{2}-1\right|\right)+\frac{3}{2}\log\left(\left|x^{2}-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(4*x^2-7)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

Mupad [B]
time = 0.06, size = 17, normalized size = 0.68

$$\frac{\ln\left(x^{2}-1\right)}{2}+\frac{3\,\ln\left(x^{2}-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(4*x^2 - 7))/(x^4 - 5*x^2 + 4),x)
[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

3.138 $\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx$

Optimal. Leaf size=25

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

```
[Out] 1/2*ln(-x^2+1)+3/2*ln(-x^2+4)
```

Rubi [A]
time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {1607, 1261, 646, 31}

$$\frac{1}{2}\log\left(1-x^2\right)+\frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Int[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 646

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> W
ith[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(c*d - e*(b/2 - q/2))/q, Int[1/(b/2 - q/
2 + c*x), x], x] - Dist[(c*d - e*(b/2 + q/2))/q, Int[1/(b/2 + q/2 + c*x), x
], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a
*c, 0] && NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\int \frac{-7x+4x^3}{4-5x^2+x^4}\,dx = \int \frac{x(-7+4x^2)}{4-5x^2+x^4}\,dx$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{-7+4x}{4-5x+x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \frac{1}{-1+x}\,dx, x, x^2\right) + \frac{3}{2}\text{Subst}\left(\int \frac{1}{-4+x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\log\left(1-x^2\right) + \frac{3}{2}\log\left(4-x^2\right)$$

Mathematica [**A**]
time = 0.00, size = 25, normalized size = 1.00

$$\frac{1}{2}\log\left(1-x^2\right) + \frac{3}{2}\log\left(4-x^2\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-7*x + 4*x^3)/(4 - 5*x^2 + x^4),x]
```

```
[Out] Log[1 - x^2]/2 + (3*Log[4 - x^2])/2
```

Maple [**A**]
time = 0.02, size = 18, normalized size = 0.72

method	result	size
default	$\frac{\ln(x^2-1)}{2} + \frac{3\ln(x^2-4)}{2}$	18
risch	$\frac{\ln(x^2-1)}{2} + \frac{3\ln(x^2-4)}{2}$	18
norman	$\frac{3\ln(x-2)}{2} + \frac{\ln(-1+x)}{2} + \frac{\ln(1+x)}{2} + \frac{3\ln(x+2)}{2}$	26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((4*x^3-7*x)/(x^4-5*x^2+4),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*ln(x^2-1)+3/2*ln(x^2-4)
```

Maxima [**A**]
time = 0.27, size = 25, normalized size = 1.00

$$\frac{3}{2}\log\left(x+2\right) + \frac{1}{2}\log\left(x+1\right) + \frac{1}{2}\log\left(x-1\right) + \frac{3}{2}\log\left(x-2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="maxima")
```

```
[Out] 3/2*log(x + 2) + 1/2*log(x + 1) + 1/2*log(x - 1) + 3/2*log(x - 2)
```

Fricas [A]
time = 0.34, size = 17, normalized size = 0.68

$$\frac{1}{2}\log\left(x^2-1\right)+\frac{3}{2}\log\left(x^2-4\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="fricas")
[Out] 1/2*log(x^2 - 1) + 3/2*log(x^2 - 4)
```

Sympy [A]
time = 0.04, size = 17, normalized size = 0.68

$$\frac{3\log\left(x^2-4\right)}{2}+\frac{\log\left(x^2-1\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x**3-7*x)/(x**4-5*x**2+4),x)
[Out] 3*log(x**2 - 4)/2 + log(x**2 - 1)/2
```

Giac [A]
time = 4.98, size = 19, normalized size = 0.76

$$\frac{1}{2}\log\left(\left|x^2-1\right|\right)+\frac{3}{2}\log\left(\left|x^2-4\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((4*x^3-7*x)/(x^4-5*x^2+4),x, algorithm="giac")
[Out] 1/2*log(abs(x^2 - 1)) + 3/2*log(abs(x^2 - 4))
```

Mupad [B]
time = 0.03, size = 17, normalized size = 0.68

$$\frac{\ln\left(x^2-1\right)}{2}+\frac{3\,\ln\left(x^2-4\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(7*x - 4*x^3)/(x^4 - 5*x^2 + 4),x)
[Out] log(x^2 - 1)/2 + (3*log(x^2 - 4))/2
```

3.139 $\int \frac{x(2+x^2)}{1+x^2+x^4} dx$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)$$

```
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 17, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {1261, 648, 632, 210, 642}

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + x^2))/(1 + x^2 + x^4),x]
```

```
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(2+x^2)}{1+x^2+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right)\\ &= \frac{1}{4}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right)+\frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right)\\ &= \frac{1}{4}\log\left(1+x^2+x^4\right)-\frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right)\\ &= \frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)\end{aligned}$$

Mathematica [A]

time = 0.01, size = 37, normalized size = 1.00

$$\frac{1}{2}\sqrt{3}\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + x^2))/(1 + x^2 + x^4),x]
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Maple [A]

time = 0.02, size = 31, normalized size = 0.84

method	result	size
default	$\frac{\ln(x^4+x^2+1)}{4}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	31
risch	$\frac{\ln(4x^4+4x^2+4)}{4}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(x^2+2)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.50, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="maxima")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Fricas [A]
time = 0.36, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Sympy [A]
time = 0.04, size = 37, normalized size = 1.00

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x**2+2)/(x**4+x**2+1),x)
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

Giac [A]
time = 4.56, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(x^2+2)/(x^4+x^2+1),x, algorithm="giac")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Mupad [B]
time = 0.21, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\,\mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(x^2 + 2))/(x^2 + x^4 + 1),x)
[Out] log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

3.140 $\int \frac{2x+x^3}{1+x^2+x^4} \, dx$

Optimal. Leaf size=37

$$\frac{1}{2}\sqrt{3} \tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right)+\frac{1}{4}\log\left(1+x^2+x^4\right)$$

[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)

Rubi [A]
time = 0.03, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {1607, 1261, 648, 632, 210, 642}

$$\frac{1}{2}\sqrt{3}\,\text{ArcTan}\left(\frac{2x^2+1}{\sqrt{3}}\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Antiderivative was successfully verified.

[In] Int[(2*x + x^3)/(1 + x^2 + x^4),x]

[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{2x+x^3}{1+x^2+x^4}\,dx &= \int \frac{x(2+x^2)}{1+x^2+x^4}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{2+x}{1+x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{4}\text{Subst}\left(\int \frac{1+2x}{1+x+x^2}\,dx, x, x^2\right) + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1+x+x^2}\,dx, x, x^2\right) \\
&= \frac{1}{4}\log\left(1+x^2+x^4\right) - \frac{3}{2}\text{Subst}\left(\int \frac{1}{-3-x^2}\,dx, x, 1+2x^2\right) \\
&= \frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1+x^2+x^4\right)
\end{aligned}$$

Mathematica [A]
time = 0.00, size = 37, normalized size = 1.00

$$\frac{1}{2}\sqrt{3}\,\tan^{-1}\left(\frac{1+2x^2}{\sqrt{3}}\right) + \frac{1}{4}\log\left(1+x^2+x^4\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2*x + x^3)/(1 + x^2 + x^4),x]
[Out] (Sqrt[3]*ArcTan[(1 + 2*x^2)/Sqrt[3]])/2 + Log[1 + x^2 + x^4]/4
```

Maple [A]
time = 0.02, size = 31, normalized size = 0.84

method	result	size

default	$\frac{\ln(x^4+x^2+1)}{4}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	31
risch	$\frac{\ln(4x^4+4x^2+4)}{4}+\frac{\arctan\left(\frac{(2x^2+1)\sqrt{3}}{3}\right)\sqrt{3}}{2}$	35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3+2*x)/(x^4+x^2+1),x,method=_RETURNVERBOSE)
[Out] 1/4*ln(x^4+x^2+1)+1/2*arctan(1/3*(2*x^2+1)*3^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.47, size = 53, normalized size = 1.43

$$-\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x+1)\right)+\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2x-1)\right)+\frac{1}{4}\log\left(x^2+x+1\right)+\frac{1}{4}\log\left(x^2-x+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="maxima")
[Out] -1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x + 1)) + 1/2*sqrt(3)*arctan(1/3*sqrt(3)
*(2*x - 1)) + 1/4*log(x^2 + x + 1) + 1/4*log(x^2 - x + 1)
```

Fricas [A]
time = 0.35, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="fricas")
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Sympy [A]
time = 0.04, size = 37, normalized size = 1.00

$$\frac{\log\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x**3+2*x)/(x**4+x**2+1),x)
[Out] log(x**4 + x**2 + 1)/4 + sqrt(3)*atan(2*sqrt(3)*x**2/3 + sqrt(3)/3)/2
```

Giac [A]
time = 3.73, size = 30, normalized size = 0.81

$$\frac{1}{2}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}\left(2x^2+1\right)\right)+\frac{1}{4}\log\left(x^4+x^2+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((x^3+2*x)/(x^4+x^2+1),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(3)*arctan(1/3*sqrt(3)*(2*x^2 + 1)) + 1/4*log(x^4 + x^2 + 1)
```

Mupad [B]
time = 0.03, size = 32, normalized size = 0.86

$$\frac{\ln\left(x^4+x^2+1\right)}{4}+\frac{\sqrt{3}\ \mathrm{atan}\left(\frac{2\sqrt{3}\,x^2}{3}+\frac{\sqrt{3}}{3}\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x + x^3)/(x^2 + x^4 + 1),x)
```

```
[Out] log(x^2 + x^4 + 1)/4 + (3^(1/2)*atan(3^(1/2)/3 + (2*3^(1/2)*x^2)/3))/2
```

3.141 $\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx$

Optimal. Leaf size=45

$$\frac{5+9x^2}{8(3+2x^2+x^4)}+\frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

```
[Out] 1/8*(9*x^2+5)/(x^4+2*x^2+3)+9/16*arctan(1/2*(x^2+1)*2^(1/2))*2^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1607, 1261, 652, 632, 210}

$$\frac{9\text{ArcTan}\left(\frac{x^2+1}{\sqrt{2}}\right)}{8\sqrt{2}}+\frac{9x^2+5}{8(x^4+2x^2+3)}$$

Antiderivative was successfully verified.

```
[In] Int[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 652

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[((b*d - 2*a*e + (2*c*d - b*e)*x)/((p + 1)*(b^2 - 4*a*c)))*(a + b*
x + c*x^2)^(p + 1), x] - Dist[(2*p + 3)*((2*c*d - b*e)/((p + 1)*(b^2 - 4*a*
c))), Int[(a + b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e}, x] &&
 NeQ[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && NeQ[p, -3/2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rule 1607

```
Int[(u_.)*((a_.)*(x_)^(p_.) + (b_.)*(x_)^(q_.))^(n_.), x_Symbol] :> Int[u*x
^(n*p)*(a + b*x^(q - p))^n, x] /; FreeQ[{a, b, p, q}, x] && IntegerQ[n] &&
PosQ[q - p]
```

Rubi steps

$$\begin{aligned}
\int \frac{11x+2x^3}{(3+2x^2+x^4)^2}\,dx &= \int \frac{x(11+2x^2)}{(3+2x^2+x^4)^2}\,dx \\
&= \frac{1}{2}\text{Subst}\left(\int \frac{11+2x}{(3+2x+x^2)^2}\,dx, x, x^2\right) \\
&= \frac{5+9x^2}{8\,(3+2x^2+x^4)} + \frac{9}{8}\text{Subst}\left(\int \frac{1}{3+2x+x^2}\,dx, x, x^2\right) \\
&= \frac{5+9x^2}{8\,(3+2x^2+x^4)} - \frac{9}{4}\text{Subst}\left(\int \frac{1}{-8-x^2}\,dx, x, 2\left(1+x^2\right)\right) \\
&= \frac{5+9x^2}{8\,(3+2x^2+x^4)} + \frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}
\end{aligned}$$

Mathematica [A]
time = 0.02, size = 45, normalized size = 1.00

$$\frac{5+9x^2}{8\,(3+2x^2+x^4)} + \frac{9\tan^{-1}\left(\frac{1+x^2}{\sqrt{2}}\right)}{8\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(11*x + 2*x^3)/(3 + 2*x^2 + x^4)^2,x]
```

```
[Out] (5 + 9*x^2)/(8*(3 + 2*x^2 + x^4)) + (9*ArcTan[(1 + x^2)/Sqrt[2]])/(8*Sqrt[2
])
```

Maple [A]
time = 0.02, size = 41, normalized size = 0.91

method	result	size
risch	$\frac{\frac{9x^2}{8}+\frac{5}{8}}{x^4+2x^2+3}+\frac{9\arctan\left(\frac{(x^2+1)\sqrt{2}}{2}\right)\sqrt{2}}{16}$	38
default	$\frac{18x^2+10}{16x^4+32x^2+48}+\frac{9\sqrt{2}\arctan\left(\frac{(2x^2+2)\sqrt{2}}{4}\right)}{16}$	41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^3+11*x)/(x^4+2*x^2+3)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/16*(18*x^2+10)/(x^4+2*x^2+3)+9/16*2^(1/2)*arctan(1/4*(2*x^2+2)*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 + 3) + 9/4*integrate(x/(x^4 + 2*x^2 + 3), x)
```

Fricas [A]
time = 0.37, size = 47, normalized size = 1.04

$$\frac{9\sqrt{2}\,(x^4+2\,x^2+3)\arctan\left(\frac{1}{2}\sqrt{2}\,(x^2+1)\right)+18\,x^2+10}{16\,(x^4+2\,x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(9*sqrt(2)*(x^4 + 2*x^2 + 3)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 18*x^2 +
10)/(x^4 + 2*x^2 + 3)
```

Sympy [A]
time = 0.05, size = 44, normalized size = 0.98

$$\frac{9x^2+5}{8x^4+16x^2+24}+\frac{9\sqrt{2}\,\operatorname{atan}\left(\frac{\sqrt{2}\,x^2}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**3+11*x)/(x**4+2*x**2+3)**2,x)
```

```
[Out] (9*x**2 + 5)/(8*x**4 + 16*x**2 + 24) + 9*sqrt(2)*atan(sqrt(2)*x**2/2 + sqrt
(2)/2)/16
```

Giac [A]
time = 4.29, size = 38, normalized size = 0.84

$$\frac{9}{16}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(x^{2}+1\right)\right)+\frac{9\,x^{2}+5}{8\left(x^{4}+2\,x^{2}+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^3+11*x)/(x^4+2*x^2+3)^2,x, algorithm="giac")
```

```
[Out] 9/16*sqrt(2)*arctan(1/2*sqrt(2)*(x^2 + 1)) + 1/8*(9*x^2 + 5)/(x^4 + 2*x^2 +
 3)
```

Mupad [B]
time = 0.05, size = 41, normalized size = 0.91

$$\frac{\frac{9\,x^{2}}{8}+\frac{5}{8}}{x^{4}+2\,x^{2}+3}+\frac{9\,\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\,x^{2}}{2}+\frac{\sqrt{2}}{2}\right)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((11*x + 2*x^3)/(2*x^2 + x^4 + 3)^2,x)
```

```
[Out] ((9*x^2)/8 + 5/8)/(2*x^2 + x^4 + 3) + (9*2^(1/2)*atan(2^(1/2)/2 + (2^(1/2)*
x^2)/2))/16
```

3.142 $\int x^5(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=102

$$-\frac{1633}{256}(5+2x^2)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4(3+5x^2+x^4)^{3/2}+\frac{1}{480}(1837-510x^2)(3+5x^2+x^4)^{3/2}+\frac{21229}{512}\text{ta}$$

```
[Out] 3/10*x^4*(x^4+5*x^2+3)^(3/2)+1/480*(-510*x^2+1837)*(x^4+5*x^2+3)^(3/2)+2122
9/512*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1633/256*(2*x^2+5)*(x^4+5*
x^2+3)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 846, 793, 626, 635, 212}

$$\frac{3}{10}(x^4+5x^2+3)^{3/2}x^4+\frac{1}{480}(1837-510x^2)(x^4+5x^2+3)^{3/2}-\frac{1633}{256}(2x^2+5)\sqrt{x^4+5x^2+3}+\frac{21229}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1633*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 + (3*x^4*(3 + 5*x^2 + x^4)^(3
/2))/10 + ((1837 - 510*x^2)*(3 + 5*x^2 + x^4)^(3/2))/480 + (21229*ArcTanh[(
5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int x^5\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\sqrt{3+5x+x^2}\,dx,x,x^2\right)\\
&= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{10}\text{Subst}\left(\int\left(-18-\frac{85x}{2}\right)x\sqrt{3+5x+x^2}\,d\right.\\
&= \frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}\left(1837-510x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1633}{64}\\
&= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}(1837\\
&= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}(1837\\
&= -\frac{1633}{256}\left(5+2x^2\right)\sqrt{3+5x^2+x^4}+\frac{3}{10}x^4\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{480}(1837
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 69, normalized size = 0.68

$$\frac{\sqrt{3+5x^2+x^4}\,(-78387+12250x^2-2248x^4+1680x^6+1152x^8)}{3840}-\frac{21229}{512}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-78387 + 12250*x^2 - 2248*x^4 + 1680*x^6 + 1152*x^8
))/3840 - (21229*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512
```

Maple [A]
time = 0.12, size = 91, normalized size = 0.89

method	result
risch	$\frac{(1152x^8+1680x^6-2248x^4+12250x^2-78387)\sqrt{x^4+5x^2+3}}{3840}+\frac{21229\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{512}$
trager	$\left(\frac{3}{10}x^8+\frac{7}{16}x^6-\frac{281}{480}x^4+\frac{1225}{384}x^2-\frac{26129}{1280}\right)\sqrt{x^4+5x^2+3}+\frac{21229\ln\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{512}$
default	$\frac{3x^4(x^4+5x^2+3)^{\frac{3}{2}}}{10}-\frac{17x^2(x^4+5x^2+3)^{\frac{3}{2}}}{16}+\frac{1837(x^4+5x^2+3)^{\frac{3}{2}}}{480}-\frac{1633(2x^2+5)\sqrt{x^4+5x^2+3}}{256}+\frac{21229\ln\left(x^2+\frac{5}{2}+\sqrt{\right.}}{5}$
elliptic	$\frac{3x^8\sqrt{x^4+5x^2+3}}{10}+\frac{7x^6\sqrt{x^4+5x^2+3}}{16}-\frac{281x^4\sqrt{x^4+5x^2+3}}{480}+\frac{1225x^2\sqrt{x^4+5x^2+3}}{384}-\frac{26129}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/10*x^4*(x^4+5*x^2+3)^(3/2)-17/16*x^2*(x^4+5*x^2+3)^(3/2)+1837/480*(x^4+5*
x^2+3)^(3/2)-1633/256*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+21229/512*ln(x^2+5/2+(x
^4+5*x^2+3)^(1/2))
```

Maxima [A]
time = 0.27, size = 104, normalized size = 1.02

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^4-\frac{17}{16}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{1633}{128}\sqrt{x^4+5x^2+3}\,x^2+\frac{1837}{480}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{8165}{256}\sqrt{x^4+5x^2+3}+\frac{21229}{512}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/10*(x^4 + 5*x^2 + 3)^(3/2)*x^4 - 17/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 1633
/128*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1837/480*(x^4 + 5*x^2 + 3)^(3/2) - 8165/25
6*sqrt(x^4 + 5*x^2 + 3) + 21229/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5
)
```

Fricas [A]
time = 0.35, size = 61, normalized size = 0.60

$$\frac{1}{3840}\left(1152\,x^{8}+1680\,x^{6}-2248\,x^{4}+12250\,x^{2}-78387\right)\sqrt{x^{4}+5\,x^{2}+3}-\frac{21229}{512}\,\log\left(-2\,x^{2}+2\,\sqrt{x^{4}+5\,x^{2}+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3840*(1152*x^8 + 1680*x^6 - 2248*x^4 + 12250*x^2 - 78387)*sqrt(x^4 + 5*x^
2 + 3) - 21229/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^{5}\cdot\left(3x^{2}+2\right)\sqrt{x^{4}+5x^{2}+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A]
time = 4.02, size = 102, normalized size = 1.00

$$\frac{1}{1280}\,\sqrt{x^{4}+5\,x^{2}+3}\,\left(2\,\left(4\,\left(6\,\left(8\,x^{2}+5\right)x^{2}-127\right)x^{2}+2635\right)x^{2}-33429\right)+\frac{1}{192}\,\sqrt{x^{4}+5\,x^{2}+3}\,\left(2\,\left(4\,\left(6\,x^{2}+5\right)x^{2}-89\right)x^{2}+1095\right)-\frac{21229}{512}\,\log\left(2\,x^{2}-2\,\sqrt{x^{4}+5\,x^{2}+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^
2 - 33429) + 1/192*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 +
1095) - 21229/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B]
time = 0.49, size = 102, normalized size = 1.00

$$\frac{21229\,\ln\left(\sqrt{x^{4}+5\,x^{2}+3}+x^{2}+\frac{5}{2}\right)}{512}-\frac{17\,x^{2}\,{\left(x^{4}+5\,x^{2}+3\right)}^{3/2}}{16}+\frac{3\,x^{4}\,{\left(x^{4}+5\,x^{2}+3\right)}^{3/2}}{10}+\frac{51\,\left(\frac{x^{2}}{2}+\frac{5}{4}\right)\,\sqrt{x^{4}+5\,x^{2}+3}}{16}+\frac{1837\,\sqrt{x^{4}+5\,x^{2}+3}\,\left(8\,x^{4}+10\,x^{2}-51\right)}{3840}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (21229*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 - (17*x^2*(5*x^2 + x^4
 + 3)^(3/2))/16 + (3*x^4*(5*x^2 + x^4 + 3)^(3/2))/10 + (51*(x^2/2 + 5/4)*(5
*x^2 + x^4 + 3)^(1/2))/16 + (1837*(5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 -
 51))/3840
```

3.143 $\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=81

$$\frac{259}{128}(5+2x^2)\sqrt{3+5x^2+x^4}-\frac{1}{48}(59-18x^2)\left(3+5x^2+x^4\right)^{3/2}-\frac{3367}{256}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -1/48*(-18*x^2+59)*(x^4+5*x^2+3)^(3/2)-3367/256*arctanh(1/2*(2*x^2+5)/(x^4+
5*x^2+3)^(1/2))+259/128*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 793, 626, 635, 212}

$$-\frac{1}{48}(59-18x^2)\left(x^4+5x^2+3\right)^{3/2}+\frac{259}{128}(2x^2+5)\sqrt{x^4+5x^2+3}-\frac{3367}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (259*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 - ((59 - 18*x^2)*(3 + 5*x^2 + x
^4)^(3/2))/48 - (3367*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
```

```
2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int x(2+3x)\sqrt{3+5x+x^2}\,dx, x, x^2\right)$$
$$= -\frac{1}{48}(59-18x^2)(3+5x^2+x^4)^{3/2} + \frac{259}{32}\text{Subst}\left(\int \sqrt{3+5x+x^2}\,dx, x\right.$$
$$= \frac{259}{128}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{1}{48}(59-18x^2)(3+5x^2+x^4)^{3/2} - \frac{33}{25}$$
$$= \frac{259}{128}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{1}{48}(59-18x^2)(3+5x^2+x^4)^{3/2} - \frac{33}{12}$$
$$= \frac{259}{128}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{1}{48}(59-18x^2)(3+5x^2+x^4)^{3/2} - \frac{33}{25}$$

Mathematica [A]
time = 0.08, size = 64, normalized size = 0.79

$$\frac{1}{384}\sqrt{3+5x^2+x^4}\,(2469-374x^2+248x^4+144x^6)+\frac{3367}{256}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(2469 - 374*x^2 + 248*x^4 + 144*x^6))/384 + (3367*Lo
g[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256
```

Maple [A]
time = 0.10, size = 74, normalized size = 0.91

method	result

risch	$\frac{(144x^6+248x^4-374x^2+2469)\sqrt{x^4+5x^2+3}}{384}-\frac{3367\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{256}$
trager	$\left(\frac{3}{8}x^6+\frac{31}{48}x^4-\frac{187}{192}x^2+\frac{823}{128}\right)\sqrt{x^4+5x^2+3}-\frac{3367\ln\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)}{256}$
default	$\frac{3x^2(x^4+5x^2+3)^{\frac{3}{2}}}{8}-\frac{59(x^4+5x^2+3)^{\frac{3}{2}}}{48}+\frac{259(2x^2+5)\sqrt{x^4+5x^2+3}}{128}-\frac{3367\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{256}$
elliptic	$\frac{3x^6\sqrt{x^4+5x^2+3}}{8}+\frac{31x^4\sqrt{x^4+5x^2+3}}{48}-\frac{187x^2\sqrt{x^4+5x^2+3}}{192}+\frac{823\sqrt{x^4+5x^2+3}}{128}-\frac{3367\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^2*(x^4+5*x^2+3)^(3/2)-59/48*(x^4+5*x^2+3)^(3/2)+259/128*(2*x^2+5)*(x^
4+5*x^2+3)^(1/2)-3367/256*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A]
time = 0.28, size = 87, normalized size = 1.07

$$\frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{259}{64}\sqrt{x^4+5x^2+3}\,x^2-\frac{59}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{1295}{128}\sqrt{x^4+5x^2+3}-\frac{3367}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 259/64*sqrt(x^4 + 5*x^2 + 3)*x^2 - 59/48*
(x^4 + 5*x^2 + 3)^(3/2) + 1295/128*sqrt(x^4 + 5*x^2 + 3) - 3367/256*log(2*x
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.37, size = 56, normalized size = 0.69

$$\frac{1}{384}\left(144x^6+248x^4-374x^2+2469\right)\sqrt{x^4+5x^2+3}+\frac{3367}{256}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 + 248*x^4 - 374*x^2 + 2469)*sqrt(x^4 + 5*x^2 + 3) + 3367/256
*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3\cdot\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A]
time = 4.43, size = 88, normalized size = 1.09

$$\frac{1}{128}\sqrt{x^4+5x^2+3}\left(2\left(4\left(6x^2+5\right)x^2-89\right)x^2+1095\right)+\frac{1}{24}\sqrt{x^4+5x^2+3}\left(2\left(4x^2+5\right)x^2-51\right)+\frac{3367}{256}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/128*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/24*
sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3367/256*log(2*x^2 - 2*sqr
t(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B]
time = 0.43, size = 85, normalized size = 1.05

$$\frac{3x^2\left(x^4+5x^2+3\right)^{3/2}}{8}-\frac{3367\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{256}-\frac{9\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{8}-\frac{59\sqrt{x^4+5x^2+3}\left(8x^4+10x^2-51\right)}{384}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (3*x^2*(5*x^2 + x^4 + 3)^(3/2))/8 - (3367*log((5*x^2 + x^4 + 3)^(1/2) + x^2
 + 5/2))/256 - (9*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2))/8 - (59*(5*x^2 + x
^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/384
```

3.144 $\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=74

$$-\frac{11}{16}(5+2x^2)\sqrt{3+5x^2+x^4}+\frac{1}{2}(3+5x^2+x^4)^{3/2}+\frac{143}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 1/2*(x^4+5*x^2+3)^(3/2)+143/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1
1/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1261, 654, 626, 635, 212}

$$\frac{1}{2}(x^4+5x^2+3)^{3/2}-\frac{11}{16}(2x^2+5)\sqrt{x^4+5x^2+3}+\frac{143}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-11*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 + (3 + 5*x^2 + x^4)^(3/2)/2 + (1
43*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
```

```
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int (2+3x)\sqrt{3+5x+x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}(3+5x^2+x^4)^{3/2} - \frac{11}{4}\text{Subst}\left(\int \sqrt{3+5x+x^2}\,dx, x, x^2\right)$$
$$= -\frac{11}{16}(5+2x^2)\sqrt{3+5x^2+x^4} + \frac{1}{2}(3+5x^2+x^4)^{3/2} + \frac{143}{32}\text{Subst}\left(\int \frac{}{\sqrt{}}\right.$$
$$= -\frac{11}{16}(5+2x^2)\sqrt{3+5x^2+x^4} + \frac{1}{2}(3+5x^2+x^4)^{3/2} + \frac{143}{16}\text{Subst}\left(\int \frac{}{4-}\right.$$
$$= -\frac{11}{16}(5+2x^2)\sqrt{3+5x^2+x^4} + \frac{1}{2}(3+5x^2+x^4)^{3/2} + \frac{143}{32}\tanh^{-1}\left(\frac{}{2\sqrt{}}\right.$$

Mathematica [A]
time = 0.08, size = 59, normalized size = 0.80

$$\frac{1}{16}\sqrt{3+5x^2+x^4}\,(-31+18x^2+8x^4) - \frac{143}{32}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-31 + 18*x^2 + 8*x^4))/16 - (143*Log[-5 - 2*x^2 + 2
*Sqrt[3 + 5*x^2 + x^4]])/32
```

Maple [A]
time = 0.10, size = 57, normalized size = 0.77

method	result	siz
risch	$\frac{(8x^4+18x^2-31)\sqrt{x^4+5x^2+3}}{16} + \frac{143\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}$	48

trager	$\left(\frac{1}{2}x^4+\frac{9}{8}x^2-\frac{31}{16}\right)\sqrt{x^4+5x^2+3}-\frac{143\ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{32}$	5[illegible]
default	$\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{2}-\frac{11(2x^2+5)\sqrt{x^4+5x^2+3}}{16}+\frac{143\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}$	5[illegible]
elliptic	$\frac{x^4\sqrt{x^4+5x^2+3}}{2}+\frac{9x^2\sqrt{x^4+5x^2+3}}{8}-\frac{31\sqrt{x^4+5x^2+3}}{16}+\frac{143\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}$	7[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*(x^4+5*x^2+3)^(3/2)-11/16*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+143/32*ln(x^2+5
/2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A]
time = 0.30, size = 70, normalized size = 0.95

$$-\frac{11}{8}\sqrt{x^4+5x^2+3}\,x^2+\frac{1}{2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{55}{16}\sqrt{x^4+5x^2+3}+\frac{143}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -11/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 55/16*sqrt(
x^4 + 5*x^2 + 3) + 143/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.35, size = 51, normalized size = 0.69

$$\frac{1}{16}\left(8x^4+18x^2-31\right)\sqrt{x^4+5x^2+3}-\frac{143}{32}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 + 18*x^2 - 31)*sqrt(x^4 + 5*x^2 + 3) - 143/32*log(-2*x^2 + 2*sq
rt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A]
time = 4.37, size = 74, normalized size = 1.00

$$\frac{1}{16}\sqrt{x^4+5x^2+3}\,(2(4x^2+5)x^2-51)+\frac{1}{4}\sqrt{x^4+5x^2+3}\,(2x^2+5)-\frac{143}{32}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 1/4*sqrt(x^4 + 5*x^2
+ 3)*(2*x^2 + 5) - 143/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B]
time = 0.29, size = 67, normalized size = 0.91

$$\frac{143\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{32}+\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}+\frac{\sqrt{x^4+5x^2+3}\,(8x^4+10x^2-51)}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (143*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/32 + (x^2/2 + 5/4)*(5*x^2 +
x^4 + 3)^(1/2) + ((5*x^2 + x^4 + 3)^(1/2)*(10*x^2 + 8*x^4 - 51))/16
```

3.145 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx$

Optimal. Leaf size=94

$$\frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4}+\frac{1}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 1/16*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-arctanh(1/6*(5*x^2+6)*3^(1/
2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/8*(6*x^2+23)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 828, 857, 635, 212, 738}

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\,(6x^2+23)+\frac{1}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*
x^2 + x^4])]/16 - Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x
^4])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x}\,dx,x,x^2\right)\\
&= \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4} - \frac{1}{8}\text{Subst}\left(\int \frac{-24-\frac{x}{2}}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4} + \frac{1}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right) + 3\\
&= \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4} + \frac{1}{8}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right.\\
&= \frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4} + \frac{1}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \sqrt{3}\,\tan
\end{aligned}$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 0.94

$$\frac{1}{8}(23+6x^2)\sqrt{3+5x^2+x^4}+2\sqrt{3}\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)-\frac{1}{16}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x,x]
```

```
[Out] ((23 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 + 2*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 +
5*x^2 + x^4])/Sqrt[3]] - Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]/16
```

Maple [A]
time = 0.30, size = 85, normalized size = 0.90

method	result
elliptic	$\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{23\sqrt{x^4+5x^2+3}}{8}+\frac{\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}-\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+}}\right.$
default	$\frac{3(2x^2+5)\sqrt{x^4+5x^2+3}}{8}+\frac{\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}+\sqrt{x^4+5x^2+3}-\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{}}{6\sqrt{x^4+5x^2}}\right.$
trager	$\left(\frac{3x^2}{4}+\frac{23}{8}\right)\sqrt{x^4+5x^2+3}+\frac{\ln\left(-2x^2-2\sqrt{x^4+5x^2+3}-5\right)}{16}-\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\operatorname{RootOf}(}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+1/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+(x^4
+5*x^2+3)^(1/2)-arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.49, size = 89, normalized size = 0.95

$$\frac{3}{4}\sqrt{x^4+5x^2+3}\,x^2-\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{23}{8}\sqrt{x^4+5x^2+3}+\frac{1}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)
/x^2 + 6/x^2 + 5) + 23/8*sqrt(x^4 + 5*x^2 + 3) + 1/16*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.38, size = 95, normalized size = 1.01

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\,(6x^2+23)+\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\,(5x^2+6)-2\sqrt{x^4+5x^2+3}\,\left(5\sqrt{3}-6\right)+30}{x^2}\right)-\frac{1}{16}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5
*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1/16*log(-
2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x, x)
```

Giac [A]
time = 4.34, size = 98, normalized size = 1.04

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\,(6x^2+23)+\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{1}{16}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 23) + sqrt(3)*log((x^2 + sqrt(3) - sqrt(
x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) - 1/16*log(2*x^2
 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B]
time = 0.43, size = 86, normalized size = 0.91

$$\frac{\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{16}-\sqrt{3}\,\ln\left(\frac{3}{x^2}+\frac{\sqrt{3}\,\sqrt{x^4+5x^2+3}}{x^2}+\frac{5}{2}\right)+\frac{3\left(\frac{x^2}{2}+\frac{5}{4}\right)\sqrt{x^4+5x^2+3}}{2}+\sqrt{x^4+5x^2+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x,x)
```

```
[Out] log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2)/16 - 3^(1/2)*log(3/x^2 + (3^(1/2)*
(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2) + (3*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1
/2))/2 + (5*x^2 + x^4 + 3)^(1/2)
```

3.146 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx$

Optimal. Leaf size=97

$$-\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{7\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}$$

```
[Out] 19/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-7/3*arctanh(1/6*(5*x^2+6)*3
^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/2*(-3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 826, 857, 635, 212, 738}

$$-\frac{\sqrt{x^4+5x^2+3}\,(2-3x^2)}{2x^2}+\frac{19}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{7\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] -1/2*((2 - 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2 + (19*ArcTanh[(5 + 2*x^2)/(2*S
qrt[3 + 5*x^2 + x^4])])/4 - (7*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^
2 + x^4])])/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^2}\,dx,x,x^2\right)$$
$$= -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{-28-19x}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$
$$= -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right) + 7$$
$$= -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right.$$
$$= -\frac{(2-3x^2)\sqrt{3+5x^2+x^4}}{2x^2} + \frac{19}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{7\tanh^{-1}}{}$$

Mathematica [A]
time = 0.15, size = 91, normalized size = 0.94

$$\frac{(-2+3x^2)\sqrt{3+5x^2+x^4}}{2x^2}+\frac{14\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{19}{4}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^3,x]
```

```
[Out] ((-2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/(2*x^2) + (14*ArcTanh[(x^2 - Sqrt[3 + 
5*x^2 + x^4])/Sqrt[3]])/Sqrt[3] - (19*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x
^4]])/4
```

Maple [A]
time = 0.29, size = 104, normalized size = 1.07

method	result
risch	$-\frac{\sqrt{x^4+5x^2+3}}{x^2}+\frac{3\sqrt{x^4+5x^2+3}}{2}+\frac{19\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{7\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right.}{3}$
elliptic	$-\frac{\sqrt{x^4+5x^2+3}}{x^2}+\frac{3\sqrt{x^4+5x^2+3}}{2}+\frac{19\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{7\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right.}{3}$
trager	$\frac{(3x^2-2)\sqrt{x^4+5x^2+3}}{2x^2}+\frac{7\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\mathrm{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{3}$
default	$-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{3x^2}+\frac{7\sqrt{x^4+5x^2+3}}{3}+\frac{19\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{7\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{}}{3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^2*(x^4+5*x^2+3)^(3/2)+7/3*(x^4+5*x^2+3)^(1/2)+19/4*ln(x^2+5/2+(x^4+5
*x^2+3)^(1/2))-7/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/
2)+1/6*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Maxima [A]
time = 0.49, size = 89, normalized size = 0.92

$$-\frac{7}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{\sqrt{x^4+5x^2+3}}{x^2}+\frac{19}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -7/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*sqr
t(x^4 + 5*x^2 + 3) - sqrt(x^4 + 5*x^2 + 3)/x^2 + 19/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.35, size = 112, normalized size = 1.15

$$\frac{56\sqrt{3}\,x^2\log\left(\frac{25x^2-2\sqrt{3}\,(5x^2+6)-2\sqrt{x^4+5x^2+3}\,\left(5\sqrt{3}-6\right)+30}{x^2}\right)-114x^2\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+21x^2+12\sqrt{x^4+5x^2+3}\,(3x^2-2)}{24x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/24*(56*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 114*x^2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) + 21*x^2 + 12*sqrt(x^4 + 5*x^2 + 3)*(3*x^2 - 2))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**3, x)
```

Giac [A]
time = 3.49, size = 138, normalized size = 1.42

$$\frac{7}{3}\sqrt{3}\,\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{3}{2}\sqrt{x^4+5x^2+3}+\frac{5x^2-5\sqrt{x^4+5x^2+3}+6}{\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3}-\frac{19}{4}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 7/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + (5*x^2 - 5*sqrt(x^4 + 5
*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3) - 19/4*log(2*x^2 - 2*s
qrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B]
time = 0.88, size = 84, normalized size = 0.87

$$\frac{19\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{4}-\frac{\sqrt{x^4+5x^2+3}}{x^2}-\frac{7\sqrt{3}\,\ln\left(\frac{3}{x^2}+\frac{\sqrt{3}\,\sqrt{x^4+5x^2+3}}{x^2}+\frac{5}{2}\right)}{3}+\frac{3\sqrt{x^4+5x^2+3}}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^3,x)
```

```
[Out] (19*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/4 - (5*x^2 + x^4 + 3)^(1/2)/x
^2 - (7*3^(1/2)*log(3/x^2 + (3^(1/2)*(5*x^2 + x^4 + 3)^(1/2))/x^2 + 5/2))/3
 + (3*(5*x^2 + x^4 + 3)^(1/2))/2
```

3.147 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx$

Optimal. Leaf size=99

$$-\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{77\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{24\sqrt{3}}$$

```
[Out] 3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-77/72*arctanh(1/6*(5*x^2+6)*
3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/12*(23*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^
4
```

Rubi [A]
time = 0.05, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 824, 857, 635, 212, 738}

$$-\frac{\sqrt{x^4+5x^2+3}\,(23x^2+6)}{12x^4}+\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{77\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
```

```
[Out] -1/12*((6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + (3*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/2 - (77*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*
x^2 + x^4])])/(24*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} - \frac{1}{24}\text{Subst}\left(\int \frac{-77-36x}{x\sqrt{3+5x+x^2}}\,dx, x, x^2\right)$$

$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right) + \frac{7}{2}$$

$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + 3\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)$$

$$= -\frac{(6+23x^2)\sqrt{3+5x^2+x^4}}{12x^4} + \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right) - \frac{77\tanh^{-1}}{}$$

Mathematica [A]
time = 0.18, size = 91, normalized size = 0.92

$$\frac{1}{36}\left(-\frac{3(6+23x^2)\sqrt{3+5x^2+x^4}}{x^4} + 77\sqrt{3}\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - 54\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^5,x]
[Out] ((-3*(6 + 23*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 + 77*Sqrt[3]*ArcTanh[(x^2 - Sq
rt[3 + 5*x^2 + x^4])/Sqrt[3]] - 54*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]
])/36
```

Maple [A]
time = 0.28, size = 121, normalized size = 1.22

method	result
risch	$-\frac{23x^6+121x^4+99x^2+18}{12x^4\sqrt{x^4+5x^2+3}} + \frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2} - \frac{77\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}$
elliptic	$\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2} - \frac{\sqrt{x^4+5x^2+3}}{2x^4} - \frac{23\sqrt{x^4+5x^2+3}}{12x^2} - \frac{77\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)}{72}$
trager	$-\frac{(23x^2+6)\sqrt{x^4+5x^2+3}}{12x^4} - \frac{77\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\,\text{RootOf}\left(\_Z^2-3\right)x^2+6\,\text{RootOf}\left(\_Z^2-3\right)+6\sqrt{x^4+5x^2+3}}{x^2}\right.}{72}$

default	$-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{6x^4}-\frac{13(x^4+5x^2+3)^{\frac{3}{2}}}{36x^2}+\frac{77\sqrt{x^4+5x^2+3}}{72}-\frac{77\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{72}+\frac{13(2x^2+5)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] -1/6/x^4*(x^4+5*x^2+3)^(3/2)-13/36/x^2*(x^4+5*x^2+3)^(3/2)+77/72*(x^4+5*x^2
+3)^(1/2)-77/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
13/72*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)+3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A]
time = 0.49, size = 106, normalized size = 1.07

$$-\frac{77}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{6}\sqrt{x^4+5x^2+3}-\frac{13\sqrt{x^4+5x^2+3}}{12x^2}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{6x^4}+\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="maxima")
```

```
[Out] -77/72*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/6*s
qrt(x^4 + 5*x^2 + 3) - 13/12*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/6*(x^4 + 5*x^2 +
 3)^(3/2)/x^4 + 3/2*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.35, size = 112, normalized size = 1.13

$$\frac{77\sqrt{3}\,x^4\log\left(\frac{25x^2-2\sqrt{3}\,(5x^2+6)-2\sqrt{x^4+5x^2+3}\,\left(5\sqrt{3}-6\right)+30}{x^2}\right)-108x^4\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)-138x^4-6\sqrt{x^4+5x^2+3}\,(23x^2+6)}{72x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/72*(77*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x
^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 108*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) - 5) - 138*x^4 - 6*sqrt(x^4 + 5*x^2 + 3)*(23*x^2 + 6))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**5, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(79) = 158.
time = 4.22, size = 169, normalized size = 1.71

$$\frac{77}{72}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{127\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+228\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-159x^2+159\sqrt{x^4+5x^2+3}-324}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2}-\frac{3}{2}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^5,x, algorithm="giac")
```

```
[Out] 77/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
sqrt(x^4 + 5*x^2 + 3))) + 1/12*(127*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 228*(
x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 159*x^2 + 159*sqrt(x^4 + 5*x^2 + 3) - 324)
/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2 - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x
^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^5, x)
```

3.148 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx$

Optimal. Leaf size=90

$$-\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4}-\frac{(3+5x^2+x^4)^{3/2}}{9x^6}+\frac{13\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{36\sqrt{3}}$$

```
[Out] -1/9*(x^4+5*x^2+3)^(3/2)/x^6+13/108*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^
2+3)^(1/2))*3^(1/2)-1/18*(5*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^4
```

Rubi [A]
time = 0.04, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 820, 734, 738, 212}

$$-\frac{(5x^2+6)\sqrt{x^4+5x^2+3}}{18x^4}+\frac{13\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{36\sqrt{3}}-\frac{(x^4+5x^2+3)^{3/2}}{9x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] -1/18*((6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 - (3 + 5*x^2 + x^4)^(3/2)/(9*
x^6) + (13*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(36*Sqrt
[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{2}{3}\text{Subst}\left(\int \frac{\sqrt{3+5x+x^2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} - \frac{13}{36}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5\ldots}}\right.$$

$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13}{18}\text{Subst}\left(\int \frac{1}{12-x^2}\,d\ldots\right.$$

$$= -\frac{(6+5x^2)\sqrt{3+5x^2+x^4}}{18x^4} - \frac{(3+5x^2+x^4)^{3/2}}{9x^6} + \frac{13\tanh^{-1}\left(\frac{6+\ldots}{2\sqrt{3}\sqrt{3+\ldots}}\right.}{36\sqrt{3}}$$

Mathematica [A]
time = 0.20, size = 70, normalized size = 0.78

$$\frac{1}{54}\left(-\frac{3\sqrt{3+5x^2+x^4}\,(6+16x^2+7x^4)}{x^6} - 13\sqrt{3}\,\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^7,x]
```

```
[Out] ((-3*Sqrt[3 + 5*x^2 + x^4]*(6 + 16*x^2 + 7*x^4))/x^6 - 13*Sqrt[3]*ArcTanh[(
x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])/54
```

Maple [A]
time = 0.20, size = 118, normalized size = 1.31

method	result
risch	$-\frac{7x^8+51x^6+107x^4+78x^2+18}{18x^6\sqrt{x^4+5x^2+3}}+\frac{13\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{108}$
trager	$-\frac{(7x^4+16x^2+6)\sqrt{x^4+5x^2+3}}{18x^6}-\frac{13\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\mathrm{RootOf}\left(\_\right.}{x^2}\right.}{108}$
elliptic	$-\frac{8\sqrt{x^4+5x^2+3}}{9x^4}-\frac{7\sqrt{x^4+5x^2+3}}{18x^2}+\frac{13\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{108}-\frac{\sqrt{x^4+5x^2+3}}{3x^6}$
default	$-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^4}+\frac{5(x^4+5x^2+3)^{\frac{3}{2}}}{54x^2}-\frac{13\sqrt{x^4+5x^2+3}}{108}+\frac{13\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{108}-\frac{5(2x^2+5)\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] -1/9/x^4*(x^4+5*x^2+3)^(3/2)+5/54/x^2*(x^4+5*x^2+3)^(3/2)-13/108*(x^4+5*x^2
+3)^(1/2)+13/108*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
-5/108*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)-1/9*(x^4+5*x^2+3)^(3/2)/x^6
```

Maxima [A]
time = 0.52, size = 99, normalized size = 1.10

$$\frac{13}{108}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{1}{9}\sqrt{x^4+5x^2+3}+\frac{5\sqrt{x^4+5x^2+3}}{18x^2}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^4}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="maxima")
```

```
[Out] 13/108*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1/9*s
qrt(x^4 + 5*x^2 + 3) + 5/18*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/9*(x^4 + 5*x^2 +
3)^(3/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(3/2)/x^6
```

Fricas [A]
time = 0.35, size = 90, normalized size = 1.00

$$\frac{13\sqrt{3}\,x^6\log\left(\frac{25\,x^2+2\,\sqrt{3}\,(5\,x^2+6)+2\,\sqrt{x^4+5\,x^2+3}\,\left(5\,\sqrt{3}+6\right)+30}{x^2}\right)-42\,x^6-6\,(7\,x^4+16\,x^2+6)\sqrt{x^4+5\,x^2+3}}{108\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/108*(13*sqrt(3)*x^6*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 42*x^6 - 6*(7*x^4 + 16*x^2 + 6)*sqrt(
x^4 + 5*x^2 + 3))/x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(72) = 144.
time = 3.49, size = 189, normalized size = 2.10

$$-\frac{13}{108}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)+\frac{67\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5+306\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^4+430\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^3+90\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-63\,x^2+63\sqrt{x^4+5\,x^2+3}+108}{18\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^7,x, algorithm="giac")
```

```
[Out] -13/108*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3)
- sqrt(x^4 + 5*x^2 + 3))) + 1/18*(67*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 306*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 + 430*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 90*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 63*x^2 + 63*sqrt(x^4 + 5*x^2 + 3) + 108)/
((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^7, x)
```

3.149 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx$

Optimal. Leaf size=111

$$\frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4}-\frac{(3+5x^2+x^4)^{3/2}}{12x^8}-\frac{11(3+5x^2+x^4)^{3/2}}{216x^6}-\frac{871\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3456\sqrt{3}}$$

```
[Out] -1/12*(x^4+5*x^2+3)^(3/2)/x^8-11/216*(x^4+5*x^2+3)^(3/2)/x^6-871/10368*arct
anh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+67/1728*(5*x^2+6)*(x
^4+5*x^2+3)^(1/2)/x^4
```

Rubi [A]
time = 0.06, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 848, 820, 734, 738, 212}

$$\frac{67(5x^2+6)\sqrt{x^4+5x^2+3}}{1728x^4}-\frac{871\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3456\sqrt{3}}-\frac{(x^4+5x^2+3)^{3/2}}{12x^8}-\frac{11(x^4+5x^2+3)^{3/2}}{216x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (67*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(1728*x^4) - (3 + 5*x^2 + x^4)^(3/2)
/(12*x^8) - (11*(3 + 5*x^2 + x^4)^(3/2))/(216*x^6) - (871*ArcTanh[(6 + 5*x^
2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(3456*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^9}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^5}\,dx, x, x^2\right)$$

$$= -\frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{1}{24}\text{Subst}\left(\int \frac{(-11+2x)\sqrt{3+5x+x^2}}{x^4}\,dx, x, x^2\right.$$

$$= -\frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6} - \frac{67}{144}\text{Subst}\left(\int \frac{\sqrt{3+5x+}}{x^3}\right.$$

$$= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6}$$

$$= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6}$$

$$= \frac{67(6+5x^2)\sqrt{3+5x^2+x^4}}{1728x^4} - \frac{(3+5x^2+x^4)^{3/2}}{12x^8} - \frac{11(3+5x^2+x^4)^{3/2}}{216x^6}$$

Mathematica [A]
time = 0.23, size = 80, normalized size = 0.72

$$\frac{\sqrt{3+5x^2+x^4}\,(-432-984x^2-182x^4+247x^6)}{1728x^8} + \frac{871\tanh^{-1}\left(\frac{x^2}{\sqrt{3}} - \frac{\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{1728\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^9,x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-432 - 984*x^2 - 182*x^4 + 247*x^6))/(1728*x^8) + (
871*ArcTanh[x^2/Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(1728*Sqrt[3])
```

Maple [A]
time = 0.20, size = 135, normalized size = 1.22

method	result
risch	$\frac{247x^{10}+1053x^8-1153x^6-5898x^4-5112x^2-1296}{1728x^8\sqrt{x^4+5x^2+3}} - \frac{871\,\text{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{10368}$

trager	$\frac{(247x^6-182x^4-984x^2-432)\sqrt{x^4+5x^2+3}}{1728x^8}-\frac{871\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\,\mathrm{RootOf}\left(\_Z^2-3\right)+6\sqrt{x}}{x^2}\right.}{10368}$
elliptic	$-\frac{91\sqrt{x^4+5x^2+3}}{864x^4}+\frac{247\sqrt{x^4+5x^2+3}}{1728x^2}-\frac{871\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{10368}-\frac{\sqrt{x^4+5x^2+3}}{4x^8}$
default	$-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{12x^8}-\frac{11(x^4+5x^2+3)^{\frac{3}{2}}}{216x^6}+\frac{67(x^4+5x^2+3)^{\frac{3}{2}}}{864x^4}-\frac{335(x^4+5x^2+3)^{\frac{3}{2}}}{5184x^2}+\frac{871\sqrt{x^4+5x^2+3}}{10368}-\frac{871\,\mathrm{arctanh}\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x,method=_RETURNVERBOSE)
```

```
[Out] -1/12*(x^4+5*x^2+3)^(3/2)/x^8-11/216*(x^4+5*x^2+3)^(3/2)/x^6+67/864/x^4*(x^
4+5*x^2+3)^(3/2)-335/5184/x^2*(x^4+5*x^2+3)^(3/2)+871/10368*(x^4+5*x^2+3)^(
1/2)-871/10368*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+3
35/10368*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Maxima [A]
time = 0.50, size = 116, normalized size = 1.05

$$-\frac{871}{10368}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{67}{864}\sqrt{x^4+5x^2+3}-\frac{335\sqrt{x^4+5x^2+3}}{1728x^2}+\frac{67(x^4+5x^2+3)^{\frac{3}{2}}}{864x^4}-\frac{11(x^4+5x^2+3)^{\frac{3}{2}}}{216x^6}-\frac{(x^4+5x^2+3)^{\frac{3}{2}}}{12x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="maxima")
```

```
[Out] -871/10368*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 6
7/864*sqrt(x^4 + 5*x^2 + 3) - 335/1728*sqrt(x^4 + 5*x^2 + 3)/x^2 + 67/864*(
x^4 + 5*x^2 + 3)^(3/2)/x^4 - 11/216*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/12*(x^4
 + 5*x^2 + 3)^(3/2)/x^8
```

Fricas [A]
time = 0.36, size = 95, normalized size = 0.86

$$\frac{871\sqrt{3}\,x^8\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)+1482x^8+6(247x^6-182x^4-984x^2-432)\sqrt{x^4+5x^2+3}}{10368x^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="fricas")
```

```
[Out] 1/10368*(871*sqrt(3)*x^8*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 1482*x^8 + 6*(247*x^6 - 182*x^4 -
984*x^2 - 432)*sqrt(x^4 + 5*x^2 + 3))/x^8
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**9,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**9, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(89) = 178.
time = 3.54, size = 233, normalized size = 2.10

$$\frac{871}{10368}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{871\left(x^2-\sqrt{x^4+5x^2+3}\right)^7-5184\left(x^2-\sqrt{x^4+5x^2+3}\right)^6-57389\left(x^2-\sqrt{x^4+5x^2+3}\right)^5-165888\left(x^2-\sqrt{x^4+5x^2+3}\right)^4-204807\left(x^2-\sqrt{x^4+5x^2+3}\right)^3-93312\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-2403x^2+2403\sqrt{x^4+5x^2+3}-5184}{1728\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^9,x, algorithm="giac")
```

```
[Out] 871/10368*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3
) - sqrt(x^4 + 5*x^2 + 3))) - 1/1728*(871*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 -
 5184*(x^2 - sqrt(x^4 + 5*x^2 + 3))^6 - 57389*(x^2 - sqrt(x^4 + 5*x^2 + 3))
^5 - 165888*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 204807*(x^2 - sqrt(x^4 + 5*x^
2 + 3))^3 - 93312*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 2403*x^2 + 2403*sqrt(x^
4 + 5*x^2 + 3) - 5184)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^4
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^9}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^9, x)
```

3.150 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx$

Optimal. Leaf size=132

$$-\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4}-\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}}-\frac{(3+5x^2+x^4)^{3/2}}{36x^8}+\frac{173(3+5x^2+x^4)^{3/2}}{3240x^6}+\frac{2093\tanh^{-1}}{}$$

```
[Out] -1/15*(x^4+5*x^2+3)^(3/2)/x^10-1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3240*(x^4+5
*x^2+3)^(3/2)/x^6+2093/31104*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1
/2))*3^(1/2)-161/5184*(5*x^2+6)*(x^4+5*x^2+3)^(1/2)/x^4
```

Rubi [A]
time = 0.07, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 848, 820, 734, 738, 212}

$$-\frac{161(5x^2+6)\sqrt{x^4+5x^2+3}}{5184x^4}+\frac{2093\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{10368\sqrt{3}}-\frac{(x^4+5x^2+3)^{3/2}}{15x^{10}}-\frac{(x^4+5x^2+3)^{3/2}}{36x^8}+\frac{173(x^4+5x^2+3)^{3/2}}{3240x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] (-161*(6 + 5*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5184*x^4) - (3 + 5*x^2 + x^4)^(3/
2)/(15*x^10) - (3 + 5*x^2 + x^4)^(3/2)/(36*x^8) + (173*(3 + 5*x^2 + x^4)^(3
/2))/(3240*x^6) + (2093*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4
])])/(10368*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 734

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(-(d + e*x)^(m + 1))*(d*b - 2*a*e + (2*c*d - b*e)*x)*((a + b
*x + c*x^2)^p/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[p*((b^2 - 4*a
*c)/(2*(m + 1)*(c*d^2 - b*d*e + a*e^2))), Int[(d + e*x)^(m + 2)*(a + b*x +
c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0]
&& NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && EqQ[m + 2*p + 2,
 0] && GtQ[p, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^{11}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)\sqrt{3+5x+x^2}}{x^6}\,dx, x, x^2\right)$$
$$= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{1}{30}\text{Subst}\left(\int \frac{(-10+4x)\sqrt{3+5x+x^2}}{x^5}\,dx, x, x^2\right)$$
$$= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{1}{360}\text{Subst}\left(\int \frac{(-173-10x)\sqrt{}}{x^4}\right.$$
$$= -\frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8} + \frac{173(3+5x^2+x^4)^{3/2}}{3240x^6} + \frac{161}{432}\text{Su}$$
$$= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8}$$
$$= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8}$$
$$= -\frac{161(6+5x^2)\sqrt{3+5x^2+x^4}}{5184x^4} - \frac{(3+5x^2+x^4)^{3/2}}{15x^{10}} - \frac{(3+5x^2+x^4)^{3/2}}{36x^8}$$

Mathematica [A]
time = 0.28, size = 80, normalized size = 0.61

$$\frac{-\frac{3\sqrt{3+5x^2+x^4}\,(5184+10800x^2+1176x^4-1370x^6+2641x^8)}{x^{10}} - 10465\sqrt{3}\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{77760}$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^11,x]
```

```
[Out] ((-3*Sqrt[3 + 5*x^2 + x^4]*(5184 + 10800*x^2 + 1176*x^4 - 1370*x^6 + 2641*x
^8))/x^10 - 10465*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])/7
7760
```

Maple [A]
time = 0.22, size = 152, normalized size = 1.15

method	result

risch	$-\frac{2641x^{12}+11835x^{10}+2249x^{8}+12570x^{6}+62712x^{4}+58320x^{2}+15552}{25920x^{10}\sqrt{x^{4}+5x^{2}+3}}+\frac{2093\,\mathrm{arctanh}\left(\frac{(5x^{2}+6)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{3}}{31104}$
trager	$-\frac{(2641x^{8}-1370x^{6}+1176x^{4}+10800x^{2}+5184)\sqrt{x^{4}+5x^{2}+3}}{25920x^{10}}-\frac{2093\,\mathrm{RootOf}\left(\_Z^{2}-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^{2}-3\right)x^{2}+6\sqrt{}}{}\right.}{31104}$
elliptic	$\frac{137\sqrt{x^{4}+5x^{2}+3}}{2592x^{4}}-\frac{2641\sqrt{x^{4}+5x^{2}+3}}{25920x^{2}}+\frac{2093\,\mathrm{arctanh}\left(\frac{(5x^{2}+6)\sqrt{3}}{6\sqrt{x^{4}+5x^{2}+3}}\right)\sqrt{3}}{31104}-\frac{\sqrt{x^{4}+5x^{2}+3}}{5x^{10}}$
default	$-\frac{(x^{4}+5x^{2}+3)^{\frac{3}{2}}}{36x^{8}}+\frac{173(x^{4}+5x^{2}+3)^{\frac{3}{2}}}{3240x^{6}}-\frac{161(x^{4}+5x^{2}+3)^{\frac{3}{2}}}{2592x^{4}}+\frac{805(x^{4}+5x^{2}+3)^{\frac{3}{2}}}{15552x^{2}}-\frac{2093\sqrt{x^{4}+5x^{2}+3}}{31104}+\frac{2093\,\mathrm{arct}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x,method=_RETURNVERBOSE)
```

```
[Out] -1/36*(x^4+5*x^2+3)^(3/2)/x^8+173/3240*(x^4+5*x^2+3)^(3/2)/x^6-161/2592/x^4
*(x^4+5*x^2+3)^(3/2)+805/15552/x^2*(x^4+5*x^2+3)^(3/2)-2093/31104*(x^4+5*x^
2+3)^(1/2)+2093/31104*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^
(1/2)-805/31104*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)-1/15*(x^4+5*x^2+3)^(3/2)/x^10
```

Maxima [A]
time = 0.48, size = 133, normalized size = 1.01

$$\frac{2093}{31104}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^{4}+5x^{2}+3}}{x^{2}}+\frac{6}{x^{2}}+5\right)+\frac{161}{2592}\sqrt{x^{4}+5x^{2}+3}+\frac{805\sqrt{x^{4}+5x^{2}+3}}{5184x^{2}}-\frac{161(x^{4}+5x^{2}+3)^{\frac{3}{2}}}{2592x^{4}}+\frac{173(x^{4}+5x^{2}+3)^{\frac{3}{2}}}{3240x^{6}}-\frac{(x^{4}+5x^{2}+3)^{\frac{3}{2}}}{36x^{8}}-\frac{(x^{4}+5x^{2}+3)^{\frac{3}{2}}}{15x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="maxima")
```

```
[Out] 2093/31104*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 1
61/2592*sqrt(x^4 + 5*x^2 + 3) + 805/5184*sqrt(x^4 + 5*x^2 + 3)/x^2 - 161/25
92*(x^4 + 5*x^2 + 3)^(3/2)/x^4 + 173/3240*(x^4 + 5*x^2 + 3)^(3/2)/x^6 - 1/3
6*(x^4 + 5*x^2 + 3)^(3/2)/x^8 - 1/15*(x^4 + 5*x^2 + 3)^(3/2)/x^10
```

Fricas [A]
time = 0.37, size = 100, normalized size = 0.76

$$\frac{10465\sqrt{3}x^{10}\log\left(\frac{25x^{2}+2\sqrt{3}(5x^{2}+6)+2\sqrt{x^{4}+5x^{2}+3}\left(5\sqrt{3}+6\right)+30}{x^{2}}\right)-15846x^{10}-6(2641x^{8}-1370x^{6}+1176x^{4}+10800x^{2}+5184)\sqrt{x^{4}+5x^{2}+3}}{155520x^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="fricas")
```

```
[Out] 1/155520*(10465*sqrt(3)*x^10*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x
^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 15846*x^10 - 6*(2641*x^8 - 137
0*x^6 + 1176*x^4 + 10800*x^2 + 5184)*sqrt(x^4 + 5*x^2 + 3))/x^10
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**11,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**11, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. 2(106) = 212.
time = 3.19, size = 255, normalized size = 1.93

$$-\frac{2093}{31104}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{10465\left(x^2-\sqrt{x^4+5x^2+3}\right)^9-42830\left(x^2-\sqrt{x^4+5x^2+3}\right)^7+1270080\left(x^2-\sqrt{x^4+5x^2+3}\right)^6+7060800\left(x^2-\sqrt{x^4+5x^2+3}\right)^5+15310080\left(x^2-\sqrt{x^4+5x^2+3}\right)^4+16095870\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+7568640\left(x^2-\sqrt{x^4+5x^2+3}\right)^2+1096335x^2-1096335\sqrt{x^4+5x^2+3}+202176}{25920\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^11,x, algorithm="giac")
```

```
[Out] -2093/31104*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt
(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/25920*(10465*(x^2 - sqrt(x^4 + 5*x^2 + 3)
)^9 - 42830*(x^2 - sqrt(x^4 + 5*x^2 + 3))^7 + 1270080*(x^2 - sqrt(x^4 + 5*x
^2 + 3))^6 + 7060800*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 + 15310080*(x^2 - sqrt
(x^4 + 5*x^2 + 3))^4 + 16095870*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 7568640*(
x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 1096335*x^2 - 1096335*sqrt(x^4 + 5*x^2 + 3
) + 202176)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^5
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^{11}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^11, x)
```

3.151 $\int x^4(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=322

$$-\frac{1924x\left(5+\sqrt{13}+2x^2\right)}{105\sqrt{3+5x^2+x^4}}+\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4}+$$

```
[Out] -1924/105*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+13/3*x*(x^4+5*x^2+3)^(1/
2)-26/35*x^3*(x^4+5*x^2+3)^(1/2)+1/21*x^5*(7*x^2+11)*(x^4+5*x^2+3)^(1/2)+96
2/315*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*Ell
ipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*1
3^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1
/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-13*(1/(36+x^2*(30+6*1
3^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))
^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(
5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3
)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1287, 1293, 1203, 1113, 1149}

$$-\frac{13\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{962\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{105\sqrt{x^4+5x^2+3}}+\frac{13}{3}\sqrt{x^4+5x^2+3}\,x-\frac{1924\left(2x^2+\sqrt{13}+5\right)x}{105\sqrt{x^4+5x^2+3}}+\frac{1}{21}\left(7x^2+11\right)\sqrt{x^4+5x^2+3}\,x^5-\frac{26}{35}\sqrt{x^4+5x^2+3}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-1924*x*(5 + Sqrt[13] + 2*x^2))/(105*Sqrt[3 + 5*x^2 + x^4]) + (13*x*Sqrt[3
 + 5*x^2 + x^4])/3 - (26*x^3*Sqrt[3 + 5*x^2 + x^4])/35 + (x^5*(11 + 7*x^2)*
Sqrt[3 + 5*x^2 + x^4])/21 + (962*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(105*Sqrt[3 + 5*x
^2 + x^4]) - (13*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
```

```
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1287

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Dist[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^4\left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx=\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{63}\int\frac{x^4(-117-234x^2)}{\sqrt{3+5x^2+x^4}}\,dx$$
$$=-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5x^2+x^4}-\frac{1}{315}\int\frac{x^2}{\quad}$$
$$=\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5}$$
$$=\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+x^4}+\frac{1}{21}x^5\left(11+7x^2\right)\sqrt{3+5}$$

$$=-\frac{1924x\left(5+\sqrt{13}+2x^2\right)}{105\sqrt{3+5x^2+x^4}}+\frac{13}{3}x\sqrt{3+5x^2+x^4}-\frac{26}{35}x^3\sqrt{3+5x^2+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.93, size = 237, normalized size = 0.74

$$\frac{2730x+4082x^3+460x^5+604x^7+460x^9+70x^{11}-1924i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+13i\sqrt{2}\left(-635+148\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{210\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2730*x + 4082*x^3 + 460*x^5 + 604*x^7 + 460*x^9 + 70*x^11 - (1924*I)*Sqrt[
2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + S
qrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*S
qrt[13])/6] + (13*I)*Sqrt[2]*(-635 + 148*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*
x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2
/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.13, size = 260, normalized size = 0.81

method	result
risch	$\frac{x(35x^6+55x^4-78x^2+455)\sqrt{x^4+5x^2+3}}{105}+\frac{46176\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(}{35\sqrt{-30}}$

default	$\frac{x^7\sqrt{x^4+5x^2+3}}{3}+\frac{11x^5\sqrt{x^4+5x^2+3}}{21}-\frac{26x^3\sqrt{x^4+5x^2+3}}{35}+\frac{13x\sqrt{x^4+5x^2+3}}{3}-\frac{78\sqrt{1-(\cdots}}{\cdots}$
elliptic	$\frac{x^7\sqrt{x^4+5x^2+3}}{3}+\frac{11x^5\sqrt{x^4+5x^2+3}}{21}-\frac{26x^3\sqrt{x^4+5x^2+3}}{35}+\frac{13x\sqrt{x^4+5x^2+3}}{3}-\frac{78\sqrt{1-(\cdots}}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^7*(x^4+5*x^2+3)^(1/2)+11/21*x^5*(x^4+5*x^2+3)^(1/2)-26/35*x^3*(x^4+5*
x^2+3)^(1/2)+13/3*x*(x^4+5*x^2+3)^(1/2)-78/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+
1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1
/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+46176/
35/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13
^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6
*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))
^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4\cdot\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

3.152 $\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=305

$$\frac{1247x\left(5+\sqrt{13}+2x^2\right)}{210\sqrt{3+5x^2+x^4}}-\frac{4}{3}x\sqrt{3+5x^2+x^4}+\frac{1}{35}x^3\left(29+15x^2\right)\sqrt{3+5x^2+x^4}-\frac{1247\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\ldots}}{\ldots}$$

```
[Out] 1247/210*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-4/3*x*(x^4+5*x^2+3)^(1/2)
+1/35*x^3*(15*x^2+29)*(x^4+5*x^2+3)^(1/2)+2/3*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)-1247/1260*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^
2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13
^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(
1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)
^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1287, 1293, 1203, 1113, 1149}

$$\frac{2\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{1247\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{210\sqrt{x^4+5x^2+3}}-\frac{4}{3}\sqrt{x^4+5x^2+3}\,x+\frac{1247\left(2x^2+\sqrt{13}+5\right)x}{210\sqrt{x^4+5x^2+3}}+\frac{1}{35}\left(15x^2+29\right)\sqrt{x^4+5x^2+3}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (1247*x*(5 + Sqrt[13] + 2*x^2))/(210*Sqrt[3 + 5*x^2 + x^4]) - (4*x*Sqrt[3 +
 5*x^2 + x^4])/3 + (x^3*(29 + 15*x^2)*Sqrt[3 + 5*x^2 + x^4])/35 - (1247*Sqr
t[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(210*Sqrt[3 + 5*x^2 + x^4]) + (2*Sqrt[2/(3*(5 + Sqrt[13])
)]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[1
3])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])
/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
```

```
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1287

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Dist[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^2(2+3x^2)\sqrt{3+5x^2+x^4}\,dx = \frac{1}{35}x^3(29+15x^2)\sqrt{3+5x^2+x^4} + \frac{1}{35}\int \frac{x^2(-51-140x^2)}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{4}{3}x\sqrt{3+5x^2+x^4} + \frac{1}{35}x^3(29+15x^2)\sqrt{3+5x^2+x^4} - \frac{1}{105}\int \frac{-42}{\sqrt{3}}$$

$$= -\frac{4}{3}x\sqrt{3+5x^2+x^4} + \frac{1}{35}x^3(29+15x^2)\sqrt{3+5x^2+x^4} + 4\int \frac{}{\sqrt{3+5}}$$

$$= \frac{1247x\left(5+\sqrt{13}+2x^2\right)}{210\sqrt{3+5x^2+x^4}} - \frac{4}{3}x\sqrt{3+5x^2+x^4} + \frac{1}{35}x^3(29+15x^2)\sqrt{3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.13, size = 234, normalized size = 0.77

$$\frac{4x(-420-439x^2+430x^4+312x^6+45x^8)+1247i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-5395+1247\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{420\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (4*x*(-420 - 439*x^2 + 430*x^4 + 312*x^6 + 45*x^8) + (1247*I)*Sqrt[2]*(-5 +
 Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13]
+ 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])
/6] - I*Sqrt[2]*(-5395 + 1247*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 +
Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[
13])]*x], 19/6 + (5*Sqrt[13])/6])/(420*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.05, size = 243, normalized size = 0.80

method	result
risch	$\frac{x(45x^4+87x^2-140)\sqrt{x^4+5x^2+3}}{105} - \frac{14964\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{Elliptic}\right.}{35\sqrt{-30+6\sqrt{}}}$
default	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7} + \frac{29x^3\sqrt{x^4+5x^2+3}}{35} - \frac{4x\sqrt{x^4+5x^2+3}}{3} + \frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1}}{\sqrt{}}$

elliptic	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7}+\frac{29x^3\sqrt{x^4+5x^2+3}}{35}-\frac{4x\sqrt{x^4+5x^2+3}}{3}+\frac{24\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*x^5*(x^4+5*x^2+3)^(1/2)+29/35*x^3*(x^4+5*x^2+3)^(1/2)-4/3*x*(x^4+5*x^2+
3)^(1/2)+24/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5
/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(
1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-14964/35/(-30+6*13^(1/2))^(1/2)*(1-(-
5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3
)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/
6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)
))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\cdot\left(3x^2+2\right)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)*x^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(3\,x^2+2\right)\,\sqrt{x^4+5\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

3.153 $\int (2+3x^2)\sqrt{3+5x^2+x^4}\,dx$

Optimal. Leaf size=279

$$-\frac{23x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}}+\frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}+\frac{23\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\right.}{}$$

```
[Out] -23/15*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+1/15*x*(9*x^2+25)*(x^4+5*x^
2+3)^(1/2)+23/90*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2))
)^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/
6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x
^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+(1/(36+x^2
*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*1
3^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*
(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4
+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1190, 1203, 1113, 1149}

$$\frac{\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{23\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{x^4+5x^2+3}}-\frac{23x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}+\frac{1}{15}x\left(9x^2+25\right)\sqrt{x^4+5x^2+3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-23*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) + (x*(25 + 9*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (23*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt
[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[Arc
Tan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sqrt[3 + 5*x^2 +
x^4]) + (Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13
])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

```
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \left(2+3x^2\right)\sqrt{3+5x^2+x^4}\,dx &= \frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{15}\int\frac{15-46x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}-\frac{46}{15}\int\frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+\int\frac{1}{\sqrt{3+5x}}\\ &= -\frac{23x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}}+\frac{1}{15}x\left(25+9x^2\right)\sqrt{3+5x^2+x^4}+\frac{23\sqrt{\frac{1}{6}\left(5+\right.}}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.68, size = 229, normalized size = 0.82

$$\frac{2x(75+152x^2+70x^4+9x^6)-23i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+i\sqrt{2}\left(-130+23\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{30\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(75 + 152*x^2 + 70*x^4 + 9*x^6) - (23*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[
(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipti
cE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*
(-130 + 23*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.05, size = 226, normalized size = 0.81

method	result
risch	$\frac{x(9x^2+25)\sqrt{x^4+5x^2+3}}{15}+\frac{552\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-\cdots}}{\cdots}\right.\right.}{5\sqrt{-30+6\sqrt{13}}\sqrt{\cdots}}$
default	$\frac{3x^3\sqrt{x^4+5x^2+3}}{5}+\frac{5x\sqrt{x^4+5x^2+3}}{3}+\frac{6\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\cdots}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4\cdots}}$
elliptic	$\frac{3x^3\sqrt{x^4+5x^2+3}}{5}+\frac{5x\sqrt{x^4+5x^2+3}}{3}+\frac{6\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\cdots}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*x^3*(x^4+5*x^2+3)^(1/2)+5/3*x*(x^4+5*x^2+3)^(1/2)+6/(-30+6*13^(1/2))^(1
/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^
4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^
(1/2))+552/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-
5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/
6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6
*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (3x^2+2)\sqrt{x^4+5x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2),x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2),x)
[Out] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2), x)
```

3.154 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$

Optimal. Leaf size=284

$$\frac{9x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}}-\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x}-\frac{3\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{1\ldots}\right.\right.}{2\sqrt{3+\ldots}}$$

```
[Out] 9/2*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-(-x^2+2)*(x^4+5*x^2+3)^(1/2)/x
+8/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*Elli
pticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13
^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-1
3^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-3/4*(1/(36+x^2*(3
0+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(
1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+
x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^
(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 284, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1285, 1203, 1113, 1149}

$$\frac{8\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{3\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}\,(2-x^2)}{x}+\frac{9x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

```
[Out] (9*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - ((2 - x^2)*Sqrt[3
+ 5*x^2 + x^4])/x - (3*Sqrt[(3*(5 + Sqrt[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) +
 (8*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt
[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6
]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

```
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx = -\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} - \frac{1}{3}\int \frac{-48-27x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} + 9\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + 16\int \frac{1}{\sqrt{3+5x^2+}$$

$$= \frac{9x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} - \frac{(2-x^2)\sqrt{3+5x^2+x^4}}{x} - \frac{3\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.51, size = 231, normalized size = 0.81

$$\frac{4(-6-7x^2+3x^4+x^6)+9i\sqrt{2}\left(-5+\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-13+9\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{4x\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2,x]
```

```
[Out] (4*(-6 - 7*x^2 + 3*x^4 + x^6) + (9*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 +
Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*A
rcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-13 +
 9*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[
13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[
13])/6])/(4*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.05, size = 225, normalized size = 0.79

method	result
default	$x\sqrt{x^4+5x^2+3}+\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{}}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$
risch	$\frac{x^6+3x^4-7x^2-6}{x\sqrt{x^4+5x^2+3}}-\frac{324\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\ \left(\mathrm{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x}}$
elliptic	$x\sqrt{x^4+5x^2+3}+\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{}}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(x^4+5*x^2+3)^(1/2)+96/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)
^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*
x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-324/(-30+6*13^(1/2))^(1/
2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^
(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2)))-2*(x^4+5*x^2+3)^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="maxima")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="fricas")
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**2,x)
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^2,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^2,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^2, x)
```

3.155 $\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$

Optimal. Leaf size=305

$$\frac{32x\left(5+\sqrt{13}+2x^2\right)}{9\sqrt{3+5x^2+x^4}}-\frac{64\sqrt{3+5x^2+x^4}}{9x}-\frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3}-\frac{16\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{\right.}}{6+\left(5+\sqrt{\right.}}}}{}$$

```
[Out] 32/9*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-64/9*(x^4+5*x^2+3)^(1/2)/x-1/
3*(-9*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^3-16/27*(1/(36+x^2*(30+6*13^(1/2))))^(1/
2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2
*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(
30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4
+5*x^2+3)^(1/2)+49/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1
/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2
),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(
6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1285, 1295, 1203, 1113, 1149}

$$\frac{49\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{16\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{x^4+5x^2+3}}-\frac{64\sqrt{x^4+5x^2+3}}{9x}+\frac{32x\left(2x^2+\sqrt{13}+5\right)}{9\sqrt{x^4+5x^2+3}}-\frac{\sqrt{x^4+5x^2+3}\,(2-9x^2)}{3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (32*x*(5 + Sqrt[13] + 2*x^2))/(9*Sqrt[3 + 5*x^2 + x^4]) - (64*Sqrt[3 + 5*x^
2 + x^4])/(9*x) - ((2 - 9*x^2)*Sqrt[3 + 5*x^2 + x^4])/(3*x^3) - (16*Sqrt[(2
*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2 + x^4]) + (49*Sqrt[(6 + (5 - Sqrt[13])*
x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sq
rt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[6*(5 + Sqrt[13])]*S
qrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
```

```
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx = -\frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3} - \frac{1}{3}\int \frac{-64-49x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$
$$= -\frac{64\sqrt{3+5x^2+x^4}}{9x} - \frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3} + \frac{1}{9}\int \frac{147+64x^2}{\sqrt{3+5x^2+x^4}}$$
$$= -\frac{64\sqrt{3+5x^2+x^4}}{9x} - \frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3} + \frac{64}{9}\int \frac{x^2}{\sqrt{3+5x^2+x}}$$

$$= \frac{32x\left(5+\sqrt{13}+2x^2\right)}{9\sqrt{3+5x^2+x^4}} - \frac{64\sqrt{3+5x^2+x^4}}{9x} - \frac{(2-9x^2)\sqrt{3+5x^2+x^4}}{3x^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.47, size = 237, normalized size = 0.78

$$\frac{-2(18+141x^2+191x^4+37x^6)+32i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-13+32\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{18x^3\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4,x]
```

```
[Out] (-2*(18 + 141*x^2 + 191*x^4 + 37*x^6) + (32*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*
Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*El
lipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqr
t[2]*(-13 + 32*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*
Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 1
9/6 + (5*Sqrt[13])/6])/(18*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.06, size = 228, normalized size = 0.75

method	result
default	$-\frac{37\sqrt{x^4+5x^2+3}}{9x} + \frac{98\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
risch	$-\frac{37x^6+191x^4+141x^2+18}{9x^3\sqrt{x^4+5x^2+3}} - \frac{256\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-30}}{}\right.\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+}}$

elliptic	$-\frac{37\sqrt{x^4+5x^2+3}}{9x}+\frac{98\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{}}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -37/9*(x^4+5*x^2+3)^(1/2)/x+98/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2)
)*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*Elliptic
F(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-256/(-30+6*13^(1/2
))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2
)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),
5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2
)+1/6*39^(1/2)))-2/3*(x^4+5*x^2+3)^(1/2)/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)\sqrt{x^4+5x^2+3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(1/2)/x**4,x)
```

```
[Out] Integral((3*x**2 + 2)*sqrt(x**4 + 5*x**2 + 3)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(1/2)/x^4,x, algorithm="giac")
[Out] integrate(sqrt(x^4 + 5*x^2 + 3)*(3*x^2 + 2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(3\,x^2+2)\,\sqrt{x^4+5\,x^2+3}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^4,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(1/2))/x^4, x)
```

3.156 $\int x^5(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$\frac{28379(5+2x^2)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{(3313-1070x^2}{1}$$

```
[Out] -2183/768*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)+3/14*x^4*(x^4+5*x^2+3)^(5/2)+1/1680
*(-1070*x^2+3313)*(x^4+5*x^2+3)^(5/2)-368927/4096*arctanh(1/2*(2*x^2+5)/(x^
4+5*x^2+3)^(1/2))+28379/2048*(2*x^2+5)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.240$, Rules used = {1265, 846, 793, 626, 635, 212}

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{5/2}x^4+\frac{(3313-1070x^2)\left(x^4+5x^2+3\right)^{5/2}}{1680}-\frac{2183}{768}(2x^2+5)\left(x^4+5x^2+3\right)^{3/2}+\frac{28379(2x^2+5)\sqrt{x^4+5x^2+3}}{2048}-\frac{368927\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In] Int[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (28379*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/2048 - (2183*(5 + 2*x^2)*(3 + 5*x
^2 + x^4)^(3/2))/768 + (3*x^4*(3 + 5*x^2 + x^4)^(5/2))/14 + ((3313 - 1070*x
^2)*(3 + 5*x^2 + x^4)^(5/2))/1680 - (368927*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 +
 5*x^2 + x^4])])/4096
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^5\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x^2(2+3x)\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{1}{14}\text{Subst}\left(\int\left(-18-\frac{107x}{2}\right)x\left(3+5x+x^2\right.\right.$$

$$=\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{\left(3313-1070x^2\right)\left(3+5x^2+x^4\right)^{5/2}}{1680}-\frac{2183}{96}\text{S}$$

$$=-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{14}x^4\left(3+5x^2+x^4\right)^{5/2}+\frac{(3313}{}$$

$$=\frac{28379(5+2x^2)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+$$

$$=\frac{28379(5+2x^2)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+$$

$$=\frac{28379(5+2x^2)\sqrt{3+5x^2+x^4}}{2048}-\frac{2183}{768}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}+$$

Mathematica [A]
time = 0.17, size = 79, normalized size = 0.62

$$\frac{\sqrt{3+5x^2+x^4}\,(9546951-1499570x^2+283304x^4+154800x^6+482944x^8+323840x^{10}+46080x^{12})}{215040}+\frac{368927\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)}{4096}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(9546951 - 1499570*x^2 + 283304*x^4 + 154800*x^6 + 4
82944*x^8 + 323840*x^10 + 46080*x^12))/215040 + (368927*Log[-5 - 2*x^2 + 2*
Sqrt[3 + 5*x^2 + x^4]])/4096
```

Maple [A]
time = 0.12, size = 138, normalized size = 1.09

method	result
risch	$\frac{(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951)\sqrt{x^4+5x^2+3}}{215040}-\frac{368927\ln\left(x^2+\frac{5}{2}+\sqrt{x^4-\right.}}{4096}$
trager	$\left(\frac{3}{14}x^{12}+\frac{253}{168}x^{10}+\frac{539}{240}x^8+\frac{645}{896}x^6+\frac{5059}{3840}x^4-\frac{149957}{21504}x^2+\frac{3182317}{71680}\right)\sqrt{x^4+5x^2+3}+\frac{368927\ln\left(-2x^2+2\sqrt{\right.}}{4}$

default	$\frac{3x^{12}\sqrt{x^4+5x^2+3}}{14}+\frac{253x^{10}\sqrt{x^4+5x^2+3}}{168}+\frac{3182317\sqrt{x^4+5x^2+3}}{71680}-\frac{149957x^2\sqrt{x^4+5x^2+3}}{21504}-$
elliptic	$\frac{3x^{12}\sqrt{x^4+5x^2+3}}{14}+\frac{253x^{10}\sqrt{x^4+5x^2+3}}{168}+\frac{3182317\sqrt{x^4+5x^2+3}}{71680}-\frac{149957x^2\sqrt{x^4+5x^2+3}}{21504}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/14*x^12*(x^4+5*x^2+3)^(1/2)+253/168*x^10*(x^4+5*x^2+3)^(1/2)+3182317/7168
0*(x^4+5*x^2+3)^(1/2)-149957/21504*x^2*(x^4+5*x^2+3)^(1/2)+539/240*x^8*(x^4
+5*x^2+3)^(1/2)+645/896*x^6*(x^4+5*x^2+3)^(1/2)-368927/4096*ln(x^2+5/2+(x^4
+5*x^2+3)^(1/2))+5059/3840*x^4*(x^4+5*x^2+3)^(1/2)
```

Maxima [A]
time = 0.27, size = 135, normalized size = 1.06

$$\frac{3}{14}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^4-\frac{107}{168}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^2-\frac{2183}{384}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{3313}{1680}\left(x^4+5x^2+3\right)^{\frac{5}{2}}+\frac{28379}{1024}\sqrt{x^4+5x^2+3}\,x^2-\frac{10915}{768}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{141895}{2048}\sqrt{x^4+5x^2+3}-\frac{368927}{4096}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/14*(x^4 + 5*x^2 + 3)^(5/2)*x^4 - 107/168*(x^4 + 5*x^2 + 3)^(5/2)*x^2 - 21
83/384*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3313/1680*(x^4 + 5*x^2 + 3)^(5/2) + 28
379/1024*sqrt(x^4 + 5*x^2 + 3)*x^2 - 10915/768*(x^4 + 5*x^2 + 3)^(3/2) + 14
1895/2048*sqrt(x^4 + 5*x^2 + 3) - 368927/4096*log(2*x^2 + 2*sqrt(x^4 + 5*x^
2 + 3) + 5)
```

Fricas [A]
time = 0.33, size = 71, normalized size = 0.56

$$\frac{1}{215040}\left(46080x^{12}+323840x^{10}+482944x^8+154800x^6+283304x^4-1499570x^2+9546951\right)\sqrt{x^4+5x^2+3}+\frac{368927}{4096}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/215040*(46080*x^12 + 323840*x^10 + 482944*x^8 + 154800*x^6 + 283304*x^4 -
 1499570*x^2 + 9546951)*sqrt(x^4 + 5*x^2 + 3) + 368927/4096*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^5\cdot\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A]
time = 3.22, size = 207, normalized size = 1.63

$$\frac{1}{71680}\sqrt{x^4+5x^2+3}\,(2(4(2(8(10(12x^2+5)x^2-203)x^2+7635)x^2-76083)x^2+1627215)x^2-20756241)+\frac{17}{3072}\sqrt{x^4+5x^2+3}\,(2(4(2(8(2x^2+1)x^2-33)x^2+321)x^2-6837)x^2+87147)+\frac{19}{3840}\sqrt{x^4+5x^2+3}\,(2(4(6(8x^2+5)x^2-127)x^2+2635)x^2-33429)+\frac{1}{64}\sqrt{x^4+5x^2+3}\,(2(4(6x^2+5)x^2-89)x^2+1095)+\frac{368927}{4096}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/71680*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(10*(12*x^2 + 5)*x^2 - 203)*x^2 +
 7635)*x^2 - 76083)*x^2 + 1627215)*x^2 - 20756241) + 17/3072*sqrt(x^4 + 5*x
^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x^2 - 6837)*x^2 + 8714
7) + 19/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 26
35)*x^2 - 33429) + 1/64*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x
^2 + 1095) + 368927/4096*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^5\,\left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int(x^5*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.157 $\int x^3(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=106

$$-\frac{4797(5+2x^2)\sqrt{3+5x^2+x^4}}{1024}+\frac{123}{128}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2}-\frac{1}{40}(27-10x^2)\left(3+5x^2+x^4\right)^{5/2}+\frac{6236}{}$$

```
[Out] 123/128*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)-1/40*(-10*x^2+27)*(x^4+5*x^2+3)^(5/2)
+62361/2048*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-4797/1024*(2*x^2+5)*
(x^4+5*x^2+3)^(1/2)
```

Rubi [A]

time = 0.05, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 793, 626, 635, 212}

$$-\frac{1}{40}(27-10x^2)\left(x^4+5x^2+3\right)^{5/2}+\frac{123}{128}(2x^2+5)\left(x^4+5x^2+3\right)^{3/2}-\frac{4797(2x^2+5)\sqrt{x^4+5x^2+3}}{1024}+\frac{62361\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)}{2048}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-4797*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/1024 + (123*(5 + 2*x^2)*(3 + 5*x^
2 + x^4)^(3/2))/128 - ((27 - 10*x^2)*(3 + 5*x^2 + x^4)^(5/2))/40 + (62361*A
rcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2048
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int x^3\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{1}{2}\text{Subst}\left(\int x(2+3x)\left(3+5x+x^2\right)^{3/2}\,dx,x,x^2\right)$$

$$=-\frac{1}{40}\left(27-10x^2\right)\left(3+5x^2+x^4\right)^{5/2}+\frac{123}{16}\text{Subst}\left(\int\left(3+5x+x^2\right)^{3/2}\right.$$

$$=\frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{1}{40}\left(27-10x^2\right)\left(3+5x^2+x^4\right)^{5/2}-$$

$$=-\frac{4797(5+2x^2)\sqrt{3+5x^2+x^4}}{1024}+\frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}-$$

$$=-\frac{4797(5+2x^2)\sqrt{3+5x^2+x^4}}{1024}+\frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}-$$

$$=-\frac{4797(5+2x^2)\sqrt{3+5x^2+x^4}}{1024}+\frac{123}{128}\left(5+2x^2\right)\left(3+5x^2+x^4\right)^{3/2}-$$

Mathematica [A]
time = 0.14, size = 74, normalized size = 0.70

$$\frac{\sqrt{3+5x^2+x^4}\left(-77229+12390x^2+5064x^4+14960x^6+9344x^8+1280x^{10}\right)}{5120}-\frac{62361\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)}{2048}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-77229 + 12390*x^2 + 5064*x^4 + 14960*x^6 + 9344*x^
8 + 1280*x^10))/5120 - (62361*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/20
48
```

Maple [A]
time = 0.12, size = 121, normalized size = 1.14

method	result
risch	$\frac{(1280x^{10}+9344x^8+14960x^6+5064x^4+12390x^2-77229)\sqrt{x^4+5x^2+3}}{5120}+\frac{62361\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2048}$
trager	$\left(\frac{1}{4}x^{10}+\frac{73}{40}x^8+\frac{187}{64}x^6+\frac{633}{640}x^4+\frac{1239}{512}x^2-\frac{77229}{5120}\right)\sqrt{x^4+5x^2+3}+\frac{62361\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right.}{2048}$
default	$\frac{x^{10}\sqrt{x^4+5x^2+3}}{4}-\frac{77229\sqrt{x^4+5x^2+3}}{5120}+\frac{1239x^2\sqrt{x^4+5x^2+3}}{512}+\frac{73x^8\sqrt{x^4+5x^2+3}}{40}+\frac{187x^6}{}$
elliptic	$\frac{x^{10}\sqrt{x^4+5x^2+3}}{4}-\frac{77229\sqrt{x^4+5x^2+3}}{5120}+\frac{1239x^2\sqrt{x^4+5x^2+3}}{512}+\frac{73x^8\sqrt{x^4+5x^2+3}}{40}+\frac{187x^6}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*x^10*(x^4+5*x^2+3)^(1/2)-77229/5120*(x^4+5*x^2+3)^(1/2)+1239/512*x^2*(x
^4+5*x^2+3)^(1/2)+73/40*x^8*(x^4+5*x^2+3)^(1/2)+187/64*x^6*(x^4+5*x^2+3)^(1
/2)+62361/2048*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+633/640*x^4*(x^4+5*x^2+3)^(1
/2)
```

Maxima [A]
time = 0.27, size = 118, normalized size = 1.11

$$\frac{1}{4}\left(x^4+5x^2+3\right)^{\frac{5}{2}}x^2+\frac{123}{64}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{27}{40}\left(x^4+5x^2+3\right)^{\frac{5}{2}}-\frac{4797}{512}\sqrt{x^4+5x^2+3}\,x^2+\frac{615}{128}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{23985}{1024}\sqrt{x^4+5x^2+3}+\frac{62361}{2048}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(x^4 + 5*x^2 + 3)^(5/2)*x^2 + 123/64*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 27/4
0*(x^4 + 5*x^2 + 3)^(5/2) - 4797/512*sqrt(x^4 + 5*x^2 + 3)*x^2 + 615/128*(x
^4 + 5*x^2 + 3)^(3/2) - 23985/1024*sqrt(x^4 + 5*x^2 + 3) + 62361/2048*log(2
*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.34, size = 66, normalized size = 0.62

$$\frac{1}{5120}\left(1280x^{10}+9344x^8+14960x^6+5064x^4+12390x^2-77229\right)\sqrt{x^4+5x^2+3}-\frac{62361}{2048}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/5120*(1280*x^10 + 9344*x^8 + 14960*x^6 + 5064*x^4 + 12390*x^2 - 77229)*sq
rt(x^4 + 5*x^2 + 3) - 62361/2048*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^3 \cdot (3x^2 + 2) (x^4 + 5x^2 + 3)^{\frac{3}{2}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(88) = 176.
time = 3.75, size = 179, normalized size = 1.69

$$\frac{1}{1024}\sqrt{x^4+5x^2+3}\,(2(4(2(8(2x^2+1)x^2-33)x^2+321)x^2-6837)x^2+87147)+\frac{17}{3840}\sqrt{x^4+5x^2+3}\,(2(4(6(8x^2+5)x^2-127)x^2+2635)x^2-33429)+\frac{19}{384}\sqrt{x^4+5x^2+3}\,(2(4(6x^2+5)x^2-89)x^2+1095)+\frac{1}{8}\sqrt{x^4+5x^2+3}\,(2(4x^2+5)x^2-51)-\frac{62361}{2048}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1024*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(2*(8*(2*x^2 + 1)*x^2 - 33)*x^2 + 321)*x
^2 - 6837)*x^2 + 87147) + 17/3840*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5
)*x^2 - 127)*x^2 + 2635)*x^2 - 33429) + 19/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*
(6*x^2 + 5)*x^2 - 89)*x^2 + 1095) + 1/8*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5
)*x^2 - 51) - 62361/2048*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int x^3\,\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int(x^3*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.158 $\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=99

$$\frac{429}{256}(5+2x^2)\sqrt{3+5x^2+x^4}-\frac{11}{32}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2}+\frac{3}{10}\left(3+5x^2+x^4\right)^{5/2}-\frac{5577}{512}\tanh^{-1}\left(\frac{5}{2\sqrt{3-}}\right.$$

```
[Out] -11/32*(2*x^2+5)*(x^4+5*x^2+3)^(3/2)+3/10*(x^4+5*x^2+3)^(5/2)-5577/512*arct
anh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+429/256*(2*x^2+5)*(x^4+5*x^2+3)^(1/2
)
```

Rubi [A]
time = 0.04, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {1261, 654, 626, 635, 212}

$$\frac{3}{10}\left(x^4+5x^2+3\right)^{5/2}-\frac{11}{32}(2x^2+5)\left(x^4+5x^2+3\right)^{3/2}+\frac{429}{256}(2x^2+5)\sqrt{x^4+5x^2+3}-\frac{5577}{512}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (429*(5 + 2*x^2)*Sqrt[3 + 5*x^2 + x^4])/256 - (11*(5 + 2*x^2)*(3 + 5*x^2 +
x^4)^(3/2))/32 + (3*(3 + 5*x^2 + x^4)^(5/2))/10 - (5577*ArcTanh[(5 + 2*x^2)
/(2*Sqrt[3 + 5*x^2 + x^4])])/512
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 626

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(b + 2*c*x)
*((a + b*x + c*x^2)^p/(2*c*(2*p + 1))), x] - Dist[p*((b^2 - 4*a*c)/(2*c*(2*
p + 1))), Int[(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && IntegerQ[4*p]
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx = \frac{1}{2}\mathrm{Subst}\left(\int (2+3x)\left(3+5x+x^2\right)^{3/2}\,dx, x, x^2\right)$$
$$= \frac{3}{10}\left(3+5x^2+x^4\right)^{5/2} - \frac{11}{4}\mathrm{Subst}\left(\int \left(3+5x+x^2\right)^{3/2}\,dx, x, x^2\right)$$
$$= -\frac{11}{32}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{10}\left(3+5x^2+x^4\right)^{5/2} + \frac{429}{64}\mathrm{Subst}\Big($$
$$= \frac{429}{256}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{11}{32}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{10}\Big($$
$$= \frac{429}{256}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{11}{32}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{10}\Big($$
$$= \frac{429}{256}(5+2x^2)\sqrt{3+5x^2+x^4} - \frac{11}{32}(5+2x^2)\left(3+5x^2+x^4\right)^{3/2} + \frac{3}{10}\Big($$

Mathematica [**A**]
time = 0.13, size = 69, normalized size = 0.70

$$\frac{\sqrt{3+5x^2+x^4}\,(7581+2170x^2+5304x^4+2960x^6+384x^8)}{1280} + \frac{5577}{512}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(7581 + 2170*x^2 + 5304*x^4 + 2960*x^6 + 384*x^8))/1
280 + (5577*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/512
```

Maple [**A**]
time = 0.11, size = 104, normalized size = 1.05

method	result
risch	$\frac{(384x^8+2960x^6+5304x^4+2170x^2+7581)\sqrt{x^4+5x^2+3}}{1280}-\frac{5577\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{512}$
trager	$\left(\frac{3}{10}x^8+\frac{37}{16}x^6+\frac{663}{160}x^4+\frac{217}{128}x^2+\frac{7581}{1280}\right)\sqrt{x^4+5x^2+3}+\frac{5577\ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{512}$
default	$\frac{3x^8\sqrt{x^4+5x^2+3}}{10}+\frac{37x^6\sqrt{x^4+5x^2+3}}{16}+\frac{663x^4\sqrt{x^4+5x^2+3}}{160}+\frac{217x^2\sqrt{x^4+5x^2+3}}{128}+\frac{7581\sqrt{}}{}$
elliptic	$\frac{3x^8\sqrt{x^4+5x^2+3}}{10}+\frac{37x^6\sqrt{x^4+5x^2+3}}{16}+\frac{663x^4\sqrt{x^4+5x^2+3}}{160}+\frac{217x^2\sqrt{x^4+5x^2+3}}{128}+\frac{7581\sqrt{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/10*x^8*(x^4+5*x^2+3)^(1/2)+37/16*x^6*(x^4+5*x^2+3)^(1/2)+663/160*x^4*(x^4
+5*x^2+3)^(1/2)+217/128*x^2*(x^4+5*x^2+3)^(1/2)+7581/1280*(x^4+5*x^2+3)^(1/
2)-5577/512*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A]
time = 0.28, size = 101, normalized size = 1.02

$$-\frac{11}{16}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2+\frac{3}{10}\left(x^4+5x^2+3\right)^{\frac{5}{2}}+\frac{429}{128}\sqrt{x^4+5x^2+3}\,x^2-\frac{55}{32}\left(x^4+5x^2+3\right)^{\frac{3}{2}}+\frac{2145}{256}\sqrt{x^4+5x^2+3}-\frac{5577}{512}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -11/16*(x^4 + 5*x^2 + 3)^(3/2)*x^2 + 3/10*(x^4 + 5*x^2 + 3)^(5/2) + 429/128
*sqrt(x^4 + 5*x^2 + 3)*x^2 - 55/32*(x^4 + 5*x^2 + 3)^(3/2) + 2145/256*sqrt(
x^4 + 5*x^2 + 3) - 5577/512*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.40, size = 61, normalized size = 0.62

$$\frac{1}{1280}\left(384x^8+2960x^6+5304x^4+2170x^2+7581\right)\sqrt{x^4+5x^2+3}+\frac{5577}{512}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/1280*(384*x^8 + 2960*x^6 + 5304*x^4 + 2170*x^2 + 7581)*sqrt(x^4 + 5*x^2 +
 3) + 5577/512*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A]
time = 3.83, size = 151, normalized size = 1.53

$$\frac{1}{1280}\sqrt{x^4+5x^2+3}\,(2(4(6(8x^2+5)x^2-127)x^2+2635)x^2-33429)+\frac{17}{384}\sqrt{x^4+5x^2+3}\,(2(4(6x^2+5)x^2-89)x^2+1095)+\frac{19}{48}\sqrt{x^4+5x^2+3}\,(2(4x^2+5)x^2-51)+\frac{3}{4}\sqrt{x^4+5x^2+3}\,(2x^2+5)+\frac{5577}{512}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/1280*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*(8*x^2 + 5)*x^2 - 127)*x^2 + 2635)*x^
2 - 33429) + 17/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(6*x^2 + 5)*x^2 - 89)*x^2 +
 1095) + 19/48*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 5)*x^2 - 51) + 3/4*sqrt(x^
4 + 5*x^2 + 3)*(2*x^2 + 5) + 5577/512*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) +
 5)
```

Mupad [B]
time = 0.53, size = 127, normalized size = 1.28

$$\frac{\left(x^2+\frac{5}{2}\right)\,\left(x^4+5x^2+3\right)^{3/2}}{4}-\frac{15x^2\left(x^4+5x^2+3\right)^{3/2}}{16}-\frac{5577\ln\left(\sqrt{x^4+5x^2+3}+x^2+\frac{5}{2}\right)}{512}+\frac{585\left(2x^2+5\right)\,\sqrt{x^4+5x^2+3}}{256}-\frac{39\left(\frac{x^2}{2}+\frac{5}{4}\right)\,\sqrt{x^4+5x^2+3}}{16}-\frac{75\left(x^4+5x^2+3\right)^{3/2}}{32}+\frac{3\left(x^4+5x^2+3\right)^{5/2}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] ((x^2 + 5/2)*(5*x^2 + x^4 + 3)^(3/2))/4 - (15*x^2*(5*x^2 + x^4 + 3)^(3/2))/
16 - (5577*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/512 + (585*(2*x^2 + 5)
*(5*x^2 + x^4 + 3)^(1/2))/256 - (39*(x^2/2 + 5/4)*(5*x^2 + x^4 + 3)^(1/2))/
16 - (75*(5*x^2 + x^4 + 3)^(3/2))/32 + (3*(5*x^2 + x^4 + 3)^(5/2))/10
```

3.159 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x} dx$

Optimal. Leaf size=119

$$\frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4}+\frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2}+\frac{2401}{256}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-3$$

```
[Out] 1/48*(18*x^2+61)*(x^4+5*x^2+3)^(3/2)+2401/256*arctanh(1/2*(2*x^2+5)/(x^4+5*
x^2+3)^(1/2))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
1/128*(-74*x^2+199)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 828, 857, 635, 212, 738}

$$\frac{1}{48}(18x^2+61)(x^4+5x^2+3)^{3/2}+\frac{1}{128}(199-74x^2)\sqrt{x^4+5x^2+3}+\frac{2401}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-3\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
```

```
[Out] ((199 - 74*x^2)*Sqrt[3 + 5*x^2 + x^4])/128 + ((61 + 18*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/48 + (2401*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/256 -
3*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} - \frac{1}{16}\text{Subst}\left(\int \frac{\left(-48+\frac{37x}{2}\right)\sqrt{3+5x}}{x}\right.$$
$$= \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} +$$
$$= \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} +$$
$$= \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} -$$
$$= \frac{1}{128}(199-74x^2)\sqrt{3+5x^2+x^4} + \frac{1}{48}(61+18x^2)(3+5x^2+x^4)^{3/2} +$$

Mathematica [A]
time = 0.19, size = 99, normalized size = 0.83

$$6\sqrt{3}\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)+\frac{1}{768}\left(2\sqrt{3+5x^2+x^4}\,(2061+2650x^2+1208x^4+144x^6)-7203\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x,x]
[Out] 6*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] + (2*Sqrt[3 + 5*x^
2 + x^4]*(2061 + 2650*x^2 + 1208*x^4 + 144*x^6) - 7203*Log[-5 - 2*x^2 + 2*S
qrt[3 + 5*x^2 + x^4]])/768
```

Maple [A]
time = 0.25, size = 117, normalized size = 0.98

method	result
trager	$\left(\frac{3}{8}x^6+\frac{151}{48}x^4+\frac{1325}{192}x^2+\frac{687}{128}\right)\sqrt{x^4+5x^2+3}+3\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\text{RootOf}\left(\_Z^2-3\right)x^2+6}{}\right.$
default	$\frac{3x^6\sqrt{x^4+5x^2+3}}{8}+\frac{151x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1325x^2\sqrt{x^4+5x^2+3}}{192}+\frac{687\sqrt{x^4+5x^2+3}}{128}+\frac{2401\ln}{}$
elliptic	$\frac{3x^6\sqrt{x^4+5x^2+3}}{8}+\frac{151x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1325x^2\sqrt{x^4+5x^2+3}}{192}+\frac{687\sqrt{x^4+5x^2+3}}{128}+\frac{2401\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 3/8*x^6*(x^4+5*x^2+3)^(1/2)+151/48*x^4*(x^4+5*x^2+3)^(1/2)+1325/192*x^2*(x^
4+5*x^2+3)^(1/2)+687/128*(x^4+5*x^2+3)^(1/2)+2401/256*ln(x^2+5/2+(x^4+5*x^2
+3)^(1/2))-3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.49, size = 120, normalized size = 1.01

$$\frac{3}{8}\left(x^4+5x^2+3\right)^{\frac{3}{2}}x^2-\frac{37}{64}\sqrt{x^4+5x^2+3}\,x^2+\frac{61}{48}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-3\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{199}{128}\sqrt{x^4+5x^2+3}+\frac{2401}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="maxima")
```

```
[Out] 3/8*(x^4 + 5*x^2 + 3)^(3/2)*x^2 - 37/64*sqrt(x^4 + 5*x^2 + 3)*x^2 + 61/48*(
x^4 + 5*x^2 + 3)^(3/2) - 3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2
+ 6/x^2 + 5) + 199/128*sqrt(x^4 + 5*x^2 + 3) + 2401/256*log(2*x^2 + 2*sqrt(
x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.35, size = 106, normalized size = 0.89

$$\frac{1}{384}\left(144x^6+1208x^4+2650x^2+2061\right)\sqrt{x^4+5x^2+3}+3\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-\frac{2401}{256}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="fricas")
```

```
[Out] 1/384*(144*x^6 + 1208*x^4 + 2650*x^2 + 2061)*sqrt(x^4 + 5*x^2 + 3) + 3*sqrt
(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3
) - 6) + 30)/x^2) - 2401/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x, x)
```

Giac [A]
time = 4.29, size = 113, normalized size = 0.95

$$\frac{1}{384}\sqrt{x^4+5x^2+3}\left(2\left(4\left(18x^2+151\right)x^2+1325\right)x^2+2061\right)+3\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)-\frac{2401}{256}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x,x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 + 151)*x^2 + 1325)*x^2 + 2061) +
3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt
(x^4 + 5*x^2 + 3))) - 2401/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x, x)
```

3.160 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx$

Optimal. Leaf size=122

$$\frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4}-\frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2}+\frac{609}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-12\sqrt{3}\,\tanh$$

```
[Out] -1/2*(-x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2+609/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x
^2+3)^(1/2))-12*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+
3/16*(18*x^2+109)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$,
Rules used = {1265, 826, 828, 857, 635, 212, 738}

$$-\frac{(2-x^2)(x^4+5x^2+3)^{3/2}}{2x^2}+\frac{3}{16}(18x^2+109)\sqrt{x^4+5x^2+3}+\frac{609}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-12\sqrt{3}\,\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] (3*(109 + 18*x^2)*Sqrt[3 + 5*x^2 + x^4])/16 - ((2 - x^2)*(3 + 5*x^2 + x^4)^
(3/2))/(2*x^2) + (609*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32 -
12*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x^2}\,dx, x, x^2\right)$$
$$= -\frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} - \frac{1}{4}\text{Subst}\left(\int \frac{(-48-27x)\sqrt{3+5x+x^2}}{x}\,d\right.$$
$$= \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{1}{16}\text{Subs}$$
$$= \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{32}\text{Sub}$$
$$= \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{16}\text{Sub}$$
$$= \frac{3}{16}(109+18x^2)\sqrt{3+5x^2+x^4} - \frac{(2-x^2)(3+5x^2+x^4)^{3/2}}{2x^2} + \frac{609}{32}\tan$$

Mathematica [A]
time = 0.23, size = 102, normalized size = 0.84

$$24\sqrt{3}\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) + \frac{1}{32}\left(\frac{2\sqrt{3+5x^2+x^4}\,(-48+271x^2+78x^4+8x^6)}{x^2} - 609\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^3,x]
```

```
[Out] 24*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] + ((2*Sqrt[3 + 5*
x^2 + x^4]*(-48 + 271*x^2 + 78*x^4 + 8*x^6))/x^2 - 609*Log[-5 - 2*x^2 + 2*S
qrt[3 + 5*x^2 + x^4]])/32
```

Maple [A]
time = 0.29, size = 117, normalized size = 0.96

method	result
trager	$\frac{(8x^6+78x^4+271x^2-48)\sqrt{x^4+5x^2+3}}{16x^2} - \frac{609\ln\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)}{32} - 12\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\right.$
default	$\frac{39x^2\sqrt{x^4+5x^2+3}}{8} + \frac{271\sqrt{x^4+5x^2+3}}{16} + \frac{609\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32} - \frac{3\sqrt{x^4+5x^2+3}}{x^2} - 12$
risch	$\frac{39x^2\sqrt{x^4+5x^2+3}}{8} + \frac{271\sqrt{x^4+5x^2+3}}{16} + \frac{609\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32} - \frac{3\sqrt{x^4+5x^2+3}}{x^2} - 12$

elliptic	$\frac{39x^2\sqrt{x^4+5x^2+3}}{8}+\frac{271\sqrt{x^4+5x^2+3}}{16}+\frac{609\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}-\frac{3\sqrt{x^4+5x^2+3}}{x^2}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 39/8*x^2*(x^4+5*x^2+3)^(1/2)+271/16*(x^4+5*x^2+3)^(1/2)+609/32*ln(x^2+5/2+(
x^4+5*x^2+3)^(1/2))-3*(x^4+5*x^2+3)^(1/2)/x^2-12*arctanh(1/6*(5*x^2+6)*3^(1
/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/2*x^4*(x^4+5*x^2+3)^(1/2)
```

Maxima [A]
time = 0.51, size = 120, normalized size = 0.98

$$\frac{27}{8}\sqrt{x^4+5x^2+3}\,x^2+\frac{1}{2}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-12\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{327}{16}\sqrt{x^4+5x^2+3}-\frac{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^2}+\frac{609}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="maxima")
```

```
[Out] 27/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/2*(x^4 + 5*x^2 + 3)^(3/2) - 12*sqrt(3)*l
og(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 327/16*sqrt(x^4 + 5*x
^2 + 3) - (x^4 + 5*x^2 + 3)^(3/2)/x^2 + 609/32*log(2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) + 5)
```

Fricas [A]
time = 0.40, size = 122, normalized size = 1.00

$$\frac{1536\sqrt{3}\,x^2\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-2436x^2\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+1541x^2+8\left(8x^6+78x^4+271x^2-48\right)\sqrt{x^4+5x^2+3}}{128x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="fricas")
```

```
[Out] 1/128*(1536*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 +
5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 2436*x^2*log(-2*x^2 + 2*sqrt(x^4 +
5*x^2 + 3) - 5) + 1541*x^2 + 8*(8*x^6 + 78*x^4 + 271*x^2 - 48)*sqrt(x^4 + 5
*x^2 + 3))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**3,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**3, x)
```

Giac [A]
time = 5.22, size = 153, normalized size = 1.25

$$\frac{1}{16}\sqrt{x^4+5x^2+3}\left(2\left(4x^2+39\right)x^2+271\right)+12\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{3\left(5x^2-5\sqrt{x^4+5x^2+3}+6\right)}{\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3}-\frac{609}{32}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 + 39)*x^2 + 271) + 12*sqrt(3)*log((x^2
 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))
) + 3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^
2 - 3) - 609/32*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3x^2+2\right)\,\left(x^4+5x^2+3\right)^{3/2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^3, x)
```

3.161 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx$

Optimal. Leaf size=127

$$-\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2}-\frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4}+\frac{453}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{127}{8}\sqrt{3}\ \mathrm{ta}$$

```
[Out] -1/4*(-3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4+453/16*arctanh(1/2*(2*x^2+5)/(x^4+5
*x^2+3)^(1/2))-127/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(
1/2)-3/8*(-19*x^2+28)*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1265, 826, 857, 635, 212, 738}

$$-\frac{(2-3x^2)(x^4+5x^2+3)^{3/2}}{4x^4}-\frac{3(28-19x^2)\sqrt{x^4+5x^2+3}}{8x^2}+\frac{453}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{127}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] (-3*(28 - 19*x^2)*Sqrt[3 + 5*x^2 + x^4])/(8*x^2) - ((2 - 3*x^2)*(3 + 5*x^2
+ x^4)^(3/2))/(4*x^4) + (453*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])]
)/16 - (127*Sqrt[3]*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])
/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^5}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x^3}\,dx, x, x^2\right)$$

$$= -\frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} - \frac{3}{16}\text{Subst}\left(\int \frac{(-56-38x)\sqrt{3+5x+x}}{x^2}\right.$$

$$= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{3}{32}\text{Sub}$$

$$= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{453}{16}\text{Su}$$

$$= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{453}{8}\text{Su}$$

$$= -\frac{3(28-19x^2)\sqrt{3+5x^2+x^4}}{8x^2} - \frac{(2-3x^2)(3+5x^2+x^4)^{3/2}}{4x^4} + \frac{453}{16}\,\text{ta}$$

Mathematica [A]
time = 0.24, size = 101, normalized size = 0.80

$$\frac{1}{16}\left(\frac{2\sqrt{3+5x^2+x^4}\,(-12-86x^2+83x^4+6x^6)}{x^4} + 508\sqrt{3}\,\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - 453\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^5,x]
```

```
[Out] ((2*Sqrt[3 + 5*x^2 + x^4]*(-12 - 86*x^2 + 83*x^4 + 6*x^6))/x^4 + 508*Sqrt[3
]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] - 453*Log[-5 - 2*x^2 + 2*S
qrt[3 + 5*x^2 + x^4]])/16
```

Maple [A]
time = 0.27, size = 117, normalized size = 0.92

method	result
trager	$\frac{(6x^6+83x^4-86x^2-12)\sqrt{x^4+5x^2+3}}{8x^4} + \frac{453\ln\left(-2x^2-2\sqrt{x^4+5x^2+3}-5\right)}{16} - \frac{127\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\,\text{Root}}{}\right.}{}$
default	$\frac{83\sqrt{x^4+5x^2+3}}{8} + \frac{453\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16} - \frac{3\sqrt{x^4+5x^2+3}}{2x^4} - \frac{43\sqrt{x^4+5x^2+3}}{4x^2} - \frac{127}{}$

risch	$-\frac{43x^6+221x^4+159x^2+18}{4x^4\sqrt{x^4+5x^2+3}}+\frac{3x^2\sqrt{x^4+5x^2+3}}{4}+\frac{83\sqrt{x^4+5x^2+3}}{8}+\frac{453\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}-$
elliptic	$\frac{83\sqrt{x^4+5x^2+3}}{8}+\frac{453\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}-\frac{3\sqrt{x^4+5x^2+3}}{2x^4}-\frac{43\sqrt{x^4+5x^2+3}}{4x^2}-\frac{127\,\mathrm{ar}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x,method=_RETURNVERBOSE)
```

```
[Out] 83/8*(x^4+5*x^2+3)^(1/2)+453/16*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-3/2*(x^4+5*
x^2+3)^(1/2)/x^4-43/4*(x^4+5*x^2+3)^(1/2)/x^2-127/8*arctanh(1/6*(5*x^2+6)*3
^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+3/4*x^2*(x^4+5*x^2+3)^(1/2)
```

Maxima [A]
time = 0.49, size = 137, normalized size = 1.08

$$\frac{7}{2}\sqrt{x^4+5x^2+3}\,x^2+\frac{1}{6}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{127}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{197}{8}\sqrt{x^4+5x^2+3}-\frac{23\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{12x^2}-\frac{\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{6x^4}+\frac{453}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="maxima")
```

```
[Out] 7/2*sqrt(x^4 + 5*x^2 + 3)*x^2 + 1/6*(x^4 + 5*x^2 + 3)^(3/2) - 127/8*sqrt(3)
*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 197/8*sqrt(x^4 + 5*
x^2 + 3) - 23/12*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 1/6*(x^4 + 5*x^2 + 3)^(5/2)/
x^4 + 453/16*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.37, size = 122, normalized size = 0.96

$$\frac{1016\sqrt{3}\,x^4\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-1812x^4\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)+67x^4+8\left(6x^6+83x^4-86x^2-12\right)\sqrt{x^4+5x^2+3}}{64x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="fricas")
```

```
[Out] 1/64*(1016*sqrt(3)*x^4*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5
*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 1812*x^4*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5) + 67*x^4 + 8*(6*x^6 + 83*x^4 - 86*x^2 - 12)*sqrt(x^4 + 5*x^2
 + 3))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**5,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**5, x)
```

Giac [A]
time = 5.66, size = 190, normalized size = 1.50

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\,(6x^2+83)+\frac{127}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{227\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+348\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-459x^2+459\sqrt{x^4+5x^2+3}-684}{4\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2}-\frac{453}{16}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5,x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 + 83) + 127/8*sqrt(3)*log((x^2 + sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sqrt(x^4 + 5*x^2 + 3))) + 1/4*(227
*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 348*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 45
9*x^2 + 459*sqrt(x^4 + 5*x^2 + 3) - 684)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 -
 3)^2 - 453/16*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{3/2}}{x^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^5, x)
```

3.162 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx$

Optimal. Leaf size=127

$$-\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2}-\frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6}+\frac{49}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{527\tanh^{-1}\Big(}{}$$

```
[Out] -1/6*(7*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6+49/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^
2+3)^(1/2))-527/72*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/
2)-1/12*(-32*x^2+67)*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A]
time = 0.07, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1265, 824, 826, 857, 635, 212, 738}

$$-\frac{(67-32x^2)\sqrt{x^4+5x^2+3}}{12x^2}+\frac{49}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{527\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{24\sqrt{3}}-\frac{(7x^2+2)(x^4+5x^2+3)^{3/2}}{6x^6}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] -1/12*((67 - 32*x^2)*Sqrt[3 + 5*x^2 + x^4])/x^2 - ((2 + 7*x^2)*(3 + 5*x^2 +
 x^4)^(3/2))/(6*x^6) + (49*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/
4 - (527*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/(24*Sqrt[3
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 824

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*((a + b*x + c*x^2
)^p/(e^2*(m + 1)*(m + 2)*(c*d^2 - b*d*e + a*e^2)))*((d*g - e*f*(m + 2))*(c*
d^2 - b*d*e + a*e^2) - d*p*(2*c*d - b*e)*(e*f - d*g) - e*(g*(m + 1)*(c*d^2
- b*d*e + a*e^2) + p*(2*c*d - b*e)*(e*f - d*g))*x), x] - Dist[p/(e^2*(m + 1
)*(m + 2)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 2)*(a + b*x + c*x^2)
^(p - 1)*Simp[2*a*c*e*(e*f - d*g)*(m + 2) + b^2*e*(d*g*(p + 1) - e*f*(m + p
 + 2)) + b*(a*e^2*g*(m + 1) - c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2))) - c*
(2*c*d*(d*g*(2*p + 1) - e*f*(m + 2*p + 2)) - e*(2*a*e*g*(m + 1) - b*(d*g*(m
 - 2*p) + e*f*(m + 2*p + 2))))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}
, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && GtQ[p, 0] &
& LtQ[m, -2] && LtQ[m + 2*p, 0] &&  !ILtQ[m + 2*p + 3, 0]
```

Rule 826

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(e*f*(m + 2*p + 2) -
 d*g*(2*p + 1) + e*g*(m + 1)*x)*((a + b*x + c*x^2)^p/(e^2*(m + 1)*(m + 2*p
+ 2))), x] + Dist[p/(e^2*(m + 1)*(m + 2*p + 2)), Int[(d + e*x)^(m + 1)*(a +
 b*x + c*x^2)^(p - 1)*Simp[g*(b*d + 2*a*e + 2*a*e*m + 2*b*d*p) - f*b*e*(m +
 2*p + 2) + (g*(2*c*d + b*e + b*e*m + 4*c*d*p) - 2*c*e*f*(m + 2*p + 2))*x,
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] && RationalQ[p] && p > 0 && (LtQ[m, -1] || Eq
Q[p, 1] || (IntegerQ[p] &&  !RationalQ[m])) && NeQ[m, -1] &&  !ILtQ[m + 2*p
 + 1, 0] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^7}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(2+3x)(3+5x+x^2)^{3/2}}{x^4}\,dx, x, x^2\right)$$

$$= -\frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} - \frac{1}{24}\text{Subst}\left(\int \frac{(-134-64x)\sqrt{3+5x+x}}{x^2}\right.$$

$$= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{1}{48}\text{Subst}$$

$$= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{4}\text{Subst}$$

$$= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{2}\text{Subst}$$

$$= -\frac{(67-32x^2)\sqrt{3+5x^2+x^4}}{12x^2} - \frac{(2+7x^2)(3+5x^2+x^4)^{3/2}}{6x^6} + \frac{49}{4}\tanh^{-}$$

Mathematica [A]
time = 0.33, size = 101, normalized size = 0.80

$$\frac{1}{36}\left(\frac{3\sqrt{3+5x^2+x^4}\,(-12-62x^2-141x^4+18x^6)}{x^6} + 527\sqrt{3}\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right) - 441\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^7,x]
```

```
[Out] ((3*Sqrt[3 + 5*x^2 + x^4]*(-12 - 62*x^2 - 141*x^4 + 18*x^6))/x^6 + 527*Sqrt
[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]] - 441*Log[-5 - 2*x^2 + 2
*Sqrt[3 + 5*x^2 + x^4]])/36
```

Maple [A]
time = 0.28, size = 117, normalized size = 0.92

method	result
risch	$-\frac{141x^8+767x^6+745x^4+246x^2+36}{12x^6\sqrt{x^4+5x^2+3}} + \frac{3\sqrt{x^4+5x^2+3}}{2} + \frac{49\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4} - \frac{527\,\text{arctanh}\left(\frac{(5x^2}{6\sqrt{x^4-}}\right.}{72}$
trager	$\frac{(18x^6-141x^4-62x^2-12)\sqrt{x^4+5x^2+3}}{12x^6} - \frac{49\ln\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)}{4} + \frac{527\,\text{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\text{RootO}}{}\right.}{}$

default	$\frac{3\sqrt{x^4+5x^2+3}}{2}+\frac{49\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{31\sqrt{x^4+5x^2+3}}{6x^4}-\frac{47\sqrt{x^4+5x^2+3}}{4x^2}-\frac{527\,\mathrm{a}}{}$
elliptic	$\frac{3\sqrt{x^4+5x^2+3}}{2}+\frac{49\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}-\frac{31\sqrt{x^4+5x^2+3}}{6x^4}-\frac{47\sqrt{x^4+5x^2+3}}{4x^2}-\frac{527\,\mathrm{a}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(x^4+5*x^2+3)^(1/2)+49/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-31/6*(x^4+5*x^
2+3)^(1/2)/x^4-47/4*(x^4+5*x^2+3)^(1/2)/x^2-527/72*arctanh(1/6*(5*x^2+6)*3^
(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-(x^4+5*x^2+3)^(1/2)/x^6
```

Maxima [A]
time = 0.49, size = 154, normalized size = 1.21

$$\frac{67}{36}\sqrt{x^4+5x^2+3}\,x^2+\frac{11}{54}\left(x^4+5x^2+3\right)^{\frac{3}{2}}-\frac{527}{72}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{431}{36}\sqrt{x^4+5x^2+3}-\frac{79\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{108x^2}-\frac{11\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{54x^4}-\frac{\left(x^4+5x^2+3\right)^{\frac{5}{2}}}{9x^6}+\frac{49}{4}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="maxima")
```

```
[Out] 67/36*sqrt(x^4 + 5*x^2 + 3)*x^2 + 11/54*(x^4 + 5*x^2 + 3)^(3/2) - 527/72*sq
rt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 431/36*sqrt(x^
4 + 5*x^2 + 3) - 79/108*(x^4 + 5*x^2 + 3)^(3/2)/x^2 - 11/54*(x^4 + 5*x^2 +
3)^(5/2)/x^4 - 1/9*(x^4 + 5*x^2 + 3)^(5/2)/x^6 + 49/4*log(2*x^2 + 2*sqrt(x^
4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.36, size = 122, normalized size = 0.96

$$\frac{527\sqrt{3}\,x^6\log\left(\frac{25x^2-2\sqrt{3}\left(5x^2+6\right)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-882x^6\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)-711x^6+6\left(18x^6-141x^4-62x^2-12\right)\sqrt{x^4+5x^2+3}}{72x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="fricas")
```

```
[Out] 1/72*(527*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 882*x^6*log(-2*x^2 + 2*sqrt(x^4 + 5*x
^2 + 3) - 5) - 711*x^6 + 6*(18*x^6 - 141*x^4 - 62*x^2 - 12)*sqrt(x^4 + 5*x^
2 + 3))/x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**7,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**7, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 227 vs. 2(103) = 206.
time = 3.82, size = 227, normalized size = 1.79

$$\frac{527}{72}\,\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)+\frac{3}{2}\,\sqrt{x^4+5\,x^2+3}+\frac{829\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5+1824\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^4-2200\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^3-5292\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2+2799\,x^2-2799\,\sqrt{x^4+5\,x^2+3}+5724}{12\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)^3}-\frac{49}{4}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^7,x, algorithm="giac")
```

```
[Out] 527/72*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))) + 3/2*sqrt(x^4 + 5*x^2 + 3) + 1/12*(829*(x^2 - sqr
t(x^4 + 5*x^2 + 3))^5 + 1824*(x^2 - sqrt(x^4 + 5*x^2 + 3))^4 - 2200*(x^2 -
sqrt(x^4 + 5*x^2 + 3))^3 - 5292*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 + 2799*x^2
- 2799*sqrt(x^4 + 5*x^2 + 3) + 5724)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)^
3 - 49/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}}{x^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^7, x)
```

3.163 $\int x^4(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=356

$$\frac{176723x\left(5+\sqrt{13}+2x^2\right)}{4290\sqrt{3+5x^2+x^4}}-\frac{4210}{429}x\sqrt{3+5x^2+x^4}+\frac{1251}{715}x^3\sqrt{3+5x^2+x^4}-\frac{1}{429}x^5\left(283+272x^2\right)\sqrt{3+5$$

```
[Out] 1/143*x^5*(33*x^2+71)*(x^4+5*x^2+3)^(3/2)+176723/4290*x*(5+2*x^2+13^(1/2))/
(x^4+5*x^2+3)^(1/2)-4210/429*x*(x^4+5*x^2+3)^(1/2)+1251/715*x^3*(x^4+5*x^2+
3)^(1/2)-1/429*x^5*(272*x^2+283)*(x^4+5*x^2+3)^(1/2)+2105/429*(1/(36+x^2*(3
0+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(
1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+
x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(
5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-176723/25740*(1/(36+x^2*(30+6*13^(1
/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/
2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13
^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^
(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 356, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1287, 1293, 1203, 1113, 1149}

$$\frac{2105\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{143\sqrt{x^4+5x^2+3}}-\frac{176723\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{4290\sqrt{x^4+5x^2+3}}-\frac{4210}{429}\sqrt{x^4+5x^2+3}\,x+\frac{176723\left(2x^2+\sqrt{13}+5\right)x}{4290\sqrt{x^4+5x^2+3}}+\frac{1}{143}\left(33x^2+71\right)\left(x^4+5x^2+3\right)^{3/2}x^5-\frac{1}{429}\left(272x^2+283\right)\sqrt{x^4+5x^2+3}\,x^5+\frac{1251}{715}\sqrt{x^4+5x^2+3}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (176723*x*(5 + Sqrt[13] + 2*x^2))/(4290*Sqrt[3 + 5*x^2 + x^4]) - (4210*x*Sq
rt[3 + 5*x^2 + x^4])/429 + (1251*x^3*Sqrt[3 + 5*x^2 + x^4])/715 - (x^5*(283
 + 272*x^2)*Sqrt[3 + 5*x^2 + x^4])/429 + (x^5*(71 + 33*x^2)*(3 + 5*x^2 + x^
4)^(3/2))/143 - (176723*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2
)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[
(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(4290*Sqrt[3 + 5*x^2 + x^4]) +
 (2105*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + S
qrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13]
)/6]*x], (-13 + 5*Sqrt[13])/6])/(143*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1287

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Dist[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^4 \left(2+3x^2\right) \left(3+5x^2+x^4\right)^{3/2} \, dx = \frac{1}{143} x^5 \left(71+33x^2\right) \left(3+5x^2+x^4\right)^{3/2} + \frac{3}{143} \int x^4 \left(-69-272x^2\right) \sqrt{3}$$

$$= -\frac{1}{429} x^5 \left(283+272x^2\right) \sqrt{3+5x^2+x^4} + \frac{1}{143} x^5 \left(71+33x^2\right) \left(3+5x\right.$$

$$= \frac{1251}{715} x^3 \sqrt{3+5x^2+x^4} - \frac{1}{429} x^5 \left(283+272x^2\right) \sqrt{3+5x^2+x^4} + \frac{1}{14}$$

$$= -\frac{4210}{429} x \sqrt{3+5x^2+x^4} + \frac{1251}{715} x^3 \sqrt{3+5x^2+x^4} - \frac{1}{429} x^5 \left(283+2\right.$$

$$= -\frac{4210}{429} x \sqrt{3+5x^2+x^4} + \frac{1251}{715} x^3 \sqrt{3+5x^2+x^4} - \frac{1}{429} x^5 \left(283+2\right.$$

$$= \frac{176723 x \left(5+\sqrt{13}+2x^2\right)}{4290 \sqrt{3+5x^2+x^4}} - \frac{4210}{429} x \sqrt{3+5x^2+x^4} + \frac{1251}{715} x^3 \sqrt{3-}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.02, size = 249, normalized size = 0.70

$$\frac{4x\left(-63150-93991x^2+3055x^4+29003x^6+39650x^8+24635x^{10}+6015x^{12}+495x^{14}\right)+176723 i \sqrt{2} \left(-5+\sqrt{13}\right) \sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2x^2}\, E\left(i \sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\, x\right) \middle| \frac{19}{6}+\frac{5\sqrt{13}}{6}\right) - i \sqrt{2} \left(-757315+176723\sqrt{13}\right) \sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}} \sqrt{5+\sqrt{13}+2x^2}\, F\left(i \sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\, x\right) \middle| \frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{8580 \sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(-63150 - 93991*x^2 + 3055*x^4 + 29003*x^6 + 39650*x^8 + 24635*x^10 +
6015*x^12 + 495*x^14) + (176723*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[
13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSin
h[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-757315 +
176723*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqr
t[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqr
t[13])/6])/(8580*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.07, size = 294, normalized size = 0.83

method	result

risch	$\frac{x(495x^{10}+3540x^8+5450x^6+1780x^4+3753x^2-21050)\sqrt{x^4+5x^2+3}}{2145} - \frac{2120676\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-}}{}$
default	$\frac{3x^{11}\sqrt{x^4+5x^2+3}}{13}+\frac{236x^9\sqrt{x^4+5x^2+3}}{143}+\frac{1090x^7\sqrt{x^4+5x^2+3}}{429}+\frac{356x^5\sqrt{x^4+5x^2+3}}{429}+\frac{1251}{}$
elliptic	$\frac{3x^{11}\sqrt{x^4+5x^2+3}}{13}+\frac{236x^9\sqrt{x^4+5x^2+3}}{143}+\frac{1090x^7\sqrt{x^4+5x^2+3}}{429}+\frac{356x^5\sqrt{x^4+5x^2+3}}{429}+\frac{1251}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/13*x^11*(x^4+5*x^2+3)^(1/2)+236/143*x^9*(x^4+5*x^2+3)^(1/2)+1090/429*x^7*
(x^4+5*x^2+3)^(1/2)+356/429*x^5*(x^4+5*x^2+3)^(1/2)+1251/715*x^3*(x^4+5*x^2
+3)^(1/2)-4210/429*x*(x^4+5*x^2+3)^(1/2)+25260/143/(-30+6*13^(1/2))^(1/2)*(
1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x
^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)
)-2120676/715/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(
-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1
/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+
6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^4 \cdot \left(3x^2+2\right) \left(x^4+5x^2+3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
[Out] Integral(x**4*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^4\,\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int(x^4*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.164 $\int x^2(2+3x^2)\left(3+5x^2+x^4\right)^{3/2}\,dx$

Optimal. Leaf size=331

$$-\frac{49949x\left(5+\sqrt{13}+2x^2\right)}{3465\sqrt{3+5x^2+x^4}}+\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3(911+890x^2)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3\left(67+27x^2\right)(3$$

```
[Out] 1/99*x^3*(27*x^2+67)*(x^4+5*x^2+3)^(3/2)-49949/3465*x*(5+2*x^2+13^(1/2))/(x
^4+5*x^2+3)^(1/2)+353/99*x*(x^4+5*x^2+3)^(1/2)-1/1155*x^3*(890*x^2+911)*(x^
4+5*x^2+3)^(1/2)+49949/20790*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30
+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2
)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))
^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2
)-353/33*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*
EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+3
0*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^
(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1287, 1293, 1203, 1113, 1149}

$$-\frac{353\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{33\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{49949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3465\sqrt{x^4+5x^2+3}}+\frac{353}{99}\sqrt{x^4+5x^2+3}\,x-\frac{49949\left(2x^2+\sqrt{13}+5\right)x}{3465\sqrt{x^4+5x^2+3}}+\frac{1}{99}\left(27x^2+67\right)\left(x^4+5x^2+3\right)^{3/2}x^3-\frac{\left(890x^2+911\right)\sqrt{x^4+5x^2+3}\,x^3}{1155}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-49949*x*(5 + Sqrt[13] + 2*x^2))/(3465*Sqrt[3 + 5*x^2 + x^4]) + (353*x*Sqr
t[3 + 5*x^2 + x^4])/99 - (x^3*(911 + 890*x^2)*Sqrt[3 + 5*x^2 + x^4])/1155 +
 (x^3*(67 + 27*x^2)*(3 + 5*x^2 + x^4)^(3/2))/99 + (49949*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(3465*Sqrt[3 + 5*x^2 + x^4]) - (353*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (
5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqr
t[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(33*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*
x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1287

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((b*e*2
*p + c*d*(m + 4*p + 3) + c*e*(4*p + m + 1)*x^2)/(c*f*(4*p + m + 1)*(m + 4*p
 + 3))), x] + Dist[2*(p/(c*(4*p + m + 1)*(m + 4*p + 3))), Int[(f*x)^m*(a +
b*x^2 + c*x^4)^(p - 1)*Simp[2*a*c*d*(m + 4*p + 3) - a*b*e*(m + 1) + (2*a*c*
e*(4*p + m + 1) + b*c*d*(m + 4*p + 3) - b^2*e*(m + 2*p + 1))*x^2, x], x], x
] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] &&
 NeQ[4*p + m + 1, 0] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p
] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int x^2\left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{1}{99}x^3\left(67+27x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{33}\int x^2\left(-3-178x^2\right)\sqrt{3+5x$$

$$=-\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3\left(67+27x^2\right)\left(3+5x^2+x^4\right.$$

$$=\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3(67$$

$$=\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3\left(911+890x^2\right)\sqrt{3+5x^2+x^4}}{1155}+\frac{1}{99}x^3(67$$

$$=-\frac{49949x\left(5+\sqrt{13}+2x^2\right)}{3465\sqrt{3+5x^2+x^4}}+\frac{353}{99}x\sqrt{3+5x^2+x^4}-\frac{x^3(911+890x$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.46, size = 244, normalized size = 0.74

$$\frac{2x\left(37065+74681x^2+69535x^4+84962x^6+50075x^8+11795x^{10}+945x^{12}\right)-49949i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+i\sqrt{2}\left(-212680+49949\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{6930\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2*(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (2*x*(37065 + 74681*x^2 + 69535*x^4 + 84962*x^6 + 50075*x^8 + 11795*x^10 +
945*x^12) - (49949*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/
(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 +
 Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-212680 + 49949*Sqrt[13
])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]
*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(69
30*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.05, size = 277, normalized size = 0.84

method	result
risch	$\frac{x\left(945x^8+7070x^6+11890x^4+4302x^2+12355\right)\sqrt{x^4+5x^2+3}}{3465}+\frac{399592\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\right.}}{}$

default	$\frac{3x^9\sqrt{x^4+5x^2+3}}{11}+\frac{202x^7\sqrt{x^4+5x^2+3}}{99}+\frac{2378x^5\sqrt{x^4+5x^2+3}}{693}+\frac{478x^3\sqrt{x^4+5x^2+3}}{385}+\frac{353}{}$
elliptic	$\frac{3x^9\sqrt{x^4+5x^2+3}}{11}+\frac{202x^7\sqrt{x^4+5x^2+3}}{99}+\frac{2378x^5\sqrt{x^4+5x^2+3}}{693}+\frac{478x^3\sqrt{x^4+5x^2+3}}{385}+\frac{353}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/11*x^9*(x^4+5*x^2+3)^(1/2)+202/99*x^7*(x^4+5*x^2+3)^(1/2)+2378/693*x^5*(x
^4+5*x^2+3)^(1/2)+478/385*x^3*(x^4+5*x^2+3)^(1/2)+353/99*x*(x^4+5*x^2+3)^(1
/2)-706/11/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/
6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1
/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+399592/385/(-30+6*13^(1/2))^(1/2)*(1-(
-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+
3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1
/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2
)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2 \cdot \left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)*x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\left(3\,x^2+2\right)\,{\left(x^4+5\,x^2+3\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int(x^2*(3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.165 $\int (2+3x^2)(3+5x^2+x^4)^{3/2}\,dx$

Optimal. Leaf size=308

$$\frac{203x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}}-\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}-\frac{203\sqrt{\frac{1}{6}\left(5+\sqrt{1\right.}}}{}$$

```
[Out] 1/3*x*(x^2+3)*(x^4+5*x^2+3)^(3/2)+203/30*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)
^(1/2)-1/15*x*(12*x^2+5)*(x^4+5*x^2+3)^(1/2)+5/3*(1/(36+x^2*(30+6*13^(1/2))
))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(
36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/
2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))
^(1/2)/(x^4+5*x^2+3)^(1/2)-203/180*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x
^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*1
3^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^
(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3
)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 308, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1190, 1203, 1113, 1149}

$$\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{203\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}+\frac{1}{3}x\left(x^2+3\right)\left(x^4+5x^2+3\right)^{3/2}-\frac{1}{15}x\left(12x^2+5\right)\sqrt{x^4+5x^2+3}+\frac{203x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (203*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (x*(5 + 12*x^2)
*Sqrt[3 + 5*x^2 + x^4])/15 + (x*(3 + x^2)*(3 + 5*x^2 + x^4)^(3/2))/3 - (203
*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x
^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (
-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[1
3]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqr
t[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/
6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
```

```
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \left(2+3x^2\right)\left(3+5x^2+x^4\right)^{3/2}\,dx=\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{21}\int \left(63-84x^2\right)\sqrt{3+5x^2+x^4}\,dx$$

$$=-\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+\frac{1}{315}$$

$$=-\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right)\left(3+5x^2+x^4\right)^{3/2}+10$$

$$=\frac{203x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}}-\frac{1}{15}x\left(5+12x^2\right)\sqrt{3+5x^2+x^4}+\frac{1}{3}x\left(3+x^2\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.57, size = 239, normalized size = 0.78

$$\frac{4x(120+434x^2+550x^4+293x^6+65x^8+5x^{10})+203i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-715+203\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{60\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(120 + 434*x^2 + 550*x^4 + 293*x^6 + 65*x^8 + 5*x^10) + (203*I)*Sqrt[2
]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sq
rt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sq
rt[13])/6] - I*Sqrt[2]*(-715 + 203*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(
-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 +
Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.06, size = 260, normalized size = 0.84

method	result
risch	$\frac{x(5x^6+40x^4+78x^2+40)\sqrt{x^4+5x^2+3}}{15}-\frac{2436\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\mathrm{Ell}\right.}{5\sqrt{-30+}}$
default	$\frac{x^7\sqrt{x^4+5x^2+3}}{3}+\frac{8x^5\sqrt{x^4+5x^2+3}}{3}+\frac{26x^3\sqrt{x^4+5x^2+3}}{5}+\frac{8x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-(}}{}$
elliptic	$\frac{x^7\sqrt{x^4+5x^2+3}}{3}+\frac{8x^5\sqrt{x^4+5x^2+3}}{3}+\frac{26x^3\sqrt{x^4+5x^2+3}}{5}+\frac{8x\sqrt{x^4+5x^2+3}}{3}+\frac{60\sqrt{1-(}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*x^7*(x^4+5*x^2+3)^(1/2)+8/3*x^5*(x^4+5*x^2+3)^(1/2)+26/5*x^3*(x^4+5*x^2
+3)^(1/2)+8/3*x*(x^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*
13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*
EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-2436/5/(-3
0+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2)
)*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1
/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2)
,5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(3x^2+2\right)\left(x^4+5x^2+3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2),x)
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int\left(3\,x^2+2\right)\,\left(x^4+5\,x^2+3\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2), x)
```

3.166 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2} \, dx$

Optimal. Leaf size=312

$$\frac{412x\left(5+\sqrt{13}+2x^2\right)}{35\sqrt{3+5x^2+x^4}}+\frac{1}{35}x\left(655+129x^2\right)\sqrt{3+5x^2+x^4}-\frac{(14-3x^2)\left(3+5x^2+x^4\right)^{3/2}}{7x}-\frac{206\sqrt{\frac{2}{3}\left(5+\right.}}{}$$

```
[Out] -1/7*(-3*x^2+14)*(x^4+5*x^2+3)^(3/2)/x+412/35*x*(5+2*x^2+13^(1/2))/(x^4+5*x
^2+3)^(1/2)+1/35*x*(129*x^2+655)*(x^4+5*x^2+3)^(1/2)+19/2*(1/(36+x^2*(30+6*
13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2)
)^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*
(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13
^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-206/105*(1/(36+x^2*(30+6*13^(1/2))))^(1
/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^
2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*
(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^
4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1285, 1190, 1203, 1113, 1149}

$$\frac{19\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{206\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{35\sqrt{x^4+5x^2+3}}-\frac{(14-3x^2)\left(x^4+5x^2+3\right)^{3/2}}{7x}+\frac{1}{35}x\left(129x^2+655\right)\sqrt{x^4+5x^2+3}+\frac{412x\left(2x^2+\sqrt{13}+5\right)}{35\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

```
[Out] (412*x*(5 + Sqrt[13] + 2*x^2))/(35*Sqrt[3 + 5*x^2 + x^4]) + (x*(655 + 129*x
^2)*Sqrt[3 + 5*x^2 + x^4])/35 - ((14 - 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(7*x
) - (206*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
 Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[1
3])/6]*x], (-13 + 5*Sqrt[13])/6])/(35*Sqrt[3 + 5*x^2 + x^4]) + (19*Sqrt[3/(
2*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*
(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
```

```
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^2}\,dx = -\frac{(14-3x^2)(3+5x^2+x^4)^{3/2}}{7x} - \frac{3}{7}\int (-88-43x^2)\sqrt{3+5x^2+x^4}\,dx$$

$$= \frac{1}{35}x(655+129x^2)\sqrt{3+5x^2+x^4} - \frac{(14-3x^2)(3+5x^2+x^4)^{3/2}}{7x} - \frac{1}{3}$$

$$= \frac{1}{35}x(655+129x^2)\sqrt{3+5x^2+x^4} - \frac{(14-3x^2)(3+5x^2+x^4)^{3/2}}{7x} + \frac{8}{3}$$

$$= \frac{412x\left(5+\sqrt{13}+2x^2\right)}{35\sqrt{3+5x^2+x^4}} + \frac{1}{35}x(655+129x^2)\sqrt{3+5x^2+x^4} - \frac{(14-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.30, size = 235, normalized size = 0.75

$$\frac{-1260+3884x^4+2130x^6+418x^8+30x^{10}+412i\sqrt{2}\left(-5+\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-65+412\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{70x\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^2,x]
```

```
[Out] (-1260 + 3884*x^4 + 2130*x^6 + 418*x^8 + 30*x^10 + (412*I)*Sqrt[2]*(-5 + Sq
rt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] +
 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/
6] - I*Sqrt[2]*(-65 + 412*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sq
rt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13
])]*x], 19/6 + (5*Sqrt[13])/6])/(70*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.06, size = 260, normalized size = 0.83

method	result
risch	$\frac{15x^{10}+209x^8+1065x^6+1942x^4-630}{35x\sqrt{x^4+5x^2+3}} - \frac{29664\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\mathrm{EllipticF}\left(\right.\right.}{35\sqrt{-30+6\sqrt{13}}}$
default	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7} + \frac{134x^3\sqrt{x^4+5x^2+3}}{35} + 10x\sqrt{x^4+5x^2+3} + \frac{342\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}}{}$

elliptic	$\frac{3x^5\sqrt{x^4+5x^2+3}}{7}+\frac{134x^3\sqrt{x^4+5x^2+3}}{35}+10x\sqrt{x^4+5x^2+3}+\frac{342\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 3/7*x^5*(x^4+5*x^2+3)^(1/2)+134/35*x^3*(x^4+5*x^2+3)^(1/2)+10*x*(x^4+5*x^2+
3)^(1/2)+342/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-
5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^
(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-29664/35/(-30+6*13^(1/2))^(1/2)*(1-(
-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+
3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1
/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2
)))-6*(x^4+5*x^2+3)^(1/2)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{\frac{3}{2}}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**2,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^2,x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(3\,x^2+2)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^2,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^2, x)
```

3.167 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx$

Optimal. Leaf size=314

$$\frac{949x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}}-\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x}-\frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3}-\frac{949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}}{}$$

```
[Out] -1/15*(-9*x^2+10)*(x^4+5*x^2+3)^(3/2)/x^3+949/30*x*(5+2*x^2+13^(1/2))/(x^4+
5*x^2+3)^(1/2)-13/15*(-5*x^2+24)*(x^4+5*x^2+3)^(1/2)/x+65/3*(1/(36+x^2*(30+
6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/
2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^
2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+
13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-949/180*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(
x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1285, 1203, 1113, 1149}

$$\frac{65\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{949\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}-\frac{13(24-5x^2)\sqrt{x^4+5x^2+3}}{15x}+\frac{949x\left(2x^2+\sqrt{13}+5\right)}{30\sqrt{x^4+5x^2+3}}-\frac{(10-9x^2)(x^4+5x^2+3)^{3/2}}{15x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

```
[Out] (949*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (13*(24 - 5*x^2
)*Sqrt[3 + 5*x^2 + x^4])/(15*x) - ((10 - 9*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(1
5*x^3) - (949*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
+ Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[
13])/6]*x], (-13 + 5*Sqrt[13])/6])/(30*Sqrt[3 + 5*x^2 + x^4]) + (65*Sqrt[2/
(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
```

```
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^4}\,dx = -\frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3} - \frac{1}{5}\int \frac{(-104-65x^2)\sqrt{3+5x^2+x^4}}{x^2}\,dx$$

$$= -\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3} + \frac{1}{15}\int$$

$$= -\frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)(3+5x^2+x^4)^{3/2}}{15x^3} + \frac{949}{15}\int$$

$$= \frac{949x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}} - \frac{13(24-5x^2)\sqrt{3+5x^2+x^4}}{15x} - \frac{(10-9x^2)(3}{1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.17, size = 247, normalized size = 0.79

$$\frac{4(-90-1155x^2-1405x^4+192x^6+145x^8+9x^{10})+949i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-13i\sqrt{2}\left(-65+73\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{60x^3\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^4,x]
```

```
[Out] (4*(-90 - 1155*x^2 - 1405*x^4 + 192*x^6 + 145*x^8 + 9*x^10) + (949*I)*Sqrt[
2]*(-5 + Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5
 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
(5*Sqrt[13])/6] - (13*I)*Sqrt[2]*(-65 + 73*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13
] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[
Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(60*x^3*Sqrt[3 + 5*x^2 +
 x^4])
```

Maple [A]
time = 0.06, size = 260, normalized size = 0.83

method	result
risch	$\frac{9x^{10}+145x^8+192x^6-1405x^4-1155x^2-90}{15x^3\sqrt{x^4+5x^2+3}} - \frac{11388\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{Elliptic}\right.}{5\sqrt{-30+6\sqrt{}}}$
default	$-\frac{67\sqrt{x^4+5x^2+3}}{3x}+\frac{3x^3\sqrt{x^4+5x^2+3}}{5}+\frac{20x\sqrt{x^4+5x^2+3}}{3}+\frac{780\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{}}{}$

elliptic	$-\frac{67\sqrt{x^4+5x^2+3}}{3x}+\frac{3x^3\sqrt{x^4+5x^2+3}}{5}+\frac{20x\sqrt{x^4+5x^2+3}}{3}+\frac{780\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x,method=_RETURNVERBOSE)
```

```
[Out] -67/3*(x^4+5*x^2+3)^(1/2)/x+3/5*x^3*(x^4+5*x^2+3)^(1/2)+20/3*x*(x^4+5*x^2+3
)^(1/2)+780/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5
/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(
1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-11388/5/(-30+6*13^(1/2))^(1/2)*(1-(-5
/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)
^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6
*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
)-2*(x^4+5*x^2+3)^(1/2)/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^4, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(3x^2+2)\,(x^4+5x^2+3)^{\frac{3}{2}}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**4,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^4,x, algorithm="giac")
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(3\,x^2+2)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^4,x)
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^4, x)
```

3.168 $\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6} \, dx$

Optimal. Leaf size=331

$$\frac{361x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}}-\frac{722\sqrt{3+5x^2+x^4}}{15x}-\frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3}-\frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5}-$$

```
[Out] -1/5*(-5*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^5+361/15*x*(5+2*x^2+13^(1/2))/(x^4+5*
x^2+3)^(1/2)-722/15*(x^4+5*x^2+3)^(1/2)/x-1/5*(-87*x^2+40)*(x^4+5*x^2+3)^(1
/2)/x^3-361/90*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+103*(1/(36+x
^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6
*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2)
)*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x
^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 331, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1285, 1295, 1203, 1113, 1149}

$$\frac{103\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{361\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{15\sqrt{x^4+5x^2+3}}-\frac{722\sqrt{x^4+5x^2+3}}{15x}+\frac{361x\left(2x^2+\sqrt{13}+5\right)}{15\sqrt{x^4+5x^2+3}}-\frac{(2-5x^2)(x^4+5x^2+3)^{3/2}}{5x^5}-\frac{(40-87x^2)\sqrt{x^4+5x^2+3}}{5x^3}$$

Antiderivative was successfully verified.

```
[In] Int[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

```
[Out] (361*x*(5 + Sqrt[13] + 2*x^2))/(15*Sqrt[3 + 5*x^2 + x^4]) - (722*Sqrt[3 + 5
*x^2 + x^4])/(15*x) - ((40 - 87*x^2)*Sqrt[3 + 5*x^2 + x^4])/(5*x^3) - ((2 -
 5*x^2)*(3 + 5*x^2 + x^4)^(3/2))/(5*x^5) - (361*Sqrt[(5 + Sqrt[13])/6]*Sqrt
[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2
)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(15*Sq
rt[3 + 5*x^2 + x^4]) + (103*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13
])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x
], (-13 + 5*Sqrt[13])/6])/(Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{(2+3x^2)(3+5x^2+x^4)^{3/2}}{x^6}\,dx = -\frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5} - \frac{1}{5}\int \frac{(-120-87x^2)\sqrt{3+5x^2+x^4}}{x^4}\,dx$$

$$= -\frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{(2-5x^2)(3+5x^2+x^4)^{3/2}}{5x^5} + \frac{1}{15}\int$$

$$= -\frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{(2-5x^2)(3+}{5x^5}$$

$$= -\frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{(40-87x^2)\sqrt{3+5x^2+x^4}}{5x^3} - \frac{(2-5x^2)(3+}{5x^5}$$

$$= \frac{361x\left(5+\sqrt{13}+2x^2\right)}{15\sqrt{3+5x^2+x^4}} - \frac{722\sqrt{3+5x^2+x^4}}{15x} - \frac{(40-87x^2)\sqrt{3+5x}}{5x^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.20, size = 244, normalized size = 0.74

$$\frac{-108-810x^2-3438x^4-4040x^6-634x^8+30x^{10}+361i\sqrt{2}\left(-5+\sqrt{13}\right)x^5\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-260+361\sqrt{13}\right)x^5\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{30x^5\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((2 + 3*x^2)*(3 + 5*x^2 + x^4)^(3/2))/x^6,x]
```

```
[Out] (-108 - 810*x^2 - 3438*x^4 - 4040*x^6 - 634*x^8 + 30*x^10 + (361*I)*Sqrt[2]
*(-5 + Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 +
 Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5
*Sqrt[13])/6] - I*Sqrt[2]*(-260 + 361*Sqrt[13])*x^5*Sqrt[(-5 + Sqrt[13] - 2
*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[
2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(30*x^5*Sqrt[3 + 5*x^2 + x^4]
)
```

Maple [A]
time = 0.07, size = 259, normalized size = 0.78

method	result
risch	$\frac{15x^{10}-317x^8-2020x^6-1719x^4-405x^2-54}{15x^5\sqrt{x^4+5x^2+3}} - \frac{8664\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{Ellipt}\right.}{5\sqrt{-30+6}}$

default	$-\frac{6\sqrt{x^4+5x^2+3}}{5x^5}-\frac{7\sqrt{x^4+5x^2+3}}{x^3}-\frac{392\sqrt{x^4+5x^2+3}}{15x}+\frac{618\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-}}{\sqrt{}}$
elliptic	$-\frac{6\sqrt{x^4+5x^2+3}}{5x^5}-\frac{7\sqrt{x^4+5x^2+3}}{x^3}-\frac{392\sqrt{x^4+5x^2+3}}{15x}+\frac{618\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-}}{\sqrt{}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x,method=_RETURNVERBOSE)
```

```
[Out] -6/5/x^5*(x^4+5*x^2+3)^(1/2)-7*(x^4+5*x^2+3)^(1/2)/x^3-392/15*(x^4+5*x^2+3)
^(1/2)/x+618/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-
5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^
(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-8664/5/(-30+6*13^(1/2))^(1/2)*(1-(-5
/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)
^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6
*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
)+x*(x^4+5*x^2+3)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="maxima")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="fricas")
```

```
[Out] integral((3*x^6 + 17*x^4 + 19*x^2 + 6)*sqrt(x^4 + 5*x^2 + 3)/x^6, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(3x^2+2)\,(x^4+5x^2+3)^{\frac{3}{2}}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)*(x**4+5*x**2+3)**(3/2)/x**6,x)
```

```
[Out] Integral((3*x**2 + 2)*(x**4 + 5*x**2 + 3)**(3/2)/x**6, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)*(x^4+5*x^2+3)^(3/2)/x^6,x, algorithm="giac")
```

```
[Out] integrate((x^4 + 5*x^2 + 3)^(3/2)*(3*x^2 + 2)/x^6, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(3\,x^2+2)\,\left(x^4+5\,x^2+3\right)^{3/2}}{x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^6,x)
```

```
[Out] int(((3*x^2 + 2)*(5*x^2 + x^4 + 3)^(3/2))/x^6, x)
```

3.169 $\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=153

$$\frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}+\frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2)\sqrt{a+bx^2+cx^4}}{48c^3}-\frac{(5b^3B-6Ab^2c-1$$

```
[Out] -1/32*(8*A*a*c^2-6*A*b^2*c-12*B*a*b*c+5*B*b^3)*arctanh(1/2*(2*c*x^2+b)/c^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)+1/6*B*x^4*(c*x^4+b*x^2+a)^(1/2)/c+1/48*(
15*b^2*B-18*A*b*c-16*a*B*c-2*c*(-6*A*c+5*B*b)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^
3
```

Rubi [A]
time = 0.14, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1265, 846, 793, 635, 212}

$$\frac{\sqrt{a+bx^2+cx^4}\,(-16aBc-2cx^2(5bB-6Ac)-18Abc+15b^2B)}{48c^3}-\frac{(8aAc^2-12abBc-6Ab^2c+5b^3B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{7/2}}+\frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x^4*Sqrt[a + b*x^2 + c*x^4])/(6*c) + ((15*b^2*B - 18*A*b*c - 16*a*B*c -
2*c*(5*b*B - 6*A*c)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(48*c^3) - ((5*b^3*B - 6*
A*b^2*c - 12*a*b*B*c + 8*a*A*c^2)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a +
 b*x^2 + c*x^4])])/(32*c^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
```

```
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\
&= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{\text{Subst}\left(\int \frac{x\left(-2aB-\frac{1}{2}(5bB-6Ac)x\right)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6c}\\
&= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2)\sqrt{a+bx^2}}{48c^3}\\
&= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2)\sqrt{a+bx^2}}{48c^3}\\
&= \frac{Bx^4\sqrt{a+bx^2+cx^4}}{6c} + \frac{(15b^2B-18Abc-16aBc-2c(5bB-6Ac)x^2)\sqrt{a+bx^2}}{48c^3}
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 135, normalized size = 0.88

$$
\frac{\sqrt{a+bx^2+cx^4}\,(15b^2B-18Abc-16aBc-10bBcx^2+12Ac^2x^2+8Bc^2x^4)}{48c^3} + \frac{(5b^3B-6Ab^2c-12abBc+8aAc^2)\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{32c^{7/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(15*b^2*B - 18*A*b*c - 16*a*B*c - 10*b*B*c*x^2 + 1
2*A*c^2*x^2 + 8*B*c^2*x^4))/(48*c^3) + ((5*b^3*B - 6*A*b^2*c - 12*a*b*B*c +
 8*a*A*c^2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(32*c^(7/
2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 281 vs. $2(135) = 270$.
time = 0.08, size = 282, normalized size = 1.84

method	result
risch	$-\frac{\left(-8 c^{2} B \,x^{4}-12 A \,c^{2} x^{2}+10 B b c \,x^{2}+18 b c A +16 a c B -15 b^{2} B \right) \sqrt{c \,x^{4}+b \,x^{2}+a}}{48 c^{3}}-\frac{\ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x^{2}+a}\right) a A}{4 c^{\frac{3}{2}}}-$
default	$B \left(\frac{x^{4} \sqrt{c \,x^{4}+b \,x^{2}+a}}{6 c}-\frac{5 b \,x^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{24 c^{2}}+\frac{5 b^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{16 c^{3}}-\frac{5 b^{3} \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b}\right.}{32 c^{\frac{7}{2}}}\right.$
elliptic	$\frac{B \,x^{4} \sqrt{c \,x^{4}+b \,x^{2}+a}}{6 c}-\frac{5 B b \,x^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{24 c^{2}}+\frac{5 B \,b^{2} \sqrt{c \,x^{4}+b \,x^{2}+a}}{16 c^{3}}-\frac{5 \ln \left(\frac{\frac{b}{2}+c \,x^{2}}{\sqrt{c}}+\sqrt{c \,x^{4}+b \,x}\right.}{32 c^{\frac{7}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/6*x^4*(c*x^4+b*x^2+a)^(1/2)/c-5/24*b/c^2*x^2*(c*x^4+b*x^2+a)^(1/2)+5/1
6*b^2/c^3*(c*x^4+b*x^2+a)^(1/2)-5/32*b^3/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(
c*x^4+b*x^2+a)^(1/2))+3/8*b/c^(5/2)*a*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))-1/3*a/c^2*(c*x^4+b*x^2+a)^(1/2))+A*(1/4*x^2*(c*x^4+b*x^2+a)^(1/2
)/c-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/
2)+(c*x^4+b*x^2+a)^(1/2))-1/4*a/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 315, normalized size = 2.06

$$\left[\frac{3\,(5\,Bb^3+8\,Aac^2-6\,(2\,Bab+Ab^2)c)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)+4\,(8\,Bc^3x^4+15\,Bb^2c-2\,(8\,Ba+9\,Ab)c^2-2\,(5\,Bbc^2-6\,Ac^3)x^2)\sqrt{cx^4+bx^2+a}}{192\,c^4}, \frac{3\,(5\,Bb^3+8\,Aac^2-6\,(2\,Bab+Ab^2)c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)+2\,(8\,Bc^3x^4+15\,Bb^2c-2\,(8\,Ba+9\,Ab)c^2-2\,(5\,Bbc^2-6\,Ac^3)x^2)\sqrt{cx^4+bx^2+a}}{96\,c^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*B*b^3 + 8*A*a*c^2 - 6*(2*B*a*b + A*b^2)*c)*sqrt(c)*log(-8*c^2*
x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4
*a*c) + 4*(8*B*c^3*x^4 + 15*B*b^2*c - 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2
- 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 + a))/c^4, 1/96*(3*(5*B*b^3 + 8*A*a*c^2
- 6*(2*B*a*b + A*b^2)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x
^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(8*B*c^3*x^4 + 15*B*b^2*c -
 2*(8*B*a + 9*A*b)*c^2 - 2*(5*B*b*c^2 - 6*A*c^3)*x^2)*sqrt(c*x^4 + b*x^2 +
a))/c^4]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 4.76, size = 138, normalized size = 0.90

$$\frac{1}{48}\,\sqrt{cx^4+bx^2+a}\left(2\left(\frac{4\,Bx^2}{c}-\frac{5\,Bbc-6\,Ac^2}{c^3}\right)x^2+\frac{15\,Bb^2-16\,Bac-18\,Abc}{c^3}\right)+\frac{(5\,Bb^3-12\,Babc-6\,Ab^2c+8\,Aac^2)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}-b\right|\right)}{32\,c^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*sqrt(c*x^4 + b*x^2 + a)*(2*(4*B*x^2/c - (5*B*b*c - 6*A*c^2)/c^3)*x^2 +
 (15*B*b^2 - 16*B*a*c - 18*A*b*c)/c^3) + 1/32*(5*B*b^3 - 12*B*a*b*c - 6*A*b
^2*c + 8*A*a*c^2)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c
) - b))/c^(7/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^5*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.170 $\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=100

$$-\frac{(3bB-4Ac-2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2}+\frac{(3b^2B-4Abc-4aBc)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}$$

```
[Out] 1/16*(-4*A*b*c-4*B*a*c+3*B*b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^
2+a)^(1/2))/c^(5/2)-1/8*(-2*B*c*x^2-4*A*c+3*B*b)*(c*x^4+b*x^2+a)^(1/2)/c^2
```

Rubi [A]
time = 0.06, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1265, 793, 635, 212}

$$\frac{(-4aBc-4Abc+3b^2B)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{5/2}}-\frac{\sqrt{a+bx^2+cx^4}\,(-4Ac+3bB-2Bcx^2)}{8c^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -1/8*((3*b*B - 4*A*c - 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c^2 + ((3*b^2*B
- 4*A*b*c - 4*a*B*c)*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(16*c^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
```

```
d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(A+Bx)}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{(3b^2B-4Abc-4aBc)\,\text{Subst}\left(\int \frac{}{\sqrt{a}\ldots}\right.}{16c^2}$$

$$= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{(3b^2B-4Abc-4aBc)\,\text{Subst}\left(\int \frac{1}{4c-x^2}\ldots\right.}{8c^2}$$

$$= -\frac{(3bB-4Ac-2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{(3b^2B-4Abc-4aBc)\tanh^{-1}\left(\frac{}{2\sqrt{c}}\ldots\right.}{16c^{5/2}}$$

Mathematica [A]
time = 0.32, size = 104, normalized size = 1.04

$$\frac{(-3bB+4Ac+2Bcx^2)\sqrt{a+bx^2+cx^4}}{8c^2} + \frac{(-3b^2B+4Abc+4aBc)\log\left(bc^2+2c^3x^2-2c^{5/2}\sqrt{a+bx^2+cx^4}\right)}{16c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((-3*b*B + 4*A*c + 2*B*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*c^2) + ((-3*b^2*B
 + 4*A*b*c + 4*a*B*c)*Log[b*c^2 + 2*c^3*x^2 - 2*c^(5/2)*Sqrt[a + b*x^2 + c*
x^4]])/(16*c^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(86) = 172$.
time = 0.07, size = 176, normalized size = 1.76

method	result

risch	$\frac{(2Bc\,x^2+4Ac-3bB)\sqrt{c\,x^4+b\,x^2+a}}{8c^2}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)bA}{4c^{\frac{3}{2}}}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right.}{4c^{\frac{3}{2}}}$
default	$B\left(\frac{x^2\sqrt{c\,x^4+b\,x^2+a}}{4c}-\frac{3b\sqrt{c\,x^4+b\,x^2+a}}{8c^2}+\frac{3b^2\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{16c^{\frac{5}{2}}}-\frac{a\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{\right.}}{}\right.$
elliptic	$\frac{B\,x^2\sqrt{c\,x^4+b\,x^2+a}}{4c}-\frac{3Bb\sqrt{c\,x^4+b\,x^2+a}}{8c^2}+\frac{3\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)b^2B}{16c^{\frac{5}{2}}}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/4*x^2*(c*x^4+b*x^2+a)^(1/2)/c-3/8*b*(c*x^4+b*x^2+a)^(1/2)/c^2+3/16*b^2
/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/4*a/c^(3/2)*ln((
1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)))+A*(1/2*(c*x^4+b*x^2+a)^(1/2)/c
-1/4*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 233, normalized size = 2.33

$$\left[-\frac{(3\,Bb^2-4\,(Ba+Ab)c)\sqrt{c}\,\log\left(-8\,c^2x^4-8\,bcx^2-b^2+4\,\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{c}-4\,ac\right)-4\,(2\,Bc^2x^2-3\,Bbc+4\,Ac^2)\sqrt{cx^4+bx^2+a}}{32\,c^3},\ -\frac{(3\,Bb^2-4\,(Ba+Ab)c)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(2\,cx^2+b)\sqrt{-c}}{2\,(c^2x^4+bcx^2+ac)}\right)-2\,(2\,Bc^2x^2-3\,Bbc+4\,Ac^2)\sqrt{cx^4+bx^2+a}}{16\,c^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*((3*B*b^2 - 4*(B*a + A*b)*c)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^
2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 4*(2*B*c^2*x
^2 - 3*B*b*c + 4*A*c^2)*sqrt(c*x^4 + b*x^2 + a))/c^3, -1/16*((3*B*b^2 - 4*(
B*a + A*b)*c)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqr
```

```
t(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*(2*B*c^2*x^2 - 3*B*b*c + 4*A*c^2)*sqrt
(c*x^4 + b*x^2 + a))/c^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(A + Bx^2)}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 5.27, size = 98, normalized size = 0.98

$$\frac{1}{8}\sqrt{cx^4 + bx^2 + a}\left(\frac{2\,Bx^2}{c} - \frac{3\,Bb - 4\,Ac}{c^2}\right) - \frac{(3\,Bb^2 - 4\,Bac - 4\,Abc)\log\left(\left|-2\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)\sqrt{c} - b\right|\right)}{16\,c^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(c*x^4 + b*x^2 + a)*(2*B*x^2/c - (3*B*b - 4*A*c)/c^2) - 1/16*(3*B*b
^2 - 4*B*a*c - 4*A*b*c)*log(abs(-2*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*
sqrt(c) - b))/c^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3\,\left(B\,x^2 + A\right)}{\sqrt{c\,x^4 + b\,x^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^3*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.171 $\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=76

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

```
[Out] -1/4*(-2*A*c+B*b)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^
(3/2)+1/2*B*(c*x^4+b*x^2+a)^(1/2)/c
```

Rubi [A]
time = 0.04, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1261, 654, 635, 212}

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) - ((b*B - 2*A*c)*ArcTanh[(b + 2*c*x^2)/(2
*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\ &= \frac{B\sqrt{a+bx^2+cx^4}}{2c}+\frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4c}\\ &= \frac{B\sqrt{a+bx^2+cx^4}}{2c}+\frac{(-bB+2Ac)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2c}\\ &= \frac{B\sqrt{a+bx^2+cx^4}}{2c}-\frac{(bB-2Ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 78, normalized size = 1.03

$$\frac{B\sqrt{a+bx^2+cx^4}}{2c}+\frac{(bB-2Ac)\log\left(bc+2c^2x^2-2c^{3/2}\sqrt{a+bx^2+cx^4}\right)}{4c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
[Out] (B*Sqrt[a + b*x^2 + c*x^4])/(2*c) + ((b*B - 2*A*c)*Log[b*c + 2*c^2*x^2 - 2*
c^(3/2)*Sqrt[a + b*x^2 + c*x^4]])/(4*c^(3/2))
```

Maple [A]
time = 0.05, size = 94, normalized size = 1.24

method	result	size
risch	$\frac{B\sqrt{c\,x^4+b\,x^2+a}}{2c}+\frac{A\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)bB}{4c^{\frac{3}{2}}}$	93
elliptic	$\frac{B\sqrt{c\,x^4+b\,x^2+a}}{2c}+\frac{A\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2\sqrt{c}}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)bB}{4c^{\frac{3}{2}}}$	93

default	$B\left(\frac{\sqrt{c\,x^{4}+b\,x^{2}+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{4c^{\frac{3}{2}}}\right)+\frac{A\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2)))+1/2*A*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^
(1/2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.38, size = 178, normalized size = 2.34

$$\left[\frac{4\sqrt{cx^{4}+bx^{2}+a}\,Bc-(Bb-2\,Ac)\sqrt{c}\,\log\left(-8\,c^{2}x^{4}-8\,bcx^{2}-b^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{c}-4\,ac\right)}{8\,c^{2}},\ \frac{2\sqrt{cx^{4}+bx^{2}+a}\,Bc+(Bb-2\,Ac)\sqrt{-c}\,\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(2\,cx^{2}+b)\sqrt{-c}}{2\,(c^{2}x^{4}+bcx^{2}+ac)}\right)}{4\,c^{2}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*B*c - (B*b - 2*A*c)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
))/c^2, 1/4*(2*sqrt(c*x^4 + b*x^2 + a)*B*c + (B*b - 2*A*c)*sqrt(-c)*arctan(
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c
)))/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 4.06, size = 69, normalized size = 0.91

$$\frac{\sqrt{cx^4+bx^2+a}\,B}{2\,c}+\frac{(Bb-2\,Ac)\log\left(\left|-2\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)\sqrt{c}\,-b\right|\right)}{4\,c^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(c*x^4 + b*x^2 + a)*B/c + 1/4*(B*b - 2*A*c)*log(abs(-2*(sqrt(c)*x^2
 - sqrt(c*x^4 + b*x^2 + a))*sqrt(c) - b))/c^(3/2)
```

Mupad [B]
time = 1.05, size = 92, normalized size = 1.21

$$\frac{A\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}}+\frac{B\,\sqrt{c\,x^4+b\,x^2+a}}{2\,c}-\frac{B\,b\,\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{4\,c^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] (A*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) + (B
*(a + b*x^2 + c*x^4)^(1/2))/(2*c) - (B*b*log((a + b*x^2 + c*x^4)^(1/2) + (b
/2 + c*x^2)/c^(1/2)))/(4*c^(3/2))
```

3.172 $\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=90

$$-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}+\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}$$

```
[Out] -1/2*A*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(1/2)+1/2*B
*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(1/2)
```

Rubi [A]

time = 0.06, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.185, Rules used = {1265, 857, 635, 212, 738}

$$\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}-\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(A*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/Sqrt[a]
 + (B*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[c
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\ &= \frac{1}{2}A\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)+\frac{1}{2}B\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x,\right.\\ &= -\left(A\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)\right)+B\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x,\right.\\ &= -\frac{A\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}}+\frac{B\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}}\end{aligned}$$

Mathematica [A]
time = 0.23, size = 85, normalized size = 0.94

$$\frac{A\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{B\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{2\sqrt{c}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (A*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/Sqrt[a] - (B*L
og[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/(2*Sqrt[c])
```

Maple [A]
time = 0.04, size = 76, normalized size = 0.84

method	result	size
default	$\frac{B\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}-\frac{A\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{a}}$	76
elliptic	$\frac{B\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2\sqrt{c}}-\frac{A\ln\left(\frac{2a+b\,x^{2}+2\sqrt{a}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}{x^{2}}\right)}{2\sqrt{a}}$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*B*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*A/a^(1/2)
*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.45, size = 517, normalized size = 5.74

$$\left[\frac{Ba\sqrt{c}\log\left(-8c^{2}x^{4}-8bcx^{2}-b^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(2cx^{2}+b)\sqrt{c}-4ac\right)+A\sqrt{a}\,c\log\left(-\frac{(b^{2}+4ac)x^{4}+8abx^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{a}+8a^{2}}{x^{4}}\right)}{4ac},\ -\frac{2Ba\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(2cx^{2}+b)\sqrt{-c}}{2(c^{2}x^{4}+bcx^{2}+ac)}\right)-A\sqrt{a}\,c\log\left(-\frac{(b^{2}+4ac)x^{4}+8abx^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{a}+8a^{2}}{x^{4}}\right)}{4ac},\ \frac{2A\sqrt{-a}\,c\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{-a}}{2(acx^{4}+abx^{2}+a^{2})}\right)+Ba\sqrt{c}\log\left(-8c^{2}x^{4}-8bcx^{2}-b^{2}-4\sqrt{cx^{4}+bx^{2}+a}\,(2cx^{2}+b)\sqrt{c}-4ac\right)}{4ac},\ \frac{A\sqrt{-a}\,c\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(bx^{2}+2a)\sqrt{-a}}{2(acx^{4}+abx^{2}+a^{2})}\right)-Ba\sqrt{-c}\arctan\left(\frac{\sqrt{cx^{4}+bx^{2}+a}\,(2cx^{2}+b)\sqrt{-c}}{2(c^{2}x^{4}+bcx^{2}+ac)}\right)}{2ac}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 +
 a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/
(a*c), -1/4*(2*B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - A*sqrt(a)*c*log(-((b^2 + 4*a*c)*x^4
 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4
))/(a*c), 1/4*(2*A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2
*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + B*a*sqrt(c)*log(-8*c^2*x^4 - 8*b*
```

```
c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(a*
c), 1/2*(A*sqrt(-a)*c*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt
(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - B*a*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^
2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(a*c)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:index.cc index_m operator + Error: Ba
d Argument Value
```

Mupad [B]
time = 0.76, size = 81, normalized size = 0.90

$$\frac{B\ln\left(\sqrt{c\,x^4+b\,x^2+a}+\frac{c\,x^2+\frac{b}{2}}{\sqrt{c}}\right)}{2\,\sqrt{c}}-\frac{A\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}-\frac{A\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (B*log((a + b*x^2 + c*x^4)^(1/2) + (b/2 + c*x^2)/c^(1/2)))/(2*c^(1/2)) - (A
*log(1/x^2))/(2*a^(1/2)) - (A*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2)
 + b*x^2))/(2*a^(1/2))
```

3.173 $\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=80

$$-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}+\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

```
[Out] 1/4*(A*b-2*B*a)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3
/2)-1/2*A*(c*x^4+b*x^2+a)^(1/2)/a/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1265, 820, 738, 212}

$$\frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2) + ((A*b - 2*a*B)*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
```

```
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^3\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^2\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} - \frac{(Ab-2aB)\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4a}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2a}$$
$$= -\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(Ab-2aB)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}}$$

Mathematica [A]
time = 0.32, size = 81, normalized size = 1.01

$$-\frac{A\sqrt{a+bx^2+cx^4}}{2ax^2} + \frac{(-Ab+2aB)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{2a^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^3*Sqrt[a + b*x^2 + c*x^4]),x]
[Out] -1/2*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^2) + ((-(A*b) + 2*a*B)*ArcTanh[(Sqrt[
c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(2*a^(3/2))
```

Maple [A]
time = 0.06, size = 105, normalized size = 1.31

method	result

risch	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}+\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)Ab}{4a^{\frac{3}{2}}}-\frac{B\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right.}{2\sqrt{a}}$
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}+\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)Ab}{4a^{\frac{3}{2}}}-\frac{B\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right.}{2\sqrt{a}}$
default	$A\left(-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2a\,x^2}+\frac{b\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{4a^{\frac{3}{2}}}\right)-\frac{B\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2}}{x^2}\right.}{2\sqrt{a}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/2*(c*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*b/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(
c*x^4+b*x^2+a)^(1/2))/x^2))-1/2*B/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.40, size = 197, normalized size = 2.46

$$\left[-\frac{(2\,Ba-Ab)\sqrt{a}\,x^2\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)+4\,\sqrt{cx^4+bx^2+a}\,Aa}{8\,a^2x^2},\ \frac{(2\,Ba-Ab)\sqrt{-a}\,x^2\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)-2\,\sqrt{cx^4+bx^2+a}\,Aa}{4\,a^2x^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*((2*B*a - A*b)*sqrt(a)*x^2*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sq
rt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*sqrt(c*x^4 +
b*x^2 + a)*A*a)/(a^2*x^2), 1/4*((2*B*a - A*b)*sqrt(-a)*x^2*arctan(1/2*sqrt(
```

```
c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*sq
rt(c*x^4 + b*x^2 + a)*A*a)/(a^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^3\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**3/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**3*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 3.70, size = 124, normalized size = 1.55

$$\frac{(2\,Ba - Ab)\arctan\left(-\frac{\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,a} + \frac{\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)Ab + 2\,Aa\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^3/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*(2*B*a - A*b)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))
/(sqrt(-a)*a) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*b + 2*A*a*sq
rt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a)
```

Mupad [B]
time = 0.78, size = 103, normalized size = 1.29

$$\frac{A\,b\,\mathrm{atanh}\left(\frac{\frac{b\,x^2}{2}+a}{\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}\right)}{4\,a^{3/2}} - \frac{B\,\ln\left(2\,a+2\,\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}+b\,x^2\right)}{2\,\sqrt{a}} - \frac{A\,\sqrt{c\,x^4+b\,x^2+a}}{2\,a\,x^2} - \frac{B\,\ln\left(\frac{1}{x^2}\right)}{2\,\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^3*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] (A*b*atanh((a + (b*x^2)/2)/(a^(1/2)*(a + b*x^2 + c*x^4)^(1/2))))/(4*a^(3/2)
) - (B*log(2*a + 2*a^(1/2)*(a + b*x^2 + c*x^4)^(1/2) + b*x^2))/(2*a^(1/2))
- (A*(a + b*x^2 + c*x^4)^(1/2))/(2*a*x^2) - (B*log(1/x^2))/(2*a^(1/2))
```

3.174 $\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=124

$$-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{(3Ab^2-4abB-4aAc)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}$$

```
[Out] -1/16*(-4*A*a*c+3*A*b^2-4*B*a*b)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x
^2+a)^(1/2))/a^(5/2)-1/4*A*(c*x^4+b*x^2+a)^(1/2)/a/x^4+1/8*(3*A*b-4*B*a)*(c
*x^4+b*x^2+a)^(1/2)/a^2/x^2
```

Rubi [A]
time = 0.10, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1265, 848, 820, 738, 212}

$$-\frac{(-4aAc-4abB+3Ab^2)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{16a^{5/2}}+\frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2}-\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/4*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^4) + ((3*A*b - 4*a*B)*Sqrt[a + b*x^2
+ c*x^4])/(8*a^2*x^2) - ((3*A*b^2 - 4*a*b*B - 4*a*A*c)*ArcTanh[(2*a + b*x^2
)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(16*a^(5/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^5\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^3\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(3Ab-4aB)+Acx}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} + \frac{(3Ab^2-4abB-4aAc)}{}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{(3Ab^2-4abB-4aAc)}{}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{4ax^4} + \frac{(3Ab-4aB)\sqrt{a+bx^2+cx^4}}{8a^2x^2} - \frac{(3Ab^2-4abB-4aAc)}{}$$

Mathematica [**A**]

time = 0.54, size = 145, normalized size = 1.17

$$\frac{\sqrt{a}\sqrt{a+bx^2+cx^4}\,(3Abx^2-2a(A+2Bx^2))+3Ab^2x^4\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)+4a(bB+Ac)x^4\tanh^{-1}\left(\frac{-\sqrt{c}\,x^2+\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^5*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a]*Sqrt[a + b*x^2 + c*x^4]*(3*A*b*x^2 - 2*a*(A + 2*B*x^2)) + 3*A*b^2*
x^4*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] + 4*a*(b*B + A
*c)*x^4*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(8*a^(
5/2)*x^4)
```

Maple [A]
time = 0.07, size = 194, normalized size = 1.56

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-3Ab\,x^2+4aB\,x^2+2aA)}{8a^2x^4}+\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)cA}{4a^{\frac{3}{2}}}-\frac{3\ln\left(\frac{2a+b\,x^2+2\sqrt{a}}{}\right.}{}$
default	$A\left(-\frac{\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}+\frac{3b\sqrt{c\,x^4+b\,x^2+a}}{8a^2x^2}-\frac{3b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{c\ln\left(\frac{2a+}{}\right.}{}\right.$
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}+\frac{3Ab\sqrt{c\,x^4+b\,x^2+a}}{8a^2x^2}-\frac{3\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)A\,b^2}{16a^{\frac{5}{2}}}+\frac{\ln\left(\frac{2a+b}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] A*(-1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^2-3/1
6*b^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/4*c/a^(
3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2))+B*(-1/2*(c*x^4+b*
x^2+a)^(1/2)/a/x^2+1/4*b/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1
/2))/x^2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.44, size = 255, normalized size = 2.06

$$\left[\frac{(4\,Bab-3\,Ab^2+4\,Aac)\sqrt{a}\,x^4\log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2+4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right)-4\,\sqrt{cx^4+bx^2+a}\,(2\,Aa^2+(4\,Ba^2-3\,Aab)x^2)}{32\,a^3x^4},-\frac{(4\,Bab-3\,Ab^2+4\,Aac)\sqrt{-a}\,x^4\arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right)+2\,\sqrt{cx^4+bx^2+a}\,(2\,Aa^2+(4\,Ba^2-3\,Aab)x^2)}{16\,a^3x^4}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/32*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(a)*x^4*log(-((b^2 + 4*a*c)*x^4 +
8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) -
 4*sqrt(c*x^4 + b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4),
-1/16*((4*B*a*b - 3*A*b^2 + 4*A*a*c)*sqrt(-a)*x^4*arctan(1/2*sqrt(c*x^4 + b
*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) + 2*sqrt(c*x^4
+ b*x^2 + a)*(2*A*a^2 + (4*B*a^2 - 3*A*a*b)*x^2))/(a^3*x^4)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^5\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**5/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**5*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(106) = 212.
time = 3.51, size = 339, normalized size = 2.73

$$-\frac{(4\,Bab-3\,Ab^2+4\,Aac)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{8\,\sqrt{-a}\,a^2}+\frac{4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3Bab-3\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3Ab^2+4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^3Aac+8\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2Ba^2\sqrt{c}-4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)Ba^2b+5\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)Aab^2+4\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)Aa^2c-8\,Ba^3\sqrt{c}+8\,Aa^2b\sqrt{c}}{8\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^5/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*(4*B*a*b - 3*A*b^2 + 4*A*a*c)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^
2 + a))/sqrt(-a))/(sqrt(-a)*a^2) + 1/8*(4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
 + a))^3*B*a*b - 3*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*b^2 + 4*(sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a*c + 8*(sqrt(c)*x^2 - sqrt(c*x^4 +
 b*x^2 + a))^2*B*a^2*sqrt(c) - 4*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*B*
a^2*b + 5*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a*b^2 + 4*(sqrt(c)*x^2
```

```
- sqrt(c*x^4 + b*x^2 + a))*A*a^2*c - 8*B*a^3*sqrt(c) + 8*A*a^2*b*sqrt(c))/(
((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^2*a^2)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^5\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((A + B*x^2)/(x^5*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.175 $\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=177

$$-\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}+\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4}-\frac{(15Ab^2-18abB-16aAc)\sqrt{a+bx^2+cx^4}}{48a^3x^2}+\frac{(5Ab^3-}{}$$

```
[Out] 1/32*(-12*A*a*b*c+5*A*b^3+8*B*a^2*c-6*B*a*b^2)*arctanh(1/2*(b*x^2+2*a)/a^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/a^(7/2)-1/6*A*(c*x^4+b*x^2+a)^(1/2)/a/x^6+1/24*(
5*A*b-6*B*a)*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-1/48*(-16*A*a*c+15*A*b^2-18*B*a*
b)*(c*x^4+b*x^2+a)^(1/2)/a^3/x^2
```

Rubi [A]

time = 0.16, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1265, 848, 820, 738, 212}

$$-\frac{\sqrt{a+bx^2+cx^4}\,(-16aAc-18abB+15Ab^2)}{48a^3x^2}+\frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4}+\frac{(8a^2Bc-12aAbc-6ab^2B+5Ab^3)\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{32a^{7/2}}-\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/6*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^6) + ((5*A*b - 6*a*B)*Sqrt[a + b*x^2
+ c*x^4])/(24*a^2*x^4) - ((15*A*b^2 - 18*a*b*B - 16*a*A*c)*Sqrt[a + b*x^2 +
 c*x^4])/(48*a^3*x^2) + ((5*A*b^3 - 6*a*b^2*B - 12*a*A*b*c + 8*a^2*B*c)*Arc
Tanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(32*a^(7/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
```

```
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{A+Bx^2}{x^7\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{A+Bx}{x^4\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(5Ab-6aB)+2Acx}{x^3\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{6a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} + \frac{\text{Subst}\left(\int \frac{\frac{1}{4}(15Ab^2-18abB}{x^2\sqrt{a}}\right.}{}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15Ab^2-18abB-16}{48a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15Ab^2-18abB-16}{48a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{6ax^6} + \frac{(5Ab-6aB)\sqrt{a+bx^2+cx^4}}{24a^2x^4} - \frac{(15Ab^2-18abB-16}{48a}$$

Mathematica [A]
time = 0.75, size = 186, normalized size = 1.05

$$\frac{\sqrt{a+bx^2+cx^4}\,(-8a^2A+10aAbx^2-12a^2Bx^2-15Ab^2x^4+18abBx^4+16aAcx^4)}{48a^3x^6}+\frac{(-5Ab^3-8a^2Bc)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{16a^{7/2}}-\frac{3b(bB+2Ac)\tanh^{-1}\left(\frac{-\sqrt{c}\,x^2+\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{8a^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^7*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (Sqrt[a + b*x^2 + c*x^4]*(-8*a^2*A + 10*a*A*b*x^2 - 12*a^2*B*x^2 - 15*A*b^2
*x^4 + 18*a*b*B*x^4 + 16*a*A*c*x^4))/(48*a^3*x^6) + ((-5*A*b^3 - 8*a^2*B*c)
*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/(16*a^(7/2)) - (
3*b*(b*B + 2*A*c)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a
]])/(8*a^(5/2))
```

Maple [A]
time = 0.09, size = 307, normalized size = 1.73

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-16Aac\,x^4+15A\,b^2x^4-18Bab\,x^4-10aAb\,x^2+12a^2B\,x^2+8a^2A)}{48a^3x^6}-\frac{3\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x}}{x^2}\right.}{8a^{\frac{5}{2}}}$
default	$B\left(-\frac{\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}+\frac{3b\sqrt{c\,x^4+b\,x^2+a}}{8a^2x^2}-\frac{3b^2\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{16a^{\frac{5}{2}}}+\frac{c\ln\left(\frac{2a+b\,x\right.}{}\right.$
elliptic	$-\frac{B\sqrt{c\,x^4+b\,x^2+a}}{4a\,x^4}+\frac{3Bb\sqrt{c\,x^4+b\,x^2+a}}{8a^2x^2}-\frac{3\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)B\,b^2}{16a^{\frac{5}{2}}}+\frac{\ln\left(\frac{2a+b\,x^2\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-1/4*(c*x^4+b*x^2+a)^(1/2)/a/x^4+3/8*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^2-3/1
6*b^2/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/4*c/a^(
3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2))+A*(-1/6*(c*x^4+b*
x^2+a)^(1/2)/a/x^6+5/24*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x^4-5/16*b^2/a^3/x^2*(c
*x^4+b*x^2+a)^(1/2)+5/32*b^3/a^(7/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a
)^(1/2))/x^2)-3/8*b/a^(5/2)*c*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2)
)/x^2)+1/3*c/a^2/x^2*(c*x^4+b*x^2+a)^(1/2))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 0.57, size = 339, normalized size = 1.92

$$\left[\frac{3\,(6\,Bab^2 - 5\,Ab^3 - 4\,(2\,Ba^2 - 3\,Aab)c)\sqrt{a}\,x^6 \log\left(-\frac{(b^2+4\,ac)x^4+8\,abx^2-4\,\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{a}+8\,a^2}{x^4}\right) + 4\,((18\,Ba^2b - 15\,Aab^2 + 16\,Aa^2c)x^4 - 8\,Aa^3 - 2\,(6\,Ba^3 - 5\,Aa^2b)x^2)\sqrt{cx^4+bx^2+a}}{192\,a^4x^6}, \frac{3\,(6\,Bab^2 - 5\,Ab^3 - 4\,(2\,Ba^2 - 3\,Aab)c)\sqrt{-a}\,x^6 \arctan\left(\frac{\sqrt{cx^4+bx^2+a}\,(bx^2+2\,a)\sqrt{-a}}{2\,(acx^4+abx^2+a^2)}\right) + 2\,((18\,Ba^2b - 15\,Aab^2 + 16\,Aa^2c)x^4 - 8\,Aa^3 - 2\,(6\,Ba^3 - 5\,Aa^2b)x^2)\sqrt{cx^4+bx^2+a}}{96\,a^4x^6}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(a)*x^6*log(-
((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(a) + 8*a^2)/x^4) + 4*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a
^3 - 2*(6*B*a^3 - 5*A*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6), 1/96*
(3*(6*B*a*b^2 - 5*A*b^3 - 4*(2*B*a^2 - 3*A*a*b)*c)*sqrt(-a)*x^6*arctan(1/2*
sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) +
 2*((18*B*a^2*b - 15*A*a*b^2 + 16*A*a^2*c)*x^4 - 8*A*a^3 - 2*(6*B*a^3 - 5*A
*a^2*b)*x^2)*sqrt(c*x^4 + b*x^2 + a))/(a^4*x^6)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^7\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**7/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**7*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 571 vs. 2(155) = 310.
time = 3.19, size = 571, normalized size = 3.23

$$\frac{(6\,Bab^2 - 5\,Ab^3 - 8\,Ba^2c + 12\,Aabc)\arctan\left(-\frac{\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{16\,\sqrt{-a}\,a^3} - \frac{18\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^5 Bab^2 - 15\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^5 Ab^3 - 24\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^5 Ba^2c + 36\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^5 Aabc - 48\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^3 Ba^2b^2 + 40\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^3 Aab^3 - 96\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^3 Aa^2bc - 48\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^2 Ba^3b\sqrt{c} - 96\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^2 Aa^3c^{\frac{3}{2}} + 30\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right) Ba^3b^2 - 33\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right) Aa^2b^3 + 24\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right) Ba^4c - 36\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right) Aa^3bc + 48\,Ba^4b\sqrt{c} - 48\,Aa^3b^2\sqrt{c} + 32\,Aa^4c^{\frac{3}{2}}}{48\left(\left(\sqrt{c}\,x^2 - \sqrt{cx^4+bx^2+a}\right)^2 - a\right)^3 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^7/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*(6*B*a*b^2 - 5*A*b^3 - 8*B*a^2*c + 12*A*a*b*c)*arctan(-(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^3) - 1/48*(18*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^5*B*a*b^2 - 15*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))^5*A*b^3 - 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*B*a^2*c + 36*(
sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^5*A*a*b*c - 48*(sqrt(c)*x^2 - sqrt(c
*x^4 + b*x^2 + a))^3*B*a^2*b^2 + 40*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))
^3*A*a*b^3 - 96*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^3*A*a^2*b*c - 48*(s
qrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2*B*a^3*b*sqrt(c) - 96*(sqrt(c)*x^2 -
 sqrt(c*x^4 + b*x^2 + a))^2*A*a^3*c^(3/2) + 30*(sqrt(c)*x^2 - sqrt(c*x^4 +
b*x^2 + a))*B*a^3*b^2 - 33*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*A*a^2*b^
3 + 24*(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*B*a^4*c - 36*(sqrt(c)*x^2 -
sqrt(c*x^4 + b*x^2 + a))*A*a^3*b*c + 48*B*a^4*b*sqrt(c) - 48*A*a^3*b^2*sqrt
(c) + 32*A*a^4*c^(3/2))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)^3*
a^3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{B\,x^2+A}{x^7\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^7*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.176 $\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=403

$$-\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2}+\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}+\frac{(8b^2B-10Abc-9aBc)\,x\sqrt{a+bx^2+cx^4}}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}}{}$$

```
[Out] -1/15*(-5*A*c+4*B*b)*x*(c*x^4+b*x^2+a)^(1/2)/c^2+1/5*B*x^3*(c*x^4+b*x^2+a)^
(1/2)/c+1/15*(-10*A*b*c-9*B*a*c+8*B*b^2)*x*(c*x^4+b*x^2+a)^(1/2)/c^(5/2)/(a
^(1/2)+x^2*c^(1/2))-1/15*a^(1/4)*(-10*A*b*c-9*B*a*c+8*B*b^2)*(cos(2*arctan(
c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin
(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2
*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(11/4)/(c*x^4+b
*x^2+a)^(1/2)+1/30*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2
*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*
(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(8*b^2*B-10*A*b*c-9*a*B*
c+(-5*A*c+4*B*b)*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)
^(1/2)/c^(11/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 403, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1293, 1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{a}\sqrt{c}(4bB-5Ac)-9aBc-10Abc+8b^2B\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{30c^{11/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}(-9aBc-10Abc+8b^2B)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{15c^{11/4}\sqrt{a+bx^2+cx^4}}+\frac{x\sqrt{a+bx^2+cx^4}(-9aBc-10Abc+8b^2B)}{15c^{5/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{x\sqrt{a+bx^2+cx^4}(4bB-5Ac)}{15c^2}+\frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] -1/15*((4*b*B - 5*A*c)*x*Sqrt[a + b*x^2 + c*x^4])/c^2 + (B*x^3*Sqrt[a + b*x
^2 + c*x^4])/(5*c) + ((8*b^2*B - 10*A*b*c - 9*a*B*c)*x*Sqrt[a + b*x^2 + c*x
^4])/(15*c^(5/2)*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*(8*b^2*B - 10*A*b*c -
9*a*B*c)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c
]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c])
)/4])/(15*c^(11/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(8*b^2*B - 10*A*b*c
- 9*a*B*c + Sqrt[a]*Sqrt[c]*(4*b*B - 5*A*c))*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(
a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x
)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(30*c^(11/4)*Sqrt[a + b*x^2 + c*x
^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\int \frac{x^2(3aB+(4bB-5Ac)x^2)}{\sqrt{a+bx^2+cx^4}}\,dx}{5c}$$

$$= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{\int \frac{a(4bB-5Ac)+(8b^2B-10}{\sqrt{a+bx^2+}}}{15c^2}$$

$$= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} - \frac{\left(\sqrt{a}\,(8b^2B-10Abc\right.}{}$$

$$= -\frac{(4bB-5Ac)x\sqrt{a+bx^2+cx^4}}{15c^2} + \frac{Bx^3\sqrt{a+bx^2+cx^4}}{5c} + \frac{(8b^2B-10Abc-9a}{15c^{5/2}\left(\sqrt{a}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.36, size = 532, normalized size = 1.32

$$\frac{4c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(-4bB+5Ac+3Bcx^2)(a+bx^2+cx^4)+i(8b^2B-10Abc-9aBc)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(-8b^3B+bc\left(17aB-10A\sqrt{b^2-4ac}\right)+2b^2\left(5Ac+4B\sqrt{b^2-4ac}\right)-ac\left(10Ac+9B\sqrt{b^2-4ac}\right)\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{60c^3\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (4*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(-4*b*B + 5*A*c + 3*B*c*x^2)*(a + b*
x^2 + c*x^4) + I*(8*b^2*B - 10*A*b*c - 9*a*B*c)*(-b + Sqrt[b^2 - 4*a*c])*Sq
rt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2
*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[
Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sq
rt[b^2 - 4*a*c])] - I*(-8*b^3*B + b*c*(17*a*B - 10*A*Sqrt[b^2 - 4*a*c]) + 2
*b^2*(5*A*c + 4*B*Sqrt[b^2 - 4*a*c]) - a*c*(10*A*c + 9*B*Sqrt[b^2 - 4*a*c])
)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b
 - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcS
inh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b
- Sqrt[b^2 - 4*a*c])])/(60*c^3*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x
^2 + c*x^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(391) = 782$.
time = 0.04, size = 815, normalized size = 2.02 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(1/5/c*x^3*(c*x^4+b*x^2+a)^(1/2)-4/15*b/c^2*x*(c*x^4+b*x^2+a)^(1/2)+1/15*
a*b/c^2*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/
2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(
1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*
(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-1/2*(-3/5*a/c+8/15*b^2/c^2)*a*2^(1/2)/((
-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(
4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^
2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-
4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+A*
(1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12*a/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)
^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b
^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/c*
a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(
b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^
(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2
)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)
^(1/2))))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(A + Bx^2)}{\sqrt{a + bx^2 + cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**4*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)*x^4/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((x^4*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.177 $\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=336

$$\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}-\frac{(2bB-3Ac)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{\sqrt[4]{a}\,(2bB-3Ac)\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}}{3c^{7/4}\sqrt{a+bx^2+}}$$

```
[Out] 1/3*B*x*(c*x^4+b*x^2+a)^(1/2)/c-1/3*(-3*A*c+2*B*b)*x*(c*x^4+b*x^2+a)^(1/2)/
c^(3/2)/(a^(1/2)+x^2*c^(1/2))+1/3*a^(1/4)*(-3*A*c+2*B*b)*(cos(2*arctan(c^(1
/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*a
rctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(
1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+
a)^(1/2)-1/6*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arcta
n(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a
^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(2*b*B-3*A*c+B*a^(1/2)*c^(1/2)
)*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(7/4)/(c*x^4+b*x^2+a)^(
1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1293, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{a}\,B\sqrt{c}-3Ac+2bB\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6c^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}(2bB-3Ac)E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3c^{7/4}\sqrt{a+bx^2+cx^4}}-\frac{x\sqrt{a+bx^2+cx^4}\,(2bB-3Ac)}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{Bx\sqrt{a+bx^2+cx^4}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(3*c) - ((2*b*B - 3*A*c)*x*Sqrt[a + b*x^2 + c
*x^4])/(3*c^(3/2)*(Sqrt[a] + Sqrt[c]*x^2)) + (a^(1/4)*(2*b*B - 3*A*c)*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*c^(7/
4)*Sqrt[a + b*x^2 + c*x^4]) - (a^(1/4)*(2*b*B + Sqrt[a]*B*Sqrt[c] - 3*A*c)*
(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]
*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*
c^(7/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^2(A+Bx^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{\int \frac{aB+(2bB-3Ac)x^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3c}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} + \frac{\left(\sqrt{a}\,(2bB-3Ac)\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{3c^{3/2}} - \frac{\left(\sqrt{a}\,(2bB+\right.}{}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{3c} - \frac{(2bB-3Ac)x\sqrt{a+bx^2+cx^4}}{3c^{3/2}\left(\sqrt{a}+\sqrt{c}\,x^2\right)} + \frac{\sqrt[4]{a}\,(2bB-3Ac)\left(\sqrt{a}\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.87, size = 479, normalized size = 1.43

$$\frac{4Bc\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x(a+bx^2+cx^4) - i(2bB-3Ac)\left(-b+\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right) + i\left(-2b^2B+3Abc+2aBc+2bB\sqrt{b^2-4ac}-3Ac\sqrt{b^2-4ac}\right)\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{12c^2\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

[In] Integrate[(x^2*(A + B*x^2))/Sqrt[a + b*x^2 + c*x^4],x]

[Out] (4*B*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*(a + b*x^2 + c*x^4) - I*(2*b*B - 3*A*c)*(-b + Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] + I*(-2*b^2*B + 3*A*b*c + 2*a*B*c + 2*b*B*Sqrt[b^2 - 4*a*c] - 3*A*c*Sqrt[b^2 - 4*a*c])*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])])/(12*c^2*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[a + b*x^2 + c*x^4])

Maple [A]
time = 0.04, size = 607, normalized size = 1.81

method	result
elliptic	$\frac{Bx\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{aB\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\ldots\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4\ldots}}$ $-\frac{(3Ac-2bB)a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\ldots\right.}{\ldots}$
risch	$\frac{Bx\sqrt{c\,x^4+b\,x^2+a}}{3c}+\frac{\ldots}{\ldots}$
default	$B\left(\frac{x\sqrt{c\,x^4+b\,x^2+a}}{3c}-\frac{a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\ldots\right.}{12c\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\ldots}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)

[Out] B*(1/3*x*(c*x^4+b*x^2+a)^(1/2)/c-1/12*a/c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/

```
2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c
+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+1/3*b/
c*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/
a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a
)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1
/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/
c)^(1/2)))-1/2*A*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*
a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a
*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-Elli
pticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*
c+b^2)^(1/2))/a/c)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(A + Bx^2)}{\sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**2*(A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((B*x^2 + A)*x^2/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(B\,x^2+A\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((x^2*(A + B*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.178 $\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=283

$$\frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{\sqrt[4]{a}\,B\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+$$

```
[Out] B*x*(c*x^4+b*x^2+a)^(1/2)/c^(1/2)/(a^(1/2)+x^2*c^(1/2))-a^(1/4)*B*(cos(2*ar
ctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*Elliptic
E(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2
)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x
^4+b*x^2+a)^(1/2)+1/2*a^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/co
s(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1
/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*(B+A*c^(1/2)/a^(1/2))
*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/c^(3/4)/(c*x^4+b*x^2+a)^(1
/2)
```

Rubi [A]
time = 0.05, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1211, 1117, 1209}

$$\frac{\sqrt[4]{a}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(\frac{A\sqrt{c}}{\sqrt{a}}+B\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2c^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{\sqrt[4]{a}\,B\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{c^{3/4}\sqrt{a+bx^2+cx^4}}+\frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (B*x*Sqrt[a + b*x^2 + c*x^4])/(Sqrt[c]*(Sqrt[a] + Sqrt[c]*x^2)) - (a^(1/4)*
B*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^
2]*EllipticE[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(
c^(3/4)*Sqrt[a + b*x^2 + c*x^4]) + (a^(1/4)*(B + (A*Sqrt[c])/Sqrt[a])*(Sqrt
[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*Elli
pticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(2*c^(3/
4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rubi steps

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx = \left(A+\frac{\sqrt{a}\,B}{\sqrt{c}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{\left(\sqrt{a}\,B\right)\int \frac{1-\frac{\sqrt{c}\,x^2}{\sqrt{a}}}{\sqrt{a+bx^2+cx^4}}\,dx}{\sqrt{c}}$$

$$= \frac{Bx\sqrt{a+bx^2+cx^4}}{\sqrt{c}\,\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{\sqrt[4]{a}\,B\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{c}}\right.\right.}{c^{3/4}\sqrt{a+bx^2+cx^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.14, size = 302, normalized size = 1.07

$$\frac{i\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(B\left(-b+\sqrt{b^2-4ac}\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)+\left(bB-2Ac-B\sqrt{b^2-4ac}\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{2\sqrt{2}\,c\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] ((I/2)*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(B*(-b + Sqrt[b^2 - 4*a*c])*Ellipti
cE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*
a*c])/(b - Sqrt[b^2 - 4*a*c])] + (b*B - 2*A*c - B*Sqrt[b^2 - 4*a*c])*Ellipt
icF[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4
*a*c])/(b - Sqrt[b^2 - 4*a*c])]))/(Sqrt[2]*c*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])
]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.02, size = 362, normalized size = 1.28

method	result
default	$-\frac{B a \sqrt{2}\, \sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\, \sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\, \left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4ac}}{a}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$
elliptic	$-\frac{B a \sqrt{2}\, \sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\, \sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\, \left(\mathrm{EllipticF}\left(\frac{x\sqrt{2}\, \sqrt{\frac{-b+\sqrt{-4ac}}{a}}}{2}\right.\right.}{2\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*B*a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1
/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^
(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/
2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x
*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2
))/a/c)^(1/2)))+1/4*A*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-
4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*
x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2
),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2),x)
```

```
[Out] int((A + B*x^2)/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.179 $\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=312

$$-\frac{A\sqrt{a+bx^2+cx^4}}{ax}+\frac{A\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\frac{\sqrt[4]{c}}{\sqrt[4]{}}\right.\right.}{a^{3/4}\sqrt{a+bx^2+cx^4}}$$

```
[Out] -A*(c*x^4+b*x^2+a)^(1/2)/a/x+A*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a/(a^(1/2)+x
^2*c^(1/2))-A*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arct
an(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/
a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2
*c^(1/2))^2)^(1/2)/a^(3/4)/(c*x^4+b*x^2+a)^(1/2)+1/2*(cos(2*arctan(c^(1/4)*
x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticF(sin(2*arcta
n(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(B*a^(1/2)+A*c^(1/2)
)*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(
3/4)/c^(1/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1295, 1211, 1117, 1209}

$$\frac{\left(\sqrt{a}+\sqrt{c}\,x^2\right)\left(\sqrt{a}\,B+A\sqrt{c}\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{2a^{3/4}\sqrt[4]{c}\sqrt{a+bx^2+cx^4}}-\frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\,E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{a^{3/4}\sqrt{a+bx^2+cx^4}}-\frac{A\sqrt{a+bx^2+cx^4}}{ax}+\frac{A\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -((A*Sqrt[a + b*x^2 + c*x^4])/(a*x)) + (A*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4]
)/(a*(Sqrt[a] + Sqrt[c]*x^2)) - (A*c^(1/4)*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a
+ b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcTan[(c^(1/4)*x)/
a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(a^(3/4)*Sqrt[a + b*x^2 + c*x^4]) +
 ((Sqrt[a]*B + A*Sqrt[c])*(Sqrt[a] + Sqrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/
(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/
(Sqrt[a]*Sqrt[c]))/4])/(2*a^(3/4)*c^(1/4)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{ax} - \frac{\int \frac{-aB-Acx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{ax} + \left(B+\frac{A\sqrt{c}}{\sqrt{a}}\right)\int \frac{1}{\sqrt{a+bx^2+cx^4}}\,dx - \frac{\left(A\sqrt{c}\right)\int \frac{1}{\sqrt{a+\cdots}}}{\sqrt{\cdots}}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{ax} + \frac{A\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{a\left(\sqrt{a}+\sqrt{c}\,x^2\right)} - \frac{A\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+\cdots}{\left(\sqrt{a}\cdots\right.}}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.66, size = 448, normalized size = 1.44

$$\frac{-4A\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\left(a+bx^2+cx^4\right)+iA\left(-b+\sqrt{b^2-4ac}\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)-i\left(2aB+A\left(-b+\sqrt{b^2-4ac}\right)\right)x\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{\frac{2b-2\sqrt{b^2-4ac}+4cx^2}{b-\sqrt{b^2-4ac}}}\,F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)\middle|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)}{4a\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^2*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-4*A*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*(a + b*x^2 + c*x^4) + I*A*(-b + Sqrt[
b^2 - 4*a*c])*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c])]*Sqrt[(2*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*El
lipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2
 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])] - I*(2*a*B + A*(-b + Sqrt[b^2 - 4*a*c])
)*x*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[(2
*b - 2*Sqrt[b^2 - 4*a*c] + 4*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*EllipticF[I*Ar
cSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(
b - Sqrt[b^2 - 4*a*c])])/(4*a*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x*Sqrt[a + b*
x^2 + c*x^4])
```

Maple [A]
time = 0.06, size = 386, normalized size = 1.24

method	result
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{a x}+\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\ldots\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x\ldots}}$
default	$\frac{B\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\mathrm{EllipticF}\left(\frac{x\sqrt{2}\,\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}}{2}\ldots\right.}{4\sqrt{\frac{-b+\sqrt{-4ac+b^2}}{a}}\,\sqrt{c\,x^4+b\,x^2+a}}$
risch	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{a x}+\frac{-\frac{A c a\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\left(\mathrm{Ellipti}\ldots\right.}{\ldots}}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*B*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2)
)/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/
```

```
2)*EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b
+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+A*(-(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2
)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/
2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*
c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/
2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2)))
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + Bx^2}{x^2 \sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**2/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((A + B*x**2)/(x**2*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^2/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^2), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{x^2\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^2*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.180 $\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=376

$$-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}+\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{(2Ab-3aB)\sqrt{c}\,x\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}+\frac{(2Ab-3aB)\sqrt[4]{c}}{}$$

```
[Out] -1/3*A*(c*x^4+b*x^2+a)^(1/2)/a/x^3+1/3*(2*A*b-3*B*a)*(c*x^4+b*x^2+a)^(1/2)/
a^2/x-1/3*(2*A*b-3*B*a)*x*c^(1/2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(a^(1/2)+x^2*c^
(1/2))+1/3*(2*A*b-3*B*a)*c^(1/4)*(cos(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)
/cos(2*arctan(c^(1/4)*x/a^(1/4)))*EllipticE(sin(2*arctan(c^(1/4)*x/a^(1/4))
),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(a^(1/2)+x^2*c^(1/2))*((c*x^4+b*x^2+a)/(
a^(1/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)-1/6*c^(1/4)*(co
s(2*arctan(c^(1/4)*x/a^(1/4)))^2)^(1/2)/cos(2*arctan(c^(1/4)*x/a^(1/4)))*El
lipticF(sin(2*arctan(c^(1/4)*x/a^(1/4))),1/2*(2-b/a^(1/2)/c^(1/2))^(1/2))*(
a^(1/2)+x^2*c^(1/2))*(2*A*b-3*a*B+A*a^(1/2)*c^(1/2))*((c*x^4+b*x^2+a)/(a^(1
/2)+x^2*c^(1/2))^2)^(1/2)/a^(7/4)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1295, 1211, 1117, 1209}

$$-\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}\left(\sqrt{a}\,A\sqrt{c}-3aB+2Ab\right)F\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{6a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{\sqrt[4]{c}\left(\sqrt{a}+\sqrt{c}\,x^2\right)(2Ab-3aB)\sqrt{\frac{a+bx^2+cx^4}{\left(\sqrt{a}+\sqrt{c}\,x^2\right)^2}}E\left(2\text{ArcTan}\left(\frac{\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)|\frac{1}{4}\left(2-\frac{b}{\sqrt{a}\sqrt{c}}\right)\right)}{3a^{7/4}\sqrt{a+bx^2+cx^4}}+\frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x}-\frac{\sqrt{c}\,x(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2\left(\sqrt{a}+\sqrt{c}\,x^2\right)}-\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/3*(A*Sqrt[a + b*x^2 + c*x^4])/(a*x^3) + ((2*A*b - 3*a*B)*Sqrt[a + b*x^2
+ c*x^4])/(3*a^2*x) - ((2*A*b - 3*a*B)*Sqrt[c]*x*Sqrt[a + b*x^2 + c*x^4])/(
3*a^2*(Sqrt[a] + Sqrt[c]*x^2)) + ((2*A*b - 3*a*B)*c^(1/4)*(Sqrt[a] + Sqrt[c
]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticE[2*ArcT
an[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(3*a^(7/4)*Sqrt[a +
b*x^2 + c*x^4]) - ((2*A*b - 3*a*B + Sqrt[a]*A*Sqrt[c])*c^(1/4)*(Sqrt[a] + S
qrt[c]*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(Sqrt[a] + Sqrt[c]*x^2)^2]*EllipticF[2
*ArcTan[(c^(1/4)*x)/a^(1/4)], (2 - b/(Sqrt[a]*Sqrt[c]))/4])/(6*a^(7/4)*Sqrt
[a + b*x^2 + c*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
```

```
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx = -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} - \frac{\int \frac{2Ab-3aB+Acx^2}{x^2\sqrt{a+bx^2+cx^4}}\,dx}{3a}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\int \frac{-aAc-(2Ab-3aB)cx^2}{\sqrt{a+bx^2+cx^4}}\,dx}{3a^2}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} + \frac{\left((2Ab-3aB)\sqrt{c}\right)\int}{3a^{3/}}$$

$$= -\frac{A\sqrt{a+bx^2+cx^4}}{3ax^3} + \frac{(2Ab-3aB)\sqrt{a+bx^2+cx^4}}{3a^2x} - \frac{(2Ab-3aB)\sqrt{c}\,x\sqrt{a}}{3a^2\left(\sqrt{a}+\sqrt{}\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.42, size = 373, normalized size = 0.99

$$\frac{-\frac{4(a+bx^2+cx^4)(-2Abx^2+a(A+3Bx^2))}{x^3}+\frac{i\sqrt{2}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\left(-\left((2Ab-3aB)\left(-b+\sqrt{b^2-4ac}\right)E\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)+\left(3aB\left(b-\sqrt{b^2-4ac}\right)+2A\left(-b^2+ac+b\sqrt{b^2-4ac}\right)\right)F\left(i\sinh^{-1}\left(\sqrt{2}\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}\,x\right)|\frac{b+\sqrt{b^2-4ac}}{b-\sqrt{b^2-4ac}}\right)\right)}{\sqrt{\frac{c}{b+\sqrt{b^2-4ac}}}}}{12a^2\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*x^2)/(x^4*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((-4*(a + b*x^2 + c*x^4)*(-2*A*b*x^2 + a*(A + 3*B*x^2)))/x^3 + (I*Sqrt[2]*S
qrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*(-((2*A*b - 3*a*B)*(-b + Sqrt[b^2 - 4*a*c])
*EllipticE[I*ArcSinh[Sqrt[2]*Sqrt[c/(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[
b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c])]) + (3*a*B*(b - Sqrt[b^2 - 4*a*c]) +
2*A*(-b^2 + a*c + b*Sqrt[b^2 - 4*a*c]))*EllipticF[I*ArcSinh[Sqrt[2]*Sqrt[c/
(b + Sqrt[b^2 - 4*a*c])]*x], (b + Sqrt[b^2 - 4*a*c])/(b - Sqrt[b^2 - 4*a*c]
)]))/Sqrt[c/(b + Sqrt[b^2 - 4*a*c])])/(12*a^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [A]
time = 0.05, size = 656, normalized size = 1.74

method	result
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}\,(-2Ab\,x^2+3aB\,x^2+aA)}{3a^2x^3}-\frac{c\left(-\frac{(2Ab-3aB)a\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\left(b\cdots\right.}{}}}{}\right.}{}$
elliptic	$-\frac{A\sqrt{c\,x^4+b\,x^2+a}}{3a\,x^3}+\frac{(2Ab-3aB)\sqrt{c\,x^4+b\,x^2+a}}{3a^2x}-\frac{cA\sqrt{2}\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\sqrt{4+\frac{2\cdots}{}}}{}$

default	$B\left(-\frac{\sqrt{c\,x^4+b\,x^2+a}}{ax}-\frac{c\sqrt{2}\,\sqrt{4-\frac{2\left(-b+\sqrt{-4ac+b^2}\right)x^2}{a}}\,\sqrt{4+\frac{2\left(b+\sqrt{-4ac+b^2}\right)x^2}{a}}\left(\mathrm{Ellipt}\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] B*(-(c*x^4+b*x^2+a)^(1/2)/a/x-1/2*c*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/
2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*
x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^
(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/
a/c)^(1/2))-EllipticE(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(
-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))))+A*(-1/3*(c*x^4+b*x^2+a)^(1/2)/a
/x^3+2/3*b*(c*x^4+b*x^2+a)^(1/2)/a^2/x-1/12*c/a*2^(1/2)/((-b+(-4*a*c+b^2)^(
1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^
2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2)*EllipticF(1/2*x*2^(1/2)*((-b+(
-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))+
1/3*c*b/a*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2)*(4-2*(-b+(-4*a*c+b^2)^(
1/2))/a*x^2)^(1/2)*(4+2*(b+(-4*a*c+b^2)^(1/2))/a*x^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))*(EllipticF(1/2*x*2^(1/2)*((-b+(-4*a*c+b^2)^(1
/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/2))/a/c)^(1/2))-EllipticE(1/2*
x*2^(1/2)*((-b+(-4*a*c+b^2)^(1/2))/a)^(1/2),1/2*(-4+2*b*(b+(-4*a*c+b^2)^(1/
2))/a/c)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+Bx^2}{x^4\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x**2+A)/x**4/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((A + B*x**2)/(x**4*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((B*x^2+A)/x^4/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*x^2 + A)/(sqrt(c*x^4 + b*x^2 + a)*x^4), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{B\,x^2+A}{x^4\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int((A + B*x^2)/(x^4*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.181 $\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}} dx$

Optimal. Leaf size=98

$$-\frac{89}{48}x^4\sqrt{3+5x^2+x^4}+\frac{3}{8}x^6\sqrt{3+5x^2+x^4}-\frac{1}{384}(24243-3802x^2)\sqrt{3+5x^2+x^4}+\frac{32801}{256}\tanh^{-1}\left(\frac{\cdot}{2\sqrt{3}\cdots}\right.$$

```
[Out] 32801/256*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-89/48*x^4*(x^4+5*x^2+3
)^(1/2)+3/8*x^6*(x^4+5*x^2+3)^(1/2)-1/384*(-3802*x^2+24243)*(x^4+5*x^2+3)^(
1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 846, 793, 635, 212}

$$-\frac{89}{48}\sqrt{x^4+5x^2+3}\,x^4-\frac{1}{384}(24243-3802x^2)\sqrt{x^4+5x^2+3}+\frac{32801}{256}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)+\frac{3}{8}\sqrt{x^4+5x^2+3}\,x^6$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-89*x^4*Sqrt[3 + 5*x^2 + x^4])/48 + (3*x^6*Sqrt[3 + 5*x^2 + x^4])/8 - ((24
243 - 3802*x^2)*Sqrt[3 + 5*x^2 + x^4])/384 + (32801*ArcTanh[(5 + 2*x^2)/(2*
Sqrt[3 + 5*x^2 + x^4])])/256
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(2+3x)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= \frac{3}{8}x^6\sqrt{3+5x^2+x^4} + \frac{1}{8}\text{Subst}\left(\int \frac{\left(-27-\frac{89x}{2}\right)x^2}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4} + \frac{3}{8}x^6\sqrt{3+5x^2+x^4} + \frac{1}{24}\text{Subst}\left(\int \frac{x\left(267+\frac{1901x}{4}\right)}{\sqrt{3+5x+x^2}}\,dx, x,\right.\\ &= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4} + \frac{3}{8}x^6\sqrt{3+5x^2+x^4} - \frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+}\\ &= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4} + \frac{3}{8}x^6\sqrt{3+5x^2+x^4} - \frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+}\\ &= -\frac{89}{48}x^4\sqrt{3+5x^2+x^4} + \frac{3}{8}x^6\sqrt{3+5x^2+x^4} - \frac{1}{384}\left(24243-3802x^2\right)\sqrt{3+5x^2+}\end{aligned}$$

Mathematica [A]
time = 0.11, size = 64, normalized size = 0.65

$$\frac{1}{384}\sqrt{3+5x^2+x^4}\left(-24243+3802x^2-712x^4+144x^6\right)-\frac{32801}{256}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

[Out] (Sqrt[3 + 5*x^2 + x^4]*(-24243 + 3802*x^2 - 712*x^4 + 144*x^6))/384 - (32801*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/256

Maple [A]
time = 0.10, size = 87, normalized size = 0.89

method	result
risch	$\frac{(144x^6-712x^4+3802x^2-24243)\sqrt{x^4+5x^2+3}}{384}+\frac{32801\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{256}$
trager	$\left(\frac{3}{8}x^6-\frac{89}{48}x^4+\frac{1901}{192}x^2-\frac{8081}{128}\right)\sqrt{x^4+5x^2+3}+\frac{32801\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{256}$
default	$\frac{3x^6\sqrt{x^4+5x^2+3}}{8}-\frac{89x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1901x^2\sqrt{x^4+5x^2+3}}{192}-\frac{8081\sqrt{x^4+5x^2+3}}{128}+\frac{32801}{}$
elliptic	$\frac{3x^6\sqrt{x^4+5x^2+3}}{8}-\frac{89x^4\sqrt{x^4+5x^2+3}}{48}+\frac{1901x^2\sqrt{x^4+5x^2+3}}{192}-\frac{8081\sqrt{x^4+5x^2+3}}{128}+\frac{32801}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)

[Out] 3/8*x^6*(x^4+5*x^2+3)^(1/2)-89/48*x^4*(x^4+5*x^2+3)^(1/2)+1901/192*x^2*(x^4+5*x^2+3)^(1/2)-8081/128*(x^4+5*x^2+3)^(1/2)+32801/256*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))

Maxima [A]
time = 0.30, size = 90, normalized size = 0.92

$$\frac{3}{8}\sqrt{x^4+5x^2+3}\,x^6-\frac{89}{48}\sqrt{x^4+5x^2+3}\,x^4+\frac{1901}{192}\sqrt{x^4+5x^2+3}\,x^2-\frac{8081}{128}\sqrt{x^4+5x^2+3}+\frac{32801}{256}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")

[Out] 3/8*sqrt(x^4 + 5*x^2 + 3)*x^6 - 89/48*sqrt(x^4 + 5*x^2 + 3)*x^4 + 1901/192*sqrt(x^4 + 5*x^2 + 3)*x^2 - 8081/128*sqrt(x^4 + 5*x^2 + 3) + 32801/256*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)

Fricas [A]
time = 0.36, size = 56, normalized size = 0.57

$$\frac{1}{384}\left(144x^6-712x^4+3802x^2-24243\right)\sqrt{x^4+5x^2+3}-\frac{32801}{256}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")

[Out] 1/384*(144*x^6 - 712*x^4 + 3802*x^2 - 24243)*sqrt(x^4 + 5*x^2 + 3) - 32801/256*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 \cdot (3x^2 + 2)}{\sqrt{x^4 + 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**7*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A]
time = 3.97, size = 60, normalized size = 0.61

$$\frac{1}{384}\,\sqrt{x^4+5\,x^2+3}\,\left(2\,\left(4\,\left(18\,x^2-89\right)x^2+1901\right)x^2-24243\right)-\frac{32801}{256}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}\,+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/384*sqrt(x^4 + 5*x^2 + 3)*(2*(4*(18*x^2 - 89)*x^2 + 1901)*x^2 - 24243) -
32801/256*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^7\,\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^7*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.182 $\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4}-\frac{1083}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -1083/32*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+1/2*x^4*(x^4+5*x^2+3)^(
1/2)+3/16*(-14*x^2+89)*(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 846, 793, 635, 212}

$$\frac{1}{2}\sqrt{x^4+5x^2+3}\,x^4+\frac{3}{16}\left(89-14x^2\right)\sqrt{x^4+5x^2+3}-\frac{1083}{32}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (x^4*Sqrt[3 + 5*x^2 + x^4])/2 + (3*(89 - 14*x^2)*Sqrt[3 + 5*x^2 + x^4])/16
- (1083*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/32
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 846

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[g*(d + e*x)^m*((a + b*x + c*x^2)^(p +
1)/(c*(m + 2*p + 2))), x] + Dist[1/(c*(m + 2*p + 2)), Int[(d + e*x)^(m - 1)
*(a + b*x + c*x^2)^p*Simp[m*(c*d*f - a*e*g) + d*(2*c*f - b*g)*(p + 1) + (m*
(c*e*f + c*d*g - b*e*g) + e*(p + 1)*(2*c*f - b*g))*x, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && GtQ[m, 0] && NeQ[m + 2*p + 2, 0] && (IntegerQ[m] || IntegerQ[p]
 || IntegersQ[2*m, 2*p]) &&  !(IGtQ[m, 0] && EqQ[f, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(2+3x)}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{1}{6}\text{Subst}\left(\int \frac{\left(-18-\frac{63x}{2}\right)x}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4}-\frac{1083}{32}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+\right.\\ &= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4}-\frac{1083}{16}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,\right.\\ &= \frac{1}{2}x^4\sqrt{3+5x^2+x^4}+\frac{3}{16}\left(89-14x^2\right)\sqrt{3+5x^2+x^4}-\frac{1083}{32}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2}\right.\end{aligned}$$

Mathematica [A]
time = 0.10, size = 59, normalized size = 0.77

$$\frac{1}{16}\sqrt{3+5x^2+x^4}\left(267-42x^2+8x^4\right)+\frac{1083}{32}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(267 - 42*x^2 + 8*x^4))/16 + (1083*Log[-5 - 2*x^2 +
2*Sqrt[3 + 5*x^2 + x^4]])/32
```

Maple [A]
time = 0.10, size = 70, normalized size = 0.91

method	result
risch	$\frac{(8x^4-42x^2+267)\sqrt{x^4+5x^2+3}}{16}-\frac{1083\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}$
trager	$\left(\frac{1}{2}x^4-\frac{21}{8}x^2+\frac{267}{16}\right)\sqrt{x^4+5x^2+3}-\frac{1083\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{32}$
default	$\frac{x^4\sqrt{x^4+5x^2+3}}{2}-\frac{21x^2\sqrt{x^4+5x^2+3}}{8}+\frac{267\sqrt{x^4+5x^2+3}}{16}-\frac{1083\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}$
elliptic	$\frac{x^4\sqrt{x^4+5x^2+3}}{2}-\frac{21x^2\sqrt{x^4+5x^2+3}}{8}+\frac{267\sqrt{x^4+5x^2+3}}{16}-\frac{1083\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{32}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^4*(x^4+5*x^2+3)^(1/2)-21/8*x^2*(x^4+5*x^2+3)^(1/2)+267/16*(x^4+5*x^2+
3)^(1/2)-1083/32*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.95

$$\frac{1}{2}\sqrt{x^4+5x^2+3}\,x^4-\frac{21}{8}\sqrt{x^4+5x^2+3}\,x^2+\frac{267}{16}\sqrt{x^4+5x^2+3}-\frac{1083}{32}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(x^4 + 5*x^2 + 3)*x^4 - 21/8*sqrt(x^4 + 5*x^2 + 3)*x^2 + 267/16*sqr
t(x^4 + 5*x^2 + 3) - 1083/32*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.36, size = 51, normalized size = 0.66

$$\frac{1}{16}\left(8x^4-42x^2+267\right)\sqrt{x^4+5x^2+3}+\frac{1083}{32}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(8*x^4 - 42*x^2 + 267)*sqrt(x^4 + 5*x^2 + 3) + 1083/32*log(-2*x^2 + 2*
sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^5\cdot(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A]
time = 4.15, size = 53, normalized size = 0.69

$$\frac{1}{16}\sqrt{x^4+5\,x^2+3}\,\left(2\,\left(4\,x^2-21\right)x^2+267\right)+\frac{1083}{32}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/16*sqrt(x^4 + 5*x^2 + 3)*(2*(4*x^2 - 21)*x^2 + 267) + 1083/32*log(2*x^2 -
 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.183 $\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=56

$$-\frac{1}{8}(37-6x^2)\sqrt{3+5x^2+x^4}+\frac{149}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 149/16*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/8*(-6*x^2+37)*(x^4+5*x^
2+3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {1265, 793, 635, 212}

$$\frac{149}{16}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{1}{8}(37-6x^2)\sqrt{x^4+5x^2+3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] -1/8*((37 - 6*x^2)*Sqrt[3 + 5*x^2 + x^4]) + (149*ArcTanh[(5 + 2*x^2)/(2*Sqr
t[3 + 5*x^2 + x^4])])/16
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 793

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(b*e*g*(p + 2) - c*(e*f + d*g)*(2*p + 3) -
 2*c*e*g*(p + 1)*x))*((a + b*x + c*x^2)^(p + 1)/(2*c^2*(p + 1)*(2*p + 3))),
 x] + Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c*(2*c*d*f - b*(e*f + d*g))*(2*p
+ 3))/(2*c^2*(2*p + 3)), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c,
 d, e, f, g, p}, x] && NeQ[b^2 - 4*a*c, 0] &&  !LeQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{1}{8}(37-6x^2)\sqrt{3+5x^2+x^4} + \frac{149}{16}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{1}{8}(37-6x^2)\sqrt{3+5x^2+x^4} + \frac{149}{8}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{1}{8}(37-6x^2)\sqrt{3+5x^2+x^4} + \frac{149}{16}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}
$$

Mathematica [A]
time = 0.09, size = 54, normalized size = 0.96

$$\frac{1}{8}(-37+6x^2)\sqrt{3+5x^2+x^4} - \frac{149}{16}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
[Out] ((-37 + 6*x^2)*Sqrt[3 + 5*x^2 + x^4])/8 - (149*Log[-5 - 2*x^2 + 2*Sqrt[3 +
5*x^2 + x^4]])/16
```

Maple [A]
time = 0.10, size = 53, normalized size = 0.95

method	result	size
risch	$\frac{(6x^2-37)\sqrt{x^4+5x^2+3}}{8}+\frac{149\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}$	43
trager	$\left(\frac{3x^2}{4}-\frac{37}{8}\right)\sqrt{x^4+5x^2+3}+\frac{149\ln\left(2x^2+5+2\sqrt{x^4+5x^2+3}\right)}{16}$	46
default	$\frac{3x^2\sqrt{x^4+5x^2+3}}{4}-\frac{37\sqrt{x^4+5x^2+3}}{8}+\frac{149\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}$	53

elliptic	$\frac{3x^2\sqrt{x^4+5x^2+3}}{4}-\frac{37\sqrt{x^4+5x^2+3}}{8}+\frac{149\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{16}$	53

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*x^2*(x^4+5*x^2+3)^(1/2)-37/8*(x^4+5*x^2+3)^(1/2)+149/16*ln(x^2+5/2+(x^4
+5*x^2+3)^(1/2))
```

Maxima [A]
time = 0.27, size = 56, normalized size = 1.00

$$\frac{3}{4}\sqrt{x^4+5x^2+3}\,x^2-\frac{37}{8}\sqrt{x^4+5x^2+3}+\frac{149}{16}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 3/4*sqrt(x^4 + 5*x^2 + 3)*x^2 - 37/8*sqrt(x^4 + 5*x^2 + 3) + 149/16*log(2*x
^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.37, size = 46, normalized size = 0.82

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2-37\right)-\frac{149}{16}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(-2*x^2 + 2*sqrt(x^4 + 5
*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\cdot(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A]
time = 4.03, size = 46, normalized size = 0.82

$$\frac{1}{8}\sqrt{x^4+5x^2+3}\left(6x^2-37\right)-\frac{149}{16}\log\left(2x^2-2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^4 + 5*x^2 + 3)*(6*x^2 - 37) - 149/16*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{x^3\,\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.184 $\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=49

$$\frac{3}{2}\sqrt{3+5x^2+x^4}-\frac{11}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -11/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+3/2*(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.02, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {1261, 654, 635, 212}

$$\frac{3}{2}\sqrt{x^4+5x^2+3}-\frac{11}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 - (11*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x
^4])])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= \frac{3}{2}\sqrt{3+5x^2+x^4} - \frac{11}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.08, size = 47, normalized size = 0.96

$$\frac{3}{2}\sqrt{3+5x^2+x^4} + \frac{11}{4}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*Sqrt[3 + 5*x^2 + x^4])/2 + (11*Log[-5 - 2*x^2 + 2*Sqrt[3 + 5*x^2 + x^4]]
)/4
```

Maple [A]
time = 0.10, size = 36, normalized size = 0.73

method	result	size
default	$\frac{3\sqrt{x^4+5x^2+3}}{2} - \frac{11\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}$	36
risch	$\frac{3\sqrt{x^4+5x^2+3}}{2} - \frac{11\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}$	36
elliptic	$\frac{3\sqrt{x^4+5x^2+3}}{2} - \frac{11\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}$	36
trager	$\frac{3\sqrt{x^4+5x^2+3}}{2} + \frac{11\ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{4}$	40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*(x^4+5*x^2+3)^(1/2)-11/4*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))
```

Maxima [A]
time = 0.28, size = 39, normalized size = 0.80

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3}-\frac{11}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) - 11/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.41, size = 39, normalized size = 0.80

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3}+\frac{11}{4}\,\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x(3x^2+2)}{\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
[Out] Integral(x*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [A]
time = 3.80, size = 39, normalized size = 0.80

$$\frac{3}{2}\sqrt{x^4+5\,x^2+3}+\frac{11}{4}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] 3/2*sqrt(x^4 + 5*x^2 + 3) + 11/4*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B]
time = 0.52, size = 35, normalized size = 0.71

$$\frac{3\,\sqrt{x^4+5\,x^2+3}}{2}-\frac{11\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] (3*(5*x^2 + x^4 + 3)^(1/2))/2 - (11*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2
))/4
```

3.185 $\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=69

$$\frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}$$

```
[Out] 3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^
(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 857, 635, 212, 738}

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/2 - ArcTanh[(6 + 5*x^2)/
(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/Sqrt[3]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= \frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx,x,x^2\right)+\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\left(2\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\right)+3\text{Subst}\left(\int \frac{1}{4-x^2}\,dx,x,\frac{5}{\sqrt{3+}}\right.\\
&= \frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)-\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{\sqrt{3}}
\end{aligned}$$

Mathematica [A]
time = 0.10, size = 63, normalized size = 0.91

$$\frac{2\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{3}{2}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (2*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 + x^4])/Sqrt[3]])/Sqrt[3] - (3*Log[-5 - 2*
x^2 + 2*Sqrt[3 + 5*x^2 + x^4]])/2
```

Maple [A]
time = 0.25, size = 52, normalized size = 0.75

method	result
default	$\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
elliptic	$\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{3}$
trager	$-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\operatorname{RootOf}\left(\_Z^2-3\right)+6\sqrt{x^4+5x^2+3}}{x^2}\right)}{3}-\frac{3\ln\left(2x^2-2\sqrt{x^4+5x^2+\right.}}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))-1/3*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+
5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.54, size = 58, normalized size = 0.84

$$-\frac{1}{3}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{3}{2}\log\left(2x^2+2\sqrt{x^4+5x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 3/2*log
(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.37, size = 75, normalized size = 1.09

$$\frac{1}{3}\sqrt{3}\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-\frac{3}{2}\log\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(3)*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(
5*sqrt(3) - 6) + 30)/x^2) - 3/2*log(-2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) - 5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{3x^2+2}{x\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [A]
time = 3.84, size = 78, normalized size = 1.13

$$\frac{1}{3}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5\,x^2+3}}\right)-\frac{3}{2}\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 1/3*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [B]
time = 1.01, size = 56, normalized size = 0.81

$$\frac{3\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{2}-\frac{\sqrt{3}\,\left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}+5\,x^2+6\right)\right)}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] (3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (3^(1/2)*(log(1/x^2) + log
(2*3^(1/2)*(5*x^2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/3
```

3.186 $\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=62

$$-\frac{\sqrt{3+5x^2+x^4}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

```
[Out] -2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5*
x^2+3)^(1/2)/x^2
```

Rubi [A]
time = 0.03, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1265, 820, 738, 212}

$$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/3*Sqrt[3 + 5*x^2 + x^4]/x^2 - (2*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 +
 5*x^2 + x^4])])/(3*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
```

```
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^3\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{3x^2}+\frac{2}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{3x^2}-\frac{4}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{3x^2}-\frac{2\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 58, normalized size = 0.94

$$\frac{1}{9}\left(-\frac{3\sqrt{3+5x^2+x^4}}{x^2}+4\sqrt{3}\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] ((-3*Sqrt[3 + 5*x^2 + x^4])/x^2 + 4*Sqrt[3]*ArcTanh[(x^2 - Sqrt[3 + 5*x^2 +
 x^4])/Sqrt[3]])/9
```

Maple [A]
time = 0.19, size = 49, normalized size = 0.79

method	result	siz

default	$-\frac{2\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$	4
risch	$-\frac{2\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$	4
elliptic	$-\frac{2\,\mathrm{arctanh}\left(\frac{\left(5x^2+6\right)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$	4
trager	$-\frac{\sqrt{x^4+5x^2+3}}{3x^2}+\frac{2\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\mathrm{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{9}$	6

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/3*(x^4+5*
x^2+3)^(1/2)/x^2
```

Maxima [A]
time = 0.49, size = 51, normalized size = 0.82

$$-\frac{2}{9}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{3x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/3*sqr
t(x^4 + 5*x^2 + 3)/x^2
```

Fricas [A]
time = 0.40, size = 78, normalized size = 1.26

$$\frac{2\sqrt{3}x^2\log\left(\frac{25x^2-2\sqrt{3}(5x^2+6)-2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}-6\right)+30}{x^2}\right)-3x^2-3\sqrt{x^4+5x^2+3}}{9x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/9*(2*sqrt(3)*x^2*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*x^2
 + 3)*(5*sqrt(3) - 6) + 30)/x^2) - 3*x^2 - 3*sqrt(x^4 + 5*x^2 + 3))/x^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^3\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(48) = 96.
time = 3.53, size = 101, normalized size = 1.63

$$\frac{2}{9}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{5x^2-5\sqrt{x^4+5x^2+3}+6}{3\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - sq
rt(x^4 + 5*x^2 + 3))) + 1/3*(5*x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - s
qrt(x^4 + 5*x^2 + 3))^2 - 3)
```

Mupad [B]
time = 0.66, size = 83, normalized size = 1.34

$$\frac{5\sqrt{3}\,\mathrm{atanh}\left(\frac{\sqrt{3}\,(5x^2+6)}{6\sqrt{x^4+5x^2+3}}\right)}{18}-\frac{\sqrt{x^4+5x^2+3}}{3x^2}-\frac{\sqrt{3}\left(\ln\left(\frac{1}{x^2}\right)+\ln\left(2\sqrt{3}\sqrt{x^4+5x^2+3}+5x^2+6\right)\right)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] (5*3^(1/2)*atanh((3^(1/2)*(5*x^2 + 6))/(6*(5*x^2 + x^4 + 3)^(1/2))))/18 - (
5*x^2 + x^4 + 3)^(1/2)/(3*x^2) - (3^(1/2)*(log(1/x^2) + log(2*3^(1/2)*(5*x^
2 + x^4 + 3)^(1/2) + 5*x^2 + 6)))/2
```

3.187 $\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 1/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/6*(x^4+5*x
^2+3)^(1/2)/x^4-1/12*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A]
time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {1265, 848, 820, 738, 212}

$$-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/6*Sqrt[3 + 5*x^2 + x^4]/x^4 - Sqrt[3 + 5*x^2 + x^4]/(12*x^2) + (Sqrt[3]*
ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])])/8
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
```

```
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{x^5\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^3\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{1}{12}\text{Subst}\left(\int \frac{-3+2x}{x^2\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}-\frac{3}{8}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}+\frac{3}{4}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\ &= -\frac{\sqrt{3+5x^2+x^4}}{6x^4}-\frac{\sqrt{3+5x^2+x^4}}{12x^2}+\frac{1}{8}\sqrt{3}\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)\end{aligned}$$

Mathematica [A]
time = 0.16, size = 63, normalized size = 0.76

$$-\frac{(2+x^2)\sqrt{3+5x^2+x^4}}{12x^4}-\frac{1}{4}\sqrt{3}\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^5*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/12*((2 + x^2)*Sqrt[3 + 5*x^2 + x^4])/x^4 - (Sqrt[3]*ArcTanh[(x^2 - Sqrt[
3 + 5*x^2 + x^4])/Sqrt[3]])/4
```

Maple [A]
time = 0.21, size = 66, normalized size = 0.80

method	result
risch	$-\frac{x^6+7x^4+13x^2+6}{12x^4\sqrt{x^4+5x^2+3}}+\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{8}$
default	$\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{8}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}-\frac{\sqrt{x^4+5x^2+3}}{12x^2}$
elliptic	$\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{8}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}-\frac{\sqrt{x^4+5x^2+3}}{12x^2}$
trager	$-\frac{(x^2+2)\sqrt{x^4+5x^2+3}}{12x^4}-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\operatorname{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/6*(x^4+5*x
^2+3)^(1/2)/x^4-1/12*(x^4+5*x^2+3)^(1/2)/x^2
```

Maxima [A]
time = 0.49, size = 68, normalized size = 0.82

$$\frac{1}{8}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{\sqrt{x^4+5x^2+3}}{12x^2}-\frac{\sqrt{x^4+5x^2+3}}{6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) - 1/12*sqr
t(x^4 + 5*x^2 + 3)/x^2 - 1/6*sqrt(x^4 + 5*x^2 + 3)/x^4
```

Fricas [A]
time = 0.35, size = 83, normalized size = 1.00

$$\frac{3\sqrt{3}x^4\log\left(\frac{25x^2+2\sqrt{3}(5x^2+6)+2\sqrt{x^4+5x^2+3}\left(5\sqrt{3}+6\right)+30}{x^2}\right)-2x^4-2\sqrt{x^4+5x^2+3}(x^2+2)}{24x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/24*(3*sqrt(3)*x^4*log((25*x^2 + 2*sqrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^
2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) - 2*x^4 - 2*sqrt(x^4 + 5*x^2 + 3)*(x^2 +
2))/x^4
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^5\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**5/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**5*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(65) = 130.
time = 3.72, size = 145, normalized size = 1.75

$$-\frac{1}{8}\sqrt{3}\log\left(\frac{x^2+\sqrt{3}-\sqrt{x^4+5x^2+3}}{x^2-\sqrt{3}-\sqrt{x^4+5x^2+3}}\right)+\frac{9\left(x^2-\sqrt{x^4+5x^2+3}\right)^3+36\left(x^2-\sqrt{x^4+5x^2+3}\right)^2+47x^2-47\sqrt{x^4+5x^2+3}+12}{12\left(\left(x^2-\sqrt{x^4+5x^2+3}\right)^2-3\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^5/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 1/12*(9*(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 + 36*(x^2
- sqrt(x^4 + 5*x^2 + 3))^2 + 47*x^2 - 47*sqrt(x^4 + 5*x^2 + 3) + 12)/((x^2
- sqrt(x^4 + 5*x^2 + 3))^2 - 3)^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{3\,x^2+2}{x^5\,\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^5*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.188 $\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=104

$$-\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{13\sqrt{3+5x^2+x^4}}{108x^2}-\frac{61\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{216\sqrt{3}}$$

```
[Out] -61/648*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/9*(x^4
+5*x^2+3)^(1/2)/x^6-1/54*(x^4+5*x^2+3)^(1/2)/x^4+13/108*(x^4+5*x^2+3)^(1/2)
/x^2
```

Rubi [A]

time = 0.05, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 848, 820, 738, 212}

$$\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{61\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{216\sqrt{3}}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] -1/9*Sqrt[3 + 5*x^2 + x^4]/x^6 - Sqrt[3 + 5*x^2 + x^4]/(54*x^4) + (13*Sqrt[
3 + 5*x^2 + x^4])/(108*x^2) - (61*ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5
*x^2 + x^4])])/(216*Sqrt[3])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 848

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(e*f - d*g)*(d + e*x)^(m + 1)*((a + b*
x + c*x^2)^(p + 1)/((m + 1)*(c*d^2 - b*d*e + a*e^2))), x] + Dist[1/((m + 1)
*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p*Simp[(
c*d*f - f*b*e + a*e*g)*(m + 1) + b*(d*g - e*f)*(p + 1) - c*(e*f - d*g)*(m +
 2*p + 3)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] ||
 IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{2+3x^2}{x^7\sqrt{3+5x^2+x^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^4\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{1}{18}\text{Subst}\left(\int \frac{-2+4x}{x^3\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{1}{108}\text{Subst}\left(\int \frac{-39-2x}{x^2\sqrt{3+5x+x^2}}\,dx,x,x^2\right.\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{13\sqrt{3+5x^2+x^4}}{108x^2}+\frac{61}{216}\text{Subst}\left(\int \frac{}{x\sqrt{3}}\right.\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{13\sqrt{3+5x^2+x^4}}{108x^2}-\frac{61}{108}\text{Subst}\left(\int \frac{1}{12-}\right.\\
&= -\frac{\sqrt{3+5x^2+x^4}}{9x^6}-\frac{\sqrt{3+5x^2+x^4}}{54x^4}+\frac{13\sqrt{3+5x^2+x^4}}{108x^2}-\frac{61\tanh^{-1}\left(\frac{}{2\sqrt{3}\sqrt{}}\right.}{216\sqrt{}}
\end{aligned}$$

Mathematica [A]

time = 0.20, size = 75, normalized size = 0.72

$$\frac{\sqrt{3+5x^2+x^4}\,(-12-2x^2+13x^4)}{108x^6}+\frac{61\tanh^{-1}\left(\frac{x^2}{\sqrt{3}}-\frac{\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{108\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^7*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (Sqrt[3 + 5*x^2 + x^4]*(-12 - 2*x^2 + 13*x^4))/(108*x^6) + (61*ArcTanh[x^2/
Sqrt[3] - Sqrt[3 + 5*x^2 + x^4]/Sqrt[3]])/(108*Sqrt[3])
```

Maple [A]
time = 0.19, size = 83, normalized size = 0.80

method	result
risch	$\frac{13x^8+63x^6+17x^4-66x^2-36}{108x^6\sqrt{x^4+5x^2+3}}-\frac{61\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{648}$
trager	$\frac{(13x^4-2x^2-12)\sqrt{x^4+5x^2+3}}{108x^6}+\frac{61\,\mathrm{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\,\mathrm{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\,\mathrm{RootOf}\left(\_\right.}{x^2}\right.}{648}$
default	$-\frac{61\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{648}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}$
elliptic	$-\frac{61\,\mathrm{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{648}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -61/648*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)-1/9*(x^4
+5*x^2+3)^(1/2)/x^6-1/54*(x^4+5*x^2+3)^(1/2)/x^4+13/108*(x^4+5*x^2+3)^(1/2)
/x^2
```

Maxima [A]
time = 0.49, size = 85, normalized size = 0.82

$$-\frac{61}{648}\sqrt{3}\log\left(\frac{2\sqrt{3}\sqrt{x^4+5x^2+3}}{x^2}+\frac{6}{x^2}+5\right)+\frac{13\sqrt{x^4+5x^2+3}}{108x^2}-\frac{\sqrt{x^4+5x^2+3}}{54x^4}-\frac{\sqrt{x^4+5x^2+3}}{9x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] -61/648*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^2 + 3)/x^2 + 6/x^2 + 5) + 13/1
08*sqrt(x^4 + 5*x^2 + 3)/x^2 - 1/54*sqrt(x^4 + 5*x^2 + 3)/x^4 - 1/9*sqrt(x^
4 + 5*x^2 + 3)/x^6
```

Fricas [A]
time = 0.35, size = 90, normalized size = 0.87

$$\frac{61\sqrt{3}\,x^6\log\left(\frac{25\,x^2-2\,\sqrt{3}\,(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\,\left(5\,\sqrt{3}\,-6\right)+30}{x^2}\right)+78\,x^6+6\,(13\,x^4-2\,x^2-12)\sqrt{x^4+5\,x^2+3}}{648\,x^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/648*(61*sqrt(3)*x^6*log((25*x^2 - 2*sqrt(3)*(5*x^2 + 6) - 2*sqrt(x^4 + 5*
x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 78*x^6 + 6*(13*x^4 - 2*x^2 - 12)*sqrt
(x^4 + 5*x^2 + 3))/x^6
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{3x^2+2}{x^7\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**7/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**7*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(82) = 164.
time = 3.63, size = 167, normalized size = 1.61

$$\frac{61}{648}\,\sqrt{3}\,\log\left(\frac{x^2+\sqrt{3}\,-\sqrt{x^4+5\,x^2+3}}{x^2-\sqrt{3}\,-\sqrt{x^4+5\,x^2+3}}\right)-\frac{61\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^5-920\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^3-2052\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-1449\,x^2+1449\,\sqrt{x^4+5\,x^2+3}\,-108}{108\left(\left(x^2-\sqrt{x^4+5\,x^2+3}\right)^2-3\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^7/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] 61/648*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) -
 sqrt(x^4 + 5*x^2 + 3))) - 1/108*(61*(x^2 - sqrt(x^4 + 5*x^2 + 3))^5 - 920*
(x^2 - sqrt(x^4 + 5*x^2 + 3))^3 - 2052*(x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 14
49*x^2 + 1449*sqrt(x^4 + 5*x^2 + 3) - 108)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2
 - 3)^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{3\,x^2+2}{x^7\,\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)),x)
[Out] int((3*x^2 + 2)/(x^7*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.189 $\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=298

$$\frac{419x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}}-\frac{10}{3}x\sqrt{3+5x^2+x^4}+\frac{3}{5}x^3\sqrt{3+5x^2+x^4}-\frac{419\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right.}{6+\left(5+\sqrt{13}\right.}}}{}$$

[Out] 419/30*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-10/3*x*(x^4+5*x^2+3)^(1/2)+3/5*x^3*(x^4+5*x^2+3)^(1/2)+5/3*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-419/180*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)

Rubi [A]
time = 0.12, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1293, 1203, 1113, 1149}

$$\frac{5\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{419\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{30\sqrt{x^4+5x^2+3}}-\frac{10}{3}\sqrt{x^4+5x^2+3}\,x+\frac{419\left(2x^2+\sqrt{13}+5\right)x}{30\sqrt{x^4+5x^2+3}}+\frac{3}{5}\sqrt{x^4+5x^2+3}\,x^3$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (419*x*(5 + Sqrt[13] + 2*x^2))/(30*Sqrt[3 + 5*x^2 + x^4]) - (10*x*Sqrt[3 +
5*x^2 + x^4])/3 + (3*x^3*Sqrt[3 + 5*x^2 + x^4])/5 - (419*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(30*Sqrt[3 + 5*x^2 + x^4]) + (5*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5
- Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipti
cF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2
+ x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
```

```
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}
\int \frac{x^4(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx &= \frac{3}{5}x^3\sqrt{3+5x^2+x^4} - \frac{1}{5}\int \frac{x^2(27+50x^2)}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{10}{3}x\sqrt{3+5x^2+x^4} + \frac{3}{5}x^3\sqrt{3+5x^2+x^4} + \frac{1}{15}\int \frac{150+419x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{10}{3}x\sqrt{3+5x^2+x^4} + \frac{3}{5}x^3\sqrt{3+5x^2+x^4} + 10\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + \frac{419}{15}\int \\
&= \frac{419x\left(5+\sqrt{13}+2x^2\right)}{30\sqrt{3+5x^2+x^4}} - \frac{10}{3}x\sqrt{3+5x^2+x^4} + \frac{3}{5}x^3\sqrt{3+5x^2+x^4} - \frac{419\sqrt{\frac{1}{6}\Big(}}{}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 229, normalized size = 0.77

$$\frac{4x(-150-223x^2-5x^4+9x^6)+419i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-1795+419\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{60\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (4*x*(-150 - 223*x^2 - 5*x^4 + 9*x^6) + (419*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqr
t[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellip
ticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2
]*(-1795 + 419*Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt
[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
+ (5*Sqrt[13])/6])/(60*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.06, size = 226, normalized size = 0.76

method	result
risch	$\frac{x(9x^2-50)\sqrt{x^4+5x^2+3}}{15} - \frac{5028\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-}}{}\right.\right.}{5\sqrt{-30+6\sqrt{13}}\sqrt{x}}$
default	$\frac{3x^3\sqrt{x^4+5x^2+3}}{5} - \frac{10x\sqrt{x^4+5x^2+3}}{3} + \frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4-}}$
elliptic	$\frac{3x^3\sqrt{x^4+5x^2+3}}{5} - \frac{10x\sqrt{x^4+5x^2+3}}{3} + \frac{60\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4-}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/5*x^3*(x^4+5*x^2+3)^(1/2)-10/3*x*(x^4+5*x^2+3)^(1/2)+60/(-30+6*13^(1/2))^
(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(
x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*3
9^(1/2))-5028/5/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1
-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF
(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-3
0+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \cdot (3x^2 + 2)}{\sqrt{x^4 + 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
[Out] Integral(x**4*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)*x^4/sqrt(x^4 + 5*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
[Out] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.190 $\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=270

$$-\frac{4x\left(5+\sqrt{13}+2x^2\right)}{\sqrt{3+5x^2+x^4}}+x\sqrt{3+5x^2+x^4}+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)}{\sqrt{3+5x^2+}}$$

```
[Out] -4*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+x*(x^4+5*x^2+3)^(1/2)-1/2*(1/(3
6+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(3
0+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1
/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/
(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+2/3*(1/(36+x^2*(30+6*13^(1/
2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2
)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^
(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(
1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1293, 1203, 1113, 1149}

$$-\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}+\sqrt{x^4+5x^2+3}\,x-\frac{4\left(2x^2+\sqrt{13}+5\right)x}{\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (-4*x*(5 + Sqrt[13] + 2*x^2))/Sqrt[3 + 5*x^2 + x^4] + x*Sqrt[3 + 5*x^2 + x^
4] + (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 +
Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13
])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4] - (Sqrt[3/(2*(5 + Sq
rt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

```
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(2+3x^2)}{\sqrt{3+5x^2+x^4}}\,dx &= x\sqrt{3+5x^2+x^4}-\frac{1}{3}\int \frac{9+24x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= x\sqrt{3+5x^2+x^4}-3\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx-8\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{4x\left(5+\sqrt{13}+2x^2\right)}{\sqrt{3+5x^2+x^4}}+x\sqrt{3+5x^2+x^4}+\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right.}{6+\left(5+\sqrt{13}\right.}}}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.16, size = 222, normalized size = 0.82

$$\frac{2x(3+5x^2+x^4)-4i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+i\sqrt{2}\left(-17+4\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{2\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(2 + 3*x^2))/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (2*x*(3 + 5*x^2 + x^4) - (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13]
- 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sq
rt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-17 + 4*Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(2
*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.05, size = 208, normalized size = 0.77

method	result
default	$x\sqrt{x^4+5x^2+3}-\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{1}}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
risch	$x\sqrt{x^4+5x^2+3}-\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{1}}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$
elliptic	$x\sqrt{x^4+5x^2+3}-\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{1}}}{6}\right.}{\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] x*(x^4+5*x^2+3)^(1/2)-18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)
^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*
x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+288/(-30+6*13^(1/2))^(1/
2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4
+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^
(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*
39^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cdot (3x^2 + 2)}{\sqrt{x^4 + 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/sqrt(x^4 + 5*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(3\,x^2+2\right)}{\sqrt{x^4+5\,x^2+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.191 $\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}} \, dx$

Optimal. Leaf size=257

$$\frac{3x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}}-\frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\tan^{-1}\left(\sqrt{\frac{1}{6}\left(5+\right.\right.\right.}{2\sqrt{3+5x^2+x^4}}$$

```
[Out] 3/2*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)+1/3*(1/(36+x^2*(30+6*13^(1/2))
))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(
36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/
2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))
^(1/2)/(x^4+5*x^2+3)^(1/2)-1/4*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(
30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1
/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2
))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1
/2)
```

Rubi [A]
time = 0.06, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1203, 1113, 1149}

$$\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{2\sqrt{x^4+5x^2+3}}+\frac{3x\left(2x^2+\sqrt{13}+5\right)}{2\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] (3*x*(5 + Sqrt[13] + 2*x^2))/(2*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[(3*(5 + Sqrt
[13]))/2]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 +
 Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[1
3])/6])/(2*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (
5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellip
ticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/Sqrt[3 + 5*x^
2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

```
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{2+3x^2}{\sqrt{3+5x^2+x^4}}\,dx = 2\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + 3\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{3x\left(5+\sqrt{13}+2x^2\right)}{2\sqrt{3+5x^2+x^4}} - \frac{\sqrt{\frac{3}{2}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right.\right.}{2\sqrt{3+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.07, size = 159, normalized size = 0.62

$$\frac{i\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\left(3\left(-5+\sqrt{13}\right)E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+\left(11-3\sqrt{13}\right)F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)\right)}{2\sqrt{2}\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/Sqrt[3 + 5*x^2 + x^4],x]
```

```
[Out] ((I/2)*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*
x^2]*(3*(-5 + Sqrt[13])*EllipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6
 + (5*Sqrt[13])/6] + (11 - 3*Sqrt[13])*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt
[13])]*x], 19/6 + (5*Sqrt[13])/6]))/(Sqrt[2]*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.04, size = 194, normalized size = 0.75

method	result
default	$-\frac{108\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right.\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}\,\left(5+\sqrt{13}\right)}$
elliptic	$-\frac{108\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right.\right.}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}\,\left(5+\sqrt{13}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -108/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*
13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30
+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2
))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))+12/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*
13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*
EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 + 2}{\sqrt{x^4 + 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/sqrt(x**4 + 5*x**2 + 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/sqrt(x^4 + 5*x^2 + 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2 + 2}{\sqrt{x^4 + 5\,x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(1/2),x)
```

```
[Out] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(1/2), x)
```

3.192 $\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=278

$$\frac{x\left(5+\sqrt{13}+2x^2\right)}{3\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{3x}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right)E\left(\right.}{3\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/3*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-2/3*(x^4+5*x^2+3)^(1/2)/x+1/2*
(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF
(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2
))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/
2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-1/18*(1/(36+x^2*(30+6*
13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2)
)^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*
(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2
))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 278, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1295, 1203, 1113, 1149}

$$\frac{\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{\sqrt{x^4+5x^2+3}}-\frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{3\sqrt{x^4+5x^2+3}}+\frac{x\left(2x^2+\sqrt{13}+5\right)}{3\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (x*(5 + Sqrt[13] + 2*x^2))/(3*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 +
x^4])/(3*x) - (Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5
 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt
[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3*Sqrt[3 + 5*x^2 + x^4]) + (Sqrt[3/(2*
(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/Sqrt[3 + 5*x^2 + x^4]
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

```
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx &= -\frac{2\sqrt{3+5x^2+x^4}}{3x} - \frac{1}{3}\int \frac{-9-2x^2}{\sqrt{3+5x^2+x^4}}\,dx \\
&= -\frac{2\sqrt{3+5x^2+x^4}}{3x} + \frac{2}{3}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx + 3\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx \\
&= \frac{x\left(5+\sqrt{13}+2x^2\right)}{3\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{3x} - \frac{\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right.}{6+\left(5+\sqrt{13}\right.}}}{}
\end{aligned}
$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 224, normalized size = 0.81

$$
\frac{-4(3+5x^2+x^4)+i\sqrt{2}\left(-5+\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(4+\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{6x\sqrt{3+5x^2+x^4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(3 + 5*x^2 + x^4) + I*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2
*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[
2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(4 + Sqrt[13])*x*S
qrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ell
ipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(6*x*Sq
rt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.05, size = 211, normalized size = 0.76

method	result
default	$\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-$
risch	$\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-$
elliptic	$\frac{18\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\ \mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{13}}}{6},\frac{5\sqrt{3}}{6}+\frac{\sqrt{39}}{6}\right)}{\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 18/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13
^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/
2),5/6*3^(1/2)+1/6*39^(1/2))-2/3*(x^4+5*x^2+3)^(1/2)/x-24/(-30+6*13^(1/2))^
(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(
x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6
*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1
/6*39^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^2\sqrt{x^4+5x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(1/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**2*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^2\,\sqrt{x^4+5\,x^2+3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(1/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.193 $\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$

Optimal. Leaf size=302

$$\frac{7x\left(5+\sqrt{13}+2x^2\right)}{54\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{9x^3}-\frac{7\sqrt{3+5x^2+x^4}}{27x}-\frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\right.}{}$$

```
[Out] 7/54*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+3)^(1/2)-2/9*(x^4+5*x^2+3)^(1/2)/x^3-7
/27*(x^4+5*x^2+3)^(1/2)/x-1/27*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(
30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1
/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13
^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+
3)^(1/2)-7/324*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 302, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1295, 1203, 1113, 1149}

$$-\frac{\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{9\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{54\sqrt{x^4+5x^2+3}}+\frac{7x\left(2x^2+\sqrt{13}+5\right)}{54\sqrt{x^4+5x^2+3}}-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{2\sqrt{x^4+5x^2+3}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (7*x*(5 + Sqrt[13] + 2*x^2))/(54*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2
 + x^4])/(9*x^3) - (7*Sqrt[3 + 5*x^2 + x^4])/(27*x) - (7*Sqrt[(5 + Sqrt[13]
)/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt
[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6
])/(54*Sqrt[3 + 5*x^2 + x^4]) - (Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 -
Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF
[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(9*Sqrt[3 + 5*x^2
 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
```

```
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{2+3x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx = -\frac{2\sqrt{3+5x^2+x^4}}{9x^3} - \frac{1}{9}\int \frac{-7+2x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3} - \frac{7\sqrt{3+5x^2+x^4}}{27x} + \frac{1}{27}\int \frac{-6+7x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{2\sqrt{3+5x^2+x^4}}{9x^3} - \frac{7\sqrt{3+5x^2+x^4}}{27x} - \frac{2}{9}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + \frac{7}{27}\int \frac{}{\sqrt{3}}$$

$$= \frac{7x\left(5+\sqrt{13}+2x^2\right)}{54\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{9x^3} - \frac{7\sqrt{3+5x^2+x^4}}{27x} - \frac{7\sqrt{\frac{1}{6}\left(5+\sqrt{1}\right.}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.18, size = 237, normalized size = 0.78

$$\frac{-4(18+51x^2+41x^4+7x^6)+7i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-47+7\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{108x^3\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^4*Sqrt[3 + 5*x^2 + x^4]),x]
```

```
[Out] (-4*(18 + 51*x^2 + 41*x^4 + 7*x^6) + (7*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3*Sqrt
[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Ellipt
icE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]
*(-47 + 7*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[
5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 +
 (5*Sqrt[13])/6])/(108*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.06, size = 228, normalized size = 0.75

method	result
default	$-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{28\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.\right.}{3\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x}}$
risch	$-\frac{7x^6+41x^4+51x^2+18}{27x^3\sqrt{x^4+5x^2+3}}-\frac{28\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-30+}}{6}\right.\right.}{3\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+}}$
elliptic	$-\frac{7\sqrt{x^4+5x^2+3}}{27x}-\frac{28\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\text{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.\right.}{3\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -7/27*(x^4+5*x^2+3)^(1/2)/x-28/3/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/
2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^
(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-El
lipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-2/9*(x^4+5*
x^2+3)^(1/2)/x^3-4/3/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/
2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-
30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2 + 2}{x^4\sqrt{x^4 + 5x^2 + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(1/2),x)
[Out] Integral((3*x**2 + 2)/(x**4*sqrt(x**4 + 5*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(1/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/(sqrt(x^4 + 5*x^2 + 3)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^4\,\sqrt{x^4+5\,x^2+3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(1/2)),x)
[Out] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(1/2)), x)
```

3.194 $\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}}+\frac{133}{26}\sqrt{3+5x^2+x^4}-\frac{41}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] -41/4*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))-1/13*x^2*(47*x^2+33)/(x^4+
5*x^2+3)^(1/2)+133/26*(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 832, 654, 635, 212}

$$-\frac{(47x^2+33)\,x^2}{13\sqrt{x^4+5x^2+3}}+\frac{133}{26}\sqrt{x^4+5x^2+3}-\frac{41}{4}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/13*(x^2*(33 + 47*x^2))/Sqrt[3 + 5*x^2 + x^4] + (133*Sqrt[3 + 5*x^2 + x^4
])/26 - (41*ArcTanh[(5 + 2*x^2)/(2*Sqrt[3 + 5*x^2 + x^4])])/4
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 654

```
Int[((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Symbol
] :> Simp[e*((a + b*x + c*x^2)^(p + 1)/(2*c*(p + 1))), x] + Dist[(2*c*d - b
*e)/(2*c), Int[(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x]
&& NeQ[2*c*d - b*e, 0] && NeQ[p, -1]
```

Rule 832

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(d + e*x)^(m - 1))*(a + b*x + c*x^2)
^(p + 1)*((2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (2*c^2*d*f + b^2*e*g - c
*(b*e*f + b*d*g + 2*a*e*g))*x)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[1/(c*(
p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^(m - 2)*(a + b*x + c*x^2)^(p + 1)*Simp
[2*c^2*d^2*f*(2*p + 3) + b*e*g*(a*e*(m - 1) + b*d*(p + 2)) - c*(2*a*e*(e*f*
(m - 1) + d*g*m) + b*d*(d*g*(2*p + 3) - e*f*(m - 2*p - 4))) + e*(b^2*e*g*(m
 + p + 1) + 2*c^2*d*f*(m + 2*p + 2) - c*(2*a*e*g*m + b*(e*f + d*g)*(m + 2*p
 + 2)))*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1] && GtQ[m, 1] && ((EqQ[m
, 2] && EqQ[p, -3] && RationalQ[a, b, c, d, e, f, g]) ||  !ILtQ[m + 2*p + 3
, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2(2+3x)}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)\\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{1}{13}\text{Subst}\left(\int \frac{33+\frac{133x}{2}}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x,\right.\\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{2}\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+}{\sqrt{3+5}}\right.\\
&= -\frac{x^2(33+47x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{133}{26}\sqrt{3+5x^2+x^4} - \frac{41}{4}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)
\end{aligned}$$

Mathematica [A]
time = 0.18, size = 59, normalized size = 0.77

$$\frac{399+599x^2+39x^4}{26\sqrt{3+5x^2+x^4}} + \frac{41}{4}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (399 + 599*x^2 + 39*x^4)/(26*Sqrt[3 + 5*x^2 + x^4]) + (41*Log[-5 - 2*x^2 +
2*Sqrt[3 + 5*x^2 + x^4]])/4
```

Maple [A]
time = 0.11, size = 91, normalized size = 1.18

method	result
risch	$\frac{39x^4+599x^2+399}{26\sqrt{x^4+5x^2+3}}-\frac{41\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{4}$
trager	$\frac{39x^4+599x^2+399}{26\sqrt{x^4+5x^2+3}}+\frac{41\ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{4}$
default	$\frac{3x^4}{2\sqrt{x^4+5x^2+3}}+\frac{41x^2}{4\sqrt{x^4+5x^2+3}}-\frac{133}{8\sqrt{x^4+5x^2+3}}+\frac{\frac{665x^2}{52}+\frac{3325}{104}}{\sqrt{x^4+5x^2+3}}-\frac{41\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\right.}{4}$
elliptic	$\frac{3x^4}{2\sqrt{x^4+5x^2+3}}+\frac{41x^2}{4\sqrt{x^4+5x^2+3}}-\frac{133}{8\sqrt{x^4+5x^2+3}}+\frac{\frac{665x^2}{52}+\frac{3325}{104}}{\sqrt{x^4+5x^2+3}}-\frac{41\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/2*x^4/(x^4+5*x^2+3)^(1/2)+41/4*x^2/(x^4+5*x^2+3)^(1/2)-133/8/(x^4+5*x^2+3
)^(1/2)+665/104*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)-41/4*ln(x^2+5/2+(x^4+5*x^2+3)
^(1/2))
```

Maxima [A]
time = 0.27, size = 73, normalized size = 0.95

$$\frac{3\,x^4}{2\,\sqrt{x^4+5\,x^2+3}}+\frac{599\,x^2}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{399}{26\,\sqrt{x^4+5\,x^2+3}}-\frac{41}{4}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/2*x^4/sqrt(x^4 + 5*x^2 + 3) + 599/26*x^2/sqrt(x^4 + 5*x^2 + 3) + 399/26/s
qrt(x^4 + 5*x^2 + 3) - 41/4*log(2*x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.38, size = 86, normalized size = 1.12

$$\frac{1811\,x^4+9055\,x^2+1066\,(x^4+5\,x^2+3)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+4\,(39\,x^4+599\,x^2+399)\sqrt{x^4+5\,x^2+3}+5433}{104\,(x^4+5\,x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/104*(1811*x^4 + 9055*x^2 + 1066*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4
 + 5*x^2 + 3) - 5) + 4*(39*x^4 + 599*x^2 + 399)*sqrt(x^4 + 5*x^2 + 3) + 543
3)/(x^4 + 5*x^2 + 3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5 \cdot (3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**5*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [**A**]
time = 4.43, size = 52, normalized size = 0.68

$$\frac{(39\,x^2+599)x^2+399}{26\,\sqrt{x^4+5\,x^2+3}}+\frac{41}{4}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/26*((39*x^2 + 599)*x^2 + 399)/sqrt(x^4 + 5*x^2 + 3) + 41/4*log(2*x^2 - 2*
sqrt(x^4 + 5*x^2 + 3) + 5)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5\,(3\,x^2+2)}{(x^4+5\,x^2+3)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int((x^5*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.195 $\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=56

$$\frac{-33-47x^2}{13\sqrt{3+5x^2+x^4}}+\frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)$$

```
[Out] 3/2*arctanh(1/2*(2*x^2+5)/(x^4+5*x^2+3)^(1/2))+1/13*(-47*x^2-33)/(x^4+5*x^2
+3)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1265, 791, 635, 212}

$$\frac{3}{2}\tanh^{-1}\left(\frac{2x^2+5}{2\sqrt{x^4+5x^2+3}}\right)-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] -1/13*(33 + 47*x^2)/Sqrt[3 + 5*x^2 + x^4] + (3*ArcTanh[(5 + 2*x^2)/(2*Sqrt[
3 + 5*x^2 + x^4])])/2
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 791

```
Int[((d_.) + (e_.)*(x_))*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(
x_)^2)^(p_), x_Symbol] :> Simp[(-(2*a*c*(e*f + d*g) - b*(c*d*f + a*e*g) - (
b^2*e*g - b*c*(e*f + d*g) + 2*c*(c*d*f - a*e*g))*x))*((a + b*x + c*x^2)^(p
+ 1)/(c*(p + 1)*(b^2 - 4*a*c))), x] - Dist[(b^2*e*g*(p + 2) - 2*a*c*e*g + c
*(2*c*d*f - b*(e*f + d*g))*(2*p + 3))/(c*(p + 1)*(b^2 - 4*a*c)), Int[(a + b
*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[p, -1]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^3(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x(2+3x)}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)\\ &= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}}+\frac{3}{2}\text{Subst}\left(\int \frac{1}{\sqrt{3+5x+x^2}}\,dx, x, x^2\right)\\ &= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}}+3\text{Subst}\left(\int \frac{1}{4-x^2}\,dx, x, \frac{5+2x^2}{\sqrt{3+5x^2+x^4}}\right)\\ &= -\frac{33+47x^2}{13\sqrt{3+5x^2+x^4}}+\frac{3}{2}\tanh^{-1}\left(\frac{5+2x^2}{2\sqrt{3+5x^2+x^4}}\right)\end{aligned}$$

Mathematica [A]
time = 0.14, size = 54, normalized size = 0.96

$$\frac{-33-47x^2}{13\sqrt{3+5x^2+x^4}}-\frac{3}{2}\log\left(-5-2x^2+2\sqrt{3+5x^2+x^4}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
[Out] (-33 - 47*x^2)/(13*Sqrt[3 + 5*x^2 + x^4]) - (3*Log[-5 - 2*x^2 + 2*Sqrt[3 +
5*x^2 + x^4]])/2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(46) = 92$.
time = 0.11, size = 95, normalized size = 1.70

method	result
risch	$-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}$
trager	$-\frac{47x^2+33}{13\sqrt{x^4+5x^2+3}}-\frac{3\ln\left(-2x^2+2\sqrt{x^4+5x^2+3}-5\right)}{2}$

elliptic	$-\frac{3x^2}{2\sqrt{x^4+5x^2+3}}+\frac{11}{4\sqrt{x^4+5x^2+3}}-\frac{55(2x^2+5)}{52\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}$
default	$-\frac{3x^2}{2\sqrt{x^4+5x^2+3}}+\frac{15}{4\sqrt{x^4+5x^2+3}}-\frac{75(2x^2+5)}{52\sqrt{x^4+5x^2+3}}+\frac{3\ln\left(x^2+\frac{5}{2}+\sqrt{x^4+5x^2+3}\right)}{2}+\frac{}{\sqrt{x^4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -3/2*x^2/(x^4+5*x^2+3)^(1/2)+15/4/(x^4+5*x^2+3)^(1/2)-75/52*(2*x^2+5)/(x^4+
5*x^2+3)^(1/2)+3/2*ln(x^2+5/2+(x^4+5*x^2+3)^(1/2))+2/13/(x^4+5*x^2+3)^(1/2)
*(5*x^2+6)
```

Maxima [A]
time = 0.29, size = 56, normalized size = 1.00

$$-\frac{47\,x^2}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{33}{13\,\sqrt{x^4+5\,x^2+3}}+\frac{3}{2}\,\log\left(2\,x^2+2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -47/13*x^2/sqrt(x^4 + 5*x^2 + 3) - 33/13/sqrt(x^4 + 5*x^2 + 3) + 3/2*log(2*
x^2 + 2*sqrt(x^4 + 5*x^2 + 3) + 5)
```

Fricas [A]
time = 0.40, size = 81, normalized size = 1.45

$$-\frac{94\,x^4+470\,x^2+39\,(x^4+5\,x^2+3)\log\left(-2\,x^2+2\,\sqrt{x^4+5\,x^2+3}-5\right)+2\,\sqrt{x^4+5\,x^2+3}\,(47\,x^2+33)+282}{26\,(x^4+5\,x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/26*(94*x^4 + 470*x^2 + 39*(x^4 + 5*x^2 + 3)*log(-2*x^2 + 2*sqrt(x^4 + 5*
x^2 + 3) - 5) + 2*sqrt(x^4 + 5*x^2 + 3)*(47*x^2 + 33) + 282)/(x^4 + 5*x^2 +
 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^3\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**3*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A]
time = 5.42, size = 46, normalized size = 0.82

$$-\frac{47\,x^2+33}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{3}{2}\,\log\left(2\,x^2-2\,\sqrt{x^4+5\,x^2+3}+5\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/13*(47*x^2 + 33)/sqrt(x^4 + 5*x^2 + 3) - 3/2*log(2*x^2 - 2*sqrt(x^4 + 5*
x^2 + 3) + 5)
```

Mupad [B]
time = 0.31, size = 52, normalized size = 0.93

$$\frac{3\,\ln\left(\sqrt{x^4+5\,x^2+3}+x^2+\frac{5}{2}\right)}{2}-\frac{47\,x^2}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{33}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] (3*log((5*x^2 + x^4 + 3)^(1/2) + x^2 + 5/2))/2 - (47*x^2)/(13*(5*x^2 + x^4
+ 3)^(1/2)) - 33/(13*(5*x^2 + x^4 + 3)^(1/2))
```

3.196 $\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=25

$$\frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}$$

```
[Out] 1/13*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.01, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1261, 650}

$$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 650

```
Int[((d_.) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(3/2), x_Symbo
l] :> Simp[-2*((b*d - 2*a*e + (2*c*d - b*e)*x)/((b^2 - 4*a*c)*Sqrt[a + b*x
+ c*x^2])), x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[2*c*d - b*e, 0] && NeQ[b
^2 - 4*a*c, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{(3+5x+x^2)^{3/2}}\,dx, x, x^2\right)\\ &= \frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}\end{aligned}$$

Mathematica [A]
time = 0.14, size = 25, normalized size = 1.00

$$\frac{8+11x^2}{13\sqrt{3+5x^2+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (8 + 11*x^2)/(13*Sqrt[3 + 5*x^2 + x^4])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 43 vs. $2(21) = 42$.
time = 0.04, size = 44, normalized size = 1.76

method	result	size
gosper	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
trager	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
risch	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
elliptic	$\frac{11x^2+8}{13\sqrt{x^4+5x^2+3}}$	22
default	$\frac{\frac{15x^2}{13}+\frac{18}{13}}{\sqrt{x^4+5x^2+3}}-\frac{2(2x^2+5)}{13\sqrt{x^4+5x^2+3}}$	44

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 3/13/(x^4+5*x^2+3)^(1/2)*(5*x^2+6)-2/13*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)
```

Maxima [A]
time = 0.28, size = 32, normalized size = 1.28

$$\frac{11\,x^2}{13\,\sqrt{x^4+5\,x^2+3}}+\frac{8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] 11/13*x^2/sqrt(x^4 + 5*x^2 + 3) + 8/13/sqrt(x^4 + 5*x^2 + 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(21) = 42$.
time = 0.35, size = 46, normalized size = 1.84

$$\frac{11\,x^4+55\,x^2+\sqrt{x^4+5\,x^2+3}\,(11\,x^2+8)+33}{13\,(x^4+5\,x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/13*(11*x^4 + 55*x^2 + sqrt(x^4 + 5*x^2 + 3)*(11*x^2 + 8) + 33)/(x^4 + 5*x
^2 + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [A]
time = 6.33, size = 21, normalized size = 0.84

$$\frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] 1/13*(11*x^2 + 8)/sqrt(x^4 + 5*x^2 + 3)
```

Mupad [B]
time = 0.24, size = 21, normalized size = 0.84

$$\frac{11\,x^2+8}{13\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] (11*x^2 + 8)/(13*(5*x^2 + x^4 + 3)^(1/2))
```

3.197 $\int \frac{2+3x^2}{x(3+5x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=66

$$\frac{-7-8x^2}{39\sqrt{3+5x^2+x^4}}-\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\,\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

[Out] -1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/39*(-8*x^2-7)/(x^4+5*x^2+3)^(1/2)

Rubi [A]
time = 0.04, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 836, 12, 738, 212}

$$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\,\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/39*(7 + 8*x^2)/Sqrt[3 + 5*x^2 + x^4] - ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !MatchQ[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (GtQ[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Symbol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{2+3x^2}{x\left(3+5x^2+x^4\right)^{3/2}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x\left(3+5x+x^2\right)^{3/2}}\,dx,x,x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}}-\frac{1}{39}\text{Subst}\left(\int -\frac{13}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}}+\frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x,x^2\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}}-\frac{2}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6+5x^2}{\sqrt{3+5x^2+x^4}}\right)\\
&= -\frac{7+8x^2}{39\sqrt{3+5x^2+x^4}}-\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}
\end{aligned}
$$

Mathematica [A]
time = 0.20, size = 62, normalized size = 0.94

$$
\frac{-7-8x^2}{39\sqrt{3+5x^2+x^4}}+\frac{2\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{3\sqrt{3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x*(3 + 5*x^2 + x^4)^(3/2)),x]
[Out] (-7 - 8*x^2)/(39*Sqrt[3 + 5*x^2 + x^4]) + (2*ArcTanh[(x^2 - Sqrt[3 + 5*x^2
+ x^4])/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A]
time = 0.20, size = 67, normalized size = 1.02

method	result	si
risch	$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	53
default	$-\frac{4(2x^2+5)}{39\sqrt{x^4+5x^2+3}}+\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	67
elliptic	$-\frac{4(2x^2+5)}{39\sqrt{x^4+5x^2+3}}+\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$	67
trager	$-\frac{8x^2+7}{39\sqrt{x^4+5x^2+3}}-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\operatorname{RootOf}\left(\_Z^2-3\right)+6\sqrt{x^4+5x^2+3}}{x^2}\right)}{9}$	71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
[Out] -4/39*(2*x^2+5)/(x^4+5*x^2+3)^(1/2)+1/3/(x^4+5*x^2+3)^(1/2)-1/9*arctanh(1/6
*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.51, size = 65, normalized size = 0.98

$$-\frac{8\,x^2}{39\,\sqrt{x^4+5\,x^2+3}}-\frac{1}{9}\,\sqrt{3}\,\log\left(\frac{2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{7}{39\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] -8/39*x^2/sqrt(x^4 + 5*x^2 + 3) - 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 7/39/sqrt(x^4 + 5*x^2 + 3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 107 vs. 2(52) = 104.
time = 0.34, size = 107, normalized size = 1.62

$$-\frac{24\,x^4-13\,\sqrt{3}\,(x^4+5\,x^2+3)\log\left(\frac{25\,x^2-2\,\sqrt{3}\,(5\,x^2+6)-2\,\sqrt{x^4+5\,x^2+3}\,\left(5\,\sqrt{3}-6\right)+30}{x^2}\right)+120\,x^2+3\,\sqrt{x^4+5\,x^2+3}\,(8\,x^2+7)+72}{117\,(x^4+5\,x^2+3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/117*(24*x^4 - 13*sqrt(3)*(x^4 + 5*x^2 + 3)*log((25*x^2 - 2*sqrt(3)*(5*x^
2 + 6) - 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) - 6) + 30)/x^2) + 120*x^2 + 3*s
qrt(x^4 + 5*x^2 + 3)*(8*x^2 + 7) + 72)/(x^4 + 5*x^2 + 3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x\,(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Giac [A]
time = 3.61, size = 78, normalized size = 1.18

$$-\frac{1}{9}\sqrt{3}\log\left(-x^2+\sqrt{3}+\sqrt{x^4+5\,x^2+3}\right)+\frac{1}{9}\sqrt{3}\log\left(-x^2-\sqrt{3}+\sqrt{x^4+5\,x^2+3}\right)-\frac{8\,x^2+7}{39\,\sqrt{x^4+5\,x^2+3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*log(-x^2 + sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) + 1/9*sqrt(3)*log(
-x^2 - sqrt(3) + sqrt(x^4 + 5*x^2 + 3)) - 1/39*(8*x^2 + 7)/sqrt(x^4 + 5*x^2
 + 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{3\,x^2+2}{x\,{\left(x^4+5\,x^2+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.198 $\int \frac{2+3x^2}{x^3(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=90

$$\frac{-7-8x^2}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{3+5x^2+x^4}}{39x^2}+\frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

```
[Out] 1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)+1/39*(-8*x^2
-7)/x^2/(x^4+5*x^2+3)^(1/2)-2/39*(x^4+5*x^2+3)^(1/2)/x^2
```

Rubi [A]
time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1265, 836, 820, 738, 212}

$$-\frac{8x^2+7}{39x^2\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{x^4+5x^2+3}}{39x^2}+\frac{\tanh^{-1}\left(\frac{5x^2+6}{2\sqrt{3}\sqrt{x^4+5x^2+3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] -1/39*(7 + 8*x^2)/(x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[3 + 5*x^2 + x^4])/(
39*x^2) + ArcTanh[(6 + 5*x^2)/(2*Sqrt[3]*Sqrt[3 + 5*x^2 + x^4])]/(3*Sqrt[3]
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
```

```
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{2+3x^2}{x^3\left(3+5x^2+x^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{2+3x}{x^2\left(3+5x+x^2\right)^{3/2}}\,dx,x,x^2\right)$$

$$= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{1}{39}\text{Subst}\left(\int \frac{-6+8x}{x^2\sqrt{3+5x+x^2}}\,dx,x,x^2\right)$$

$$= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{39x^2} - \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{3+5x+x^2}}\,dx,x$$

$$= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{39x^2} + \frac{2}{3}\text{Subst}\left(\int \frac{1}{12-x^2}\,dx,x,\frac{6-}{\sqrt{3+5}}$$

$$= -\frac{7+8x^2}{39x^2\sqrt{3+5x^2+x^4}} - \frac{2\sqrt{3+5x^2+x^4}}{39x^2} + \frac{\tanh^{-1}\left(\frac{6+5x^2}{2\sqrt{3}\sqrt{3+5x^2+x^4}}\right)}{3\sqrt{3}}$$

Mathematica [A]

time = 0.23, size = 70, normalized size = 0.78

$$\frac{-13-18x^2-2x^4}{39x^2\sqrt{3+5x^2+x^4}}-\frac{2\tanh^{-1}\left(\frac{x^2-\sqrt{3+5x^2+x^4}}{\sqrt{3}}\right)}{3\sqrt{3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(2 + 3*x^2)/(x^3*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-13 - 18*x^2 - 2*x^4)/(39*x^2*Sqrt[3 + 5*x^2 + x^4]) - (2*ArcTanh[(x^2 - S
qrt[3 + 5*x^2 + x^4])/Sqrt[3]])/(3*Sqrt[3])
```

Maple [A]
time = 0.20, size = 84, normalized size = 0.93

method	result
risch	$-\frac{2x^4+18x^2+13}{39x^2\sqrt{x^4+5x^2+3}}+\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$
trager	$-\frac{2x^4+18x^2+13}{39x^2\sqrt{x^4+5x^2+3}}-\frac{\operatorname{RootOf}\left(\_Z^2-3\right)\ln\left(-\frac{-5\operatorname{RootOf}\left(\_Z^2-3\right)x^2+6\sqrt{x^4+5x^2+3}-6\operatorname{RootOf}\left(\_Z^2-3\right)}{x^2}\right)}{9}$
default	$-\frac{1}{3x^2\sqrt{x^4+5x^2+3}}-\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39\sqrt{x^4+5x^2+3}}+\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$
elliptic	$-\frac{1}{3x^2\sqrt{x^4+5x^2+3}}-\frac{1}{3\sqrt{x^4+5x^2+3}}-\frac{2x^2+5}{39\sqrt{x^4+5x^2+3}}+\frac{\operatorname{arctanh}\left(\frac{(5x^2+6)\sqrt{3}}{6\sqrt{x^4+5x^2+3}}\right)\sqrt{3}}{9}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3/x^2/(x^4+5*x^2+3)^(1/2)-1/3/(x^4+5*x^2+3)^(1/2)-1/39*(2*x^2+5)/(x^4+5*
x^2+3)^(1/2)+1/9*arctanh(1/6*(5*x^2+6)*3^(1/2)/(x^4+5*x^2+3)^(1/2))*3^(1/2)
```

Maxima [A]
time = 0.50, size = 82, normalized size = 0.91

$$-\frac{2\,x^2}{39\,\sqrt{x^4+5\,x^2+3}}+\frac{1}{9}\,\sqrt{3}\,\log\left(\frac{2\,\sqrt{3}\,\sqrt{x^4+5\,x^2+3}}{x^2}+\frac{6}{x^2}+5\right)-\frac{6}{13\,\sqrt{x^4+5\,x^2+3}}-\frac{1}{3\,\sqrt{x^4+5\,x^2+3}\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] -2/39*x^2/sqrt(x^4 + 5*x^2 + 3) + 1/9*sqrt(3)*log(2*sqrt(3)*sqrt(x^4 + 5*x^
2 + 3)/x^2 + 6/x^2 + 5) - 6/13/sqrt(x^4 + 5*x^2 + 3) - 1/3/(sqrt(x^4 + 5*x^
2 + 3)*x^2)
```

Fricas [A]
time = 0.41, size = 124, normalized size = 1.38

$$-\frac{6\,x^{6}+30\,x^{4}-13\,\sqrt{3}\,(x^{6}+5\,x^{4}+3\,x^{2})\log\left(\frac{25\,x^{2}+2\,\sqrt{3}\,(5\,x^{2}+6)+2\,\sqrt{x^{4}+5\,x^{2}+3}\,\left(5\,\sqrt{3}+6\right)+30}{x^{2}}\right)+18\,x^{2}+3\,(2\,x^{4}+18\,x^{2}+13)\sqrt{x^{4}+5\,x^{2}+3}}{117\,(x^{6}+5\,x^{4}+3\,x^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/117*(6*x^6 + 30*x^4 - 13*sqrt(3)*(x^6 + 5*x^4 + 3*x^2)*log((25*x^2 + 2*s
qrt(3)*(5*x^2 + 6) + 2*sqrt(x^4 + 5*x^2 + 3)*(5*sqrt(3) + 6) + 30)/x^2) + 1
8*x^2 + 3*(2*x^4 + 18*x^2 + 13)*sqrt(x^4 + 5*x^2 + 3))/(x^6 + 5*x^4 + 3*x^2
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{3x^{2}+2}{x^{3}\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**3/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**3*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Giac [A]
time = 3.47, size = 122, normalized size = 1.36

$$-\frac{1}{9}\,\sqrt{3}\,\log\left(\frac{x^{2}+\sqrt{3}-\sqrt{x^{4}+5\,x^{2}+3}}{x^{2}-\sqrt{3}-\sqrt{x^{4}+5\,x^{2}+3}}\right)+\frac{7\,x^{2}+11}{117\,\sqrt{x^{4}+5\,x^{2}+3}}+\frac{5\,x^{2}-5\,\sqrt{x^{4}+5\,x^{2}+3}+6}{9\left(\left(x^{2}-\sqrt{x^{4}+5\,x^{2}+3}\right)^{2}-3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^3/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] -1/9*sqrt(3)*log((x^2 + sqrt(3) - sqrt(x^4 + 5*x^2 + 3))/(x^2 - sqrt(3) - s
qrt(x^4 + 5*x^2 + 3))) + 1/117*(7*x^2 + 11)/sqrt(x^4 + 5*x^2 + 3) + 1/9*(5*
x^2 - 5*sqrt(x^4 + 5*x^2 + 3) + 6)/((x^2 - sqrt(x^4 + 5*x^2 + 3))^2 - 3)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{3\,x^{2}+2}{x^{3}\,{\left(x^{4}+5\,x^{2}+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)),x)
[Out] int((3*x^2 + 2)/(x^3*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.199 $\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=307

$$\frac{43x\left(5+\sqrt{13}+2x^2\right)}{13\sqrt{3+5x^2+x^4}}+\frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}}-\frac{11}{13}x\sqrt{3+5x^2+x^4}-\frac{43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}$$

```
[Out] 1/13*x^3*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)+43/13*x*(5+2*x^2+13^(1/2))/(x^4+5*x
^2+3)^(1/2)-11/13*x*(x^4+5*x^2+3)^(1/2)+11/26*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)-43/78*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(3
0+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/
2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2)
)^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/
2)
```

Rubi [A]
time = 0.11, antiderivative size = 307, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1289, 1293, 1203, 1113, 1149}

$$\frac{11\sqrt{\frac{3}{2\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}-\frac{43\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}-\frac{11}{13}\sqrt{x^4+5x^2+3}\,x+\frac{43\left(2x^2+\sqrt{13}+5\right)x}{13\sqrt{x^4+5x^2+3}}+\frac{(11x^2+8)x^3}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (43*x*(5 + Sqrt[13] + 2*x^2))/(13*Sqrt[3 + 5*x^2 + x^4]) + (x^3*(8 + 11*x^2
))/(13*Sqrt[3 + 5*x^2 + x^4]) - (11*x*Sqrt[3 + 5*x^2 + x^4])/13 - (43*Sqrt[
(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(
6 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 +
5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[3/(2*(5 + Sqrt[13]))]
*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13]
)*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(
13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
```

```
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rubi steps

$$\int \frac{x^4(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx = \frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}} + \frac{1}{13}\int \frac{x^2(-24-33x^2)}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}} - \frac{11}{13}x\sqrt{3+5x^2+x^4} - \frac{1}{39}\int \frac{-99-258x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= \frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}} - \frac{11}{13}x\sqrt{3+5x^2+x^4} + \frac{33}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx + \frac{86}{13}\int \frac{}{\sqrt{3}}$$

$$= \frac{43x\left(5+\sqrt{13}+2x^2\right)}{13\sqrt{3+5x^2+x^4}} + \frac{x^3(8+11x^2)}{13\sqrt{3+5x^2+x^4}} - \frac{11}{13}x\sqrt{3+5x^2+x^4} - \frac{43\sqrt{\frac{1}{6}\left(5+\right.}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 219, normalized size = 0.71

$$\frac{-2x(33+47x^2)+43i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-182+43\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{26\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^4*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-2*x*(33 + 47*x^2) + (43*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] -
2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt
[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-182 + 43*Sqrt[1
3])*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2
]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(2
6*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.07, size = 240, normalized size = 0.78

method	result
risch	$-\frac{x(47x^2+33)}{13\sqrt{x^4+5x^2+3}} - \frac{3096\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-30+}}{6}\right.\right.}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+}}$
elliptic	$-\frac{2\left(\frac{47}{26}x^3+\frac{33}{26}x\right)}{\sqrt{x^4+5x^2+3}} - \frac{3096\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.\right.}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x}}$

default	$-\frac{6\left(\frac{19}{26}x^3+\frac{15}{26}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{198\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4+5x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6*(19/26*x^3+15/26*x)/(x^4+5*x^2+3)^(1/2)+198/13/(-30+6*13^(1/2))^(1/2)*(1
-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^
2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))
-3096/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-
1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*
(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^
(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-4*(-5/26*x^3-3/13*x)/(x^4+5*x^2+3)^
(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Fricas [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\cdot(3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**4*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^4/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\left(3\,x^2+2\right)}{{\left(x^4+5\,x^2+3\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int((x^4*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.200 $\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}} \, dx$

Optimal. Leaf size=286

$$-\frac{11x\left(5+\sqrt{13}+2x^2\right)}{26\sqrt{3+5x^2+x^4}}+\frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}}+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right.\right.}{26\sqrt{3+5}}$$

```
[Out] 1/13*x*(11*x^2+8)/(x^4+5*x^2+3)^(1/2)-11/26*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2
+3)^(1/2)-4/39*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*6^(1/2)/(5+13^(1/2))^(1/2)*((
6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+11/156*
(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticE
(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2
))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/
(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1289, 1203, 1113, 1149}

$$-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{x^4+5x^2+3}}+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{26\sqrt{x^4+5x^2+3}}-\frac{11x\left(2x^2+\sqrt{13}+5\right)}{26\sqrt{x^4+5x^2+3}}+\frac{x(11x^2+8)}{13\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-11*x*(5 + Sqrt[13] + 2*x^2))/(26*Sqrt[3 + 5*x^2 + x^4]) + (x*(8 + 11*x^2)
)/(13*Sqrt[3 + 5*x^2 + x^4]) + (11*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(26*Sqrt[3 + 5*x^2
+ x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 +
(5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sq
rt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(13*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
```

```
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(2+3x^2)}{(3+5x^2+x^4)^{3/2}}\,dx &= \frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}}+\frac{1}{13}\int \frac{-8-11x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}}-\frac{8}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx-\frac{11}{13}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{11x\left(5+\sqrt{13}+2x^2\right)}{26\sqrt{3+5x^2+x^4}}+\frac{x(8+11x^2)}{13\sqrt{3+5x^2+x^4}}+\frac{11\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\right.}{6+\left(5+\right.}}}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.

time = 10.16, size = 219, normalized size = 0.77

$$\frac{4x(8+11x^2)-11i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+i\sqrt{2}\left(-39+11\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{52\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(x^2*(2 + 3*x^2))/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(8 + 11*x^2) - (11*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*
x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2
/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sqrt[2]*(-39 + 11*Sqrt[13])
*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*E
llipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(52*S
qrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.06, size = 240, normalized size = 0.84

method	result
risch	$\frac{x(11x^2+8)}{13\sqrt{x^4+5x^2+3}}+\frac{396\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.\right.}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5}}$
elliptic	$-\frac{2\left(-\frac{11}{26}x^3-\frac{4}{13}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{396\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-30+}}{6}\right.\right.}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5}}$
default	$-\frac{6\left(-\frac{5}{26}x^3-\frac{3}{13}x\right)}{\sqrt{x^4+5x^2+3}}-\frac{48\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-30+6}}{6}\right.}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6*(-5/26*x^3-3/13*x)/(x^4+5*x^2+3)^(1/2)-48/13/(-30+6*13^(1/2))^(1/2)*(1-(
-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+
3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))+3
96/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6
*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-3
0+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/
2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-4*(1/13*x^3+5/26*x)/(x^4+5*x^2+3)^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \cdot (3x^2+2)}{(x^4+5x^2+3)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral(x**2*(3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)*x^2/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\left(3\,x^2+2\right)}{{\left(x^4+5\,x^2+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2),x)
```

```
[Out] int((x^2*(3*x^2 + 2))/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.201 $\int \frac{2+3x^2}{\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=282

$$\frac{4x\left(5+\sqrt{13}+2x^2\right)}{39\sqrt{3+5x^2+x^4}}-\frac{x(7+8x^2)}{39\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)x^2}{6+\left(5+\sqrt{13}\right)x^2}}\left(6+\left(5+\sqrt{13}\right)x^2\right.}{39\sqrt{3+5x^2}}$$

```
[Out] -1/39*x*(8*x^2+7)/(x^4+5*x^2+3)^(1/2)+4/39*x*(5+2*x^2+13^(1/2))/(x^4+5*x^2+
3)^(1/2)-2/117*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^
(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*
(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2
*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)+11/13*(1/(36
+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30
+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/
2))*(6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/
(x^4+5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1192, 1203, 1113, 1149}

$$\frac{11\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{13\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}-\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{x^4+5x^2+3}}+\frac{4x\left(2x^2+\sqrt{13}+5\right)}{39\sqrt{x^4+5x^2+3}}-\frac{x(8x^2+7)}{39\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (4*x*(5 + Sqrt[13] + 2*x^2))/(39*Sqrt[3 + 5*x^2 + x^4]) - (x*(7 + 8*x^2))/(
39*Sqrt[3 + 5*x^2 + x^4]) - (2*Sqrt[(2*(5 + Sqrt[13]))/3]*Sqrt[(6 + (5 - Sq
rt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*EllipticE[A
rcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(39*Sqrt[3 + 5*x^2
+ x^4]) + (11*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 +
(5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sq
rt[13])/6])/(13*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
```

```
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{2+3x^2}{\left(3+5x^2+x^4\right)^{3/2}}\,dx &= -\frac{x(7+8x^2)}{39\sqrt{3+5x^2+x^4}}-\frac{1}{39}\int \frac{-33-8x^2}{\sqrt{3+5x^2+x^4}}\,dx\\ &= -\frac{x(7+8x^2)}{39\sqrt{3+5x^2+x^4}}+\frac{8}{39}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx+\frac{11}{13}\int \frac{1}{\sqrt{3+5x^2+x^4}}\,dx\\ &= \frac{4x\left(5+\sqrt{13}+2x^2\right)}{39\sqrt{3+5x^2+x^4}}-\frac{x(7+8x^2)}{39\sqrt{3+5x^2+x^4}}-\frac{2\sqrt{\frac{2}{3}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right.}{6+\left(5+\sqrt{13}\right.}}}{}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.20, size = 219, normalized size = 0.78

$$\frac{-2x(7+8x^2)+4i\sqrt{2}\left(-5+\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(13+4\sqrt{13}\right)\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{78\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(3 + 5*x^2 + x^4)^(3/2),x]
```

```
[Out] (-2*x*(7 + 8*x^2) + (4*I)*Sqrt[2]*(-5 + Sqrt[13])*Sqrt[(-5 + Sqrt[13] - 2*x
^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*ArcSinh[Sqrt[2/
(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(13 + 4*Sqrt[13])*Sq
rt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*Elli
pticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6])/(78*Sqrt
[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.05, size = 240, normalized size = 0.85

method	result
risch	$-\frac{x(8x^2+7)}{39\sqrt{x^4+5x^2+3}}-\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-30+\cdots}}{6}\cdots\right.\right.}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+\cdots}}$
elliptic	$-\frac{2\left(\frac{4}{39}x^3+\frac{7}{78}x\right)}{\sqrt{x^4+5x^2+3}}-\frac{96\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\text{EllipticF}\left(\frac{x\sqrt{-30+6\cdots}}{6}\cdots\right.\right.}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5\cdots}}$
default	$-\frac{6\left(\frac{1}{13}x^3+\frac{5}{26}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{66\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\text{EllipticF}\left(\frac{x\sqrt{-30+6\cdots}}{6}\cdots\right.}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4+5x^2+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6*(1/13*x^3+5/26*x)/(x^4+5*x^2+3)^(1/2)+66/13/(-30+6*13^(1/2))^(1/2)*(1-(-
5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3
)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-96
/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-30+
6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/2)
)^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-4*(-19/78*x-5/78*x^3)/(x^4+5*x^2+3)^(1/2
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/(x**4+5*x**2+3)**(3/2),x)
[Out] Integral((3*x**2 + 2)/(x**4 + 5*x**2 + 3)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/(x^4 + 5*x^2 + 3)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{{\left(x^4+5\,x^2+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(3/2),x)
[Out] int((3*x^2 + 2)/(5*x^2 + x^4 + 3)^(3/2), x)
```

3.202 $\int \frac{2+3x^2}{x^2(3+5x^2+x^4)^{3/2}}\,dx$

Optimal. Leaf size=309

$$\frac{19x\left(5+\sqrt{13}+2x^2\right)}{234\sqrt{3+5x^2+x^4}}-\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}}-\frac{19\sqrt{3+5x^2+x^4}}{117x}-\frac{19\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{6+\left(5-\sqrt{13}\right)}{6+\left(5+\sqrt{13}\right)}}}{}$$

```
[Out] 1/39*(-8*x^2-7)/x/(x^4+5*x^2+3)^(1/2)+19/234*x*(5+2*x^2+13^(1/2))/(x^4+5*x^
2+3)^(1/2)-19/117*(x^4+5*x^2+3)^(1/2)/x-4/117*(1/(36+x^2*(30+6*13^(1/2))))^
(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13^(1/2))^(1/2)/(36+
x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2))
)*6^(1/2)/(5+13^(1/2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1
/2)/(x^4+5*x^2+3)^(1/2)-19/1404*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*
(30+6*13^(1/2)))^(1/2)*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(
1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/
2))^(1/2)*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(
1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1291, 1295, 1203, 1113, 1149}

$$-\frac{4\sqrt{\frac{2}{3\left(5+\sqrt{13}\right)}}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{39\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{234\sqrt{x^4+5x^2+3}}+\frac{19x\left(2x^2+\sqrt{13}+5\right)}{234\sqrt{x^4+5x^2+3}}-\frac{19\sqrt{x^4+5x^2+3}}{117x}-\frac{8x^2+7}{39x\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (19*x*(5 + Sqrt[13] + 2*x^2))/(234*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/(39
*x*Sqrt[3 + 5*x^2 + x^4]) - (19*Sqrt[3 + 5*x^2 + x^4])/(117*x) - (19*Sqrt[(
5 + Sqrt[13])/6]*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6
 + (5 + Sqrt[13])*x^2)*EllipticE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5
*Sqrt[13])/6])/(234*Sqrt[3 + 5*x^2 + x^4]) - (4*Sqrt[2/(3*(5 + Sqrt[13]))]*
Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])
*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(3
9*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
```

```
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{2+3x^2}{x^2\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{1}{39}\int \frac{-19+8x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{19\sqrt{3+5x^2+x^4}}{117x} + \frac{1}{117}\int \frac{-24+19x^2}{\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{19\sqrt{3+5x^2+x^4}}{117x} + \frac{19}{117}\int \frac{x^2}{\sqrt{3+5x^2+x^4}}\,dx - \frac{8}{39}$$

$$= \frac{19x\left(5+\sqrt{13}+2x^2\right)}{234\sqrt{3+5x^2+x^4}} - \frac{7+8x^2}{39x\sqrt{3+5x^2+x^4}} - \frac{19\sqrt{3+5x^2+x^4}}{117x} - \frac{19\sqrt{\frac{1}{6}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 228, normalized size = 0.74

$$\frac{-4(78+119x^2+19x^4)+19i\sqrt{2}\left(-5+\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)-i\sqrt{2}\left(-143+19\sqrt{13}\right)x\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{468x\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^2*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-4*(78 + 119*x^2 + 19*x^4) + (19*I)*Sqrt[2]*(-5 + Sqrt[13])*x*Sqrt[(-5 + S
qrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticE[I*Ar
cSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] - I*Sqrt[2]*(-143 +
 19*Sqrt[13])*x*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt
[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt
[13])/6])/(468*x*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.08, size = 257, normalized size = 0.83

method	result
risch	$-\frac{19x^4+119x^2+78}{117x\sqrt{x^4+5x^2+3}} - \frac{76\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-30}}{}\right.\right.}{13\sqrt{-30+6\sqrt{13}}\sqrt{x^4}}$
elliptic	$-\frac{2\sqrt{x^4+5x^2+3}}{9x} - \frac{2\left(-\frac{7}{234}x^3-\frac{11}{234}x\right)}{\sqrt{x^4+5x^2+3}} - \frac{76\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\left(\right.}{13\sqrt{-30}}$

default	$-\frac{6\left(-\frac{19}{78}x-\frac{5}{78}x^{3}\right)}{\sqrt{x^{4}+5x^{2}+3}}-\frac{16\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^{2}}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^{2}}\;\mathrm{EllipticF}\left(\frac{x\sqrt{-30+6\sqrt{}}}{6}\right.}{13\sqrt{-30+6\sqrt{13}}\,\sqrt{x^{4}+5x^{2}+3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -6*(-19/78*x-5/78*x^3)/(x^4+5*x^2+3)^(1/2)-16/13/(-30+6*13^(1/2))^(1/2)*(1-
(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2
+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-
76/13/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6
*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/(5+13^(1/2))*(EllipticF(1/6*x*(-3
0+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2))-EllipticE(1/6*x*(-30+6*13^(1/
2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-2/9*(x^4+5*x^2+3)^(1/2)/x-4*(19/234*x^
3+40/117*x)/(x^4+5*x^2+3)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^{2}+2}{x^{2}\left(x^{4}+5x^{2}+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**2/(x**4+5*x**2+3)**(3/2),x)
[Out] Integral((3*x**2 + 2)/(x**2*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^2/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^2\,{\left(x^4+5\,x^2+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(3/2)),x)
[Out] int((3*x^2 + 2)/(x^2*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.203 $\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=326

$$-\frac{133x\left(5+\sqrt{13}+2x^2\right)}{1053\sqrt{3+5x^2+x^4}}-\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}}-\frac{5\sqrt{3+5x^2+x^4}}{351x^3}+\frac{266\sqrt{3+5x^2+x^4}}{1053x}+\frac{133\sqrt{\frac{1}{6}\left(5+\sqrt{1\ldots\right.}}}{\ldots}$$

```
[Out] 1/39*(-8*x^2-7)/x^3/(x^4+5*x^2+3)^(1/2)-133/1053*x*(5+2*x^2+13^(1/2))/(x^4+
5*x^2+3)^(1/2)-5/351*(x^4+5*x^2+3)^(1/2)/x^3+266/1053*(x^4+5*x^2+3)^(1/2)/x
+133/6318*(1/(36+x^2*(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)
*EllipticE(x*(30+6*13^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+
30*13^(1/2))^(1/2))*(6+x^2*(5+13^(1/2)))*(30+6*13^(1/2))^(1/2)*((6+x^2*(5-1
3^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+5*x^2+3)^(1/2)-5/351*(1/(36+x^2*
(30+6*13^(1/2))))^(1/2)*(36+x^2*(30+6*13^(1/2)))^(1/2)*EllipticF(x*(30+6*13
^(1/2))^(1/2)/(36+x^2*(30+6*13^(1/2)))^(1/2),1/6*(-78+30*13^(1/2))^(1/2))*(
6+x^2*(5+13^(1/2)))*((6+x^2*(5-13^(1/2)))/(6+x^2*(5+13^(1/2))))^(1/2)/(x^4+
5*x^2+3)^(1/2)/(30+6*13^(1/2))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1291, 1295, 1203, 1113, 1149}

$$-\frac{5\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)F\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{351\sqrt{6\left(5+\sqrt{13}\right)}\sqrt{x^4+5x^2+3}}+\frac{133\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}\sqrt{\frac{\left(5-\sqrt{13}\right)x^2+6}{\left(5+\sqrt{13}\right)x^2+6}}\left(\left(5+\sqrt{13}\right)x^2+6\right)E\left(\text{ArcTan}\left(\sqrt{\frac{1}{6}\left(5+\sqrt{13}\right)}x\right)|\frac{1}{6}\left(-13+5\sqrt{13}\right)\right)}{1053\sqrt{x^4+5x^2+3}}-\frac{133x\left(2x^2+\sqrt{13}+5\right)}{1053\sqrt{x^4+5x^2+3}}+\frac{266\sqrt{x^4+5x^2+3}}{1053x}-\frac{5\sqrt{x^4+5x^2+3}}{351x^3}-\frac{8x^2+7}{39x^3\sqrt{x^4+5x^2+3}}$$

Antiderivative was successfully verified.

```
[In] Int[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-133*x*(5 + Sqrt[13] + 2*x^2))/(1053*Sqrt[3 + 5*x^2 + x^4]) - (7 + 8*x^2)/
(39*x^3*Sqrt[3 + 5*x^2 + x^4]) - (5*Sqrt[3 + 5*x^2 + x^4])/(351*x^3) + (266
*Sqrt[3 + 5*x^2 + x^4])/(1053*x) + (133*Sqrt[(5 + Sqrt[13])/6]*Sqrt[(6 + (5
 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]*(6 + (5 + Sqrt[13])*x^2)*Ellipt
icE[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13 + 5*Sqrt[13])/6])/(1053*Sqrt[3 +
 5*x^2 + x^4]) - (5*Sqrt[(6 + (5 - Sqrt[13])*x^2)/(6 + (5 + Sqrt[13])*x^2)]
*(6 + (5 + Sqrt[13])*x^2)*EllipticF[ArcTan[Sqrt[(5 + Sqrt[13])/6]*x], (-13
+ 5*Sqrt[13])/6])/(351*Sqrt[6*(5 + Sqrt[13])]*Sqrt[3 + 5*x^2 + x^4])
```

Rule 1113

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b
^2 - 4*a*c, 2]}, Simp[(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q)*x^2)/(2*a +
```

```
(b + q)*x^2)]/(2*a*Rt[(b + q)/(2*a), 2]*Sqrt[a + b*x^2 + c*x^4]))*EllipticF
[ArcTan[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&
!(PosQ[(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[
{a, b, c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1149

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[b^2 - 4*a*c, 2]}, Simp[x*((b + q + 2*c*x^2)/(2*c*Sqrt[a + b*x^2 + c*x^4
])), x] - Simp[Rt[(b + q)/(2*a), 2]*(2*a + (b + q)*x^2)*(Sqrt[(2*a + (b - q
)*x^2)/(2*a + (b + q)*x^2)]/(2*c*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[ArcTan
[Rt[(b + q)/(2*a), 2]*x], 2*(q/(b + q))], x] /; PosQ[(b + q)/a] &&  !(PosQ[
(b - q)/a] && SimplerSqrtQ[(b - q)/(2*a), (b + q)/(2*a)])] /; FreeQ[{a, b,
c}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1203

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[d, Int[1/Sqrt[a + b*x^2 + c*x^4],
 x], x] + Dist[e, Int[x^2/Sqrt[a + b*x^2 + c*x^4], x], x] /; PosQ[(b + q)/a
] || PosQ[(b - q)/a]] /; FreeQ[{a, b, c, d, e}, x] && GtQ[b^2 - 4*a*c, 0]
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rubi steps

$$\int \frac{2+3x^2}{x^4\left(3+5x^2+x^4\right)^{3/2}}\,dx = -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}} - \frac{1}{39}\int \frac{-5+24x^2}{x^4\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}} - \frac{5\sqrt{3+5x^2+x^4}}{351x^3} + \frac{1}{351}\int \frac{-266-5x^2}{x^2\sqrt{3+5x^2+x^4}}\,dx$$

$$= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}} - \frac{5\sqrt{3+5x^2+x^4}}{351x^3} + \frac{266\sqrt{3+5x^2+x^4}}{1053x} - \frac{\int \frac{15+2\cdots}{\sqrt{3+5x\cdots}}}{10\cdots}$$

$$= -\frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}} - \frac{5\sqrt{3+5x^2+x^4}}{351x^3} + \frac{266\sqrt{3+5x^2+x^4}}{1053x} - \frac{5}{351}\int \frac{\cdots}{\sqrt{3\cdots}}$$

$$= -\frac{133x\left(5+\sqrt{13}+2x^2\right)}{1053\sqrt{3+5x^2+x^4}} - \frac{7+8x^2}{39x^3\sqrt{3+5x^2+x^4}} - \frac{5\sqrt{3+5x^2+x^4}}{351x^3} + \frac{266\sqrt{\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.17, size = 234, normalized size = 0.72

$$\frac{-468+1014x^2+2630x^4+532x^6-133i\sqrt{2}\left(-5+\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,E\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)+i\sqrt{2}\left(-650+133\sqrt{13}\right)x^3\sqrt{\frac{-5+\sqrt{13}-2x^2}{-5+\sqrt{13}}}\sqrt{5+\sqrt{13}+2x^2}\,F\left(i\sinh^{-1}\left(\sqrt{\frac{2}{5+\sqrt{13}}}\,x\right)\middle|\frac{19}{6}+\frac{5\sqrt{13}}{6}\right)}{2106x^3\sqrt{3+5x^2+x^4}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(2 + 3*x^2)/(x^4*(3 + 5*x^2 + x^4)^(3/2)),x]
```

```
[Out] (-468 + 1014*x^2 + 2630*x^4 + 532*x^6 - (133*I)*Sqrt[2]*(-5 + Sqrt[13])*x^3
*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13])]*Sqrt[5 + Sqrt[13] + 2*x^2]*E
llipticE[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x], 19/6 + (5*Sqrt[13])/6] + I*Sq
rt[2]*(-650 + 133*Sqrt[13])*x^3*Sqrt[(-5 + Sqrt[13] - 2*x^2)/(-5 + Sqrt[13]
)]*Sqrt[5 + Sqrt[13] + 2*x^2]*EllipticF[I*ArcSinh[Sqrt[2/(5 + Sqrt[13])]*x]
, 19/6 + (5*Sqrt[13])/6])/(2106*x^3*Sqrt[3 + 5*x^2 + x^4])
```

Maple [A]
time = 0.06, size = 274, normalized size = 0.84

method	result
risch	$\frac{266x^6+1315x^4+507x^2-234}{1053x^3\sqrt{x^4+5x^2+3}} + \frac{1064\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\,\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\,\left(\mathrm{EllipticF}\left(\frac{x\sqrt{-30\cdots}}{\cdots}\right.\right.}{117\sqrt{-30+6\sqrt{13}}\,\sqrt{x^4\cdots}}$

elliptic	$-\frac{2\sqrt{x^4+5x^2+3}}{27x^3}+\frac{23\sqrt{x^4+5x^2+3}}{81x}-\frac{2\left(\frac{11}{702}x^3+\frac{17}{351}x\right)}{\sqrt{x^4+5x^2+3}}+\frac{1064\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-}}{}$
default	$\frac{23\sqrt{x^4+5x^2+3}}{81x}-\frac{6\left(\frac{19}{234}x^3+\frac{40}{117}x\right)}{\sqrt{x^4+5x^2+3}}-\frac{10\sqrt{1-\left(-\frac{5}{6}+\frac{\sqrt{13}}{6}\right)x^2}\sqrt{1-\left(-\frac{5}{6}-\frac{\sqrt{13}}{6}\right)x^2}\ \mathrm{Ell}}{117\sqrt{-30+6\sqrt{13}}\sqrt{x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 23/81*(x^4+5*x^2+3)^(1/2)/x-6*(19/234*x^3+40/117*x)/(x^4+5*x^2+3)^(1/2)-10/
117/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*1
3^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)*EllipticF(1/6*x*(-30+6*13^(1/2))^(1
/2),5/6*3^(1/2)+1/6*39^(1/2))+1064/117/(-30+6*13^(1/2))^(1/2)*(1-(-5/6+1/6*
13^(1/2))*x^2)^(1/2)*(1-(-5/6-1/6*13^(1/2))*x^2)^(1/2)/(x^4+5*x^2+3)^(1/2)/
(5+13^(1/2))*(EllipticF(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/
2))-EllipticE(1/6*x*(-30+6*13^(1/2))^(1/2),5/6*3^(1/2)+1/6*39^(1/2)))-2/27*
(x^4+5*x^2+3)^(1/2)/x^3-4*(-40/351*x^3-343/702*x)/(x^4+5*x^2+3)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{3x^2+2}{x^4\left(x^4+5x^2+3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x**2+2)/x**4/(x**4+5*x**2+3)**(3/2),x)
```

```
[Out] Integral((3*x**2 + 2)/(x**4*(x**4 + 5*x**2 + 3)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3*x^2+2)/x^4/(x^4+5*x^2+3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((3*x^2 + 2)/((x^4 + 5*x^2 + 3)^(3/2)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{3\,x^2+2}{x^4\,{\left(x^4+5\,x^2+3\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(3/2)),x)
```

```
[Out] int((3*x^2 + 2)/(x^4*(5*x^2 + x^4 + 3)^(3/2)), x)
```

3.204 $\int (fx)^{3/2} (d+ex^2) \sqrt{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,F}{9f^3\sqrt{1+\frac{}{b-}}}$$

```
[Out] 2/5*d*(f*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/9*e*(f*
x)^(9/2)*AppellF1(9/4,-1/2,-1/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{9}{4};-\frac{1}{2},-\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1/2, 9/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4, -1/2
, -1/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int (fx)^{3/2}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx=\int\left(d(fx)^{3/2}\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{7/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$=d\int(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx+\frac{e\int(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$=\frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{\cdots}{b+\sqrt{\cdots}}}\cdots}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{2d(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{\cdots}{b+\sqrt{\cdots}}\cdots\right.}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A]
time = 10.65, size = 430, normalized size = 1.45

$$\frac{2f\sqrt{fx}\left(5\left(a+bx^2+cx^4\right)\left(-14b^2e+2bc\left(13d+5ex^2\right)+c\left(36ae+65cdx^2+45cex^4\right)\right)+10a\left(-13bcd+7b^2e-18ace\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+2\left(-39b^2cd+130ac^2d+21b^3e-79abce\right)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{2925c^2\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(3/2)*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*(a + b*x^2 + c*x^4)*(-14*b^2*e + 2*b*c*(13*d + 5*e*x^2) +
 c*(36*a*e + 65*c*d*x^2 + 45*c*e*x^4)) + 10*a*(-13*b*c*d + 7*b^2*e - 18*a*c
*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
```

```
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2,
 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4
*a*c])] + 2*(-39*b^2*c*d + 130*a*c^2*d + 21*b^3*e - 79*a*b*c*e)*x^2*Sqrt[(b
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4,
(-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(
2925*c^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (fx)^{\frac{3}{2}} \left(e\,x^{2}+d\right) \sqrt{c\,x^{4}+b\,x^{2}+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*(f*x)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(f*x^3*e + d*f*x)*sqrt(f*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{\frac{3}{2}} \left(d+ex^{2}\right) \sqrt{a+bx^{2}+cx^{4}}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((f*x)**(3/2)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*(f*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^{3/2}\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.205 $\int \sqrt{fx}\,(d+ex^2)\sqrt{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,F}{7f^3\sqrt{1+\frac{}{b-}}}$$

```
[Out] 2/3*d*(f*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/7*e*(f*
x)^(7/2)*AppellF1(7/4,-1/2,-1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x
^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{7}{4};-\frac{1}{2},-\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2, -1/2, 7/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4, -1/2
, -1/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*
c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{fx}\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}\,dx &= \int \left(d\sqrt{fx}\sqrt{a+bx^2+cx^4}+\frac{e(fx)^{5/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx \\
&= d\int \sqrt{fx}\sqrt{a+bx^2+cx^4}\,dx+\frac{e\int (fx)^{5/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2} \\
&= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c}{b+\sqrt{b}}}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&= \frac{2d(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{}{b+\sqrt{}}\right.}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}
$$

Mathematica [A]
time = 15.52, size = 386, normalized size = 1.30

$$
\frac{2x\sqrt{fx}\left(21(11cd+2be+7cex^2)\left(a+bx^2+cx^4\right)+14a(22cd-3be)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+6(11bcd-5b^2e+14ace)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{1617c\sqrt{a+bx^2+cx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f*x]*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*x*Sqrt[f*x]*(21*(11*c*d + 2*b*e + 7*c*e*x^2)*(a + b*x^2 + c*x^4) + 14*a*
(22*c*d - 3*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
```

```
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])] + 6*(11*b*c*d - 5*b^2*e + 14*a*c*e)*x^2*Sqrt[(b - Sqrt
[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(1617*c
*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{fx}\, \left(e\,x^{2}+d\right) \sqrt{c\,x^{4}+b\,x^{2}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{fx}\, \left(d + ex^{2}\right) \sqrt{a + bx^{2} + cx^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral(sqrt(f*x)*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f\,x}\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.206 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx$

Optimal. Leaf size=295

$$\frac{2d\sqrt{fx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F}{5f^3\sqrt{1+\frac{}{b-}}}$$

```
[Out] 2/5*e*(f*x)^(5/2)*AppellF1(5/4,-1/2,-1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2*d*App
ellF1(1/4,-1/2,-1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c
+b^2)^(1/2)))*(f*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-4*a*c+b^2
)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2d\sqrt{fx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2e(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{1}{2},-\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -1/2, -1/2, 5/4, (-2*c
*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt
[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 -
4*a*c])]) + (2*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -1/2, -1
/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx = \int \left(\frac{d\sqrt{a+bx^2+cx^4}}{\sqrt{fx}} + \frac{e(fx)^{3/2}\sqrt{a+bx^2+cx^4}}{f^2}\right)dx$$

$$= d\int \frac{\sqrt{a+bx^2+cx^4}}{\sqrt{fx}}\,dx + \frac{e\int (fx)^{3/2}\sqrt{a+bx^2+cx^4}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{\sqrt{fx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2d\sqrt{fx}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A]
time = 10.48, size = 386, normalized size = 1.31

$$\frac{2x\left(5(9cd+2be+5cex^2)(a+bx^2+cx^4)+10a(18cd-be)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+2(9bcd-3b^2e+10ace)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{225c\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/Sqrt[f*x],x]
```

```
[Out] (2*x*(5*(9*c*d + 2*b*e + 5*c*e*x^2)*(a + b*x^2 + c*x^4) + 10*a*(18*c*d - b*
e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2,
1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*
a*c])] + 2*(9*b*c*d - 3*b^2*e + 10*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b
^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(225*c*Sqrt[f*x]*Sqrt[a
 + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{\sqrt{f x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)/sqrt(f*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x)/(f*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^{2}\right)\sqrt{a+bx^{2}+cx^{4}}}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/sqrt(f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)/sqrt(f*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}{\sqrt{f\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(1/2),x)
```

```
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(1/2), x)
```

3.207 $\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx$

Optimal. Leaf size=295

$$-\frac{2d\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\Big(}{3f^3\sqrt{1+\frac{2}{b-\sqrt{}}}}$$

```
[Out] 2/3*e*(f*x)^(3/2)*AppellF1(3/4,-1/2,-1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)
),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-2*d*App
ellF1(-1/4,-1/2,-1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*
c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(f*x)^(1/2)/(1+2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2e(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{1}{2},-\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2d\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (-2*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -1/2, -1/2, 3/4, (-2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -1/2,
-1/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x
^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(d+ex^2)\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx &= \int \left(\frac{d\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}} + \frac{e\sqrt{fx}\,\sqrt{a+bx^2+cx^4}}{f^2}\right) dx \\
&= d\int \frac{\sqrt{a+bx^2+cx^4}}{(fx)^{3/2}}\,dx + \frac{e\int \sqrt{fx}\,\sqrt{a+bx^2+cx^4}\,dx}{f^2} \\
&= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int \frac{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}{(fx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} \\
&= -\frac{2d\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{1}{2},-\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\,\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}
$$

Mathematica [A]
time = 10.57, size = 370, normalized size = 1.25

$$
\frac{x\left(-42(7d-ex^2)\left(a+bx^2+cx^4\right)+28(7bd+2ae)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+12(14cd+be)x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{147(fx)^{3/2}\sqrt{a+bx^2+cx^4}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(f*x)^(3/2),x]
```

```
[Out] (x*(-42*(7*d - e*x^2)*(a + b*x^2 + c*x^4) + 28*(7*b*d + 2*a*e)*x^2*Sqrt[(b
- Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (
-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 12
*(14*c*d + b*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Ap
pellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])]))/(147*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)/(f*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x)/(f^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d+ex^{2}\right)\sqrt{a+bx^{2}+cx^{4}}}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(1/2)/(f*x)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)/(f*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(1/2)/(f*x)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)/(f*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}{{\left(f\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(3/2),x)
```

```
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2))/(f*x)^(3/2), x)
```

3.208 $\int (fx)^{3/2} (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx$

Optimal. Leaf size=299

$$\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx}}{9f^3\sqrt{1+}}$$

```
[Out] 2/5*a*d*(f*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/9*a*e
*(f*x)^(9/2)*AppellF1(9/4,-3/2,-3/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{9/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{9}{4};-\frac{3}{2},-\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*d*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2, -3/2, 9/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(9/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[9/4,
-3/2, -3/2, 13/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(9*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int (fx)^{3/2}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx=\int\left(d(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{7/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right.$$

$$=d\int (fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}\,dx+\frac{e\int (fx)^{7/2}\left(a+bx^2+cx^4\right)^{3}}{f^2}$$

$$=\frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int (fx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\right.}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$=\frac{2ad(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{5f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A]
time = 10.94, size = 567, normalized size = 1.90

$$\frac{2f\sqrt{fx}\left(5(a+bx^2+cx^4)(308b^4e-4b^3c(147d+55ex^2)+12b^2c(-167ae+5cx^2(7d+3ex^2))+3bc^2(16a(77d+25ex^2)+5cx^4(399d+299ex^2))+3c^2(816a^2e+65c^2x^6(21d+17ex^2)+5acx^2(637d+425ex^2)))-20a(-147b^3cd+924abc^2d+77b^4e-501ab^2ce+612a^2c^2e)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)-4(-441b^4cd+3297ab^2c^2d-5460a^2c^3d+231b^5e-1778ab^3ce+3336a^2bc^2e)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{348075c^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*(a + b*x^2 + c*x^4)*(308*b^4*e - 4*b^3*c*(147*d + 55*e*x^
2) + 12*b^2*c*(-167*a*e + 5*c*x^2*(7*d + 3*e*x^2)) + 3*b*c^2*(16*a*(77*d +
25*e*x^2) + 5*c*x^4*(399*d + 299*e*x^2)) + 3*c^2*(816*a^2*e + 65*c^2*x^6*(2
```

```
1*d + 17*e*x^2) + 5*a*c*x^2*(637*d + 425*e*x^2))) - 20*a*(-147*b^3*c*d + 92
4*a*b*c^2*d + 77*b^4*e - 501*a*b^2*c*e + 612*a^2*c^2*e)*Sqrt[(b - Sqrt[b^2
- 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(
b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 4*(-441*b^4*c
*d + 3297*a*b^2*c^2*d - 5460*a^2*c^3*d + 231*b^5*e - 1778*a*b^3*c*e + 3336*
a^2*b*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*Appel
lF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b +
 Sqrt[b^2 - 4*a*c])]))/(348075*c^3*Sqrt[a + b*x^2 + c*x^4])
```

Maple [**F**]
time = 0.01, size = 0, normalized size = 0.00

$$\int (fx)^{\frac{3}{2}} \left(e\,x^2+d\right) \left(c\,x^4+b\,x^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)*(f*x)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral((c*d*f*x^5 + b*d*f*x^3 + a*d*f*x + (c*f*x^7 + b*f*x^5 + a*f*x^3)*e
)*sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^{\frac{3}{2}} \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)*(f*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^{3/2}\,\left(e\,x^2+d\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((f*x)^(3/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.209 $\int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx$

Optimal. Leaf size=299

$$\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx}}{7f^3\sqrt{1+}}$$

```
[Out] 2/3*a*d*(f*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-
(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2/7*a*e
*(f*x)^(7/2)*AppellF1(7/4,-3/2,-3/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2
*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(b-(-4*
a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 299, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {1349, 1155, 524}

$$\frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{7/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{7}{4};-\frac{3}{2},-\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*a*d*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3/2, -3/2, 7/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*
f*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])]) + (2*a*e*(f*x)^(7/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[7/4,
-3/2, -3/2, 11/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c])])/(7*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 +
 (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \sqrt{fx}\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}\,dx = \int \left(d\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}+\frac{e(fx)^{5/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right) d$$

$$= d\int \sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}\,dx+\frac{e\int (fx)^{5/2}\left(a+bx^2+cx^4\right)^{3/2}}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \sqrt{fx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b+\sqrt{}}\right.}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{2ad(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{3f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A]
time = 15.78, size = 490, normalized size = 1.64

$$\frac{2x\sqrt{fx}\left(7(a+bx^2+cx^4)\left(-108b^3e+12b^2c(19d+7ex^2)+bc(624ae+7cx^2(323d+231ex^2))+c^2(77cx^4(19d+15ex^2)+a(3971d+2415ex^2))\right)+28a(-57b^2cd+836ac^2d+27b^3e-156abce)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+12(-95b^3cd+684abc^2d+45b^4e-309ab^2ce+420a^2c^2e)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{153615c^2\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*x*Sqrt[f*x]*(7*(a + b*x^2 + c*x^4)*(-108*b^3*e + 12*b^2*c*(19*d + 7*e*x^
2) + b*c*(624*a*e + 7*c*x^2*(323*d + 231*e*x^2)) + c^2*(77*c*x^4*(19*d + 15
*e*x^2) + a*(3971*d + 2415*e*x^2))) + 28*a*(-57*b^2*c*d + 836*a*c^2*d + 27*
```

```
b^3*e - 156*a*b*c*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*A
ppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(
-b + Sqrt[b^2 - 4*a*c])] + 12*(-95*b^3*c*d + 684*a*b*c^2*d + 45*b^4*e - 309
*a*b^2*c*e + 420*a^2*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b -
 Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
 (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(153615*c^2*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \sqrt{fx}\,\left(e\,x^{2}+d\right)\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)*sqrt(f*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral((c*d*x^4 + b*d*x^2 + a*d + (c*x^6 + b*x^4 + a*x^2)*e)*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(f*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{fx}\left(d+ex^{2}\right)\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)*sqrt(f*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{f\,x}\,\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.210 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx$

Optimal. Leaf size=297

$$\frac{2ad\sqrt{fx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}}{5f^3\sqrt{1+\frac{}{b}}}$$

```
[Out] 2/5*a*e*(f*x)^(5/2)*AppellF1(5/4,-3/2,-3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)+2*a*d
*AppellF1(1/4,-3/2,-3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4
*a*c+b^2)^(1/2)))*(f*x)^(1/2)*(c*x^4+b*x^2+a)^(1/2)/f/(1+2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2ad\sqrt{fx}\,\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{2ae(fx)^{5/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{5}{4};-\frac{3}{2},-\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\,\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

```
[Out] (2*a*d*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]*AppellF1[1/4, -3/2, -3/2, 5/4, (-2
*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sq
rt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]) + (2*a*e*(f*x)^(5/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[5/4, -3/2
, -3/2, 9/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(5*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}
\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx &= \int \left(\frac{d(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}} + \frac{e(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}{f^2}\right)dx \\
&= d\int \frac{(a+bx^2+cx^4)^{3/2}}{\sqrt{fx}}\,dx + \frac{e\int (fx)^{3/2}(a+bx^2+cx^4)^{3/2}\,dx}{f^2} \\
&= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{\sqrt{fx}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} + \\
&= \frac{2ad\sqrt{fx}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-}\right.}{f\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}
\end{aligned}$$

Mathematica [A]
time = 10.80, size = 487, normalized size = 1.64

$$\frac{2x\left(5(a+bx^2+cx^4)\left(-28b^3e+4b^2c(17d+5ex^2)+c^2(867ad+455aex^2+255cdx^4+195cex^6)+bc(176ae+5cx^2(85d+57ex^2))\right)+20a(-17b^2cd+612ac^2d+7b^3e-44abce)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+4(-51b^3cd+476abc^2d+21b^4e-157ab^2ce+260a^2c^2e)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{16575c^2\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/Sqrt[f*x],x]
```

```
[Out] (2*x*(5*(a + b*x^2 + c*x^4)*(-28*b^3*e + 4*b^2*c*(17*d + 5*e*x^2) + c^2*(86
7*a*d + 455*a*e*x^2 + 255*c*d*x^4 + 195*c*e*x^6) + b*c*(176*a*e + 5*c*x^2*(
85*d + 57*e*x^2))) + 20*a*(-17*b^2*c*d + 612*a*c^2*d + 7*b^3*e - 44*a*b*c*e
)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1
/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])] + 4*(-51*b^3*c*d + 476*a*b*c^2*d + 21*b^4*e - 157*a*b^2*c*e + 260*a^2
*c^2*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]
*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5
/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))/(16575*c^2*Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{\sqrt{f x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)/sqrt(f*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((c*d*x^4 + b*d*x^2 + a*d + (c*x^6 + b*x^4 + a*x^2)*e)*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(f*x)/(f*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}{\sqrt{fx}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(1/2),x)
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/sqrt(f*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(1/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)/sqrt(f*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)\,{(c\,x^4+b\,x^2+a)}^{3/2}}{\sqrt{f\,x}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(1/2),x)
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(1/2), x)
```

3.211 $\int \frac{(d+ex^2)(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx$

Optimal. Leaf size=297

$$-\frac{2ad\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F}{3f^3\sqrt{1+\frac{}{b-}}}$$

```
[Out] 2/3*a*e*(f*x)^(3/2)*AppellF1(3/4,-3/2,-3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/
2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(1+2*c*x^2/(
b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)-2*a*d
*AppellF1(-1/4,-3/2,-3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-
4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(f*x)^(1/2)/(1+2*c*x^2/(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2ae(fx)^{3/2}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{3}{4};-\frac{3}{2},-\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}-\frac{2ad\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

```
[Out] (-2*a*d*Sqrt[a + b*x^2 + c*x^4]*AppellF1[-1/4, -3/2, -3/2, 3/4, (-2*c*x^2)/
(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*
Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^
2 - 4*a*c])]) + (2*a*e*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[3/4, -3
/2, -3/2, 7/4, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2
 - 4*a*c])])/(3*f^3*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx = \int \left(\frac{d(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}} + \frac{e\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}{f^2}\right) dx$$

$$= d\int \frac{(a+bx^2+cx^4)^{3/2}}{(fx)^{3/2}}\,dx + \frac{e\int \sqrt{fx}\,(a+bx^2+cx^4)^{3/2}\,dx}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int \frac{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}}{(fx)^{3/2}}\,dx}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}} +$$

$$= -\frac{2ad\sqrt{a+bx^2+cx^4}\,F_1\left(-\frac{1}{4};-\frac{3}{2},-\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4a}}\right.}{f\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Mathematica [A]
time = 10.68, size = 447, normalized size = 1.51

$$\frac{x\left(14(a+bx^2+cx^4)\left(ac(-1155d+209ex^2)+x^2(12b^2e+7c^2x^2(15d+11ex^2)+bc(195d+119ex^2))\right)-56a(-240bcd+3b^2e-44ace)\,x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+24(15b^2cd+420ac^2d-5b^3e+36abce)\,x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{8085c(fx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x]
```

```
[Out] (x*(14*(a + b*x^2 + c*x^4)*(a*c*(-1155*d + 209*e*x^2) + x^2*(12*b^2*e + 7*c
^2*x^2*(15*d + 11*e*x^2) + b*c*(195*d + 119*e*x^2))) - 56*a*(-240*b*c*d + 3
```

```
*b^2*e - 44*a*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2
 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]
*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)
/(-b + Sqrt[b^2 - 4*a*c])] + 24*(15*b^2*c*d + 420*a*c^2*d - 5*b^3*e + 36*a*
b*c*e)*x^4*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/
4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt
[b^2 - 4*a*c])]))/(8085*c*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\left(e\,x^{2}+d\right)\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}{\left(fx\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
```

```
[Out] int((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)/(f*x)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral((c*d*x^4 + b*d*x^2 + a*d + (c*x^6 + b*x^4 + a*x^2)*e)*sqrt(c*x^4 +
 b*x^2 + a)*sqrt(f*x)/(f^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(d + ex^{2}\right)\left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}}{\left(fx\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)*(c*x**4+b*x**2+a)**(3/2)/(f*x)**(3/2),x)
[Out] Integral((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)/(f*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)*(c*x^4+b*x^2+a)^(3/2)/(f*x)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)/(f*x)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}{(f\,x)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2),x)
[Out] int(((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(f*x)^(3/2), x)
```

3.212 $\int \frac{(fx)^{3/2}(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\cdots$$

```
[Out] 2/5*d*(f*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)+2/9*e*(f*x)
^(9/2)*AppellF1(9/4,1/2,1/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{9}{4};\frac{1}{2},\frac{1}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9f^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 1/2, 1/2, 13/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(9*
f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
```

```
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \left(\frac{d(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{7/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$= d\int \frac{(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{(fx)^{7/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1 \cdots}}}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4 \cdots}}\right.}{5f\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 10.40, size = 354, normalized size = 1.19

$$\frac{2f\sqrt{fx}\left(5e\left(a+bx^2+cx^4\right)-5ae\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+(5cd-3be)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{25c\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^(3/2)*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*f*Sqrt[f*x]*(5*e*(a + b*x^2 + c*x^4) - 5*a*e*Sqrt[(b - Sqrt[b^2 - 4*a*c]
 + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt
[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (5*c*d - 3*b*e)*x^2*S
qrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sq
```

```
rt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2,
 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]
)]))/(25*c*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{\frac{3}{2}} (e\,x^2+d)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x^2*e + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((f*x^3*e + d*f*x)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{\frac{3}{2}} (d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)*(f*x)^(3/2)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^{3/2}\,\left(e\,x^2+d\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.213 $\int \frac{\sqrt{fx}\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=297

$$\frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\cdots$$

```
[Out] 2/3*d*(f*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)+2/7*e*(f*x)
^(7/2)*AppellF1(7/4,1/2,1/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/
(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7f^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (
-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(7*
f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{\sqrt{fx}\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx = \int \left(\frac{d\sqrt{fx}}{\sqrt{a+bx^2+cx^4}} + \frac{e(fx)^{5/2}}{f^2\sqrt{a+bx^2+cx^4}}\right) dx$$

$$= d\int \frac{\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}\,dx + \frac{e\int \frac{(fx)^{5/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{fx}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\cdots}}}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4\cdots}}\right.}{3f\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 15.14, size = 242, normalized size = 0.81

$$\frac{2\sqrt{fx}\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(7dxF_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+3ex^3F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{21\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f*x]*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (2*Sqrt[f*x]*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(7*d*x*Ap
```

```
pellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-
b + Sqrt[b^2 - 4*a*c])] + 3*e*x^3*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*Sqrt[a +
 b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f x}\,\left(e\,x^{2}+d\right)}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x^2*e + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral((x^2*e + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{fx}\,(d+ex^{2})}{\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(sqrt(f*x)*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)*sqrt(f*x)/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f\,x}\,\left(e\,x^2+d\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.214 $\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=295

$$\frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}+\frac{2e$$

```
[Out] 2/5*e*(f*x)^(5/2)*AppellF1(5/4,1/2,1/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)+2*d*Appel
lF1(1/4,1/2,1/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2
)^(1/2)))*(f*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(
b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5f^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[a + b*x^2 +
c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt
[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*
x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*f^3*S
qrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
```

```
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx = \int \left(\frac{d}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{3/2}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx$$

$$= d\int \frac{1}{\sqrt{fx}\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{(fx)^{3/2}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{}{\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-}}}}}{\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2}{b-\sqrt{b}}\right.}{f\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 10.13, size = 241, normalized size = 0.82

$$\frac{2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(5dxF_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+ex^3F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{5\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(Sqrt[f*x]*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(5*d*x*AppellF1[1/4
, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b
^2 - 4*a*c])] + e*x^3*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2
```

```
 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*Sqrt[f*x]*Sqrt[a + b*x
^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e\,x^{2}+d}{\sqrt{f x}\,\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x^2*e + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x)/(c*f*x^5 + b*f*x^3 +
 a*f*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^{2}}{\sqrt{fx}\,\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral((d + e*x**2)/(sqrt(f*x)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)/(sqrt(c*x^4 + b*x^2 + a)*sqrt(f*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{f\,x}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.215 $\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=295

$$-\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}}+\frac{2e(f}{}$$

```
[Out] 2/3*e*(f*x)^(3/2)*AppellF1(3/4,1/2,1/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(c*x^4+b*x^2+a)^(1/2)-2*d*Appel
lF1(-1/4,1/2,1/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^
2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))^(1/2)/f/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3f^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 1/2, 1/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*Sqrt[f*x]*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*f^3
*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
```

```
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}
\int \frac{d+ex^2}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx &= \int \left(\frac{d}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}+\frac{e\sqrt{fx}}{f^2\sqrt{a+bx^2+cx^4}}\right)dx \\
&= d\int \frac{1}{(fx)^{3/2}\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{\sqrt{fx}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2} \\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{}{(fx)^{3/2}\sqrt{1+\frac{2cx}{b-\sqrt{b^2}}}}}{\sqrt{a+bx^2+cx^4}} \\
&= -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{3}{4};-\frac{2c}{b-\sqrt{b^2}}\right.}{f\sqrt{fx}\sqrt{a+bx^2+cx^4}}
\end{aligned}$$

Mathematica [A]
time = 10.42, size = 356, normalized size = 1.21

$$\frac{2x\left(-21d(a+bx^2+cx^4)+7(bd+ae)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+9cdx^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{21a(fx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/((f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (2*x*(-21*d*(a + b*x^2 + c*x^4) + 7*(b*d + a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*d*x^4*Sqrt[(b
```

```
 - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4,
 (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/
(21*a*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{e\,x^{2}+d}{\left(f x\right)^{\frac{3}{2}} \sqrt{c\,x^{4}+b\,x^{2}+a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x^2*e + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x)/(c*f^2*x^6 + b*f^2*x
^4 + a*f^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d + ex^{2}}{\left(fx\right)^{\frac{3}{2}} \sqrt{a + bx^{2} + cx^{4}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((d + e*x**2)/((f*x)**(3/2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)/(sqrt(c*x^4 + b*x^2 + a)*(f*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{{\left(f\,x\right)}^{3/2}\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.216 $\int \frac{(fx)^{3/2}(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\cdots 2$$

```
[Out] 2/5*d*(f*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)+2/9*e*(f*
x)^(9/2)*AppellF1(9/4,3/2,3/2,13/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2d(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{9/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{9}{4};\frac{3}{2},\frac{3}{2};\frac{13}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{9af^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(9/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[9/4, 3/2, 3/2, 13/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
9*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
```

```
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{(fx)^{3/2}\left(d+ex^2\right)}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d(fx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e(fx)^{7/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{(fx)^{3/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{7/2}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^{3/2}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{\quad}{b+\ldots}\right.}}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2d(fx)^{5/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-\ldots}}\right.}{5af\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 10.50, size = 375, normalized size = 1.24

$$-\frac{f\sqrt{fx}\left(5(bd-2ae+2cdx^2-bex^2)-5(bd-2ae)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+(-2cd+be)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{5\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] -1/5*(f*Sqrt[f*x]*(5*(b*d - 2*a*e + 2*c*d*x^2 - b*e*x^2) - 5*(b*d - 2*a*e)*
Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 1/2, 1/2
, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
])] + (-2*c*d + b*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
```

```
)]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
2)/(-b + Sqrt[b^2 - 4*a*c])]))/((b^2 - 4*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{\frac{3}{2}} (e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((x^2*e + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(f*x^3*e + d*f*x)*sqrt(f*x)/(c^2*x^8 + 2*b
*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{\frac{3}{2}} (d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(3/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((f*x)**(3/2)*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(3/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)*(f*x)^(3/2)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^{3/2}\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((f*x)^(3/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.217 $\int \frac{\sqrt{fx}\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=303

$$\frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}}+\cdots$$

```
[Out] 2/3*d*(f*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)+2/7*e*(f*
x)^(7/2)*AppellF1(7/4,3/2,3/2,11/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^
2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2d(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{7/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{7}{4};\frac{3}{2},\frac{3}{2};\frac{11}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{7af^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (2*d*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a*f*Sqrt[a + b*
x^2 + c*x^4]) + (2*e*(f*x)^(7/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])
]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 3/2, 3/2, 11/4,
 (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(
7*a*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{fx}\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx &= \int \left(\frac{d\sqrt{fx}}{(a+bx^2+cx^4)^{3/2}}+\frac{e(fx)^{5/2}}{f^2\,(a+bx^2+cx^4)^{3/2}}\right)dx\\ &= d\int \frac{\sqrt{fx}}{(a+bx^2+cx^4)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{5/2}}{(a+bx^2+cx^4)^{3/2}}\,dx}{f^2}\\ &= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{\sqrt{fx}}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{\quad}{b+\cdots}\right.}\cdots}{a\sqrt{a+bx^2+cx^4}}\\ &= \frac{2d(fx)^{3/2}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-\cdots}}\right.\cdots}{3af\sqrt{a+bx^2+cx^4}}\end{aligned}$$

Mathematica [A]
time = 15.52, size = 397, normalized size = 1.31

$$\frac{x\sqrt{fx}\left(\left(-21b^2d+21b(ae-cdx^2)+42ac(d+ex^2)+7(b^2d+2acd-3abe)\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+9c(bd-2ae)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{21a\left(-b^2+4ac\right)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[f*x]*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*Sqrt[f*x]*(-21*b^2*d + 21*b*(a*e - c*d*x^2) + 42*a*c*(d + e*x^2) + 7*(b^
2*d + 2*a*c*d - 3*a*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b
^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^
```

```
2)/(-b + Sqrt[b^2 - 4*a*c])] + 9*c*(b*d - 2*a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4
*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/2, 11/4, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(21*a*(-b^2 + 4
*a*c)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{f x}\,\left(e\,x^{2}+d\right)}{\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x^2*e + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x)/(c^2*x^8 + 2*b*c*x^6
 + (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**(1/2)*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^(1/2)*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)*sqrt(f*x)/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{f\,x}\,\left(e\,x^2+d\right)}{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
```

```
[Out] int(((f*x)^(1/2)*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.218 $\int \frac{d+ex^2}{\sqrt{fx}\,(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=301

$$\frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}}+\cdots 2e$$

```
[Out] 2/5*e*(f*x)^(5/2)*AppellF1(5/4,3/2,3/2,9/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)+2*d*App
ellF1(1/4,3/2,3/2,5/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b
^2)^(1/2)))*(f*x)^(1/2)*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2
/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2d\sqrt{fx}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{a+bx^2+cx^4}}+\frac{2e(fx)^{5/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{5}{4};\frac{3}{2},\frac{3}{2};\frac{9}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{5af^3\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (2*d*Sqrt[f*x]*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^
2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/4, 3/2, 3/2, 5/4, (-2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[a + b*x^2
+ c*x^4]) + (2*e*(f*x)^(5/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sq
rt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[5/4, 3/2, 3/2, 9/4, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(5*a*f
^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
```

```
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{d+ex^2}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e(fx)^{3/2}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{1}{\sqrt{fx}\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{3/2}}{(a+bx^2+cx^4)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{}{\sqrt{fx}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4a}}\right.}}{a\sqrt{a+bx^2+cx^4}}$$

$$= \frac{2d\sqrt{fx}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{5}{4};-\frac{}{b-\sqrt{}}\right.}{af\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 10.47, size = 395, normalized size = 1.31

$$\frac{x\left(-5b^2d+5b(ae-cdx^2)+10ac(d+ex^2)-5(b^2d-6acd+abe)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1}{4};\frac{1}{2},\frac{1}{2};\frac{5}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+c(bd-2ae)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{5}{4};\frac{1}{2},\frac{1}{2};\frac{9}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{5a\left(-b^2+4ac\right)\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/(Sqrt[f*x]*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (x*(-5*b^2*d + 5*b*(a*e - c*d*x^2) + 10*a*c*(d + e*x^2) - 5*(b^2*d - 6*a*c*
d + a*b*e)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[1/
4, 1/2, 1/2, 5/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[
b^2 - 4*a*c])] + c*(b*d - 2*a*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)
/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[
```

```
b^2 - 4*a*c])]*AppellF1[5/4, 1/2, 1/2, 9/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*
c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(5*a*(-b^2 + 4*a*c)*Sqrt[f*x]*Sq
rt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e\,x^{2}+d}{\sqrt{f x}\,\left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
[Out] integrate((x^2*e + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x)/(c^2*f*x^9 + 2*b*c*f
*x^7 + (b^2 + 2*a*c)*f*x^5 + 2*a*b*f*x^3 + a^2*f*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^{2}}{\sqrt{fx}\,\left(a+bx^{2}+cx^{4}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(1/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/(sqrt(f*x)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)/((c*x^4 + b*x^2 + a)^(3/2)*sqrt(f*x)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{\sqrt{f\,x}\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int((d + e*x^2)/((f*x)^(1/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.219 $\int \frac{d+ex^2}{(fx)^{3/2}(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=301

$$-\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}}+\frac{2e(f}{}$$

```
[Out] 2/3*e*(f*x)^(3/2)*AppellF1(3/4,3/2,3/2,7/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(
1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(c*x^4+b*x^2+a)^(1/2)-2*d*App
ellF1(-1/4,3/2,3/2,3/4,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+
b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c
+b^2)^(1/2)))^(1/2)/a/f/(f*x)^(1/2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.25, antiderivative size = 301, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {1349, 1155, 524}

$$\frac{2e(fx)^{3/2}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{3}{4};\frac{3}{2},\frac{3}{2};\frac{7}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{3af^3\sqrt{a+bx^2+cx^4}}-\frac{2d\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*d*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + S
qrt[b^2 - 4*a*c])]*AppellF1[-1/4, 3/2, 3/2, 3/4, (-2*c*x^2)/(b - Sqrt[b^2 -
 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*Sqrt[f*x]*Sqrt[a + b*x^
2 + c*x^4]) + (2*e*(f*x)^(3/2)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[3/4, 3/2, 3/2, 7/4, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(3*a
*f^3*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
```

```
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int \frac{d+ex^2}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \int \left(\frac{d}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}+\frac{e\sqrt{fx}}{f^2\left(a+bx^2+cx^4\right)^{3/2}}\right)dx$$

$$= d\int \frac{1}{(fx)^{3/2}\left(a+bx^2+cx^4\right)^{3/2}}\,dx+\frac{e\int \frac{\sqrt{fx}}{\left(a+bx^2+cx^4\right)^{3/2}}\,dx}{f^2}$$

$$= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{}{(fx)^{3/2}\left(1+\frac{2cx^2}{b-\sqrt{b^2-4}}\right.}}{a\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{2d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(-\frac{1}{4};\frac{3}{2},\frac{3}{2};\frac{3}{4};-\frac{}{b-\sqrt{b}}\right.}{af\sqrt{fx}\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 10.66, size = 460, normalized size = 1.53

$$-\frac{x\left(-21\left(-3b^2dx^2(b+cx^2)+a^2c(8d-2ex^2)+a(10c^2dx^4+b^2(-2d+ex^2)+bcx^2(11d+ex^2))\right)+7(-3b^3d+9abcd+ab^2e+2a^2ce)x^2\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{3}{4};\frac{1}{2},\frac{1}{2};\frac{7}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)-9c(3b^2d-10acd-abe)x^4\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}F_1\left(\frac{7}{4};\frac{1}{2},\frac{1}{2};\frac{11}{4};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{21a^2\left(b^2-4ac\right)(fx)^{3/2}\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/21*(x*(-21*(-3*b^2*d*x^2*(b + c*x^2) + a^2*c*(8*d - 2*e*x^2) + a*(10*c^2
*d*x^4 + b^2*(-2*d + e*x^2) + b*c*x^2*(11*d + e*x^2))) + 7*(-3*b^3*d + 9*a*
b*c*d + a*b^2*e + 2*a^2*c*e)*x^2*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b
- Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2
- 4*a*c])]*AppellF1[3/4, 1/2, 1/2, 7/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
```

```
(2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] - 9*c*(3*b^2*d - 10*a*c*d - a*b*e)*x^4
*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[7/4, 1/2, 1/
2, 11/4, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a
*c])]))/(a^2*(b^2 - 4*a*c)*(f*x)^(3/2)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{e\,x^2+d}{(fx)^{\frac{3}{2}}\,(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

```
[Out] int((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima"
)
```

```
[Out] integrate((x^2*e + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas"
)
```

```
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*sqrt(f*x)/(c^2*f^2*x^10 + 2*b*
c*f^2*x^8 + (b^2 + 2*a*c)*f^2*x^6 + 2*a*b*f^2*x^4 + a^2*f^2*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{d+ex^2}{(fx)^{\frac{3}{2}}\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)/(f*x)**(3/2)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((d + e*x**2)/((f*x)**(3/2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)/(f*x)^(3/2)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)/((c*x^4 + b*x^2 + a)^(3/2)*(f*x)^(3/2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{e\,x^2+d}{{\left(f\,x\right)}^{3/2}\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int((d + e*x^2)/((f*x)^(3/2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.220 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx$

Optimal. Leaf size=243

$$\frac{a^3d(fx)^{1+m}}{f(1+m)}+\frac{a^2(3bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{3a(b^2d+acd+abe)\,(fx)^{5+m}}{f^5(5+m)}+\frac{(b^3d+6abcd+3ab^2e+3a^2ce)\,(fx)^{7+m}}{f^7(7+m)}$$

```
[Out] a^3*d*(f*x)^(1+m)/f/(1+m)+a^2*(a*e+3*b*d)*(f*x)^(3+m)/f^3/(3+m)+3*a*(a*b*e+
a*c*d+b^2*d)*(f*x)^(5+m)/f^5/(5+m)+(3*a^2*c*e+3*a*b^2*e+6*a*b*c*d+b^3*d)*(f
*x)^(7+m)/f^7/(7+m)+(6*a*b*c*e+3*a*c^2*d+b^3*e+3*b^2*c*d)*(f*x)^(9+m)/f^9/(
9+m)+3*c*(a*c*e+b^2*e+b*c*d)*(f*x)^(11+m)/f^11/(11+m)+c^2*(3*b*e+c*d)*(f*x)
^(13+m)/f^13/(13+m)+c^3*e*(f*x)^(15+m)/f^15/(15+m)
```

Rubi [A]

time = 0.11, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = {1275}

$$\frac{a^3d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\,(3a^2ce+3ab^2e+6abcd+b^3d)}{f^7(m+7)}+\frac{a^2(fx)^{m+3}(ae+3bd)}{f^3(m+3)}+\frac{3c(fx)^{m+11}\,(ace+b^2e+bcd)}{f^{11}(m+11)}+\frac{3a(fx)^{m+5}\,(abe+acd+b^2d)}{f^5(m+5)}+\frac{(fx)^{m+9}\,(6abce+3ac^2d+b^3e+3b^2cd)}{f^9(m+9)}+\frac{c^2(fx)^{m+13}(3be+cd)}{f^{13}(m+13)}+\frac{c^3e(fx)^{m+15}}{f^{15}(m+15)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] (a^3*d*(f*x)^(1 + m))/(f*(1 + m)) + (a^2*(3*b*d + a*e)*(f*x)^(3 + m))/(f^3*
(3 + m)) + (3*a*(b^2*d + a*c*d + a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((b^
3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + ((3
*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*(f*x)^(9 + m))/(f^9*(9 + m)) + (3
*c*(b*c*d + b^2*e + a*c*e)*(f*x)^(11 + m))/(f^11*(11 + m)) + (c^2*(c*d + 3*
b*e)*(f*x)^(13 + m))/(f^13*(13 + m)) + (c^3*e*(f*x)^(15 + m))/(f^15*(15 + m
))
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^3 \, dx &= \int \left(a^3d(fx)^m+\frac{a^2(3bd+ae)(fx)^{2+m}}{f^2}+\frac{3a(b^2d+acd+abe)\,(f}{f^4}\right.\\ &= \frac{a^3d(fx)^{1+m}}{f(1+m)}+\frac{a^2(3bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{3a(b^2d+acd+abe)\,(fx)}{f^5(5+m)}\end{aligned}$$

Mathematica [A]
time = 0.77, size = 191, normalized size = 0.79

$$x(fx)^m\left(\frac{a^3d}{1+m}+\frac{a^2(3bd+ae)x^2}{3+m}+\frac{3a(b^2d+acd+abe)\,x^4}{5+m}+\frac{(b^3d+6abcd+3ab^2e+3a^2ce)\,x^6}{7+m}+\frac{(3b^2cd+3ac^2d+b^3e+6abce)\,x^8}{9+m}+\frac{3c(bcd+b^2e+ace)\,x^{10}}{11+m}+\frac{c^2(cd+3be)x^{12}}{13+m}+\frac{c^3ex^{14}}{15+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x]
```

```
[Out] x*(f*x)^m*((a^3*d)/(1 + m) + (a^2*(3*b*d + a*e)*x^2)/(3 + m) + (3*a*(b^2*d
+ a*c*d + a*b*e)*x^4)/(5 + m) + ((b^3*d + 6*a*b*c*d + 3*a*b^2*e + 3*a^2*c*e
)*x^6)/(7 + m) + ((3*b^2*c*d + 3*a*c^2*d + b^3*e + 6*a*b*c*e)*x^8)/(9 + m)
+ (3*c*(b*c*d + b^2*e + a*c*e)*x^10)/(11 + m) + (c^2*(c*d + 3*b*e)*x^12)/(1
3 + m) + (c^3*e*x^14)/(15 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1934 vs. 2(243) = 486.
time = 0.03, size = 1935, normalized size = 7.96

method	result	size
gosper	Expression too large to display	1935
risch	Expression too large to display	1935

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

```
[Out] x*(c^3*e*m^7*x^14+49*c^3*e*m^6*x^14+3*b*c^2*e*m^7*x^12+c^3*d*m^7*x^12+973*c
^3*e*m^5*x^14+153*b*c^2*e*m^6*x^12+51*c^3*d*m^6*x^12+10045*c^3*e*m^4*x^14+3
*a*c^2*e*m^7*x^10+3*b^2*c*e*m^7*x^10+3*b*c^2*d*m^7*x^10+3135*b*c^2*e*m^5*x^
12+1045*c^3*d*m^5*x^12+57379*c^3*e*m^3*x^14+159*a*c^2*e*m^6*x^10+159*b^2*c*
e*m^6*x^10+159*b*c^2*d*m^6*x^10+33165*b*c^2*e*m^4*x^12+11055*c^3*d*m^4*x^12
+177331*c^3*e*m^2*x^14+6*a*b*c*e*m^7*x^8+3*a*c^2*d*m^7*x^8+3375*a*c^2*e*m^5
*x^10+b^3*e*m^7*x^8+3*b^2*c*d*m^7*x^8+3375*b^2*c*e*m^5*x^10+3375*b*c^2*d*m^
5*x^10+193017*b*c^2*e*m^3*x^12+64339*c^3*d*m^3*x^12+264207*c^3*e*m*x^14+330
*a*b*c*e*m^6*x^8+165*a*c^2*d*m^6*x^8+36795*a*c^2*e*m^4*x^10+55*b^3*e*m^6*x^
8+165*b^2*c*d*m^6*x^8+36795*b^2*c*e*m^4*x^10+36795*b*c^2*d*m^4*x^10+604827*
b*c^2*e*m^2*x^12+201609*c^3*d*m^2*x^12+135135*c^3*e*x^14+3*a^2*c*e*m^7*x^6+
3*a*b^2*e*m^7*x^6+6*a*b*c*d*m^7*x^6+7278*a*b*c*e*m^5*x^8+3639*a*c^2*d*m^5*x
^8+219417*a*c^2*e*m^3*x^10+b^3*d*m^7*x^6+1213*b^3*e*m^5*x^8+3639*b^2*c*d*m^
5*x^8+219417*b^2*c*e*m^3*x^10+219417*b*c^2*d*m^3*x^10+909765*b*c^2*e*m*x^12
+303255*c^3*d*m*x^12+171*a^2*c*e*m^6*x^6+171*a*b^2*e*m^6*x^6+342*a*b*c*d*m^
6*x^6+82338*a*b*c*e*m^4*x^8+41169*a*c^2*d*m^4*x^8+700461*a*c^2*e*m^2*x^10+5
7*b^3*d*m^6*x^6+13723*b^3*e*m^4*x^8+41169*b^2*c*d*m^4*x^8+700461*b^2*c*e*m^
2*x^10+700461*b*c^2*d*m^2*x^10+467775*b*c^2*e*x^12+155925*c^3*d*x^12+3*a^2*
b*e*m^7*x^4+3*a^2*c*d*m^7*x^4+3927*a^2*c*e*m^5*x^6+3*a*b^2*d*m^7*x^4+3927*a
*b^2*e*m^5*x^6+7854*a*b*c*d*m^5*x^6+507282*a*b*c*e*m^3*x^8+253641*a*c^2*d*m
```

```
^3*x^8+1067445*a*c^2*e*m*x^10+1309*b^3*d*m^5*x^6+84547*b^3*e*m^3*x^8+253641
*b^2*c*d*m^3*x^8+1067445*b^2*c*e*m*x^10+1067445*b*c^2*d*m*x^10+177*a^2*b*e*
m^6*x^4+177*a^2*c*d*m^6*x^4+46431*a^2*c*e*m^4*x^6+177*a*b^2*d*m^6*x^4+46431
*a*b^2*e*m^4*x^6+92862*a*b*c*d*m^4*x^6+1662558*a*b*c*e*m^2*x^8+831279*a*c^2
*d*m^2*x^8+552825*a*c^2*e*x^10+15477*b^3*d*m^4*x^6+277093*b^3*e*m^2*x^8+831
279*b^2*c*d*m^2*x^8+552825*b^2*c*e*x^10+552825*b*c^2*d*x^10+a^3*e*m^7*x^2+3
*a^2*b*d*m^7*x^2+4239*a^2*b*e*m^5*x^4+4239*a^2*c*d*m^5*x^4+299145*a^2*c*e*m
^3*x^6+4239*a*b^2*d*m^5*x^4+299145*a*b^2*e*m^3*x^6+598290*a*b*c*d*m^3*x^6+2
582010*a*b*c*e*m*x^8+1291005*a*c^2*d*m*x^8+99715*b^3*d*m^3*x^6+430335*b^3*e
*m*x^8+1291005*b^2*c*d*m*x^8+61*a^3*e*m^6*x^2+183*a^2*b*d*m^6*x^2+52725*a^2
*b*e*m^4*x^4+52725*a^2*c*d*m^4*x^4+1020033*a^2*c*e*m^2*x^6+52725*a*b^2*d*m^
4*x^4+1020033*a*b^2*e*m^2*x^6+2040066*a*b*c*d*m^2*x^6+1351350*a*b*c*e*x^8+6
75675*a*c^2*d*x^8+340011*b^3*d*m^2*x^6+225225*b^3*e*x^8+675675*b^2*c*d*x^8+
a^3*d*m^7+1525*a^3*e*m^5*x^2+4575*a^2*b*d*m^5*x^2+360537*a^2*b*e*m^3*x^4+36
0537*a^2*c*d*m^3*x^4+1632285*a^2*c*e*m*x^6+360537*a*b^2*d*m^3*x^4+1632285*a
*b^2*e*m*x^6+3264570*a*b*c*d*m*x^6+544095*b^3*d*m*x^6+63*a^3*d*m^6+20065*a^
3*e*m^4*x^2+60195*a^2*b*d*m^4*x^2+1311363*a^2*b*e*m^2*x^4+1311363*a^2*c*d*m
^2*x^4+868725*a^2*c*e*x^6+1311363*a*b^2*d*m^2*x^4+868725*a*b^2*e*x^6+173745
0*a*b*c*d*x^6+289575*b^3*d*x^6+1645*a^3*d*m^5+147859*a^3*e*m^3*x^2+443577*a
^2*b*d*m^3*x^2+2215701*a^2*b*e*m*x^4+2215701*a^2*c*d*m*x^4+2215701*a*b^2*d*
m*x^4+22995*a^3*d*m^4+594439*a^3*e*m^2*x^2+1783317*a^2*b*d*m^2*x^2+1216215*
a^2*b*e*x^4+1216215*a^2*c*d*x^4+1216215*a*b^2*d*x^4+185059*a^3*d*m^3+114085
5*a^3*e*m*x^2+3422565*a^2*b*d*m*x^2+852957*a^3*d*m^2+675675*a^3*e*x^2+20270
25*a^2*b*d*x^2+2071215*a^3*d*m+2027025*a^3*d)*(f*x)^m/(1+m)/(3+m)/(5+m)/(7+
m)/(9+m)/(11+m)/(13+m)/(15+m)
```

Maxima [A]
time = 0.31, size = 438, normalized size = 1.80

$$\frac{c^3 f^m x^{15} e^{(m\log(x)+1)}}{m+15}+\frac{c^3 d f^m x^{13} x^m}{m+13}+\frac{3\,bc^2 f^m x^{13} e^{(m\log(x)+1)}}{m+13}+\frac{3\,bc^2 d f^m x^{11} x^m}{m+11}+\frac{3\,b^2 c f^m x^{11} e^{(m\log(x)+1)}}{m+11}+\frac{3\,ac^2 f^m x^{11} e^{(m\log(x)+1)}}{m+11}+\frac{3\,b^2 c d f^m x^9 x^m}{m+9}+\frac{3\,ac^2 d f^m x^9 x^m}{m+9}+\frac{b^3 f^m x^9 e^{(m\log(x)+1)}}{m+9}+\frac{6\,abc f^m x^9 e^{(m\log(x)+1)}}{m+9}+\frac{b^3 d f^m x^7 x^m}{m+7}+\frac{6\,abcd f^m x^7 x^m}{m+7}+\frac{3\,ab^2 f^m x^7 e^{(m\log(x)+1)}}{m+7}+\frac{3\,a^2 c f^m x^7 e^{(m\log(x)+1)}}{m+7}+\frac{3\,ab^2 d f^m x^5 x^m}{m+5}+\frac{3\,a^2 c d f^m x^5 x^m}{m+5}+\frac{3\,a^2 b f^m x^5 e^{(m\log(x)+1)}}{m+5}+\frac{3\,a^2 b d f^m x^3 x^m}{m+3}+\frac{a^3 f^m x^3 e^{(m\log(x)+1)}}{m+3}+\frac{(fx)^{m+1} a^3 d}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="maxima")
```

```
[Out] c^3*f^m*x^15*e^(m*log(x) + 1)/(m + 15) + c^3*d*f^m*x^13*x^m/(m + 13) + 3*b*
c^2*f^m*x^13*e^(m*log(x) + 1)/(m + 13) + 3*b*c^2*d*f^m*x^11*x^m/(m + 11) +
3*b^2*c*f^m*x^11*e^(m*log(x) + 1)/(m + 11) + 3*a*c^2*f^m*x^11*e^(m*log(x) +
 1)/(m + 11) + 3*b^2*c*d*f^m*x^9*x^m/(m + 9) + 3*a*c^2*d*f^m*x^9*x^m/(m + 9
) + b^3*f^m*x^9*e^(m*log(x) + 1)/(m + 9) + 6*a*b*c*f^m*x^9*e^(m*log(x) + 1)
/(m + 9) + b^3*d*f^m*x^7*x^m/(m + 7) + 6*a*b*c*d*f^m*x^7*x^m/(m + 7) + 3*a*
b^2*f^m*x^7*e^(m*log(x) + 1)/(m + 7) + 3*a^2*c*f^m*x^7*e^(m*log(x) + 1)/(m
+ 7) + 3*a*b^2*d*f^m*x^5*x^m/(m + 5) + 3*a^2*c*d*f^m*x^5*x^m/(m + 5) + 3*a^
2*b*f^m*x^5*e^(m*log(x) + 1)/(m + 5) + 3*a^2*b*d*f^m*x^3*x^m/(m + 3) + a^3*
f^m*x^3*e^(m*log(x) + 1)/(m + 3) + (f*x)^(m + 1)*a^3*d/(f*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1376 vs. $2(253) = 506$.

time = 0.39, size = 1376, normalized size = 5.66

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="fricas")
```

```
[Out] ((c^3*d*m^7 + 51*c^3*d*m^6 + 1045*c^3*d*m^5 + 11055*c^3*d*m^4 + 64339*c^3*d
*m^3 + 201609*c^3*d*m^2 + 303255*c^3*d*m + 155925*c^3*d)*x^13 + 3*(b*c^2*d*
m^7 + 53*b*c^2*d*m^6 + 1125*b*c^2*d*m^5 + 12265*b*c^2*d*m^4 + 73139*b*c^2*d
*m^3 + 233487*b*c^2*d*m^2 + 355815*b*c^2*d*m + 184275*b*c^2*d)*x^11 + 3*((b
^2*c + a*c^2)*d*m^7 + 55*(b^2*c + a*c^2)*d*m^6 + 1213*(b^2*c + a*c^2)*d*m^5
 + 13723*(b^2*c + a*c^2)*d*m^4 + 84547*(b^2*c + a*c^2)*d*m^3 + 277093*(b^2*
c + a*c^2)*d*m^2 + 430335*(b^2*c + a*c^2)*d*m + 225225*(b^2*c + a*c^2)*d)*x
^9 + ((b^3 + 6*a*b*c)*d*m^7 + 57*(b^3 + 6*a*b*c)*d*m^6 + 1309*(b^3 + 6*a*b*
c)*d*m^5 + 15477*(b^3 + 6*a*b*c)*d*m^4 + 99715*(b^3 + 6*a*b*c)*d*m^3 + 3400
11*(b^3 + 6*a*b*c)*d*m^2 + 544095*(b^3 + 6*a*b*c)*d*m + 289575*(b^3 + 6*a*b
*c)*d)*x^7 + 3*((a*b^2 + a^2*c)*d*m^7 + 59*(a*b^2 + a^2*c)*d*m^6 + 1413*(a*
b^2 + a^2*c)*d*m^5 + 17575*(a*b^2 + a^2*c)*d*m^4 + 120179*(a*b^2 + a^2*c)*d
*m^3 + 437121*(a*b^2 + a^2*c)*d*m^2 + 738567*(a*b^2 + a^2*c)*d*m + 405405*(
a*b^2 + a^2*c)*d)*x^5 + 3*(a^2*b*d*m^7 + 61*a^2*b*d*m^6 + 1525*a^2*b*d*m^5
+ 20065*a^2*b*d*m^4 + 147859*a^2*b*d*m^3 + 594439*a^2*b*d*m^2 + 1140855*a^2
*b*d*m + 675675*a^2*b*d)*x^3 + (a^3*d*m^7 + 63*a^3*d*m^6 + 1645*a^3*d*m^5 +
 22995*a^3*d*m^4 + 185059*a^3*d*m^3 + 852957*a^3*d*m^2 + 2071215*a^3*d*m +
2027025*a^3*d)*x + ((c^3*m^7 + 49*c^3*m^6 + 973*c^3*m^5 + 10045*c^3*m^4 + 5
7379*c^3*m^3 + 177331*c^3*m^2 + 264207*c^3*m + 135135*c^3)*x^15 + 3*(b*c^2*
m^7 + 51*b*c^2*m^6 + 1045*b*c^2*m^5 + 11055*b*c^2*m^4 + 64339*b*c^2*m^3 + 2
01609*b*c^2*m^2 + 303255*b*c^2*m + 155925*b*c^2)*x^13 + 3*((b^2*c + a*c^2)*
m^7 + 53*(b^2*c + a*c^2)*m^6 + 1125*(b^2*c + a*c^2)*m^5 + 12265*(b^2*c + a*
c^2)*m^4 + 73139*(b^2*c + a*c^2)*m^3 + 184275*b^2*c + 184275*a*c^2 + 233487
*(b^2*c + a*c^2)*m^2 + 355815*(b^2*c + a*c^2)*m)*x^11 + ((b^3 + 6*a*b*c)*m^
7 + 55*(b^3 + 6*a*b*c)*m^6 + 1213*(b^3 + 6*a*b*c)*m^5 + 13723*(b^3 + 6*a*b*
c)*m^4 + 84547*(b^3 + 6*a*b*c)*m^3 + 225225*b^3 + 1351350*a*b*c + 277093*(b
^3 + 6*a*b*c)*m^2 + 430335*(b^3 + 6*a*b*c)*m)*x^9 + 3*((a*b^2 + a^2*c)*m^7
+ 57*(a*b^2 + a^2*c)*m^6 + 1309*(a*b^2 + a^2*c)*m^5 + 15477*(a*b^2 + a^2*c)
*m^4 + 99715*(a*b^2 + a^2*c)*m^3 + 289575*a*b^2 + 289575*a^2*c + 340011*(a*
b^2 + a^2*c)*m^2 + 544095*(a*b^2 + a^2*c)*m)*x^7 + 3*(a^2*b*m^7 + 59*a^2*b*
m^6 + 1413*a^2*b*m^5 + 17575*a^2*b*m^4 + 120179*a^2*b*m^3 + 437121*a^2*b*m^
2 + 738567*a^2*b*m + 405405*a^2*b)*x^5 + (a^3*m^7 + 61*a^3*m^6 + 1525*a^3*m
^5 + 20065*a^3*m^4 + 147859*a^3*m^3 + 594439*a^3*m^2 + 1140855*a^3*m + 6756
75*a^3)*x^3)*e)*(f*x)^m/(m^8 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 +
 1038016*m^3 + 2924172*m^2 + 4098240*m + 2027025)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 11266 vs. $2(238) = 476$.

time = 1.62, size = 11266, normalized size = 46.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**3,x)
```

```
[Out] Piecewise(((-a**3*d/(14*x**14) - a**3*e/(12*x**12) - a**2*b*d/(4*x**12) - 3
*a**2*b*e/(10*x**10) - 3*a**2*c*d/(10*x**10) - 3*a**2*c*e/(8*x**8) - 3*a*b*
*2*d/(10*x**10) - 3*a*b**2*e/(8*x**8) - 3*a*b*c*d/(4*x**8) - a*b*c*e/x**6 -
 a*c**2*d/(2*x**6) - 3*a*c**2*e/(4*x**4) - b**3*d/(8*x**8) - b**3*e/(6*x**6
) - b**2*c*d/(2*x**6) - 3*b**2*c*e/(4*x**4) - 3*b*c**2*d/(4*x**4) - 3*b*c**
2*e/(2*x**2) - c**3*d/(2*x**2) + c**3*e*log(x))/f**15, Eq(m, -15)), ((-a**3
*d/(12*x**12) - a**3*e/(10*x**10) - 3*a**2*b*d/(10*x**10) - 3*a**2*b*e/(8*x
**8) - 3*a**2*c*d/(8*x**8) - a**2*c*e/(2*x**6) - 3*a*b**2*d/(8*x**8) - a*b*
*2*e/(2*x**6) - a*b*c*d/x**6 - 3*a*b*c*e/(2*x**4) - 3*a*c**2*d/(4*x**4) - 3
*a*c**2*e/(2*x**2) - b**3*d/(6*x**6) - b**3*e/(4*x**4) - 3*b**2*c*d/(4*x**4
) - 3*b**2*c*e/(2*x**2) - 3*b*c**2*d/(2*x**2) + 3*b*c**2*e*log(x) + c**3*d*
log(x) + c**3*e*x**2/2)/f**13, Eq(m, -13)), ((-a**3*d/(10*x**10) - a**3*e/(
8*x**8) - 3*a**2*b*d/(8*x**8) - a**2*b*e/(2*x**6) - a**2*c*d/(2*x**6) - 3*a
**2*c*e/(4*x**4) - a*b**2*d/(2*x**6) - 3*a*b**2*e/(4*x**4) - 3*a*b*c*d/(2*x
**4) - 3*a*b*c*e/x**2 - 3*a*c**2*d/(2*x**2) + 3*a*c**2*e*log(x) - b**3*d/(4
*x**4) - b**3*e/(2*x**2) - 3*b**2*c*d/(2*x**2) + 3*b**2*c*e*log(x) + 3*b*c*
*2*d*log(x) + 3*b*c**2*e*x**2/2 + c**3*d*x**2/2 + c**3*e*x**4/4)/f**11, Eq(
m, -11)), ((-a**3*d/(8*x**8) - a**3*e/(6*x**6) - a**2*b*d/(2*x**6) - 3*a**2
*b*e/(4*x**4) - 3*a**2*c*d/(4*x**4) - 3*a**2*c*e/(2*x**2) - 3*a*b**2*d/(4*x
**4) - 3*a*b**2*e/(2*x**2) - 3*a*b*c*d/x**2 + 6*a*b*c*e*log(x) + 3*a*c**2*d
*log(x) + 3*a*c**2*e*x**2/2 - b**3*d/(2*x**2) + b**3*e*log(x) + 3*b**2*c*d*
log(x) + 3*b**2*c*e*x**2/2 + 3*b*c**2*d*x**2/2 + 3*b*c**2*e*x**4/4 + c**3*d
*x**4/4 + c**3*e*x**6/6)/f**9, Eq(m, -9)), ((-a**3*d/(6*x**6) - a**3*e/(4*x
**4) - 3*a**2*b*d/(4*x**4) - 3*a**2*b*e/(2*x**2) - 3*a**2*c*d/(2*x**2) + 3*
a**2*c*e*log(x) - 3*a*b**2*d/(2*x**2) + 3*a*b**2*e*log(x) + 6*a*b*c*d*log(x
) + 3*a*b*c*e*x**2 + 3*a*c**2*d*x**2/2 + 3*a*c**2*e*x**4/4 + b**3*d*log(x)
+ b**3*e*x**2/2 + 3*b**2*c*d*x**2/2 + 3*b**2*c*e*x**4/4 + 3*b*c**2*d*x**4/4
 + b*c**2*e*x**6/2 + c**3*d*x**6/6 + c**3*e*x**8/8)/f**7, Eq(m, -7)), ((-a*
*3*d/(4*x**4) - a**3*e/(2*x**2) - 3*a**2*b*d/(2*x**2) + 3*a**2*b*e*log(x) +
 3*a**2*c*d*log(x) + 3*a**2*c*e*x**2/2 + 3*a*b**2*d*log(x) + 3*a*b**2*e*x**
2/2 + 3*a*b*c*d*x**2 + 3*a*b*c*e*x**4/2 + 3*a*c**2*d*x**4/4 + a*c**2*e*x**6
/2 + b**3*d*x**2/2 + b**3*e*x**4/4 + 3*b**2*c*d*x**4/4 + b**2*c*e*x**6/2 +
b*c**2*d*x**6/2 + 3*b*c**2*e*x**8/8 + c**3*d*x**8/8 + c**3*e*x**10/10)/f**5
, Eq(m, -5)), ((-a**3*d/(2*x**2) + a**3*e*log(x) + 3*a**2*b*d*log(x) + 3*a*
*2*b*e*x**2/2 + 3*a**2*c*d*x**2/2 + 3*a**2*c*e*x**4/4 + 3*a*b**2*d*x**2/2 +
 3*a*b**2*e*x**4/4 + 3*a*b*c*d*x**4/2 + a*b*c*e*x**6 + a*c**2*d*x**6/2 + 3*
a*c**2*e*x**8/8 + b**3*d*x**4/4 + b**3*e*x**6/6 + b**2*c*d*x**6/2 + 3*b**2*
c*e*x**8/8 + 3*b*c**2*d*x**8/8 + 3*b*c**2*e*x**10/10 + c**3*d*x**10/10 + c*
```

```
*3*e*x**12/12)/f**3, Eq(m, -3)), ((a**3*d*log(x) + a**3*e*x**2/2 + 3*a**2*b
*d*x**2/2 + 3*a**2*b*e*x**4/4 + 3*a**2*c*d*x**4/4 + a**2*c*e*x**6/2 + 3*a*b
**2*d*x**4/4 + a*b**2*e*x**6/2 + a*b*c*d*x**6 + 3*a*b*c*e*x**8/4 + 3*a*c**2
*d*x**8/8 + 3*a*c**2*e*x**10/10 + b**3*d*x**6/6 + b**3*e*x**8/8 + 3*b**2*c*
d*x**8/8 + 3*b**2*c*e*x**10/10 + 3*b*c**2*d*x**10/10 + b*c**2*e*x**12/4 + c
**3*d*x**12/12 + c**3*e*x**14/14)/f, Eq(m, -1)), (a**3*d*m**7*x*(f*x)**m/(m
**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 29241
72*m**2 + 4098240*m + 2027025) + 63*a**3*d*m**6*x*(f*x)**m/(m**8 + 64*m**7
+ 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098
240*m + 2027025) + 1645*a**3*d*m**5*x*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6
+ 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027
025) + 22995*a**3*d*m**4*x*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**
5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 1850
59*a**3*d*m**3*x*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054
*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 852957*a**3*d*
m**2*x*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 10
38016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 2071215*a**3*d*m*x*(f*x)
**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 +
 2924172*m**2 + 4098240*m + 2027025) + 2027025*a**3*d*x*(f*x)**m/(m**8 + 64
*m**7 + 1708*m**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2
+ 4098240*m + 2027025) + a**3*e*m**7*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m
**6 + 24640*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m +
2027025) + 61*a**3*e*m**6*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640
*m**5 + 208054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) +
1525*a**3*e*m**5*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 2
08054*m**4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 20065*a**
3*e*m**4*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 24640*m**5 + 208054*m*
*4 + 1038016*m**3 + 2924172*m**2 + 4098240*m + 2027025) + 147859*a**3*e*m**
3*x**3*(f*x)**m/(m**8 + 64*m**7 + 1708*m**6 + 2...
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 2816 vs. $2(253) = 506$.
time = 5.87, size = 2816, normalized size = 11.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^3,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^3*m^7*x^15*e + 49*(f*x)^m*c^3*m^6*x^15*e + (f*x)^m*c^3*d*m^7*x^1
3 + 3*(f*x)^m*b*c^2*m^7*x^13*e + 973*(f*x)^m*c^3*m^5*x^15*e + 51*(f*x)^m*c^
3*d*m^6*x^13 + 153*(f*x)^m*b*c^2*m^6*x^13*e + 10045*(f*x)^m*c^3*m^4*x^15*e
+ 3*(f*x)^m*b*c^2*d*m^7*x^11 + 1045*(f*x)^m*c^3*d*m^5*x^13 + 3*(f*x)^m*b^2*
c*m^7*x^11*e + 3*(f*x)^m*a*c^2*m^7*x^11*e + 3135*(f*x)^m*b*c^2*m^5*x^13*e +
 57379*(f*x)^m*c^3*m^3*x^15*e + 159*(f*x)^m*b*c^2*d*m^6*x^11 + 11055*(f*x)^
```

m*c^3*d*m^4*x^13 + 159*(f*x)^m*b^2*c*m^6*x^11*e + 159*(f*x)^m*a*c^2*m^6*x^1
1*e + 33165*(f*x)^m*b*c^2*m^4*x^13*e + 177331*(f*x)^m*c^3*m^2*x^15*e + 3*(f
*x)^m*b^2*c*d*m^7*x^9 + 3*(f*x)^m*a*c^2*d*m^7*x^9 + 3375*(f*x)^m*b*c^2*d*m^
5*x^11 + 64339*(f*x)^m*c^3*d*m^3*x^13 + (f*x)^m*b^3*m^7*x^9*e + 6*(f*x)^m*a
*b*c*m^7*x^9*e + 3375*(f*x)^m*b^2*c*m^5*x^11*e + 3375*(f*x)^m*a*c^2*m^5*x^1
1*e + 193017*(f*x)^m*b*c^2*m^3*x^13*e + 264207*(f*x)^m*c^3*m*x^15*e + 165*(
f*x)^m*b^2*c*d*m^6*x^9 + 165*(f*x)^m*a*c^2*d*m^6*x^9 + 36795*(f*x)^m*b*c^2*
d*m^4*x^11 + 201609*(f*x)^m*c^3*d*m^2*x^13 + 55*(f*x)^m*b^3*m^6*x^9*e + 330
*(f*x)^m*a*b*c*m^6*x^9*e + 36795*(f*x)^m*b^2*c*m^4*x^11*e + 36795*(f*x)^m*a
*c^2*m^4*x^11*e + 604827*(f*x)^m*b*c^2*m^2*x^13*e + 135135*(f*x)^m*c^3*x^15
*e + (f*x)^m*b^3*d*m^7*x^7 + 6*(f*x)^m*a*b*c*d*m^7*x^7 + 3639*(f*x)^m*b^2*c
*d*m^5*x^9 + 3639*(f*x)^m*a*c^2*d*m^5*x^9 + 219417*(f*x)^m*b*c^2*d*m^3*x^11
 + 303255*(f*x)^m*c^3*d*m*x^13 + 3*(f*x)^m*a*b^2*m^7*x^7*e + 3*(f*x)^m*a^2*
c*m^7*x^7*e + 1213*(f*x)^m*b^3*m^5*x^9*e + 7278*(f*x)^m*a*b*c*m^5*x^9*e + 2
19417*(f*x)^m*b^2*c*m^3*x^11*e + 219417*(f*x)^m*a*c^2*m^3*x^11*e + 909765*(
f*x)^m*b*c^2*m*x^13*e + 57*(f*x)^m*b^3*d*m^6*x^7 + 342*(f*x)^m*a*b*c*d*m^6*
x^7 + 41169*(f*x)^m*b^2*c*d*m^4*x^9 + 41169*(f*x)^m*a*c^2*d*m^4*x^9 + 70046
1*(f*x)^m*b*c^2*d*m^2*x^11 + 155925*(f*x)^m*c^3*d*x^13 + 171*(f*x)^m*a*b^2*
m^6*x^7*e + 171*(f*x)^m*a^2*c*m^6*x^7*e + 13723*(f*x)^m*b^3*m^4*x^9*e + 823
38*(f*x)^m*a*b*c*m^4*x^9*e + 700461*(f*x)^m*b^2*c*m^2*x^11*e + 700461*(f*x)
^m*a*c^2*m^2*x^11*e + 467775*(f*x)^m*b*c^2*x^13*e + 3*(f*x)^m*a*b^2*d*m^7*x
^5 + 3*(f*x)^m*a^2*c*d*m^7*x^5 + 1309*(f*x)^m*b^3*d*m^5*x^7 + 7854*(f*x)^m*
a*b*c*d*m^5*x^7 + 253641*(f*x)^m*b^2*c*d*m^3*x^9 + 253641*(f*x)^m*a*c^2*d*m
^3*x^9 + 1067445*(f*x)^m*b*c^2*d*m*x^11 + 3*(f*x)^m*a^2*b*m^7*x^5*e + 3927*
(f*x)^m*a*b^2*m^5*x^7*e + 3927*(f*x)^m*a^2*c*m^5*x^7*e + 84547*(f*x)^m*b^3*
m^3*x^9*e + 507282*(f*x)^m*a*b*c*m^3*x^9*e + 1067445*(f*x)^m*b^2*c*m*x^11*e
 + 1067445*(f*x)^m*a*c^2*m*x^11*e + 177*(f*x)^m*a*b^2*d*m^6*x^5 + 177*(f*x)
^m*a^2*c*d*m^6*x^5 + 15477*(f*x)^m*b^3*d*m^4*x^7 + 92862*(f*x)^m*a*b*c*d*m^
4*x^7 + 831279*(f*x)^m*b^2*c*d*m^2*x^9 + 831279*(f*x)^m*a*c^2*d*m^2*x^9 + 5
52825*(f*x)^m*b*c^2*d*x^11 + 177*(f*x)^m*a^2*b*m^6*x^5*e + 46431*(f*x)^m*a*
b^2*m^4*x^7*e + 46431*(f*x)^m*a^2*c*m^4*x^7*e + 277093*(f*x)^m*b^3*m^2*x^9*
e + 1662558*(f*x)^m*a*b*c*m^2*x^9*e + 552825*(f*x)^m*b^2*c*x^11*e + 552825*
(f*x)^m*a*c^2*x^11*e + 3*(f*x)^m*a^2*b*d*m^7*x^3 + 4239*(f*x)^m*a*b^2*d*m^5
x^5 + 4239(f*x)^m*a^2*c*d*m^5*x^5 + 99715*(f*x)^m*b^3*d*m^3*x^7 + 598290*
(f*x)^m*a*b*c*d*m^3*x^7 + 1291005*(f*x)^m*b^2*c*d*m*x^9 + 1291005*(f*x)^m*a
*c^2*d*m*x^9 + (f*x)^m*a^3*m^7*x^3*e + 4239*(f*x)^m*a^2*b*m^5*x^5*e + 29914
5*(f*x)^m*a*b^2*m^3*x^7*e + 299145*(f*x)^m*a^2*c*m^3*x^7*e + 430335*(f*x)^m
*b^3*m*x^9*e + 2582010*(f*x)^m*a*b*c*m*x^9*e + 183*(f*x)^m*a^2*b*d*m^6*x^3
+ 52725*(f*x)^m*a*b^2*d*m^4*x^5 + 52725*(f*x)^m*a^2*c*d*m^4*x^5 + 340011*(f
*x)^m*b^3*d*m^2*x^7 + 2040066*(f*x)^m*a*b*c*d*m^2*x^7 + 675675*(f*x)^m*b^2*
c*d*x^9 + 675675*(f*x)^m*a*c^2*d*x^9 + 61*(f*x)^m*a^3*m^6*x^3*e + 52725*(f*
x)^m*a^2*b*m^4*x^5*e + 1020033*(f*x)^m*a*b^2*m^2*x^7*e + 1020033*(f*x)^m*a^
2*c*m^2*x^7*e + 225225*(f*x)^m*b^3*x^9*e + 1351350*(f*x)^m*a*b*c*x^9*e + (f
*x)^m*a^3*d*m^7*x + 4575*(f*x)^m*a^2*b*d*m^5*x^3 + 360537*(f*x)^m*a*b^2*d*m
^3*x^5 + 360537*(f*x)^m*a^2*c*d*m^3*x^5 + 544095*(f*x)^m*b^3*d*m*x^7 + 3264

570*(f*x)^m*a*b*c*d*m*x^7 + 1525*(f*x)^m*a^3*m^5*x^3*e + 360537*(f*x)^m*a^2
*b*m^3*x^5*e + 1632285*(f*x)^m*a*b^2*m*x^7*e + 1632285*(f*x)^m*a^2*c*m*x^7*
e + 63*(f*x)^m*a^3*d*m^6*x + 60195*(f*x)^m*a^2*b*d*m^4*x^3 + 1311363*(f*x)^
m*a*b^2*d*m^2*x^5 + 1311363*(f*x)^m*a^2*c*d*m^2*x^5 + 289575*(f*x)^m*b^3*d*
x^7 + 1737450*(f*x)^m*a*b*c*d*x^7 + 20065*(f*x)^m*a^3*m^4*x^3*e + 1311363*(
f*x)^m*a^2*b*m^2*x^5*e + 868725*(f*x)^m*a*b^2*x^7*e + 868725*(f*x)^m*a^2*c*
x^7*e + 1645*(f*x)^m*a^3*d*m^5*x + 443577*(f*x)^m*a^2*b*d*m^3*x^3 + 2215701
*(f*x)^m*a*b^2*d*m*x^5 + 2215701*(f*x)^m*a^2*c*d*m*x^5 + 147859*(f*x)^m*a^3
*m^3*x^3*e + 2215701*(f*x)^m*a^2*b*m*x^5*e + 22995*(f*x)^m*a^3*d*m^4*x + 17
83317*(f*x)^m*a^2*b*d*m^2*x^3 + 1216215*(f*x)^m*a*b^2*d*x^5 + 1216215*(f*x)
^m*a^2*c*d*x^5 + 594439*(f*x)^m*a^3*m^2*x^3*e + 1216215*(f*x)^m*a^2*b*x^5*e
 + 185059*(f*x)^m*a^3*d*m^3*x + 3422565*(f*x)^m*a^2*b*d*m*x^3 + 1140855*(f*
x)^m*a^3*m*x^3*e + 852957*(f*x)^m*a^3*d*m^2*x + 2027025*(f*x)^m*a^2*b*d*x^3
 + 675675*(f*x)^m*a^3*x^3*e + 2071215*(f*x)^m*a^3*d*m*x + 2027025*(f*x)^m*a
^3*d*x)/(m^8 + 64*m^7 + 1708*m^6 + 24640*m^5 + 208054*m^4 + 1038016*m^3 + 2
924172*m^2 + 4098240*m + 2027025)

Mupad [B]
time = 1.06, size = 769, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^3,x)

[Out] (x^7*(f*x)^m*(b^3*d + 3*a*b^2*e + 3*a^2*c*e + 6*a*b*c*d)*(544095*m + 340011
*m^2 + 99715*m^3 + 15477*m^4 + 1309*m^5 + 57*m^6 + m^7 + 289575))/(4098240*
m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7
+ m^8 + 2027025) + (x^9*(f*x)^m*(b^3*e + 3*a*c^2*d + 3*b^2*c*d + 6*a*b*c*e)
*(430335*m + 277093*m^2 + 84547*m^3 + 13723*m^4 + 1213*m^5 + 55*m^6 + m^7 +
 225225))/(4098240*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 +
 1708*m^6 + 64*m^7 + m^8 + 2027025) + (a^3*d*x*(f*x)^m*(2071215*m + 852957*
m^2 + 185059*m^3 + 22995*m^4 + 1645*m^5 + 63*m^6 + m^7 + 2027025))/(4098240
*m + 2924172*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7
 + m^8 + 2027025) + (c^3*e*x^15*(f*x)^m*(264207*m + 177331*m^2 + 57379*m^3
+ 10045*m^4 + 973*m^5 + 49*m^6 + m^7 + 135135))/(4098240*m + 2924172*m^2 +
1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025) +
 (3*a*x^5*(f*x)^m*(b^2*d + a*b*e + a*c*d)*(738567*m + 437121*m^2 + 120179*m
^3 + 17575*m^4 + 1413*m^5 + 59*m^6 + m^7 + 405405))/(4098240*m + 2924172*m^
2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 202702
5) + (3*c*x^11*(f*x)^m*(b^2*e + a*c*e + b*c*d)*(355815*m + 233487*m^2 + 731
39*m^3 + 12265*m^4 + 1125*m^5 + 53*m^6 + m^7 + 184275))/(4098240*m + 292417
2*m^2 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 20
27025) + (a^2*x^3*(f*x)^m*(a*e + 3*b*d)*(1140855*m + 594439*m^2 + 147859*m^
3 + 20065*m^4 + 1525*m^5 + 61*m^6 + m^7 + 675675))/(4098240*m + 2924172*m^2

```
 + 1038016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025
) + (c^2*x^13*(f*x)^m*(3*b*e + c*d)*(303255*m + 201609*m^2 + 64339*m^3 + 11
055*m^4 + 1045*m^5 + 51*m^6 + m^7 + 155925))/(4098240*m + 2924172*m^2 + 103
8016*m^3 + 208054*m^4 + 24640*m^5 + 1708*m^6 + 64*m^7 + m^8 + 2027025)
```

3.221 $\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx$

Optimal. Leaf size=155

$$\frac{a^2d(fx)^{1+m}}{f(1+m)}+\frac{a(2bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{\left(b^2d+2acd+2abe\right)(fx)^{5+m}}{f^5(5+m)}+\frac{\left(2bcd+b^2e+2ace\right)(fx)^{7+m}}{f^7(7+m)}+\frac{c(cd+2be\ldots}{f^9(9\ldots}$$

```
[Out] a^2*d*(f*x)^(1+m)/f/(1+m)+a*(a*e+2*b*d)*(f*x)^(3+m)/f^3/(3+m)+(2*a*b*e+2*a*
c*d+b^2*d)*(f*x)^(5+m)/f^5/(5+m)+(2*a*c*e+b^2*e+2*b*c*d)*(f*x)^(7+m)/f^7/(7
+m)+c*(2*b*e+c*d)*(f*x)^(9+m)/f^9/(9+m)+c^2*e*(f*x)^(11+m)/f^11/(11+m)
```

Rubi [A]
time = 0.07, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.037$, Rules used = $\{1275\}$

$$\frac{a^2d(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+7}\left(2ace+b^2e+2bcd\right)}{f^7(m+7)}+\frac{(fx)^{m+5}\left(2abe+2acd+b^2d\right)}{f^5(m+5)}+\frac{a(fx)^{m+3}(ae+2bd)}{f^3(m+3)}+\frac{c(fx)^{m+9}(2be+cd)}{f^9(m+9)}+\frac{c^2e(fx)^{m+11}}{f^{11}(m+11)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (a^2*d*(f*x)^(1 + m))/(f*(1 + m)) + (a*(2*b*d + a*e)*(f*x)^(3 + m))/(f^3*(3
 + m)) + ((b^2*d + 2*a*c*d + 2*a*b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + ((2*b*
c*d + b^2*e + 2*a*c*e)*(f*x)^(7 + m))/(f^7*(7 + m)) + (c*(c*d + 2*b*e)*(f*x
)^(9 + m))/(f^9*(9 + m)) + (c^2*e*(f*x)^(11 + m))/(f^11*(11 + m))
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^2 \, dx &= \int \left(a^2d(fx)^m+\frac{a(2bd+ae)(fx)^{2+m}}{f^2}+\frac{\left(b^2d+2acd+2abe\right)(fx)^{4\ldots}}{f^4}\ldots\right.\\ &= \frac{a^2d(fx)^{1+m}}{f(1+m)}+\frac{a(2bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{\left(b^2d+2acd+2abe\right)(fx)^{5+\ldots}}{f^5(5+m)}\ldots\end{aligned}$$

Mathematica [A]
time = 0.45, size = 117, normalized size = 0.75

$$x(fx)^m\left(\frac{a^2d}{1+m}+\frac{a(2bd+ae)x^2}{3+m}+\frac{\left(b^2d+2acd+2abe\right)x^4}{5+m}+\frac{\left(2bcd+b^2e+2ace\right)x^6}{7+m}+\frac{c(cd+2be)x^8}{9+m}+\frac{c^2ex^{10}}{11+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] x*(f*x)^m*((a^2*d)/(1 + m) + (a*(2*b*d + a*e)*x^2)/(3 + m) + ((b^2*d + 2*a*
c*d + 2*a*b*e)*x^4)/(5 + m) + ((2*b*c*d + b^2*e + 2*a*c*e)*x^6)/(7 + m) + (
c*(c*d + 2*b*e)*x^8)/(9 + m) + (c^2*e*x^10)/(11 + m))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 782 vs. 2(155) = 310.
time = 0.02, size = 783, normalized size = 5.05

method	result
gosper	$\frac{x\left(c^{2} e\,m^{5} x^{10}+25 c^{2} e\,m^{4} x^{10}+2 b c e\,m^{5} x^{8}+c^{2} d\,m^{5} x^{8}+230 c^{2} e\,m^{3} x^{10}+54 b c e\,m^{4} x^{8}+27 c^{2} d\,m^{4} x^{8}+950 c^{2} e\,m^{2} x^{10}+2 a c e\,m^{5} x^{6}+b^{2} e\,m^{5} x\right.}{}$
risch	$\frac{x\left(c^{2} e\,m^{5} x^{10}+25 c^{2} e\,m^{4} x^{10}+2 b c e\,m^{5} x^{8}+c^{2} d\,m^{5} x^{8}+230 c^{2} e\,m^{3} x^{10}+54 b c e\,m^{4} x^{8}+27 c^{2} d\,m^{4} x^{8}+950 c^{2} e\,m^{2} x^{10}+2 a c e\,m^{5} x^{6}+b^{2} e\,m^{5} x\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] x*(c^2*e*m^5*x^10+25*c^2*e*m^4*x^10+2*b*c*e*m^5*x^8+c^2*d*m^5*x^8+230*c^2*e
*m^3*x^10+54*b*c*e*m^4*x^8+27*c^2*d*m^4*x^8+950*c^2*e*m^2*x^10+2*a*c*e*m^5*
x^6+b^2*e*m^5*x^6+2*b*c*d*m^5*x^6+524*b*c*e*m^3*x^8+262*c^2*d*m^3*x^8+1689*
c^2*e*m*x^10+58*a*c*e*m^4*x^6+29*b^2*e*m^4*x^6+58*b*c*d*m^4*x^6+2244*b*c*e*
m^2*x^8+1122*c^2*d*m^2*x^8+945*c^2*e*x^10+2*a*b*e*m^5*x^4+2*a*c*d*m^5*x^4+6
04*a*c*e*m^3*x^6+b^2*d*m^5*x^4+302*b^2*e*m^3*x^6+604*b*c*d*m^3*x^6+4082*b*c
*e*m*x^8+2041*c^2*d*m*x^8+62*a*b*e*m^4*x^4+62*a*c*d*m^4*x^4+2732*a*c*e*m^2*
x^6+31*b^2*d*m^4*x^4+1366*b^2*e*m^2*x^6+2732*b*c*d*m^2*x^6+2310*b*c*e*x^8+1
155*c^2*d*x^8+a^2*e*m^5*x^2+2*a*b*d*m^5*x^2+700*a*b*e*m^3*x^4+700*a*c*d*m^3
*x^4+5154*a*c*e*m*x^6+350*b^2*d*m^3*x^4+2577*b^2*e*m*x^6+5154*b*c*d*m*x^6+3
3*a^2*e*m^4*x^2+66*a*b*d*m^4*x^2+3460*a*b*e*m^2*x^4+3460*a*c*d*m^2*x^4+2970
*a*c*e*x^6+1730*b^2*d*m^2*x^4+1485*b^2*e*x^6+2970*b*c*d*x^6+a^2*d*m^5+406*a
^2*e*m^3*x^2+812*a*b*d*m^3*x^2+6978*a*b*e*m*x^4+6978*a*c*d*m*x^4+3489*b^2*d
*m*x^4+35*a^2*d*m^4+2262*a^2*e*m^2*x^2+4524*a*b*d*m^2*x^2+4158*a*b*e*x^4+41
58*a*c*d*x^4+2079*b^2*d*x^4+470*a^2*d*m^3+5353*a^2*e*m*x^2+10706*a*b*d*m*x^
2+3010*a^2*d*m^2+3465*a^2*e*x^2+6930*a*b*d*x^2+9129*a^2*d*m+10395*a^2*d)*(f
*x)^m/(11+m)/(9+m)/(7+m)/(5+m)/(3+m)/(1+m)
```

Maxima [A]
time = 0.29, size = 248, normalized size = 1.60

$$\frac{c^{2} f^{m} x^{11} e^{(m \log(x)+1)}}{m+11} + \frac{c^{2} d f^{m} x^{9} x^{m}}{m+9} + \frac{2 \, b c f^{m} x^{9} e^{(m \log(x)+1)}}{m+9} + \frac{2 \, b c d f^{m} x^{7} x^{m}}{m+7} + \frac{b^{2} f^{m} x^{7} e^{(m \log(x)+1)}}{m+7} + \frac{2 \, a c f^{m} x^{7} e^{(m \log(x)+1)}}{m+7} + \frac{b^{2} d f^{m} x^{5} x^{m}}{m+5} + \frac{2 \, a c d f^{m} x^{5} x^{m}}{m+5} + \frac{2 \, a b f^{m} x^{5} e^{(m \log(x)+1)}}{m+5} + \frac{2 \, a b d f^{m} x^{3} x^{m}}{m+3} + \frac{a^{2} f^{m} x^{3} e^{(m \log(x)+1)}}{m+3} + \frac{\left(f x\right)^{m+1} a^{2} d}{f (m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
```

```
[Out] c^2*f^m*x^11*e^(m*log(x) + 1)/(m + 11) + c^2*d*f^m*x^9*x^m/(m + 9) + 2*b*c*
f^m*x^9*e^(m*log(x) + 1)/(m + 9) + 2*b*c*d*f^m*x^7*x^m/(m + 7) + b^2*f^m*x^
7*e^(m*log(x) + 1)/(m + 7) + 2*a*c*f^m*x^7*e^(m*log(x) + 1)/(m + 7) + b^2*d
*f^m*x^5*x^m/(m + 5) + 2*a*c*d*f^m*x^5*x^m/(m + 5) + 2*a*b*f^m*x^5*e^(m*log
(x) + 1)/(m + 5) + 2*a*b*d*f^m*x^3*x^m/(m + 3) + a^2*f^m*x^3*e^(m*log(x) +
1)/(m + 3) + (f*x)^(m + 1)*a^2*d/(f*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(161) = 322$.
time = 0.41, size = 578, normalized size = 3.73

$$\frac{\left(\left(c^{2} d m^{5} + 27 \, c^{2} d m^{4} + 262 \, c^{2} d m^{3} + 1122 \, c^{2} d m^{2} + 2041 \, c^{2} d m + 1155 \, c^{2} d\right)x^{9} + 2 \, \left(b c d m^{5} + 29 \, b c d m^{4} + 302 \, b c d m^{3} + 1366 \, b c d m^{2} + 2577 \, b c d m + 1485 \, b c d\right)x^{7} + \left(\left(b^{2} + 2 \, a c\right) d m^{5} + 31 \, \left(b^{2} + 2 \, a c\right) d m^{4} + 350 \, \left(b^{2} + 2 \, a c\right) d m^{3} + 1730 \, \left(b^{2} + 2 \, a c\right) d m^{2} + 3489 \, \left(b^{2} + 2 \, a c\right) d m + 2079 \, \left(b^{2} + 2 \, a c\right) d\right)x^{5} + 2 \, \left(a b d m^{5} + 33 \, a b d m^{4} + 406 \, a b d m^{3} + 2262 \, a b d m^{2} + 5353 \, a b d m + 3465 \, a b d\right)x^{3} + \left(a^{2} d m^{5} + 35 \, a^{2} d m^{4} + 470 \, a^{2} d m^{3} + 3010 \, a^{2} d m^{2} + 9129 \, a^{2} d m + 10395 \, a^{2} d\right)x + \left(\left(c^{2} m^{5} + 25 \, c^{2} m^{4} + 230 \, c^{2} m^{3} + 950 \, c^{2} m^{2} + 1689 \, c^{2} m + 945 \, c^{2}\right)x^{11} + 2 \, \left(b c m^{5} + 27 \, b c m^{4} + 262 \, b c m^{3} + 1122 \, b c m^{2} + 2041 \, b c m + 1155 \, b c\right)x^{9} + \left(\left(b^{2} + 2 \, a c\right) m^{5} + 29 \, \left(b^{2} + 2 \, a c\right) m^{4} + 302 \, \left(b^{2} + 2 \, a c\right) m^{3} + 1366 \, \left(b^{2} + 2 \, a c\right) m^{2} + 1485 \, b^{2} + 2970 \, a c + 2577 \, \left(b^{2} + 2 \, a c\right) m\right)x^{7} + 2 \, \left(a b m^{5} + 31 \, a b m^{4} + 350 \, a b m^{3} + 1730 \, a b m^{2} + 3489 \, a b m + 2079 \, a b\right)x^{5} + \left(a^{2} m^{5} + 33 \, a^{2} m^{4} + 406 \, a^{2} m^{3} + 2262 \, a^{2} m^{2} + 5353 \, a^{2} m + 3465 \, a^{2}\right)x^{3}\right)e\right)\left(f x\right)^{m}}{m^{6} + 36 \, m^{5} + 505 \, m^{4} + 3480 \, m^{3} + 12139 \, m^{2} + 19524 \, m + 10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
```

```
[Out] ((c^2*d*m^5 + 27*c^2*d*m^4 + 262*c^2*d*m^3 + 1122*c^2*d*m^2 + 2041*c^2*d*m
+ 1155*c^2*d)*x^9 + 2*(b*c*d*m^5 + 29*b*c*d*m^4 + 302*b*c*d*m^3 + 1366*b*c*
d*m^2 + 2577*b*c*d*m + 1485*b*c*d)*x^7 + ((b^2 + 2*a*c)*d*m^5 + 31*(b^2 + 2
*a*c)*d*m^4 + 350*(b^2 + 2*a*c)*d*m^3 + 1730*(b^2 + 2*a*c)*d*m^2 + 3489*(b^
2 + 2*a*c)*d*m + 2079*(b^2 + 2*a*c)*d)*x^5 + 2*(a*b*d*m^5 + 33*a*b*d*m^4 +
406*a*b*d*m^3 + 2262*a*b*d*m^2 + 5353*a*b*d*m + 3465*a*b*d)*x^3 + (a^2*d*m^
5 + 35*a^2*d*m^4 + 470*a^2*d*m^3 + 3010*a^2*d*m^2 + 9129*a^2*d*m + 10395*a^
2*d)*x + ((c^2*m^5 + 25*c^2*m^4 + 230*c^2*m^3 + 950*c^2*m^2 + 1689*c^2*m +
945*c^2)*x^11 + 2*(b*c*m^5 + 27*b*c*m^4 + 262*b*c*m^3 + 1122*b*c*m^2 + 2041
*b*c*m + 1155*b*c)*x^9 + ((b^2 + 2*a*c)*m^5 + 29*(b^2 + 2*a*c)*m^4 + 302*(b
^2 + 2*a*c)*m^3 + 1366*(b^2 + 2*a*c)*m^2 + 1485*b^2 + 2970*a*c + 2577*(b^2
+ 2*a*c)*m)*x^7 + 2*(a*b*m^5 + 31*a*b*m^4 + 350*a*b*m^3 + 1730*a*b*m^2 + 34
89*a*b*m + 2079*a*b)*x^5 + (a^2*m^5 + 33*a^2*m^4 + 406*a^2*m^3 + 2262*a^2*m
^2 + 5353*a^2*m + 3465*a^2)*x^3)*e)*(f*x)^m/(m^6 + 36*m^5 + 505*m^4 + 3480*
m^3 + 12139*m^2 + 19524*m + 10395)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 4068 vs. $2(146) = 292$.
time = 0.86, size = 4068, normalized size = 26.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**2,x)
```

```
[Out] Piecewise(((-a**2*d/(10*x**10) - a**2*e/(8*x**8) - a*b*d/(4*x**8) - a*b*e/(
3*x**6) - a*c*d/(3*x**6) - a*c*e/(2*x**4) - b**2*d/(6*x**6) - b**2*e/(4*x**
4) - b*c*d/(2*x**4) - b*c*e/x**2 - c**2*d/(2*x**2) + c**2*e*log(x))/f**11,
Eq(m, -11)), ((-a**2*d/(8*x**8) - a**2*e/(6*x**6) - a*b*d/(3*x**6) - a*b*e/
(2*x**4) - a*c*d/(2*x**4) - a*c*e/x**2 - b**2*d/(4*x**4) - b**2*e/(2*x**2)
```

```
- b*c*d/x**2 + 2*b*c*e*log(x) + c**2*d*log(x) + c**2*e*x**2/2)/f**9, Eq(m,
-9)), ((-a**2*d/(6*x**6) - a**2*e/(4*x**4) - a*b*d/(2*x**4) - a*b*e/x**2 -
a*c*d/x**2 + 2*a*c*e*log(x) - b**2*d/(2*x**2) + b**2*e*log(x) + 2*b*c*d*log
(x) + b*c*e*x**2 + c**2*d*x**2/2 + c**2*e*x**4/4)/f**7, Eq(m, -7)), ((-a**2
*d/(4*x**4) - a**2*e/(2*x**2) - a*b*d/x**2 + 2*a*b*e*log(x) + 2*a*c*d*log(x
) + a*c*e*x**2 + b**2*d*log(x) + b**2*e*x**2/2 + b*c*d*x**2 + b*c*e*x**4/2
+ c**2*d*x**4/4 + c**2*e*x**6/6)/f**5, Eq(m, -5)), ((-a**2*d/(2*x**2) + a**
2*e*log(x) + 2*a*b*d*log(x) + a*b*e*x**2 + a*c*d*x**2 + a*c*e*x**4/2 + b**2
*d*x**2/2 + b**2*e*x**4/4 + b*c*d*x**4/2 + b*c*e*x**6/3 + c**2*d*x**6/6 + c
**2*e*x**8/8)/f**3, Eq(m, -3)), ((a**2*d*log(x) + a**2*e*x**2/2 + a*b*d*x**
2 + a*b*e*x**4/2 + a*c*d*x**4/2 + a*c*e*x**6/3 + b**2*d*x**4/4 + b**2*e*x**
6/6 + b*c*d*x**6/3 + b*c*e*x**8/4 + c**2*d*x**8/8 + c**2*e*x**10/10)/f, Eq(
m, -1)), (a**2*d*m**5*x*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 35*a**2*d*m**4*x*(f*x)**m/(m**6 + 36*m**5 +
505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 470*a**2*d*m**3*x*(f
*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5) + 3010*a**2*d*m**2*x*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 1
2139*m**2 + 19524*m + 10395) + 9129*a**2*d*m*x*(f*x)**m/(m**6 + 36*m**5 + 5
05*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 10395*a**2*d*x*(f*x)*
*m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) +
 a**2*e*m**5*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 33*a**2*e*m**4*x**3*(f*x)**m/(m**6 + 36*m**5 + 505
*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 406*a**2*e*m**3*x**3*(f
*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 1039
5) + 2262*a**2*e*m**2*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 5353*a**2*e*m*x**3*(f*x)**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 3465*a**2*e*x**
3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m +
10395) + 2*a*b*d*m**5*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3
+ 12139*m**2 + 19524*m + 10395) + 66*a*b*d*m**4*x**3*(f*x)**m/(m**6 + 36*m*
*5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 812*a*b*d*m**3*
x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m
 + 10395) + 4524*a*b*d*m**2*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480
*m**3 + 12139*m**2 + 19524*m + 10395) + 10706*a*b*d*m*x**3*(f*x)**m/(m**6 +
 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6930*a*b*
d*x**3*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524
*m + 10395) + 2*a*b*e*m**5*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*
m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*b*e*m**4*x**5*(f*x)**m/(m**6 +
36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 700*a*b*e*
m**3*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19
524*m + 10395) + 3460*a*b*e*m**2*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 +
 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6978*a*b*e*m*x**5*(f*x)**m/(m*
*6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4158*
a*b*e*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 1
9524*m + 10395) + 2*a*c*d*m**5*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3
```

```
480*m**3 + 12139*m**2 + 19524*m + 10395) + 62*a*c*d*m**4*x**5*(f*x)**m/(m**
6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 700*a*
c*d*m**3*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
+ 19524*m + 10395) + 3460*a*c*d*m**2*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m*
*4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 6978*a*c*d*m*x**5*(f*x)**m
/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 4
158*a*c*d*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2
 + 19524*m + 10395) + 2*a*c*e*m**5*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4
 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 58*a*c*e*m**4*x**7*(f*x)**m/
(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 60
4*a*c*e*m**3*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m
**2 + 19524*m + 10395) + 2732*a*c*e*m**2*x**7*(f*x)**m/(m**6 + 36*m**5 + 50
5*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395) + 5154*a*c*e*m*x**7*(f*x
)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)
 + 2970*a*c*e*x**7*(f*x)**m/(m**6 + 36*m**5 + 505*m**4 + 3480*m**3 + 12139*
m**2 + 19524*m + 10395) + b**2*d*m**5*x**5*(f*x)**m/(m**6 + 36*m**5 + 505*m
**4 + 3480*m**3 + 12139*m**2 + 19524*m + 10395)...
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1178 vs. $2(161) = 322$.
time = 3.83, size = 1178, normalized size = 7.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] ((f*x)^m*c^2*m^5*x^11*e + 25*(f*x)^m*c^2*m^4*x^11*e + (f*x)^m*c^2*d*m^5*x^9
 + 2*(f*x)^m*b*c*m^5*x^9*e + 230*(f*x)^m*c^2*m^3*x^11*e + 27*(f*x)^m*c^2*d*
m^4*x^9 + 54*(f*x)^m*b*c*m^4*x^9*e + 950*(f*x)^m*c^2*m^2*x^11*e + 2*(f*x)^m
*b*c*d*m^5*x^7 + 262*(f*x)^m*c^2*d*m^3*x^9 + (f*x)^m*b^2*m^5*x^7*e + 2*(f*x
)^m*a*c*m^5*x^7*e + 524*(f*x)^m*b*c*m^3*x^9*e + 1689*(f*x)^m*c^2*m*x^11*e +
 58*(f*x)^m*b*c*d*m^4*x^7 + 1122*(f*x)^m*c^2*d*m^2*x^9 + 29*(f*x)^m*b^2*m^4
*x^7*e + 58*(f*x)^m*a*c*m^4*x^7*e + 2244*(f*x)^m*b*c*m^2*x^9*e + 945*(f*x)^
m*c^2*x^11*e + (f*x)^m*b^2*d*m^5*x^5 + 2*(f*x)^m*a*c*d*m^5*x^5 + 604*(f*x)^
m*b*c*d*m^3*x^7 + 2041*(f*x)^m*c^2*d*m*x^9 + 2*(f*x)^m*a*b*m^5*x^5*e + 302*
(f*x)^m*b^2*m^3*x^7*e + 604*(f*x)^m*a*c*m^3*x^7*e + 4082*(f*x)^m*b*c*m*x^9*
e + 31*(f*x)^m*b^2*d*m^4*x^5 + 62*(f*x)^m*a*c*d*m^4*x^5 + 2732*(f*x)^m*b*c*
d*m^2*x^7 + 1155*(f*x)^m*c^2*d*x^9 + 62*(f*x)^m*a*b*m^4*x^5*e + 1366*(f*x)^
m*b^2*m^2*x^7*e + 2732*(f*x)^m*a*c*m^2*x^7*e + 2310*(f*x)^m*b*c*x^9*e + 2*(
f*x)^m*a*b*d*m^5*x^3 + 350*(f*x)^m*b^2*d*m^3*x^5 + 700*(f*x)^m*a*c*d*m^3*x^
5 + 5154*(f*x)^m*b*c*d*m*x^7 + (f*x)^m*a^2*m^5*x^3*e + 700*(f*x)^m*a*b*m^3*
x^5*e + 2577*(f*x)^m*b^2*m*x^7*e + 5154*(f*x)^m*a*c*m*x^7*e + 66*(f*x)^m*a*
b*d*m^4*x^3 + 1730*(f*x)^m*b^2*d*m^2*x^5 + 3460*(f*x)^m*a*c*d*m^2*x^5 + 297
0*(f*x)^m*b*c*d*x^7 + 33*(f*x)^m*a^2*m^4*x^3*e + 3460*(f*x)^m*a*b*m^2*x^5*e
```

```
 + 1485*(f*x)^m*b^2*x^7*e + 2970*(f*x)^m*a*c*x^7*e + (f*x)^m*a^2*d*m^5*x +
812*(f*x)^m*a*b*d*m^3*x^3 + 3489*(f*x)^m*b^2*d*m*x^5 + 6978*(f*x)^m*a*c*d*m
*x^5 + 406*(f*x)^m*a^2*m^3*x^3*e + 6978*(f*x)^m*a*b*m*x^5*e + 35*(f*x)^m*a^
2*d*m^4*x + 4524*(f*x)^m*a*b*d*m^2*x^3 + 2079*(f*x)^m*b^2*d*x^5 + 4158*(f*x
)^m*a*c*d*x^5 + 2262*(f*x)^m*a^2*m^2*x^3*e + 4158*(f*x)^m*a*b*x^5*e + 470*(
f*x)^m*a^2*d*m^3*x + 10706*(f*x)^m*a*b*d*m*x^3 + 5353*(f*x)^m*a^2*m*x^3*e +
 3010*(f*x)^m*a^2*d*m^2*x + 6930*(f*x)^m*a*b*d*x^3 + 3465*(f*x)^m*a^2*x^3*e
 + 9129*(f*x)^m*a^2*d*m*x + 10395*(f*x)^m*a^2*d*x)/(m^6 + 36*m^5 + 505*m^4
+ 3480*m^3 + 12139*m^2 + 19524*m + 10395)
```

Mupad [B]
time = 0.60, size = 429, normalized size = 2.77

$$\frac{x^5\,{\left(f\,x\right)}^m\,\left(d\,b^2+2\,a\,e\,b+2\,a\,c\,d\right)\,\left(m^5+31\,m^4+350\,m^3+1730\,m^2+3489\,m+2079\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{x^7\,{\left(f\,x\right)}^m\,\left(e\,b^2+2\,c\,d\,b+2\,a\,c\,e\right)\,\left(m^5+29\,m^4+302\,m^3+1366\,m^2+2577\,m+1485\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{a^2\,d\,x\,{\left(f\,x\right)}^m\,\left(m^5+35\,m^4+470\,m^3+3010\,m^2+9129\,m+10395\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{a\,x^3\,{\left(f\,x\right)}^m\,\left(a\,e+2\,b\,d\right)\,\left(m^5+33\,m^4+406\,m^3+2262\,m^2+5353\,m+3465\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{c\,x^9\,{\left(f\,x\right)}^m\,\left(2\,b\,e+c\,d\right)\,\left(m^5+27\,m^4+262\,m^3+1122\,m^2+2041\,m+1155\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}+\frac{c^2\,e\,x^{11}\,{\left(f\,x\right)}^m\,\left(m^5+25\,m^4+230\,m^3+950\,m^2+1689\,m+945\right)}{m^6+36\,m^5+505\,m^4+3480\,m^3+12139\,m^2+19524\,m+10395}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] (x^5*(f*x)^m*(b^2*d + 2*a*b*e + 2*a*c*d)*(3489*m + 1730*m^2 + 350*m^3 + 31*
m^4 + m^5 + 2079))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6
 + 10395) + (x^7*(f*x)^m*(b^2*e + 2*a*c*e + 2*b*c*d)*(2577*m + 1366*m^2 + 3
02*m^3 + 29*m^4 + m^5 + 1485))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 +
36*m^5 + m^6 + 10395) + (a^2*d*x*(f*x)^m*(9129*m + 3010*m^2 + 470*m^3 + 35*
m^4 + m^5 + 10395))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (a*x^3*(f*x)^m*(a*e + 2*b*d)*(5353*m + 2262*m^2 + 406*m^3 + 33
*m^4 + m^5 + 3465))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (c*x^9*(f*x)^m*(2*b*e + c*d)*(2041*m + 1122*m^2 + 262*m^3 + 27
*m^4 + m^5 + 1155))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^
6 + 10395) + (c^2*e*x^11*(f*x)^m*(1689*m + 950*m^2 + 230*m^3 + 25*m^4 + m^5
 + 945))/(19524*m + 12139*m^2 + 3480*m^3 + 505*m^4 + 36*m^5 + m^6 + 10395)
```

3.222 $\int (fx)^m (d+ex^2)(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=83

$$\frac{ad(fx)^{1+m}}{f(1+m)}+\frac{(bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{(cd+be)(fx)^{5+m}}{f^5(5+m)}+\frac{ce(fx)^{7+m}}{f^7(7+m)}$$

```
[Out] a*d*(f*x)^(1+m)/f/(1+m)+(a*e+b*d)*(f*x)^(3+m)/f^3/(3+m)+(b*e+c*d)*(f*x)^(5+
m)/f^5/(5+m)+c*e*(f*x)^(7+m)/f^7/(7+m)
```

Rubi [A]
time = 0.03, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.040$, Rules used = {1275}

$$\frac{(fx)^{m+3}(ae+bd)}{f^3(m+3)}+\frac{ad(fx)^{m+1}}{f(m+1)}+\frac{(fx)^{m+5}(be+cd)}{f^5(m+5)}+\frac{ce(fx)^{m+7}}{f^7(m+7)}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d*(f*x)^(1 + m))/(f*(1 + m)) + ((b*d + a*e)*(f*x)^(3 + m))/(f^3*(3 + m))
 + ((c*d + b*e)*(f*x)^(5 + m))/(f^5*(5 + m)) + (c*e*(f*x)^(7 + m))/(f^7*(7
+ m))
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int (fx)^m \left(d+ex^2\right)\left(a+bx^2+cx^4\right)\,dx &= \int \left(ad(fx)^m+\frac{(bd+ae)(fx)^{2+m}}{f^2}+\frac{(cd+be)(fx)^{4+m}}{f^4}+\frac{ce(fx)^{6+}}{f^6}\right.\\ &= \frac{ad(fx)^{1+m}}{f(1+m)}+\frac{(bd+ae)(fx)^{3+m}}{f^3(3+m)}+\frac{(cd+be)(fx)^{5+m}}{f^5(5+m)}+\frac{ce(fx)^{7+m}}{f^7(7+m)}\end{aligned}$$

Mathematica [A]
time = 0.07, size = 59, normalized size = 0.71

$$x(fx)^m\left(\frac{ad}{1+m}+\frac{(bd+ae)x^2}{3+m}+\frac{(cd+be)x^4}{5+m}+\frac{cex^6}{7+m}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x]
```

```
[Out] x*(f*x)^m*((a*d)/(1 + m) + ((b*d + a*e)*x^2)/(3 + m) + ((c*d + b*e)*x^4)/(5
 + m) + (c*e*x^6)/(7 + m))
```

Maple [A]
time = 0.02, size = 82, normalized size = 0.99

method	result
norman	$\frac{(ae+bd)x^3 e^{m\ln(fx)}}{3+m}+\frac{(eb+cd)x^5 e^{m\ln(fx)}}{5+m}+\frac{adx\,e^{m\ln(fx)}}{1+m}+\frac{ce\,x^7 e^{m\ln(fx)}}{7+m}$
gosper	$\frac{x(ce\,m^3x^6+9ce\,m^2x^6+be\,m^3x^4+cd\,m^3x^4+23cem\,x^6+11be\,m^2x^4+11cd\,m^2x^4+15ce\,x^6+ae\,m^3x^2+bd\,m^3x^2+31bem\,x^4+31cdm\,x^4+}{(7+m)(5+m)(3-}$
risch	$\frac{x(ce\,m^3x^6+9ce\,m^2x^6+be\,m^3x^4+cd\,m^3x^4+23cem\,x^6+11be\,m^2x^4+11cd\,m^2x^4+15ce\,x^6+ae\,m^3x^2+bd\,m^3x^2+31bem\,x^4+31cdm\,x^4+}{(7+m)(5+m)(3-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] (a*e+b*d)/(3+m)*x^3*exp(m*ln(f*x))+(b*e+c*d)/(5+m)*x^5*exp(m*ln(f*x))+a*d/(
1+m)*x*exp(m*ln(f*x))+c*e/(7+m)*x^7*exp(m*ln(f*x))
```

Maxima [A]
time = 0.31, size = 113, normalized size = 1.36

$$\frac{cf^m x^7 e^{(m\log(x)+1)}}{m+7}+\frac{cdf^m x^5 x^m}{m+5}+\frac{bf^m x^5 e^{(m\log(x)+1)}}{m+5}+\frac{bdf^m x^3 x^m}{m+3}+\frac{af^m x^3 e^{(m\log(x)+1)}}{m+3}+\frac{(fx)^{m+1}ad}{f(m+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] c*f^m*x^7*e^(m*log(x) + 1)/(m + 7) + c*d*f^m*x^5*x^m/(m + 5) + b*f^m*x^5*e^
(m*log(x) + 1)/(m + 5) + b*d*f^m*x^3*x^m/(m + 3) + a*f^m*x^3*e^(m*log(x) +
1)/(m + 3) + (f*x)^(m + 1)*a*d/(f*(m + 1))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 179 vs. $2(86) = 172$.
time = 0.38, size = 179, normalized size = 2.16

$$\frac{((cdm^3+11\,cdm^2+31\,cdm+21\,cd)x^5+(bdm^3+13\,bdm^2+47\,bdm+35\,bd)x^3+(adm^3+15\,adm^2+71\,adm+105\,ad)x+((cm^3+9\,cm^2+23\,cm+15\,c)x^7+(bm^3+11\,bm^2+31\,bm+21\,b)x^5+(am^3+13\,am^2+47\,am+35\,a)x^3)e)(fx)^m}{m^4+16\,m^3+86\,m^2+176\,m+105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] ((c*d*m^3 + 11*c*d*m^2 + 31*c*d*m + 21*c*d)*x^5 + (b*d*m^3 + 13*b*d*m^2 + 4
7*b*d*m + 35*b*d)*x^3 + (a*d*m^3 + 15*a*d*m^2 + 71*a*d*m + 105*a*d)*x + ((c
```

```
*m^3 + 9*c*m^2 + 23*c*m + 15*c)*x^7 + (b*m^3 + 11*b*m^2 + 31*b*m + 21*b)*x^
5 + (a*m^3 + 13*a*m^2 + 47*a*m + 35*a)*x^3)*e)*(f*x)^m/(m^4 + 16*m^3 + 86*m
^2 + 176*m + 105)
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 1015 vs. $2(71) = 142$.
time = 0.39, size = 1015, normalized size = 12.23

$$\begin{cases} \frac{-\frac{ad}{6x^6}-\frac{ae}{4x^4}-\frac{bd}{4x^4}-\frac{be}{2x^2}-\frac{cd}{2x^2}+ce\log(x)}{f^7} & \text{for } m=-7 \\ \frac{-\frac{ad}{4x^4}-\frac{ae}{2x^2}-\frac{bd}{2x^2}+be\log(x)+cd\log(x)+\frac{cex^2}{2}}{f^5} & \text{for } m=-5 \\ \frac{-\frac{ad}{2x^2}+ae\log(x)+bd\log(x)+\frac{bex^2}{2}+\frac{cdx^2}{2}+\frac{cex^4}{4}}{f^3} & \text{for } m=-3 \\ \frac{ad\log(x)+\frac{aex^2}{2}+\frac{bdx^2}{2}+\frac{bex^4}{4}+\frac{cdx^4}{4}+\frac{cex^6}{6}}{f} & \text{for } m=-1 \\ \frac{adm^3x(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{15adm^2x(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{71admx(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{105adx(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{aem^3x^3(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{13aem^2x^3(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{47aemx^3(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{35aex^3(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{bdm^3x^3(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{13bdm^2x^3(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{47bdmx^3(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{35bdx^3(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{bem^3x^5(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{11bem^2x^5(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{31bemx^5(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{21bex^5(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{cdm^3x^5(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{11cdm^2x^5(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{31cdmx^5(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{21cdx^5(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{cem^3x^7(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{9cem^2x^7(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{23cemx^7(fx)^m}{m^4+16m^3+86m^2+176m+105}+\frac{15cex^7(fx)^m}{m^4+16m^3+86m^2+176m+105} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a),x)
```

```
[Out] Piecewise(((-a*d/(6*x**6) - a*e/(4*x**4) - b*d/(4*x**4) - b*e/(2*x**2) - c*
d/(2*x**2) + c*e*log(x))/f**7, Eq(m, -7)), ((-a*d/(4*x**4) - a*e/(2*x**2) -
 b*d/(2*x**2) + b*e*log(x) + c*d*log(x) + c*e*x**2/2)/f**5, Eq(m, -5)), ((-
a*d/(2*x**2) + a*e*log(x) + b*d*log(x) + b*e*x**2/2 + c*d*x**2/2 + c*e*x**4
/4)/f**3, Eq(m, -3)), ((a*d*log(x) + a*e*x**2/2 + b*d*x**2/2 + b*e*x**4/4 +
 c*d*x**4/4 + c*e*x**6/6)/f, Eq(m, -1)), (a*d*m**3*x*(f*x)**m/(m**4 + 16*m*
*3 + 86*m**2 + 176*m + 105) + 15*a*d*m**2*x*(f*x)**m/(m**4 + 16*m**3 + 86*m
**2 + 176*m + 105) + 71*a*d*m*x*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m
+ 105) + 105*a*d*x*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + a*e*
m**3*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 13*a*e*m**2*x
**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 47*a*e*m*x**3*(f*x)
**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 35*a*e*x**3*(f*x)**m/(m**4 +
 16*m**3 + 86*m**2 + 176*m + 105) + b*d*m**3*x**3*(f*x)**m/(m**4 + 16*m**3
+ 86*m**2 + 176*m + 105) + 13*b*d*m**2*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m
**2 + 176*m + 105) + 47*b*d*m*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176
*m + 105) + 35*b*d*x**3*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) +
 b*e*m**3*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 11*b*e*m
**2*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 31*b*e*m*x**5*
(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 21*b*e*x**5*(f*x)**m/(m
**4 + 16*m**3 + 86*m**2 + 176*m + 105) + c*d*m**3*x**5*(f*x)**m/(m**4 + 16*
m**3 + 86*m**2 + 176*m + 105) + 11*c*d*m**2*x**5*(f*x)**m/(m**4 + 16*m**3 +
 86*m**2 + 176*m + 105) + 31*c*d*m*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2
+ 176*m + 105) + 21*c*d*x**5*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 1
05) + c*e*m**3*x**7*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 9*c
*e*m**2*x**7*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 23*c*e*m*x
**7*(f*x)**m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105) + 15*c*e*x**7*(f*x)**
m/(m**4 + 16*m**3 + 86*m**2 + 176*m + 105), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(86) = 172$.
time = 3.33, size = 350, normalized size = 4.22

$$\frac{(fx)^m cm^3x^7e + 9(fx)^m cm^2x^7e + (fx)^m cdm^3x^5 + (fx)^m bm^3x^5e + 23(fx)^m cmx^7e + 11(fx)^m cdm^2x^5 + 11(fx)^m bm^2x^5e + 15(fx)^m cx^7e + (fx)^m bdm^3x^3 + 31(fx)^m cdmx^5 + (fx)^m am^3x^3e + 31(fx)^m bmx^5e + 13(fx)^m bdm^2x^3 + 21(fx)^m cdx^5 + 13(fx)^m am^2x^3e + 21(fx)^m bx^5e + (fx)^m adm^3x + 47(fx)^m bdmx^3 + 47(fx)^m amx^3e + 15(fx)^m adm^2x + 35(fx)^m bdx^3 + 35(fx)^m ax^3e + 71(fx)^m admx + 105(fx)^m adx}{m^4 + 16m^3 + 86m^2 + 176m + 105}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] ((f*x)^m*c*m^3*x^7*e + 9*(f*x)^m*c*m^2*x^7*e + (f*x)^m*c*d*m^3*x^5 + (f*x)^
m*b*m^3*x^5*e + 23*(f*x)^m*c*m*x^7*e + 11*(f*x)^m*c*d*m^2*x^5 + 11*(f*x)^m*
b*m^2*x^5*e + 15*(f*x)^m*c*x^7*e + (f*x)^m*b*d*m^3*x^3 + 31*(f*x)^m*c*d*m*x
^5 + (f*x)^m*a*m^3*x^3*e + 31*(f*x)^m*b*m*x^5*e + 13*(f*x)^m*b*d*m^2*x^3 +
21*(f*x)^m*c*d*x^5 + 13*(f*x)^m*a*m^2*x^3*e + 21*(f*x)^m*b*x^5*e + (f*x)^m*
a*d*m^3*x + 47*(f*x)^m*b*d*m*x^3 + 47*(f*x)^m*a*m*x^3*e + 15*(f*x)^m*a*d*m^
2*x + 35*(f*x)^m*b*d*x^3 + 35*(f*x)^m*a*x^3*e + 71*(f*x)^m*a*d*m*x + 105*(f
*x)^m*a*d*x)/(m^4 + 16*m^3 + 86*m^2 + 176*m + 105)
```

Mupad [B]
time = 0.34, size = 171, normalized size = 2.06

$$(f\,x)^m\,\left(\frac{x^3\,\left(a\,e+b\,d\right)\,\left(m^3+13\,m^2+47\,m+35\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{x^5\,\left(b\,e+c\,d\right)\,\left(m^3+11\,m^2+31\,m+21\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{a\,d\,x\,\left(m^3+15\,m^2+71\,m+105\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}+\frac{c\,e\,x^7\,\left(m^3+9\,m^2+23\,m+15\right)}{m^4+16\,m^3+86\,m^2+176\,m+105}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4),x)
```

```
[Out] (f*x)^m*((x^3*(a*e + b*d)*(47*m + 13*m^2 + m^3 + 35))/(176*m + 86*m^2 + 16*
m^3 + m^4 + 105) + (x^5*(b*e + c*d)*(31*m + 11*m^2 + m^3 + 21))/(176*m + 86
*m^2 + 16*m^3 + m^4 + 105) + (a*d*x*(71*m + 15*m^2 + m^3 + 105))/(176*m + 8
6*m^2 + 16*m^3 + m^4 + 105) + (c*e*x^7*(23*m + 9*m^2 + m^3 + 15))/(176*m +
86*m^2 + 16*m^3 + m^4 + 105))
```

3.223 $\int \frac{(fx)^m(d+ex^2)}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=194

$$\frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)}+\frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+}{}\right.}{\left(b+\sqrt{b^2-4ac}\right)f(1+}$$

```
[Out] (f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(
1/2)))*(e+(-b*e+2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))+(
f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))*(e+(b*e-2*c*d)/(-4*a*c+b^2)^(1/2))/f/(1+m)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.21, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {1299, 371}

$$\frac{(fx)^{m+1}\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{f(m+1)\left(b-\sqrt{b^2-4ac}\right)}+\frac{(fx)^{m+1}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*Hypergeometric2F1[1, (
1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/((b - Sqrt[b^2 -
4*a*c])*f*(1 + m)) + ((e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*(f*x)^(1 + m)*H
ypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
)])/((b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^2)}{a+bx^2+cx^4}\,dx = \frac{1}{2}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{(fx)^m}{\frac{b}{2}+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx + \frac{1}{2}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\int \frac{\quad}{\frac{b}{2}-}$$

$$= \frac{\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)(fx)^{1+m}\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{\left(b-\sqrt{b^2-4ac}\right)f(1+m)} + \frac{\left(e-\frac{2cd-be}{\sqrt{b^2-4a}}\right.}{}$$

Mathematica [A]
time = 0.33, size = 156, normalized size = 0.80

$$\frac{x(fx)^m\left(\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+\left(-bd+\sqrt{b^2-4ac}\,d+2ae\right)\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)\right)}{2a\sqrt{b^2-4ac}\,(1+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*(f*x)^m*((b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, (1 + m
)/2, (3 + m)/2, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (-(b*d) + Sqrt[b^2 -
4*a*c]*d + 2*a*e)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b
+ Sqrt[b^2 - 4*a*c])]))/(2*a*Sqrt[b^2 - 4*a*c]*(1 + m))
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m\,(e\,x^2+d)}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((x^2*e + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d + ex^2)}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a),x)
[Out] Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((x^2*e + d)*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(f\,x)^m\,(e\,x^2 + d)}{c\,x^4 + b\,x^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4), x)
```

3.224 $\int \frac{(fx)^m(d+ex^2)}{(a+bx^2+cx^4)^2}\,dx$

Optimal. Leaf size=392

$$\frac{(fx)^{1+m}\left(b^2d-2acd-abe+c(bd-2ae)x^2\right)}{2a\left(b^2-4ac\right)f\left(a+bx^2+cx^4\right)}+\frac{c\left(b\left(4ae+\sqrt{b^2-4ac}\,d(1-m)\right)-2a\left(\sqrt{b^2-4ac}\,e(1-m)\right.\right.}{2a\left(b^2-4ac\right)^{3/}}$$

```
[Out] 1/2*(f*x)^(1+m)*(b^2*d-2*a*c*d-a*b*e+c*(-2*a*e+b*d)*x^2)/a/(-4*a*c+b^2)/f/(
c*x^4+b*x^2+a)-1/2*c*(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-2*c*
x^2/(b+(-4*a*c+b^2)^(1/2)))*(b^2*d*(1-m)+b*(4*a*e-d*(1-m)*(-4*a*c+b^2)^(1/2
))+2*a*(-2*c*d*(3-m)+e*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2)^(3/2)/f/(1
+m)/(b+(-4*a*c+b^2)^(1/2))+1/2*c*(f*x)^(1+m)*hypergeom([1, 1/2+1/2*m],[3/2+
1/2*m],-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))*(b^2*(-d*m+d)+b*(4*a*e+d*(1-m)*(-4*
a*c+b^2)^(1/2))-2*a*(2*c*d*(3-m)+e*(1-m)*(-4*a*c+b^2)^(1/2)))/a/(-4*a*c+b^2
)^(3/2)/f/(1+m)/(b-(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 1.78, antiderivative size = 358, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1291, 1299, 371}

$$\frac{c(fx)^{m+1}\left((1-m)\sqrt{b^2-4ac}\,(bd-2ae)+4abe-4acd(3-m)+b^2(d-dm)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(b-\sqrt{b^2-4ac}\right)}-\frac{c(fx)^{m+1}\left(-(1-m)\sqrt{b^2-4ac}\,(bd-2ae)+4abe-4acd(3-m)+b^2(d-dm)\right)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{2af(m+1)\left(b^2-4ac\right)^{3/2}\left(\sqrt{b^2-4ac}+b\right)}+\frac{(fx)^{m+1}\left(cx^2(bd-2ae)-abe-2acd+b^2d\right)}{2af\left(b^2-4ac\right)\left(a+bx^2+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] ((f*x)^(1 + m)*(b^2*d - 2*a*c*d - a*b*e + c*(b*d - 2*a*e)*x^2))/(2*a*(b^2 -
 4*a*c)*f*(a + b*x^2 + c*x^4)) + (c*(4*a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - 2*a
*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m))*(f*x)^(1 + m)*Hypergeometric
2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])])/(2*a*(b^2
 - 4*a*c)^(3/2)*(b - Sqrt[b^2 - 4*a*c])*f*(1 + m)) - (c*(4*a*b*e - Sqrt[b^2
 - 4*a*c]*(b*d - 2*a*e)*(1 - m) - 4*a*c*d*(3 - m) + b^2*(d - d*m))*(f*x)^(1
 + m)*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 -
 4*a*c])])/(2*a*(b^2 - 4*a*c)^(3/2)*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m))
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 1291

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(a + b*x^2 + c*x^4)^(p + 1)
*((d*(b^2 - 2*a*c) - a*b*e + (b*d - 2*a*e)*c*x^2)/(2*a*f*(p + 1)*(b^2 - 4*a
*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^m*(a + b*x^2 + c*
x^4)^(p + 1)*Simp[d*(b^2*(m + 2*(p + 1) + 1) - 2*a*c*(m + 4*(p + 1) + 1)) -
 a*b*e*(m + 1) + c*(m + 2*(2*p + 3) + 1)*(b*d - 2*a*e)*x^2, x], x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && Inte
gerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1299

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2))/((a_) + (b_.)*(x_)^2 + (c_.)
*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b
*e)/(2*q), Int[(f*x)^m/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*
e)/(2*q), Int[(f*x)^m/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e,
 f, m}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^2}\,dx = \frac{(fx)^{1+m}(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a(b^2-4ac)f(a+bx^2+cx^4)} - \frac{\int \frac{(fx)^m(-b^2d(1-m)+2acd(3-m)-abe(1+m)}{a+bx^2+cx^4}}{2a(b^2-4ac)}$$

$$= \frac{(fx)^{1+m}(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a(b^2-4ac)f(a+bx^2+cx^4)} + \frac{\left(c\left(4abe+b^2d(1-m)+\sqrt{b^2-4a}\right.\right.}{}$$

$$= \frac{(fx)^{1+m}(b^2d-2acd-abe+c(bd-2ae)x^2)}{2a(b^2-4ac)f(a+bx^2+cx^4)} + \frac{c\left(4abe+b^2d(1-m)+\sqrt{b^2-4ac}\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 6 vs. order 5 in optimal.
time = 0.57, size = 160, normalized size = 0.41

$$\frac{x(fx)^m\left(d(3+m)F_1\left(\frac{1+m}{2};2,2;\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^2F_1\left(\frac{3+m}{2};2,2;\frac{5+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{a^2(1+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x]
```

```
[Out] (x*(f*x)^m*(d*(3 + m)*AppellF1[(1 + m)/2, 2, 2, (3 + m)/2, (-2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*App
```

```
ellF1[(3 + m)/2, 2, 2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*
x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/(a^2*(1 + m)*(3 + m))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{m} (e\,x^{2}+d)}{(c\,x^{4}+b\,x^{2}+a)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="maxima")
[Out] integrate((x^2*e + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="fricas")
[Out] integral((x^2*e + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 + (b^2 + 2*a*c)*x^4 + 2*a
*b*x^2 + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**2,x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^2,x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^m\,\left(e\,x^2+d\right)}{{\left(c\,x^4+b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2,x)
```

```
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^2, x)
```

3.225 $\int (fx)^m (d+ex^2) (a+bx^2+cx^4)^{3/2} \, dx$

Optimal. Leaf size=319

$$\frac{ad(fx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{ae(fx)^{3+m}\sqrt{a+bx^2}}{f^3(3+}$$

```
[Out] a*d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,-3/2,-3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+
b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+m)/
(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))
^(1/2)+a*e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,-3/2,-3/2,5/2+1/2*m,-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f
^3/(3+m)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2
)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1349, 1155, 524}

$$\frac{ad(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{ae(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+3}{2};-\frac{3}{2},-\frac{3}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (a*d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -3/2, -3/2,
(3 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4
*a*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2
*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (a*e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x
^4]*AppellF1[(3 + m)/2, -3/2, -3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4
*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2
)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right) \left(a+bx^2+cx^4\right)^{3/2} \, dx = \int \left(d(fx)^m \left(a+bx^2+cx^4\right)^{3/2} + \frac{e(fx)^{2+m} \left(a+bx^2+cx^4\right)^{3/2}}{f^2}\right)$$

$$= d\int (fx)^m \left(a+bx^2+cx^4\right)^{3/2} \, dx + \frac{e\int (fx)^{2+m} \left(a+bx^2+cx^4\right)^{3/}}{f^2}$$

$$= \frac{\left(ad\sqrt{a+bx^2+cx^4}\right)\int (fx)^m \left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2} \left(1+\frac{}{b+\sqrt{}}\right.}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{ad(fx)^{1+m}\sqrt{a+bx^2+cx^4}\, F_1\left(\frac{1+m}{2};-\frac{3}{2},-\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4a}}\right.}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-}}}}$$

Mathematica [A]
time = 1.76, size = 466, normalized size = 1.46

$$\frac{x(fx)^m\sqrt{a+bx^2+cx^4}\left(ad(105+71m+15m^2+m^3)F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+(1+m)x^2\left((bd+ae)(35+12m+m^2)F_1\left(\frac{3+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{5+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+(3+m)x^2\left((cd+be)(7+m)F_1\left(\frac{5+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{7+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+ce(5+m)x^2F_1\left(\frac{7+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{9+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)\right)\right)}{(1+m)(3+m)(5+m)(7+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(a*d*(105 + 71*m + 15*m^2 + m^3)*AppellF
1[(1 + m)/2, -1/2, -1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*
c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (1 + m)*x^2*((b*d + a*e)*(35 + 12*m + m^
```

```
2)*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + (3 + m)*x^2*((c*d + b*e)*(7 +
m)*AppellF1[(5 + m)/2, -1/2, -1/2, (7 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*
a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + c*e*(5 + m)*x^2*AppellF1[(7 +
m)/2, -1/2, -1/2, (9 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/
(-b + Sqrt[b^2 - 4*a*c])])))/((1 + m)*(3 + m)*(5 + m)*(7 + m)*Sqrt[(b - Sq
rt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \left(f x\right)^{m} \left(e\,x^{2}+d\right) \left(c\,x^{4}+b\,x^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral((c*d*x^4 + b*d*x^2 + a*d + (c*x^6 + b*x^4 + a*x^2)*e)*sqrt(c*x^4 +
 b*x^2 + a)*(f*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(fx\right)^{m} \left(d + ex^{2}\right) \left(a + bx^{2} + cx^{4}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)*(f*x)^m, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2), x)
```

3.226 $\int (fx)^m (d+ex^2) \sqrt{a+bx^2+cx^4}\, dx$

Optimal. Leaf size=317

$$\frac{d(fx)^{1+m}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}+\frac{e(fx)^{3+m}\sqrt{a+bx^2+}}{f^3(3+}$$

```
[Out] d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,-1/2,-1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^
2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f/(1+m)/(1
+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(
1/2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,-1/2,-1/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a
*c+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(c*x^4+b*x^2+a)^(1/2)/f^3/(
3+m)/(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(1+2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)))^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1349, 1155, 524}

$$\frac{d(fx)^{m+1}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+1}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}+\frac{e(fx)^{m+3}\sqrt{a+bx^2+cx^4}\,F_1\left(\frac{m+3}{2};-\frac{1}{2},-\frac{1}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[a + b*x^2 + c*x^4]*AppellF1[(1 + m)/2, -1/2, -1/2, (3
 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a
*c])])/(f*(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c])]) + (e*(f*x)^(3 + m)*Sqrt[a + b*x^2 + c*x^4]*
AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c
]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[1 + (2*c*x^2)/(b
 - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\, dx = \int \left(d(fx)^m \sqrt{a+bx^2+cx^4} + \frac{e(fx)^{2+m}\sqrt{a+bx^2+cx^4}}{f^2}\right) dx$$

$$= d\int (fx)^m \sqrt{a+bx^2+cx^4}\, dx + \frac{e\int (fx)^{2+m}\sqrt{a+bx^2+cx^4}\, dx}{f^2}$$

$$= \frac{\left(d\sqrt{a+bx^2+cx^4}\right)\int (fx)^m \sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2c}{b+\sqrt{b^2}}}}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}}$$

$$= \frac{d(fx)^{1+m}\sqrt{a+bx^2+cx^4}\, F_1\left(\frac{1+m}{2}; -\frac{1}{2}, -\frac{1}{2}; \frac{3+m}{2}; -\frac{2cx^2}{b-\sqrt{b^2-4ac}},\right.}{f(1+m)\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4a}}}}$$

Mathematica [A]
time = 0.86, size = 267, normalized size = 0.84

$$\frac{x(fx)^m\sqrt{a+bx^2+cx^4}\left(d(3+m)F_1\left(\frac{1+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^2F_1\left(\frac{3+m}{2};-\frac{1}{2},-\frac{1}{2};\frac{5+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{(1+m)(3+m)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(f*x)^m*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[a + b*x^2 + c*x^4]*(d*(3 + m)*AppellF1[(1 + m)/2, -1/2, -1/
2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2
```

```
- 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, -1/2, -1/2, (5 + m)/2, (-2*c
*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((1 +
m)*(3 + m)*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int (fx)^m \left(e\,x^2+d\right) \sqrt{c\,x^4+b\,x^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*(f*x)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*(f*x)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (fx)^m \left(d+ex^2\right) \sqrt{a+bx^2+cx^4}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)*(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)*(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*(f*x)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int (f\,x)^m\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int((f*x)^m*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2), x)
```

3.227 $\int \frac{(fx)^m(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=317

$$\frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right.}{f(1+m)\sqrt{a+bx^2+cx^4}}$$

```
[Out] d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,1/2,1/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f/(1+m)/(c*x^4+b*x^2+a)^(1/
2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,1/2,1/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b
^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2
)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/f^3/(3+m)/(c*x^4+b*x^2+a
)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1349, 1155, 524}

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{2};\frac{1}{2},\frac{1}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+3}{2};\frac{1}{2},\frac{1}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{f^3(m+3)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(f*(1
+ m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b - Sq
rt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3 +
 m)/2, 1/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/
(b + Sqrt[b^2 - 4*a*c])])/(f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(fx)^m\,(d+ex^2)}{\sqrt{a+bx^2+cx^4}}\,dx &= \int \left(\frac{d(fx)^m}{\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{2+m}}{f^2\sqrt{a+bx^2+cx^4}}\right) dx \\
&= d\int \frac{(fx)^m}{\sqrt{a+bx^2+cx^4}}\,dx+\frac{e\int \frac{(fx)^{2+m}}{\sqrt{a+bx^2+cx^4}}\,dx}{f^2} \\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^m}{\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+}}}{\sqrt{a+bx^2+cx^4}} \\
&= \frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{}{b-\sqrt{b}}\right.}{f(1+m)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [A]
time = 1.59, size = 267, normalized size = 0.84

$$
\frac{x(fx)^m\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\,\sqrt{\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}}\left(d(3+m)F_1\left(\frac{1+m}{2};\frac{1}{2},\frac{1}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^2F_1\left(\frac{3+m}{2};\frac{1}{2},\frac{1}{2};\frac{5+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{(1+m)(3+m)\sqrt{a+bx^2+cx^4}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/Sqrt[a + b*x^2 + c*x^4],x]
```

```
[Out] (x*(f*x)^m*Sqrt[(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*
Sqrt[(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*(d*(3 + m)*
AppellF1[(1 + m)/2, 1/2, 1/2, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])
```

```
, (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])] + e*(1 + m)*x^2*AppellF1[(3 + m)/2, 1
/2, 1/2, (5 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqr
t[b^2 - 4*a*c])]))/((1 + m)*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{m} (e\,x^{2}+d)}{\sqrt{c\,x^{4}+b\,x^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
[Out] integrate((x^2*e + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
[Out] integral((x^2*e + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^{m} (d+ex^{2})}{\sqrt{a+bx^{2}+cx^{4}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
[Out] Integral((f*x)**m*(d + e*x**2)/sqrt(a + b*x**2 + c*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)*(f*x)^m/sqrt(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^m\,\left(e\,x^2+d\right)}{\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(1/2), x)
```

3.228 $\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{3/2}} \, dx$

Optimal. Leaf size=323

$$\frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(1+m)\sqrt{a+bx^2+cx^4}}$$

```
[Out] d*(f*x)^(1+m)*AppellF1(1/2+1/2*m,3/2,3/2,3/2+1/2*m,-2*c*x^2/(b-(-4*a*c+b^2)
^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1/2)))
^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f/(1+m)/(c*x^4+b*x^2+a)^(
1/2)+e*(f*x)^(3+m)*AppellF1(3/2+1/2*m,3/2,3/2,5/2+1/2*m,-2*c*x^2/(b-(-4*a*c
+b^2)^(1/2)),-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))*(1+2*c*x^2/(b-(-4*a*c+b^2)^(1
/2)))^(1/2)*(1+2*c*x^2/(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a/f^3/(3+m)/(c*x^4+b*x
^2+a)^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1349, 1155, 524}

$$\frac{d(fx)^{m+1}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+1}{2};\frac{3}{2},\frac{3}{2};\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af(m+1)\sqrt{a+bx^2+cx^4}}+\frac{e(fx)^{m+3}\sqrt{\frac{2cx^2}{b-\sqrt{b^2-4ac}}+1}\sqrt{\frac{2cx^2}{\sqrt{b^2-4ac}+b}+1}\,F_1\left(\frac{m+3}{2};\frac{3}{2},\frac{3}{2};\frac{m+5}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{2cx^2}{b+\sqrt{b^2-4ac}}\right)}{af^3(m+3)\sqrt{a+bx^2+cx^4}}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (d*(f*x)^(1 + m)*Sqrt[1 + (2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*
x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])])/(a*f*(
1 + m)*Sqrt[a + b*x^2 + c*x^4]) + (e*(f*x)^(3 + m)*Sqrt[1 + (2*c*x^2)/(b -
Sqrt[b^2 - 4*a*c])]*Sqrt[1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c])]*AppellF1[(3
 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), (-2*c*x^2
)/(b + Sqrt[b^2 - 4*a*c])])/(a*f^3*(3 + m)*Sqrt[a + b*x^2 + c*x^4])
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 1155

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Dist[a^IntPart[p]*((a + b*x^2 + c*x^4)^FracPart[p]/((1 + 2*c*(x^2/(b +
Rt[b^2 - 4*a*c, 2])))^FracPart[p]*(1 + 2*c*(x^2/(b - Rt[b^2 - 4*a*c, 2])))^
FracPart[p])), Int[(d*x)^m*(1 + 2*c*(x^2/(b + Sqrt[b^2 - 4*a*c])))^p*(1 + 2
*c*(x^2/(b - Sqrt[b^2 - 4*a*c])))^p, x], x] /; FreeQ[{a, b, c, d, m, p}, x]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rubi steps

$$
\begin{aligned}
\int \frac{(fx)^m\,(d+ex^2)}{(a+bx^2+cx^4)^{3/2}}\,dx &= \int \left(\frac{d(fx)^m}{(a+bx^2+cx^4)^{3/2}}+\frac{e(fx)^{2+m}}{f^2\,(a+bx^2+cx^4)^{3/2}}\right)dx \\
&= d\int \frac{(fx)^m}{(a+bx^2+cx^4)^{3/2}}\,dx+\frac{e\int \frac{(fx)^{2+m}}{(a+bx^2+cx^4)^{3/2}}\,dx}{f^2} \\
&= \frac{\left(d\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\right)\int \frac{(fx)^m}{\left(1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}\right)^{3/2}\left(1+\frac{}{b+}\right.}}{a\sqrt{a+bx^2+cx^4}} \\
&= \frac{d(fx)^{1+m}\sqrt{1+\frac{2cx^2}{b-\sqrt{b^2-4ac}}}\sqrt{1+\frac{2cx^2}{b+\sqrt{b^2-4ac}}}\,F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{}{b-\sqrt{}}\right.}{af(1+m)\sqrt{a+bx^2+cx^4}}
\end{aligned}
$$

Mathematica [A]
time = 10.26, size = 307, normalized size = 0.95

$$
\frac{x(fx)^m\left(-b+\sqrt{b^2-4ac}-2cx^2\right)\sqrt{\frac{b-\sqrt{b^2-4ac}+2cx^2}{b-\sqrt{b^2-4ac}}}\left(\frac{b+\sqrt{b^2-4ac}+2cx^2}{b+\sqrt{b^2-4ac}}\right)^{3/2}\left(d(3+m)F_1\left(\frac{1+m}{2};\frac{3}{2},\frac{3}{2};\frac{3+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)+e(1+m)x^2F_1\left(\frac{3+m}{2};\frac{3}{2},\frac{3}{2};\frac{5+m}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},\frac{2cx^2}{-b+\sqrt{b^2-4ac}}\right)\right)}{\left(-b+\sqrt{b^2-4ac}\right)(1+m)(3+m)\left(a+bx^2+cx^4\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x]
```

```
[Out] (x*(f*x)^m*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)*Sqrt[(b - Sqrt[b^2 - 4*a*c] +
 2*c*x^2)/(b - Sqrt[b^2 - 4*a*c])]*((b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)/(b +
Sqrt[b^2 - 4*a*c]))^(3/2)*(d*(3 + m)*AppellF1[(1 + m)/2, 3/2, 3/2, (3 + m)/
```

```
2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]
+ e*(1 + m)*x^2*AppellF1[(3 + m)/2, 3/2, 3/2, (5 + m)/2, (-2*c*x^2)/(b + Sq
rt[b^2 - 4*a*c]), (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c])]))/((-b + Sqrt[b^2 - 4
*a*c])*(1 + m)*(3 + m)*(a + b*x^2 + c*x^4)^(3/2))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
[Out] int((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate((x^2*e + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*(f*x)^m/(c^2*x^8 + 2*b*c*x^6 +
 (b^2 + 2*a*c)*x^4 + 2*a*b*x^2 + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d+ex^2)}{(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral((f*x)**m*(d + e*x**2)/(a + b*x**2 + c*x**4)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] integrate((x^2*e + d)*(f*x)^m/(c*x^4 + b*x^2 + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^m\,(e\,x^2+d)}{(c\,x^4+b\,x^2+a)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2),x)
[Out] int(((f*x)^m*(d + e*x^2))/(a + b*x^2 + c*x^4)^(3/2), x)
```

3.229 $\int \frac{x^9}{(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=134

$$-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(cd^2+ae^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(cd^2+ae^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(cd^2+ae^2\right)}$$

```
[Out] -1/2*d*x^2/c/e^2+1/4*x^4/c/e+1/2*a^(3/2)*d*arctan(x^2*c^(1/2)/a^(1/2))/c^(3
/2)/(a*e^2+c*d^2)+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2+c*d^2)-1/4*a^2*e*ln(c*x^4+
a)/c^2/(a*e^2+c*d^2)
```

Rubi [A]
time = 0.12, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {1266, 1643, 649, 211, 266}

$$\frac{a^{3/2}d\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}\left(ae^2+cd^2\right)}-\frac{a^2e\log\left(a+cx^4\right)}{4c^2\left(ae^2+cd^2\right)}+\frac{d^4\log\left(d+ex^2\right)}{2e^3\left(ae^2+cd^2\right)}-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(d*x^2)/(c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^
2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^9}{(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{d}{ce^2}+\frac{x}{ce}+\frac{d^4}{e^2(cd^2+ae^2)(d+ex)}+\frac{a^2(d-ex)}{c(cd^2+ae^2)(a+cx^2)}\right) dx\right.\\
&= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2+ae^2)}+\frac{a^2\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx, x, x^2\right)}{2c(cd^2+ae^2)}\\
&= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2+ae^2)}+\frac{(a^2d)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2c(cd^2+ae^2)}-\frac{(a^2e)\,\text{Subst}(}{2c(c}\\
&= -\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2+ae^2)}-\frac{a^2e\log(a+cx^4)}{4c^2(cd^2+ae^2)}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 134, normalized size = 1.00

$$
-\frac{dx^2}{2ce^2}+\frac{x^4}{4ce}+\frac{a^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2+ae^2)}-\frac{a^2e\log(a+cx^4)}{4c^2(cd^2+ae^2)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(d*x^2)/(c*e^2) + x^4/(4*c*e) + (a^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*c^(3/2)*(c*d^2 + a*e^2)) + (d^4*Log[d + e*x^2])/(2*e^3*(c*d^2 + a*e^
2)) - (a^2*e*Log[a + c*x^4])/(4*c^2*(c*d^2 + a*e^2))
```

Maple [A]
time = 0.18, size = 103, normalized size = 0.77

method	result
default	$\frac{(-e\,x^{2}+d)^{2}}{4c\,e^{3}}+\frac{d^{4}\ln(e\,x^{2}+d)}{2e^{3}(a\,e^{2}+c\,d^{2})}+\frac{a^{2}\left(-\frac{e\ln(c\,x^{4}+a)}{2c}+\frac{d\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a\,e^{2}+c\,d^{2})c}$
risch	$\frac{x^{4}}{4ce}-\frac{d\,x^{2}}{2c\,e^{2}}+\frac{d^{2}}{4c\,e^{3}}+\frac{d^{4}\ln(e\,x^{2}+d)}{2e^{3}(a\,e^{2}+c\,d^{2})}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(a\,c^{2}e^{2}+c^{3}d^{2}\right)\_Z^{2}+2a^{2}c\,e^{3}\_Z+e^{4}a^{3}\right)}\_R\ln\left(\left(\left(2a\,c^{2}e^{4}-2c^{3}d^{2}e^{2}\right)\_R^{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-e*x^2+d)^2/c/e^3+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2+c*d^2)+1/2*a^2/(a*e^2
+c*d^2)/c*(-1/2*e/c*ln(c*x^4+a)+d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 118, normalized size = 0.88

$$\frac{d^{4}\log\left(x^{2}e+d\right)}{2\left(cd^{2}e^{3}+ae^{5}\right)}-\frac{a^{2}e\log\left(cx^{4}+a\right)}{4\left(c^{3}d^{2}+ac^{2}e^{2}\right)}+\frac{a^{2}d\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{2\left(c^{2}d^{2}+ace^{2}\right)\sqrt{ac}}+\frac{\left(x^{4}e-2\,dx^{2}\right)e^{(-2)}}{4\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*d^4*log(x^2*e + d)/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/(c^3*
d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e^2)*s
qrt(a*c)) + 1/4*(x^4*e - 2*d*x^2)*e^(-2)/c
```

Fricas [A]
time = 3.27, size = 273, normalized size = 2.04

$$\left[\frac{c^{2}d^{2}x^{4}e^{2}-2\,c^{2}d^{3}x^{2}e+acx^{4}e^{4}+2\,c^{2}d^{4}\log\left(x^{2}e+d\right)-2\,acdx^{2}e^{3}+acd\sqrt{-\frac{a}{c}}\,e^{3}\log\left(\frac{cx^{4}+2\,cx^{2}\sqrt{-\frac{a}{c}}-a}{cx^{4}+a}\right)-a^{2}e^{4}\log\left(cx^{4}+a\right)}{4\left(c^{3}d^{2}e^{3}+ac^{2}e^{5}\right)},\ \frac{c^{2}d^{2}x^{4}e^{2}-2\,c^{2}d^{3}x^{2}e+acx^{4}e^{4}+2\,c^{2}d^{4}\log\left(x^{2}e+d\right)-2\,acdx^{2}e^{3}+2\,acd\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^{2}\sqrt{\frac{a}{c}}}{a}\right)e^{3}-a^{2}e^{4}\log\left(cx^{4}+a\right)}{4\left(c^{3}d^{2}e^{3}+ac^{2}e^{5}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c^2*d^2*x^4*e^2 - 2*c^2*d^3*x^2*e + a*c*x^4*e^4 + 2*c^2*d^4*log(x^2*e
 + d) - 2*a*c*d*x^2*e^3 + a*c*d*sqrt(-a/c)*e^3*log((c*x^4 + 2*c*x^2*sqrt(-a
/c) - a)/(c*x^4 + a)) - a^2*e^4*log(c*x^4 + a))/(c^3*d^2*e^3 + a*c^2*e^5),
1/4*(c^2*d^2*x^4*e^2 - 2*c^2*d^3*x^2*e + a*c*x^4*e^4 + 2*c^2*d^4*log(x^2*e
```

```
+ d) - 2*a*c*d*x^2*e^3 + 2*a*c*d*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a)*e^3 -
a^2*e^4*log(c*x^4 + a))/(c^3*d^2*e^3 + a*c^2*e^5)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.69, size = 121, normalized size = 0.90

$$\frac{d^4\log\left(|x^2e+d|\right)}{2\left(cd^2e^3+ae^5\right)}-\frac{a^2e\log\left(cx^4+a\right)}{4\left(c^3d^2+ac^2e^2\right)}+\frac{a^2d\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{(cx^4e-2\,cdx^2)e^{(-2)}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 + a*e^5) - 1/4*a^2*e*log(c*x^4 + a)/
(c^3*d^2 + a*c^2*e^2) + 1/2*a^2*d*arctan(c*x^2/sqrt(a*c))/((c^2*d^2 + a*c*e
^2)*sqrt(a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2)*e^(-2)/c^2
```

Mupad [**B**]
time = 0.87, size = 181, normalized size = 1.35

$$\frac{\ln\left(\sqrt{-a^3\,c^5}+a\,c^3\,x^2\right)\,\left(d\,\sqrt{-a^3\,c^5}-a^2\,c^2\,e\right)}{4\,c^5\,d^2+4\,a\,c^4\,e^2}-\frac{\ln\left(\sqrt{-a^3\,c^5}-a\,c^3\,x^2\right)\,\left(d\,\sqrt{-a^3\,c^5}+a^2\,c^2\,e\right)}{4\,\left(c^5\,d^2+a\,c^4\,e^2\right)}+\frac{d^4\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e^3+2\,a\,e^5}+\frac{x^4}{4\,c\,e}-\frac{d\,x^2}{2\,c\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log((-a^3*c^5)^(1/2) + a*c^3*x^2)*(d*(-a^3*c^5)^(1/2) - a^2*c^2*e))/(4*c^5
*d^2 + 4*a*c^4*e^2) - (log((-a^3*c^5)^(1/2) - a*c^3*x^2)*(d*(-a^3*c^5)^(1/2
) + a^2*c^2*e))/(4*(c^5*d^2 + a*c^4*e^2)) + (d^4*log(d + e*x^2))/(2*a*e^5 +
 2*c*d^2*e^3) + x^4/(4*c*e) - (d*x^2)/(2*c*e^2)
```

3.230 $\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=118

$$\frac{x^2}{2ce}-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)}-\frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)}-\frac{ad\log(a+cx^4)}{4c(cd^2+ae^2)}$$

```
[Out] 1/2*x^2/c/e-1/2*a^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/c^(3/2)/(a*e^2+c*d^2)
-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2)-1/4*a*d*ln(c*x^4+a)/c/(a*e^2+c*d^2)
```

Rubi [A]
time = 0.10, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1266, 1643, 649, 211, 266}

$$-\frac{a^{3/2}e\,\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}(ae^2+cd^2)}-\frac{ad\log(a+cx^4)}{4c(ae^2+cd^2)}-\frac{d^3\log(d+ex^2)}{2e^2(ae^2+cd^2)}+\frac{x^2}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) - (a^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*c^(3/2)*(c*d^2 +
 a*e^2)) - (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 + a*e^2)) - (a*d*Log[a + c*x^
4])/(4*c*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7}{(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ce}-\frac{d^3}{e(cd^2+ae^2)(d+ex)}-\frac{a(ae+cdx)}{c(cd^2+ae^2)(a+cx^2)}\right)\,dx, x, x^2\right)\\
&= \frac{x^2}{2ce}-\frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)}-\frac{a\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2c(cd^2+ae^2)}\\
&= \frac{x^2}{2ce}-\frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)}-\frac{(ad)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}-\frac{(a^2e)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, \right.}{2c(cd^2+ae^2)}\\
&= \frac{x^2}{2ce}-\frac{a^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2c^{3/2}(cd^2+ae^2)}-\frac{d^3\log(d+ex^2)}{2e^2(cd^2+ae^2)}-\frac{ad\log(a+cx^4)}{4c(cd^2+ae^2)}\end{aligned}$$

Mathematica [A]
time = 0.06, size = 99, normalized size = 0.84

$$\frac{-\frac{2a^{3/2}e^3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{c^{3/2}}-2d^3\log(d+ex^2)+\frac{e(2(cd^2+ae^2)x^2-ade\log(a+cx^4))}{c}}{4e^2(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((-2*a^(3/2)*e^3*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) - 2*d^3*Log[d + e*x
^2] + (e*(2*(c*d^2 + a*e^2)*x^2 - a*d*e*Log[a + c*x^4]))/c)/(4*e^2*(c*d^2 +
 a*e^2))
```

Maple [A]
time = 0.17, size = 92, normalized size = 0.78

method	result

default	$\frac{x^2}{2ce} - \frac{d^3 \ln(e x^2+d)}{2e^2(a e^2+c d^2)} - \frac{a\left(\frac{d \ln(c x^4+a)}{2} + \frac{ae \arctan\left(\frac{c x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a e^2+c d^2)c}$
risch	$\frac{x^2}{2ce} + \frac{a \ln\left(\left(-\sqrt{-ac}\, a^2 e^5+3\sqrt{-ac}\, ac d^2 e^3-4\sqrt{-ac}\, c^2 d^4 e+3a^2 cd e^4-3a c^2 d^3 e^2+2c^3 d^5\right)x^2-3\sqrt{-ac}\, a^2 d e^4+3\sqrt{-}\right.}{4c^2(a e^2+c d^2)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c/e-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2+c*d^2)-1/2*a/(a*e^2+c*d^2)/c*(1/
2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))
```

Maxima [A]
time = 0.49, size = 104, normalized size = 0.88

$$-\frac{d^3 \log\left(x^2 e+d\right)}{2\left(cd^2e^2+ae^4\right)} - \frac{a^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right) e}{2\left(c^2d^2+ace^2\right)\sqrt{ac}} + \frac{x^2 e^{(-1)}}{2\,c} - \frac{ad \log\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*d^3*log(x^2*e + d)/(c*d^2*e^2 + a*e^4) - 1/2*a^2*arctan(c*x^2/sqrt(a*c
))*e/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) + 1/2*x^2*e^(-1)/c - 1/4*a*d*log(c*x^4
 + a)/(c^2*d^2 + a*c*e^2)
```

Fricas [A]
time = 1.39, size = 211, normalized size = 1.79

$$\left[\frac{2\,cd^2x^2e - 2\,cd^3 \log\left(x^2e+d\right) + 2\,ax^2e^3 - ade^2 \log\left(cx^4+a\right) + a\sqrt{-\frac{a}{c}}\,e^3 \log\left(\frac{cx^4 - 2\,cx^2\sqrt{-\frac{a}{c}} - a}{cx^4+a}\right)}{4\left(c^2d^2e^2+ace^4\right)}, \frac{2\,cd^2x^2e - 2\,cd^3 \log\left(x^2e+d\right) + 2\,ax^2e^3 - ade^2 \log\left(cx^4+a\right) - 2\,a\sqrt{\frac{a}{c}} \arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) e^3}{4\left(c^2d^2e^2+ace^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*c*d^2*x^2*e - 2*c*d^3*log(x^2*e + d) + 2*a*x^2*e^3 - a*d*e^2*log(c*
x^4 + a) + a*sqrt(-a/c)*e^3*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a
)))/(c^2*d^2*e^2 + a*c*e^4), 1/4*(2*c*d^2*x^2*e - 2*c*d^3*log(x^2*e + d) +
2*a*x^2*e^3 - a*d*e^2*log(c*x^4 + a) - 2*a*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)
/a)*e^3)/(c^2*d^2*e^2 + a*c*e^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.39, size = 105, normalized size = 0.89

$$-\frac{d^3\log\left(|x^2e+d|\right)}{2\left(cd^2e^2+ae^4\right)}-\frac{a^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(c^2d^2+ace^2\right)\sqrt{ac}}+\frac{x^2e^{(-1)}}{2\,c}-\frac{ad\log\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 + a*e^4) - 1/2*a^2*arctan(c*x^2/sqr
t(a*c))*e/((c^2*d^2 + a*c*e^2)*sqrt(a*c)) + 1/2*x^2*e^(-1)/c - 1/4*a*d*log(
c*x^4 + a)/(c^2*d^2 + a*c*e^2)
```

Mupad [**B**]
time = 0.72, size = 166, normalized size = 1.41

$$\frac{x^2}{2\,c\,e}-\frac{d^3\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e^2+2\,a\,e^4}-\frac{\ln\left(\sqrt{-a^3\,c^3}+a\,c^2\,x^2\right)\,\left(e\,\sqrt{-a^3\,c^3}+a\,c^2\,d\right)}{4\,\left(c^4\,d^2+a\,c^3\,e^2\right)}+\frac{\ln\left(\sqrt{-a^3\,c^3}-a\,c^2\,x^2\right)\,\left(e\,\sqrt{-a^3\,c^3}-a\,c^2\,d\right)}{4\,c^4\,d^2+4\,a\,c^3\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*a*e^4 + 2*c*d^2*e^2) - (log((-a^3*c^3
)^(1/2) + a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) + a*c^2*d))/(4*(c^4*d^2 + a*c^3*e^
2)) + (log((-a^3*c^3)^(1/2) - a*c^2*x^2)*(e*(-a^3*c^3)^(1/2) - a*c^2*d))/(4
*c^4*d^2 + 4*a*c^3*e^2)
```

3.231 $\int \frac{x^5}{(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=105

$$-\frac{\sqrt{a}\, d \tan^{-1}\left(\frac{\sqrt{c}\, x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)}+\frac{d^2\log(d+ex^2)}{2e(cd^2+ae^2)}+\frac{ae\log(a+cx^4)}{4c(cd^2+ae^2)}$$

```
[Out] 1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)+1/4*a*e*ln(c*x^4+a)/c/(a*e^2+c*d^2)-1/2
*d*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+c*d^2)/c^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1266, 1643, 649, 211, 266}

$$-\frac{\sqrt{a}\, d\text{ArcTan}\left(\frac{\sqrt{c}\, x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(ae^2+cd^2)}+\frac{ae\log(a+cx^4)}{4c(ae^2+cd^2)}+\frac{d^2\log(d+ex^2)}{2e(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(Sqrt[a]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[c]*(c*d^2 + a*e^2)) +
(d^2*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)) + (a*e*Log[a + c*x^4])/(4*c*(c*d
^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
```

```
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^5}{(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{(cd^2+ae^2)(d+ex)} - \frac{a(d-ex)}{(cd^2+ae^2)(a+cx^2)}\right)\,dx, x, x^2\right)\\ &= \frac{d^2\log(d+ex^2)}{2e(cd^2+ae^2)} - \frac{a\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\ &= \frac{d^2\log(d+ex^2)}{2e(cd^2+ae^2)} - \frac{(ad)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)} + \frac{(ae)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\ &= -\frac{\sqrt{a}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)} + \frac{d^2\log(d+ex^2)}{2e(cd^2+ae^2)} + \frac{ae\log(a+cx^4)}{4c(cd^2+ae^2)}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 77, normalized size = 0.73

$$\frac{-2\sqrt{a}\,\sqrt{c}\,de\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right) + 2cd^2\log(d+ex^2) + ae^2\log(a+cx^4)}{4c^2d^2e + 4ace^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + 2*c*d^2*Log[d + e*x
^2] + a*e^2*Log[a + c*x^4])/(4*c^2*d^2*e + 4*a*c*e^3)
```

Maple [A]
time = 0.18, size = 80, normalized size = 0.76

method	result

default	$\frac{d^{2} \ln \left(e \,x^{2}+d \right)}{2 e \left(a \,e^{2}+c \,d^{2}\right)}-\frac{a \left(-\frac{e \ln \left(c \,x^{4}+a \right)}{2 c}+\frac{d \arctan \left(\frac{c \,x^{2}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{2 \left(a \,e^{2}+c \,d^{2}\right)}$
risch	$\frac{d^{2} \ln \left(e \,x^{2}+d \right)}{2 e \left(a \,e^{2}+c \,d^{2}\right)}+\frac{\left(\sum_{\_R =\operatorname{RootOf}\left(\left(a \,c^{2} e^{2}+c^{3} d^{2}\right) \_Z^{2}-2 \_Z e c a +a \right)}\_R \ln \left(\left(\left(2 a \,c^{2} e^{4}-2 c^{3} d^{2} e^{2}\right) \_R^{2}+\left(-4 e^{3} a c +5 d^{2} e \,c^{2}\right) \_R +2 a \,e^{2}\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^5/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] 1/2*d^2*ln(e*x^2+d)/e/(a*e^2+c*d^2)-1/2*a/(a*e^2+c*d^2)*(-1/2*e/c*ln(c*x^4+a)+d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))

Maxima [A]
time = 0.50, size = 89, normalized size = 0.85

$$\frac{ae\log\left(cx^{4}+a\right)}{4\left(c^{2}d^{2}+ace^{2}\right)}+\frac{d^{2}\log\left(x^{2}e+d\right)}{2\left(cd^{2}e+ae^{3}\right)}-\frac{ad\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{2\left(cd^{2}+ae^{2}\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(x^2*e + d)/(c*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c))

Fricas [A]
time = 0.95, size = 172, normalized size = 1.64

$$\left[\frac{cd\sqrt{-\frac{a}{c}}\,e\log\left(\frac{cx^{4}-2\,cx^{2}\sqrt{-\frac{a}{c}}-a}{cx^{4}+a}\right)+2\,cd^{2}\log\left(x^{2}e+d\right)+ae^{2}\log\left(cx^{4}+a\right)}{4\left(c^{2}d^{2}e+ace^{3}\right)},\,-\frac{2\,cd\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^{2}\sqrt{\frac{a}{c}}}{a}\right)e-2\,cd^{2}\log\left(x^{2}e+d\right)-ae^{2}\log\left(cx^{4}+a\right)}{4\left(c^{2}d^{2}e+ace^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] [1/4*(c*d*sqrt(-a/c)*e*log((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*c*d^2*log(x^2*e + d) + a*e^2*log(c*x^4 + a))/(c^2*d^2*e + a*c*e^3), -1/4*(2*c*d*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a)*e - 2*c*d^2*log(x^2*e + d) - a*e^2*log(c*x^4 + a))/(c^2*d^2*e + a*c*e^3)]

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.82, size = 90, normalized size = 0.86

$$\frac{ae\log\left(cx^4+a\right)}{4\left(c^2d^2+ace^2\right)}+\frac{d^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e+ae^3\right)}-\frac{ad\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*a*e*log(c*x^4 + a)/(c^2*d^2 + a*c*e^2) + 1/2*d^2*log(abs(x^2*e + d))/(c
*d^2*e + a*e^3) - 1/2*a*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c
))
```

Mupad [B]
time = 0.99, size = 138, normalized size = 1.31

$$\frac{d^2\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2\,e+2\,a\,e^3}-\frac{\ln\left(\sqrt{-a\,c^3}+c^2\,x^2\right)\,\left(d\,\sqrt{-a\,c^3}-a\,c\,e\right)}{4\,\left(c^3\,d^2+a\,c^2\,e^2\right)}+\frac{\ln\left(\sqrt{-a\,c^3}-c^2\,x^2\right)\,\left(d\,\sqrt{-a\,c^3}+a\,c\,e\right)}{4\,c^3\,d^2+4\,a\,c^2\,e^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (d^2*log(d + e*x^2))/(2*a*e^3 + 2*c*d^2*e) - (log((-a*c^3)^(1/2) + c^2*x^2)
*(d*(-a*c^3)^(1/2) - a*c*e))/(4*(c^3*d^2 + a*c^2*e^2)) + (log((-a*c^3)^(1/2
) - c^2*x^2)*(d*(-a*c^3)^(1/2) + a*c*e))/(4*c^3*d^2 + 4*a*c^2*e^2)
```

3.232 $\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=96

$$\frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)}-\frac{d\log(d+ex^2)}{2(cd^2+ae^2)}+\frac{d\log(a+cx^4)}{4(cd^2+ae^2)}$$

```
[Out] -1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)+1/4*d*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*e*arct
an(x^2*c^(1/2)/a^(1/2))*a^(1/2)/(a*e^2+c*d^2)/c^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1266, 815, 649, 211, 266}

$$\frac{\sqrt{a}\,e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(ae^2+cd^2)}+\frac{d\log(a+cx^4)}{4(ae^2+cd^2)}-\frac{d\log(d+ex^2)}{2(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[c]*(c*d^2 + a*e^2)) - (d*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) + (d*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
```

```
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3}{(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{de}{(cd^2+ae^2)(d+ex)}+\frac{ae+cdx}{(cd^2+ae^2)(a+cx^2)}\right)dx, x, x^2\right)\\
&= -\frac{d\log(d+ex^2)}{2(cd^2+ae^2)}+\frac{\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\
&= -\frac{d\log(d+ex^2)}{2(cd^2+ae^2)}+\frac{(cd)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}+\frac{(ae)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\
&= \frac{\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{c}\,(cd^2+ae^2)}-\frac{d\log(d+ex^2)}{2(cd^2+ae^2)}+\frac{d\log(a+cx^4)}{4(cd^2+ae^2)}
\end{aligned}
$$

Mathematica [A]
time = 0.03, size = 66, normalized size = 0.69

$$
\frac{\frac{2\sqrt{a}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{c}}-2d\log(d+ex^2)+d\log(a+cx^4)}{4cd^2+4ae^2}
$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)),x]
[Out] ((2*Sqrt[a]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[c] - 2*d*Log[d + e*x^2] +
 d*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

Maple [A]
time = 0.17, size = 72, normalized size = 0.75

method	result

default	$-\frac{d\ln(e\,x^2+d)}{2(a\,e^2+c\,d^2)}+\frac{\frac{d\ln(c\,x^4+a)}{2}+\frac{ae\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2a\,e^2+2c\,d^2}$
risch	$-\frac{d\ln(e\,x^2+d)}{2(a\,e^2+c\,d^2)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(1+\left(ac\,e^2+c^2d^2\right)\_Z^2-2\_Zcd\right)}\_R\ln\left(\left(\left(2e^3ac-2d^2e\,c^2\right)\_R^2-3\_Rcde+2e\right)x^2+\left(3acd\,e^2-c^2d^3\right.\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*ln(e*x^2+d)/(a*e^2+c*d^2)+1/2/(a*e^2+c*d^2)*(1/2*d*ln(c*x^4+a)+a*e/(
a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))
```

Maxima [A]
time = 0.50, size = 81, normalized size = 0.84

$$\frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}+\frac{d\log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}-\frac{d\log\left(x^2e+d\right)}{2\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*a*arctan(c*x^2/sqrt(a*c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x
^4 + a)/(c*d^2 + a*e^2) - 1/2*d*log(x^2*e + d)/(c*d^2 + a*e^2)
```

Fricas [A]
time = 0.51, size = 147, normalized size = 1.53

$$\left[\frac{\sqrt{-\frac{a}{c}}\,e\log\left(\frac{cx^4+2\,cx^2\sqrt{-\frac{a}{c}}-a}{cx^4+a}\right)+d\log\left(cx^4+a\right)-2\,d\log\left(x^2e+d\right)}{4\left(cd^2+ae^2\right)},\ \frac{2\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right)e+d\log\left(cx^4+a\right)-2\,d\log\left(x^2e+d\right)}{4\left(cd^2+ae^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/c)*e*log((c*x^4 + 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + d*lo
g(c*x^4 + a) - 2*d*log(x^2*e + d))/(c*d^2 + a*e^2), 1/4*(2*sqrt(a/c)*arctan
(c*x^2*sqrt(a/c)/a)*e + d*log(c*x^4 + a) - 2*d*log(x^2*e + d))/(c*d^2 + a*e
^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.58, size = 86, normalized size = 0.90

$$-\frac{de\log\left(|x^2e+d|\right)}{2\left(cd^2e+ae^3\right)}+\frac{a\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}+\frac{d\log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(x^2*e + d))/(c*d^2*e + a*e^3) + 1/2*a*arctan(c*x^2/sqrt(a*
c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) + 1/4*d*log(c*x^4 + a)/(c*d^2 + a*e^2)
```

Mupad [**B**]
time = 1.94, size = 944, normalized size = 9.83

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (c*d*log(a^4*e^6 - 9*a*c^3*d^6 - 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2
) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 79*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/
2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 10*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3
*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c
)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^(3/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (d*log(
d + e*x^2))/(2*(a*e^2 + c*d^2)) + (c*d*log(9*a*c^3*d^6 - a^4*e^6 + 39*a^3*c
*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) - 79*a^2*c
^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(1/2) - 76*a
^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 76*a^2*c*d^3
*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c*d^2*e^4*x^
2*(-a*c)^(1/2)))/(4*c^2*d^2 + 4*a*c*e^2) - (e*log(a^4*e^6 - 9*a*c^3*d^6 - 3
9*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2) + 7
9*a^2*c^2*d^4*e^2 - 42*c*d^5*e*(-a*c)^(3/2) + 76*a*d^3*e^3*(-a*c)^(3/2) + 1
0*a^3*d*e^5*(-a*c)^(1/2) + 76*a^2*c^2*d^3*e^3*x^2 - 42*a*c^3*d^5*e*x^2 - 10
```

```
*a^3*c*d*e^5*x^2 + 39*a*d^2*e^4*x^2*(-a*c)^(3/2) - 79*c*d^4*e^2*x^2*(-a*c)^
(3/2))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2) + (e*log(9*a*c^3*d^6 - a^4*e^6
 + 39*a^3*c*d^2*e^4 + a^3*e^6*x^2*(-a*c)^(1/2) - 9*c^3*d^6*x^2*(-a*c)^(1/2)
 - 79*a^2*c^2*d^4*e^2 + 10*a^3*d*e^5*(-a*c)^(1/2) + 42*a*c^2*d^5*e*(-a*c)^(
1/2) - 76*a^2*c^2*d^3*e^3*x^2 + 42*a*c^3*d^5*e*x^2 + 10*a^3*c*d*e^5*x^2 - 7
6*a^2*c*d^3*e^3*(-a*c)^(1/2) + 79*a*c^2*d^4*e^2*x^2*(-a*c)^(1/2) - 39*a^2*c
*d^2*e^4*x^2*(-a*c)^(1/2))*(-a*c)^(1/2))/(4*c^2*d^2 + 4*a*c*e^2)
```

3.233 $\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=96

$$\frac{\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^2+ae^2)}+\frac{e\log(d+ex^2)}{2(cd^2+ae^2)}-\frac{e\log(a+cx^4)}{4(cd^2+ae^2)}$$

```
[Out] 1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)-1/4*e*ln(c*x^4+a)/(a*e^2+c*d^2)+1/2*d*arcta
n(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)/a^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1262, 720, 31, 649, 211, 266}

$$\frac{\sqrt{c}\,d\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(ae^2+cd^2)}-\frac{e\log(a+cx^4)}{4(ae^2+cd^2)}+\frac{e\log(d+ex^2)}{2(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)) + (e*
Log[d + e*x^2])/(2*(c*d^2 + a*e^2)) - (e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)
)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 720

```
Int[1/(((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)), x_Symbol] :> Dist[e^2/(c
*d^2 + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d^2 + a*e^2), Int[(c*d -
 c*e*x)/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2,
 0]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{cd-cex}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)} + \frac{e^2\text{Subst}\left(\int \frac{1}{d+ex}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\
&= \frac{e\log(d+ex^2)}{2(cd^2+ae^2)} + \frac{(cd)\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)} - \frac{(ce)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)}\\
&= \frac{\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^2+ae^2)} + \frac{e\log(d+ex^2)}{2(cd^2+ae^2)} - \frac{e\log(a+cx^4)}{4(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A]
time = 0.03, size = 67, normalized size = 0.70

$$\frac{\frac{2\sqrt{c}\,d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}} + 2e\log(d+ex^2) - e\log(a+cx^4)}{4cd^2+4ae^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] ((2*Sqrt[c]*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/Sqrt[a] + 2*e*Log[d + e*x^2] -
 e*Log[a + c*x^4])/(4*c*d^2 + 4*a*e^2)
```

Maple [A]
time = 0.19, size = 75, normalized size = 0.78

method	result
default	$\frac{e\ln(e\,x^{2}+d)}{2a\,e^{2}+2c\,d^{2}}+\frac{c\left(-\frac{e\ln\left(c\,x^{4}+a\right)}{2c}+\frac{d\arctan\left(\frac{c\,x^{2}}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2a\,e^{2}+2c\,d^{2}}$
risch	$\frac{\ln\left(\left(\left(-3e^{3}c\,a^{2}+5a\,c^{2}d^{2}e+7\sqrt{-ac}\,acd\,e^{2}-\sqrt{-ac}\,c^{2}d^{3}\right)x^{2}-7a^{2}cd\,e^{2}+a\,c^{2}d^{3}-3\sqrt{-ac}\,a^{2}e^{3}+5\sqrt{-ac}\,ac\,d^{2}e\right)d\sqrt{-ac}\right.}{4a(a\,e^{2}+c\,d^{2})}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e*ln(e*x^2+d)/(a*e^2+c*d^2)+1/2*c/(a*e^2+c*d^2)*(-1/2*e/c*ln(c*x^4+a)+d
/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))
```

Maxima [A]
time = 0.53, size = 82, normalized size = 0.85

$$\frac{cd\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{2\left(cd^{2}+ae^{2}\right)\sqrt{ac}}-\frac{e\log\left(cx^{4}+a\right)}{4\left(cd^{2}+ae^{2}\right)}+\frac{e\log\left(x^{2}e+d\right)}{2\left(cd^{2}+ae^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x
^4 + a)/(c*d^2 + a*e^2) + 1/2*e*log(x^2*e + d)/(c*d^2 + a*e^2)
```

Fricas [A]
time = 0.60, size = 150, normalized size = 1.56

$$\left[\frac{d\sqrt{-\frac{c}{a}}\log\left(\frac{cx^{4}+2\,ax^{2}\sqrt{-\frac{c}{a}}-a}{cx^{4}+a}\right)-e\log\left(cx^{4}+a\right)+2\,e\log\left(x^{2}e+d\right)}{4\left(cd^{2}+ae^{2}\right)},-\frac{2\,d\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^{2}}\right)+e\log\left(cx^{4}+a\right)-2\,e\log\left(x^{2}e+d\right)}{4\left(cd^{2}+ae^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(d*sqrt(-c/a)*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - e*lo
g(c*x^4 + a) + 2*e*log(x^2*e + d))/(c*d^2 + a*e^2), -1/4*(2*d*sqrt(c/a)*arc
tan(a*sqrt(c/a)/(c*x^2)) + e*log(c*x^4 + a) - 2*e*log(x^2*e + d))/(c*d^2 +
a*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.10, size = 85, normalized size = 0.89

$$\frac{cd\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e\log\left(cx^4+a\right)}{4\left(cd^2+ae^2\right)}+\frac{e^2\log\left(|x^2e+d|\right)}{2\left(cd^2e+ae^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*c*d*arctan(c*x^2/sqrt(a*c))/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/4*e*log(c*x
^4 + a)/(c*d^2 + a*e^2) + 1/2*e^2*log(abs(x^2*e + d))/(c*d^2*e + a*e^3)
```

Mupad [**B**]
time = 1.02, size = 328, normalized size = 3.42

$$\frac{e\,\ln\left(e\,x^2+d\right)}{2\,c\,d^2+2\,a\,e^2}-\frac{\ln\left(a\,c^5\,d^6\,x^2-c^3\,d^6\,{\left(-a\,c\right)}^{3/2}-9\,a^3\,e^6\,{\left(-a\,c\right)}^{3/2}+9\,a^4\,c^2\,e^6\,x^2+19\,a\,d^2\,e^4\,{\left(-a\,c\right)}^{5/2}+11\,c\,d^4\,e^2\,{\left(-a\,c\right)}^{5/2}+11\,a^2\,c^4\,d^4\,e^2\,x^2+19\,a^3\,c^3\,d^2\,e^4\,x^2\right)\,\left(a\,e-d\,\sqrt{-a\,c}\right)}{4\,\left(a^2\,e^2+c\,a\,d^2\right)}-\frac{\ln\left(9\,a^3\,e^6\,{\left(-a\,c\right)}^{3/2}+c^3\,d^6\,{\left(-a\,c\right)}^{3/2}+a\,c^5\,d^6\,x^2+9\,a^4\,c^2\,e^6\,x^2-19\,a\,d^2\,e^4\,{\left(-a\,c\right)}^{5/2}-11\,c\,d^4\,e^2\,{\left(-a\,c\right)}^{5/2}+11\,a^2\,c^4\,d^4\,e^2\,x^2+19\,a^3\,c^3\,d^2\,e^4\,x^2\right)\,\left(a\,e+d\,\sqrt{-a\,c}\right)}{4\,\left(a^2\,e^2+c\,a\,d^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2) - (log(a*c^5*d^6*x^2 - c^3*d^6*(-a*c
)^(3/2) - 9*a^3*e^6*(-a*c)^(3/2) + 9*a^4*c^2*e^6*x^2 + 19*a*d^2*e^4*(-a*c)^
(5/2) + 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4*e^2*x^2 + 19*a^3*c^3*d^2
*e^4*x^2)*(a*e - d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a*c*d^2)) - (log(9*a^3*e^6*
(-a*c)^(3/2) + c^3*d^6*(-a*c)^(3/2) + a*c^5*d^6*x^2 + 9*a^4*c^2*e^6*x^2 - 1
9*a*d^2*e^4*(-a*c)^(5/2) - 11*c*d^4*e^2*(-a*c)^(5/2) + 11*a^2*c^4*d^4*e^2*x
^2 + 19*a^3*c^3*d^2*e^4*x^2)*(a*e + d*(-a*c)^(1/2)))/(4*(a^2*e^2 + a*c*d^2)
)
```

3.234 $\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=114

$$-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^2+ae^2\right)}+\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}$$

```
[Out] ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)-1/4*c*d*ln(c*x^4+a)/a/(a*e^2+
c*d^2)-1/2*e*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)/a^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}=0.227$, Rules used = {1266, 908, 649, 211, 266}

$$-\frac{\sqrt{c}\,e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(ae^2+cd^2\right)}-\frac{cd\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*(Sqrt[c]*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*(c*d^2 + a*e^2)) +
Log[x]/(a*d) - (e^2*Log[d + e*x^2])/(2*d*(c*d^2 + a*e^2)) - (c*d*Log[a + c*
x^4])/(4*a*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
```

```
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx} - \frac{e^3}{d(cd^2+ae^2)(d+ex)} - \frac{c(ae+cdx)}{a(cd^2+ae^2)(a+cx^2)}\right) dx, x, \right.\\ &= \frac{\log(x)}{ad} - \frac{e^2\log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{c\text{Subst}\left(\int \frac{ae+cdx}{a+cx^2}\,dx, x, x^2\right)}{2a(cd^2+ae^2)}\\ &= \frac{\log(x)}{ad} - \frac{e^2\log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{(c^2d)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2a(cd^2+ae^2)} - \frac{(ce)\text{Subst}\left(\int \frac{1}{a+c}\right.}{2(cd^2+}\\ &= -\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,(cd^2+ae^2)} + \frac{\log(x)}{ad} - \frac{e^2\log(d+ex^2)}{2d(cd^2+ae^2)} - \frac{cd\log(a+cx^4)}{4a(cd^2+ae^2)}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 134, normalized size = 1.18

$$\frac{2\sqrt{a}\,\sqrt{c}\,de\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt{a}\,\sqrt{c}\,de\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+4cd^2\log(x)+4ae^2\log(x)-2ae^2\log(d+ex^2)-cd^2\log(a+cx^4)}{4acd^3+4a^2de^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*Sqrt[a]*Sqrt[c]*d*e*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*
Sqrt[c]*d*e*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c*d^2*Log[x] + 4*a*
e^2*Log[x] - 2*a*e^2*Log[d + e*x^2] - c*d^2*Log[a + c*x^4])/(4*a*c*d^3 + 4*
a^2*d*e^2)
```

Maple [A]
time = 0.19, size = 90, normalized size = 0.79

method	result
default	$-\frac{e^{2} \ln\left(e\,x^{2}+d\right)}{2 d \left(a\,e^{2}+c\,d^{2}\right)}-\frac{c \left(\frac{d \ln\left(c\,x^{4}+a\right)}{2}+\frac{a e \arctan\left(\frac{c\,x^{2}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{2 \left(a\,e^{2}+c\,d^{2}\right) a}+\frac{\ln\left(x\right)}{a d}$
risch	$\frac{\ln\left(x\right)}{a d}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^{3} e^{2}+a^{2} c\,d^{2}\right)\_Z^{2}+2 a d\_Zc+c\right)}\_R \ln\left(\left(\left(-6 e^{4} a^{3}-7 a^{2} c\,d^{2} e^{2}-5 a\,c^{2} d^{4}\right)\_R^{2}+\left(-22 a c d\,e^{2}-5 c^{2} d^{3}\right)\_R-14 c\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)

[Out] -1/2*e^2*ln(e*x^2+d)/d/(a*e^2+c*d^2)-1/2*c/(a*e^2+c*d^2)/a*(1/2*d*ln(c*x^4+a)+a*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))+ln(x)/a/d

Maxima [A]
time = 0.50, size = 99, normalized size = 0.87

$$-\frac{cd\log\left(cx^{4}+a\right)}{4\left(acd^{2}+a^{2}e^{2}\right)}-\frac{c\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)e}{2\left(cd^{2}+ae^{2}\right)\sqrt{ac}}-\frac{e^{2}\log\left(x^{2}e+d\right)}{2\left(cd^{3}+ade^{2}\right)}+\frac{\log\left(x^{2}\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")

[Out] -1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*c*arctan(c*x^2/sqrt(a*c))*e/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/2*e^2*log(x^2*e + d)/(c*d^3 + a*d*e^2) + 1/2*log(x^2)/(a*d)

Fricas [A]
time = 4.72, size = 199, normalized size = 1.75

$$\left[\frac{ad\sqrt{-\frac{c}{a}}\,e\log\left(\frac{cx^{4}-2\,ax^{2}\sqrt{-\frac{c}{a}}-a}{cx^{4}+a}\right)-cd^{2}\log\left(cx^{4}+a\right)-2\,ae^{2}\log\left(x^{2}e+d\right)+4\left(cd^{2}+ae^{2}\right)\log\left(x\right)}{4\left(acd^{3}+a^{2}de^{2}\right)},\ \frac{2\,ad\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^{2}}\right)e-cd^{2}\log\left(cx^{4}+a\right)-2\,ae^{2}\log\left(x^{2}e+d\right)+4\left(cd^{2}+ae^{2}\right)\log\left(x\right)}{4\left(acd^{3}+a^{2}de^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] [1/4*(a*d*sqrt(-c/a)*e*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) - c*d^2*log(c*x^4 + a) - 2*a*e^2*log(x^2*e + d) + 4*(c*d^2 + a*e^2)*log(x))/(a*c*d^3 + a^2*d*e^2), 1/4*(2*a*d*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2))*e -

```
c*d^2*log(c*x^4 + a) - 2*a*e^2*log(x^2*e + d) + 4*(c*d^2 + a*e^2)*log(x))/(
a*c*d^3 + a^2*d*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.58, size = 102, normalized size = 0.89

$$-\frac{cd\log\left(cx^4+a\right)}{4\left(acd^2+a^2e^2\right)}-\frac{c\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(cd^2+ae^2\right)\sqrt{ac}}-\frac{e^3\log\left(\left|x^2e+d\right|\right)}{2\left(cd^3e+ade^3\right)}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/4*c*d*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) - 1/2*c*arctan(c*x^2/sqrt(a*c))
*e/((c*d^2 + a*e^2)*sqrt(a*c)) - 1/2*e^3*log(abs(x^2*e + d))/(c*d^3*e + a*d
*e^3) + 1/2*log(x^2)/(a*d)
```

Mupad [**B**]
time = 0.96, size = 527, normalized size = 4.62

$$\frac{\ln\left(64\,a^7\,c\,e^{10}\,x^2-64\,a^6\,e^{10}\,\sqrt{-a^3\,c}-25\,a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+25\,a^2\,c^6\,d^{10}\,x^2+180\,a^2\,d^2\,e^8\,{\left(-a^3\,c\right)}^{3/2}-41\,c^2\,d^6\,e^4\,{\left(-a^3\,c\right)}^{3/2}-9\,a^3\,c^5\,d^8\,e^2\,x^2-41\,a^4\,c^4\,d^6\,e^4\,x^2+109\,a^5\,c^3\,d^4\,e^6\,x^2+180\,a^6\,c^2\,d^2\,e^8\,x^2+9\,a^2\,c^4\,d^8\,e^2\,\sqrt{-a^3\,c}+109\,a\,c\,d^4\,e^6\,{\left(-a^3\,c\right)}^{3/2}\right)\,\left(e\,\sqrt{-a^3\,c}-a\,c\,d\right)}{4\,a^3\,e^2+4\,c\,a^2\,d^2}-\frac{\ln\left(64\,a^6\,e^{10}\,\sqrt{-a^3\,c}+64\,a^7\,c\,e^{10}\,x^2+25\,a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+25\,a^2\,c^6\,d^{10}\,x^2-180\,a^2\,d^2\,e^8\,{\left(-a^3\,c\right)}^{3/2}+41\,c^2\,d^6\,e^4\,{\left(-a^3\,c\right)}^{3/2}-9\,a^3\,c^5\,d^8\,e^2\,x^2-41\,a^4\,c^4\,d^6\,e^4\,x^2+109\,a^5\,c^3\,d^4\,e^6\,x^2+180\,a^6\,c^2\,d^2\,e^8\,x^2-9\,a^2\,c^4\,d^8\,e^2\,\sqrt{-a^3\,c}-109\,a\,c\,d^4\,e^6\,{\left(-a^3\,c\right)}^{3/2}\right)\,\left(e\,\sqrt{-a^3\,c}+a\,c\,d\right)}{4\,\left(a^3\,e^2+c\,a^2\,d^2\right)}-\frac{e^2\,\ln\left(e\,x^2+d\right)}{2\,c\,d^3+2\,a\,d\,e^2}+\frac{\ln\left(x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(64*a^7*c*e^10*x^2 - 64*a^6*e^10*(-a^3*c)^(1/2) - 25*a*c^5*d^10*(-a^3*c
)^(1/2) + 25*a^2*c^6*d^10*x^2 + 180*a^2*d^2*e^8*(-a^3*c)^(3/2) - 41*c^2*d^6
*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c^4*d^6*e^4*x^2 + 109*
a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 + 9*a^2*c^4*d^8*e^2*(-a^3*c)^
(1/2) + 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(-a^3*c)^(1/2) - a*c*d))/(4*a^3*
e^2 + 4*a^2*c*d^2) - (log(64*a^6*e^10*(-a^3*c)^(1/2) + 64*a^7*c*e^10*x^2 +
25*a*c^5*d^10*(-a^3*c)^(1/2) + 25*a^2*c^6*d^10*x^2 - 180*a^2*d^2*e^8*(-a^3*
c)^(3/2) + 41*c^2*d^6*e^4*(-a^3*c)^(3/2) - 9*a^3*c^5*d^8*e^2*x^2 - 41*a^4*c
^4*d^6*e^4*x^2 + 109*a^5*c^3*d^4*e^6*x^2 + 180*a^6*c^2*d^2*e^8*x^2 - 9*a^2*
c^4*d^8*e^2*(-a^3*c)^(1/2) - 109*a*c*d^4*e^6*(-a^3*c)^(3/2))*(e*(-a^3*c)^(1
/2) + a*c*d))/(4*(a^3*e^2 + a^2*c*d^2)) - (e^2*log(d + e*x^2))/(2*c*d^3 + 2
*a*d*e^2) + log(x)/(a*d)
```

3.235 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=129

$$-\frac{1}{2adx^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)}-\frac{e\log(x)}{ad^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(cd^2+ae^2\right)}$$

```
[Out] -1/2/a/d/x^2-1/2*c^(3/2)*d*arctan(x^2*c^(1/2)/a^(1/2))/a^(3/2)/(a*e^2+c*d^2
)-e*ln(x)/a/d^2+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)+1/4*c*e*ln(c*x^4+a)/a
/(a*e^2+c*d^2)
```

Rubi [A]
time = 0.10, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1266, 908, 649, 211, 266}

$$-\frac{c^{3/2}d\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{ce\log\left(a+cx^4\right)}{4a\left(ae^2+cd^2\right)}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(ae^2+cd^2\right)}-\frac{e\log(x)}{ad^2}-\frac{1}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/2*1/(a*d*x^2) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(3/2)*(c*
d^2 + a*e^2)) - (e*Log[x])/(a*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 + a
*e^2)) + (c*e*Log[a + c*x^4])/(4*a*(c*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x^3(d+ex^2)(a+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)(a+cx^2)}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^2}-\frac{e}{ad^2x}+\frac{e^4}{d^2(cd^2+ae^2)(d+ex)}-\frac{c^2(d-ex)}{a(cd^2+ae^2)(a+c\right.\right.\\ &= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log(d+ex^2)}{2d^2(cd^2+ae^2)}-\frac{c^2\text{Subst}\left(\int \frac{d-ex}{a+cx^2}\,dx, x, x^2\right)}{2a(cd^2+ae^2)}\\ &= -\frac{1}{2adx^2}-\frac{e\log(x)}{ad^2}+\frac{e^3\log(d+ex^2)}{2d^2(cd^2+ae^2)}-\frac{(c^2d)\,\text{Subst}\left(\int \frac{1}{a+cx^2}\,dx, x, x^2\right)}{2a(cd^2+ae^2)}+\frac{(c\\ &= -\frac{1}{2adx^2}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}(cd^2+ae^2)}-\frac{e\log(x)}{ad^2}+\frac{e^3\log(d+ex^2)}{2d^2(cd^2+ae^2)}+\frac{ce\log(a+}{4a(cd^2+\end{aligned}$$

Mathematica [A]
time = 0.07, size = 169, normalized size = 1.31

$$\frac{2c^{3/2}d^3x^2\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2c^{3/2}d^3x^2\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+\sqrt{a}\left(-2cd^3-2ade^2-4e(cd^2+ae^2)x^2\log(x)+2ae^3x^2\log(d+ex^2)+cd^2ex^2\log(a+cx^4)\right)}{4a^{3/2}d^2(cd^2+ae^2)x^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (2*c^(3/2)*d^3*x^2*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*c^(3/2)*d^3*
x^2*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*(-2*c*d^3 - 2*a*d*e^2
 - 4*e*(c*d^2 + a*e^2)*x^2*Log[x] + 2*a*e^3*x^2*Log[d + e*x^2] + c*d^2*e*x^
2*Log[a + c*x^4]))/(4*a^(3/2)*d^2*(c*d^2 + a*e^2)*x^2)
```

Maple [A]
time = 0.18, size = 107, normalized size = 0.83

method	result
default	$\frac{e^{3} \ln \left(e \,x^{2}+d \right)}{2 d^{2} \left(a \,e^{2}+c \,d^{2}\right)}-\frac{c^{2} \left(-\frac{e \ln \left(c \,x^{4}+a \right)}{2 c}+\frac{d \arctan \left(\frac{c \,x^{2}}{\sqrt{a c}}\right)}{\sqrt{a c}}\right)}{2 \left(a \,e^{2}+c \,d^{2}\right) a}-\frac{1}{2 a d \,x^{2}}-\frac{e \ln \left(x \right)}{a \,d^{2}}$
risch	$-\frac{1}{2 a d \,x^{2}}-\frac{e \ln \left(x \right)}{a \,d^{2}}+\frac{e^{3} \ln \left(-e \,x^{2}-d \right)}{2 d^{2} \left(a \,e^{2}+c \,d^{2}\right)}+\frac{\left(\sum_{\textit{\_R} =\operatorname{RootOf}\left(\left(e^{2} a^{4}+a^{3} c \,d^{2}\right) \textit{\_Z}^{2}-2 a^{2} c e \textit{\_Z} +c^{2}\right)}\textit{\_R} \ln \left(\left(\left(-6 a^{5} d^{2} e^{4}-7 a^{4} c \,d^{4} e^{2}-5 a^{3} c^{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)-1/2*c^2/(a*e^2+c*d^2)/a*(-1/2*e/c*ln(
c*x^4+a)+d/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))-1/2/a/d/x^2-e*ln(x)/a/d^2
```

Maxima [A]
time = 0.50, size = 119, normalized size = 0.92

$$-\frac{c^{2} d \arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{2 \left(acd^{2} + a^{2} e^{2}\right) \sqrt{ac}} + \frac{ce \log\left(cx^{4} + a\right)}{4 \left(acd^{2} + a^{2} e^{2}\right)} + \frac{e^{3} \log\left(x^{2} e + d\right)}{2 \left(cd^{4} + ad^{2} e^{2}\right)} - \frac{e \log\left(x^{2}\right)}{2 \, ad^{2}} - \frac{1}{2 \, adx^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/2*c^2*d*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*
e*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) + 1/2*e^3*log(x^2*e + d)/(c*d^4 + a*d^
2*e^2) - 1/2*e*log(x^2)/(a*d^2) - 1/2/(a*d*x^2)
```

Fricas [A]
time = 22.84, size = 275, normalized size = 2.13

$$\left[\frac{cd^{3}x^{2}\sqrt{-\frac{c}{a}}\log\left(\frac{cx^{4} - 2\,ax^{2}\sqrt{-\frac{c}{a}} - a}{cx^{4} + a}\right) + cd^{2}x^{2}e\log\left(cx^{4} + a\right) + 2\,ax^{2}e^{3}\log\left(x^{2}e + d\right) - 2\,cd^{3} - 2\,ade^{2} - 4\left(cd^{2}x^{2}e + ax^{2}e^{3}\right)\log\left(x\right)}{4\left(acd^{4}x^{2} + a^{2}d^{2}x^{2}e^{2}\right)}, \frac{2\,cd^{3}x^{2}\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^{2}}\right) + cd^{2}x^{2}e\log\left(cx^{4} + a\right) + 2\,ax^{2}e^{3}\log\left(x^{2}e + d\right) - 2\,cd^{3} - 2\,ade^{2} - 4\left(cd^{2}x^{2}e + ax^{2}e^{3}\right)\log\left(x\right)}{4\left(acd^{4}x^{2} + a^{2}d^{2}x^{2}e^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(c*d^3*x^2*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a) - a)/(c*x^4 + a)
) + c*d^2*x^2*e*log(c*x^4 + a) + 2*a*x^2*e^3*log(x^2*e + d) - 2*c*d^3 - 2*a
*d*e^2 - 4*(c*d^2*x^2*e + a*x^2*e^3)*log(x))/(a*c*d^4*x^2 + a^2*d^2*x^2*e^2
), 1/4*(2*c*d^3*x^2*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + c*d^2*x^2*e*log
```

```
(c*x^4 + a) + 2*a*x^2*e^3*log(x^2*e + d) - 2*c*d^3 - 2*a*d*e^2 - 4*(c*d^2*x
^2*e + a*x^2*e^3)*log(x))/(a*c*d^4*x^2 + a^2*d^2*x^2*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a),x)
[Out] Timed out
```

Giac [**A**]
time = 3.92, size = 132, normalized size = 1.02

$$-\frac{c^2 d \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{2\left(acd^2 + a^2e^2\right)\sqrt{ac}} + \frac{ce \log\left(cx^4 + a\right)}{4\left(acd^2 + a^2e^2\right)} + \frac{e^4 \log\left(|x^2e + d|\right)}{2\left(cd^4e + ad^2e^3\right)} - \frac{e \log\left(x^2\right)}{2\,ad^2} + \frac{x^2e - d}{2\,ad^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
[Out] -1/2*c^2*d*arctan(c*x^2/sqrt(a*c))/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) + 1/4*c*
e*log(c*x^4 + a)/(a*c*d^2 + a^2*e^2) + 1/2*e^4*log(abs(x^2*e + d))/(c*d^4*e
 + a*d^2*e^3) - 1/2*e*log(x^2)/(a*d^2) + 1/2*(x^2*e - d)/(a*d^2*x^2)
```

Mupad [**B**]
time = 1.38, size = 820, normalized size = 6.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)*(d + e*x^2)),x)
[Out] (log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*x^2 + a^2*c^7*d^16*(-a^3*c^3)^(3/
2) - 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) + 63*a^3*d^8*e^8*(-a^3*c^3)^(5/2) +
224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) - 28*c^3*d^14*e^2*(-a^3*c^3)^(5/2) + 28*a
^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^12*e^4*x^2 + 108*a^9*c^9*d^10*e^6*x^2
 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c^7*d^6*e^10*x^2 + 212*a^12*c^6*d^4*e^
12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 - 114*a*c^2*d^12*e^4*(-a^3*c^3)^(5/2) -
108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2) + 212*a^8*c*d^4*e^12*(-a^3*c^3)^(3/2) -
 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/2))*(d*(-a^3*c^3)^(1/2) + a^2*c*e))/(4*a
^4*e^2 + 4*a^3*c*d^2) - (log(a^6*c^12*d^16*x^2 + 64*a^14*c^4*e^16*x^2 - a^2
*c^7*d^16*(-a^3*c^3)^(3/2) + 64*a^13*c^2*e^16*(-a^3*c^3)^(1/2) - 63*a^3*d^8
*e^8*(-a^3*c^3)^(5/2) - 224*a^9*d^2*e^14*(-a^3*c^3)^(3/2) + 28*c^3*d^14*e^2
```

```
*(-a^3*c^3)^(5/2) + 28*a^7*c^11*d^14*e^2*x^2 + 114*a^8*c^10*d^12*e^4*x^2 +
108*a^9*c^9*d^10*e^6*x^2 - 63*a^10*c^8*d^8*e^8*x^2 - 32*a^11*c^7*d^6*e^10*x
^2 + 212*a^12*c^6*d^4*e^12*x^2 + 224*a^13*c^5*d^2*e^14*x^2 + 114*a*c^2*d^12
*e^4*(-a^3*c^3)^(5/2) + 108*a^2*c*d^10*e^6*(-a^3*c^3)^(5/2) - 212*a^8*c*d^4
*e^12*(-a^3*c^3)^(3/2) + 32*a^7*c^2*d^6*e^10*(-a^3*c^3)^(3/2))*(d*(-a^3*c^3
)^(1/2) - a^2*c*e))/(4*(a^4*e^2 + a^3*c*d^2)) + (e^3*log(d + e*x^2))/(2*c*d
^4 + 2*a*d^2*e^2) - 1/(2*a*d*x^2) - (e*log(x))/(a*d^2)
```

3.236 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=156

$$-\frac{1}{4adx^4}+\frac{e}{2ad^2x^2}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(cd^2+ae^2\right)}-\frac{\left(cd^2-ae^2\right)\log(x)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)}+\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(cd^2+ae^2\right)}$$

```
[Out] -1/4/a/d/x^4+1/2*e/a/d^2/x^2+1/2*c^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(3
/2)/(a*e^2+c*d^2)-(-a*e^2+c*d^2)*ln(x)/a^2/d^3-1/2*e^4*ln(e*x^2+d)/d^3/(a*e
^2+c*d^2)+1/4*c^2*d*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)
```

Rubi [A]

time = 0.12, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {1266, 908, 649, 211, 266}

$$\frac{c^{3/2}e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\left(ae^2+cd^2\right)}+\frac{c^2d\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)}-\frac{\log(x)\left(cd^2-ae^2\right)}{a^2d^3}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2+cd^2\right)}+\frac{e}{2ad^2x^2}-\frac{1}{4adx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/4*1/(a*d*x^4) + e/(2*a*d^2*x^2) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*a^(3/2)*(c*d^2 + a*e^2)) - ((c*d^2 - a*e^2)*Log[x])/(a^2*d^3) - (e^4
*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)) + (c^2*d*Log[a + c*x^4])/(4*a^2*(c
*d^2 + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^5\,(d+ex^2)\,(a+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(d+ex)\,(a+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^3} - \frac{e}{ad^2x^2} + \frac{-cd^2+ae^2}{a^2d^3x} - \frac{e^5}{d^3\,(cd^2+ae^2)\,(d+ex)} + \frac{\ldots}{a^2\,(cd\ldots}\right.\right.\\
&= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} - \frac{(cd^2-ae^2)\log(x)}{a^2d^3} - \frac{e^4\log(d+ex^2)}{2d^3\,(cd^2+ae^2)} + \frac{c^2\text{Subst}\left(\int \frac{ae+cd\ldots}{a+cx^2}\ldots\right.}{2a^2\,(cd^2+\ldots}\\
&= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} - \frac{(cd^2-ae^2)\log(x)}{a^2d^3} - \frac{e^4\log(d+ex^2)}{2d^3\,(cd^2+ae^2)} + \frac{(c^3d)\,\text{Subst}\left(\int \frac{\ldots}{a\ldots}\right.}{2a^2\,(cd^2\ldots}\\
&= -\frac{1}{4adx^4} + \frac{e}{2ad^2x^2} + \frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{3/2}\,(cd^2+ae^2)} - \frac{(cd^2-ae^2)\log(x)}{a^2d^3} - \frac{e^4\log(d+\ldots}{2d^3\,(cd^2+\ldots}
\end{aligned}$$

Mathematica [A]
time = 0.06, size = 209, normalized size = 1.34

$$-\frac{acd^4+a^2d^2e^2-2acd^3ex^2-2a^2de^3x^2+2\sqrt{a}\,c^{3/2}d^3ex^4\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt{a}\,c^{3/2}d^3ex^4\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+4c^2d^4x^4\log(x)-4a^2e^4x^4\log(x)+2a^2e^4x^4\log(d+ex^2)-c^2d^4x^4\log(a+cx^4)}{4a^2d^3\,(cd^2+ae^2)\,x^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/4*(a*c*d^4 + a^2*d^2*e^2 - 2*a*c*d^3*e*x^2 - 2*a^2*d*e^3*x^2 + 2*Sqrt[a]
*c^(3/2)*d^3*e*x^4*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[a]*c^(3
/2)*d^3*e*x^4*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*c^2*d^4*x^4*Log[x
] - 4*a^2*e^4*x^4*Log[x] + 2*a^2*e^4*x^4*Log[d + e*x^2] - c^2*d^4*x^4*Log[a
 + c*x^4])/(a^2*d^3*(c*d^2 + a*e^2)*x^4)
```

Maple [A]
time = 0.18, size = 127, normalized size = 0.81

method	result
default	$-\frac{e^4 \ln(e x^2+d)}{2d^3(a e^2+c d^2)}+\frac{c^2\left(\frac{d \ln(c x^4+a)}{2}+\frac{a e \arctan\left(\frac{c x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2(a e^2+c d^2)a^2}-\frac{1}{4ad x^4}+\frac{(a e^2-c d^2)\ln(x)}{d^3 a^2}+\frac{e}{2a d^2 x^2}$
risch	$\frac{\frac{e x^2}{2d^2 a}-\frac{1}{4da}}{x^4}+\frac{\ln(x)e^2}{d^3 a}-\frac{\ln(x)c}{d a^2}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5 e^2+d^2 a^4 c\right)\_Z^2-2a^2 c^2 d\_Z+c^3\right)} \_R \ln\left(\left(\left(-6a^6 d^4 e^4-7a^5 c d^6 e^2-5a^4 c^2 d^8\right)\_\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)+1/2*c^2/(a*e^2+c*d^2)/a^2*(1/2*d*ln(
c*x^4+a)+a*e/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2)))-1/4/a/d/x^4+(a*e^2-c*d^
2)/d^3/a^2*ln(x)+1/2*e/a/d^2/x^2
```

Maxima [A]
time = 0.49, size = 143, normalized size = 0.92

$$\frac{c^2 d \log\left(cx^4+a\right)}{4\left(a^2cd^2+a^3e^2\right)}+\frac{c^2 \arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(acd^2+a^2e^2\right)\sqrt{ac}}-\frac{e^4 \log\left(x^2e+d\right)}{2\left(cd^5+ad^3e^2\right)}-\frac{\left(cd^2-ae^2\right)\log\left(x^2\right)}{2\,a^2d^3}+\frac{2\,x^2e-d}{4\,ad^2x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/4*c^2*d*log(c*x^4 + a)/(a^2*c*d^2 + a^3*e^2) + 1/2*c^2*arctan(c*x^2/sqrt(
a*c))*e/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) - 1/2*e^4*log(x^2*e + d)/(c*d^5 + a
*d^3*e^2) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(2*x^2*e - d)/(a*d
^2*x^4)
```

Fricas [A]
time = 54.53, size = 350, normalized size = 2.24

$$\left[\frac{acd^3x^4\sqrt{-\frac{c}{a}}\,e\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+c^2d^4x^4\log\left(cx^4+a\right)+2\,acd^3x^2e-2\,a^2x^4e^4\log\left(x^2e+d\right)-acd^4+2\,a^2dx^2e^3-a^2d^2e^2-4\left(c^2d^4x^4-a^2x^4e^4\right)\log\left(x\right)}{4\left(a^2cd^5x^4+a^3d^3x^4e^2\right)},\ -\frac{2\,acd^3x^4\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)e-c^2d^4x^4\log\left(cx^4+a\right)-2\,acd^3x^2e+2\,a^2x^4e^4\log\left(x^2e+d\right)+acd^4-2\,a^2dx^2e^3+a^2d^2e^2+4\left(c^2d^4x^4-a^2x^4e^4\right)\log\left(x\right)}{4\left(a^2cd^5x^4+a^3d^3x^4e^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(a*c*d^3*x^4*sqrt(-c/a)*e*log((c*x^4 + 2*a*x^2*sqrt(-c/a) - a)/(c*x^4
+ a)) + c^2*d^4*x^4*log(c*x^4 + a) + 2*a*c*d^3*x^2*e - 2*a^2*x^4*e^4*log(x^
2*e + d) - a*c*d^4 + 2*a^2*d*x^2*e^3 - a^2*d^2*e^2 - 4*(c^2*d^4*x^4 - a^2*x
```

```
^4*e^4)*log(x))/(a^2*c*d^5*x^4 + a^3*d^3*x^4*e^2), -1/4*(2*a*c*d^3*x^4*sqrt
(c/a)*arctan(a*sqrt(c/a)/(c*x^2))*e - c^2*d^4*x^4*log(c*x^4 + a) - 2*a*c*d^
3*x^2*e + 2*a^2*x^4*e^4*log(x^2*e + d) + a*c*d^4 - 2*a^2*d*x^2*e^3 + a^2*d^
2*e^2 + 4*(c^2*d^4*x^4 - a^2*x^4*e^4)*log(x))/(a^2*c*d^5*x^4 + a^3*d^3*x^4*
e^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.42, size = 168, normalized size = 1.08

$$\frac{c^2 d\log\left(cx^4+a\right)}{4\left(a^2cd^2+a^3e^2\right)}+\frac{c^2\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)e}{2\left(acd^2+a^2e^2\right)\sqrt{ac}}-\frac{e^5\log\left(|x^2e+d|\right)}{2\left(cd^5e+ad^3e^3\right)}-\frac{\left(cd^2-ae^2\right)\log\left(x^2\right)}{2\,a^2d^3}+\frac{3\,cd^2x^4-3\,ax^4e^2+2\,adx^2e-ad^2}{4\,a^2d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/4*c^2*d*log(c*x^4 + a)/(a^2*c*d^2 + a^3*e^2) + 1/2*c^2*arctan(c*x^2/sqrt(
a*c))*e/((a*c*d^2 + a^2*e^2)*sqrt(a*c)) - 1/2*e^5*log(abs(x^2*e + d))/(c*d^
5*e + a*d^3*e^3) - 1/2*(c*d^2 - a*e^2)*log(x^2)/(a^2*d^3) + 1/4*(3*c*d^2*x^
4 - 3*a*x^4*e^2 + 2*a*d*x^2*e - a*d^2)/(a^2*d^3*x^4)
```

Mupad [B]
time = 1.87, size = 1017, normalized size = 6.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(25*a^2*c^9*d^20*(-a^5*c^3)^(3/2) - 64*a^19*c^4*e^20*x^2 - 25*a^9*c^14*
d^20*x^2 - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^12*(-a^5*c^3)^
(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^6*(-a^5*c^3)^(5
/2) - 76*a^10*c^13*d^18*e^2*x^2 - 138*a^11*c^12*d^16*e^4*x^2 - 112*a^12*c^1
1*d^14*e^6*x^2 + 55*a^13*c^10*d^12*e^8*x^2 + 104*a^14*c^9*d^10*e^10*x^2 + 1
00*a^15*c^8*d^8*e^12*x^2 + 172*a^16*c^7*d^6*e^14*x^2 + 32*a^17*c^6*d^4*e^16
*x^2 - 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a^5*c^3)^(5/2) + 104
*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*(-a^5*c^3)^(3/2) + 7
```

```
6*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d^16*e^4*(-a^5*c^3)^(3/2)
 - 172*a^9*c^2*d^6*e^14*(-a^5*c^3)^(3/2))*(e*(-a^5*c^3)^(1/2) + a^2*c^2*d))
/(4*a^5*e^2 + 4*a^4*c*d^2) - (e^4*log(d + e*x^2))/(2*(c*d^5 + a*d^3*e^2)) -
 (log(25*a^9*c^14*d^20*x^2 + 64*a^19*c^4*e^20*x^2 + 25*a^2*c^9*d^20*(-a^5*c
^3)^(3/2) - 64*a^17*c^2*e^20*(-a^5*c^3)^(1/2) + 100*a^3*d^8*e^12*(-a^5*c^3)
^(5/2) + 128*a^11*d^2*e^18*(-a^5*c^3)^(3/2) - 112*c^3*d^14*e^6*(-a^5*c^3)^(
5/2) + 76*a^10*c^13*d^18*e^2*x^2 + 138*a^11*c^12*d^16*e^4*x^2 + 112*a^12*c^
11*d^14*e^6*x^2 - 55*a^13*c^10*d^12*e^8*x^2 - 104*a^14*c^9*d^10*e^10*x^2 -
100*a^15*c^8*d^8*e^12*x^2 - 172*a^16*c^7*d^6*e^14*x^2 - 32*a^17*c^6*d^4*e^1
6*x^2 + 128*a^18*c^5*d^2*e^18*x^2 + 55*a*c^2*d^12*e^8*(-a^5*c^3)^(5/2) + 10
4*a^2*c*d^10*e^10*(-a^5*c^3)^(5/2) - 32*a^10*c*d^4*e^16*(-a^5*c^3)^(3/2) +
76*a^3*c^8*d^18*e^2*(-a^5*c^3)^(3/2) + 138*a^4*c^7*d^16*e^4*(-a^5*c^3)^(3/2
) - 172*a^9*c^2*d^6*e^14*(-a^5*c^3)^(3/2))*(e*(-a^5*c^3)^(1/2) - a^2*c^2*d)
)/(4*(a^5*e^2 + a^4*c*d^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2))/x^4 + (log(x)
*(a*e^2 - c*d^2))/(a^2*d^3)
```

3.237 $\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=359

$$-\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\,(cd^2+ae^2)}-\frac{a^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{7/4}\,(cd^2+ae^2)}+\frac{a^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1\right.}{2\sqrt{2}\,c^{7/4}\,(cd^2+a}$$

```
[Out] -d*x/c/e^2+1/3*x^3/c/e+d^(7/2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/(a*e^2+c*d
^2)+1/4*a^(5/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))
/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*a^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(-e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*a^(5/4)*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(7/4)/
(a*e^2+c*d^2)*2^(1/2)+1/8*a^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{a^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,c^{7/4}\,(ae^2+cd^2)}+\frac{a^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,c^{7/4}\,(ae^2+cd^2)}-\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\,(ae^2+cd^2)}+\frac{a^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}\,(ae^2+cd^2)}+\frac{d^{7/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}\,(ae^2+cd^2)}-\frac{dx}{ce^2}+\frac{x^3}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -((d*x)/(c*e^2)) + x^3/(3*c*e) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(
5/2)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) + (a^(5/4)*(Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(7
/4)*(c*d^2 + a*e^2)) - (a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) +
(a^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^8}{(d+ex^2)(a+cx^4)}\,dx &= \int \left(-\frac{d}{ce^2}+\frac{x^2}{ce}+\frac{d^4}{e^2(cd^2+ae^2)(d+ex^2)}+\frac{a^2(d-ex^2)}{c(cd^2+ae^2)(a+cx^4)}\right)dx \\
&= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{a^2\int\frac{d-ex^2}{a+cx^4}\,dx}{c(cd^2+ae^2)}+\frac{d^4\int\frac{1}{d+ex^2}\,dx}{e^2(cd^2+ae^2)} \\
&= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int\frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c^2(cd^2+ae^2)}+\frac{(a \\
&= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)}+\frac{\left(a^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int\frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c^2(cd^2+ae^2)} \\
&= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)}-\frac{a^{5/4}(\sqrt{c}\,d+\sqrt{a}\,e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{}\right.}{4\sqrt{2}\,c^{7/4}(cd^2+ae^2)} \\
&= -\frac{dx}{ce^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}(cd^2+ae^2)}-\frac{a^{7/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{7/4}(cd^2+ae^2)}+
\end{aligned}$$

Mathematica [A]
time = 0.26, size = 344, normalized size = 0.96

$$\frac{-24c^{3/4}d\sqrt{e}\,(cd^2+ae^2)\,x+8c^{3/4}e^{3/2}(cd^2+ae^2)\,x^3+24c^{7/4}d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+6\sqrt{2}\,a^{5/4}e^{5/2}(-\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-6\sqrt{2}\,a^{5/4}e^{5/2}(-\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)-3\sqrt{2}\,ae^{5/2}(\sqrt[4]{a}\sqrt{c}\,d+a^{3/4}e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+3\sqrt{2}\,ae^{5/2}(\sqrt[4]{a}\sqrt{c}\,d+a^{3/4}e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{24c^{7/4}e^{5/2}(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-24*c^(3/4)*d*Sqrt[e]*(c*d^2 + a*e^2)*x + 8*c^(3/4)*e^(3/2)*(c*d^2 + a*e^2
)*x^3 + 24*c^(7/4)*d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(5/4)*
e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)]
- 6*Sqrt[2]*a^(5/4)*e^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*
c^(1/4)*x)/a^(1/4)] - 3*Sqrt[2]*a*e^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + 3*Sqrt[2]*a*e^(5/2)
*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(24*c^(7/4)*e^(5/2)*(c*d^2 + a*e^2))
```

Maple [A]
time = 0.20, size = 279, normalized size = 0.78

method	result
default	$-\frac{-\frac{1}{3}e\,x^3+dx}{c\,e^2}+\frac{d^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e^2(a\,e^2+c\,d^2)\sqrt{de}}+\frac{a^2\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}}{\left(\frac{a}{c}\right)}\right.\right.}{8a}\right.}{(}$
risch	$\frac{x^3}{3ce}-\frac{dx}{c\,e^2}+\frac{\sqrt{-de}\,d^3\ln\left(\left(-16(-de)^{\frac{5}{2}}a\,c^7d^{12}e^2+16(-de)^{\frac{5}{2}}c^8d^{14}+14a^4c^4d^7e^9(-de)^{\frac{3}{2}}-4a^3c^5d^9e^7(-de)^{\frac{3}{2}}-2a^2c^6d^{11}e^5(-d\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c/e^2*(-1/3*e*x^3+d*x)+1/e^2*d^4/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d
*e)^(1/2))+a^2/(a*e^2+c*d^2)/c*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^
(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/
c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/
4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/
2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]

time = 0.50, size = 291, normalized size = 0.81

$$\frac{d^{\frac{7}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e^2+ae^4}+\frac{a^2\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\left(c^2d^2+ace^2\right)}+\frac{\left(x^3e-3\,dx\right)e^{(-2)}}{3\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 + a*e^4) + 1/8*a^2*(2
*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*
a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt
(c)) + 2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x -
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)
*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)
))/(c^2*d^2 + a*c*e^2) + 1/3*(x^3*e - 3*d*x)*e^(-2)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2061 vs. $2(263) = 526$.

time = 5.53, size = 4154, normalized size = 11.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(4*c*d^2*x^3*e + 6*sqrt(-d*e^(-1))*c*d^3*log((x^2*e + 2*sqrt(-d*e^(-1
))*x*e - d)/(x^2*e + d)) - 12*c*d^3*x + 4*a*x^3*e^3 - 12*a*d*x*e^2 - 3*(c^2
*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*
e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d
^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 +
2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-a^3*c*d^2*x + a^4*x*e^2 + (a^2*c^3*d^3
 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c
^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^
9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 +
 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*
e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 +
 a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^
2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sq
rt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2
+ 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4
*d^2*e^2 + a^2*c^3*e^4))*log(-a^3*c*d^2*x + a^4*x*e^2 - (a^2*c^3*d^3 - a^3*
c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4
- 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e
^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^
4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c
^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^
7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c
*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^
5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2
*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^
2 + a^2*c^3*e^4))*log(-a^3*c*d^2*x + a^4*x*e^2 + (a^2*c^3*d^3 - a^3*c^2*d*e
^2 - (c^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6
*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*
a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e
^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8
 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8))
)/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*s
qrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d
^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^
4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2
*c^3*e^4))*log(-a^3*c*d^2*x + a^4*x*e^2 - (a^2*c^3*d^3 - a^3*c^2*d*e^2 - (c
^7*d^4*e + 2*a*c^6*d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*
e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8
*d^2*e^6 + a^4*c^7*e^8)))*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^
2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*
c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*
```

```
d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))))/(c^2*d^2*e^2 + a*c*e^4), 1/12*(4*c*
d^2*x^3*e + 12*c*d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) - 12*c*d^3*x +
4*a*x^3*e^3 - 12*a*d*x*e^2 - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c
^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^
2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d
^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-a^3
*c*d^2*x + a^4*x*e^2 + (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*
d^2*e^3 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^1
1*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*
e^8)))*sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a
^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^
2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e
^2 + a^2*c^3*e^4))) + 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e + (c^5*d^4
+ 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7
*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6
+ a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4))*log(-a^3*c*d^2*
x + a^4*x*e^2 - (a^2*c^3*d^3 - a^3*c^2*d*e^2 + (c^7*d^4*e + 2*a*c^6*d^2*e^3
 + a^2*c^5*e^5)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 +
 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))*
sqrt((2*a^3*d*e + (c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*
d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d
^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^
2*c^3*e^4))) - 3*(c^2*d^2*e^2 + a*c*e^4)*sqrt((2*a^3*d*e - (c^5*d^4 + 2*a*c
^4*d^2*e^2 + a^2*c^3*e^4)*sqrt(-(a^5*c^2*d^4 - 2*a^6*c*d^2*e^2 + a^7*e^4)/(
c^11*d^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6 + a^4*c
^7*e^8)))/(c^5*d^4 + 2*a*c^4*d^2*e^2 + a^2*c^3*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.00, size = 363, normalized size = 1.01

$$\frac{d^{\frac{7}{2}} \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)}}{cd^2e^2 + ae^4} + \frac{\left(\left(ac^3\right)^{\frac{1}{4}} ac^2d - \left(ac^3\right)^{\frac{3}{4}} ae\right) \arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^5d^2 + \sqrt{2}ac^4e^2\right)} + \frac{\left(\left(ac^3\right)^{\frac{1}{4}} ac^2d - \left(ac^3\right)^{\frac{3}{4}} ae\right) \arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^5d^2 + \sqrt{2}ac^4e^2\right)} + \frac{\left(\left(ac^3\right)^{\frac{1}{4}} ac^2d + \left(ac^3\right)^{\frac{3}{4}} ae\right) \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2 + \sqrt{2}ac^4e^2\right)} - \frac{\left(\left(ac^3\right)^{\frac{1}{4}} ac^2d + \left(ac^3\right)^{\frac{3}{4}} ae\right) \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^5d^2 + \sqrt{2}ac^4e^2\right)} + \frac{\left(c^2x^3e^2 - 3c^2dxe\right)e^{(-3)}}{3c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 + a*e^4) + 1/2*((a*c^
3)^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/2*((a*c^3)
^(1/4)*a*c^2*d - (a*c^3)^(3/4)*a*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) + 1/4*((a*c^3)^(
1/4)*a*c^2*d + (a*c^3)^(3/4)*a*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(sqrt(2)*c^5*d^2 + sqrt(2)*a*c^4*e^2) - 1/4*((a*c^3)^(1/4)*a*c^2*d + (a
*c^3)^(3/4)*a*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^5*
d^2 + sqrt(2)*a*c^4*e^2) + 1/3*(c^2*x^3*e^2 - 3*c^2*d*x*e)*e^(-3)/c^3
```

Mupad [B]
time = 2.06, size = 2500, normalized size = 6.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^7*d^4*e^26 + 16*c^7*d^18*e^12 - 16*c^7*x*(-d^7*e^5)^(5/2) + 2*a^6*c*
d^6*e^24 + 16*a^3*c^4*d^12*e^18 + a^5*c^2*d^8*e^22 - a^7*e^24*x*(-d^7*e^5)^
(1/2) - a^5*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2) + 16*a^3*c^4*d*e^11*x*(-d^7*e^5
)^(3/2) - 2*a^6*c*d^2*e^22*x*(-d^7*e^5)^(1/2))*(-d^7*e^5)^(1/2))/(2*a*e^7 +
 2*c*d^2*e^5) - atan((((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*
a^5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^
7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(
1/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a
^4*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1
/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)
+ (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^
5*d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*
a^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4
*a^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1
/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)
- (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e
^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d
^2*e^2)))^(1/2)*1i - (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a
^5*c^6*d^2*e^9)/(c^3*e^3) + (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7
)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1
/2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^
4*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) -
 (2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5
*d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2
*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a
```

```
^3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*
a^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) +
 (2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^
2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^
2*e^2)))^(1/2)*1i)/((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^
5*c^6*d^2*e^9)/(c^3*e^3) - (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)
^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/
2)*(256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4
*c^8*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2
) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) +
(2*x*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*
d^3*e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*
a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^
3*c^6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a
^6*c^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2
) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) -
(2*x*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2
*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2
*e^2)))^(1/2) + (((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^
6*d^2*e^9)/(c^3*e^3) + (2*x*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/
2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2)*(
256*a^5*c^7*e^12 - 256*a^2*c^10*d^6*e^6 - 256*a^3*c^9*d^4*e^8 + 256*a^4*c^8
*d^2*e^10))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (2*x
*(64*a^2*c^8*d^9*e + 56*a^6*c^4*d*e^9 - 8*a^4*c^6*d^5*e^5 - 16*a^5*c^5*d^3*
e^7))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*
c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) - (16*a^3*c^
6*d^9 + 4*a^7*c^2*d*e^8 - 64*a^4*c^5*d^7*e^2 + 64*a^5*c^4*d^5*e^4 + 4*a^6*c
^3*d^3*e^6)/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a^5*c^7)^(1/2) +
2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2)))^(1/2) + (2*x
*(a^8*e^8 + 2*a^4*c^4*d^8))/(c^3*e^3))*((c*d^2*(-a^5*c^7)^(1/2) - a*e^2*(-a
^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4 + 2*a*c^8*d^2*e^2
)))^(1/2) + (2*(a^7*d^4*e^3 - a^6*c*d^6*e))/(c^3*e^3)))*((c*d^2*(-a^5*c^7)^
(1/2) - a*e^2*(-a^5*c^7)^(1/2) + 2*a^3*c^4*d*e)/(16*(c^9*d^4 + a^2*c^7*e^4
+ 2*a*c^8*d^2*e^2)))^(1/2)*2i - (log(a^7*d^4*e^26 + 16*c^7*d^18*e^12 + 16*c
^7*x*(-d^7*e^5)^(5/2) + 2*a^6*c*d^6*e^24 + 16*a^3*c^4*d^12*e^18 + a^5*c^2*d
^8*e^22 + a^7*e^24*x*(-d^7*e^5)^(1/2) + a^5*c^2*d^4*e^20*x*(-d^7*e^5)^(1/2)
 - 16*a^3*c^4*d*e^11*x*(-d^7*e^5)^(3/2) + 2*a^6*c*d^2*e^22*x*(-d^7*e^5)^(1/
2))*(-d^7*e^5)^(1/2))/(2*(a*e^7 + c*d^2*e^5)) -...
```

3.238 $\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=345

$$\frac{x}{ce}-\frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}\left(cd^2+ae^2\right)}+\frac{a^{3/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{5/4}\left(cd^2+ae^2\right)}-\frac{a^{3/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\cdots}{\sqrt[4]{\cdots}}\right.}{2\sqrt{2}\,c^{5/4}\left(cd^2+ae^2\right)}$$

```
[Out] x/c/e-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)/(a*e^2+c*d^2)-1/8*a^(3/4)*l
n(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/c^
(5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*a^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2
)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(
3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/c^(5/4)/(a*
e^2+c*d^2)*2^(1/2)-1/4*a^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/
2)+d*c^(1/2))/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\frac{a^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{2\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}-\frac{a^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{2\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}-\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}+\frac{a^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{5/4}\left(ae^2+cd^2\right)}-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}\left(ae^2+cd^2\right)}+\frac{x}{ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) +
 (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/
(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) - (a^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcT
an[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) -
(a^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*c^(5/4)*(c*d^2 + a*e^2)) + (a^(3/4)*(Sqrt[c]*d - S
qrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2
]*c^(5/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)}\,dx = \int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2+ae^2)(d+ex^2)} - \frac{a(ae+cdx^2)}{c(cd^2+ae^2)(a+cx^4)}\right) dx$$

$$= \frac{x}{ce} - \frac{a\int \frac{ae+cdx^2}{a+cx^4}\,dx}{c(cd^2+ae^2)} - \frac{d^3\int \frac{1}{d+ex^2}\,dx}{e(cd^2+ae^2)}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} + \frac{\left(a\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2c(cd^2+ae^2)} - \frac{\left(a\left(d+\frac{\sqrt{a}}{\sqrt{c}}\right.\right.}{2c}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} - \frac{\left(a^{3/4}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,c^{3/4}(cd^2+ae^2)} - \cdots$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} - \frac{a^{3/4}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right.}{4\sqrt{2}\,c^{3/4}(cd^2+ae^2)}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} + \frac{a^{3/4}(\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{5/4}(cd^2+ae^2)} - \frac{a^{3/4}(\cdots}{}$$

Mathematica [A]
time = 0.17, size = 373, normalized size = 1.08

$$\frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(cd^2+ae^2)} - \frac{(a^{3/4}cd+a^{5/4}\sqrt{c}\,e)\tan^{-1}\left(\frac{-\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{7/4}(cd^2+ae^2)} - \frac{(a^{3/4}cd+a^{5/4}\sqrt{c}\,e)\tan^{-1}\left(\frac{\sqrt{2}\sqrt[4]{a}+2\sqrt[4]{c}\,x}{\sqrt{2}\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{7/4}(cd^2+ae^2)} - \frac{(a^{3/4}cd-a^{5/4}\sqrt{c}\,e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}(cd^2+ae^2)} + \frac{(a^{3/4}cd-a^{5/4}\sqrt{c}\,e)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{7/4}(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] x/(c*e) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 + a*e^2)) -
 ((a^(3/4)*c*d + a^(5/4)*Sqrt[c]*e)*ArcTan[(-(Sqrt[2]*a^(1/4)) + 2*c^(1/4)*
x)/(Sqrt[2]*a^(1/4))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d
+ a^(5/4)*Sqrt[c]*e)*ArcTan[(Sqrt[2]*a^(1/4) + 2*c^(1/4)*x)/(Sqrt[2]*a^(1/4
))])/(2*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*
e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/
4)*(c*d^2 + a*e^2)) + ((a^(3/4)*c*d - a^(5/4)*Sqrt[c]*e)*Log[Sqrt[a] + Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2))
```

Maple [A]
time = 0.18, size = 263, normalized size = 0.76

method	result
default	$\frac{x}{ce}-\frac{d^{3}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{e(a\,e^{2}+c\,d^{2})\sqrt{de}}-\frac{a\left(\frac{e\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d\sqrt{\ }}{}\right.}{(a\,e^{2}+c\,d^{2})c}$
risch	$\frac{x}{ce}+\frac{\sum_{\_R=\mathrm{RootOf}\left(\left(a^{2}c\,e^{4}+2a\,c^{2}d^{2}e^{2}+c^{3}d^{4}\right)\_Z^{4}+4e^{3}cd\,a^{2}\_Z^{2}+e^{4}a^{3}\right)}\_R\ln\left(\left(\left(-2a^{3}c\,e^{7}-2a^{2}c^{2}d^{2}e^{5}+2a\,c^{3}d^{4}e^{3}+2c^{4}d^{6}e\right)\_R^{5}+(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] x/c/e-1/e*d^3/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/(a*e^2+c*
d^2)/c*(1/8*e*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*d/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a
/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2
*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 287, normalized size = 0.83

$$-\frac{d^{\frac{5}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^{2}e+ae^{3}}-\frac{a\left(\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}\,cd-a\sqrt{c}\,e\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}\,cd-a\sqrt{c}\,e\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\left(c^{2}d^{2}+ace^{2}\right)}+\frac{xe^{(-1)}}{c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e + a*e^3) - 1/8*a*(2*sq
rt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)
*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqr
t(c)) + 2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)
*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(c))*sqrt(c)) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 + s
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(sqrt(a)*c*
d - a*sqrt(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^
(3/4)*c^(3/4)))/(c^2*d^2 + a*c*e^2) + x*e^(-1)/c
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2039 vs. $2(252) = 504$.
time = 1.13, size = 4110, normalized size = 11.91

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] [1/4*(2*sqrt(-d*e^(-1))*c*d^2*log((x^2*e - 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e + d)) + 4*c*d^2*x + 4*a*x*e^2 + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-a^2*c*d^2*x + a^3*x*e^2 + (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-a^2*c*d^2*x + a^3*x*e^2 - (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-a^2*c*d^2*x + a^3*x*e^2 + (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-a^2*c*d^2*x + a^3*x*e^2 - (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e - (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))))/(c^2*d^2*e + a*c*e^3), -1/4*(4*c*d^

```
(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) - 4*c*d^2*x - 4*a*x*e^2 - (c^2*d^2
*e + a*c*e^3)*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*
sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2
+ 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3
*d^2*e^2 + a^2*c^2*e^4))*log(-a^2*c*d^2*x + a^3*x*e^2 + (a^2*c^2*d^2*e - a^
3*c*e^3 - (c^6*d^5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 -
2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 +
 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d
^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*
d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8
)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))) + (c^2*d^2*e + a*c*e^3)*sqr
t(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^
4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e
^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^
2*e^4))*log(-a^2*c*d^2*x + a^3*x*e^2 - (a^2*c^2*d^2*e - a^3*c*e^3 - (c^6*d^
5 + 2*a*c^5*d^3*e^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 +
 a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^
6 + a^4*c^5*e^8)))*sqrt(-(2*a^2*d*e + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*
e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6
*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*
a*c^3*d^2*e^2 + a^2*c^2*e^4))) - (c^2*d^2*e + a*c*e^3)*sqrt(-(2*a^2*d*e - (
c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*c*d^2*e
^2 + a^5*e^4)/(c^9*d^8 + 4*a*c^8*d^6*e^2 + 6*a^2*c^7*d^4*e^4 + 4*a^3*c^6*d^
2*e^6 + a^4*c^5*e^8)))/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4))*log(-a^2*
c*d^2*x + a^3*x*e^2 + (a^2*c^2*d^2*e - a^3*c*e^3 + (c^6*d^5 + 2*a*c^5*d^3*e
^2 + a^2*c^4*d*e^4)*sqrt(-(a^3*c^2*d^4 - 2*a^4*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.64, size = 333, normalized size = 0.97

$$-\frac{d^{\frac{5}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2e+ae^3}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}+\frac{xe^{(-1)}}{c}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2+\sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e + a*e^3) - 1/2*((a*c^3
)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4
)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(
a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) + x*e^(-1)/c - 1/4*((a*c^
3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/
c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*a*c*e - (a*c
^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2
+ sqrt(2)*a*c^3*e^2)
```

Mupad [B]
time = 1.83, size = 2500, normalized size = 7.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*
e) - (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*
e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10
- 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*
(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^
7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 5
6*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*
(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a
^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*
e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5
)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1
/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*
d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*
d^2*e^2)))^(1/2)*1i - (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*
c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2
) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(2
56*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d
^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*
c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(64*a
^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/
(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/
(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e -
 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) -
 c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c
^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3
*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^
5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*1i)/((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^
```

5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) + (2*x*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*(a^4*c*d^5 - a^5*d^3*e^2))/(c*e)))*(-(a*e^2*(-a^3*c^5)^(1/2) - c*d^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*2i + atan((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^3*e^6)/(c*e) - (2*x*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2)*(256*a^5*c^5*e^10 - 256*a^2*c^8*d^6*e^4 - 256*a^3*c^7*d^4*e^6 + 256*a^4*c^6*d^2*e^8))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) + (2*x*(64*a^2*c^6*d^7*e - 56*a^5*c^3*d*e^7 + 8*a^3*c^5*d^5*e^3 + 16*a^4*c^4*d^3*e^5))/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (48*a^3*c^4*d^6*e - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5)/(c*e))*(-(c*d^2*(-a^3*c^5)^(1/2) - a*e^2*(-a^3*c^5)^(1/2) + 2*a^2*c^3*d*e)/(16*(c^7*d^4 + a^2*c^5*e^4 + 2*a*c^6*d^2*e^2)))^(1/2) - (2*x*(a^6*e^6 - 2*a^3*c^3*d^6))/(c*e)...

3.239 $\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$\frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2+ae^2)}+\frac{\sqrt[4]{a}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{3/4}(cd^2+ae^2)}-\frac{\sqrt[4]{a}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{3/4}(cd^2+ae^2)}$$

```
[Out] -1/4*a^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/c^
(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(-e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*a^(1/4)*ln(-a^(1/
4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*
e^2+c*d^2)*2^(1/2)-1/8*a^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(
1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)+d^(3/2)*arctan(x*
e^(1/2)/d^(1/2))/(a*e^2+c*d^2)/e^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\frac{\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,c^{3/4}(ae^2+cd^2)}-\frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,c^{3/4}(ae^2+cd^2)}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(ae^2+cd^2)}+\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(ae^2+cd^2)}-\frac{\sqrt[4]{a}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)) + (a^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) + (a^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)) - (a^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*
(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx = \int \left(\frac{d^2}{(cd^2+ae^2)(d+ex^2)} - \frac{a(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right)\,dx$$

$$= -\frac{a\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2+ae^2)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2c\,(cd^2+ae^2)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}+e\right)\right)\int}{2c\,(cd^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2+ae^2)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}+x^2}\,dx}{4c\,(cd^2+ae^2)} - \frac{\left(a\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-\right.\right.}{}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2+ae^2)} + \frac{\sqrt[4]{a}\,(\sqrt{c}\,d+\sqrt{a}\,e)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}\,(cd^2+ae^2)}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2+ae^2)} + \frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,c^{3/4}\,(cd^2+ae^2)} - \frac{a^{3/4}\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-\right.}{2\sqrt{2}}$$

Mathematica [A]
time = 0.10, size = 233, normalized size = 0.69

$$\frac{8c^{3/4}d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{2}\sqrt[4]{a}\sqrt{e}\left(2(\sqrt{c}\,d-\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+(-2\sqrt{c}\,d+2\sqrt{a}\,e)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+(\sqrt{c}\,d+\sqrt{a}\,e)\left(\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)\right)\right)}{8c^{3/4}\sqrt{e}\,(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*c^(3/4)*d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*a^(1/4)*Sqrt[e]*(2
*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (-2*Sqrt
[c]*d + 2*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*c^(3/4)*Sqrt[e]*(c*
d^2 + a*e^2))
```

Maple [A]
time = 0.19, size = 254, normalized size = 0.76

method	result

default	$\frac{d^2\arctan\left(\frac{ex}{\sqrt{de}}\right)}{(a\,e^2+c\,d^2)\sqrt{de}} - \frac{a\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a} - \frac{e\sqrt{2}\,(\mathrm{l}}{}\right.}{a\,e^2+c\,d^2}$
risch	$\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^2c^3e^4+2a\,c^4d^2e^2+c^5d^4\right)\_Z^4-4a\,c^2de\_Z^2+a\right)} \_R\ln\left(\left(\left(-2a^3c^3e^8-2a^2c^4d^2e^6+2a\,c^5d^4e^4+2c^6d^6e^2\right)\_R^5+(7a^2c^2\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] d^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/(a*e^2+c*d^2)*(1/8*
d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a
/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arcta
n(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4
)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(
2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 266, normalized size = 0.79

$$\frac{d^{\frac{3}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{cd^2+ae^2} - \frac{a\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\,(cd^2+ae^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 + a*e^2) - 1/8*a*(2*sqrt(
2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4
)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) +
 2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2
)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sq
rt(c)) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*
c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*lo
g(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c*
d^2 + a*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1880 vs. $2(244) = 488$.
time = 0.59, size = 3790, normalized size = 11.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] [1/4*((c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-c*d^2*x + a*x*e^2 + (c^2*d^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-c*d^2*x + a*x*e^2 - (c^2*d^3 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-c*d^2*x + a*x*e^2 + (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-c*d^2*x + a*x*e^2 - (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e - (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) + 2*sqrt(-d*e^(-1))*d*log((x^2*e + 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e + d)))/(c*d^2 + a*e^2), 1/4*(4*d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) + (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 +

```
6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*
d^2*e^2 + a^2*c*e^4))*log(-c*d^2*x + a*x*e^2 + (c^2*d^3 - a*c*d*e^2 + (c^4*
d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 +
 a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^
6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*
sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 +
6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d
^2*e^2 + a^2*c*e^4))) - (c*d^2 + a*e^2)*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*
d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8
 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))
/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-c*d^2*x + a*x*e^2 - (c^2*d^3
 - a*c*d*e^2 + (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e^5)*sqrt(-(a*c^2*d^4
 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^
4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e + (c^3*d^4 + 2*a*c^2*d
^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8
+ 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))/
(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))) + (c*d^2 + a*e^2)*sqrt((2*a*d*e -
 (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2
 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*
e^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4))*log(-c*d^2*x
+ a*x*e^2 + (c^2*d^3 - a*c*d*e^2 - (c^4*d^4*e + 2*a*c^3*d^2*e^3 + a^2*c^2*e
^5)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2 + a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^
2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)))*sqrt((2*a*d*e -
(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(-(a*c^2*d^4 - 2*a^2*c*d^2*e^2
+ a^3*e^4)/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e
^6 + a^4*c^3*e^8)))/(c^3*d^4 + 2*a*c^2*d^2*e^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.65, size = 327, normalized size = 0.97

$$\frac{d^{\frac{3}{2}} \arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\left(-\frac{1}{2}\right)}}{cd^2 + ae^2} - \frac{\left((ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2}\left(2x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2 + \sqrt{2}ac^3e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}} c^2 d - (ac^3)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2}\left(2x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}c^4d^2 + \sqrt{2}ac^3e^2\right)} - \frac{\left((ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e\right) \log\left(x^2 + \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2 + \sqrt{2}ac^3e^2\right)} + \frac{\left((ac^3)^{\frac{1}{4}} c^2 d + (ac^3)^{\frac{3}{4}} e\right) \log\left(x^2 - \sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}c^4d^2 + \sqrt{2}ac^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 + a*e^2) - 1/2*((a*c^3)^(
1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
)/(a/c)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/2*((a*c^3)^(1/4)*c
^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c
)^(1/4))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*e^2) - 1/4*((a*c^3)^(1/4)*c^2*d +
 (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4
*d^2 + sqrt(2)*a*c^3*e^2) + 1/4*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log
(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^4*d^2 + sqrt(2)*a*c^3*
e^2)
```

Mupad [B]
time = 2.20, size = 2500, normalized size = 7.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(-((((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c
^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 11
2*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-
a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))
^(1/2)*(64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2
) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512
*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*
e^7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) -
c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^
2*e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) -
 x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)
^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
*1i + (((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^
5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112
*a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a
*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^
(1/2)*(x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(
c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*
e^2 + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c
*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2
*e^2)))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) -
x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^
(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5
*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*
a^2*c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) + ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*
```

c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(
1/2)*(64*a^2*c^6*d^6*e^2 - x*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2)
+ 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a
^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^
7) + 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c*
d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*
e^2)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x
*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(
1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) -
 (((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4
 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*
c^5*d^5*e^2 - 32*a^3*c^4*d^3*e^4) - ((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)
^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
(x((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d
^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*
d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 64*a^2*c^6*d^6*e^2 +
 128*a^3*c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*
(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)
))^(1/2) - 16*a^2*c^4*d^5*e - 4*a^4*c^2*d*e^5 + 60*a^3*c^3*d^3*e^3) - x*(2*
a^4*c*e^5 + 4*a^2*c^3*d^4*e))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2)
 + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2) + 2*a
^3*c*d^2*e^2))*((a*e^2*(-a*c^3)^(1/2) - c*d^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)
/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*2i + atan(-((((c*d^2
*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^
3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e
^2 - 32*a^3*c^4*d^3*e^4) + ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) +
2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(64*a^2*
c^6*d^6*e^2 - x*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e
)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 -
512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 128*a^3*
c^5*d^4*e^4 + 64*a^4*c^4*d^2*e^6))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^
(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)
+ 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3) - x*(2*a^4*c*e^5
 + 4*a^2*c^3*d^4*e))*((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^
2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*1i + (((c*d^2*
(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3
*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*((x*(112*a^4*c^3*d*e^6 + 112*a^2*c^5*d^5*e^
2 - 32*a^3*c^4*d^3*e^4) - ((c*d^2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2
*a*c^2*d*e)/(16*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)))^(1/2)*(x*((c*d^
2*(-a*c^3)^(1/2) - a*e^2*(-a*c^3)^(1/2) + 2*a*c...

3.240 $\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=337

$$-\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2+ae^2}-\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(cd^2+ae^2\right)}+\frac{\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(cd^2+ae^2\right)}$$

```
[Out] 1/8*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2
))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a
^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^
(1/2)+1/4*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4
)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*
a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-arctan(x*e^(1/2)/d
^(1/2))*d^(1/2)*e^(1/2)/(a*e^2+c*d^2)
```

Rubi [A]
time = 0.17, antiderivative size = 337, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt{d}\,\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{ae^2+cd^2}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}+\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}-\frac{\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,\left(ae^2+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -((Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)) - ((Sqrt[c
]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4
)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c
^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) + ((Sqrt[c]
*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*
Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)) - ((Sqrt[c]*d - Sqrt[a]*e)*Log[Sqr
t[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)
*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx = \int \left(-\frac{de}{(cd^2+ae^2)(d+ex^2)} + \frac{ae+cdx^2}{(cd^2+ae^2)(a+cx^4)}\right) dx$$

$$= \frac{\int \frac{ae+cdx^2}{a+cx^4}\,dx}{cd^2+ae^2} - \frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2}$$

$$= -\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2+ae^2} - \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \frac{\sqrt{a}\,\sqrt{c}-cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)} + \frac{\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\int \ldots}{2(cd^2+\ldots}$$

$$= -\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2+ae^2} + \frac{\left(\sqrt[4]{c}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\,\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,\sqrt[4]{a}\,(cd^2+ae^2)} + \ldots$$

$$= -\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2+ae^2} + \frac{\sqrt[4]{c}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x \ldots\right.}{4\sqrt{2}\,\sqrt[4]{a}\,(cd^2+ae^2)} \ldots$$

$$= -\frac{\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2+ae^2} - \frac{\sqrt[4]{c}\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,(cd^2+ae^2)} + \frac{\sqrt[4]{c}\,(d \ldots}{\ldots}$$

Mathematica [A]
time = 0.08, size = 232, normalized size = 0.69

$$\frac{-8\sqrt[4]{a}\,\sqrt[4]{c}\,\sqrt{d}\,\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+\sqrt{2}\left(-2(\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2(\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+(\sqrt{c}\,d-\sqrt{a}\,e)\left(\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)\right)\right)}{8\sqrt[4]{a}\,\sqrt[4]{c}\,(cd^2+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(1/4)*c^(1/4)*Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*(
-2*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sqr
t[c]*d + Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + (Sqrt[c]*d -
Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[Sq
rt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(1/4)*c^(1/4)*(c*d
^2 + a*e^2))
```

Maple [A]
time = 0.18, size = 246, normalized size = 0.73

method	result

default	$-\frac{de\arctan\left(\frac{ex}{\sqrt{de}}\right)}{\left(a\,e^{2}+c\,d^{2}\right)\sqrt{de}}+\frac{\frac{e\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d\sqrt{2}\,(\ln}{}}{a\,e^{2}+c\,d^{2}}$
risch	$\frac{\sqrt{-de}\,\ln\left(\left(-16(-de)^{\frac{5}{2}}a^{2}c\,e^{3}+16(-de)^{\frac{5}{2}}a\,c^{2}d^{2}e-14a^{2}c\,d\,e^{4}(-de)^{\frac{3}{2}}+20a\,c^{2}d^{3}e^{2}(-de)^{\frac{3}{2}}+2d^{5}(-de)^{\frac{3}{2}}c^{3}-\sqrt{-de}\,a^{3}e^{7}+3\sqrt{-}\right.\right.}{2a\,e^{2}+2c\,d^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -d*e/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/(a*e^2+c*d^2)*(1/8
*e*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/
c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan
(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*d/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x
*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(
1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.50, size = 274, normalized size = 0.81

$$-\frac{\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{cd^{2}+ae^{2}}+\frac{\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}\,cd-a\sqrt{c}\,e\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}\,cd-a\sqrt{c}\,e\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{8\left(cd^{2}+ae^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 + a*e^2) + 1/8*(2*sqrt(2)
*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1
/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c))
 + 2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x -
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 + sqrt(2
)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*(sqrt(a)*c*d - a
*sqrt(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)
*c^(3/4)))/(c*d^2 + a*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1812 vs. $2(245) = 490$.
time = 0.44, size = 3651, normalized size = 10.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*s
qrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6
*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^
2 + a^2*e^4))*log(-c*d^2*x + a*x*e^2 + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 +
2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^
4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4
*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
)) - (c*d^2 + a*e^2)*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqr
t(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a
^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2
+ a^2*e^4))*log(-c*d^2*x + a*x*e^2 - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*
a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))*sqrt(-(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4
- 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e
^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)))
 + (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(
-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4))*log(-c*d^2*x + a*x*e^2 + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^
2*c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 -
2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4
 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) -
 (c*d^2 + a*e^2)*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(
c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c
^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^
2*e^4))*log(-c*d^2*x + a*x*e^2 - (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*
c^2*d^3*e^2 + a^3*c*d*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5
*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^
8)))*sqrt(-(2*d*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*
a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) - 2
*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)))/(c*d^2 + a*e^2),
 -1/4*(4*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + (c*d^2 + a*e^2)*sqrt(-
(2*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2
 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*
d^2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*log(-c*d^2*x +
a*x*e^2 + (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e
^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^
```

```
2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))*sqrt(-(2*d*e + (c^
2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/
(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^
5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) - (c*d^2 + a*e^2)*sqrt(-(2
*d*e + (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 +
 a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^
2*e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*log(-c*d^2*x + a*
x*e^2 - (a*c*d^2*e - a^2*e^3 - (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4
)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2
+ 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))*sqrt(-(2*d*e + (c^2*
d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a
*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*
c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) + (c*d^2 + a*e^2)*sqrt(-(2*d
*e - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a
^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*
e^6 + a^5*c*e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))*log(-c*d^2*x + a*x*
e^2 + (a*c*d^2*e - a^2*e^3 + (a*c^3*d^5 + 2*a^2*c^2*d^3*e^2 + a^3*c*d*e^4)*
sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 +
6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)))*sqrt(-(2*d*e - (c^2*d^
4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/(a*c
^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*
e^8)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4))) - (c*d^2 + a*e^2)*sqrt(-(2*d*e
 - (c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(-(c^2*d^4 - 2*a*c*d^2*e^2 + a^2
*e^4)/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.00, size = 336, normalized size = 1.00

$$-\frac{\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{cd^2+ae^2}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace+\left(ac^3\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ace-\left(ac^3\right)^{\frac{3}{4}}d\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 + a*e^2) + 1/2*((a*c^3)^(
1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1/4)
```

```
)/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/2*((a*c^3)^(1/
4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)*(a/c)^(1/4))/
(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*((a*c^3)^(1/4)
*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqr
t(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*a*c*e - (a*c^3)^
(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^3*d^2 +
sqrt(2)*a^2*c^2*e^2)
```

Mupad [B]
time = 1.59, size = 2500, normalized size = 7.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] (log(a^2*d*e^7 + c^2*d^5*e^3 - c^2*d*x*(-d*e)^(7/2) + 2*a*c*d^3*e^5 + a^2*e
^7*x*(-d*e)^(1/2) + 2*a*c*e^3*x*(-d*e)^(5/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*c*
d^2) - atan((((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(
a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2)
- a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2)*(x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 5
12*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c
^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2
 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*
(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))
^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4
*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c
^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i - ((-(c*d^2*(-a*c)^(1/2)
 - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d
^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7 -
 x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 +
 a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*
e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 + 38
4*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*
d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^
3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c
^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) -
a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*
e^2)))^(1/2)*1i)/(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/
(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(
1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c
^2*d^2*e^2)))^(1/2)*(x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d
```

```
*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^
9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*
a^4*c^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^
5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a
*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e
^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*
a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16
*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + ((-(c*d^2*(-a*c)^(1/
2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2
*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)
/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(192*a^4*c^4*d*e^7
 - x*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4
 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^
6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^2*c^6*d^5*e^3 +
384*a^3*c^5*d^3*e^5) + x*(16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 + 160*a^2*c^
5*d^3*e^4))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*
c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2
*c^4*d^2*e^4) - x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3))*(-(c*d^2*(-a*c)^(1/2)
- a*e^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2) + 2*a*c^3*d*e^3))*(-(c*d^2*(-a*c)^(1/2) - a*e^2*(-a*c)^(1/2)
 + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*2i -
(log(a^2*d*e^7 + c^2*d^5*e^3 + c^2*d*x*(-d*e)^(7/2) + 2*a*c*d^3*e^5 - a^2*e
^7*x*(-d*e)^(1/2) - 2*a*c*e^3*x*(-d*e)^(5/2))*(-d*e)^(1/2))/(2*(a*e^2 + c*d
^2)) - atan((((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(
a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2)
- c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^
2*e^2)))^(1/2)*(x*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 5
12*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) + 192*a^4*c
^4*d*e^7 + 192*a^2*c^6*d^5*e^3 + 384*a^3*c^5*d^3*e^5) - x*(16*a*c^6*d^5*e^2
 - 112*a^3*c^4*d*e^6 + 160*a^2*c^5*d^3*e^4))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*
(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))
^(1/2) + 4*a*c^5*d^4*e^2 + 52*a^2*c^4*d^2*e^4) + x*(2*a^2*c^3*e^5 - 4*a*c^4
*d^2*e^3))*(-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c
^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2)))^(1/2)*1i - ((-(a*e^2*(-a*c)^(1/2)
 - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d
^2*e^2)))^(1/2)*(((-(a*e^2*(-a*c)^(1/2) - c*d^2*(-a*c)^(1/2) + 2*a*c*d*e)/(
16*(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2))...
```

3.241 $\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=336

$$\frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)}-\frac{\sqrt[4]{c}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}\,(cd^2+ae^2)}+\frac{\sqrt[4]{c}\,\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,a^{3/4}\,(cd^2+ae^2)}$$

```
[Out] 1/4*c^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(
3/4)/(a*e^2+c*d^2)*2^(1/2)+1/4*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*
(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/8*c^(1/4)*ln(-a^(1/4
)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e
^2+c*d^2)*2^(1/2)+1/8*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)*2^(1/2)+e^(3/2)*arctan(x*e
^(1/2)/d^(1/2))/(a*e^2+c*d^2)/d^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.421$, Rules used = {1185, 211, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{3/4}\,(ae^2+cd^2)}+\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{3/4}\,(ae^2+cd^2)}-\frac{\sqrt[4]{c}\,\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\,(ae^2+cd^2)}+\frac{\sqrt[4]{c}\,\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}\,(ae^2+cd^2)}+\frac{e^{3/2}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)) - (c^(1/4)*
(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]
*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sq
rt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*(Sqrt[c]*d + Sqrt[a]*e)*
Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*
(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx &= \int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)}\right) dx \\
&= \frac{c\int \frac{d-ex^2}{a+cx^4}\,dx}{cd^2+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2+ae^2} \\
&= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}} - e\right)\int \frac{\sqrt{a}\sqrt{c}+cx^2}{a+cx^4}\,dx}{2(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}} + e\right)\int \frac{\sqrt{a}\sqrt{c}}{a+cx^4}
\end{aligned}
$$

$$
\begin{aligned}
&= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}} - e\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}} + x^2}\,dx}{4(cd^2+ae^2)} + \frac{\left(\frac{\sqrt{c}\,d}{\sqrt{a}} - e\right)\int}{4(cd}\\
&= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(cd^2+ae^2)} \\
&= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)} - \frac{\sqrt[4]{c}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(cd^2+ae^2)} + \frac{\sqrt[4]{c}\left(\sqrt{c}\,d\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.10, size = 234, normalized size = 0.70

$$
\frac{8a^{3/4}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) + \sqrt{2}\sqrt[4]{c}\sqrt{d}\left(\left(-2\sqrt{c}\,d+2\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right) + 2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right) - \left(\sqrt{c}\,d+\sqrt{a}\,e\right)\left(\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right) - \log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)\right)\right)}{8a^{3/4}\sqrt{d}\,(cd^2+ae^2)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (8*a^(3/4)*e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + Sqrt[2]*c^(1/4)*Sqrt[d]*((
-2*Sqrt[c]*d + 2*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*(Sq
rt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] - (Sqrt[c]*d +
 Sqrt[a]*e)*(Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Log[S
qrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])))/(8*a^(3/4)*Sqrt[d]*(c*
d^2 + a*e^2))
```

Maple [A]
time = 0.16, size = 253, normalized size = 0.75

method	result

default	$\frac{e^2 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(a e^2+c d^2)\sqrt{de}}+\frac{c\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a}-\frac{e\sqrt{2}\,\Big(\ln}{}\right.}{a e^2+c d^2}$
risch	$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^5e^4+2a^4c\,d^2e^2+a^3c^2d^4\right)\_Z^4-4a^2cde\_Z^2+c\right)}\_R\ln\Big(\Big(\big(-2a^5e^7-2a^4c\,d^2e^5+2a^3c^2d^4e^3+2a^2c^3d^6e\big)\_R^4+\big(15a^2cd}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+c/(a*e^2+c*d^2)*(1/8*
d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a
/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arcta
n(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4
)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(
2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.49, size = 266, normalized size = 0.79

$$\frac{c\left(\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\,(cd^2+ae^2)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{(cd^2+ae^2)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] 1/8*c*(2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x +
sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(
c))*sqrt(c)) + 2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt
(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(
a)*sqrt(c))*sqrt(c)) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 + sq
rt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*d +
 sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)
*c^(3/4)))/(c*d^2 + a*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 + a*
e^2)*sqrt(d))
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1904 vs. $2(244) = 488$.
time = 0.90, size = 3842, normalized size = 11.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")

[Out] [-1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c*x*e^2 + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c*x*e^2 - (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c*x*e^2 + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c*x*e^2 - (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e - (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) - 2*sqrt(-e/d)*e*log((x^2*e + 2*d*x*sqrt(-e/d) - d)/(x^2*e + d)))/(c*d^2 + a*e^2), -1/4*((c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^

```
4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*l
og(-c^2*d^2*x + a*c*x*e^2 + (a*c^2*d^3 - a^2*c*d*e^2 + (a^3*c^2*d^4*e + 2*a
^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*
c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8
)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4
- 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2
*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*
e^4))) - (c*d^2 + a*e^2)*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3
*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^
3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2
*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c*x*e^2 - (a*c^2*d^3 - a^2*c*
d*e^2 + (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*e^5)*sqrt(-(c^3*d^4 - 2*a*c^
2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4
 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e + (a*c^2*d^4 + 2*a^2*c*d^2*e^
2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4
*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))/(a*c^2*
d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))) + (c*d^2 + a*e^2)*sqrt((2*c*d*e - (a*c^2
*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*
e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6
 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4))*log(-c^2*d^2*x + a*c
*x*e^2 + (a*c^2*d^3 - a^2*c*d*e^2 - (a^3*c^2*d^4*e + 2*a^4*c*d^2*e^3 + a^5*
e^5)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 + a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3
*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)))*sqrt((2*c*d*e -
 (a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(-(c^3*d^4 - 2*a*c^2*d^2*e^2 +
 a^2*c*e^4)/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*
d^2*e^6 + a^7*e^8)))/(a*c^2*d^4 + 2*a^2*c*d^2*e...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.61, size = 339, normalized size = 1.01

$$\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d-\left(ac^3\right)^{\frac{3}{4}}e\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}c^2d+\left(ac^3\right)^{\frac{3}{4}}e\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}ac^3d^2+\sqrt{2}a^2c^2e^2\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{\left(cd^2+ae^2\right)\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] 1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt(
2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/
2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2)
*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + 1/4*
((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + s
qrt(a/c))/(sqrt(2)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) - 1/4*((a*c^3)^(1/4)*c^
2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2
)*a*c^3*d^2 + sqrt(2)*a^2*c^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*
d^2 + a*e^2)*sqrt(d))
```

Mupad [B]
time = 1.67, size = 2500, normalized size = 7.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-
a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
 c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d
^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^
5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3
*c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*
((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
 a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*1i - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*
(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)
))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) +
2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4
*c^4*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(
16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512
*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^
6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32
*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c
)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2
) + 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^
(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*
1i)/((((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5
*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a
^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 + x*((a*e^2*(-a^3*c)^(1/2) -
```

```
c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5
 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*
c^5*d^2*e^6) + x*(16*c^7*d^5*e^2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*(
(a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 +
a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((a
*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^
3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) + (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^
3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(
1/2)*(4*c^6*d^3*e^3 - (((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^
2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4
*e^8 - x*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(
a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2
*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^
2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c
^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2) +
20*a*c^5*d*e^5) + 6*c^5*e^5*x)*((a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2
) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)))*((
a*e^2*(-a^3*c)^(1/2) - c*d^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a
^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*2i + atan(((((c*d^2*(-a^3*c)^(1/2) -
a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^
2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1
/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(2
56*a^4*c^4*e^8 + x*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*
d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9
 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*
c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 448*a^3*c^5*d^2*e^6) + x*(16*c^7*d^5*e^
2 + 32*a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6))*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(
-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
)^(1/2) + 20*a*c^5*d*e^5) - 6*c^5*e^5*x)*((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a
^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^
(1/2)*1i - (((c*d^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(1
6*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(4*c^6*d^3*e^3 - (((c*d
^2*(-a^3*c)^(1/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*
c^2*d^4 + 2*a^4*c*d^2*e^2)))^(1/2)*(256*a^4*c^4*e^8 - x*((c*d^2*(-a^3*c)^(1
/2) - a*e^2*(-a^3*c)^(1/2) + 2*a^2*c*d*e)/(16*(a^5*e^4 + a^3*c^2*d^4 + 2*a^
4*c*d^2*e^2)))^(1/2)*(512*a^5*c^4*e^9 - 512*a^2*c^7*d^6*e^3 - 512*a^3*c^6*d
^4*e^5 + 512*a^4*c^5*d^2*e^7) - 64*a*c^7*d^6*e^2 + 128*a^2*c^6*d^4*e^4 + 44
8*a^3*c^5*d^2*e^6) - x*(16*c^7*d^5*e^2 + 32*a*c...
```

3.242 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)}\,dx$

Optimal. Leaf size=348

$$-\frac{1}{adx}-\frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)}+\frac{c^{3/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}-\frac{c^{3/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1+\sqrt{\ }\right.}{2\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}$$

```
[Out] -1/a/d/x-e^(5/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)-1/8*c^(3/4
)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))
/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*
c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(5/4)/
(a*e^2+c*d^2)*2^(1/2)-1/4*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^
(1/2)+d*c^(1/2))/a^(5/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\frac{c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{2\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{a}\,e+\sqrt{c}\,d\right)}{2\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}-\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}+\frac{c^{3/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{5/4}\left(ae^2+cd^2\right)}-\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(ae^2+cd^2\right)}-\frac{1}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^
2)) + (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(Sqrt[c]*d + Sqrt[a]*e)
*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2
)) - (c^(3/4)*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(5/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(Sqrt[c]*
d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*S
qrt[2]*a^(5/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)}\,dx = \int \left(\frac{1}{adx^2} - \frac{e^3}{d\left(cd^2+ae^2\right)\left(d+ex^2\right)} - \frac{c\left(ae+cdx^2\right)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)}\right)dx$$

$$= -\frac{1}{adx} - \frac{c\int \frac{ae+cdx^2}{a+cx^4}\,dx}{a\left(cd^2+ae^2\right)} - \frac{e^3\int \frac{1}{d+ex^2}\,dx}{d\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} + \frac{\left(c\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\sqrt{a}\sqrt{c}-cx^2}{a+cx^4}\,dx}{2a\left(cd^2+ae^2\right)} - \frac{\left(c\left(d-\right.\right.}{}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} - \frac{\left(c^{5/4}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \frac{\frac{\sqrt{2}\sqrt[4]{a}}{\sqrt[4]{c}}+2x}{-\frac{\sqrt{a}}{\sqrt{c}}-\frac{\sqrt{2}\sqrt[4]{a}\,x}{\sqrt[4]{c}}-x^2}\,dx}{4\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} - \frac{c^{5/4}\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\right.}{4\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)} + \frac{c^{3/4}\left(\sqrt{c}\,d+\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{5/4}\left(cd^2+ae^2\right)} -$$

Mathematica [**A**]
time = 0.17, size = 389, normalized size = 1.12

$$\frac{-8a^{5/4}e^{5/2}x\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)-\sqrt{d}\left(8\sqrt[4]{a}\,cd^2+8a^{5/4}e^2-2\sqrt{2}\,c^{3/4}d\left(\sqrt{c}\,d+\sqrt{a}\,e\right)x\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+2\sqrt{2}\,c^{3/4}d\left(\sqrt{c}\,d+\sqrt{a}\,e\right)x\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+\sqrt{2}\,c^{5/4}d^2x\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-\sqrt{2}\sqrt{a}\,c^{3/4}dex\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-\sqrt{2}\,c^{5/4}d^2x\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)+\sqrt{2}\sqrt{a}\,c^{3/4}dex\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)\right)}{8a^{5/4}d^{3/2}\left(cd^2+ae^2\right)x}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a^(5/4)*e^(5/2)*x*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - Sqrt[d]*(8*a^(1/4)*c*d^
2 + 8*a^(5/4)*e^2 - 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e)*x*ArcTan[1
- (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 2*Sqrt[2]*c^(3/4)*d*(Sqrt[c]*d + Sqrt[a]*e
)*x*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[2]*c^(5/4)*d^2*x*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*Sqrt[a]*c^(3/4)*d
*e*x*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - Sqrt[2]*c^(5/
4)*d^2*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] + Sqrt[2]*S
qrt[a]*c^(3/4)*d*e*x*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
))/(8*a^(5/4)*d^(3/2)*(c*d^2 + a*e^2)*x)
```

Maple [**A**]
time = 0.19, size = 266, normalized size = 0.76

method	result
default	$-\frac{e^{3}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\left(a\,e^{2}+c\,d^{2}\right)\sqrt{de}}-\frac{c\left(\frac{e\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8}+\frac{d\sqrt{2}}{}\right.}{\left(a\,e^{2}+c\,d^{2}\right)a}$
risch	$-\frac{1}{adx}+\frac{\sqrt{-de}\,e^{2}\ln\left(\left(-16d^{2}e^{10}a^{4}+16d^{4}c\,e^{8}a^{3}-c^{3}d^{8}e^{4}a-c^{4}d^{10}e^{2}\right)x+16(-de)^{\frac{5}{2}}a^{4}e^{7}-16(-de)^{\frac{5}{2}}a^{3}c\,d^{2}e^{5}+4(-de)^{\frac{5}{2}}a^{2}c^{2}d^{4}e^{3}\right.}{2d^{2}\left(a\,e^{2}+c\,d^{2}\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*e^3/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-c/(a*e^2+c*d^2)/
a*(1/8*e*(a/c)^(1/4)*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x
^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*
arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*d/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(
1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arct
an(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))-1/a/d/x
```

Maxima [A]
time = 0.50, size = 290, normalized size = 0.83

$$-\frac{c\left(\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{a}\,cd+a\sqrt{c}\,e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\left(\sqrt{a}\,cd-a\sqrt{c}\,e\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(\sqrt{a}\,cd-a\sqrt{c}\,e\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{8\left(acd^{2}+a^{2}e^{2}\right)}-\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{5}{2}}}{\left(cd^{3}+ade^{2}\right)\sqrt{d}}-\frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*c*(2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)
*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*
sqrt(c))*sqrt(c)) + 2*sqrt(2)*(sqrt(a)*c*d + a*sqrt(c)*e)*arctan(1/2*sqrt(2
)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*s
qrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqr
t(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)
*(sqrt(a)*c*d - a*sqrt(c)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x +
sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c*d^2 + a^2*e^2) - arctan(x*e^(1/2)/sqrt(d))
*e^(5/2)/((c*d^3 + a*d*e^2)*sqrt(d)) - 1/(a*d*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2043 vs. $2(256) = 512$.
time = 1.96, size = 4120, normalized size = 11.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*a*x*sqrt(-e/d)*e^2*log((x^2*e - 2*d*x*sqrt(-e/d) - d)/(x^2*e + d))
- 4*c*d^2 + (a*c*d^3*x + a^2*d*x*e^2)*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a
^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^
5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e
^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-c^3*d^2*x + a*c^2*x*e
^2 + (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^
4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3
*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*
e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*
e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4
*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - (
a*c*d^3*x + a^2*d*x*e^2)*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4
*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^
2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*log(-c^3*d^2*x + a*c^2*x*e^2 - (a^2*c^2
*d^2*e - a^3*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5
*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*
a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*
d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3
*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^
6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3*x + a
^2*d*x*e^2)*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sq
rt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*
e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3
*c*d^2*e^2 + a^4*e^4))*log(-c^3*d^2*x + a*c^2*x*e^2 + (a^2*c^2*d^2*e - a^3*
c*e^3 + (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^
4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e
^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c
*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^
4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))
)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3*x + a^2*d*x*e^2)*s
qrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4
- 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c
^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
a^4*e^4))*log(-c^3*d^2*x + a*c^2*x*e^2 - (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*
c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a
^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*
d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^
4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6
*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^
4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - 4*a*e^2)/(a*c*d^3*x + a^2*d*x*e^2), -1/4
```

```
*(4*a*x*arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/sqrt(d) + 4*c*d^2 - (a*c*d^3*x +
a^2*d*x*e^2)*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*s
qrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6
*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^
3*c*d^2*e^2 + a^4*e^4))*log(-c^3*d^2*x + a*c^2*x*e^2 + (a^2*c^2*d^2*e - a^3
*c*e^3 - (a^4*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c
^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*
e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*
c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c
^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)
))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) + (a*c*d^3*x + a^2*d*x*e^2)*
sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4
 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*
c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4))*log(-c^3*d^2*x + a*c^2*x*e^2 - (a^2*c^2*d^2*e - a^3*c*e^3 - (a^4
*c^2*d^5 + 2*a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 +
a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c
*d^2*e^6 + a^9*e^8)))*sqrt(-(2*c^2*d*e + (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a
^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^
6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d
^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))) - (a*c*d^3*x + a^2*d*x*e^2)*sqrt(-(2*c^2*
d*e - (a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(-(c^5*d^4 - 2*a*c^4*d^
2*e^2 + a^2*c^3*e^4)/(a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 +
 4*a^8*c*d^2*e^6 + a^9*e^8)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4))*lo
g(-c^3*d^2*x + a*c^2*x*e^2 + (a^2*c^2*d^2*e - a^3*c*e^3 + (a^4*c^2*d^5 + 2*
a^5*c*d^3*e^2 + a^6*d*e^4)*sqrt(-(c^5*d^4 - 2*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.26, size = 348, normalized size = 1.00

$$-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}ace+\left(ac^{3}\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}ace+\left(ac^{3}\right)^{\frac{3}{4}}d\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}ace-\left(ac^{3}\right)^{\frac{3}{4}}d\right)\log\left(x^{2}+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}ace-\left(ac^{3}\right)^{\frac{3}{4}}d\right)\log\left(x^{2}-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2}a^{2}c^{2}d^{2}+\sqrt{2}a^{3}ce^{2}\right)}-\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{5}{2}}}{\left(cd^{3}+ade^{2}\right)\sqrt{d}}-\frac{1}{adx}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1
/2*((a*c^3)^(1/4)*a*c*e + (a*c^3)^(3/4)*d)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4
*((a*c^3)^(1/4)*a*c*e - (a*c^3)^(3/4)*d)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*a
*c*e - (a*c^3)^(3/4)*d)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/((c
*d^3 + a*d*e^2)*sqrt(d)) - 1/(a*d*x)
```

Mupad [B]
time = 2.00, size = 2500, normalized size = 7.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2
) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*
a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2)
 + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*
(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^
7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512
*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^
10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*
c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*
d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c
^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^
(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^7*e^8))*(-(
a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e
^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*1i + (x*(2*a^7*c^7*d^9*e^5 - 4*
a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a
^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^
2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 +
 a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^
2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^
2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^
5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^11*c^6*d
^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^1
4*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8)
)*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(
a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e^2 + 4*a
^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(
-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e
```

```
^2)))^(1/2)*1i)/((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^
5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c
^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5
*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2))
)^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d
*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^1
5*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^
9) - 192*a^10*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7
+ 256*a^13*c^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 11
2*a^10*c^6*d^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*
d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*
d^2*e^2)))^(1/2) - 4*a^7*c^8*d^13*e^2 - 4*a^8*c^7*d^11*e^4 + 16*a^10*c^5*d^
7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)
/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) - (x*(2*a^7*c^7*d^9*
e^5 - 4*a^8*c^6*d^7*e^7) - (-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/
2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(
((-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a
^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(-(a*e^2*(-a^5*c^3)^(1/2
) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*
a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512
*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) + 192*a^10*c^7*d^14*e^3 + 128*a^
11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d^8*e^9) + x*(16*a^8
*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d^10*e^6 + 128*a^11*c^5*
d^8*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*
e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2) + 4*a^7*c^8*d^13*e
^2 + 4*a^8*c^7*d^11*e^4 - 16*a^10*c^5*d^7*e^8))*(-(a*e^2*(-a^5*c^3)^(1/2) -
 c*d^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6
*c*d^2*e^2)))^(1/2)))*(-(a*e^2*(-a^5*c^3)^(1/2) - c*d^2*(-a^5*c^3)^(1/2) +
2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*2i + a
tan(((x*(2*a^7*c^7*d^9*e^5 - 4*a^8*c^6*d^7*e^7) - (-(c*d^2*(-a^5*c^3)^(1/2)
 - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a
^6*c*d^2*e^2)))^(1/2)*(((-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2)
+ 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(x*(
-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7
*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^15*e^3 + 512*
a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14*c^4*d^9*e^9) - 192*a^1
0*c^7*d^14*e^3 - 128*a^11*c^6*d^12*e^5 + 320*a^12*c^5*d^10*e^7 + 256*a^13*c
^4*d^8*e^9) + x*(16*a^8*c^8*d^14*e^2 + 32*a^9*c^7*d^12*e^4 - 112*a^10*c^6*d
^10*e^6 + 128*a^11*c^5*d^8*e^8))*(-(c*d^2*(-a^5*c^3)^(1/2) - a*e^2*(-a^5*c^
3)^(1/2) + 2*a^3*c^2*d*e)/(16*(a^7*e^4 + a^5*c^...
```

3.243 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)} dx$

Optimal. Leaf size=360

$$-\frac{1}{3adx^3}+\frac{e}{ad^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2+ae^2\right)}+\frac{c^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{7/4}\left(cd^2+ae^2\right)}-\frac{c^{5/4}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}}{2\sqrt{2}\,a^{7/4}\left(cd^2\right.}$$

```
[Out] -1/3/a/d/x^3+e/a/d^2/x+e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(a*e^2+c*d
^2)-1/4*c^(5/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))
/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*c^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/
4))*(-e*a^(1/2)+d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(5/4)*ln(-a^
(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/
(a*e^2+c*d^2)*2^(1/2)-1/8*c^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*
c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {1302, 211, 1182, 1176, 631, 210, 1179, 642}

$$\frac{c^{5/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}-\frac{c^{5/4}\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{7/4}\left(ae^2+cd^2\right)}+\frac{e^{7/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(ae^2+cd^2\right)}+\frac{e}{ad^2x}-\frac{1}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] -1/3*1/(a*d*x^3) + e/(a*d^2*x) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(
5/2)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt
[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(7
/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[
2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) -
(c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
Sqrt[c]*x^2])/(4*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^4(d+ex^2)(a+cx^4)}\,dx = \int \left(\frac{1}{adx^4} - \frac{e}{ad^2x^2} + \frac{e^4}{d^2(cd^2+ae^2)(d+ex^2)} - \frac{c^2(d-ex^2)}{a(cd^2+ae^2)(a+cx^4)}\right) dx$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} - \frac{c^2\int \frac{d-ex^2}{a+cx^4}\,dx}{a(cd^2+ae^2)} + \frac{e^4\int \frac{1}{d+ex^2}\,dx}{d^2(cd^2+ae^2)}$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)} - \frac{\left(c\left(\frac{\sqrt{c}\,d}{\sqrt{a}} - e\right)\right)\int \frac{\sqrt{a}\,\sqrt{c}+cx^2}{a+cx^4}\,dx}{2a(cd^2+ae^2)}$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)} - \frac{\left(c\left(\frac{\sqrt{c}\,d}{\sqrt{a}} - e\right)\right)\int \frac{1}{\frac{\sqrt{a}}{\sqrt{c}} - \frac{\sqrt{2}\,\sqrt[4]{a}\,x}{\sqrt[4]{c}}}}{4a(cd^2+ae^2)}$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)} + \frac{c^{5/4}\left(\sqrt{c}\,d + \sqrt{a}\,e\right)\log\left(\sqrt{a} - \sqrt{2}\right.}{4\sqrt{2}\,a^{7/4}(cd^2+a}$$

$$= -\frac{1}{3adx^3} + \frac{e}{ad^2x} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)} + \frac{c^{5/4}\left(\sqrt{c}\,d - \sqrt{a}\,e\right)\tan^{-1}\left(1 - \frac{\sqrt{2}}{\sqrt[4]{}}\right.}{2\sqrt{2}\,a^{7/4}(cd^2+ae^2)}$$

Mathematica [A]
time = 0.29, size = 367, normalized size = 1.02

$$\frac{-8ad^{3/2}(cd^2+ae^2)+24a\sqrt{d}\,e(cd^2+ae^2)x^2+24a^2e^{7/2}x^3\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)+6\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}d^{5/2}\left(\sqrt{c}\,d-\sqrt{a}\,e\right)x^3\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+6\sqrt{2}\,\sqrt[4]{a}\,c^{5/4}d^{5/2}\left(-\sqrt{c}\,d+\sqrt{a}\,e\right)x^3\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+3\sqrt{2}\,c^{5/4}d^{5/2}\left(\sqrt[4]{a}\,\sqrt{c}\,d+a^{3/4}e\right)x^3\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)-3\sqrt{2}\,c^{5/4}d^{5/2}\left(\sqrt[4]{a}\,\sqrt{c}\,d+a^{3/4}e\right)x^3\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{24a^2d^{5/2}(cd^2+ae^2)x^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)),x]
```

```
[Out] (-8*a*d^(3/2)*(c*d^2 + a*e^2) + 24*a*Sqrt[d]*e*(c*d^2 + a*e^2)*x^2 + 24*a^2
*e^(7/2)*x^3*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + 6*Sqrt[2]*a^(1/4)*c^(5/4)*d^(5/2
)*(Sqrt[c]*d - Sqrt[a]*e)*x^3*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 6*S
qrt[2]*a^(1/4)*c^(5/4)*d^(5/2)*(-(Sqrt[c]*d) + Sqrt[a]*e)*x^3*ArcTan[1 + (S
qrt[2]*c^(1/4)*x)/a^(1/4)] + 3*Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d +
 a^(3/4)*e)*x^3*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2] - 3*
Sqrt[2]*c^(5/4)*d^(5/2)*(a^(1/4)*Sqrt[c]*d + a^(3/4)*e)*x^3*Log[Sqrt[a] + S
qrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(24*a^2*d^(5/2)*(c*d^2 + a*e^2)*x^
3)
```

Maple [A]
time = 0.18, size = 284, normalized size = 0.79

method	result
default	$\frac{e^4\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d^2(ae^2+cd^2)\sqrt{de}} - \frac{c^2\left(\frac{d\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}+1\right)+2\arctan\left(\frac{\sqrt{2}\,x}{\left(\frac{a}{c}\right)^{\frac{1}{4}}}-1\right)\right)}{8a} - e\sqrt{2}\,(\cdots\right)}{(ae^2+cd^2)a}$
risch	$\frac{\frac{ex^2}{d^2a}-\frac{1}{3da}}{x^3} + \frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^9e^4+2a^8cd^2e^2+c^2a^7d^4\right)\_Z^4-4a^4c^3de\_Z^2+c^5\right)} \_R\ln\left(\left(\left(6a^{11}d^5e^8+19a^{10}cd^7e^6+25a^9c^2d^9e^4+17a^8\cdots\right.\right.\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*e^4/(a*e^2+c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-c^2/(a*e^2+c*d^
2)/a*(1/8*d*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2
))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+
1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))-1/8*e/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))
+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))-1/3/
a/d/x^3+e/a/d^2/x
```

Maxima [A]
time = 0.51, size = 296, normalized size = 0.82

$$-\frac{c^2\left(\frac{2\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac14}c^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(\sqrt{c}d-\sqrt{a}e\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac14}c^{\frac14}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac14}c^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}c^{\frac34}}-\frac{\sqrt{2}\left(\sqrt{c}d+\sqrt{a}e\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac14}c^{\frac14}x+\sqrt{a}\right)}{a^{\frac34}c^{\frac34}}\right)}{8\left(acd^2+a^2e^2\right)}+\frac{\arctan\left(\frac{xe^{\frac12}}{\sqrt{d}}\right)e^{\frac72}}{\left(cd^4+ad^2e^2\right)\sqrt{d}}+\frac{3x^2e-d}{3ad^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="maxima")
```

```
[Out] -1/8*c^2*(2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x
 + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(c))*sqrt(c)) + 2*sqrt(2)*(sqrt(c)*d - sqrt(a)*e)*arctan(1/2*sqrt(2)*(2*s
qrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sq
rt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(sqrt(c)*d + sqrt(a)*e)*log(sqrt(c)*x^2 +
 sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(sqrt(c)*
d + sqrt(a)*e)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3
/4)*c^(3/4)))/(a*c*d^2 + a^2*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c*d
^4 + a*d^2*e^2)*sqrt(d)) + 1/3*(3*x^2*e - d)/(a*d^2*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2097 vs. $2(267) = 534$.

time = 5.61, size = 4228, normalized size = 11.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="fricas")
```

```
[Out] [1/12*(6*a*x^3*sqrt(-e/d)*e^3*log((x^2*e + 2*d*x*sqrt(-e/d) - d)/(x^2*e + d
)) + 12*c*d^2*x^2*e - 4*c*d^3 + 12*a*x^2*e^3 - 4*a*d*e^2 + 3*(a*c*d^4*x^3 +
 a^2*d^2*x^3*e^2)*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^
4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3
*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4
+ 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-c^5*d^2*x + a*c^4*x*e^2 + (a^2*c^4*d^3 -
 a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4
 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*
c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4
 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^
4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6
+ a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4*x^3
+ a^2*d^2*x^3*e^2)*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e
^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4
 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-c^5*d^2*x + a*c^4*x*e^2 - (a^2*c^4*d^3
- a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^
4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9
*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e
^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6
 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3*(a*c*d^4*x^3
 + a^2*d^2*x^3*e^2)*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*
e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c
^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^
4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-c^5*d^2*x + a*c^4*x*e^2 + (a^2*c^4*d^3
 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d
^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*
e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^
6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4*x^
3 + a^2*d^2*x^3*e^2)*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5
*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*
c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d
^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-c^5*d^2*x + a*c^4*x*e^2 - (a^2*c^4*d^
3 - a^3*c^3*d*e^2 - (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*
d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a
```

```
^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e - (a^3*c^2*
d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5
*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e
^6 + a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))))/(a*c*d^4*x^3
+ a^2*d^2*x^3*e^2), 1/12*(12*c*d^2*x^2*e + 12*a*x^3*arctan(x*e^(1/2)/sqrt(d
))*e^(7/2)/sqrt(d) - 4*c*d^3 + 12*a*x^2*e^3 - 4*a*d*e^2 + 3*(a*c*d^4*x^3 +
a^2*d^2*x^3*e^2)*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4
)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4 +
 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-c^5*d^2*x + a*c^4*x*e^2 + (a^2*c^4*d^3 -
a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4
- 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c
^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4
+ 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4
)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 +
 a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) - 3*(a*c*d^4*x^3 +
 a^2*d^2*x^3*e^2)*sqrt((2*c^3*d*e + (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^
4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3
*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))/(a^3*c^2*d^4
+ 2*a^4*c*d^2*e^2 + a^5*e^4))*log(-c^5*d^2*x + a*c^4*x*e^2 - (a^2*c^4*d^3 -
 a^3*c^3*d*e^2 + (a^6*c^2*d^4*e + 2*a^7*c*d^2*e^3 + a^8*e^5)*sqrt(-(c^7*d^4
 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*
c^2*d^4*e^4 + 4*a^10*c*d^2*e^6 + a^11*e^8)))*sqrt((2*c^3*d*e + (a^3*c^2*d^4
 + 2*a^4*c*d^2*e^2 + a^5*e^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^
4)/(a^7*c^4*d^8 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^6
+ a^11*e^8)))/(a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4))) + 3*(a*c*d^4*x^3
+ a^2*d^2*x^3*e^2)*sqrt((2*c^3*d*e - (a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e
^4)*sqrt(-(c^7*d^4 - 2*a*c^6*d^2*e^2 + a^2*c^5*e^4)/(a^7*c^4*d^8 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4 + 4*a^10*c*d^2*e^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.14, size = 364, normalized size = 1.01

$$-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}} c^{2} d - \left(ac^{3}\right)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2}\left(2 x + \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2} a^{2} c^{2} d^{2} + \sqrt{2} a^{3} c e^{2}\right)} - \frac{\left(\left(ac^{3}\right)^{\frac{1}{4}} c^{2} d - \left(ac^{3}\right)^{\frac{3}{4}} e\right) \arctan\left(\frac{\sqrt{2}\left(2 x - \sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{2\left(\sqrt{2} a^{2} c^{2} d^{2} + \sqrt{2} a^{3} c e^{2}\right)} - \frac{\left(\left(ac^{3}\right)^{\frac{1}{4}} c^{2} d + \left(ac^{3}\right)^{\frac{3}{4}} e\right) \log\left(x^{2} + \sqrt{2} x \left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2} a^{2} c^{2} d^{2} + \sqrt{2} a^{3} c e^{2}\right)} + \frac{\left(\left(ac^{3}\right)^{\frac{1}{4}} c^{2} d + \left(ac^{3}\right)^{\frac{3}{4}} e\right) \log\left(x^{2} - \sqrt{2} x \left(\frac{a}{c}\right)^{\frac{1}{4}} + \sqrt{\frac{a}{c}}\right)}{4\left(\sqrt{2} a^{2} c^{2} d^{2} + \sqrt{2} a^{3} c e^{2}\right)} + \frac{\arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\frac{7}{2}}}{\left(c d^{4} + a d^{2} e^{2}\right)\sqrt{d}} + \frac{3 x^{2} e - d}{3 a d^{2} x^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a),x, algorithm="giac")
```

```
[Out] -1/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x + sqrt
(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1
/2*((a*c^3)^(1/4)*c^2*d - (a*c^3)^(3/4)*e)*arctan(1/2*sqrt(2)*(2*x - sqrt(2
)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) - 1/4
*((a*c^3)^(1/4)*c^2*d + (a*c^3)^(3/4)*e)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) +
sqrt(a/c))/(sqrt(2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + 1/4*((a*c^3)^(1/4)*c
^2*d + (a*c^3)^(3/4)*e)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(
2)*a^2*c^2*d^2 + sqrt(2)*a^3*c*e^2) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c
*d^4 + a*d^2*e^2)*sqrt(d)) + 1/3*(3*x^2*e - d)/(a*d^2*x^3)
```

Mupad [B]
time = 2.26, size = 2500, normalized size = 6.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)*(d + e*x^2)),x)
```

```
[Out] atan(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/
2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2
*a^8*c*d^2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2)
 + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*
((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9
*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*
a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9
*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^
5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*
d^21*e^4 - 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^
5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d
^4 + 2*a^8*c*d^2*e^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 4
8*a^9*c^6*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*
a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*1i + (x*
(2*a^5*c^9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^
2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^
2*e^2)))^(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*
c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(
-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^
7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*
d^22*e^5 - 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24
*e^2 - 128*a^10*c^7*d^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^
8 + 256*a^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4
- 112*a^9*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2)
- c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^
8*c*d^2*e^2)))^(1/2) + 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6
```

```
*d^15*e^9))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d
*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*1i)/((x*(2*a^5*c^
9*d^18*e^5 + 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c
^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^
(1/2)*((((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/
(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(-a^7*c^5)
^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4
 + 2*a^8*c*d^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5
- 512*a^13*c^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 12
8*a^10*c^7*d^22*e^4 + 192*a^11*c^6*d^20*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a
^13*c^4*d^16*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9
*c^7*d^19*e^6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(
-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e
^2)))^(1/2) - 4*a^6*c^9*d^21*e^3 - 4*a^7*c^8*d^19*e^5 + 48*a^9*c^6*d^15*e^9
))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(
a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) - (x*(2*a^5*c^9*d^18*e^5 +
 4*a^7*c^7*d^14*e^9) - ((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) +
2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((a*
e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4
 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((a*e^2*(-a^7*c^5)^(1/2) - c*d
^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d
^2*e^2)))^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c
^5*d^20*e^7 - 512*a^14*c^4*d^18*e^9) + 64*a^9*c^8*d^24*e^2 - 128*a^10*c^7*d
^22*e^4 - 192*a^11*c^6*d^20*e^6 + 256*a^12*c^5*d^18*e^8 + 256*a^13*c^4*d^16
*e^10) - x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - 112*a^9*c^7*d^19*e^
6 - 128*a^11*c^5*d^15*e^10))*((a*e^2*(-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1
/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)
+ 4*a^6*c^9*d^21*e^3 + 4*a^7*c^8*d^19*e^5 - 48*a^9*c^6*d^15*e^9))*((a*e^2*(
-a^7*c^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^
7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2) + 2*a^5*c^8*d^14*e^8))*((a*e^2*(-a^7*c
^5)^(1/2) - c*d^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*
d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*2i + atan(((x*(2*a^5*c^9*d^18*e^5 + 4*a^7*c^
7*d^14*e^9) - ((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3
*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*((((c*d^2*(-a^7
*c^5)^(1/2) - a*e^2*(-a^7*c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^
2*d^4 + 2*a^8*c*d^2*e^2)))^(1/2)*(x*((c*d^2*(-a^7*c^5)^(1/2) - a*e^2*(-a^7*
c^5)^(1/2) + 2*a^4*c^3*d*e)/(16*(a^9*e^4 + a^7*c^2*d^4 + 2*a^8*c*d^2*e^2)))
^(1/2)*(512*a^11*c^7*d^24*e^3 + 512*a^12*c^6*d^22*e^5 - 512*a^13*c^5*d^20*e
^7 - 512*a^14*c^4*d^18*e^9) - 64*a^9*c^8*d^24*e^2 + 128*a^10*c^7*d^22*e^4 +
 192*a^11*c^6*d^20*e^6 - 256*a^12*c^5*d^18*e^8 - 256*a^13*c^4*d^16*e^10) -
x*(16*a^7*c^9*d^23*e^2 + 32*a^8*c^8*d^21*e^4 - ...
```

3.244 $\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=169

$$\frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)}-\frac{\sqrt{a}\,d(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2}+\frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2}+\frac{ae(2cd^2+ae^2)\log(a+cx^4)}{4c^2(cd^2+ae^2)^2}$$

```
[Out] 1/4*a*(c*d*x^2+a*e)/c^2/(a*e^2+c*d^2)/(c*x^4+a)+1/2*d^4*ln(e*x^2+d)/e/(a*e^
2+c*d^2)^2+1/4*a*e*(a*e^2+2*c*d^2)*ln(c*x^4+a)/c^2/(a*e^2+c*d^2)^2-1/4*d*(a
*e^2+3*c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))*a^(1/2)/c^(3/2)/(a*e^2+c*d^2)^2
```

Rubi [A]
time = 0.24, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1266, 1661, 1643, 649, 211, 266}

$$-\frac{\sqrt{a}\,d\,\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)(ae^2+3cd^2)}{4c^{3/2}(ae^2+cd^2)^2}+\frac{ae(ae^2+2cd^2)\log(a+cx^4)}{4c^2(ae^2+cd^2)^2}+\frac{a(ae+cdx^2)}{4c^2(a+cx^4)(ae^2+cd^2)}+\frac{d^4\log(d+ex^2)}{2e(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(a*e + c*d*x^2))/(4*c^2*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d
^2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) +
(d^4*Log[d + e*x^2])/(2*e*(c*d^2 + a*e^2)^2) + (a*e*(2*c*d^2 + a*e^2)*Log[a
 + c*x^4])/(4*c^2*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1643

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol]
:> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
 d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^9}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{\frac{a^2d^2}{cd^2+ae^2}-\frac{a^2dex}{cd^2+ae^2}-2ax^2}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2acd^4}{(cd^2+ae^2)^2(d+ex)} + \frac{a^2\left(d(3cd^2+ae^2)-2e(2cd^2+\right.}{(cd^2+ae^2)^2(a+cx^2)}\right.\right.}{4ac}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} - \frac{a\text{Subst}\left(\int \frac{d(3cd^2+ae^2)-2e(2cd^2+ae^2}{a+cx^2}\right.}{4c(cd^2+ae^2)^2}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} + \frac{d^4\log(d+ex^2)}{2e(cd^2+ae^2)^2} + \frac{(ae(2cd^2+ae^2))\,\text{Subst}\left(\int \frac{x}{a+cx^2}\right.}{2c(cd^2+ae^2)^2}$$

$$= \frac{a(ae+cdx^2)}{4c^2(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{a}\,d(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2} + \frac{d^4\log(d+e}{2e(cd^2+ae}$$

Mathematica [A]

time = 0.14, size = 135, normalized size = 0.80

$$\frac{\frac{a(cd^2+ae^2)(ae+cdx^2)}{c^2(a+cx^4)}-\frac{\sqrt{a}\,d(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{c^{3/2}}+\frac{2d^4\log(d+ex^2)}{e}+\frac{ae(2cd^2+ae^2)\log(a+cx^4)}{c^2}}{4\left(cd^2+ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((a*(c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c^2*(a + c*x^4)) - (Sqrt[a]*d*(3*c*d^
2 + a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/c^(3/2) + (2*d^4*Log[d + e*x^2])/
e + (a*e*(2*c*d^2 + a*e^2)*Log[a + c*x^4])/c^2)/(4*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.23, size = 160, normalized size = 0.95

method	result	size
default	$\frac{d^4\ln(e\,x^2+d)}{2e(a\,e^2+c\,d^2)^2}-\frac{a\left(\frac{-\frac{d\left(a\,e^2+c\,d^2\right)x^2}{2c}-\frac{ae\left(a\,e^2+c\,d^2\right)}{2c^2}}{c\,x^4+a}+\frac{\frac{\left(-2a\,e^3-4c\,d^2e\right)\ln\left(c\,x^4+a\right)}{2c}+\frac{\left(d\,e^2a+3c\,d^3\right)\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2c}\right)}{2(a\,e^2+c\,d^2)^2}$	160
risch	Expression too large to display	2233

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d^4*ln(e*x^2+d)/e/(a*e^2+c*d^2)^2-1/2*a/(a*e^2+c*d^2)^2*((-1/2*d*(a*e^2
+c*d^2)/c*x^2-1/2*a*e*(a*e^2+c*d^2)/c^2)/(c*x^4+a)+1/2/c*(1/2*(-2*a*e^3-4*c
*d^2*e)/c*ln(c*x^4+a)+(a*d*e^2+3*c*d^3)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2
))))
```

Maxima [A]
time = 0.50, size = 214, normalized size = 1.27

$$\frac{d^4\log\left(x^2e+d\right)}{2\left(c^2d^4e+2\,acd^2e^3+a^2e^5\right)}+\frac{\left(2\,acd^2e+a^2e^3\right)\log\left(cx^4+a\right)}{4\left(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4\right)}-\frac{\left(3\,acd^3+a^2de^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}+\frac{acdx^2+a^2e}{4\left(ac^3d^2+\left(c^4d^2+ac^3e^2\right)x^4+a^2c^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/2*d^4*log(x^2*e + d)/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*a*c*d
^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4) -
```

```
1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2
*e^2 + a^2*c*e^4)*sqrt(a*c)) + 1/4*(a*c*d*x^2 + a^2*e)/(a*c^3*d^2 + (c^4*d^
2 + a*c^3*e^2)*x^4 + a^2*c^2*e^2)
```

Fricas [A]
time = 9.53, size = 548, normalized size = 3.24

$$\left[\frac{2\,ac^2d^3x^2e + 2\,a^2cdx^2e^3 + 2\,a^2cd^2e^2 + 2\,a^3e^4 + \left((ac^2dx^4 + a^2cd)e^3 + 3\,(c^3d^3x^4 + ac^2d^3)e\right)\sqrt{-\frac{a}{c}}\log\left(\frac{cx^4 - 2\,cx^2\sqrt{-\frac{a}{c}} - a}{cx^4 + a}\right) + 2\,\left((a^2cx^4 + a^3)e^4 + 2\,(ac^2d^2x^4 + a^2cd^2)e^2\right)\log\left(cx^4 + a\right) + 4\,(c^3d^4x^4 + ac^2d^4)\log\left(x^2e + d\right)}{8\,\left((a^2c^3x^4 + a^3c^2)e^5 + 2\,(ac^4d^2x^4 + a^2c^3d^2)e^3 + (c^5d^4x^4 + ac^4d^4)e\right)}, \frac{ac^2d^3x^2e + a^2cdx^2e^3 + a^2cd^2e^2 + a^3e^4 - \left((ac^2dx^4 + a^2cd)e^3 + 3\,(c^3d^3x^4 + ac^2d^3)e\right)\sqrt{\frac{a}{c}}\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) + \left((a^2cx^4 + a^3)e^4 + 2\,(ac^2d^2x^4 + a^2cd^2)e^2\right)\log\left(cx^4 + a\right) + 2\,(c^3d^4x^4 + ac^2d^4)\log\left(x^2e + d\right)}{4\,\left((a^2c^3x^4 + a^3c^2)e^5 + 2\,(ac^4d^2x^4 + a^2c^3d^2)e^3 + (c^5d^4x^4 + ac^4d^4)e\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c^2*d^3*x^2*e + 2*a^2*c*d*x^2*e^3 + 2*a^2*c*d^2*e^2 + 2*a^3*e^4 +
 ((a*c^2*d*x^4 + a^2*c*d)*e^3 + 3*(c^3*d^3*x^4 + a*c^2*d^3)*e)*sqrt(-a/c)*l
og((c*x^4 - 2*c*x^2*sqrt(-a/c) - a)/(c*x^4 + a)) + 2*((a^2*c*x^4 + a^3)*e^4
 + 2*(a*c^2*d^2*x^4 + a^2*c*d^2)*e^2)*log(c*x^4 + a) + 4*(c^3*d^4*x^4 + a*c
^2*d^4)*log(x^2*e + d))/((a^2*c^3*x^4 + a^3*c^2)*e^5 + 2*(a*c^4*d^2*x^4 + a
^2*c^3*d^2)*e^3 + (c^5*d^4*x^4 + a*c^4*d^4)*e), 1/4*(a*c^2*d^3*x^2*e + a^2*
c*d*x^2*e^3 + a^2*c*d^2*e^2 + a^3*e^4 - ((a*c^2*d*x^4 + a^2*c*d)*e^3 + 3*(c
^3*d^3*x^4 + a*c^2*d^3)*e)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) + ((a^2*c*x^
4 + a^3)*e^4 + 2*(a*c^2*d^2*x^4 + a^2*c*d^2)*e^2)*log(c*x^4 + a) + 2*(c^3*d
^4*x^4 + a*c^2*d^4)*log(x^2*e + d))/((a^2*c^3*x^4 + a^3*c^2)*e^5 + 2*(a*c^4
*d^2*x^4 + a^2*c^3*d^2)*e^3 + (c^5*d^4*x^4 + a*c^4*d^4)*e)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.62, size = 251, normalized size = 1.49

$$\frac{d^4\log\left(|x^2e + d|\right)}{2\,(c^2d^4e + 2\,acd^2e^3 + a^2e^5)} + \frac{(2\,acd^2e + a^2e^3)\log\left(cx^4 + a\right)}{4\,(c^4d^4 + 2\,ac^3d^2e^2 + a^2c^2e^4)} - \frac{(3\,acd^3 + a^2de^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,(c^3d^4 + 2\,ac^2d^2e^2 + a^2ce^4)\sqrt{ac}} - \frac{2\,acd^2x^4e - acd^3x^2 + a^2x^4e^3 - a^2dx^2e^2 + a^2d^2e}{4\,(c^3d^4 + 2\,ac^2d^2e^2 + a^2ce^4)(cx^4 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(2*
a*c*d^2*e + a^2*e^3)*log(c*x^4 + a)/(c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^
```

```
4) - 1/4*(3*a*c*d^3 + a^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^
2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c)) - 1/4*(2*a*c*d^2*x^4*e - a*c*d^3*x^2 + a^
2*x^4*e^3 - a^2*d*x^2*e^2 + a^2*d^2*e)/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*
e^4)*(c*x^4 + a))
```

Mupad [B]
time = 1.30, size = 305, normalized size = 1.80

$$\frac{\frac{a^2\,e}{4\,c^2\,\left(c\,d^2+a\,e^2\right)}+\frac{a\,d\,x^2}{4\,c\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}-\frac{\ln\left(\sqrt{-a\,c^5}+c^3\,x^2\right)\,\left(3\,c\,d^3\,\sqrt{-a\,c^5}-2\,a^2\,c^2\,e^3-4\,a\,c^3\,d^2\,e+a\,d\,e^2\,\sqrt{-a\,c^5}\right)}{8\,\left(a^2\,c^4\,e^4+2\,a\,c^5\,d^2\,e^2+c^6\,d^4\right)}+\frac{\ln\left(\sqrt{-a\,c^5}-c^3\,x^2\right)\,\left(3\,c\,d^3\,\sqrt{-a\,c^5}+2\,a^2\,c^2\,e^3+4\,a\,c^3\,d^2\,e+a\,d\,e^2\,\sqrt{-a\,c^5}\right)}{8\,\left(a^2\,c^4\,e^4+2\,a\,c^5\,d^2\,e^2+c^6\,d^4\right)}+\frac{d^4\,\ln\left(e\,x^2+d\right)}{2\,a^2\,e^5+4\,a\,c\,d^2\,e^3+2\,c^2\,d^4\,e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a^2*e)/(4*c^2*(a*e^2 + c*d^2)) + (a*d*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*
x^4) - (log((-a*c^5)^(1/2) + c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/2) - 2*a^2*c^2*e
^3 - 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4 + a^2*c^4*e^4 + 2
*a*c^5*d^2*e^2)) + (log((-a*c^5)^(1/2) - c^3*x^2)*(3*c*d^3*(-a*c^5)^(1/2) +
 2*a^2*c^2*e^3 + 4*a*c^3*d^2*e + a*d*e^2*(-a*c^5)^(1/2)))/(8*(c^6*d^4 + a^2
*c^4*e^4 + 2*a*c^5*d^2*e^2)) + (d^4*log(d + e*x^2))/(2*a^2*e^5 + 2*c^2*d^4*
e + 4*a*c*d^2*e^3)
```

3.245 $\int \frac{x^7}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=150

$$\frac{a(d-ex^2)}{4c(cd^2+ae^2)(a+cx^4)}+\frac{\sqrt{a}\,e(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}(cd^2+ae^2)^2}-\frac{d^3\log(d+ex^2)}{2(cd^2+ae^2)^2}+\frac{d^3\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

```
[Out] 1/4*a*(-e*x^2+d)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2
)^2+1/4*d^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*e*(a*e^2+3*c*d^2)*arctan(x^2*c^
(1/2)/a^(1/2))*a^(1/2)/c^(3/2)/(a*e^2+c*d^2)^2
```

Rubi [A]
time = 0.16, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1266, 1661, 815, 649, 211, 266}

$$\frac{\sqrt{a}\,e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)(ae^2+3cd^2)}{4c^{3/2}(ae^2+cd^2)^2}+\frac{a(d-ex^2)}{4c(a+cx^4)(ae^2+cd^2)}+\frac{d^3\log(a+cx^4)}{4(ae^2+cd^2)^2}-\frac{d^3\log(d+ex^2)}{2(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*(d - e*x^2))/(4*c*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[a]*e*(3*c*d^2 + a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*c^(3/2)*(c*d^2 + a*e^2)^2) - (d^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^7}{(d+ex^2)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)\left(a+cx^2\right)^2}\,dx,x,x^2\right)$$

$$= \frac{a(d-ex^2)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{a^2de}{cd^2+ae^2}-\frac{a\left(2cd^2+ae^2\right)x}{cd^2+ae^2}}{(d+ex)(a+cx^2)}\,dx,x,x^2\right)}{4ac}$$

$$= \frac{a(d-ex^2)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{\text{Subst}\left(\int \left(\frac{2acd^3e}{\left(cd^2+ae^2\right)^2(d+ex)} - \frac{a\left(3acd^2e+a^2e^3+2c^2d^3x\right)}{\left(cd^2+ae^2\right)^2(a+cx^2)}\right)\,dx,\right.}{4ac}$$

$$= \frac{a(d-ex^2)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^3\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{\text{Subst}\left(\int \frac{3acd^2e+a^2e^3+2c^2d^3x}{a+cx^2}\,dx,x,x\right.}{4c\left(cd^2+ae^2\right)^2}$$

$$= \frac{a(d-ex^2)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} - \frac{d^3\log\left(d+ex^2\right)}{2\left(cd^2+ae^2\right)^2} + \frac{\left(cd^3\right)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2\left(cd^2+ae^2\right)^2} + \cdots$$

$$= \frac{a(d-ex^2)}{4c\left(cd^2+ae^2\right)\left(a+cx^4\right)} + \frac{\sqrt{a}\,e(3cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4c^{3/2}\left(cd^2+ae^2\right)^2} - \frac{d^3\log\left(d+ex^2\right.}{2\left(cd^2+ae^2\right)^2}$$

Mathematica [A]
time = 0.08, size = 142, normalized size = 0.95

$$\frac{\sqrt{a}\, e(3cd^2+ae^2)\,(a+cx^4)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)+\sqrt{c}\,(a(cd^2+ae^2)\,(d-ex^2)-2cd^3(a+cx^4)\log\left(d+ex^2\right)+cd^3(a+cx^4)\log\left(a+cx^4\right))}{4c^{3/2}\left(cd^2+ae^2\right)^2(a+cx^4)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (Sqrt[a]*e*(3*c*d^2 + a*e^2)*(a + c*x^4)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]] + Sq
rt[c]*(a*(c*d^2 + a*e^2)*(d - e*x^2) - 2*c*d^3*(a + c*x^4)*Log[d + e*x^2] +
 c*d^3*(a + c*x^4)*Log[a + c*x^4]))/(4*c^(3/2)*(c*d^2 + a*e^2)^2*(a + c*x^4
))
```

Maple [A]
time = 0.19, size = 146, normalized size = 0.97

method	result
default	$-\frac{d^3\ln\left(e\,x^2+d\right)}{2(a\,e^2+c\,d^2)^2}+\frac{\frac{-\frac{ae\left(a\,e^2+c\,d^2\right)x^2}{2c}+\frac{ad\left(a\,e^2+c\,d^2\right)}{2c}}{c\,x^4+a}+\frac{c\,d^3\ln\left(c\,x^4+a\right)+\frac{\left(a^2e^3+3a\,d^2ec\right)\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2c}}{2(a\,e^2+c\,d^2)^2}$
risch	$\frac{-\frac{ae\,x^2}{4c\left(a\,e^2+c\,d^2\right)}+\frac{ad}{4c\left(a\,e^2+c\,d^2\right)}}{c\,x^4+a}-\frac{d^3\ln\left(e\,x^2+d\right)}{2(a^2e^4+2ac\,d^2e^2+c^2d^4)}+\frac{\left(\sum_{\_R=\text{RootOf}\left(\left(a^2c^3e^4+2a\,c^4d^2e^2+c^5d^4\right)\_Z^2-4c^3d^3\_Z+a\,e^2+4c\,d^2\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2+1/2/(a*e^2+c*d^2)^2*((-1/2*a*e*(a*e^2+
c*d^2)/c*x^2+1/2*a*d*(a*e^2+c*d^2)/c)/(c*x^4+a)+1/2/c*(c*d^3*ln(c*x^4+a)+(a
^2*e^3+3*a*c*d^2*e)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))))
```

Maxima [A]
time = 0.49, size = 191, normalized size = 1.27

$$\frac{d^3\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{d^3\log\left(x^2e+d\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{\left(3\,acd^2e+a^2e^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)\sqrt{ac}}-\frac{ax^2e-ad}{4\left(ac^2d^2+(c^3d^2+ac^2e^2)x^4+a^2ce^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d^3*log(x^
2*e + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e + a^2*e^3)*
```

```
arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*sqrt(a*c))
 - 1/4*(a*x^2*e - a*d)/(a*c^2*d^2 + (c^3*d^2 + a*c^2*e^2)*x^4 + a^2*c*e^2)
```

Fricas [A]
time = 4.63, size = 460, normalized size = 3.07

$$\left[-\frac{2\,acd^2x^2e - 2\,acd^3 + 2\,a^2x^2e^3 - 2\,a^2de^2 - ((acx^4 + a^2)e^3 + 3\,(c^2d^2x^4 + acd^2)e)\sqrt{-\frac{a}{c}}\,\log\left(\frac{cx^4 + 2\,cx^2\sqrt{-\frac{a}{c}} - a}{cx^4 + a}\right) - 2\,(c^2d^3x^4 + acd^3)\log\left(cx^4 + a\right) + 4\,(c^2d^3x^4 + acd^3)\log\left(x^2e + d\right)}{8\,(c^4d^4x^4 + ac^3d^4 + (a^2c^2x^4 + a^3c)e^4 + 2\,(ac^3d^2x^4 + a^2c^2d^2)e^2)}, -\frac{acd^2x^2e - acd^3 + a^2x^2e^3 - a^2de^2 - ((acx^4 + a^2)e^3 + 3\,(c^2d^2x^4 + acd^2)e)\sqrt{\frac{a}{c}}\,\arctan\left(\frac{cx^2\sqrt{\frac{a}{c}}}{a}\right) - (c^2d^3x^4 + acd^3)\log\left(cx^4 + a\right) + 2\,(c^2d^3x^4 + acd^3)\log\left(x^2e + d\right)}{4\,(c^4d^4x^4 + ac^3d^4 + (a^2c^2x^4 + a^3c)e^4 + 2\,(ac^3d^2x^4 + a^2c^2d^2)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*c*d^2*x^2*e - 2*a*c*d^3 + 2*a^2*x^2*e^3 - 2*a^2*d*e^2 - ((a*c*x^
4 + a^2)*e^3 + 3*(c^2*d^2*x^4 + a*c*d^2)*e)*sqrt(-a/c)*log((c*x^4 + 2*c*x^2
*sqrt(-a/c) - a)/(c*x^4 + a)) - 2*(c^2*d^3*x^4 + a*c*d^3)*log(c*x^4 + a) +
4*(c^2*d^3*x^4 + a*c*d^3)*log(x^2*e + d))/(c^4*d^4*x^4 + a*c^3*d^4 + (a^2*c
^2*x^4 + a^3*c)*e^4 + 2*(a*c^3*d^2*x^4 + a^2*c^2*d^2)*e^2), -1/4*(a*c*d^2*x
^2*e - a*c*d^3 + a^2*x^2*e^3 - a^2*d*e^2 - ((a*c*x^4 + a^2)*e^3 + 3*(c^2*d^
2*x^4 + a*c*d^2)*e)*sqrt(a/c)*arctan(c*x^2*sqrt(a/c)/a) - (c^2*d^3*x^4 + a*
c*d^3)*log(c*x^4 + a) + 2*(c^2*d^3*x^4 + a*c*d^3)*log(x^2*e + d))/(c^4*d^4*
x^4 + a*c^3*d^4 + (a^2*c^2*x^4 + a^3*c)*e^4 + 2*(a*c^3*d^2*x^4 + a^2*c^2*d^
2)*e^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.38, size = 223, normalized size = 1.49

$$-\frac{d^3e\log\left(|x^2e + d|\right)}{2\,(c^2d^4e + 2\,acd^2e^3 + a^2e^5)} + \frac{d^3\log\left(cx^4 + a\right)}{4\,(c^2d^4 + 2\,acd^2e^2 + a^2e^4)} + \frac{(3\,acd^2e + a^2e^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,(c^3d^4 + 2\,ac^2d^2e^2 + a^2ce^4)\sqrt{ac}} - \frac{c^2d^3x^4 + acd^2x^2e + a^2x^2e^3 - a^2de^2}{4\,(c^3d^4 + 2\,ac^2d^2e^2 + a^2ce^4)(cx^4 + a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/2*d^3*e*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*
d^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(3*a*c*d^2*e +
 a^2*e^3)*arctan(c*x^2/sqrt(a*c))/((c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*
```

```
sqrt(a*c)) - 1/4*(c^2*d^3*x^4 + a*c*d^2*x^2*e + a^2*x^2*e^3 - a^2*d*e^2)/((
c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

Mupad [B]
time = 1.49, size = 647, normalized size = 4.31

$$\frac{\frac{a\,d}{4\,c\,(c\,d^2+a\,e^2)}-\frac{a\,e\,x^2}{4\,c\,(c\,d^2+a\,e^2)}}{c\,x^4+a}-\frac{d^3\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}+\frac{\ln\left(36\,c^8\,d^{10}\,x^2+36\,c^6\,d^{10}\,\sqrt{-a\,c^3}+a^5\,c\,e^{10}\,\sqrt{-a\,c^3}+a^5\,c^3\,e^{10}\,x^2-22\,a^2\,d^4\,e^6\,{\left(-a\,c^3\right)}^{3/2}-81\,c^2\,d^8\,e^2\,{\left(-a\,c^3\right)}^{3/2}+60\,a^2\,c^6\,d^6\,e^4\,x^2+22\,a^3\,c^5\,d^4\,e^6\,x^2+8\,a^4\,c^4\,d^2\,e^8\,x^2+8\,a^4\,c^2\,d^2\,e^8\,\sqrt{-a\,c^3}-60\,a\,c\,d^6\,e^4\,{\left(-a\,c^3\right)}^{3/2}+81\,a\,c^7\,d^8\,e^2\,x^2\right)\,\left(2\,c^3\,d^3+a\,e^3\,\sqrt{-a\,c^3}+3\,c\,d^2\,e\,\sqrt{-a\,c^3}\right)}{8\,\left(a^2\,c^3\,e^4+2\,a\,c^4\,d^2\,e^2+c^5\,d^4\right)}-\frac{\ln\left(36\,c^8\,d^{10}\,x^2-36\,c^6\,d^{10}\,\sqrt{-a\,c^3}-a^5\,c\,e^{10}\,\sqrt{-a\,c^3}+a^5\,c^3\,e^{10}\,x^2+22\,a^2\,d^4\,e^6\,{\left(-a\,c^3\right)}^{3/2}+81\,c^2\,d^8\,e^2\,{\left(-a\,c^3\right)}^{3/2}+60\,a^2\,c^6\,d^6\,e^4\,x^2+22\,a^3\,c^5\,d^4\,e^6\,x^2+8\,a^4\,c^4\,d^2\,e^8\,x^2-8\,a^4\,c^2\,d^2\,e^8\,\sqrt{-a\,c^3}+60\,a\,c\,d^6\,e^4\,{\left(-a\,c^3\right)}^{3/2}+81\,a\,c^7\,d^8\,e^2\,x^2\right)\,\left(a\,e^3\,\sqrt{-a\,c^3}-2\,c^3\,d^3+3\,c\,d^2\,e\,\sqrt{-a\,c^3}\right)}{8\,\left(a^2\,c^3\,e^4+2\,a\,c^4\,d^2\,e^2+c^5\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a*d)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^2)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4)
 - (d^3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36*c
^8*d^10*x^2 + 36*c^6*d^10*(-a*c^3)^(1/2) + a^5*c*e^10*(-a*c^3)^(1/2) + a^5*
c^3*e^10*x^2 - 22*a^2*d^4*e^6*(-a*c^3)^(3/2) - 81*c^2*d^8*e^2*(-a*c^3)^(3/2
) + 60*a^2*c^6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2
 + 8*a^4*c^2*d^2*e^8*(-a*c^3)^(1/2) - 60*a*c*d^6*e^4*(-a*c^3)^(3/2) + 81*a*
c^7*d^8*e^2*x^2)*(2*c^3*d^3 + a*e^3*(-a*c^3)^(1/2) + 3*c*d^2*e*(-a*c^3)^(1/
2)))/(8*(c^5*d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2)) - (log(36*c^8*d^10*x^2 -
 36*c^6*d^10*(-a*c^3)^(1/2) - a^5*c*e^10*(-a*c^3)^(1/2) + a^5*c^3*e^10*x^2
+ 22*a^2*d^4*e^6*(-a*c^3)^(3/2) + 81*c^2*d^8*e^2*(-a*c^3)^(3/2) + 60*a^2*c^
6*d^6*e^4*x^2 + 22*a^3*c^5*d^4*e^6*x^2 + 8*a^4*c^4*d^2*e^8*x^2 - 8*a^4*c^2*
d^2*e^8*(-a*c^3)^(1/2) + 60*a*c*d^6*e^4*(-a*c^3)^(3/2) + 81*a*c^7*d^8*e^2*x
^2)*(a*e^3*(-a*c^3)^(1/2) - 2*c^3*d^3 + 3*c*d^2*e*(-a*c^3)^(1/2)))/(8*(c^5*
d^4 + a^2*c^3*e^4 + 2*a*c^4*d^2*e^2))
```

3.246 $\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=155

$$\frac{-ae-cdx^2}{4c(cd^2+ae^2)(a+cx^4)}+\frac{d(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{c}\,(cd^2+ae^2)^2}+\frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2}-\frac{d^2e\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

```
[Out] 1/4*(-c*d*x^2-a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)+1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c
*d^2)^2-1/4*d^2*e*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*d*(-a*e^2+c*d^2)*arctan(x
^2*c^(1/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 153, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1266, 1661, 815, 649, 211, 266}

$$\frac{d\,\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)(cd^2-ae^2)}{4\sqrt{a}\,\sqrt{c}\,(ae^2+cd^2)^2}-\frac{d^2e\log(a+cx^4)}{4(ae^2+cd^2)^2}+\frac{d^2e\log(d+ex^2)}{2(ae^2+cd^2)^2}-\frac{ae+cdx^2}{4c(a+cx^4)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*(a*e + c*d*x^2)/(c*(c*d^2 + a*e^2)*(a + c*x^4)) + (d*(c*d^2 - a*e^2)*A
rcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) + (d^2*
e*Log[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (d^2*e*Log[a + c*x^4])/(4*(c*d^2
+ a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1661

```
Int[(Pq_)*((d_) + (e_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :
> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + c*x^2, x], f = Coeff[Pol
ynomialRemainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 0], g = Coeff[Polynomial
Remainder[(d + e*x)^m*Pq, a + c*x^2, x], x, 1]}, Simp[(a*g - c*f*x)*((a + c
*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Dist[1/(2*a*c*(p + 1)), Int[(d + e*x)^
m*(a + c*x^2)^(p + 1)*ExpandToSum[(2*a*c*(p + 1)*Q)/(d + e*x)^m + (c*f*(2*p
 + 3))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, c, d, e}, x] && PolyQ[Pq, x] &
& NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)\\
&= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{-\frac{acd^2}{cd^2+ae^2}+\frac{acdex}{cd^2+ae^2}}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4ac}\\
&= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2acd^2e^2}{(cd^2+ae^2)^2(d+ex)}+\frac{acd(-cd^2+ae^2+2cdex)}{(cd^2+ae^2)^2(a+cx^2)}\right)\right.}{4ac}\\
&= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{d\text{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx, x\right.}{4(cd^2+ae^2)^2}\\
&= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{(cd^2e)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right.}{2(cd^2+ae^2)^2}\\
&= -\frac{ae+cdx^2}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{c}\,(cd^2+ae^2)^2} + \frac{d^2e\log(d+ex^2)}{2(cd^2+ae^2)^2}
\end{aligned}
$$

Mathematica [A]
time = 0.11, size = 120, normalized size = 0.77

$$\frac{-\frac{(cd^2+ae^2)(ae+cdx^2)}{c(a+cx^4)}+\frac{d(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}+2d^2e\log(d+ex^2)-d^2e\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(c*(a + c*x^4))) + (d*(c*d^2 - a*e^2)*
ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) + 2*d^2*e*Log[d + e*x^2] -
 d^2*e*Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.18, size = 136, normalized size = 0.88

method	result
default	$\frac{d^2 e\ln(e\,x^2+d)}{2(a\,e^2+c\,d^2)^2}-\frac{\frac{\left(\frac{1}{2}d\,e^2 a+\frac{1}{2}c\,d^3\right)x^2+\frac{ae\left(a\,e^2+c\,d^2\right)}{2c}}{c\,x^4+a}+\frac{d\left(de\ln\left(c\,x^4+a\right)+\frac{\left(a\,e^2-c\,d^2\right)\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2}}{2(a\,e^2+c\,d^2)^2}$
risch	$\frac{-\frac{d\,x^2}{4\left(a\,e^2+c\,d^2\right)}-\frac{ae}{4c\left(a\,e^2+c\,d^2\right)}}{c\,x^4+a}+\frac{d^2 e\ln\left(e\,x^2+d\right)}{2a^2e^4+4ac\,d^2e^2+2c^2d^4}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(a^3c\,e^4+2a^2c^2d^2e^2+c^3a\,d^4\right)\_Z^2+4ac\,d^2e\_Z+d^2\right)}\_R\ln\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*d^2*e*ln(e*x^2+d)/(a*e^2+c*d^2)^2-1/2/(a*e^2+c*d^2)^2*(((1/2*d*e^2*a+1/
2*c*d^3)*x^2+1/2*a*e*(a*e^2+c*d^2)/c)/(c*x^4+a)+1/2*d*(d*e*ln(c*x^4+a)+(a*e
^2-c*d^2)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))))
```

Maxima [A]
time = 0.50, size = 187, normalized size = 1.21

$$-\frac{d^2e\log(cx^4+a)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)}+\frac{d^2e\log(x^2e+d)}{2(c^2d^4+2\,acd^2e^2+a^2e^4)}+\frac{(cd^3-ade^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{ac}}-\frac{cdx^2+ae}{4(ac^2d^2+(c^3d^2+ac^2e^2)x^4+a^2ce^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e*l
og(x^2*e + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c*d^3 - a*d*e^2)*a
rctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) - 1/
4*(c*d*x^2 + a*e)/(a*c^2*d^2 + (c^3*d^2 + a*c^2*e^2)*x^4 + a^2*c*e^2)
```

Fricas [**A**]
time = 2.00, size = 484, normalized size = 3.12

$$\left[-\frac{2\,ac^{2}d^{3}x^{2}+2\,a^{2}cdx^{2}e^{2}+2\,a^{2}cd^{2}e+2\,a^{3}e^{3}+2\,(ac^{2}d^{2}x^{4}+a^{2}cd^{2})e\log\left(cx^{4}+a\right)-4\,(ac^{2}d^{2}x^{4}+a^{2}cd^{2})e\log\left(x^{2}e+d\right)-(c^{2}d^{3}x^{4}+acd^{3}-(acdx^{4}+a^{2}d)e^{2})\sqrt{-ac}\log\left(\frac{cx^{4}+2\sqrt{-ac}\,x^{2}-a}{cx^{4}+a}\right)}{8\,(ac^{4}d^{4}x^{4}+a^{2}c^{3}d^{4}+(a^{3}c^{2}x^{4}+a^{4}c)e^{4}+2\,(a^{2}c^{3}d^{2}x^{4}+a^{3}c^{2}d^{2})e^{2})},\ -\frac{ac^{2}d^{3}x^{2}+a^{2}cdx^{2}e^{2}+a^{2}cd^{2}e+a^{3}e^{3}+(ac^{2}d^{2}x^{4}+a^{2}cd^{2})e\log\left(cx^{4}+a\right)-2\,(ac^{2}d^{2}x^{4}+a^{2}cd^{2})e\log\left(x^{2}e+d\right)+(c^{2}d^{3}x^{4}+acd^{3}-(acdx^{4}+a^{2}d)e^{2})\sqrt{ac}\arctan\left(\frac{\sqrt{ac}}{cx^{2}}\right)}{4\,(ac^{4}d^{4}x^{4}+a^{2}c^{3}d^{4}+(a^{3}c^{2}x^{4}+a^{4}c)e^{4}+2\,(a^{2}c^{3}d^{2}x^{4}+a^{3}c^{2}d^{2})e^{2})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*c^2*d^3*x^2 + 2*a^2*c*d*x^2*e^2 + 2*a^2*c*d^2*e + 2*a^3*e^3 + 2*
(a*c^2*d^2*x^4 + a^2*c*d^2)*e*log(c*x^4 + a) - 4*(a*c^2*d^2*x^4 + a^2*c*d^2
)*e*log(x^2*e + d) - (c^2*d^3*x^4 + a*c*d^3 - (a*c*d*x^4 + a^2*d)*e^2)*sqrt
(-a*c)*log((c*x^4 + 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)))/(a*c^4*d^4*x^4 + a^
2*c^3*d^4 + (a^3*c^2*x^4 + a^4*c)*e^4 + 2*(a^2*c^3*d^2*x^4 + a^3*c^2*d^2)*e
^2), -1/4*(a*c^2*d^3*x^2 + a^2*c*d*x^2*e^2 + a^2*c*d^2*e + a^3*e^3 + (a*c^2
*d^2*x^4 + a^2*c*d^2)*e*log(c*x^4 + a) - 2*(a*c^2*d^2*x^4 + a^2*c*d^2)*e*lo
g(x^2*e + d) + (c^2*d^3*x^4 + a*c*d^3 - (a*c*d*x^4 + a^2*d)*e^2)*sqrt(a*c)*
arctan(sqrt(a*c)/(c*x^2)))/(a*c^4*d^4*x^4 + a^2*c^3*d^4 + (a^3*c^2*x^4 + a^
4*c)*e^4 + 2*(a^2*c^3*d^2*x^4 + a^3*c^2*d^2)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.22, size = 220, normalized size = 1.42

$$-\frac{d^{2}e\log\left(cx^{4}+a\right)}{4\,(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4})}+\frac{d^{2}e^{2}\log\left(|x^{2}e+d|\right)}{2\,(c^{2}d^{4}e+2\,acd^{2}e^{3}+a^{2}e^{5})}+\frac{(cd^{3}-ade^{2})\arctan\left(\frac{cx^{2}}{\sqrt{ac}}\right)}{4\,(c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4})\sqrt{ac}}+\frac{c^{2}d^{2}x^{4}e-c^{2}d^{3}x^{2}-acdx^{2}e^{2}-a^{2}e^{3}}{4\,(c^{3}d^{4}+2\,ac^{2}d^{2}e^{2}+a^{2}ce^{4})(cx^{4}+a)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*d^2*e*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*d^2*e^2
*log(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c*d^3 - a
*d*e^2)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a
```

```
*c)) + 1/4*(c^2*d^2*x^4*e - c^2*d^3*x^2 - a*c*d*x^2*e^2 - a^2*e^3)/((c^3*d^
4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*(c*x^4 + a))
```

Mupad [B]
time = 1.52, size = 528, normalized size = 3.41

$$\frac{\ln\left(a^4\,e^8\,\sqrt{-a\,c}+c^4\,d^8\,\sqrt{-a\,c}+70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}+c^5\,d^8\,x^2+a^4\,c\,e^8\,x^2-36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}-36\,c^2\,d^6\,e^2\,{\left(-a\,c\right)}^{3/2}+70\,a^2\,c^3\,d^4\,e^4\,x^2+36\,a^3\,c^2\,d^2\,e^6\,x^2+36\,a\,c^4\,d^6\,e^2\,x^2\right)\,\left(c\,\left(\frac{d^3\,\sqrt{-a\,c}}{8}-\frac{a\,d^2\,e}{4}\right)-\frac{a\,d\,e^2\,\sqrt{-a\,c}}{8}\right)}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a\,c^3\,d^4}-\frac{\frac{d\,x^2}{4\,\left(c\,d^2+a\,e^2\right)}+\frac{a\,e}{4\,c\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}-\frac{\ln\left(c^5\,d^8\,x^2-c^4\,d^8\,\sqrt{-a\,c}-70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}-a^4\,e^8\,\sqrt{-a\,c}+a^4\,c\,e^8\,x^2+36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}+36\,c^2\,d^6\,e^2\,{\left(-a\,c\right)}^{3/2}+70\,a^2\,c^3\,d^4\,e^4\,x^2+36\,a^3\,c^2\,d^2\,e^6\,x^2+36\,a\,c^4\,d^6\,e^2\,x^2\right)\,\left(c\,\left(\frac{d^3\,\sqrt{-a\,c}}{8}+\frac{a\,d^2\,e}{4}\right)-\frac{a\,d\,e^2\,\sqrt{-a\,c}}{8}\right)}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a\,c^3\,d^4}+\frac{d^2\,e\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
+ c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*e^
2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4
*d^6*e^2*x^2)*(c*((d^3*(-a*c)^(1/2))/8 - (a*d^2*e)/4) - (a*d*e^2*(-a*c)^(1/
2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - ((d*x^2)/(4*(a*e^2 +
c*d^2)) + (a*e)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4
*d^8*(-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c*
e^8*x^2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^
2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(c*((d^3
*(-a*c)^(1/2))/8 + (a*d^2*e)/4) - (a*d*e^2*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a
^3*c*e^4 + 2*a^2*c^2*d^2*e^2) + (d^2*e*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^
4 + 2*a*c*d^2*e^2))
```

3.247 $\int \frac{x^3}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=149

$$\frac{-d+ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{e(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\,\sqrt{c}\,(cd^2+ae^2)^2} - \frac{de^2\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{de^2\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

```
[Out] 1/4*(e*x^2-d)/(a*e^2+c*d^2)/(c*x^4+a)-1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2
+1/4*d*e^2*ln(c*x^4+a)/(a*e^2+c*d^2)^2-1/4*e*(-a*e^2+c*d^2)*arctan(x^2*c^(1
/2)/a^(1/2))/(a*e^2+c*d^2)^2/a^(1/2)/c^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 148, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1266, 837, 815, 649, 211, 266}

$$-\frac{e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)(cd^2-ae^2)}{4\sqrt{a}\,\sqrt{c}\,(ae^2+cd^2)^2} + \frac{de^2\log(a+cx^4)}{4(ae^2+cd^2)^2} - \frac{de^2\log(d+ex^2)}{2(ae^2+cd^2)^2} - \frac{d-ex^2}{4(a+cx^4)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*(d - e*x^2)/((c*d^2 + a*e^2)*(a + c*x^4)) - (e*(c*d^2 - a*e^2)*ArcTan[
(Sqrt[c]*x^2)/Sqrt[a]])/(4*Sqrt[a]*Sqrt[c]*(c*d^2 + a*e^2)^2) - (d*e^2*Log[
d + e*x^2])/(2*(c*d^2 + a*e^2)^2) + (d*e^2*Log[a + c*x^4])/(4*(c*d^2 + a*e^
2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 837

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Dist[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Simp
[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f +
 a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[
c*d^2 + a*e^2, 0] && LtQ[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ
[2*m, 2*p])
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)(a+cx^2)^2}\,dx,x,x^2\right)\\
&= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{acde-ace^2x}{(d+ex)(a+cx^2)}\,dx,x,x^2\right)}{4ac(cd^2+ae^2)}\\
&= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(\frac{2acde^3}{(cd^2+ae^2)(d+ex)} - \frac{ace(-cd^2+ae^2+2cdex)}{(cd^2+ae^2)(a+cx^2)}\right)dx,x\right.}{4ac(cd^2+ae^2)}\\
&= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{de^2\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{e\text{Subst}\left(\int \frac{-cd^2+ae^2+2cdex}{a+cx^2}\,dx,x,x\right.}{4(cd^2+ae^2)^2}\\
&= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{de^2\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{(cde^2)\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx,x,x^2\right)}{2(cd^2+ae^2)^2}\\
&= -\frac{d-ex^2}{4(cd^2+ae^2)(a+cx^4)} - \frac{e(cd^2-ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4\sqrt{a}\sqrt{c}\,(cd^2+ae^2)^2} - \frac{de^2\log(d+ex^2)}{2(cd^2+ae^2)^2} +
\end{aligned}$$

Mathematica [A]

time = 0.10, size = 114, normalized size = 0.77

$$\frac{\frac{(cd^2+ae^2)(-d+ex^2)}{a+cx^4}+\frac{e(-cd^2+ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{c}}-2de^2\log\left(d+ex^2\right)+de^2\log\left(a+cx^4\right)}{4\left(cd^2+ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(-d + e*x^2))/(a + c*x^4) + (e*(-(c*d^2) + a*e^2)*ArcTan[
(Sqrt[c]*x^2)/Sqrt[a]])/(Sqrt[a]*Sqrt[c]) - 2*d*e^2*Log[d + e*x^2] + d*e^2*
Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.20, size = 132, normalized size = 0.89

method	result
default	$-\frac{d\,e^2\ln(e\,x^2+d)}{2(a\,e^2+c\,d^2)^2}+\frac{\frac{\left(\frac{1}{2}a\,e^3+\frac{1}{2}c\,d^2e\right)x^2-\frac{d\left(a\,e^2+c\,d^2\right)}{2}}{c\,x^4+a}+\frac{e\left(d e\ln\left(c\,x^4+a\right)+\frac{\left(a\,e^2-c\,d^2\right)\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}\right)}{2}}{2(a\,e^2+c\,d^2)^2}$
risch	$\frac{\frac{e\,x^2}{4a\,e^2+4c\,d^2}-\frac{d}{4\left(a\,e^2+c\,d^2\right)}}{c\,x^4+a}+\frac{e^2\ln\left(\left(\left(8a^3cd\,e^5-16a^2c^2d^3e^3+8c^3d^5ea+\sqrt{-ac\left(a\,e^2-c\,d^2\right)^2}\,a^2e^4-14\sqrt{-ac\left(a\,e^2-c\right.}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*d*e^2*ln(e*x^2+d)/(a*e^2+c*d^2)^2+1/2/(a*e^2+c*d^2)^2*(((1/2*a*e^3+1/2
*c*d^2*e)*x^2-1/2*d*(a*e^2+c*d^2))/(c*x^4+a)+1/2*e*(d*e*ln(c*x^4+a)+(a*e^2-
c*d^2)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))))
```

Maxima [A]
time = 0.51, size = 178, normalized size = 1.19

$$\frac{de^2\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{de^2\log\left(x^2e+d\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}-\frac{\left(cd^2e-ae^3\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)\sqrt{ac}}+\frac{x^2e-d}{4\left(\left(c^2d^2+ace^2\right)x^4+acd^2+a^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^2*lo
g(x^2*e + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/4*(c*d^2*e - a*e^3)*ar
```

```
ctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)) + 1/4
*(x^2*e - d)/((c^2*d^2 + a*c*e^2)*x^4 + a*c*d^2 + a^2*e^2)
```

Fricas [A]
time = 1.93, size = 475, normalized size = 3.19

$$\left[\frac{2\,ac^2d^2x^2e - 2\,ac^2d^3 + 2\,a^2cx^2e^3 - 2\,a^2cde^2 + 2\,(ac^2dx^4 + a^2cd)e^2\log\left(cx^4 + a\right) - 4\,(ac^2dx^4 + a^2cd)e^2\log\left(x^2e + d\right) - \sqrt{-ac}\,((acx^4 + a^2)e^3 - (c^2d^2x^4 + acd^2)e)\log\left(\frac{cx^4 - 2\,\sqrt{-ac}\,x^2 - a}{cx^4 + a}\right)}{8\,(ac^4d^4x^4 + a^2c^3d^4 + (a^3c^2x^4 + a^4c)e^4 + 2\,(a^2c^3d^2x^4 + a^3c^2d^2)e^2)}, \frac{ac^2d^2x^2e - ac^2d^3 + a^2cx^2e^3 - a^2cde^2 + (ac^2dx^4 + a^2cd)e^2\log\left(cx^4 + a\right) - 2\,(ac^2dx^4 + a^2cd)e^2\log\left(x^2e + d\right) - \sqrt{ac}\,((acx^4 + a^2)e^3 - (c^2d^2x^4 + acd^2)e)\arctan\left(\frac{\sqrt{ac}}{cx^2}\right)}{4\,(ac^4d^4x^4 + a^2c^3d^4 + (a^3c^2x^4 + a^4c)e^4 + 2\,(a^2c^3d^2x^4 + a^3c^2d^2)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*a*c^2*d^2*x^2*e - 2*a*c^2*d^3 + 2*a^2*c*x^2*e^3 - 2*a^2*c*d*e^2 + 2
*(a*c^2*d*x^4 + a^2*c*d)*e^2*log(c*x^4 + a) - 4*(a*c^2*d*x^4 + a^2*c*d)*e^2
*log(x^2*e + d) - sqrt(-a*c)*((a*c*x^4 + a^2)*e^3 - (c^2*d^2*x^4 + a*c*d^2)
*e)*log((c*x^4 - 2*sqrt(-a*c)*x^2 - a)/(c*x^4 + a)))/(a*c^4*d^4*x^4 + a^2*c
^3*d^4 + (a^3*c^2*x^4 + a^4*c)*e^4 + 2*(a^2*c^3*d^2*x^4 + a^3*c^2*d^2)*e^2)
, 1/4*(a*c^2*d^2*x^2*e - a*c^2*d^3 + a^2*c*x^2*e^3 - a^2*c*d*e^2 + (a*c^2*d
*x^4 + a^2*c*d)*e^2*log(c*x^4 + a) - 2*(a*c^2*d*x^4 + a^2*c*d)*e^2*log(x^2*
e + d) - sqrt(a*c)*((a*c*x^4 + a^2)*e^3 - (c^2*d^2*x^4 + a*c*d^2)*e)*arctan
(sqrt(a*c)/(c*x^2)))/(a*c^4*d^4*x^4 + a^2*c^3*d^4 + (a^3*c^2*x^4 + a^4*c)*e
^4 + 2*(a^2*c^3*d^2*x^4 + a^3*c^2*d^2)*e^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.15, size = 188, normalized size = 1.26

$$\frac{de^2\log\left(cx^4 + a\right)}{4\,(c^2d^4 + 2\,acd^2e^2 + a^2e^4)} - \frac{de^3\log\left(|x^2e + d|\right)}{2\,(c^2d^4e + 2\,acd^2e^3 + a^2e^5)} - \frac{(cd^2e - ae^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,(c^2d^4 + 2\,acd^2e^2 + a^2e^4)\sqrt{ac}} - \frac{cd^3 - (cd^2e + ae^3)x^2 + ade^2}{4\,(cx^4 + a)(cd^2 + ae^2)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*d*e^2*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) - 1/2*d*e^3*lo
g(abs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) - 1/4*(c*d^2*e - a*
e^3)*arctan(c*x^2/sqrt(a*c))/((c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(a*c)
) - 1/4*(c*d^3 - (c*d^2*e + a*e^3)*x^2 + a*d*e^2)/((c*x^4 + a)*(c*d^2 + a*e
^2)^2)
```

Mupad [B]
time = 1.41, size = 527, normalized size = 3.54

$$\frac{\ln\left(a^4\,e^8\,\sqrt{-a\,c}+c^4\,d^8\,\sqrt{-a\,c}+70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}+c^5\,d^8\,x^2+a^4\,c\,e^8\,x^2-36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}-36\,c^2\,d^6\,e^2\,{\left(-a\,c\right)}^{3/2}+70\,a^2\,c^3\,d^4\,e^4\,x^2+36\,a^3\,c^2\,d^2\,e^6\,x^2+36\,a\,c^4\,d^6\,e^2\,x^2\right)\,\left(a\,\left(\frac{e^3\,\sqrt{-a\,c}}{8}+\frac{c\,d\,e^2}{4}\right)-\frac{c\,d^2\,e\,\sqrt{-a\,c}}{8}\right)}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a\,c^3\,d^4}-\frac{\frac{d}{4\,\left(c\,d^2+a\,e^2\right)}-\frac{e\,x^2}{4\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}-\frac{\ln\left(c^5\,d^8\,x^2-c^4\,d^8\,\sqrt{-a\,c}-70\,d^4\,e^4\,{\left(-a\,c\right)}^{5/2}-a^4\,e^8\,\sqrt{-a\,c}+a^4\,c\,e^8\,x^2+36\,a^2\,d^2\,e^6\,{\left(-a\,c\right)}^{3/2}+36\,c^2\,d^6\,e^2\,{\left(-a\,c\right)}^{3/2}+70\,a^2\,c^3\,d^4\,e^4\,x^2+36\,a^3\,c^2\,d^2\,e^6\,x^2+36\,a\,c^4\,d^6\,e^2\,x^2\right)\,\left(a\,\left(\frac{e^3\,\sqrt{-a\,c}}{8}-\frac{c\,d\,e^2}{4}\right)-\frac{c\,d^2\,e\,\sqrt{-a\,c}}{8}\right)}{a^3\,c\,e^4+2\,a^2\,c^2\,d^2\,e^2+a\,c^3\,d^4}-\frac{d\,e^2\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(a^4*e^8*(-a*c)^(1/2) + c^4*d^8*(-a*c)^(1/2) + 70*d^4*e^4*(-a*c)^(5/2)
+ c^5*d^8*x^2 + a^4*c*e^8*x^2 - 36*a^2*d^2*e^6*(-a*c)^(3/2) - 36*c^2*d^6*e^
2*(-a*c)^(3/2) + 70*a^2*c^3*d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4
*d^6*e^2*x^2)*(a*((e^3*(-a*c)^(1/2))/8 + (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/
2))/8))/(a*c^3*d^4 + a^3*c*e^4 + 2*a^2*c^2*d^2*e^2) - (d/(4*(a*e^2 + c*d^2)
) - (e*x^2)/(4*(a*e^2 + c*d^2)))/(a + c*x^4) - (log(c^5*d^8*x^2 - c^4*d^8*(
-a*c)^(1/2) - 70*d^4*e^4*(-a*c)^(5/2) - a^4*e^8*(-a*c)^(1/2) + a^4*c*e^8*x^
2 + 36*a^2*d^2*e^6*(-a*c)^(3/2) + 36*c^2*d^6*e^2*(-a*c)^(3/2) + 70*a^2*c^3*
d^4*e^4*x^2 + 36*a^3*c^2*d^2*e^6*x^2 + 36*a*c^4*d^6*e^2*x^2)*(a*((e^3*(-a*c
)^(1/2))/8 - (c*d*e^2)/4) - (c*d^2*e*(-a*c)^(1/2))/8))/(a*c^3*d^4 + a^3*c*e
^4 + 2*a^2*c^2*d^2*e^2) - (d*e^2*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*
a*c*d^2*e^2))
```

3.248 $\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=151

$$\frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)}+\frac{\sqrt{c}\,d(cd^2+3ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^2+ae^2)^2}+\frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2}-\frac{e^3\log(a+cx^4)}{4(cd^2+ae^2)^2}$$

```
[Out] 1/4*(c*d*x^2+a*e)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^
2)^2-1/4*e^3*ln(c*x^4+a)/(a*e^2+c*d^2)^2+1/4*d*(3*a*e^2+c*d^2)*arctan(x^2*c
^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^2+c*d^2)^2
```

Rubi [A]
time = 0.12, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {1262, 755, 815, 649, 211, 266}

$$\frac{\sqrt{c}\,d\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)(3ae^2+cd^2)}{4a^{3/2}(ae^2+cd^2)^2}+\frac{ae+cdx^2}{4a(a+cx^4)(ae^2+cd^2)}-\frac{e^3\log(a+cx^4)}{4(ae^2+cd^2)^2}+\frac{e^3\log(d+ex^2)}{2(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*e + c*d*x^2)/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*a
*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)^2) + (e^3*L
og[d + e*x^2])/(2*(c*d^2 + a*e^2)^2) - (e^3*Log[a + c*x^4])/(4*(c*d^2 + a*e
^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 755

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(d + e*x)^(m + 1))*(a*e + c*d*x)*((a + c*x^2)^(p + 1)/(2*a*(p + 1)*(c*d^2
 + a*e^2))), x] + Dist[1/(2*a*(p + 1)*(c*d^2 + a*e^2)), Int[(d + e*x)^m*Sim
p[c*d^2*(2*p + 3) + a*e^2*(m + 2*p + 3) + c*e*d*(m + 2*p + 4)*x, x]*(a + c*
x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e, m}, x] && NeQ[c*d^2 + a*e^2, 0]
&& LtQ[p, -1] && IntQuadraticQ[a, 0, c, d, e, m, p, x]
```

Rule 815

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_) + (c_.)*(x_)^2),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a + c*x^2)), x],
x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[c*d^2 + a*e^2, 0] && IntegerQ[m]
```

Rule 1262

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex^2)(a+cx^4)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)(a+cx^2)^2}\,dx, x, x^2\right)\\
&= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \frac{-cd^2-2ae^2-cdex}{(d+ex)(a+cx^2)}\,dx, x, x^2\right)}{4a(cd^2+ae^2)}\\
&= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\text{Subst}\left(\int \left(-\frac{2ae^4}{(cd^2+ae^2)(d+ex)} - \frac{c(cd^3+3ade^2-2ae^3x)}{(cd^2+ae^2)(a+cx^2)}\right) dx,\right.}{4a(cd^2+ae^2)}\\
&= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} + \frac{c\text{Subst}\left(\int \frac{cd^3+3ade^2-2ae^3x}{a+cx^2}\,dx, x, x\right.}{4a(cd^2+ae^2)^2}\\
&= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^3\log(d+ex^2)}{2(cd^2+ae^2)^2} - \frac{(ce^3)\,\text{Subst}\left(\int \frac{x}{a+cx^2}\,dx, x, x^2\right)}{2(cd^2+ae^2)^2} +\\
&= \frac{ae+cdx^2}{4a(cd^2+ae^2)(a+cx^4)} + \frac{\sqrt{c}\,d(cd^2+3ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}(cd^2+ae^2)^2} + \frac{e^3\log(d+ex}{2(cd^2+ae^2)}
\end{aligned}$$

Mathematica [A]

time = 0.09, size = 117, normalized size = 0.77

$$\frac{\frac{(cd^2+ae^2)(ae+cdx^2)}{a(a+cx^4)}+\frac{\sqrt{c}\,d(cd^2+3ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{a^{3/2}}+2e^3\log\left(d+ex^2\right)-e^3\log\left(a+cx^4\right)}{4\left(cd^2+ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (((c*d^2 + a*e^2)*(a*e + c*d*x^2))/(a*(a + c*x^4)) + (Sqrt[c]*d*(c*d^2 + 3*
a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/a^(3/2) + 2*e^3*Log[d + e*x^2] - e^3*
Log[a + c*x^4])/(4*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.20, size = 146, normalized size = 0.97

method	result
default	$\frac{e^3\ln(e\,x^2+d)}{2(a\,e^2+c\,d^2)^2}+\frac{c\left(\frac{\frac{d(a\,e^2+c\,d^2)x^2}{2a}+\frac{e(a\,e^2+c\,d^2)}{2c}}{c\,x^4+a}+\frac{-\frac{a\,e^3\ln(c\,x^4+a)}{c}+\frac{(3d\,e^2a+c\,d^3)\arctan\left(\frac{c\,x^2}{\sqrt{ac}}\right)}{\sqrt{ac}}}{2a}\right)}{2(a\,e^2+c\,d^2)^2}$
risch	$\frac{\frac{cd\,x^2}{4a(a\,e^2+c\,d^2)}+\frac{e}{4a\,e^2+4c\,d^2}}{c\,x^4+a}-\frac{\ln\left(\left(\left(-18a^4cd\,e^7+30a^3c^2d^3e^5+18a^2c^3d^5e^3+2a\,c^4d^7e+15\sqrt{-ac\,d^2\left(3a\,e^2+c\,d^2\right)^2}\,a^2cd\,e^4-\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^3*ln(e*x^2+d)/(a*e^2+c*d^2)^2+1/2*c/(a*e^2+c*d^2)^2*((1/2*d*(a*e^2+c*
d^2)/a*x^2+1/2*e*(a*e^2+c*d^2)/c)/(c*x^4+a)+1/2/a*(-a*e^3/c*ln(c*x^4+a)+(3*
a*d*e^2+c*d^3)/(a*c)^(1/2)*arctan(c*x^2/(a*c)^(1/2))))
```

Maxima [A]
time = 0.51, size = 187, normalized size = 1.24

$$-\frac{e^3\log\left(cx^4+a\right)}{4\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{e^3\log\left(x^2e+d\right)}{2\left(c^2d^4+2\,acd^2e^2+a^2e^4\right)}+\frac{\left(c^2d^3+3\,acde^2\right)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}+\frac{cdx^2+ae}{4\left(a^2cd^2+\left(ac^2d^2+a^2ce^2\right)x^4+a^3e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*e^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^3*log(x
^2*e + d)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/4*(c^2*d^3 + 3*a*c*d*e^2)
```

```
*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)
) + 1/4*(c*d*x^2 + a*e)/(a^2*c*d^2 + (a*c^2*d^2 + a^2*c*e^2)*x^4 + a^3*e^2)
```

Fricas [A]
time = 4.45, size = 433, normalized size = 2.87

$$\left[\frac{2\,c^2d^3x^2+2\,acdx^2e^2+2\,acd^2e+2\,a^2e^3-2\,(acx^4+a^2)e^3\log\left(cx^4+a\right)+4\,(acx^4+a^2)e^3\log\left(x^2e+d\right)+(c^2d^3x^4+acd^3+3\,(acdx^4+a^2d)e^2)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4+2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)}{8\,(ac^3d^4x^4+a^2c^2d^4+(a^3cx^4+a^4)e^4+2\,(a^2c^2d^2x^4+a^3cd^2)e^2)}, \frac{c^2d^3x^2+acdx^2e^2+acd^2e+a^2e^3-(acx^4+a^2)e^3\log\left(cx^4+a\right)+2\,(acx^4+a^2)e^3\log\left(x^2e+d\right)-(c^2d^3x^4+acd^3+3\,(acdx^4+a^2d)e^2)\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)}{4\,(ac^3d^4x^4+a^2c^2d^4+(a^3cx^4+a^4)e^4+2\,(a^2c^2d^2x^4+a^3cd^2)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(2*c^2*d^3*x^2 + 2*a*c*d*x^2*e^2 + 2*a*c*d^2*e + 2*a^2*e^3 - 2*(a*c*x^
4 + a^2)*e^3*log(c*x^4 + a) + 4*(a*c*x^4 + a^2)*e^3*log(x^2*e + d) + (c^2*d
^3*x^4 + a*c*d^3 + 3*(a*c*d*x^4 + a^2*d)*e^2)*sqrt(-c/a)*log((c*x^4 + 2*a*x
^2*sqrt(-c/a) - a)/(c*x^4 + a)))/(a*c^3*d^4*x^4 + a^2*c^2*d^4 + (a^3*c*x^4
+ a^4)*e^4 + 2*(a^2*c^2*d^2*x^4 + a^3*c*d^2)*e^2), 1/4*(c^2*d^3*x^2 + a*c*d
*x^2*e^2 + a*c*d^2*e + a^2*e^3 - (a*c*x^4 + a^2)*e^3*log(c*x^4 + a) + 2*(a*
c*x^4 + a^2)*e^3*log(x^2*e + d) - (c^2*d^3*x^4 + a*c*d^3 + 3*(a*c*d*x^4 + a
^2*d)*e^2)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)))/(a*c^3*d^4*x^4 + a^2*c^2*
d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^4 + a^3*c*d^2)*e^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.77, size = 199, normalized size = 1.32

$$-\frac{e^3\log\left(cx^4+a\right)}{4\,(c^2d^4+2\,acd^2e^2+a^2e^4)}+\frac{e^4\log\left(|x^2e+d|\right)}{2\,(c^2d^4e+2\,acd^2e^3+a^2e^5)}+\frac{(c^2d^3+3\,acde^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,(ac^2d^4+2\,a^2cd^2e^2+a^3e^4)\sqrt{ac}}+\frac{acd^2e+(c^2d^3+acde^2)x^2+a^2e^3}{4\,(cx^4+a)(cd^2+ae^2)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*e^3*log(c*x^4 + a)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4) + 1/2*e^4*log(a
bs(x^2*e + d))/(c^2*d^4*e + 2*a*c*d^2*e^3 + a^2*e^5) + 1/4*(c^2*d^3 + 3*a*c
*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)*sq
rt(a*c)) + 1/4*(a*c*d^2*e + (c^2*d^3 + a*c*d*e^2)*x^2 + a^2*e^3)/((c*x^4 +
a)*(c*d^2 + a*e^2)^2*a)
```

Mupad [B]
time = 1.49, size = 649, normalized size = 4.30

$$\frac{\frac{e}{4\,\left(c\,d^2+a\,e^2\right)}+\frac{c\,d\,x^2}{4\,a\,\left(c\,d^2+a\,e^2\right)}}{c\,x^4+a}+\frac{e^3\,\ln\left(e\,x^2+d\right)}{2\,\left(a^2\,e^4+2\,a\,c\,d^2\,e^2+c^2\,d^4\right)}+\frac{\ln\left(36\,a^6\,e^{10}\,\sqrt{-a^3\,c}+36\,a^7\,c\,e^{10}\,x^2+a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+a^2\,c^6\,d^{10}\,x^2-81\,a^2\,d^2\,e^8\,{\left(-a^3\,c\right)}^{3/2}-22\,c^2\,d^6\,e^4\,{\left(-a^3\,c\right)}^{3/2}+8\,a^3\,c^5\,d^8\,e^2\,x^2+22\,a^4\,c^4\,d^6\,e^4\,x^2+60\,a^5\,c^3\,d^4\,e^6\,x^2+81\,a^6\,c^2\,d^2\,e^8\,x^2+8\,a^2\,c^4\,d^8\,e^2\,\sqrt{-a^3\,c}-60\,a\,c\,d^4\,e^6\,{\left(-a^3\,c\right)}^{3/2}\right)\,\left(c\,d^3\,\sqrt{-a^3\,c}-2\,a^3\,e^3+3\,a\,d\,e^2\,\sqrt{-a^3\,c}\right)}{8\,\left(a^5\,e^4+2\,a^4\,c\,d^2\,e^2+a^3\,c^2\,d^4\right)}-\frac{\ln\left(36\,a^7\,c\,e^{10}\,x^2-36\,a^6\,e^{10}\,\sqrt{-a^3\,c}-a\,c^5\,d^{10}\,\sqrt{-a^3\,c}+a^2\,c^6\,d^{10}\,x^2+81\,a^2\,d^2\,e^8\,{\left(-a^3\,c\right)}^{3/2}+22\,c^2\,d^6\,e^4\,{\left(-a^3\,c\right)}^{3/2}+8\,a^3\,c^5\,d^8\,e^2\,x^2+22\,a^4\,c^4\,d^6\,e^4\,x^2+60\,a^5\,c^3\,d^4\,e^6\,x^2+81\,a^6\,c^2\,d^2\,e^8\,x^2-8\,a^2\,c^4\,d^8\,e^2\,\sqrt{-a^3\,c}+60\,a\,c\,d^4\,e^6\,{\left(-a^3\,c\right)}^{3/2}\right)\,\left(2\,a^3\,e^3+c\,d^3\,\sqrt{-a^3\,c}+3\,a\,d\,e^2\,\sqrt{-a^3\,c}\right)}{8\,\left(a^5\,e^4+2\,a^4\,c\,d^2\,e^2+a^3\,c^2\,d^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (e/(4*(a*e^2 + c*d^2)) + (c*d*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) + (e^
3*log(d + e*x^2))/(2*(a^2*e^4 + c^2*d^4 + 2*a*c*d^2*e^2)) + (log(36*a^6*e^1
0*(-a^3*c)^(1/2) + 36*a^7*c*e^10*x^2 + a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c^6*
d^10*x^2 - 81*a^2*d^2*e^8*(-a^3*c)^(3/2) - 22*c^2*d^6*e^4*(-a^3*c)^(3/2) +
8*a^3*c^5*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 8
1*a^6*c^2*d^2*e^8*x^2 + 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) - 60*a*c*d^4*e^6*(
-a^3*c)^(3/2))*(c*d^3*(-a^3*c)^(1/2) - 2*a^3*e^3 + 3*a*d*e^2*(-a^3*c)^(1/2)
))/(8*(a^5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2)) - (log(36*a^7*c*e^10*x^2 -
 36*a^6*e^10*(-a^3*c)^(1/2) - a*c^5*d^10*(-a^3*c)^(1/2) + a^2*c^6*d^10*x^2
+ 81*a^2*d^2*e^8*(-a^3*c)^(3/2) + 22*c^2*d^6*e^4*(-a^3*c)^(3/2) + 8*a^3*c^5
*d^8*e^2*x^2 + 22*a^4*c^4*d^6*e^4*x^2 + 60*a^5*c^3*d^4*e^6*x^2 + 81*a^6*c^2
*d^2*e^8*x^2 - 8*a^2*c^4*d^8*e^2*(-a^3*c)^(1/2) + 60*a*c*d^4*e^6*(-a^3*c)^(
3/2))*(2*a^3*e^3 + c*d^3*(-a^3*c)^(1/2) + 3*a*d*e^2*(-a^3*c)^(1/2)))/(8*(a^
5*e^4 + a^3*c^2*d^4 + 2*a^4*c*d^2*e^2))
```

3.249 $\int \frac{1}{x(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=209

$$\frac{c(d-ex^2)}{4a\left(cd^2+ae^2\right)(a+cx^4)}-\frac{\sqrt{c}\,e^3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^2+ae^2\right)^2}-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^2+ae^2\right)}+\frac{\log(x)}{a^2d}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2}-\frac{cd(cd^2}{4}$$

```
[Out] 1/4*c*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+ln(x)/a^2/d-1/2*e^4*ln(e*x^2+d)/
d/(a*e^2+c*d^2)^2-1/4*c*d*(2*a*e^2+c*d^2)*ln(c*x^4+a)/a^2/(a*e^2+c*d^2)^2-1
/4*e*arctan(x^2*c^(1/2)/a^(1/2))*c^(1/2)/a^(3/2)/(a*e^2+c*d^2)-1/2*e^3*arct
an(x^2*c^(1/2)/a^(1/2))*c^(1/2)/(a*e^2+c*d^2)^2/a^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {1266, 908, 653, 211, 649, 266}

$$-\frac{\sqrt{c}\,e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(ae^2+cd^2\right)}-\frac{cd(2ae^2+cd^2)\log\left(a+cx^4\right)}{4a^2\left(ae^2+cd^2\right)^2}+\frac{\log(x)}{a^2d}-\frac{\sqrt{c}\,e^3\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(ae^2+cd^2\right)^2}+\frac{c(d-ex^2)}{4a\left(a+cx^4\right)\left(ae^2+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(ae^2+cd^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[c]*e^3*ArcTan[(Sq
rt[c]*x^2)/Sqrt[a]])/(2*Sqrt[a]*(c*d^2 + a*e^2)^2) - (Sqrt[c]*e*ArcTan[(Sqr
t[c]*x^2)/Sqrt[a]])/(4*a^(3/2)*(c*d^2 + a*e^2)) + Log[x]/(a^2*d) - (e^4*Log
[d + e*x^2])/(2*d*(c*d^2 + a*e^2)^2) - (c*d*(c*d^2 + 2*a*e^2)*Log[a + c*x^4
])/(4*a^2*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] && !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{x\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2dx}-\frac{e^5}{d\left(cd^2+ae^2\right)^2(d+ex)}-\frac{c(ae+cdx)}{a\left(cd^2+ae^2\right)\left(a+cx^2\right)^2}+\right.\right.\\ &= \frac{\log(x)}{a^2d}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2}+\frac{c\text{Subst}\left(\int \frac{-a^2e^3-cd\left(cd^2+2ae^2\right)x}{a+cx^2}\,dx, x, x^2\right)}{2a^2\left(cd^2+ae^2\right)^2}-\\ &= \frac{c\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{\log(x)}{a^2d}-\frac{e^4\log\left(d+ex^2\right)}{2d\left(cd^2+ae^2\right)^2}-\frac{\left(ce^3\right)\text{Subst}\left(\int \frac{1}{a+cx^2}\right.}{2\left(cd^2+ae^2\right)^2}\\ &= \frac{c\left(d-ex^2\right)}{4a\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{\sqrt{c}\,e^3\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2\sqrt{a}\,\left(cd^2+ae^2\right)^2}-\frac{\sqrt{c}\,e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{3/2}\left(cd^2+ae^2\right)}+\end{aligned}$$

Mathematica [A]
time = 0.12, size = 241, normalized size = 1.15

$$\frac{acd\left(cd^2+ae^2\right)\left(d-ex^2\right)+\sqrt{a}\,\sqrt{c}\,de\left(cd^2+3ae^2\right)\left(a+cx^4\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+\sqrt{a}\,\sqrt{c}\,de\left(cd^2+3ae^2\right)\left(a+cx^4\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)+4\left(cd^2+ae^2\right)^2\left(a+cx^4\right)\log(x)-2a^2e^4\left(a+cx^4\right)\log\left(d+ex^2\right)-cd^2\left(cd^2+2ae^2\right)\left(a+cx^4\right)\log\left(a+cx^4\right)}{4a^2d\left(cd^2+ae^2\right)^2\left(a+cx^4\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (a*c*d*(c*d^2 + a*e^2)*(d - e*x^2) + Sqrt[a]*Sqrt[c]*d*e*(c*d^2 + 3*a*e^2)*
(a + c*x^4)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + Sqrt[a]*Sqrt[c]*d*e*(
c*d^2 + 3*a*e^2)*(a + c*x^4)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)] + 4*(c
*d^2 + a*e^2)^2*(a + c*x^4)*Log[x] - 2*a^2*e^4*(a + c*x^4)*Log[d + e*x^2] -
 c*d^2*(c*d^2 + 2*a*e^2)*(a + c*x^4)*Log[a + c*x^4])/(4*a^2*d*(c*d^2 + a*e^
2)^2*(a + c*x^4))
```

Maple [A]
time = 0.20, size = 171, normalized size = 0.82

method	result
default	$-\frac{e^{4} \ln \left(e\,x^{2}+d\right)}{2 d \left(a\,e^{2}+c\,d^{2}\right)^{2}}-\frac{c \left(\frac{\left(\frac{1}{2} a^{2} e^{3}+\frac{1}{2} a\,d^{2} e c\right) x^{2}-\frac{a d \left(a\,e^{2}+c\,d^{2}\right)}{2}}{c\,x^{4}+a}+\frac{\left(4 a c d\,e^{2}+2 c^{2} d^{3}\right) \ln \left(c\,x^{4}+a\right)}{4 c}+\frac{\left(3 a^{2} e^{3}+a\,d^{2} e c\right) \arctan \left(\frac{c\,x^{2}}{\sqrt{a c}}\right)}{2 \sqrt{a c}}\right)}{2 \left(a\,e^{2}+c\,d^{2}\right)^{2} a^{2}}+$
risch	$\frac{-\frac{c e\,x^{2}}{4 a \left(a\,e^{2}+c\,d^{2}\right)}+\frac{c d}{4 a \left(a\,e^{2}+c\,d^{2}\right)}}{c\,x^{4}+a}+\frac{\ln \left(x \right)}{d\,a^{2}}-\frac{e^{4} \ln \left(e\,x^{2}+d\right)}{2 d \left(a^{2} e^{4}+2 a c\,d^{2} e^{2}+c^{2} d^{4}\right)}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^4*ln(e*x^2+d)/d/(a*e^2+c*d^2)^2-1/2*c/(a*e^2+c*d^2)^2/a^2*(((1/2*a^2
*e^3+1/2*a*d^2*e*c)*x^2-1/2*a*d*(a*e^2+c*d^2))/(c*x^4+a)+1/4*(4*a*c*d*e^2+2
*c^2*d^3)/c*ln(c*x^4+a)+1/2*(3*a^2*e^3+a*c*d^2*e)/(a*c)^(1/2)*arctan(c*x^2/
(a*c)^(1/2)))+ln(x)/d/a^2
```

Maxima [A]
time = 0.50, size = 220, normalized size = 1.05

$$-\frac{\left(c^{2} d^{3}+2 \, a c d e^{2}\right) \log \left(c x^{4}+a\right)}{4 \left(a^{2} c^{2} d^{4}+2 \, a^{3} c d^{2} e^{2}+a^{4} e^{4}\right)}-\frac{e^{4} \log \left(x^{2} e+d\right)}{2 \left(c^{2} d^{5}+2 \, a c d^{3} e^{2}+a^{2} d e^{4}\right)}-\frac{\left(c^{2} d^{2} e+3 \, a c e^{3}\right) \arctan \left(\frac{c x^{2}}{\sqrt{a c}}\right)}{4 \left(a c^{2} d^{4}+2 \, a^{2} c d^{2} e^{2}+a^{3} e^{4}\right) \sqrt{a c}}-\frac{c x^{2} e-c d}{4 \left(a^{2} c d^{2}+\left(a c^{2} d^{2}+a^{2} c e^{2}\right) x^{4}+a^{3} e^{2}\right)}+\frac{\log \left(x^{2}\right)}{2 \, a^{2} d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(c^2*d^3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4) - 1/2*e^4*log(x^2*e + d)/(c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4) -
 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2*a^2*c*
d^2*e^2 + a^3*e^4)*sqrt(a*c)) - 1/4*(c*x^2*e - c*d)/(a^2*c*d^2 + (a*c^2*d^2
 + a^2*c*e^2)*x^4 + a^3*e^2) + 1/2*log(x^2)/(a^2*d)
```

Fricas [A]
time = 72.45, size = 671, normalized size = 3.21

$$\left[-\frac{2 \, a c^{2} d^{3} x^{2} e-2 \, a c^{2} d^{4}+2 \, a^{2} c d x^{2} e^{3}-2 \, a^{2} c d^{2} e^{2}+4 \left(a^{2} c x^{4}+a^{3}\right) e^{4} \log \left(x^{2} e+d\right)-\left(3 \left(a^{2} c d x^{4}+a^{3} d\right) e^{3}+\left(a c^{2} d^{3} x^{4}+a^{2} c d^{3}\right) e\right) \sqrt{-\frac{c}{a}} \log \left(\frac{c x^{4}-2 \, a x^{2} \sqrt{-\frac{c}{a}}-a}{c x^{4}+a}\right)+2 \left(c^{3} d^{4} x^{4}+a c^{2} d^{4}+2 \left(a c^{2} d^{2} x^{4}+a^{2} c d^{2}\right) e^{2}\right) \log \left(c x^{4}+a\right)-8 \left(c^{3} d^{4} x^{4}+a c^{2} d^{4}+\left(a^{2} c x^{4}+a^{3}\right) e^{4}+2 \left(a c^{2} d^{2} x^{4}+a^{2} c d^{2}\right) e^{2}\right) \log \left(x\right)}{8 \left(a^{2} c^{3} d^{5} x^{4}+a^{3} c^{2} d^{5}+\left(a^{4} c d x^{4}+a^{5} d\right) e^{4}+2 \left(a^{3} c^{2} d^{3} x^{4}+a^{4} c d^{3}\right) e^{2}\right)}, -\frac{a c^{2} d^{3} x^{2} e-a c^{2} d^{4}+a^{2} c d x^{2} e^{3}-a^{2} c d^{2} e^{2}+2 \left(a^{2} c x^{4}+a^{3}\right) e^{4} \log \left(x^{2} e+d\right)-\left(3 \left(a^{2} c d x^{4}+a^{3} d\right) e^{3}+\left(a c^{2} d^{3} x^{4}+a^{2} c d^{3}\right) e\right) \sqrt{\frac{c}{a}} \arctan \left(\frac{a \sqrt{\frac{c}{a}}}{c x^{2}}\right)+\left(c^{3} d^{4} x^{4}+a c^{2} d^{4}+2 \left(a c^{2} d^{2} x^{4}+a^{2} c d^{2}\right) e^{2}\right) \log \left(c x^{4}+a\right)-4 \left(c^{3} d^{4} x^{4}+a c^{2} d^{4}+\left(a^{2} c x^{4}+a^{3}\right) e^{4}+2 \left(a c^{2} d^{2} x^{4}+a^{2} c d^{2}\right) e^{2}\right) \log \left(x\right)}{4 \left(a^{2} c^{3} d^{5} x^{4}+a^{3} c^{2} d^{5}+\left(a^{4} c d x^{4}+a^{5} d\right) e^{4}+2 \left(a^{3} c^{2} d^{3} x^{4}+a^{4} c d^{3}\right) e^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(2*a*c^2*d^3*x^2*e - 2*a*c^2*d^4 + 2*a^2*c*d*x^2*e^3 - 2*a^2*c*d^2*e^
2 + 4*(a^2*c*x^4 + a^3)*e^4*log(x^2*e + d) - (3*(a^2*c*d*x^4 + a^3*d)*e^3 +
 (a*c^2*d^3*x^4 + a^2*c*d^3)*e)*sqrt(-c/a)*log((c*x^4 - 2*a*x^2*sqrt(-c/a)
- a)/(c*x^4 + a)) + 2*(c^3*d^4*x^4 + a*c^2*d^4 + 2*(a*c^2*d^2*x^4 + a^2*c*d
^2)*e^2)*log(c*x^4 + a) - 8*(c^3*d^4*x^4 + a*c^2*d^4 + (a^2*c*x^4 + a^3)*e^
4 + 2*(a*c^2*d^2*x^4 + a^2*c*d^2)*e^2)*log(x))/(a^2*c^3*d^5*x^4 + a^3*c^2*d
^5 + (a^4*c*d*x^4 + a^5*d)*e^4 + 2*(a^3*c^2*d^3*x^4 + a^4*c*d^3)*e^2), -1/4
*(a*c^2*d^3*x^2*e - a*c^2*d^4 + a^2*c*d*x^2*e^3 - a^2*c*d^2*e^2 + 2*(a^2*c*
x^4 + a^3)*e^4*log(x^2*e + d) - (3*(a^2*c*d*x^4 + a^3*d)*e^3 + (a*c^2*d^3*x
^4 + a^2*c*d^3)*e)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + (c^3*d^4*x^4 + a
*c^2*d^4 + 2*(a*c^2*d^2*x^4 + a^2*c*d^2)*e^2)*log(c*x^4 + a) - 4*(c^3*d^4*x
^4 + a*c^2*d^4 + (a^2*c*x^4 + a^3)*e^4 + 2*(a*c^2*d^2*x^4 + a^2*c*d^2)*e^2)
*log(x))/(a^2*c^3*d^5*x^4 + a^3*c^2*d^5 + (a^4*c*d*x^4 + a^5*d)*e^4 + 2*(a^
3*c^2*d^3*x^4 + a^4*c*d^3)*e^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.19, size = 279, normalized size = 1.33

$$-\frac{(c^2d^3+2\,acde^2)\log\left(cx^4+a\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)}-\frac{e^5\log\left(|x^2e+d|\right)}{2\left(c^2d^5e+2\,acd^3e^3+a^2de^5\right)}-\frac{(c^2d^2e+3\,ace^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(ac^2d^4+2\,a^2cd^2e^2+a^3e^4\right)\sqrt{ac}}+\frac{c^3d^3x^4+2\,ac^2dx^4e^2-ac^2d^2x^2e+2\,ac^2d^3-a^2cx^2e^3+3\,a^2cde^2}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\left(cx^4+a\right)}+\frac{\log\left(x^2\right)}{2\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/4*(c^2*d^3 + 2*a*c*d*e^2)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2
+ a^4*e^4) - 1/2*e^5*log(abs(x^2*e + d))/(c^2*d^5*e + 2*a*c*d^3*e^3 + a^2*d
*e^5) - 1/4*(c^2*d^2*e + 3*a*c*e^3)*arctan(c*x^2/sqrt(a*c))/((a*c^2*d^4 + 2
*a^2*c*d^2*e^2 + a^3*e^4)*sqrt(a*c)) + 1/4*(c^3*d^3*x^4 + 2*a*c^2*d*x^4*e^2
 - a*c^2*d^2*x^2*e + 2*a*c^2*d^3 - a^2*c*x^2*e^3 + 3*a^2*c*d*e^2)/((a^2*c^2
*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*(c*x^4 + a)) + 1/2*log(x^2)/(a^2*d)
```

Mupad [B]
time = 2.58, size = 1082, normalized size = 5.18

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^2)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4)
 - (log(400*a^9*c^12*d^20*x^2 - 10481*d^4*e^16*(-a^5*c)^(7/2) - 1024*a^12*e
^20*(-a^5*c)^(3/2) + 1024*a^19*c^2*e^20*x^2 - 400*a^2*c^10*d^20*(-a^5*c)^(3
/2) + 5840*a^6*d^2*e^18*(-a^5*c)^(5/2) + 33710*c^6*d^14*e^6*(-a^5*c)^(5/2)
+ 4104*a^10*c^11*d^18*e^2*x^2 + 16689*a^11*c^10*d^16*e^4*x^2 + 33710*a^12*c
^9*d^14*e^6*x^2 + 33391*a^13*c^8*d^12*e^8*x^2 + 10748*a^14*c^7*d^10*e^10*x^
2 - 3585*a^15*c^6*d^8*e^12*x^2 + 3998*a^16*c^5*d^6*e^14*x^2 + 10481*a^17*c^
4*d^4*e^16*x^2 + 5840*a^18*c^3*d^2*e^18*x^2 + 10748*a^2*c^4*d^10*e^10*(-a^5
*c)^(5/2) - 3585*a^3*c^3*d^8*e^12*(-a^5*c)^(5/2) + 3998*a^4*c^2*d^6*e^14*(-
a^5*c)^(5/2) - 4104*a^3*c^9*d^18*e^2*(-a^5*c)^(3/2) - 16689*a^4*c^8*d^16*e^
4*(-a^5*c)^(3/2) + 33391*a*c^5*d^12*e^8*(-a^5*c)^(5/2))*(3*a*e^3*(-a^5*c)^(
1/2) + 2*a^2*c^2*d^3 + 4*a^3*c*d*e^2 + c*d^2*e*(-a^5*c)^(1/2)))/(8*(a^6*e^4
 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) + (log(1024*a^12*e^20*(-a^5*c)^(3/2) + 1
0481*d^4*e^16*(-a^5*c)^(7/2) + 400*a^9*c^12*d^20*x^2 + 1024*a^19*c^2*e^20*x
^2 + 400*a^2*c^10*d^20*(-a^5*c)^(3/2) - 5840*a^6*d^2*e^18*(-a^5*c)^(5/2) -
33710*c^6*d^14*e^6*(-a^5*c)^(5/2) + 4104*a^10*c^11*d^18*e^2*x^2 + 16689*a^1
1*c^10*d^16*e^4*x^2 + 33710*a^12*c^9*d^14*e^6*x^2 + 33391*a^13*c^8*d^12*e^8
*x^2 + 10748*a^14*c^7*d^10*e^10*x^2 - 3585*a^15*c^6*d^8*e^12*x^2 + 3998*a^1
6*c^5*d^6*e^14*x^2 + 10481*a^17*c^4*d^4*e^16*x^2 + 5840*a^18*c^3*d^2*e^18*x
^2 - 10748*a^2*c^4*d^10*e^10*(-a^5*c)^(5/2) + 3585*a^3*c^3*d^8*e^12*(-a^5*c
)^(5/2) - 3998*a^4*c^2*d^6*e^14*(-a^5*c)^(5/2) + 4104*a^3*c^9*d^18*e^2*(-a^
5*c)^(3/2) + 16689*a^4*c^8*d^16*e^4*(-a^5*c)^(3/2) - 33391*a*c^5*d^12*e^8*(
-a^5*c)^(5/2))*(3*a*e^3*(-a^5*c)^(1/2) - 2*a^2*c^2*d^3 - 4*a^3*c*d*e^2 + c*
d^2*e*(-a^5*c)^(1/2)))/(8*(a^6*e^4 + a^4*c^2*d^4 + 2*a^5*c*d^2*e^2)) - (e^4
*log(d + e*x^2))/(2*c^2*d^5 + 2*a^2*d*e^4 + 4*a*c*d^3*e^2) + log(x)/(a^2*d)
```

3.250 $\int \frac{1}{x^3(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=236

$$-\frac{1}{2a^2dx^2}-\frac{c(ae+cdx^2)}{4a^2(cd^2+ae^2)(a+cx^4)}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}(cd^2+ae^2)}-\frac{c^{3/2}d(cd^2+2ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}(cd^2+ae^2)^2}-\frac{e\log(x)}{a^2d^2}+\frac{e}{2}$$

```
[Out] -1/2/a^2/d/x^2-1/4*c*(c*d*x^2+a*e)/a^2/(a*e^2+c*d^2)/(c*x^4+a)-1/4*c^(3/2)*
d*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)-1/2*c^(3/2)*d*(2*a*e^2+
c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)^2-e*ln(x)/a^2/d^2+
1/2*e^5*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)^2+1/4*c*e*(2*a*e^2+c*d^2)*ln(c*x^4+a)
/a^2/(a*e^2+c*d^2)^2
```

Rubi [A]
time = 0.18, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1266, 908, 653, 211, 649, 266}

$$-\frac{c^{3/2}d\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)(2ae^2+cd^2)}{2a^{5/2}(ae^2+cd^2)^2}-\frac{c^{3/2}d\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}(ae^2+cd^2)}+\frac{ce(2ae^2+cd^2)\log(a+cx^4)}{4a^2(ae^2+cd^2)^2}-\frac{c(ae+cdx^2)}{4a^2(a+cx^4)(ae^2+cd^2)}-\frac{e\log(x)}{a^2d^2}-\frac{1}{2a^2dx^2}+\frac{e^5\log(d+ex^2)}{2d^2(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/2*1/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4)
) - (c^(3/2)*d*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*(c*d^2 + a*e^2)) -
 (c^(3/2)*d*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(2*a^(5/2)*(c*
d^2 + a*e^2)^2) - (e*Log[x])/(a^2*d^2) + (e^5*Log[d + e*x^2])/(2*d^2*(c*d^2
 + a*e^2)^2) + (c*e*(c*d^2 + 2*a*e^2)*Log[a + c*x^4])/(4*a^2*(c*d^2 + a*e^2
)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x^3\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+cx^2\right)^2}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2dx^2}-\frac{e}{a^2d^2x}+\frac{e^6}{d^2\left(cd^2+ae^2\right)^2(d+ex)}-\frac{c^2(d-ex}{a\left(cd^2+ae^2\right)(a}\right.\right.\\
&= -\frac{1}{2a^2dx^2}-\frac{e\log(x)}{a^2d^2}+\frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2}-\frac{c^2\text{Subst}\left(\int \frac{d-ex}{(a+cx^2)^2}\,dx,x,x^2\right)}{2a\left(cd^2+ae^2\right)}-\\
&= -\frac{1}{2a^2dx^2}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e\log(x)}{a^2d^2}+\frac{e^5\log\left(d+ex^2\right)}{2d^2\left(cd^2+ae^2\right)^2}-\frac{(c^2d}{}\\
&= -\frac{1}{2a^2dx^2}-\frac{c\left(ae+cdx^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{c^{3/2}d\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)}-\frac{c^{3/2}d(cd^2}{2a}
\end{aligned}$$

Mathematica [A]
time = 0.29, size = 248, normalized size = 1.05

$$\frac{1}{4}\left(-\frac{2}{a^2dx^2}-\frac{c\left(ae+cdx^2\right)}{a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{c^{3/2}d\left(3cd^2+5ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(cd^2+ae^2\right)^2}+\frac{c^{3/2}d\left(3cd^2+5ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(cd^2+ae^2\right)^2}-\frac{4e\log(x)}{a^2d^2}+\frac{2e^5\log\left(d+ex^2\right)}{\left(cd^3+ade^2\right)^2}+\frac{c\left(cd^2e+2ae^3\right)\log\left(a+cx^4\right)}{a^2\left(cd^2+ae^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-2/(a^2*d*x^2) - (c*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4)) + (
c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
5/2)*(c*d^2 + a*e^2)^2) + (c^(3/2)*d*(3*c*d^2 + 5*a*e^2)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (4*e*Log[x])/(a^2*d^2)
 + (2*e^5*Log[d + e*x^2])/(c*d^3 + a*d*e^2)^2 + (c*(c*d^2*e + 2*a*e^3)*Log[
a + c*x^4])/(a^2*(c*d^2 + a*e^2)^2))/4
```

Maple [A]

time = 0.20, size = 181, normalized size = 0.77

method	result
default	$\frac{e^{5} \ln \left(e \,x^{2}+d \right)}{2 d^{2} \left(a \,e^{2}+c \,d^{2}\right)^{2}}-\frac{c^{2} \left(\frac{\left(\frac{1}{2} d \,e^{2} a +\frac{1}{2} c \,d^{3}\right) x^{2}+\frac{a e \left(a \,e^{2}+c \,d^{2}\right)}{2 c}}{c \,x^{4}+a}+\frac{\left(-4 a \,e^{3}-2 c \,d^{2} e \right) \ln \left(c \,x^{4}+a \right)}{4 c}+\frac{\left(5 d \,e^{2} a +3 c \,d^{3}\right) \arctan \left(\frac{c \,x^{2}}{\sqrt{a c}}\right)}{2 \sqrt{a c}}\right)}{2 \left(a \,e^{2}+c \,d^{2}\right)^{2} a^{2}}-\frac{1}{2 a^{2} d \,x}$
risch	$\frac{-\frac{c \left(2 a \,e^{2}+3 c \,d^{2}\right) x^{4}}{4 d \,a^{2} \left(a \,e^{2}+c \,d^{2}\right)}-\frac{c e \,x^{2}}{4 a \left(a \,e^{2}+c \,d^{2}\right)}-\frac{1}{2 d a}}{x^{2} \left(c \,x^{4}+a \right)}-\frac{e \ln \left(x \right)}{a^{2} d^{2}}+\frac{e^{5} \ln \left(-e \,x^{2}-d \right)}{2 d^{2} \left(a^{2} e^{4}+2 a c \,d^{2} e^{2}+c^{2} d^{4}\right)}+\frac{\left(\sum_{\_R =\mathit{RootOf}\left(\left(e^{4} a^{7}+2 a^{6} c \,d^{2} e^{2}+a^{5} c^{2} d^{4}\right) \_Z^{2}+\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*e^5*ln(e*x^2+d)/d^2/(a*e^2+c*d^2)^2-1/2*c^2/(a*e^2+c*d^2)^2/a^2*(((1/2*
d*e^2*a+1/2*c*d^3)*x^2+1/2*a*e*(a*e^2+c*d^2)/c)/(c*x^4+a)+1/4*(-4*a*e^3-2*c
*d^2*e)/c*ln(c*x^4+a)+1/2*(5*a*d*e^2+3*c*d^3)/(a*c)^(1/2)*arctan(c*x^2/(a*c
)^(1/2)))-1/2/a^2/d/x^2-e*ln(x)/a^2/d^2
```

Maxima [A]

time = 0.51, size = 269, normalized size = 1.14

$$\frac{(c^2d^2e + 2\,ace^3)\log\left(cx^4 + a\right)}{4\,(a^2c^2d^4 + 2\,a^3cd^2e^2 + a^4e^4)} + \frac{e^5\log\left(x^2e + d\right)}{2\,(c^2d^6 + 2\,acd^4e^2 + a^2d^2e^4)} - \frac{(3\,c^3d^3 + 5\,ac^2de^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,(a^2c^2d^4 + 2\,a^3cd^2e^2 + a^4e^4)\sqrt{ac}} - \frac{acdx^2e + (3\,c^2d^2 + 2\,ace^2)x^4 + 2\,acd^2 + 2\,a^2e^2}{4\,((a^2c^2d^3 + a^3cde^2)x^6 + (a^3cd^3 + a^4de^2)x^2)} - \frac{e\log\left(x^2\right)}{2\,a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(c^2*d^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4) + 1/2*e^5*log(x^2*e + d)/(c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)
- 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^4 + 2
*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(a*c*d*x^2*e + (3*c^2*d^2 + 2*a*
c*e^2)*x^4 + 2*a*c*d^2 + 2*a^2*e^2)/((a^2*c^2*d^3 + a^3*c*d*e^2)*x^6 + (a^3
*c*d^3 + a^4*d*e^2)*x^2) - 1/2*e*log(x^2)/(a^2*d^2)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(204) = 408$.
time = 210.30, size = 863, normalized size = 3.66

$$\left[-\frac{6\,c^3d^5x^4+2\,ac^2d^4x^2e+4\,ac^2d^5+2\,a^2cd^2x^2e^3-4\,(a^2cx^6+a^3x^2)e^5\log\left(x^2e+d\right)-(3\,c^3d^5x^6+3\,ac^2d^5x^2+5\,(ac^2d^3x^6+a^2cd^3x^2)e^2)\sqrt{-\frac{c}{a}}\log\left(\frac{cx^4-2\,ax^2\sqrt{-\frac{c}{a}}-a}{cx^4+a}\right)+4\,(a^2cdx^4+a^3d)e^4+2\,(5\,ac^2d^3x^4+4\,a^2cd^3)e^2-2\,(2\,(ac^2d^2x^6+a^2cd^2x^2)e^3+(c^3d^4x^6+ac^2d^4x^2)e)\log\left(cx^4+a\right)+8\,((a^2cx^6+a^3x^2)e^5+2\,(ac^2d^2x^6+a^2cd^2x^2)e^3+(c^3d^4x^6+ac^2d^4x^2)e)\log\left(x\right)}{8\,(a^2c^3d^6x^6+a^3c^2d^6x^2+(a^4cd^2x^6+a^5d^2x^2)e^4+2\,(a^3c^2d^4x^6+a^4cd^4x^2)e^2)},\ -\frac{3\,c^3d^5x^4+ac^2d^4x^2e+2\,ac^2d^5+a^2cd^2x^2e^3-2\,(a^2cx^6+a^3x^2)e^5\log\left(x^2e+d\right)-(3\,c^3d^5x^6+3\,ac^2d^5x^2+5\,(ac^2d^3x^6+a^2cd^3x^2)e^2)\sqrt{\frac{c}{a}}\arctan\left(\frac{a\sqrt{\frac{c}{a}}}{cx^2}\right)+2\,(a^2cdx^4+a^3d)e^4+(5\,ac^2d^3x^4+4\,a^2cd^3)e^2-(2\,(ac^2d^2x^6+a^2cd^2x^2)e^3+(c^3d^4x^6+ac^2d^4x^2)e)\log\left(cx^4+a\right)+4\,((a^2cx^6+a^3x^2)e^5+2\,(ac^2d^2x^6+a^2cd^2x^2)e^3+(c^3d^4x^6+ac^2d^4x^2)e)\log\left(x\right)}{4\,(a^2c^3d^6x^6+a^3c^2d^6x^2+(a^4cd^2x^6+a^5d^2x^2)e^4+2\,(a^3c^2d^4x^6+a^4cd^4x^2)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(6*c^3*d^5*x^4 + 2*a*c^2*d^4*x^2*e + 4*a*c^2*d^5 + 2*a^2*c*d^2*x^2*e^
3 - 4*(a^2*c*x^6 + a^3*x^2)*e^5*log(x^2*e + d) - (3*c^3*d^5*x^6 + 3*a*c^2*d
^5*x^2 + 5*(a*c^2*d^3*x^6 + a^2*c*d^3*x^2)*e^2)*sqrt(-c/a)*log((c*x^4 - 2*a
*x^2*sqrt(-c/a) - a)/(c*x^4 + a)) + 4*(a^2*c*d*x^4 + a^3*d)*e^4 + 2*(5*a*c^
2*d^3*x^4 + 4*a^2*c*d^3)*e^2 - 2*(2*(a*c^2*d^2*x^6 + a^2*c*d^2*x^2)*e^3 + (
c^3*d^4*x^6 + a*c^2*d^4*x^2)*e)*log(c*x^4 + a) + 8*((a^2*c*x^6 + a^3*x^2)*e
^5 + 2*(a*c^2*d^2*x^6 + a^2*c*d^2*x^2)*e^3 + (c^3*d^4*x^6 + a*c^2*d^4*x^2)*
e)*log(x))/(a^2*c^3*d^6*x^6 + a^3*c^2*d^6*x^2 + (a^4*c*d^2*x^6 + a^5*d^2*x^
2)*e^4 + 2*(a^3*c^2*d^4*x^6 + a^4*c*d^4*x^2)*e^2), -1/4*(3*c^3*d^5*x^4 + a*
c^2*d^4*x^2*e + 2*a*c^2*d^5 + a^2*c*d^2*x^2*e^3 - 2*(a^2*c*x^6 + a^3*x^2)*e
^5*log(x^2*e + d) - (3*c^3*d^5*x^6 + 3*a*c^2*d^5*x^2 + 5*(a*c^2*d^3*x^6 + a
^2*c*d^3*x^2)*e^2)*sqrt(c/a)*arctan(a*sqrt(c/a)/(c*x^2)) + 2*(a^2*c*d*x^4 +
 a^3*d)*e^4 + (5*a*c^2*d^3*x^4 + 4*a^2*c*d^3)*e^2 - (2*(a*c^2*d^2*x^6 + a^2
*c*d^2*x^2)*e^3 + (c^3*d^4*x^6 + a*c^2*d^4*x^2)*e)*log(c*x^4 + a) + 4*((a^2
*c*x^6 + a^3*x^2)*e^5 + 2*(a*c^2*d^2*x^6 + a^2*c*d^2*x^2)*e^3 + (c^3*d^4*x^
6 + a*c^2*d^4*x^2)*e)*log(x))/(a^2*c^3*d^6*x^6 + a^3*c^2*d^6*x^2 + (a^4*c*d
^2*x^6 + a^5*d^2*x^2)*e^4 + 2*(a^3*c^2*d^4*x^6 + a^4*c*d^4*x^2)*e^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 3.10, size = 344, normalized size = 1.46

$$\frac{(c^2d^2e+2\,ace^3)\log\left(cx^4+a\right)}{4\,(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4)}+\frac{e^6\log\left(|x^2e+d|\right)}{2\,(c^2d^6e+2\,acd^4e^3+a^2d^2e^5)}-\frac{(3\,c^3d^3+5\,ac^2de^2)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4)\sqrt{ac}}-\frac{9\,c^3d^5x^4+15\,ac^2d^3x^4e^2-2\,a^2cx^6e^5+3\,ac^2d^4x^2e+6\,a^2cdx^4e^4+6\,ac^2d^5+3\,a^2cd^2x^2e^3+12\,a^2cd^3e^2-2\,a^3x^2e^5+6\,a^3de^4}{12\,(a^2c^2d^6+2\,a^3cd^4e^2+a^4d^2e^4)(cx^6+ax^2)}-\frac{e\log\left(x^2\right)}{2\,a^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(c^2*d^2*e + 2*a*c*e^3)*log(c*x^4 + a)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 +
 a^4*e^4) + 1/2*e^6*log(abs(x^2*e + d))/(c^2*d^6*e + 2*a*c*d^4*e^3 + a^2*d^
2*e^5) - 1/4*(3*c^3*d^3 + 5*a*c^2*d*e^2)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*
d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/12*(9*c^3*d^5*x^4 + 15*a*c^
2*d^3*x^4*e^2 - 2*a^2*c*x^6*e^5 + 3*a*c^2*d^4*x^2*e + 6*a^2*c*d*x^4*e^4 + 6
*a*c^2*d^5 + 3*a^2*c*d^2*x^2*e^3 + 12*a^2*c*d^3*e^2 - 2*a^3*x^2*e^5 + 6*a^3
*d*e^4)/((a^2*c^2*d^6 + 2*a^3*c*d^4*e^2 + a^4*d^2*e^4)*(c*x^6 + a*x^2)) - 1
/2*e*log(x^2)/(a^2*d^2)
```

Mupad [B]
time = 2.94, size = 1337, normalized size = 5.67

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] (log(81*a^10*c^16*d^24*x^2 + 1024*a^22*c^4*e^24*x^2 - 81*a^3*c^11*d^24*(-a^
5*c^3)^(3/2) + 1024*a^20*c^2*e^24*(-a^5*c^3)^(1/2) - 14496*a^6*d^8*e^16*(-a
^5*c^3)^(5/2) - 5120*a^14*d^2*e^22*(-a^5*c^3)^(3/2) + 11647*c^6*d^20*e^4*(-
a^5*c^3)^(5/2) + 1638*a^11*c^15*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4*x^2
 + 43524*a^13*c^13*d^18*e^6*x^2 + 97311*a^14*c^12*d^16*e^8*x^2 + 133334*a^1
5*c^11*d^14*e^10*x^2 + 103633*a^16*c^10*d^12*e^12*x^2 + 29456*a^17*c^9*d^10
*e^14*x^2 - 14496*a^18*c^8*d^8*e^16*x^2 - 7984*a^19*c^7*d^6*e^18*x^2 + 5888
*a^20*c^6*d^4*e^20*x^2 + 5120*a^21*c^5*d^2*e^22*x^2 + 43524*a*c^5*d^18*e^6*
(-a^5*c^3)^(5/2) + 29456*a^5*c*d^10*e^14*(-a^5*c^3)^(5/2) - 5888*a^13*c*d^4
*e^20*(-a^5*c^3)^(3/2) + 97311*a^2*c^4*d^16*e^8*(-a^5*c^3)^(5/2) + 133334*a
^3*c^3*d^14*e^10*(-a^5*c^3)^(5/2) + 103633*a^4*c^2*d^12*e^12*(-a^5*c^3)^(5/
2) - 1638*a^4*c^10*d^22*e^2*(-a^5*c^3)^(3/2) + 7984*a^12*c^2*d^6*e^18*(-a^5
*c^3)^(3/2))*(4*a^4*c*e^3 - 3*c*d^3*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d^2*e - 5*
a*d*e^2*(-a^5*c^3)^(1/2)))/(8*(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) -
(1/(2*a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(2*a*e^2 + 3*c*d^2))/
(4*a^2*d*(a*e^2 + c*d^2)))/(a*x^2 + c*x^6) + (log(81*a^10*c^16*d^24*x^2 + 1
024*a^22*c^4*e^24*x^2 + 81*a^3*c^11*d^24*(-a^5*c^3)^(3/2) - 1024*a^20*c^2*e
^24*(-a^5*c^3)^(1/2) + 14496*a^6*d^8*e^16*(-a^5*c^3)^(5/2) + 5120*a^14*d^2*
e^22*(-a^5*c^3)^(3/2) - 11647*c^6*d^20*e^4*(-a^5*c^3)^(5/2) + 1638*a^11*c^1
5*d^22*e^2*x^2 + 11647*a^12*c^14*d^20*e^4*x^2 + 43524*a^13*c^13*d^18*e^6*x^
2 + 97311*a^14*c^12*d^16*e^8*x^2 + 133334*a^15*c^11*d^14*e^10*x^2 + 103633*
a^16*c^10*d^12*e^12*x^2 + 29456*a^17*c^9*d^10*e^14*x^2 - 14496*a^18*c^8*d^8
*e^16*x^2 - 7984*a^19*c^7*d^6*e^18*x^2 + 5888*a^20*c^6*d^4*e^20*x^2 + 5120*
a^21*c^5*d^2*e^22*x^2 - 43524*a*c^5*d^18*e^6*(-a^5*c^3)^(5/2) - 29456*a^5*c
*d^10*e^14*(-a^5*c^3)^(5/2) + 5888*a^13*c*d^4*e^20*(-a^5*c^3)^(3/2) - 97311
*a^2*c^4*d^16*e^8*(-a^5*c^3)^(5/2) - 133334*a^3*c^3*d^14*e^10*(-a^5*c^3)^(5
/2) - 103633*a^4*c^2*d^12*e^12*(-a^5*c^3)^(5/2) + 1638*a^4*c^10*d^22*e^2*(-
a^5*c^3)^(3/2) - 7984*a^12*c^2*d^6*e^18*(-a^5*c^3)^(3/2))*(4*a^4*c*e^3 + 3*
```

```
c*d^3*(-a^5*c^3)^(1/2) + 2*a^3*c^2*d^2*e + 5*a*d*e^2*(-a^5*c^3)^(1/2)))/(8*
(a^7*e^4 + a^5*c^2*d^4 + 2*a^6*c*d^2*e^2)) + (e^5*log(d + e*x^2))/(2*c^2*d^
6 + 2*a^2*d^2*e^4 + 4*a*c*d^4*e^2) - (e*log(x))/(a^2*d^2)
```

3.251 $\int \frac{1}{x^5(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=265

$$-\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{c^2(d-ex^2)}{4a^2(cd^2+ae^2)(a+cx^4)}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}(cd^2+ae^2)}+\frac{c^{3/2}e(cd^2+2ae^2)\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{2a^{5/2}(cd^2+ae^2)^2}-(2$$

```
[Out] -1/4/a^2/d/x^4+1/2*e/a^2/d^2/x^2-1/4*c^2*(-e*x^2+d)/a^2/(a*e^2+c*d^2)/(c*x^
4+a)+1/4*c^(3/2)*e*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)+1/2*c^
(3/2)*e*(2*a*e^2+c*d^2)*arctan(x^2*c^(1/2)/a^(1/2))/a^(5/2)/(a*e^2+c*d^2)^2
-(-a*e^2+2*c*d^2)*ln(x)/a^3/d^3-1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2+1/4
*c^2*d*(3*a*e^2+2*c*d^2)*ln(c*x^4+a)/a^3/(a*e^2+c*d^2)^2
```

Rubi [A]

time = 0.22, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1266, 908, 653, 211, 649, 266}

$$\frac{c^{3/2}e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)(2ae^2+cd^2)}{2a^{5/2}(ae^2+cd^2)^2}+\frac{c^{3/2}e\text{ArcTan}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}(ae^2+cd^2)}+\frac{c^2d(3ae^2+2cd^2)\log(a+cx^4)}{4a^3(ae^2+cd^2)^2}-\frac{\log(x)(2cd^2-ae^2)}{a^3d^3}-\frac{c^2(d-ex^2)}{4a^2(a+cx^4)(ae^2+cd^2)}+\frac{e}{2a^2d^2x^2}-\frac{1}{4a^2dx^4}-\frac{e^6\log(d+ex^2)}{2d^3(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*1/(a^2*d*x^4) + e/(2*a^2*d^2*x^2) - (c^2*(d - e*x^2))/(4*a^2*(c*d^2 +
a*e^2)*(a + c*x^4)) + (c^(3/2)*e*ArcTan[(Sqrt[c]*x^2)/Sqrt[a]])/(4*a^(5/2)*
(c*d^2 + a*e^2)) + (c^(3/2)*e*(c*d^2 + 2*a*e^2)*ArcTan[(Sqrt[c]*x^2)/Sqrt[a
]])/(2*a^(5/2)*(c*d^2 + a*e^2)^2) - ((2*c*d^2 - a*e^2)*Log[x])/(a^3*d^3) -
(e^6*Log[d + e*x^2])/(2*d^3*(c*d^2 + a*e^2)^2) + (c^2*d*(2*c*d^2 + 3*a*e^2)
*Log[a + c*x^4])/(4*a^3*(c*d^2 + a*e^2)^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 649

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
```

```
}, x] &&  !NiceSqrtQ[(-a)*c]
```

Rule 653

```
Int[((d_) + (e_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Simp[((a*e
 - c*d*x)/(2*a*c*(p + 1)))*(a + c*x^2)^(p + 1), x] + Dist[d*((2*p + 3)/(2*a
*(p + 1))), Int[(a + c*x^2)^(p + 1), x], x] /; FreeQ[{a, c, d, e}, x] && Lt
Q[p, -1] && NeQ[p, -3/2]
```

Rule 908

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^
2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)^n*(a + c*x
^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ[c
*d^2 + a*e^2, 0] && IntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ
[m, 0] && ILtQ[n, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+cx^2\right)^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{a^2dx^3}-\frac{e}{a^2d^2x^2}+\frac{-2cd^2+ae^2}{a^3d^3x}-\frac{e^7}{d^3\left(cd^2+ae^2\right)^2(d+ex)}+\right.\right.$$

$$= -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{\left(2cd^2-ae^2\right)\log(x)}{a^3d^3}-\frac{e^6\log\left(d+ex^2\right)}{2d^3\left(cd^2+ae^2\right)^2}+\frac{c^2\text{Subst}\left(\right.}{}$$

$$= -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{c^2\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{\left(2cd^2-ae^2\right)\log(x)}{a^3d^3}-\frac{e}{2}$$

$$= -\frac{1}{4a^2dx^4}+\frac{e}{2a^2d^2x^2}-\frac{c^2\left(d-ex^2\right)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}+\frac{c^{3/2}e\tan^{-1}\left(\frac{\sqrt{c}\,x^2}{\sqrt{a}}\right)}{4a^{5/2}\left(cd^2+ae^2\right)}+$$

Mathematica [A]
time = 0.27, size = 278, normalized size = 1.05

$$\frac{1}{4}\left(-\frac{1}{a^2dx^4}+\frac{2e}{a^2d^2x^2}+\frac{c^2\left(-d+ex^2\right)}{a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{c^{3/2}e\left(3cd^2+5ae^2\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(cd^2+ae^2\right)^2}-\frac{c^{3/2}e\left(3cd^2+5ae^2\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{5/2}\left(cd^2+ae^2\right)^2}+\frac{4\left(-2cd^2+ae^2\right)\log(x)}{a^3d^3}-\frac{2e^6\log\left(d+ex^2\right)}{d^3\left(cd^2+ae^2\right)^2}+\frac{c^2\left(2cd^3+3ade^2\right)\log\left(a+cx^4\right)}{a^3\left(cd^2+ae^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-(1/(a^2*d*x^4)) + (2*e)/(a^2*d^2*x^2) + (c^2*(-d + e*x^2))/(a^2*(c*d^2 +
a*e^2)*(a + c*x^4)) - (c^(3/2)*e*(3*c*d^2 + 5*a*e^2)*ArcTan[1 - (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) - (c^(3/2)*e*(3*c*d^2 + 5*a*
e^2)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/2)*(c*d^2 + a*e^2)^2) +
 (4*(-2*c*d^2 + a*e^2)*Log[x])/(a^3*d^3) - (2*e^6*Log[d + e*x^2])/(d^3*(c*d
^2 + a*e^2)^2) + (c^2*(2*c*d^3 + 3*a*d*e^2)*Log[a + c*x^4])/(a^3*(c*d^2 + a
*e^2)^2))/4
```

Maple [A]
time = 0.21, size = 209, normalized size = 0.79

method	result
default	$-\frac{e^6 \ln(e x^2+d)}{2d^3(a e^2+c d^2)^2}+\frac{c^2\left(\frac{\left(\frac{1}{2}a^2e^3+\frac{1}{2}a\,d^2ec\right)x^2-\frac{ad\left(a e^2+c d^2\right)}{2}}{c x^4+a}+\frac{\left(6acd\,e^2+4c^2d^3\right)\ln\left(c x^4+a\right)}{4c}+\frac{\left(5a^2e^3+3a\,d^2ec\right)\arctan\left(\frac{c x^2}{\sqrt{ac}}\right)}{2\sqrt{ac}}\right)}{2(a e^2+c d^2)^2a^3}-$
risch	$\frac{\frac{ce\left(2a\,e^2+3c\,d^2\right)x^6}{4\left(a\,e^2+c\,d^2\right)a^2d^2}-\frac{c\left(a\,e^2+2c\,d^2\right)x^4}{4d\,a^2\left(a\,e^2+c\,d^2\right)}+\frac{e\,x^2}{2d^2a}-\frac{1}{4da}}{x^4(c\,x^4+a)}+\frac{\ln(x)e^2}{a^2d^3}-\frac{2\ln(x)c}{a^3d}-\frac{e^6\ln\left(e\,x^2+d\right)}{2d^3(a^2e^4+2ac\,d^2e^2+c^2d^4)}+\Big(\_R{=}\mathrm{RootOf}\big(\big(a^8e^4+2a^7$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^6*ln(e*x^2+d)/d^3/(a*e^2+c*d^2)^2+1/2*c^2/(a*e^2+c*d^2)^2/a^3*(((1/2
*a^2*e^3+1/2*a*d^2*e*c)*x^2-1/2*a*d*(a*e^2+c*d^2))/(c*x^4+a)+1/4*(6*a*c*d*e
^2+4*c^2*d^3)/c*ln(c*x^4+a)+1/2*(5*a^2*e^3+3*a*c*d^2*e)/(a*c)^(1/2)*arctan(
c*x^2/(a*c)^(1/2)))-1/4/a^2/d/x^4+(a*e^2-2*c*d^2)/a^3/d^3*ln(x)+1/2*e/a^2/d
^2/x^2
```

Maxima [A]
time = 0.52, size = 320, normalized size = 1.21

$$\frac{(2\,c^3d^3+3\,ac^2de^2)\log\left(cx^4+a\right)}{4\left(a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4\right)}-\frac{e^6\log\left(x^2e+d\right)}{2\left(c^2d^7+2\,acd^5e^2+a^2d^3e^4\right)}+\frac{(3\,c^3d^2e+5\,ac^2e^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\left(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4\right)\sqrt{ac}}+\frac{(3\,c^2d^2e+2\,ace^3)x^6-acd^3-(2\,c^2d^3+acde^2)x^4-a^2de^2+2\left(acd^2e+a^2e^3\right)x^2}{4\left((a^2c^2d^4+a^3cd^2e^2)x^8+(a^3cd^4+a^4d^2e^2)x^4\right)}-\frac{(2\,cd^2-ae^2)\log\left(x^2\right)}{2\,a^3d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4) - 1/2*e^6*log(x^2*e + d)/(c^2*d^7 + 2*a*c*d^5*e^2 + a^2*d^3*e
^4) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2*c^2*d^4
 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) + 1/4*((3*c^2*d^2*e + 2*a*c*e^3)*x
```

```
^6 - a*c*d^3 - (2*c^2*d^3 + a*c*d*e^2)*x^4 - a^2*d*e^2 + 2*(a*c*d^2*e + a^2
*e^3)*x^2)/((a^2*c^2*d^4 + a^3*c*d^2*e^2)*x^8 + (a^3*c*d^4 + a^4*d^2*e^2)*x
^4) - 1/2*(2*c*d^2 - a*e^2)*log(x^2)/(a^3*d^3)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.05, size = 350, normalized size = 1.32

$$\frac{(2\,c^3d^3+3\,ac^2de^2)\log\left(cx^4+a\right)}{4\,(a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4)}-\frac{e^7\log\left(|x^2e+d|\right)}{2\,(c^2d^7e+2\,acd^5e^3+a^2d^3e^5)}+\frac{(3\,c^3d^2e+5\,ac^2e^3)\arctan\left(\frac{cx^2}{\sqrt{ac}}\right)}{4\,(a^2c^2d^4+2\,a^3cd^2e^2+a^4e^4)\sqrt{ac}}-\frac{2\,c^4d^3x^4+3\,ac^3dx^4e^2-ac^3d^2x^2e+3\,ac^3d^3-a^2c^2x^2e^3+4\,a^2c^2de^2}{4\,(a^3c^2d^4+2\,a^4cd^2e^2+a^5e^4)(cx^4+a)}-\frac{(2\,cd^2-ae^2)\log\left(x^2\right)}{2\,a^3d^3}+\frac{6\,cd^2x^4-3\,ax^4e^2+2\,adx^2e-ad^2}{4\,a^3d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/4*(2*c^3*d^3 + 3*a*c^2*d*e^2)*log(c*x^4 + a)/(a^3*c^2*d^4 + 2*a^4*c*d^2*e
^2 + a^5*e^4) - 1/2*e^7*log(abs(x^2*e + d))/(c^2*d^7*e + 2*a*c*d^5*e^3 + a^
2*d^3*e^5) + 1/4*(3*c^3*d^2*e + 5*a*c^2*e^3)*arctan(c*x^2/sqrt(a*c))/((a^2*
c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4)*sqrt(a*c)) - 1/4*(2*c^4*d^3*x^4 + 3*a*
c^3*d*x^4*e^2 - a*c^3*d^2*x^2*e + 3*a*c^3*d^3 - a^2*c^2*x^2*e^3 + 4*a^2*c^2
*d*e^2)/((a^3*c^2*d^4 + 2*a^4*c*d^2*e^2 + a^5*e^4)*(c*x^4 + a)) - 1/2*(2*c*
d^2 - a*e^2)*log(x^2)/(a^3*d^3) + 1/4*(6*c*d^2*x^4 - 3*a*x^4*e^2 + 2*a*d*x^
2*e - a*d^2)/(a^3*d^3*x^4)
```

Mupad [**B**]
time = 3.48, size = 1545, normalized size = 5.83

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(x^5*(a + c*x^4)^2*(d + e*x^2)),x)

[Out] (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28*x^2 - 6400*a^3*c^13*d^28*(-a^7*c^3)^(3/2) + 1024*a^24*c^2*e^28*(-a^7*c^3)^(1/2) - 10688*a^6*d^8*e^20*(-a^7*c^3)^(5/2) - 2048*a^16*d^2*e^26*(-a^7*c^3)^(3/2) + 536959*c^6*d^20*e^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^2 + 200881*a^15*c^16*d^24*e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*a^17*c^14*d^20*e^8*x^2 + 465092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*d^16*e^12*x^2 + 52822*a^20*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x^2 - 27472*a^22*c^9*d^10*e^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^24*c^7*d^6*e^22*x^2 - 3584*a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^2 + 465092*a*c^5*d^18*e^10*(-a^7*c^3)^(5/2) - 27472*a^5*c*d^10*e^18*(-a^7*c^3)^(5/2) + 3584*a^15*c*d^4*e^24*(-a^7*c^3)^(3/2) + 256991*a^2*c^4*d^16*e^12*(-a^7*c^3)^(5/2) + 52822*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) - 37423*a^4*c^2*d^12*e^16*(-a^7*c^3)^(5/2) - 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) - 200881*a^5*c^11*d^24*e^4*(-a^7*c^3)^(3/2) - 413414*a^6*c^10*d^22*e^6*(-a^7*c^3)^(3/2) + 10288*a^14*c^2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 + 5*a*e^3*(-a^7*c^3)^(1/2) + 6*a^4*c^2*d*e^2 + 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(a^8*e^4 + a^6*c^2*d^4 + 2*a^7*c*d^2*e^2)) - (e^6*log(d + e*x^2))/(2*(c^2*d^7 + a^2*d^3*e^4 + 2*a*c*d^5*e^2)) - (1/(4*a*d) - (e*x^2)/(2*a*d^2) + (x^4*(2*c^2*d^2 + a*c*e^2))/(4*a^2*d*(a*e^2 + c*d^2)) - (c*e*x^6*(2*a*e^2 + 3*c*d^2))/(4*a^2*d^2*(a*e^2 + c*d^2)))/(a*x^4 + c*x^8) + (log(6400*a^13*c^18*d^28*x^2 + 1024*a^27*c^4*e^28*x^2 + 6400*a^3*c^13*d^28*(-a^7*c^3)^(3/2) - 1024*a^24*c^2*e^28*(-a^7*c^3)^(1/2) + 10688*a^6*d^8*e^20*(-a^7*c^3)^(5/2) + 2048*a^16*d^2*e^26*(-a^7*c^3)^(3/2) - 536959*c^6*d^20*e^8*(-a^7*c^3)^(5/2) + 54944*a^14*c^17*d^26*e^2*x^2 + 200881*a^15*c^16*d^24*e^4*x^2 + 413414*a^16*c^15*d^22*e^6*x^2 + 536959*a^17*c^14*d^20*e^8*x^2 + 465092*a^18*c^13*d^18*e^10*x^2 + 256991*a^19*c^12*d^16*e^12*x^2 + 52822*a^20*c^11*d^14*e^14*x^2 - 37423*a^21*c^10*d^12*e^16*x^2 - 27472*a^22*c^9*d^10*e^18*x^2 - 10688*a^23*c^8*d^8*e^20*x^2 - 10288*a^24*c^7*d^6*e^22*x^2 - 3584*a^25*c^6*d^4*e^24*x^2 + 2048*a^26*c^5*d^2*e^26*x^2 - 465092*a*c^5*d^18*e^10*(-a^7*c^3)^(5/2) + 27472*a^5*c*d^10*e^18*(-a^7*c^3)^(5/2) - 3584*a^15*c*d^4*e^24*(-a^7*c^3)^(3/2) - 256991*a^2*c^4*d^16*e^12*(-a^7*c^3)^(5/2) - 52822*a^3*c^3*d^14*e^14*(-a^7*c^3)^(5/2) + 37423*a^4*c^2*d^12*e^16*(-a^7*c^3)^(5/2) + 54944*a^4*c^12*d^26*e^2*(-a^7*c^3)^(3/2) + 200881*a^5*c^11*d^24*e^4*(-a^7*c^3)^(3/2) + 413414*a^6*c^10*d^22*e^6*(-a^7*c^3)^(3/2) - 10288*a^14*c^2*d^6*e^22*(-a^7*c^3)^(3/2))*(4*a^3*c^3*d^3 - 5*a*e^3*(-a^7*c^3)^(1/2) + 6*a^4*c^2*d*e^2 - 3*c*d^2*e*(-a^7*c^3)^(1/2)))/(8*(a^8*e^4 + a^6*c^2*d^4 + 2*a^7*c*d^2*e^2)) + (log(x)*(a*e^2 - 2*c*d^2))/(a^3*d^3)

3.252 $\int \frac{x^8}{(d+ex^2)(a+cx^4)^2} \, dx$

Optimal. Leaf size=712

$$\frac{dx}{4c(cd^2+ae^2)} - \frac{x^3(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2+ae^2)^2} + \frac{\sqrt[4]{a}\,d^2(\sqrt{c}\,d-\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right.}{2\sqrt{2}\,c^{3/4}(cd^2+ae^2)^2}$$

```
[Out] 1/4*d*x/c/(a*e^2+c*d^2)-1/4*x^3*(c*d*x^2+a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/1
6*a^(1/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-3*e*a^(1/2)+d*c^(1/2))/c^(
7/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*a^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))
*(-3*e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*a^(1/4)*d^2*arc
tan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d
^2)^2*2^(1/2)-1/4*a^(1/4)*d^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/
2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*a^(1/4)*d^2*ln(-a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+
c*d^2)^2*2^(1/2)-1/8*a^(1/4)*d^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c
^(1/2))*(e*a^(1/2)+d*c^(1/2))/c^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/32*a^(1/4)*
ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(3*e*a^(1/2)+d*c^(1/2))/
c^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*a^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(3*e*a^(1/2)+d*c^(1/2))/c^(7/4)/(a*e^2+c*d^2)*2^(1/2)+d^(
7/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2+c*d^2)^2/e^(1/2)
```

Rubi [A]
time = 0.44, antiderivative size = 712, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 11, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {1328, 1290, 1294, 1182, 1176, 631, 210, 1179, 642, 1302, 211}

$$\frac{\sqrt[4]{a}\,d^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(\sqrt{c}\,d-\sqrt{a}\,e)}{2\sqrt{2}\,c^{3/4}(ae^2+cd^2)^2} - \frac{\sqrt[4]{a}\,d^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(\sqrt{c}\,d-\sqrt{a}\,e)}{2\sqrt{2}\,c^{3/4}(ae^2+cd^2)^2} + \frac{\sqrt[4]{a}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(\sqrt{c}\,d-3\sqrt{a}\,e)}{8\sqrt{2}\,c^{7/4}(ae^2+cd^2)} - \frac{\sqrt[4]{a}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(\sqrt{c}\,d-3\sqrt{a}\,e)}{8\sqrt{2}\,c^{7/4}(ae^2+cd^2)} + \frac{d^{7/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(ae^2+cd^2)^2} + \frac{\sqrt[4]{a}\,d^2(\sqrt{a}\,e+\sqrt{c}\,d)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(ae^2+cd^2)^2} - \frac{\sqrt[4]{a}\,d^2(\sqrt{a}\,e+\sqrt{c}\,d)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,c^{3/4}(ae^2+cd^2)^2} + \frac{\sqrt[4]{a}\,(3\sqrt{a}\,e+\sqrt{c}\,d)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,c^{7/4}(ae^2+cd^2)} - \frac{\sqrt[4]{a}\,(3\sqrt{a}\,e+\sqrt{c}\,d)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,c^{7/4}(ae^2+cd^2)} - \frac{x^3(ae+cdx^2)}{4c(a+cx^4)(ae^2+cd^2)} + \frac{dx}{4c(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (d*x)/(4*c*(c*d^2 + a*e^2)) - (x^3*(a*e + c*d*x^2))/(4*c*(c*d^2 + a*e^2)*(a
 + c*x^4)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 + a*e^2)
^2) + (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) + (a^(1/4)*(Sqrt[c]*d - 3*Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)*(c*d^2
 + a*e^2)) - (a^(1/4)*d^2*(Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(2*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d
 - 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*c^(7/4)
*(c*d^2 + a*e^2)) + (a^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt
[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2)
 + (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)
```

```
*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4)*(c*d^2 + a*e^2)) - (a^(1/4)*d^2*(Sqr
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*c^(3/4)*(c*d^2 + a*e^2)^2) - (a^(1/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)
*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*c^(7/4
)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1290

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1294

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_
Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + c*x^4)^(p + 1)/(c*(m + 4*p + 3))),
x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(a + c*x^4)^p*(a*e*(m -
1) - c*d*(m + 4*p + 3)*x^2), x], x] /; FreeQ[{a, c, d, e, f, p}, x] && GtQ[
m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m
])
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 1328

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[(-a)*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)*(d - e*x^2
)*(a + c*x^4)^p, x], x] + Dist[d^2*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)
*((a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] &&
LtQ[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\int \frac{x^8}{(d+ex^2)\left(a+cx^4\right)^2}\,dx = -\frac{a\int \frac{x^4(d-ex^2)}{(a+cx^4)^2}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{x^4}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)(a+cx^4)} - \frac{\int \frac{x^2(-3ae-cdx^2)}{a+cx^4}\,dx}{4c\left(cd^2+ae^2\right)} + \frac{d^2\int \left(\frac{d^2}{(cd^2+ae^2)(d+ex^2)} - \frac{a(d\cdots}{(cd^2+ae\cdots}\right.}{cd^2+ae^2}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)(a+cx^4)} - \frac{(ad^2)\int \frac{d-ex^2}{a+cx^4}\,dx}{\left(cd^2+ae^2\right)^2} + \frac{d^4\int \frac{1}{d+ex^2}\,dx}{\left(cd^2+ae^2\right)^2} + \cdots$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)(a+cx^4)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} - \frac{\left(ad^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}\cdots\right.\right.}{2\cdots}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)(a+cx^4)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} - \frac{\left(ad^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}}\cdots\right.\right.}{\cdots}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)(a+cx^4)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} + \frac{\sqrt[4]{a}\,d^2\left(\sqrt{c}\,\cdots\right.}{\cdots}$$

$$= \frac{dx}{4c\left(cd^2+ae^2\right)} - \frac{x^3(ae+cdx^2)}{4c\left(cd^2+ae^2\right)(a+cx^4)} + \frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\left(cd^2+ae^2\right)^2} + \frac{a^{3/4}d^2\left(\frac{\sqrt{c}\,\cdots}{\sqrt{a}}\right.}{2\cdots}$$

<u>Mathematica</u> [A]
time = 0.20, size = 431, normalized size = 0.61

$$\frac{\frac{8a(cd^2+ae^2)x(d-ex^2)}{c(a+cx^4)} + \frac{32d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}} - \frac{2\sqrt{2}\sqrt[4]{a}\left(-5c^{3/2}d^3+7\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2+3a^{3/2}e^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{7/4}} + \frac{2\sqrt{2}\sqrt[4]{a}\left(-5c^{3/2}d^3+7\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2+3a^{3/2}e^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{c^{7/4}} + \frac{\sqrt{2}\sqrt[4]{a}\left(5c^{3/2}d^3+7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+3a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{7/4}} - \frac{\sqrt{2}\sqrt[4]{a}\left(5c^{3/2}d^3+7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+3a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{c^{7/4}}}{32\left(cd^2+ae^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*a*(c*d^2 + a*e^2)*x*(d - e*x^2))/(c*(a + c*x^4)) + (32*d^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[e] - (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[
a]*c*d^2*e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/c^(7/4) + (2*Sqrt[2]*a^(1/4)*(-5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*
e - a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/c^(7/4) + (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/c^(7/4) - (Sqrt[2]*a^(1/4)*(5*c^(3/2)*d^3 + 7*Sqrt[a]*c*d^2*e + a*Sqr
```

```
t[c]*d*e^2 + 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/c^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.20, size = 334, normalized size = 0.47

method	result
default	$\frac{d^4 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{(a e^2+c d^2)^2 \sqrt{de}} - \frac{a\left(\frac{\frac{e\left(a e^2+c d^2\right)x^3}{4c}-\frac{d\left(a e^2+c d^2\right)x}{4c}}{c x^4+a}+\frac{\frac{\left(d e^2 a+5c d^3\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\arctan\left(\sqrt{\right.\right.}{8a}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] d^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-a/(a*e^2+c*d^2)^2*(
(1/4*e*(a*e^2+c*d^2)/c*x^3-1/4*d*(a*e^2+c*d^2)/c*x)/(c*x^4+a)+1/4/c*(1/8*(a
*d*e^2+5*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^
(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4
)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-3*a*e^3-7*c*d^2*e)/c/(a/c)^
(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)*
x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/
(a/c)^(1/4)*x-1))))
```

Maxima [A]
time = 0.53, size = 487, normalized size = 0.68

$$\frac{d^{\frac{7}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{c^2d^4+2\,acd^2e^2+a^2e^4} - \frac{a\left(\frac{2\sqrt{2}\left(5c^{\frac{3}{2}}d^3-7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2-3a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}+\frac{2\sqrt{2}\left(5c^{\frac{3}{2}}d^3-7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2-3a^{\frac{3}{2}}e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\,\sqrt{c}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{c}}\,\sqrt{c}}+\frac{\sqrt{2}\left(5c^{\frac{3}{2}}d^3+7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+3a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}\,x^2+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(5c^{\frac{3}{2}}d^3+7\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+3a^{\frac{3}{2}}e^3\right)\log\left(\sqrt{c}\,x^2-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{32\left(c^3d^4+2\,ac^2d^2e^2+a^2ce^4\right)} - \frac{(acd^2e+a^2e^3)x^3-(acd^3+a^2de^2)x}{4\left(ac^3d^4+2\,a^2c^2d^2e^2+(c^4d^4+2\,ac^3d^2e^2+a^2c^2e^4)x^4+a^3ce^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/32*a*(2*sqrt(2)*(5*c^(3/2)*d^3 - 7*sqrt(a)*c*d^2*e + a*sqrt(c)*d*e^
2 - 3*a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(
2)*(5*c^(3/2)*d^3 - 7*sqrt(a)*c*d^2*e + a*sqrt(c)*d*e^2 - 3*a^(3/2)*e^3)*ar
ctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(
c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(5*c^(3/2)*d^3 + 7*s
```

```
qrt(a)*c*d^2*e + a*sqrt(c)*d*e^2 + 3*a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)
*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(5*c^(3/2)*d^3 +
7*sqrt(a)*c*d^2*e + a*sqrt(c)*d*e^2 + 3*a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt
(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(c^3*d^4 + 2*a*c^2*d^2*
e^2 + a^2*c*e^4) - 1/4*((a*c*d^2*e + a^2*e^3)*x^3 - (a*c*d^3 + a^2*d*e^2)*x
)/(a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + (c^4*d^4 + 2*a*c^3*d^2*e^2 + a^2*c^2*e^4
)*x^4 + a^3*c*e^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4622 vs. $2(534) = 1068$.
time = 10.10, size = 9273, normalized size = 13.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*a*c*d^2*x^3*e - 4*a*c*d^3*x + 4*a^2*x^3*e^3 - 4*a^2*d*x*e^2 - (c^
4*d^4*x^4 + a*c^3*d^4 + (a^2*c^2*x^4 + a^3*c)*e^4 + 2*(a*c^3*d^2*x^4 + a^2*
c^2*d^2)*e^2)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*
d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8
)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 274
8*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12
)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*
e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*
a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d
^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log(-625*c^4*d^8*x + 750*a*c^3*d
^6*x*e^2 + 1376*a^2*c^2*d^4*x*e^4 + 594*a^3*c*d^2*x*e^6 + 81*a^4*x*e^8 + (1
25*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 -
 9*a^4*c^2*d*e^8 + (7*c^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 +
 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6
*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6
+ 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a
*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*
d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 +
 a^8*c^7*e^16)))*sqrt((70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c
^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*
e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 +
2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e
^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^
10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 +
 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^
5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))) + (c^4*d^4*x^4 + a*c^3*d^4 +
 (a^2*c^2*x^4 + a^3*c)*e^4 + 2*(a*c^3*d^2*x^4 + a^2*c^2*d^2)*e^2)*sqrt((70*
a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^2 +
```

```
 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12
 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 238
3*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14
*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e
^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*
c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*
e^6 + a^4*c^3*e^8))*log(-625*c^4*d^8*x + 750*a*c^3*d^6*x*e^2 + 1376*a^2*c^2
*d^4*x*e^4 + 594*a^3*c*d^2*x*e^6 + 81*a^4*x*e^8 - (125*c^6*d^9 - 170*a*c^5*
d^7*e^2 - 244*a^2*c^4*d^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 + (7*c
^10*d^10*e + 31*a*c^9*d^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 1
9*a^4*c^6*d^2*e^9 + 3*a^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^
10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8
+ 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2
*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d
^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))*sqrt((
70*a*c^2*d^5*e + 44*a^2*c*d^3*e^3 + 6*a^3*d*e^5 + (c^7*d^8 + 4*a*c^6*d^6*e^
2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d
^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 +
2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c
^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^
8*e^8 + 56*a^5*c^10*d^6*e^10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a
^8*c^7*e^16)))/(c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d
^2*e^6 + a^4*c^3*e^8))) - (c^4*d^4*x^4 + a*c^3*d^4 + (a^2*c^2*x^4 + a^3*c)*
e^4 + 2*(a*c^3*d^2*x^4 + a^2*c^2*d^2)*e^2)*sqrt((70*a*c^2*d^5*e + 44*a^2*c*
d^3*e^3 + 6*a^3*d*e^5 - (c^7*d^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*
a^3*c^4*d^2*e^6 + a^4*c^3*e^8)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^
2 - 529*a^3*c^4*d^8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738
*a^6*c*d^2*e^10 + 81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13
*d^12*e^4 + 56*a^3*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c^10*d^6*e^
10 + 28*a^6*c^9*d^4*e^12 + 8*a^7*c^8*d^2*e^14 + a^8*c^7*e^16)))/(c^7*d^8 +
4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6 + a^4*c^3*e^8))*log
(-625*c^4*d^8*x + 750*a*c^3*d^6*x*e^2 + 1376*a^2*c^2*d^4*x*e^4 + 594*a^3*c*
d^2*x*e^6 + 81*a^4*x*e^8 + (125*c^6*d^9 - 170*a*c^5*d^7*e^2 - 244*a^2*c^4*d
^5*e^4 - 86*a^3*c^3*d^3*e^6 - 9*a^4*c^2*d*e^8 - (7*c^10*d^10*e + 31*a*c^9*d
^8*e^3 + 54*a^2*c^8*d^6*e^5 + 46*a^3*c^7*d^4*e^7 + 19*a^4*c^6*d^2*e^9 + 3*a
^5*c^5*e^11)*sqrt(-(625*a*c^6*d^12 - 1950*a^2*c^5*d^10*e^2 - 529*a^3*c^4*d^
8*e^4 + 2748*a^4*c^3*d^6*e^6 + 2383*a^5*c^2*d^4*e^8 + 738*a^6*c*d^2*e^10 +
81*a^7*e^12)/(c^15*d^16 + 8*a*c^14*d^14*e^2 + 28*a^2*c^13*d^12*e^4 + 56*a^3
*c^12*d^10*e^6 + 70*a^4*c^11*d^8*e^8 + 56*a^5*c...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.89, size = 581, normalized size = 0.82

$$\frac{d^{\frac{7}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{\left(5\,(ac^3)^{\frac{1}{4}}\,c^3d^3+(ac^3)^{\frac{1}{4}}\,ac^2de^2-7\,(ac^3)^{\frac{3}{4}}\,cd^2e-3\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,c^6d^4+2\,\sqrt{2}\,ac^5d^2e^2+\sqrt{2}\,a^2c^4e^4\right)}-\frac{\left(5\,(ac^3)^{\frac{1}{4}}\,c^3d^3+(ac^3)^{\frac{1}{4}}\,ac^2de^2-7\,(ac^3)^{\frac{3}{4}}\,cd^2e-3\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2\,x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,c^6d^4+2\,\sqrt{2}\,ac^5d^2e^2+\sqrt{2}\,a^2c^4e^4\right)}-\frac{\left(5\,(ac^3)^{\frac{1}{4}}\,c^3d^3+(ac^3)^{\frac{1}{4}}\,ac^2de^2+7\,(ac^3)^{\frac{3}{4}}\,cd^2e+3\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,c^6d^4+2\,\sqrt{2}\,ac^5d^2e^2+\sqrt{2}\,a^2c^4e^4\right)}+\frac{\left(5\,(ac^3)^{\frac{1}{4}}\,c^3d^3+(ac^3)^{\frac{1}{4}}\,ac^2de^2+7\,(ac^3)^{\frac{3}{4}}\,cd^2e+3\,(ac^3)^{\frac{3}{4}}\,ae^3\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,c^6d^4+2\,\sqrt{2}\,ac^5d^2e^2+\sqrt{2}\,a^2c^4e^4\right)}-\frac{ax^3e-adx}{4\,(cx^4+a)(c^2d^2+ace^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 - 7*(a*c^3)^
(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + sqrt(2
)*a^2*c^4*e^4) - 1/8*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^2*d*e^2 -
 7*(a*c^3)^(3/4)*c*d^2*e - 3*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e
^2 + sqrt(2)*a^2*c^4*e^4) - 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a
*c^2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 + sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2
+ sqrt(2)*a^2*c^4*e^4) + 1/16*(5*(a*c^3)^(1/4)*c^3*d^3 + (a*c^3)^(1/4)*a*c^
2*d*e^2 + 7*(a*c^3)^(3/4)*c*d^2*e + 3*(a*c^3)^(3/4)*a*e^3)*log(x^2 - sqrt(2
)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*c^6*d^4 + 2*sqrt(2)*a*c^5*d^2*e^2 + s
qrt(2)*a^2*c^4*e^4) - 1/4*(a*x^3*e - a*d*x)/((c*x^4 + a)*(c^2*d^2 + a*c*e^2
))
```

Mupad [B]
time = 2.86, size = 2500, normalized size = 3.51

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((a*d*x)/(4*c*(a*e^2 + c*d^2)) - (a*e*x^3)/(4*c*(a*e^2 + c*d^2)))/(a + c*x^
4) + atan(((((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11
*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9
 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6
*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a
^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 1843
20*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(25
6*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*
d^2*e^6)) - (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a
^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c
```

^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(1920*a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) + (x*(81*a^8*e^13 + 800*a^2*c^6*d^12*e + 612*a^7*c*d^2*e^11 + 832*a^3*c^5*d^10*e^3 + 913*a^4*c^4*d^8*e^5 + 1700*a^5*c^3*d^6*e^7 + 1606*a^6*c^2*d^4*e^9))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*1i - (((5120*a^2*c^8*d^13*e + 432*a^8*c^2*d*e^13 - 17232*a^3*c^7*d^11*e^3 - 37776*a^4*c^6*d^9*e^5 - 13600*a^5*c^5*d^7*e^7 + 4320*a^6*c^4*d^5*e^9 + 2928*a^7*c^3*d^3*e^11)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) - (((81920*a^5*c^9*d^8*e^8 - 73728*a^3*c^11*d^12*e^4 - 61440*a^4*c^10*d^10*e^6 - 20480*a^2*c^12*d^14*e^2 + 184320*a^6*c^8*d^6*e^10 + 122880*a^7*c^7*d^4*e^12 + 28672*a^8*c^6*d^2*e^14)/(256*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)) + (x*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2)*(65536*a^9*c^7*e^17 - 65536*a^2*c^14*d^14*e^3 - 327680*a^3*c^13*d^12*e^5 - 589824*a^4*c^12*d^10*e^7 - 327680*a^5*c^11*d^8*e^9 + 327680*a^6*c^10*d^6*e^11 + 589824*a^7*c^9*d^4*e^13 + 327680*a^8*c^8*d^2*e^15))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*

a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))^(1/2) - (x*(1920*
a^8*c^4*d*e^14 + 13184*a^2*c^10*d^13*e^2 + 16640*a^3*c^9*d^11*e^4 + 18560*a
^4*c^8*d^9*e^6 + 56832*a^5*c^7*d^7*e^8 + 60544*a^6*c^6*d^5*e^10 + 20736*a^7
*c^5*d^3*e^12))/(128*(c^7*d^8 + a^4*c^3*e^8 + 4*a*c^6*d^6*e^2 + 6*a^2*c^5*d
^4*e^4 + 4*a^3*c^4*d^2*e^6)))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c
^7)^(1/2) + 6*a^3*c^4*d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^
2*d^4*e^2*(-a*c^7)^(1/2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8
+ a^4*c^7*e^8 + 4*a*c^10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*e^6)))
^(1/2))*((25*c^3*d^6*(-a*c^7)^(1/2) - 9*a^3*e^6*(-a*c^7)^(1/2) + 6*a^3*c^4*
d*e^5 + 44*a^2*c^5*d^3*e^3 + 70*a*c^6*d^5*e - 39*a*c^2*d^4*e^2*(-a*c^7)^(1/
2) - 41*a^2*c*d^2*e^4*(-a*c^7)^(1/2))/(256*(c^11*d^8 + a^4*c^7*e^8 + 4*a*c^
10*d^6*e^2 + 6*a^2*c^9*d^4*e^4 + 4*a^3*c^8*d^2*...

3.253 $\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=687

$$-\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)}-\frac{d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}-\frac{d^2(\sqrt{c}\,d+\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,(cd^2+ae^2)^2}+\frac{(\sqrt{c}\,d-\ldots}{8\sqrt{\ldots}}$$

```
[Out] -1/4*x*(c*d*x^2+a*e)/c/(a*e^2+c*d^2)/(c*x^4+a)-1/16*arctan(-1+c^(1/4)*x*2^(
1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-
1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(
5/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*d^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^
2*c^(1/2))*(-e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)-1
/8*d^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-e*a^(1/2)+d*c^(1
/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*d^2*arctan(-1+c^(1/4)*x*2^
(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2
)+1/4*d^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)
/c^(1/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)
+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)+1
/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))
/a^(1/4)/c^(5/4)/(a*e^2+c*d^2)*2^(1/2)-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))*e^
(1/2)/(a*e^2+c*d^2)^2
```

Rubi [A]

time = 0.38, antiderivative size = 687, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1328, 1290, 1182, 1176, 631, 210, 1179, 642, 1302, 211}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(\sqrt{c}\,d-\sqrt{a}\,e)}{8\sqrt{2}\sqrt[4]{a}\,c^{5/4}(ae^2+cd^2)}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(\sqrt{c}\,d-\sqrt{a}\,e)}{8\sqrt{2}\sqrt[4]{a}\,c^{5/4}(ae^2+cd^2)}-\frac{d^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(\sqrt{a}\,e+\sqrt{c}\,d)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2}+\frac{d^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(\sqrt{a}\,e+\sqrt{c}\,d)}{2\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2}-\frac{d^{5/2}\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2}-\frac{(\sqrt{a}\,e+\sqrt{c}\,d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\sqrt[4]{a}\,c^{5/4}(ae^2+cd^2)}+\frac{(\sqrt{a}\,e+\sqrt{c}\,d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\sqrt[4]{a}\,c^{5/4}(ae^2+cd^2)}+\frac{d^2(\sqrt{c}\,d-\sqrt{a}\,e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2}-\frac{d^2(\sqrt{c}\,d-\sqrt{a}\,e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,(ae^2+cd^2)^2}-\frac{x(ae+cdx^2)}{4c(a+cx^4)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*(x*(a*e + c*d*x^2))/(c*(c*d^2 + a*e^2)*(a + c*x^4)) - (d^(5/2)*Sqrt[e]
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (d^2*(Sqrt[c]*d + Sqrt[a]
*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4)*(c*
d^2 + a*e^2)^2) + ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a
^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d + Sq
rt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(1/4)*c^(1/4
)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)
*x)/a^(1/4)])/(8*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2)) + (d^2*(Sqrt[c]*d
 - Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sq
rt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2) - ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4
```

```
)*(c*d^2 + a*e^2)) - (d^2*(Sqrt[c]*d - Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(
1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(1/4)*c^(1/4)*(c*d^2 + a*e^2)^2
) + ((Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt
[c]*x^2])/(16*Sqrt[2]*a^(1/4)*c^(5/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1290

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_.), x
_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + c*x^4)^(p + 1)*((a*e - c*d*x^2)/(4*a*
c*(p + 1))), x] - Dist[f^2/(4*a*c*(p + 1)), Int[(f*x)^(m - 2)*(a + c*x^4)^(
p + 1)*(a*e*(m - 1) - c*d*(4*p + 4 + m + 1)*x^2), x], x] /; FreeQ[{a, c, d,
 e, f}, x] && LtQ[p, -1] && GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || I
ntegerQ[m])
```

Rule 1302

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (c_.)*(x_)^4),
 x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a + c*x^4)), x],
x] /; FreeQ[{a, c, d, e, f, m}, x] && IntegerQ[q] && IntegerQ[m]
```

Rule 1328

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[(-a)*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)*(d - e*x^2
)*(a + c*x^4)^p, x], x] + Dist[d^2*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)
*((a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] &&
LtQ[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\int \frac{x^6}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{a\int \frac{x^2(d-ex^2)}{(a+cx^4)^2}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{x^2}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{\int \frac{-ae+cdx^2}{a+cx^4}\,dx}{4c(cd^2+ae^2)} + \frac{d^2\int \left(-\frac{de}{(cd^2+ae^2)(d+ex^2)} + \frac{ae+c\cdots}{(cd^2+ae^2)\cdots}\right.}{cd^2+ae^2}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} + \frac{d^2\int \frac{ae+cdx^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2} - \frac{(d^3e)\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2} - \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\cdots}{8c(c\cdots}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\left(d^2\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\right)\int \sqrt{\cdots}}{2(cd^2+ae^2\cdots}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{(\sqrt{c}\,d+\sqrt{a}\,e)\log\left(\sqrt{c}\cdots\right.}{16\sqrt{2}\,\sqrt[4]{a}\cdots}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\left(d-\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(1\cdots\right.}{8\sqrt{2}\,\sqrt[4]{a}\,c^{3/4}(cd^2\cdots}$$

$$= -\frac{x(ae+cdx^2)}{4c(cd^2+ae^2)(a+cx^4)} - \frac{d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\,d^2\left(d+\frac{\sqrt{a}\,e}{\sqrt{c}}\right)\tan\cdots}{2\sqrt{2}\,\sqrt[4]{a}\,(cd\cdots}$$

Mathematica [A]
time = 0.25, size = 428, normalized size = 0.62

$$-\frac{\frac{8(cd^2+ae^2)(aex+cdx^3)}{c(a+cx^4)} + 32d^{5/2}\sqrt{e}\,\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) + \frac{2\sqrt{2}\left(3c^{3/2}d^3+5\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2+a^{3/2}e^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,c^{5/4}} - \frac{2\sqrt{2}\left(3c^{3/2}d^3+5\sqrt{a}\,cd^2e-a\sqrt{c}\,de^2+a^{3/2}e^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{\sqrt[4]{a}\,c^{5/4}} + \frac{\sqrt{2}\left(-3c^{3/2}d^3+5\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{a}\,c^{5/4}} - \frac{\sqrt{2}\left(-3c^{3/2}d^3+5\sqrt{a}\,cd^2e+a\sqrt{c}\,de^2+a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{\sqrt[4]{a}\,c^{5/4}}}{32(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/32*((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(c*(a + c*x^4)) + 32*d^(5/2)*S
qrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]] + (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*
c*d^2*e - a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(
1/4)])/(a^(1/4)*c^(5/4)) - (2*Sqrt[2]*(3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e -
a*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^
(1/4)*c^(5/4)) + (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[c]*d
```

```
*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(a^(1/4)*c^(5/4)) - (Sqrt[2]*(-3*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + a*Sqrt[
c]*d*e^2 + a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x
^2])/(a^(1/4)*c^(5/4)))/(c*d^2 + a*e^2)^2
```

Maple [A]
time = 0.21, size = 339, normalized size = 0.49

method	result
default	$-\frac{d^{3} e \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{\left(a e^{2}+c d^{2}\right)^{2} \sqrt{d e}}+\frac{\frac{\left(-\frac{1}{4} d e^{2} a-\frac{1}{4} c d^{3}\right) x^{3}-\frac{a e \left(a e^{2}+c d^{2}\right) x}{4 c}}{c x^{4}+a}+\frac{\frac{\left(a^{2} e^{3}+5 a d^{2} e c\right) \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \mathrm{arc}\cdots\right.}{8 a}}{\cdots}}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d^3*e/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/(a*e^2+c*d^2)^
2*(((-1/4*d*e^2*a-1/4*c*d^3)*x^3-1/4*a*e*(a*e^2+c*d^2)/c*x)/(c*x^4+a)+1/4/c
*(1/8*(a^2*e^3+5*a*c*d^2*e)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^
(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2
)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-a*c*d*e^2+3*c^2
*d^3)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^
2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*a
rctan(2^(1/2)/(a/c)^(1/4)*x-1))))
```

Maxima [A]
time = 0.51, size = 462, normalized size = 0.67

$$-\frac{d^{\frac{5}{2}} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\frac{1}{2}}}{c^{2} d^{4}+2 a c d^{2} e^{2}+a^{2} e^{4}}-\frac{c d x^{3}+a x e}{4 \left(a c^{2} d^{2}+\left(c^{3} d^{2}+a c^{2} e^{2}\right) x^{4}+a^{2} c e^{2}\right)}+\frac{\frac{2 \sqrt{2} \left(3 \sqrt{a} c^{2} d^{3}+5 a c^{\frac{3}{2}} d^{2} e-a^{\frac{3}{2}} c d e^{2}+a^{2} \sqrt{c} e^{3}\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{c} x+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{2 \sqrt{2} \left(3 \sqrt{a} c^{2} d^{3}+5 a c^{\frac{3}{2}} d^{2} e-a^{\frac{3}{2}} c d e^{2}+a^{2} \sqrt{c} e^{3}\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{c} x-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}-\frac{\sqrt{2} \left(3 \sqrt{a} c^{2} d^{3}-5 a c^{\frac{3}{2}} d^{2} e-a^{\frac{3}{2}} c d e^{2}-a^{2} \sqrt{c} e^{3}\right) \log\left(\sqrt{c} x^{2}+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}+\frac{\sqrt{2} \left(3 \sqrt{a} c^{2} d^{3}-5 a c^{\frac{3}{2}} d^{2} e-a^{\frac{3}{2}} c d e^{2}-a^{2} \sqrt{c} e^{3}\right) \log\left(\sqrt{c} x^{2}-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}}{32 \left(c^{3} d^{4}+2 a c^{2} d^{2} e^{2}+a^{2} c e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/4*(c*d*x^3 + a*x*e)/(a*c^2*d^2 + (c^3*d^2 + a*c^2*e^2)*x^4 + a^2*c*
e^2) + 1/32*(2*sqrt(2)*(3*sqrt(a)*c^2*d^3 + 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d
*e^2 + a^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c
^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*
sqrt(2)*(3*sqrt(a)*c^2*d^3 + 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e^2 + a^2*sqrt
(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sq
rt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(3*sqrt(a
```

```
)*c^2*d^3 - 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e^2 - a^2*sqrt(c)*e^3)*log(sqrt
(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*
(3*sqrt(a)*c^2*d^3 - 5*a*c^(3/2)*d^2*e - a^(3/2)*c*d*e^2 - a^2*sqrt(c)*e^3)
*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/
(c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4606 vs. 2(512) = 1024.
time = 7.14, size = 9239, normalized size = 13.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*c^2*d^3*x^3 + 4*a*c*d*x^3*e^2 + 4*a*c*d^2*x*e + 4*a^2*x*e^3 + (c^
4*d^4*x^4 + a*c^3*d^4 + (a^2*c^2*x^4 + a^3*c)*e^4 + 2*(a*c^3*d^2*x^4 + a^2*
c^2*d^2)*e^2)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8
+ 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sq
rt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d
^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 +
 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5
*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^
14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3
*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-81*c^4*d^8*x + 270*a*c^3*d^6*x*e^2 + 112*
a^2*c^2*d^4*x*e^4 + 18*a^3*c*d^2*x*e^6 + a^4*x*e^8 + (45*a*c^5*d^8*e - 146*
a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 - (3*
a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6 -
a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 +
 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c
*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12
*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28
*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^2*d^5*
e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^
4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^1
0*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 1
8*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^
11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^
10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 +
4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))) -
(c^4*d^4*x^4 + a*c^3*d^4 + (a^2*c^2*x^4 + a^3*c)*e^4 + 2*(a*c^3*d^2*x^4 + a
^2*c^2*d^2)*e^2)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d
^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)
*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^
3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^1
```

```
6 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*
a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2
*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*
a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-81*c^4*d^8*x + 270*a*c^3*d^6*x*e^2 + 1
12*a^2*c^2*d^4*x*e^4 + 18*a^3*c*d^2*x*e^6 + a^4*x*e^8 - (45*a*c^5*d^8*e - 1
46*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9 -
(3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*e^6
 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^
2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^
5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d
^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 +
 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^2*d
^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 + (c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4
*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*
d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8
+ 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3
*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6
*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8
 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8)))
 + (c^4*d^4*x^4 + a*c^3*d^4 + (a^2*c^2*x^4 + a^3*c)*e^4 + 2*(a*c^3*d^2*x^4
+ a^2*c^2*d^2)*e^2)*sqrt(-(30*c^2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^
6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 + 4*a^3*c^3*d^2*e^6 + a^4*c^2*e
^8)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3
*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*
d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^11*d^12*e^4 + 56*a^4*c^10*d^10*e^6 +
70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^10 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*
d^2*e^14 + a^9*c^5*e^16)))/(c^6*d^8 + 4*a*c^5*d^6*e^2 + 6*a^2*c^4*d^4*e^4 +
 4*a^3*c^3*d^2*e^6 + a^4*c^2*e^8))*log(-81*c^4*d^8*x + 270*a*c^3*d^6*x*e^2
+ 112*a^2*c^2*d^4*x*e^4 + 18*a^3*c*d^2*x*e^6 + a^4*x*e^8 + (45*a*c^5*d^8*e
- 146*a^2*c^4*d^6*e^3 - 76*a^3*c^3*d^4*e^5 - 14*a^4*c^2*d^2*e^7 - a^5*c*e^9
 + (3*a*c^9*d^11 + 11*a^2*c^8*d^9*e^2 + 14*a^3*c^7*d^7*e^4 + 6*a^4*c^6*d^5*
e^6 - a^5*c^5*d^3*e^8 - a^6*c^4*d*e^10)*sqrt(-(81*c^6*d^12 - 558*a*c^5*d^10
*e^2 + 799*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 143*a^4*c^2*d^4*e^8 + 18
*a^5*c*d^2*e^10 + a^6*e^12)/(a*c^13*d^16 + 8*a^2*c^12*d^14*e^2 + 28*a^3*c^1
1*d^12*e^4 + 56*a^4*c^10*d^10*e^6 + 70*a^5*c^9*d^8*e^8 + 56*a^6*c^8*d^6*e^1
0 + 28*a^7*c^7*d^4*e^12 + 8*a^8*c^6*d^2*e^14 + a^9*c^5*e^16)))*sqrt(-(30*c^
2*d^5*e - 4*a*c*d^3*e^3 - 2*a^2*d*e^5 - (c^6*d^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+a)**2,x)
```

[Out] Timed out

Giac [A]
time = 3.57, size = 595, normalized size = 0.87

$$-\frac{d^{\frac{5}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}}+\frac{\left(5\,(ac^{3})^{\frac{1}{4}}\,ac^{2}d^{2}e+3\,(ac^{3})^{\frac{3}{4}}\,cd^{3}+(ac^{3})^{\frac{1}{4}}\,a^{2}ce^{3}-(ac^{3})^{\frac{3}{4}}\,ade^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^{5}d^{4}+2\sqrt{2}\,a^{2}c^{4}d^{2}e^{2}+\sqrt{2}\,a^{3}c^{3}e^{4}\right)}+\frac{\left(5\,(ac^{3})^{\frac{1}{4}}\,ac^{2}d^{2}e+3\,(ac^{3})^{\frac{3}{4}}\,cd^{3}+(ac^{3})^{\frac{1}{4}}\,a^{2}ce^{3}-(ac^{3})^{\frac{3}{4}}\,ade^{2}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^{5}d^{4}+2\sqrt{2}\,a^{2}c^{4}d^{2}e^{2}+\sqrt{2}\,a^{3}c^{3}e^{4}\right)}+\frac{\left(5\,(ac^{3})^{\frac{1}{4}}\,ac^{2}d^{2}e-3\,(ac^{3})^{\frac{3}{4}}\,cd^{3}+(ac^{3})^{\frac{1}{4}}\,a^{2}ce^{3}+(ac^{3})^{\frac{3}{4}}\,ade^{2}\right)\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,ac^{5}d^{4}+2\sqrt{2}\,a^{2}c^{4}d^{2}e^{2}+\sqrt{2}\,a^{3}c^{3}e^{4}\right)}-\frac{\left(5\,(ac^{3})^{\frac{1}{4}}\,ac^{2}d^{2}e-3\,(ac^{3})^{\frac{3}{4}}\,cd^{3}+(ac^{3})^{\frac{1}{4}}\,a^{2}ce^{3}+(ac^{3})^{\frac{3}{4}}\,ade^{2}\right)\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,ac^{5}d^{4}+2\sqrt{2}\,a^{2}c^{4}d^{2}e^{2}+\sqrt{2}\,a^{3}c^{3}e^{4}\right)}-\frac{cdx^{3}+axe}{4\,(cx^{4}+a)(c^{2}d^{2}+ace^{2})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1
/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sq
rt(2)*a^3*c^3*e^4) + 1/8*(5*(a*c^3)^(1/4)*a*c^2*d^2*e + 3*(a*c^3)^(3/4)*c*d
^3 + (a*c^3)^(1/4)*a^2*c*e^3 - (a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2
*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c
^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e - 3*(
a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*a*d*e^2)*log(x
^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*
c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*(5*(a*c^3)^(1/4)*a*c^2*d^2*e - 3*
(a*c^3)^(3/4)*c*d^3 + (a*c^3)^(1/4)*a^2*c*e^3 + (a*c^3)^(3/4)*a*d*e^2)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2
*c^4*d^2*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/4*(c*d*x^3 + a*x*e)/((c*x^4 + a)*(c
^2*d^2 + a*c*e^2))
```

Mupad [B]
time = 2.82, size = 2500, normalized size = 3.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] atan(((((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^
6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256
*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2
*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*
d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6
*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6
*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) - (x*(-(a^3*e^6*(-a*c^5)^(1/
2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*
c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2
))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 +
4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 -
327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^
```

9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(a^6*e^13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*1i - (((432*a*c^7*d^12*e^2 + 13040*a^2*c^6*d^10*e^4 + 12000*a^3*c^5*d^8*e^6 - 1056*a^4*c^4*d^6*e^8 - 400*a^5*c^3*d^4*e^10 + 48*a^6*c^2*d^2*e^12)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (((45056*a^2*c^10*d^13*e^3 - 4096*a^8*c^4*d*e^15 + 221184*a^3*c^9*d^11*e^5 + 430080*a^4*c^8*d^9*e^7 + 409600*a^5*c^7*d^7*e^9 + 184320*a^6*c^6*d^5*e^11 + 24576*a^7*c^5*d^3*e^13)/(256*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)) + (x*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2)*(65536*a^9*c^5*e^17 - 65536*a^2*c^12*d^14*e^3 - 327680*a^3*c^11*d^12*e^5 - 589824*a^4*c^10*d^10*e^7 - 327680*a^5*c^9*d^8*e^9 + 327680*a^6*c^8*d^6*e^11 + 589824*a^7*c^7*d^4*e^13 + 327680*a^8*c^6*d^2*e^15))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) - (x*(1152*a*c^9*d^13*e^2 + 1152*a^7*c^3*d*e^14 + 21248*a^2*c^8*d^11*e^4 + 25472*a^3*c^7*d^9*e^6 - 5632*a^4*c^6*d^7*e^8 - 7296*a^5*c^5*d^5*e^10 + 4864*a^6*c^4*d^3*e^12))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*

```
(-a*c^5)^(1/2) - 2*a^3*c^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*
a*c^2*d^4*e^2*(-a*c^5)^(1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*
d^8 + a^5*c^5*e^8 + 4*a^2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e
^6)))^(1/2))*(-(a^3*e^6*(-a*c^5)^(1/2) - 9*c^3*d^6*(-a*c^5)^(1/2) - 2*a^3*c
^3*d*e^5 - 4*a^2*c^4*d^3*e^3 + 30*a*c^5*d^5*e + 31*a*c^2*d^4*e^2*(-a*c^5)^(
1/2) + 9*a^2*c*d^2*e^4*(-a*c^5)^(1/2))/(256*(a*c^9*d^8 + a^5*c^5*e^8 + 4*a^
2*c^8*d^6*e^2 + 6*a^3*c^7*d^4*e^4 + 4*a^4*c^6*d^2*e^6)))^(1/2) + (x*(a^6*e^
13 - 288*a*c^5*d^10*e^3 + 20*a^5*c*d^2*e^11 + 17*a^2*c^4*d^8*e^5 + 148*a^3*
c^3*d^6*e^7 + 118*a^4*c^2*d^4*e^9))/(128*(c^5*d^8 + a^4*c*e^8 + 4*a*c^4*d^6
*e^2 + 6*a^2*c^3*d^4*e^4 + 4*a^3*c^2*d^2*e^6)))...
```

3.254 $\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=685

$$-\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)}+\frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}-\frac{\sqrt[4]{c}\,d^2(\sqrt{c}\,d-\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(cd^2+ae^2)^2}+\frac{(3\sqrt{c}\,d}{}$$

```
[Out] -1/4*x*(-e*x^2+d)/(a*e^2+c*d^2)/(c*x^4+a)+d^(3/2)*e^(3/2)*arctan(x*e^(1/2)/
d^(1/2))/(a*e^2+c*d^2)^2+1/4*c^(1/4)*d^2*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4
))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/4*c^(1/4)*d^2*a
rctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*
d^2)^2*2^(1/2)-1/8*c^(1/4)*d^2*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^
(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*c^(1/4)*d^
2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a
^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/16*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-
e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*arctan(1+
c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+
c*d^2)*2^(1/2)+1/32*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a
^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*ln(a^(1/4)*c
^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(3/4)/c^(3/
4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1328, 1193, 1182, 1176, 631, 210, 1179, 642, 1185, 211}

$$\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(3\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}a^{3/4}c^{3/4}(ae^2+cd^2)}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(3\sqrt{c}d-\sqrt{a}e)}{8\sqrt{2}a^{3/4}c^{3/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}d^2\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}d^2\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}x}{\sqrt[4]{a}}+1\right)(\sqrt{c}d-\sqrt{a}e)}{2\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{(\sqrt{a}e+3\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(ae^2+cd^2)}-\frac{(\sqrt{a}e+3\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{16\sqrt{2}a^{3/4}c^{3/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}d^2(\sqrt{a}e+\sqrt{c}d)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}d^2(\sqrt{a}e+\sqrt{c}d)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}x+\sqrt{a}+\sqrt{c}x^2\right)}{4\sqrt{2}a^{3/4}(ae^2+cd^2)^2}+\frac{d^{3/2}e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2}-\frac{x(d-ex^2)}{4(a+cx^4)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/4*(x*(d - e*x^2))/((c*d^2 + a*e^2)*(a + c*x^4)) + (d^(3/2)*e^(3/2)*ArcTa
n[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 - (c^(1/4)*d^2*(Sqrt[c]*d - Sqrt[
a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2 +
a*e^2)^2) + ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/
4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d
- Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c
*d^2 + a*e^2)^2) - ((3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/(8*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d^2*(Sqr
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqr
t[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4
```

```
)*(c*d^2 + a*e^2)) + (c^(1/4)*d^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqr
t[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2
) - ((3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sq
rt[c]*x^2])/(16*Sqrt[2]*a^(3/4)*c^(3/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1328

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[(-a)*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)*(d - e*x^2
)*(a + c*x^4)^p, x], x] + Dist[d^2*(f^4/(c*d^2 + a*e^2)), Int[(f*x)^(m - 4)
*((a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] &&
LtQ[p, -1] && GtQ[m, 2]
```

Rubi steps

$$\int \frac{x^4}{(d+ex^2)(a+cx^4)^2}\,dx = -\frac{a\int \frac{d-ex^2}{(a+cx^4)^2}\,dx}{cd^2+ae^2} + \frac{d^2\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}$$

$$= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{\int \frac{-3d+ex^2}{a+cx^4}\,dx}{4(cd^2+ae^2)} + \frac{d^2\int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx}\right.}{cd^2+ae^2}$$

$$= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{(cd^2)\int \frac{d-ex^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2} + \frac{(d^2e^2)\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2} - \frac{\left(\frac{3\sqrt{c}\,d}{\sqrt{a}} - e\right.}{8c(}$$

$$= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\left(d^2\left(\frac{\sqrt{c}\,d}{\sqrt{a}} - e\right)\right)\int \frac{\sqrt{a}}{}}{2(cd^2+ae^2)^2}$$

$$= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{(3\sqrt{c}\,d+\sqrt{a}\,e)\log\left(\sqrt{a}\right.}{16\sqrt{2}\,a^{3/4}c}$$

$$= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{(3\sqrt{c}\,d-\sqrt{a}\,e)\tan^{-1}\left(\right.}{8\sqrt{2}\,a^{3/4}c^{3/4}(cd^2}$$

$$= -\frac{x(d-ex^2)}{4(cd^2+ae^2)(a+cx^4)} + \frac{d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\,d^2(\sqrt{c}\,d-\sqrt{a}\,e)\tan}{2\sqrt{2}\,a^{3/4}(cd^2}$$

Mathematica [A]
time = 0.18, size = 423, normalized size = 0.62

$$\frac{\frac{8(cd^2+ae^2)(-dx+ex^3)}{a+cx^4} + 32d^{3/2}e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) - \frac{2\sqrt{2}\left(c^{3/2}d^3-3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2+a^{3/2}e^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}c^{3/4}} + \frac{2\sqrt{2}\left(c^{3/2}d^3-3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2+a^{3/2}e^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{3/4}c^{3/4}} + \frac{\sqrt{2}\left(-c^{3/2}d^3-3\sqrt{a}\,cd^2e+3a\sqrt{c}\,de^2+a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{3/4}c^{3/4}} + \frac{\sqrt{2}\left(c^{3/2}d^3+3\sqrt{a}\,cd^2e-3a\sqrt{c}\,de^2-a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{3/4}c^{3/4}}}{32(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(-(d*x) + e*x^3))/(a + c*x^4) + 32*d^(3/2)*e^(3/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a
*Sqrt[c]*d*e^2 + a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(
3/4)*c^(3/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d
*e^2 + a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(3/4)*c^(3/
4)) + (Sqrt[2]*(-(c^(3/2)*d^3) - 3*Sqrt[a]*c*d^2*e + 3*a*Sqrt[c]*d*e^2 + a^
```

```
(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3/4)
*c^(3/4)) + (Sqrt[2]*(c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e - 3*a*Sqrt[c]*d*e^2 -
 a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(3
/4)*c^(3/4)))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.21, size = 327, normalized size = 0.48

method	result
default	$\frac{e^{2} d^{2} \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{\left(a e^{2}+c d^{2}\right)^{2} \sqrt{d e}}-\frac{\frac{\left(-\frac{1}{4} a e^{3}-\frac{1}{4} c d^{2} e\right) x^{3}+\left(\frac{1}{4} d e^{2} a+\frac{1}{4} c d^{3}\right) x}{c x^{4}+a}+\frac{\left(3 d e^{2} a-c d^{3}\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \operatorname{arcta}\right.}{32 a}}{}$
risch	$\frac{\frac{e x^{3}}{4 a e^{2}+4 c d^{2}}-\frac{d x}{4\left(a e^{2}+c d^{2}\right)}}{c x^{4}+a}+\left(\sum_{\_R=\operatorname{RootOf}\left(\left(a^{7} c^{3} e^{8}+4 a^{6} c^{4} d^{2} e^{6}+6 a^{5} c^{5} d^{4} e^{4}+4 a^{4} c^{6} d^{6} e^{2}+c^{7} a^{3} d^{8}\right) \_Z^{4}+\left(-12 a^{4} c^{2} d e^{5}+40 a^{3} c^{3} d^{3} e^{3}-12 a\right.\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^2*d^2/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-1/(a*e^2+c*d^2)
^2*(((-1/4*a*e^3-1/4*c*d^2*e)*x^3+(1/4*d*e^2*a+1/4*c*d^3)*x)/(c*x^4+a)+1/32
*(3*a*d*e^2-c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/
c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/32*(-a*e^3+3*c*d^2*e)/c/(a/c
)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4
)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2
)/(a/c)^(1/4)*x-1)))
```

Maxima [A]
time = 0.51, size = 470, normalized size = 0.69

$$\frac{d^{\frac{3}{2}} \arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\frac{3}{2}}}{c^{2} d^{4}+2 a c d^{2} e^{2}+a^{2} e^{4}}+\frac{(c d^{2} e+a e^{3}) x^{3}-(c d^{3}+a d e^{2}) x}{4\left(a c^{2} d^{4}+2 a^{2} c d^{2} e^{2}+(c^{3} d^{4}+2 a c^{2} d^{2} e^{2}+a^{2} c e^{4}) x^{4}+a^{3} e^{4}\right)}+\frac{\frac{2 \sqrt{2}\left(c^{\frac{3}{2}} d^{3}-3 \sqrt{a} c d^{2} e-3 a \sqrt{c} d e^{2}+a^{\frac{3}{2}} e^{3}\right) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c} x+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{2 \sqrt{2}\left(c^{\frac{3}{2}} d^{3}-3 \sqrt{a} c d^{2} e-3 a \sqrt{c} d e^{2}+a^{\frac{3}{2}} e^{3}\right) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c} x-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{\sqrt{2}\left(c^{\frac{3}{2}} d^{3}+3 \sqrt{a} c d^{2} e-3 a \sqrt{c} d e^{2}-a^{\frac{3}{2}} e^{3}\right) \log\left(\sqrt{c} x^{2}+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(c^{\frac{3}{2}} d^{3}+3 \sqrt{a} c d^{2} e-3 a \sqrt{c} d e^{2}-a^{\frac{3}{2}} e^{3}\right) \log\left(\sqrt{c} x^{2}-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}}{32\left(c^{2} d^{4}+2 a c d^{2} e^{2}+a^{2} e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^
4) + 1/4*((c*d^2*e + a*e^3)*x^3 - (c*d^3 + a*d*e^2)*x)/(a*c^2*d^4 + 2*a^2*c
*d^2*e^2 + (c^3*d^4 + 2*a*c^2*d^2*e^2 + a^2*c*e^4)*x^4 + a^3*e^4) + 1/32*(2
*sqrt(2)*(c^(3/2)*d^3 - 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3
)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*s
qrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(c^(3/2)*d^3 -
 3*sqrt(a)*c*d^2*e - 3*a*sqrt(c)*d*e^2 + a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2
```

```
*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(
sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a*
sqrt(c)*d*e^2 - a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x +
sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(c^(3/2)*d^3 + 3*sqrt(a)*c*d^2*e - 3*a
*sqrt(c)*d*e^2 - a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x +
 sqrt(a))/(a^(3/4)*c^(3/4)))/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4531 vs. $2(511) = 1022$.
time = 6.22, size = 9091, normalized size = 13.27

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*c*d^2*x^3*e - 4*c*d^3*x + 4*a*x^3*e^3 - 4*a*d*x*e^2 + 8*(c*d*x^4 +
 a*d)*sqrt(-d*e)*e*log((x^2*e + 2*sqrt(-d*e)*x - d)/(x^2*e + d)) - (c^3*d^4
*x^4 + a*c^2*d^4 + (a^2*c*x^4 + a^3)*e^4 + 2*(a*c^2*d^2*x^4 + a^2*c*d^2)*e^
2)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^
4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d
^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a
^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*
d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 +
 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3
*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2
*e^6 + a^5*c*e^8))*log(-c^4*d^8*x + 14*a*c^3*d^6*x*e^2 - 14*a^3*c*d^2*x*e^6
 + a^4*x*e^8 + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^
4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 + (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14
*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt
(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6
 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a
^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d
^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 +
a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*
d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8
)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d
^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16
 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7
*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e
^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))) + (c^3*d^4*x^4 + a*c^2*d^4 + (a^2*c*x^4 +
 a^3)*e^4 + 2*(a*c^2*d^2*x^4 + a^2*c*d^2)*e^2)*sqrt((6*c^2*d^5*e - 20*a*c*d
^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a
```

```
^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e
^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4
+ 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c
^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4
*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-c^4*d^8
*x + 14*a*c^3*d^6*x*e^2 - 14*a^3*c*d^2*x*e^6 + a^4*x*e^8 - (a*c^5*d^9 - 18*
a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 +
 (3*a^3*c^7*d^10*e + 11*a^4*c^6*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^
4*e^7 - a^7*c^3*d^2*e^9 - a^8*c^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2
 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5
*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d
^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 +
28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^
5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 + (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3
*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)*sqrt(-(c^6*d^12 - 30*a*c^5*d^
10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 -
30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5
*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e
^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^14 + a^11*c^3*e^16)))/(a*c^5*d
^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)
)) - (c^3*d^4*x^4 + a*c^2*d^4 + (a^2*c*x^4 + a^3)*e^4 + 2*(a*c^2*d^2*x^4 +
a^2*c*d^2)*e^2)*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*e^5 - (a*c^5*d
^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 + 4*a^4*c^2*d^2*e^6 + a^5*c*e^8)
*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^3*c^3*d^
6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^11*d^16
+ 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 + 70*a^7*
c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^4*d^2*e^
14 + a^11*c^3*e^16)))/(a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^3*d^4*e^4 +
4*a^4*c^2*d^2*e^6 + a^5*c*e^8))*log(-c^4*d^8*x + 14*a*c^3*d^6*x*e^2 - 14*a^
3*c*d^2*x*e^6 + a^4*x*e^8 + (a*c^5*d^9 - 18*a^2*c^4*d^7*e^2 + 60*a^3*c^3*d^
5*e^4 - 46*a^4*c^2*d^3*e^6 + 3*a^5*c*d*e^8 - (3*a^3*c^7*d^10*e + 11*a^4*c^6
*d^8*e^3 + 14*a^5*c^5*d^6*e^5 + 6*a^6*c^4*d^4*e^7 - a^7*c^3*d^2*e^9 - a^8*c
^2*e^11)*sqrt(-(c^6*d^12 - 30*a*c^5*d^10*e^2 + 255*a^2*c^4*d^8*e^4 - 452*a^
3*c^3*d^6*e^6 + 255*a^4*c^2*d^4*e^8 - 30*a^5*c*d^2*e^10 + a^6*e^12)/(a^3*c^
11*d^16 + 8*a^4*c^10*d^14*e^2 + 28*a^5*c^9*d^12*e^4 + 56*a^6*c^8*d^10*e^6 +
 70*a^7*c^7*d^8*e^8 + 56*a^8*c^6*d^6*e^10 + 28*a^9*c^5*d^4*e^12 + 8*a^10*c^
4*d^2*e^14 + a^11*c^3*e^16)))*sqrt((6*c^2*d^5*e - 20*a*c*d^3*e^3 + 6*a^2*d*
e^5 - (a*c^5*d^8 + 4*a^2*c^4*d^6*e^2 + 6*a^3*c^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.42, size = 586, normalized size = 0.86

$$\frac{d^{\frac{3}{2}}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{3}{2}}}{c^{2}d^{4}+2\,acd^{2}e^{2}+a^{2}e^{4}}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{3}-3\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}de^{2}-3\left(ac^{3}\right)^{\frac{3}{4}}cd^{2}e+\left(ac^{3}\right)^{\frac{3}{4}}ae^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^{5}d^{4}+2\sqrt{2}\,a^{2}c^{4}d^{2}e^{2}+\sqrt{2}\,a^{3}c^{3}e^{4}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{3}-3\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}de^{2}-3\left(ac^{3}\right)^{\frac{3}{4}}cd^{2}e+\left(ac^{3}\right)^{\frac{3}{4}}ae^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,ac^{5}d^{4}+2\sqrt{2}\,a^{2}c^{4}d^{2}e^{2}+\sqrt{2}\,a^{3}c^{3}e^{4}\right)}+\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{3}-3\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}de^{2}+3\left(ac^{3}\right)^{\frac{3}{4}}cd^{2}e-\left(ac^{3}\right)^{\frac{3}{4}}ae^{3}\right)\log\left(x^{2}+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,ac^{5}d^{4}+2\sqrt{2}\,a^{2}c^{4}d^{2}e^{2}+\sqrt{2}\,a^{3}c^{3}e^{4}\right)}-\frac{\left(\left(ac^{3}\right)^{\frac{1}{4}}c^{3}d^{3}-3\left(ac^{3}\right)^{\frac{1}{4}}ac^{2}de^{2}+3\left(ac^{3}\right)^{\frac{3}{4}}cd^{2}e-\left(ac^{3}\right)^{\frac{3}{4}}ae^{3}\right)\log\left(x^{2}-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,ac^{5}d^{4}+2\sqrt{2}\,a^{2}c^{4}d^{2}e^{2}+\sqrt{2}\,a^{3}c^{3}e^{4}\right)}+\frac{x^{3}e-dx}{4\left(cx^{4}+a\right)\left(cd^{2}+ae^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^
4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2 - 3*(a*c^3)^(
3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)
^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*e^2 + sqrt(
2)*a^3*c^3*e^4) + 1/8*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)*a*c^2*d*e^2
- 3*(a*c^3)^(3/4)*c*d^2*e + (a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^
2*e^2 + sqrt(2)*a^3*c^3*e^4) + 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4
)*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 + sq
rt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2
*e^2 + sqrt(2)*a^3*c^3*e^4) - 1/16*((a*c^3)^(1/4)*c^3*d^3 - 3*(a*c^3)^(1/4)
*a*c^2*d*e^2 + 3*(a*c^3)^(3/4)*c*d^2*e - (a*c^3)^(3/4)*a*e^3)*log(x^2 - sqr
t(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a*c^5*d^4 + 2*sqrt(2)*a^2*c^4*d^2*
e^2 + sqrt(2)*a^3*c^3*e^4) + 1/4*(x^3*e - d*x)/((c*x^4 + a)*(c*d^2 + a*e^2)
)
```

Mupad [B]
time = 4.87, size = 2500, normalized size = 3.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] - atan(((((((28672*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*
d^8*e^6 + 253952*a^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*
d^2*e^12)/(256*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) - (
x*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e +
 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) -
 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4
*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^
4*e^17 - 65536*a^2*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^
9*d^10*e^7 - 327680*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*
c^6*d^4*e^13 + 327680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*
```

d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2)
 - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^
3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)
^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4
*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e
^2 + 2944*a^6*c^3*d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 +
4224*a^4*c^5*d^5*e^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4
*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3
)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20
*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a
^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*
c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*
e^5 + 16*a^4*c^2*d*e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256
*(a^3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*
c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 -
 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*
(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a
^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2) - (x*(a^4*c*e^13 + 33*c^5*d^8*e
^5 - 188*a*c^4*d^6*e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^
4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*
((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6
*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 1
5*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c
^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*1i - (((((28672
*a^2*c^8*d^10*e^4 - 4096*a*c^9*d^12*e^2 + 155648*a^3*c^7*d^8*e^6 + 253952*a
^4*c^6*d^6*e^8 + 176128*a^5*c^5*d^4*e^10 + 45056*a^6*c^4*d^2*e^12)/(256*(a^
3*e^6 + c^3*d^6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)) + (x*((a^3*e^6*(-a^3*
c^3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 -
 20*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*
(-a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a
^5*c^5*d^4*e^4 + 4*a^6*c^4*d^2*e^6)))^(1/2)*(65536*a^9*c^4*e^17 - 65536*a^2
*c^11*d^14*e^3 - 327680*a^3*c^10*d^12*e^5 - 589824*a^4*c^9*d^10*e^7 - 32768
0*a^5*c^8*d^8*e^9 + 327680*a^6*c^7*d^6*e^11 + 589824*a^7*c^6*d^4*e^13 + 327
680*a^8*c^5*d^2*e^15))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*
d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*(-a^3*c
^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 + 15*a*c
^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(256*(a^3*
c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^6*c^4*d
^2*e^6)))^(1/2) - (x*(256*a*c^8*d^11*e^4 - 128*c^9*d^13*e^2 + 2944*a^6*c^3*
d*e^14 + 21632*a^2*c^7*d^9*e^6 + 32256*a^3*c^6*d^7*e^8 + 4224*a^4*c^5*d^5*e
^10 - 3840*a^5*c^4*d^3*e^12))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4
*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d^6*
(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^3 +
 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/(25
6*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4*a^

6*c^4*d^2*e^6)))^(1/2) + (16*c^6*d^9*e^3 - 960*a*c^5*d^7*e^5 + 16*a^4*c^2*d *e^11 + 8288*a^2*c^4*d^5*e^7 - 3008*a^3*c^3*d^3*e^9)/(256*(a^3*e^6 + c^3*d^ 6 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4)))*((a^3*e^6*(-a^3*c^3)^(1/2) - c^3*d ^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 20*a^3*c^3*d^3*e^ 3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(-a^3*c^3)^(1/2))/ (256*(a^3*c^7*d^8 + a^7*c^3*e^8 + 4*a^4*c^6*d^6*e^2 + 6*a^5*c^5*d^4*e^4 + 4 *a^6*c^4*d^2*e^6)))^(1/2) + (x*(a^4*c*e^13 + 33*c^5*d^8*e^5 - 188*a*c^4*d^6 *e^7 + 38*a^2*c^3*d^4*e^9 + 4*a^3*c^2*d^2*e^11))/(128*(a^4*e^8 + c^4*d^8 + 4*a*c^3*d^6*e^2 + 4*a^3*c*d^2*e^6 + 6*a^2*c^2*d^4*e^4)))*((a^3*e^6*(-a^3*c^ 3)^(1/2) - c^3*d^6*(-a^3*c^3)^(1/2) + 6*a^2*c^4*d^5*e + 6*a^4*c^2*d*e^5 - 2 0*a^3*c^3*d^3*e^3 + 15*a*c^2*d^4*e^2*(-a^3*c^3)^(1/2) - 15*a^2*c*d^2*e^4*(- a^3*c^3)^(1/2))/(256*(a^3*c^7*d^8 + a^7*c^3*e^8...

3.255 $\int \frac{x^2}{(d+ex^2)(a+cx^4)^2} dx$

Optimal. Leaf size=685

$$\frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)}-\frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2}+\frac{\sqrt[4]{c}\,de\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(cd^2+ae^2)^2}-\frac{\left(\sqrt{c}\,d\right.}{}$$

```
[Out] 1/4*x*(c*d*x^2+a*e)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/32*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+
c*d^2)*2^(1/2)-1/32*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e
*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-1/4*c^(1/4)*d*e*a
rctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c
*d^2)^2*2^(1/2)-1/4*c^(1/4)*d*e*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(
1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/8*c^(1/4)*d*e*ln(-a^(1/4)
*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^
2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*d*e*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*arctan
(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c^(1/2))/a^(5/4)/c^(1/4)/(a*e
^2+c*d^2)*2^(1/2)+1/16*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c
^(1/2))/a^(5/4)/c^(1/4)/(a*e^2+c*d^2)*2^(1/2)-e^(5/2)*arctan(x*e^(1/2)/d^(1
/2))*d^(1/2)/(a*e^2+c*d^2)^2
```

Rubi [A]
time = 0.36, antiderivative size = 685, normalized size of antiderivative = 1.00, number of steps used = 23, number of rules used = 10, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.454$, Rules used = {1330, 1193, 1182, 1176, 631, 210, 1179, 642, 1185, 211}

$$\frac{\sqrt[4]{c}\,de\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}\,de\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}-\frac{\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(3\sqrt{a}\,e+\sqrt{c}\,d\right)}{8\sqrt{2}\,a^{5/4}\sqrt[4]{c}\,(ae^2+cd^2)}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{a}\,e+\sqrt{c}\,d\right)}{8\sqrt{2}\,a^{5/4}\sqrt[4]{c}\,(ae^2+cd^2)}+\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}\,de\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}+\frac{\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}\sqrt[4]{c}\,(ae^2+cd^2)}-\frac{\left(\sqrt{c}\,d-3\sqrt{a}\,e\right)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{5/4}\sqrt[4]{c}\,(ae^2+cd^2)}-\frac{\sqrt{d}\,e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(ae^2+cd^2)^2}+\frac{x(ae+cdx^2)}{4a(a+cx^4)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (x*(a*e + c*d*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) - (Sqrt[d]*e^(5/2)*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 + a*e^2)^2 + (c^(1/4)*d*e*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^
(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2)) + (c^(1/4)*d*e*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) + ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[
Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(
```

```
1/4)*(c*d^2 + a*e^2)) - (c^(1/4)*d*e*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) - ((Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x +
 Sqrt[c]*x^2])/(16*Sqrt[2]*a^(5/4)*c^(1/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1185

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1330

```
Int[(((f_.)*(x_))^(m_.)*((a_) + (c_.)*(x_)^4)^(p_))/((d_.) + (e_.)*(x_)^2),
 x_Symbol] :> Dist[f^2/(c*d^2 + a*e^2), Int[(f*x)^(m - 2)*(a*e + c*d*x^2)*(
a + c*x^4)^p, x], x] - Dist[d*e*(f^2/(c*d^2 + a*e^2)), Int[(f*x)^(m - 2)*((
a + c*x^4)^(p + 1)/(d + e*x^2)), x], x] /; FreeQ[{a, c, d, e, f}, x] && LtQ
[p, -1] && GtQ[m, 0]
```

Rubi steps

$$\int \frac{x^2}{(d+ex^2)(a+cx^4)^2}\,dx = \frac{\int \frac{ae+cdx^2}{(a+cx^4)^2}\,dx}{cd^2+ae^2} - \frac{(de)\int \frac{1}{(d+ex^2)(a+cx^4)}\,dx}{cd^2+ae^2}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\int \frac{-3ae-cdx^2}{a+cx^4}\,dx}{4a(cd^2+ae^2)} - \frac{(de)\int \left(\frac{e^2}{(cd^2+ae^2)(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a\cdots}\right.}{cd^2+ae^2}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{(cde)\int \frac{d-ex^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2} - \frac{(de^3)\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2} - \frac{\left(d-\frac{3\sqrt{a}\,e}{\sqrt{c}}\right)\cdots}{8a(c\cdots}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\left(d\left(\frac{\sqrt{c}\,d}{\sqrt{a}}-e\right)e\right)\int \frac{\sqrt{a}\cdots}{\cdots}}{2(cd^2+ae^2)^2\cdots}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\sqrt[4]{c}\left(d-\frac{3\sqrt{a}\,e}{\sqrt{c}}\right)\log\left(\sqrt{\cdots}\right.}{16\sqrt{2}\,a^{5\cdots}}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\left(d+\frac{3\sqrt{a}\,e}{\sqrt{c}}\right)\tan^{-1}\left(\cdots\right.}{8\sqrt{2}\,a^{5/4}(cd^2\cdots}$$

$$= \frac{x(ae+cdx^2)}{4a(cd^2+ae^2)(a+cx^4)} - \frac{\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{(cd^2+ae^2)^2} + \frac{\sqrt[4]{c}\,de\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan\cdots}{2\sqrt{2}\,a^{3/4}(cd^2\cdots}$$

Mathematica [A]
time = 0.19, size = 428, normalized size = 0.62

$$\frac{\frac{8(cd^2+ae^2)(aex+cdx^3)}{a(a+cx^4)} - 32\sqrt{d}\,e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right) - \frac{2\sqrt{2}\left(c^{3/2}d^3-\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2+3a^{3/2}e^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{5/4}\sqrt[4]{c}} + \frac{2\sqrt{2}\left(c^{3/2}d^3-\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2+3a^{3/2}e^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{5/4}\sqrt[4]{c}} + \frac{\sqrt{2}\left(c^{3/2}d^3+\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2-3a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{5/4}\sqrt[4]{c}} - \frac{\sqrt{2}\left(c^{3/2}d^3+\sqrt{a}\,cd^2e+5a\sqrt{c}\,de^2-3a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{5/4}\sqrt[4]{c}}}{32(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*(c*d^2 + a*e^2)*(a*e*x + c*d*x^3))/(a*(a + c*x^4)) - 32*Sqrt[d]*e^(5/2)
*ArcTan[(Sqrt[e]*x)/Sqrt[d]] - (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e +
5*a*Sqrt[c]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(a^(5/4)*c^(1/4)) + (2*Sqrt[2]*(c^(3/2)*d^3 - Sqrt[a]*c*d^2*e + 5*a*Sqrt[c
]*d*e^2 + 3*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(5/4)*
c^(1/4)) + (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 - 3*
```

```
a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(5/
4)*c^(1/4)) - (Sqrt[2]*(c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 5*a*Sqrt[c]*d*e^2 -
 3*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^
(5/4)*c^(1/4)))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.23, size = 339, normalized size = 0.49

method	result
default	$-\frac{d e^3 \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{(a e^2+c d^2)^2 \sqrt{d e}} + \frac{\frac{c d (a e^2+c d^2) x^3}{4a} + \left(\frac{1}{4} a e^3+\frac{1}{4} c d^2 e\right) x}{c x^4+a} + \frac{\frac{(3a^2 e^3 - a d^2 e c)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}} x\sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2\,\mathrm{arct}\right.}{8a}}{}$
risch	$\frac{\frac{c d x^3}{4a\left(a e^2+c d^2\right)}+\frac{e x}{4a e^2+4c d^2}}{c x^4+a} + \frac{\sqrt{-de}\, e^2 \ln\left(\left(-4096(-de)^{\frac{5}{2}} a^5 c e^8+4096(-de)^{\frac{5}{2}} a^4 c^2 d^2 e^6-3552(-de)^{\frac{3}{2}} a^5 c d e^9+5248(-de)^{\frac{3}{2}} a^4\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -d*e^3/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+1/(a*e^2+c*d^2)^
2*((1/4*c*d*(a*e^2+c*d^2)/a*x^3+(1/4*a*e^3+1/4*c*d^2*e)*x)/(c*x^4+a)+1/4/a*
(1/8*(3*a^2*e^3-a*c*d^2*e)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(
1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)
/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(5*a*c*d*e^2+c^2*d
^3)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+
(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arc
tan(2^(1/2)/(a/c)^(1/4)*x-1))))
```

Maxima [A]
time = 0.53, size = 458, normalized size = 0.67

$$-\frac{\sqrt{d}\arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\frac{5}{2}}}{c^2 d^4+2\,a c d^2 e^2+a^2 e^4} + \frac{c d x^3+a x e}{4\,(a^2 c d^2+(a c^2 d^2+a^2 c e^2)x^4+a^3 e^2)} + \frac{\frac{2\sqrt{2}\left(\sqrt{a}c^2 d^3 - a c^{\frac{3}{2}} d^2 e + 5 a^{\frac{3}{2}} c d e^2 + 3 a^2\sqrt{c} e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} + \frac{2\sqrt{2}\left(\sqrt{a}c^2 d^3 - a c^{\frac{3}{2}} d^2 e + 5 a^{\frac{3}{2}} c d e^2 + 3 a^2\sqrt{c} e^3\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}x-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}} - \frac{\sqrt{2}\left(\sqrt{a}c^2 d^3 + a c^{\frac{3}{2}} d^2 e + 5 a^{\frac{3}{2}} c d e^2 - 3 a^2\sqrt{c} e^3\right)\log\left(\sqrt{c}x^2+\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}} + \frac{\sqrt{2}\left(\sqrt{a}c^2 d^3 + a c^{\frac{3}{2}} d^2 e + 5 a^{\frac{3}{2}} c d e^2 - 3 a^2\sqrt{c} e^3\right)\log\left(\sqrt{c}x^2-\sqrt{2}a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}}{32\,(a c^2 d^4+2\,a^2 c d^2 e^2+a^3 e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) + 1/4*(c*d*x^3 + a*x*e)/(a^2*c*d^2 + (a*c^2*d^2 + a^2*c*e^2)*x^4 + a^3*
e^2) + 1/32*(2*sqrt(2)*(sqrt(a)*c^2*d^3 - a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e
^2 + 3*a^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c
^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*
sqrt(2)*(sqrt(a)*c^2*d^3 - a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2 + 3*a^2*sqrt
```

```
(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sq
rt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(sqrt(a)*
c^2*d^3 + a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2 - 3*a^2*sqrt(c)*e^3)*log(sqrt
(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) + sqrt(2)*
(sqrt(a)*c^2*d^3 + a*c^(3/2)*d^2*e + 5*a^(3/2)*c*d*e^2 - 3*a^2*sqrt(c)*e^3)
*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/
(a*c^2*d^4 + 2*a^2*c*d^2*e^2 + a^3*e^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4578 vs. $2(514) = 1028$.
time = 6.79, size = 9185, normalized size = 13.41

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [1/16*(4*c^2*d^3*x^3 + 4*a*c*d*x^3*e^2 + 4*a*c*d^2*x*e + 4*a^2*x*e^3 + 8*(a
*c*x^4 + a^2)*sqrt(-d*e)*e^2*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d))
+ (a*c^3*d^4*x^4 + a^2*c^2*d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^4
 + a^3*c*d^2)*e^2)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*
c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8
)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d
^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d
^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a
^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d
^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-c^4*d^8*x - 18*a*c^3*d^6*x*e^2 - 112*
a^2*c^2*d^4*x*e^4 - 270*a^3*c*d^2*x*e^6 + 81*a^4*x*e^8 + (a^2*c^4*d^8*e + 6
*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 - (a^
4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 +
21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c
*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^
12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +
28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5
*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^
4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*
e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558
*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*
c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e
^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*
d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)))
- (a*c^3*d^4*x^4 + a^2*c^2*d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^4
 + a^3*c*d^2)*e^2)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*
```

```
c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8
)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d
^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d
^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a
^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d
^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-c^4*d^8*x - 18*a*c^3*d^6*x*e^2 - 112*
a^2*c^2*d^4*x*e^4 - 270*a^3*c*d^2*x*e^6 + 81*a^4*x*e^8 - (a^2*c^4*d^8*e + 6
*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 - (a^
4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 +
21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c
*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^
12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +
28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))*sqrt((2*c^2*d^5
*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 + (a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^
4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*
e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558
*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*
c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e
^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^13*c*e^16)))/(a^2*c^4*
d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8)))
+ (a*c^3*d^4*x^4 + a^2*c^2*d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^4
 + a^3*c*d^2)*e^2)*sqrt((2*c^2*d^5*e + 4*a*c*d^3*e^3 - 30*a^2*d*e^5 - (a^2*
c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4 + 4*a^5*c*d^2*e^6 + a^6*e^8
)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 + 143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d
^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d
^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a
^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 + 28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d
^2*e^14 + a^13*c*e^16)))/(a^2*c^4*d^8 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e
^4 + 4*a^5*c*d^2*e^6 + a^6*e^8))*log(-c^4*d^8*x - 18*a*c^3*d^6*x*e^2 - 112*
a^2*c^2*d^4*x*e^4 - 270*a^3*c*d^2*x*e^6 + 81*a^4*x*e^8 + (a^2*c^4*d^8*e + 6
*a^3*c^3*d^6*e^3 + 4*a^4*c^2*d^4*e^5 - 102*a^5*c*d^2*e^7 + 27*a^6*e^9 + (a^
4*c^6*d^11 + 9*a^5*c^5*d^9*e^2 + 26*a^6*c^4*d^7*e^4 + 34*a^7*c^3*d^5*e^6 +
21*a^8*c^2*d^3*e^8 + 5*a^9*c*d*e^10)*sqrt(-(c^6*d^12 + 18*a*c^5*d^10*e^2 +
143*a^2*c^4*d^8*e^4 + 540*a^3*c^3*d^6*e^6 + 799*a^4*c^2*d^4*e^8 - 558*a^5*c
*d^2*e^10 + 81*a^6*e^12)/(a^5*c^9*d^16 + 8*a^6*c^8*d^14*e^2 + 28*a^7*c^7*d^
12*e^4 + 56*a^8*c^6*d^10*e^6 + 70*a^9*c^5*d^8*e^8 + 56*a^10*c^4*d^6*e^10 +
28*a^11*c^3*d^4*e^12 + 8*a^12*c^2*d^2*e^14 + a^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 3.89, size = 603, normalized size = 0.88

$$-\frac{\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{5}{2}}}{c^2d^4+2\,acd^2e^2+a^2e^4}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e-\left(ac^3\right)^{\frac{3}{4}}cd^3-3\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-5\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e-\left(ac^3\right)^{\frac{3}{4}}cd^3-3\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-5\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}-\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+\left(ac^3\right)^{\frac{3}{4}}cd^3-3\left(ac^3\right)^{\frac{1}{4}}a^2ce^3+5\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{\left(\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+\left(ac^3\right)^{\frac{3}{4}}cd^3-3\left(ac^3\right)^{\frac{1}{4}}a^2ce^3+5\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^2c^4d^4+2\sqrt{2}a^3c^3d^2e^2+\sqrt{2}a^4c^2e^4\right)}+\frac{cdx^3+axe}{4\left(cx^4+a\right)\left(acd^2+a^2e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e
^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4
)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a
/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 +
sqrt(2)*a^4*c^2*e^4) - 1/8*((a*c^3)^(1/4)*a*c^2*d^2*e - (a*c^3)^(3/4)*c*d^3
 - 3*(a*c^3)^(1/4)*a^2*c*e^3 - 5*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*
(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e + (
a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*a*d*e^2)*l
og(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2
)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*((a*c^3)^(1/4)*a*c^2*d^2*e
+ (a*c^3)^(3/4)*c*d^3 - 3*(a*c^3)^(1/4)*a^2*c*e^3 + 5*(a*c^3)^(3/4)*a*d*e^2
)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqr
t(2)*a^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/4*(c*d*x^3 + a*x*e)/((c*x^4
 + a)*(a*c*d^2 + a^2*e^2))
```

Mupad [B]
time = 2.87, size = 2500, normalized size = 3.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((e*x)/(4*(a*e^2 + c*d^2)) + (c*d*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^4) +
 atan((((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d
^11*e^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*
d^5*e^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d
^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) - (x*(-(c^3*d^6*(-a^5*c)^(
1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*
a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1
/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4
+ 4*a^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3
 - 327680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*
```

e^9 + 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d
^2*e^15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2
 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2
) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*
(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5
*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) +
 (x*(128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 39
68*a^3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*
a^6*c^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^
3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^
5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2
*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8
+ a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6))
)^(1/2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928
*a^3*c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a
^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*
e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5
*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) +
31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*
d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(81*a^4*c^3*e
^13 + c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2
e^11))/(128(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 +
 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2)
- 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-
a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d
^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2)*1i
- (((((53248*a^9*c^4*d*e^15 + 4096*a^3*c^10*d^13*e^3 + 73728*a^4*c^9*d^11*e
^5 + 307200*a^5*c^8*d^9*e^7 + 573440*a^6*c^7*d^7*e^9 + 552960*a^7*c^6*d^5*e
^11 + 270336*a^8*c^5*d^3*e^13)/(256*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^
6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4)) + (x*(-(c^3*d^6*(-a^5*c)^(1/2)
- 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c
*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/
(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a
^8*c^2*d^2*e^6)))^(1/2)*(65536*a^11*c^4*e^17 - 65536*a^4*c^11*d^14*e^3 - 32
7680*a^5*c^10*d^12*e^5 - 589824*a^6*c^9*d^10*e^7 - 327680*a^7*c^8*d^8*e^9 +
 327680*a^8*c^7*d^6*e^11 + 589824*a^9*c^6*d^4*e^13 + 327680*a^10*c^5*d^2*e^
15))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*
a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2
*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5
*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8
+ 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) - (x*(
128*a*c^10*d^13*e^2 - 14208*a^7*c^4*d*e^14 + 768*a^2*c^9*d^11*e^4 + 3968*a^
3*c^8*d^9*e^6 + 27136*a^4*c^7*d^7*e^8 + 30592*a^5*c^6*d^5*e^10 - 7424*a^6*c
^5*d^3*e^12))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6
*e^2 + 6*a^4*c^2*d^4*e^4)))*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^

```
(1/2) - 2*a^3*c^3*d^5*e - 4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*
e^2*(-a^5*c)^(1/2) + 31*a^2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5
*c^5*d^8 + 4*a^6*c^4*d^6*e^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/
2) + (16*c^9*d^12*e^2 + 208*a*c^8*d^10*e^4 + 672*a^2*c^7*d^8*e^6 + 928*a^3*
c^6*d^6*e^8 + 12880*a^4*c^5*d^4*e^10 + 12432*a^5*c^4*d^2*e^12)/(256*(a^6*e^
8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^6 + 4*a^3*c^3*d^6*e^2 + 6*a^4*c^2*d^4*e^4))
)*(-(c^3*d^6*(-a^5*c)^(1/2) - 9*a^3*e^6*(-a^5*c)^(1/2) - 2*a^3*c^3*d^5*e -
4*a^4*c^2*d^3*e^3 + 30*a^5*c*d*e^5 + 9*a*c^2*d^4*e^2*(-a^5*c)^(1/2) + 31*a^
2*c*d^2*e^4*(-a^5*c)^(1/2))/(256*(a^9*c*e^8 + a^5*c^5*d^8 + 4*a^6*c^4*d^6*e
^2 + 6*a^7*c^3*d^4*e^4 + 4*a^8*c^2*d^2*e^6)))^(1/2) + (x*(81*a^4*c^3*e^13 +
 c^7*d^8*e^5 - 12*a*c^6*d^6*e^7 + 54*a^2*c^5*d^4*e^9 - 108*a^3*c^4*d^2*e^11
))/(128*(a^6*e^8 + a^2*c^4*d^8 + 4*a^5*c*d^2*e^...
```

3.256 $\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=689

$$\frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2}-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{c}\,d-\sqrt{a}\,e\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{2\sqrt{2}\,a^{3/4}(cd^2+ae^2)^2}-\frac{\sqrt[4]{c}\,\left(3\sqrt{c}\,d\right.}{}$$

```
[Out] 1/4*c*x*(-e*x^2+d)/a/(a*e^2+c*d^2)/(c*x^4+a)+1/4*c^(1/4)*e^2*arctan(-1+c^(1
/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/
2)+1/4*c^(1/4)*e^2*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2
))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(1/4)*e^2*ln(-a^(1/4)*c^(1/4)*x*2^
(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^
(1/2)+1/8*c^(1/4)*e^2*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*
a^(1/2)+d*c^(1/2))/a^(3/4)/(a*e^2+c*d^2)^2*2^(1/2)+1/16*c^(1/4)*arctan(-1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^
(1/2)+1/16*c^(1/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1
/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)-1/32*c^(1/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/
2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/
2)+1/32*c^(1/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2
)+3*d*c^(1/2))/a^(7/4)/(a*e^2+c*d^2)*2^(1/2)+e^(7/2)*arctan(x*e^(1/2)/d^(1/
2))/(a*e^2+c*d^2)^2/d^(1/2)
```

Rubi [A]
time = 0.38, antiderivative size = 689, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {1253, 211, 1193, 1182, 1176, 631, 210, 1179, 642}

$$-\frac{\sqrt[4]{c}\,e^2\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}\,e^2\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(\sqrt{c}\,d-\sqrt{a}\,e\right)}{2\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}\,\text{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)}{8\sqrt{2}\,a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)\left(3\sqrt{c}\,d-\sqrt{a}\,e\right)}{8\sqrt{2}\,a^{7/4}(ae^2+cd^2)}-\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}+\frac{\sqrt[4]{c}\,e^2\left(\sqrt{a}\,e+\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{3/4}(ae^2+cd^2)^2}-\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(ae^2+cd^2)}+\frac{\sqrt[4]{c}\left(\sqrt{a}\,e+3\sqrt{c}\,d\right)\log\left(\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{7/4}(ae^2+cd^2)}+\frac{e^{7/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2+cd^2)^2}+\frac{cx(d-ex^2)}{4a(a+cx^4)(ae^2+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (c*x*(d - e*x^2))/(4*a*(c*d^2 + a*e^2)*(a + c*x^4)) + (e^(7/2)*ArcTan[(Sqrt
[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 + a*e^2)^2) - (c^(1/4)*e^2*(Sqrt[c]*d - Sq
rt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)*(c*d^2
 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/
4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]
*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(3/4)
*(c*d^2 + a*e^2)^2) + (c^(1/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 + (Sqrt[2
]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2)) - (c^(1/4)*e^2*(
Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^
2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2)^2) - (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]
*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(
```

```
7/4)*(c*d^2 + a*e^2)) + (c^(1/4)*e^2*(Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] +
Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(3/4)*(c*d^2 + a*e^2
)^2) + (c^(1/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(
1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(7/4)*(c*d^2 + a*e^2))
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
```

```
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1253

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(d + e*x^2)^q*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e,
p, q}, x] && ((IntegerQ[p] && IntegerQ[q]) || IGtQ[p, 0])
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+cx^4)^2}\,dx = \int \left(\frac{e^4}{(cd^2+ae^2)^2(d+ex^2)} + \frac{c(d-ex^2)}{(cd^2+ae^2)(a+cx^4)^2} - \frac{ce^2(-d+ex^2)}{(cd^2+ae^2)^2(a+cx^4)}\right)$$

$$= -\frac{(ce^2)\int \frac{-d+ex^2}{a+cx^4}\,dx}{(cd^2+ae^2)^2} + \frac{e^4\int \frac{1}{d+ex^2}\,dx}{(cd^2+ae^2)^2} + \frac{c\int \frac{d-ex^2}{(a+cx^4)^2}\,dx}{cd^2+ae^2}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} + \frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}} - e\right)e^2\right)\int \frac{\sqrt{a}\,\sqrt{c}}{a+cx^4}}{2(cd^2+ae^2)^2}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} + \frac{\left(\left(\frac{\sqrt{c}\,d}{\sqrt{a}} - e\right)e^2\right)\int \frac{}{\frac{\sqrt{a}}{\sqrt{c}} - \sqrt{}}}{4(cd^2+ae^2)^2}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\,e^2(\sqrt{c}\,d+\sqrt{a}\,e)\log\left(\sqrt{\right.}}{4\sqrt{2}\,a^{3/4}(}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\,e^2(\sqrt{c}\,d-\sqrt{a}\,e)\tan^{-1}\left(\right.}{2\sqrt{2}\,a^{3/4}(cd^2+a}$$

$$= \frac{cx(d-ex^2)}{4a(cd^2+ae^2)(a+cx^4)} + \frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2+ae^2)^2} - \frac{\sqrt[4]{c}\,e^2(\sqrt{c}\,d-\sqrt{a}\,e)\tan^{-1}\left(\right.}{2\sqrt{2}\,a^{3/4}(cd^2+a}$$

Mathematica [A]
time = 0.17, size = 429, normalized size = 0.62

$$\frac{\frac{8c(cd^2+ae^2)x(d-ex^2)}{a(a+cx^4)} + \frac{32e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}} + \frac{2\sqrt{2}\,\sqrt[4]{c}\left(-3c^{3/2}d^3+\sqrt{a}\,cd^2e-7a\sqrt{c}\,de^2+5a^{3/2}e^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{2\sqrt{2}\,\sqrt[4]{c}\left(-3c^{3/2}d^3+\sqrt{a}\,cd^2e-7a\sqrt{c}\,de^2+5a^{3/2}e^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{7/4}} - \frac{\sqrt{2}\,\sqrt[4]{c}\left(3c^{3/2}d^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+5a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{7/4}} + \frac{\sqrt{2}\,\sqrt[4]{c}\left(3c^{3/2}d^3+\sqrt{a}\,cd^2e+7a\sqrt{c}\,de^2+5a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{7/4}}}{32(cd^2+ae^2)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] ((8*c*(c*d^2 + a*e^2)*x*(d - e*x^2))/(a*(a + c*x^4)) + (32*e^(7/2)*ArcTan[(
Sqrt[e]*x)/Sqrt[d]])/Sqrt[d] + (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]
*c*d^2*e - 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x
)/a^(1/4)])/a^(7/4) - (2*Sqrt[2]*c^(1/4)*(-3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e
- 7*a*Sqrt[c]*d*e^2 + 5*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)
])/a^(7/4) - (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqrt[c
```

```
]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*
x^2])/a^(7/4) + (Sqrt[2]*c^(1/4)*(3*c^(3/2)*d^3 + Sqrt[a]*c*d^2*e + 7*a*Sqr
t[c]*d*e^2 + 5*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[
c]*x^2])/a^(7/4))/(32*(c*d^2 + a*e^2)^2)
```

Maple [A]
time = 0.17, size = 334, normalized size = 0.48

method	result
default	$\frac{e^{4} \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{\left(a \,e^{2}+c \,d^{2}\right)^{2} \sqrt{d e}}+\frac{c \left(\frac{-\frac{e \left(a \,e^{2}+c \,d^{2}\right) x^{3}}{4 a}+\frac{d \left(a \,e^{2}+c \,d^{2}\right) x}{4 a}}{c \,x^{4}+a}+\frac{\frac{\left(7 d \,e^{2} a +3 c \,d^{3}\right) \left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\, \left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+2 \arctan\left(\right.\right.}{8 a}}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] e^4/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))+c/(a*e^2+c*d^2)^2*(
(-1/4*e*(a*e^2+c*d^2)/a*x^3+1/4*d*(a*e^2+c*d^2)/a*x)/(c*x^4+a)+1/4/a*(1/8*(
7*a*d*e^2+3*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*2^(1/2)+(a/
c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(
1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/8*(-5*a*e^3-c*d^2*e)/c/(a/c)
^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2+(a/c)^(1/4)
*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)
/(a/c)^(1/4)*x-1))))
```

Maxima [A]
time = 0.52, size = 489, normalized size = 0.71

$$\frac{c \left(\frac{2 \sqrt{2} \left(3 c^{\frac{3}{2}} d^{3}-\sqrt{a} c d^{2} e+7 a \sqrt{c} d e^{2}-5 a^{\frac{3}{2}} e^{3}\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{c} x+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{2 \sqrt{2} \left(3 c^{\frac{3}{2}} d^{3}-\sqrt{a} c d^{2} e+7 a \sqrt{c} d e^{2}-5 a^{\frac{3}{2}} e^{3}\right) \arctan\left(\frac{\sqrt{2} \left(2 \sqrt{c} x-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{\sqrt{2} \left(3 c^{\frac{3}{2}} d^{3}+\sqrt{a} c d^{2} e+7 a \sqrt{c} d e^{2}+5 a^{\frac{3}{2}} e^{3}\right) \log\left(\sqrt{c} x^{2}+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}-\frac{\sqrt{2} \left(3 c^{\frac{3}{2}} d^{3}+\sqrt{a} c d^{2} e+7 a \sqrt{c} d e^{2}+5 a^{\frac{3}{2}} e^{3}\right) \log\left(\sqrt{c} x^{2}-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}\right)}{32 \left(a c^{2} d^{4}+2 a^{2} c d^{2} e^{2}+a^{3} e^{4}\right)}+\frac{\arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\frac{7}{2}}}{\left(c^{2} d^{4}+2 a c d^{2} e^{2}+a^{2} e^{4}\right) \sqrt{d}}-\frac{\left(c^{2} d^{2} e+a c e^{3}\right) x^{3}-\left(c^{2} d^{3}+a c d e^{2}\right) x}{4 \left(a^{2} c^{2} d^{4}+2 a^{3} c d^{2} e^{2}+\left(a c^{3} d^{4}+2 a^{2} c^{2} d^{2} e^{2}+a^{3} c e^{4}\right) x^{4}+a^{4} e^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] 1/32*c*(2*sqrt(2)*(3*c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5*
a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(2)*(3*
c^(3/2)*d^3 - sqrt(a)*c*d^2*e + 7*a*sqrt(c)*d*e^2 - 5*a^(3/2)*e^3)*arctan(1
/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(
```

```
sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(3*c^(3/2)*d^3 + sqrt(a)*c
*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 + sqrt(2)*a^(1/
4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(3*c^(3/2)*d^3 + sqrt(a
)*c*d^2*e + 7*a*sqrt(c)*d*e^2 + 5*a^(3/2)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a^
(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a*c^2*d^4 + 2*a^2*c*d^2*e^2
+ a^3*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c^2*d^4 + 2*a*c*d^2*e^2 +
a^2*e^4)*sqrt(d)) - 1/4*((c^2*d^2*e + a*c*e^3)*x^3 - (c^2*d^3 + a*c*d*e^2)*
x)/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + (a*c^3*d^4 + 2*a^2*c^2*d^2*e^2 + a^3*c*
e^4)*x^4 + a^4*e^4)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4630 vs. $2(511) = 1022$.
time = 12.50, size = 9294, normalized size = 13.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*c^2*d^2*x^3*e - 4*c^2*d^3*x + 4*a*c*x^3*e^3 - 4*a*c*d*x*e^2 - 8*(
a*c*x^4 + a^2)*sqrt(-e/d)*e^3*log((x^2*e + 2*d*x*sqrt(-e/d) - d)/(x^2*e + d
)) + (a*c^3*d^4*x^4 + a^2*c^2*d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*
x^4 + a^3*c*d^2)*e^2)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5
 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 +
 a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 +
2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^
6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^1
0*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d
^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2
 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-81*c^5*d^8*x - 594*
a*c^4*d^6*x*e^2 - 1376*a^2*c^3*d^4*x*e^4 - 750*a^3*c^2*d^2*x*e^6 + 625*a^4*
c*x*e^8 + (27*a^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198
*a^5*c^2*d^3*e^6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 +
26*a^8*c^3*d^6*e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*
sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c
^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/
(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10
*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 +
8*a^14*c*d^2*e^14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*
a^2*c*d*e^5 + (a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*
c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5
*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^
10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e
^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28
*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*
```

```
c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) - (a*c^3*d^4
*x^4 + a^2*c^2*d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^4 + a^3*c*d^2
)*e^2)*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (a^3*c^4*d^8
 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(
-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^
6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*
c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6
+ 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^1
4*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^
4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))*log(-81*c^5*d^8*x - 594*a*c^4*d^6*x*e^2
 - 1376*a^2*c^3*d^4*x*e^4 - 750*a^3*c^2*d^2*x*e^6 + 625*a^4*c*x*e^8 - (27*a
^2*c^5*d^9 + 186*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^
6 - 175*a^6*c*d*e^8 + (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*
e^5 + 34*a^9*c^2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d
^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 52
9*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 +
 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*
c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^
14 + a^15*e^16)))*sqrt((6*c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 + (
a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7
*e^8)*sqrt(-(81*c^7*d^12 + 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748
*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*
e^12)/(a^7*c^8*d^16 + 8*a^8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^
5*d^10*e^6 + 70*a^11*c^4*d^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e
^12 + 8*a^14*c*d^2*e^14 + a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6
*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8))) + (a*c^3*d^4*x^4 + a^2*c^2*
d^4 + (a^3*c*x^4 + a^4)*e^4 + 2*(a^2*c^2*d^2*x^4 + a^3*c*d^2)*e^2)*sqrt((6*
c^3*d^5*e + 44*a*c^2*d^3*e^3 + 70*a^2*c*d*e^5 - (a^3*c^4*d^8 + 4*a^4*c^3*d^
6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c*d^2*e^6 + a^7*e^8)*sqrt(-(81*c^7*d^12 +
 738*a*c^6*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4
*c^3*d^4*e^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a^7*c^8*d^16 + 8*a^
8*c^7*d^14*e^2 + 28*a^9*c^6*d^12*e^4 + 56*a^10*c^5*d^10*e^6 + 70*a^11*c^4*d
^8*e^8 + 56*a^12*c^3*d^6*e^10 + 28*a^13*c^2*d^4*e^12 + 8*a^14*c*d^2*e^14 +
a^15*e^16)))/(a^3*c^4*d^8 + 4*a^4*c^3*d^6*e^2 + 6*a^5*c^2*d^4*e^4 + 4*a^6*c
*d^2*e^6 + a^7*e^8))*log(-81*c^5*d^8*x - 594*a*c^4*d^6*x*e^2 - 1376*a^2*c^3
*d^4*x*e^4 - 750*a^3*c^2*d^2*x*e^6 + 625*a^4*c*x*e^8 + (27*a^2*c^5*d^9 + 18
6*a^3*c^4*d^7*e^2 + 404*a^4*c^3*d^5*e^4 + 198*a^5*c^2*d^3*e^6 - 175*a^6*c*d
*e^8 - (a^6*c^5*d^10*e + 9*a^7*c^4*d^8*e^3 + 26*a^8*c^3*d^6*e^5 + 34*a^9*c^
2*d^4*e^7 + 21*a^10*c*d^2*e^9 + 5*a^11*e^11)*sqrt(-(81*c^7*d^12 + 738*a*c^6
*d^10*e^2 + 2383*a^2*c^5*d^8*e^4 + 2748*a^3*c^4*d^6*e^6 - 529*a^4*c^3*d^4*e
^8 - 1950*a^5*c^2*d^2*e^10 + 625*a^6*c*e^12)/(a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.46, size = 603, normalized size = 0.88

$$\frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^3+7\,(ac^3)^{\frac{1}{4}}ac^2de^2-(ac^3)^{\frac{3}{4}}cd^2e-5\,(ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2c^4d^4+2\sqrt{2}\,a^3c^3d^2e^2+\sqrt{2}\,a^4c^2e^4\right)}+\frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^3+7\,(ac^3)^{\frac{1}{4}}ac^2de^2-(ac^3)^{\frac{3}{4}}cd^2e-5\,(ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^2c^4d^4+2\sqrt{2}\,a^3c^3d^2e^2+\sqrt{2}\,a^4c^2e^4\right)}+\frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^3+7\,(ac^3)^{\frac{1}{4}}ac^2de^2+(ac^3)^{\frac{3}{4}}cd^2e+5\,(ac^3)^{\frac{3}{4}}ae^3\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,a^2c^4d^4+2\sqrt{2}\,a^3c^3d^2e^2+\sqrt{2}\,a^4c^2e^4\right)}-\frac{\left(3\,(ac^3)^{\frac{1}{4}}c^3d^3+7\,(ac^3)^{\frac{1}{4}}ac^2de^2+(ac^3)^{\frac{3}{4}}cd^2e+5\,(ac^3)^{\frac{3}{4}}ae^3\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,a^2c^4d^4+2\sqrt{2}\,a^3c^3d^2e^2+\sqrt{2}\,a^4c^2e^4\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{7}{2}}}{(c^2d^4+2\,acd^2e^2+a^2e^4)\sqrt{d}}-\frac{cx^3e-cdx}{4\,(cx^4+a)(acd^2+a^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2 - (a*c^3)^(3/4)*
c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c)^(1
/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3*d^2*e^2 + sqrt(2
)*a^4*c^2*e^4) + 1/8*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)^(1/4)*a*c^2*d*e^2
 - (a*c^3)^(3/4)*c*d^2*e - 5*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x -
 sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*c^3
*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c^3)
^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(x^2
 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a^3*
c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) - 1/16*(3*(a*c^3)^(1/4)*c^3*d^3 + 7*(a*c
^3)^(1/4)*a*c^2*d*e^2 + (a*c^3)^(3/4)*c*d^2*e + 5*(a*c^3)^(3/4)*a*e^3)*log(
x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^2*c^4*d^4 + 2*sqrt(2)*a
^3*c^3*d^2*e^2 + sqrt(2)*a^4*c^2*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/(
(c^2*d^4 + 2*a*c*d^2*e^2 + a^2*e^4)*sqrt(d)) - 1/4*(c*x^3*e - c*d*x)/((c*x^
4 + a)*(a*c*d^2 + a^2*e^2))
```

Mupad [B]
time = 2.73, size = 2500, normalized size = 3.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] ((c*d*x)/(4*a*(a*e^2 + c*d^2)) - (c*e*x^3)/(4*a*(a*e^2 + c*d^2)))/(a + c*x^
4) - atan((((((65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c
^10*d^12*e^4 - 36864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8
*c^7*d^6*e^10 + 663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a
^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*
e^4)) - (x*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c
^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^
(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a
```

```
^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*
c^4*e^17 - 65536*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*
c^9*d^10*e^7 - 327680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a
^11*c^6*d^4*e^13 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 +
 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a
^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*
e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(
-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*
d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1152*a^2*c^11*d^13*e^2 - 49024*a
^8*c^5*d*e^14 + 7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c
^8*d^7*e^8 - 66688*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e
^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)
))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e
 + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) +
39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^
2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e
^3 + 20432*a^6*c^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 +
21024*a^4*c^7*d^5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (x*(1425*a^4*c^5*e^13 + 81*c^9*d^8
*e^5 + 612*a*c^8*d^6*e^7 + 1894*a^2*c^7*d^4*e^9 + 2532*a^3*c^6*d^2*e^11))/(
128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^
2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4
*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c
)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4
*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*1i - (((((
65536*a^11*c^4*e^16 - 12288*a^4*c^11*d^14*e^2 - 57344*a^5*c^10*d^12*e^4 - 3
6864*a^6*c^9*d^10*e^6 + 245760*a^7*c^8*d^8*e^8 + 634880*a^8*c^7*d^6*e^10 +
663552*a^9*c^6*d^4*e^12 + 331776*a^10*c^5*d^2*e^14)/(256*(a^8*e^8 + a^4*c^4
*d^8 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)) + (x*((9*c
^3*d^6*(-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^
5*c^2*d^3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c
*d^2*e^4*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 +
4*a^8*c^3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2)*(65536*a^13*c^4*e^17 - 65536
*a^6*c^11*d^14*e^3 - 327680*a^7*c^10*d^12*e^5 - 589824*a^8*c^9*d^10*e^7 - 3
27680*a^9*c^8*d^8*e^9 + 327680*a^10*c^7*d^6*e^11 + 589824*a^11*c^6*d^4*e^13
 + 327680*a^12*c^5*d^2*e^15))/(128*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e^6
 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) - 25
*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c*d
*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))/(
256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^9*
c^2*d^4*e^4)))^(1/2) + (x*(1152*a^2*c^11*d^13*e^2 - 49024*a^8*c^5*d*e^14 +
```

```
7936*a^3*c^10*d^11*e^4 + 20352*a^4*c^9*d^9*e^6 + 8704*a^5*c^8*d^7*e^8 - 666
88*a^6*c^7*d^5*e^10 - 110848*a^7*c^6*d^3*e^12))/(128*(a^8*e^8 + a^4*c^4*d^8
 + 4*a^7*c*d^2*e^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(
-a^7*c)^(1/2) - 25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^
3*e^3 + 70*a^6*c*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4
*(-a^7*c)^(1/2))/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^
3*d^6*e^2 + 6*a^9*c^2*d^4*e^4)))^(1/2) - (720*a*c^10*d^11*e^3 + 20432*a^6*c
^5*d*e^13 + 4880*a^2*c^9*d^9*e^5 + 12320*a^3*c^8*d^7*e^7 + 21024*a^4*c^7*d^
5*e^9 + 33296*a^5*c^6*d^3*e^11)/(256*(a^8*e^8 + a^4*c^4*d^8 + 4*a^7*c*d^2*e
^6 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4)))*((9*c^3*d^6*(-a^7*c)^(1/2) -
25*a^3*e^6*(-a^7*c)^(1/2) + 6*a^4*c^3*d^5*e + 44*a^5*c^2*d^3*e^3 + 70*a^6*c
*d*e^5 + 41*a*c^2*d^4*e^2*(-a^7*c)^(1/2) + 39*a^2*c*d^2*e^4*(-a^7*c)^(1/2))
/(256*(a^11*e^8 + a^7*c^4*d^8 + 4*a^10*c*d^2*e^6 + 4*a^8*c^3*d^6*e^2 + 6*a^
9*c^2*d^4*e^4)))^(1/2) + (x*(1425*a^4*c^5*e^13 ...
```

3.257 $\int \frac{1}{x^2(d+ex^2)(a+cx^4)^2}\,dx$

Optimal. Leaf size=745

$$-\frac{1}{a^2dx}-\frac{cx(ae+cdx^2)}{4a^2(cd^2+ae^2)(a+cx^4)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}(cd^2+ae^2)^2}+\frac{c^{3/4}(\sqrt{c}\,d+3\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{8\sqrt{2}\,a^{9/4}(cd^2+ae^2)}+\frac{c^{3/4}\cdots}{}$$

```
[Out] -1/a^2/d/x-1/4*c*x*(c*d*x^2+a*e)/a^2/(a*e^2+c*d^2)/(c*x^4+a)-e^(9/2)*arctan
(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2+c*d^2)^2-1/32*c^(3/4)*ln(-a^(1/4)*c^(1/4
)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(-3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*
d^2)*2^(1/2)+1/32*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))
*(-3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)*2^(1/2)-1/16*c^(3/4)*arctan
(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c^(1/2))/a^(9/4)/(a*e^2+c*d^2
)*2^(1/2)-1/16*c^(3/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(3*e*a^(1/2)+d*c
^(1/2))/a^(9/4)/(a*e^2+c*d^2)*2^(1/2)+1/8*c^(3/4)*ln(-a^(1/4)*c^(1/4)*x*2^(
1/2)+a^(1/2)+x^2*c^(1/2))*(a^(3/2)*e^3-d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(
a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(3/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2
*c^(1/2))*(a^(3/2)*e^3-d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)^2*2
^(1/2)-1/4*c^(3/4)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(a^(3/2)*e^3+d*(2*a
*e^2+c*d^2)*c^(1/2))/a^(9/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/4*c^(3/4)*arctan(1+c
^(1/4)*x*2^(1/2)/a^(1/4))*(a^(3/2)*e^3+d*(2*a*e^2+c*d^2)*c^(1/2))/a^(9/4)/(
a*e^2+c*d^2)^2*2^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 745, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1350, 211, 1193, 1182, 1176, 631, 210, 1179, 642}

$$\frac{c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(3\sqrt{a}\,e+\sqrt{c}\,d)}{8\sqrt{2}\,a^{9/4}(ae^2+cd^2)}-\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(3\sqrt{a}\,e+\sqrt{c}\,d)}{8\sqrt{2}\,a^{9/4}(ae^2+cd^2)}+\frac{c^{3/4}\text{ArcTan}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)(a^{3/2}e^3+\sqrt{c}\,d(2ae^2+cd^2))}{2\sqrt{2}\,a^{9/4}(ae^2+cd^2)^2}-\frac{c^{3/4}\text{ArcTan}\left(\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}+1\right)(a^{3/2}e^3+\sqrt{c}\,d(2ae^2+cd^2))}{2\sqrt{2}\,a^{9/4}(ae^2+cd^2)^2}-\frac{c^{3/4}(\sqrt{c}\,d-3\sqrt{a}\,e)\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}(ae^2+cd^2)}+\frac{c^{3/4}(\sqrt{c}\,d-3\sqrt{a}\,e)\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{16\sqrt{2}\,a^{9/4}(ae^2+cd^2)}+\frac{c^{3/4}(a^{3/2}e^3-\sqrt{c}\,d(2ae^2+cd^2))\log\left(-\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(ae^2+cd^2)^2}-\frac{c^{3/4}(a^{3/2}e^3-\sqrt{c}\,d(2ae^2+cd^2))\log\left(\sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x+\sqrt{a}+\sqrt{c}\,x^2\right)}{4\sqrt{2}\,a^{9/4}(ae^2+cd^2)^2}-\frac{cx(ae+cdx^2)}{4a^2(a+cx^4)(ae^2+cd^2)}-\frac{1}{a^2dx}-\frac{e^{9/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}(ae^2+cd^2)^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -(1/(a^2*d*x)) - (c*x*(a*e + c*d*x^2))/(4*a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (c^(3
/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*S
qrt[2]*a^(9/4)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 + Sqrt[c]*d*(c*d^2
+ 2*a*e^2))*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(9/4)*(c*
d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d + 3*Sqrt[a]*e)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) - (c^(3/4)*(a^(3/2)*
e^3 + Sqrt[c]*d*(c*d^2 + 2*a*e^2))*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])
/(2*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2) - (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)
*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4
```

```
)*(c*d^2 + a*e^2)) + (c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*L
og[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(
c*d^2 + a*e^2)^2) + (c^(3/4)*(Sqrt[c]*d - 3*Sqrt[a]*e)*Log[Sqrt[a] + Sqrt[2
]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)) -
(c^(3/4)*(a^(3/2)*e^3 - Sqrt[c]*d*(c*d^2 + 2*a*e^2))*Log[Sqrt[a] + Sqrt[2]*
a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(9/4)*(c*d^2 + a*e^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\int \frac{1}{x^2\left(d+ex^2\right)\left(a+cx^4\right)^2}\,dx=\int \left(\frac{1}{a^2dx^2}-\frac{e^5}{d\left(cd^2+ae^2\right)^2\left(d+ex^2\right)}-\frac{c(ae+cdx^2)}{a\left(cd^2+ae^2\right)\left(a+cx^4\right)^2}+\frac{c(-a^2e^3\cdots}{a^2\left(cd^2\cdots\right.}\right.$$

$$=-\frac{1}{a^2dx}+\frac{c\int \frac{-a^2e^3-cd\left(cd^2+2ae^2\right)x^2}{a+cx^4}\,dx}{a^2\left(cd^2+ae^2\right)^2}-\frac{e^5\int \frac{1}{d+ex^2}\,dx}{d\left(cd^2+ae^2\right)^2}-\frac{c\int \frac{ae+cdx^2}{\left(a+cx^4\right)^2}\,dx}{a\left(cd^2+ae^2\right)}$$

$$=-\frac{1}{a^2dx}-\frac{cx(ae+cdx^2)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{c\int \frac{-3ae-cdx^2}{a+cx^4}\,dx}{4a^2\left(cd^2+ae^2\right)\cdots}$$

$$=-\frac{1}{a^2dx}-\frac{cx(ae+cdx^2)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{\left(c\left(d-\frac{3\sqrt{a}\,e}{\sqrt{c}}\cdots\right.\right.}{8a^2\left(\cdots\right.}$$

$$=-\frac{1}{a^2dx}-\frac{cx(ae+cdx^2)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}-\frac{c^{5/4}\left(cd^3+2ade\cdots\right.}{\cdots}$$

$$=-\frac{1}{a^2dx}-\frac{cx(ae+cdx^2)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{c^{5/4}\left(cd^3+2ade\cdots\right.}{2\cdots}$$

$$=-\frac{1}{a^2dx}-\frac{cx(ae+cdx^2)}{4a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{c^{3/4}\left(\sqrt{c}\,d+3\sqrt{\cdots}\right.}{8\sqrt{2}\cdots}$$

Mathematica [A]
time = 0.25, size = 499, normalized size = 0.67

$$\frac{1}{32}\left(-\frac{32}{a^2dx}-\frac{8cx(ae+cdx^2)}{a^2\left(cd^2+ae^2\right)\left(a+cx^4\right)}-\frac{32e^{9/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}\left(cd^2+ae^2\right)^2}+\frac{2\sqrt{2}\,c^{3/4}\left(5c^{3/2}d^3+3\sqrt{a}\,cd^2e+9a\sqrt{c}\,de^2+7a^{3/2}e^3\right)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{9/4}\left(cd^2+ae^2\right)^2}-\frac{2\sqrt{2}\,c^{3/4}\left(5c^{3/2}d^3+3\sqrt{a}\,cd^2e+9a\sqrt{c}\,de^2+7a^{3/2}e^3\right)\tan^{-1}\left(1+\frac{\sqrt{2}\,\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{9/4}\left(cd^2+ae^2\right)^2}+\frac{\sqrt{2}\,c^{3/4}\left(-5c^{3/2}d^3+3\sqrt{a}\,cd^2e-9a\sqrt{c}\,de^2+7a^{3/2}e^3\right)\log\left(\sqrt{a}-\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{9/4}\left(cd^2+ae^2\right)^2}+\frac{\sqrt{2}\,c^{3/4}\left(5c^{3/2}d^3-3\sqrt{a}\,cd^2e+9a\sqrt{c}\,de^2-7a^{3/2}e^3\right)\log\left(\sqrt{a}+\sqrt{2}\,\sqrt[4]{a}\,\sqrt[4]{c}\,x+\sqrt{c}\,x^2\right)}{a^{9/4}\left(cd^2+ae^2\right)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x) - (8*c*x*(a*e + c*d*x^2))/(a^2*(c*d^2 + a*e^2)*(a + c*x^4))
- (32*e^(9/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 + a*e^2)^2) + (2
*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 + 7
*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)*(c*d^2 + a*
e^2)^2) - (2*Sqrt[2]*c^(3/4)*(5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[
c]*d*e^2 + 7*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(a^(9/4)
*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(-5*c^(3/2)*d^3 + 3*Sqrt[a]*c*d^2*e
```

```
- 9*a*Sqrt[c]*d*e^2 + 7*a^(3/2)*e^3)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*
x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2) + (Sqrt[2]*c^(3/4)*(5*c^(3/2)
*d^3 - 3*Sqrt[a]*c*d^2*e + 9*a*Sqrt[c]*d*e^2 - 7*a^(3/2)*e^3)*Log[Sqrt[a] +
 Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(9/4)*(c*d^2 + a*e^2)^2))/32
```

Maple [A]
time = 0.20, size = 355, normalized size = 0.48

method	result
default	$-\frac{e^{5}\arctan\left(\frac{ex}{\sqrt{de}}\right)}{d\left(a\,e^{2}+c\,d^{2}\right)^{2}\sqrt{de}}-\frac{c\left(\frac{\left(\frac{1}{4}acd\,e^{2}+\frac{1}{4}c^{2}d^{3}\right)x^{3}+\left(\frac{1}{4}a^{2}e^{3}+\frac{1}{4}a\,d^{2}ec\right)x}{c\,x^{4}+a}+\frac{\left(7a^{2}e^{3}+3a\,d^{2}ec\right)\left(\frac{a}{c}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{x^{2}+\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}{x^{2}-\left(\frac{a}{c}\right)^{\frac{1}{4}}x\sqrt{2}+\sqrt{\frac{a}{c}}}\right.\right.}{32a}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*e^5/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-c/(a*e^2+c*d^2
)^2/a^2*(((1/4*a*c*d*e^2+1/4*c^2*d^3)*x^3+(1/4*a^2*e^3+1/4*a*d^2*e*c)*x)/(c
*x^4+a)+1/32*(7*a^2*e^3+3*a*c*d^2*e)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(
1/4)*x*2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arct
an(2^(1/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/32*(9*a*c*
d*e^2+5*c^2*d^3)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)
^(1/2))/(x^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/
4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1)))-1/a^2/d/x
```

Maxima [A]
time = 0.52, size = 506, normalized size = 0.68

$$-\frac{c\left(\frac{2\sqrt{2}\left(5\sqrt{a}\,c^{2}d^{3}+3\,ac^{\frac{3}{2}}d^{2}e+9\,a^{\frac{3}{2}}cde^{2}+7\,a^{2}\sqrt{c}\,e^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}+\frac{2\sqrt{2}\left(5\sqrt{a}\,c^{2}d^{3}+3\,ac^{\frac{3}{2}}d^{2}e+9\,a^{\frac{3}{2}}cde^{2}+7\,a^{2}\sqrt{c}\,e^{3}\right)\arctan\left(\frac{\sqrt{2}\left(2\sqrt{c}\,x-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}\right)}{2\sqrt{\sqrt{a}\sqrt{c}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{c}}\sqrt{c}}-\frac{\sqrt{2}\left(5\sqrt{a}\,c^{2}d^{3}-3\,ac^{\frac{3}{2}}d^{2}e+9\,a^{\frac{3}{2}}cde^{2}-7\,a^{2}\sqrt{c}\,e^{3}\right)\log\left(\sqrt{c}\,x^{2}+\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}+\frac{\sqrt{2}\left(5\sqrt{a}\,c^{2}d^{3}-3\,ac^{\frac{3}{2}}d^{2}e+9\,a^{\frac{3}{2}}cde^{2}-7\,a^{2}\sqrt{c}\,e^{3}\right)\log\left(\sqrt{c}\,x^{2}-\sqrt{2}\,a^{\frac{1}{4}}c^{\frac{1}{4}}x+\sqrt{a}\right)}{a^{\frac{3}{4}}c^{\frac{3}{4}}}\right)}{32\left(a^{2}c^{2}d^{4}+2\,a^{3}cd^{2}e^{2}+a^{4}e^{4}\right)}-\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{9}{2}}}{\left(c^{2}d^{5}+2\,acd^{3}e^{2}+a^{2}de^{4}\right)\sqrt{d}}-\frac{acdx^{2}e+\left(5\,c^{2}d^{2}+4\,ace^{2}\right)x^{4}+4\,acd^{2}+4\,a^{2}e^{2}}{4\left(\left(a^{2}c^{2}d^{3}+a^{3}cde^{2}\right)x^{5}+\left(a^{3}cd^{3}+a^{4}de^{2}\right)x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*c*(2*sqrt(2)*(5*sqrt(a)*c^2*d^3 + 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e
^2 + 7*a^2*sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c
^(1/4))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*
sqrt(2)*(5*sqrt(a)*c^2*d^3 + 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e^2 + 7*a^2*
sqrt(c)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqr
t(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) - sqrt(2)*(5*sq
rt(a)*c^2*d^3 - 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e^2 - 7*a^2*sqrt(c)*e^3)*
log(sqrt(c)*x^2 + sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) +
```

```
sqrt(2)*(5*sqrt(a)*c^2*d^3 - 3*a*c^(3/2)*d^2*e + 9*a^(3/2)*c*d*e^2 - 7*a^2*
sqrt(c)*e^3)*log(sqrt(c)*x^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4
)*c^(3/4)))/(a^2*c^2*d^4 + 2*a^3*c*d^2*e^2 + a^4*e^4) - arctan(x*e^(1/2)/sq
rt(d))*e^(9/2)/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) - 1/4*(a*c*d
*x^2*e + (5*c^2*d^2 + 4*a*c*e^2)*x^4 + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c^2*d^3
 + a^3*c*d*e^2)*x^5 + (a^3*c*d^3 + a^4*d*e^2)*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4783 vs. $2(558) = 1116$.

time = 28.27, size = 9600, normalized size = 12.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/16*(20*c^3*d^4*x^4 + 4*a*c^2*d^3*x^2*e + 16*a*c^2*d^4 + 4*a^2*c*d*x^2*e
^3 - 8*(a^2*c*x^5 + a^3*x)*sqrt(-e/d)*e^4*log((x^2*e - 2*d*x*sqrt(-e/d) - d
)/(x^2*e + d)) - (a^2*c^3*d^5*x^5 + a^3*c^2*d^5*x + (a^4*c*d*x^5 + a^5*d*x)
*e^4 + 2*(a^3*c^2*d^3*x^5 + a^4*c*d^3*x)*e^2)*sqrt(-(30*c^4*d^5*e + 124*a*c
^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c
^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^
10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8
 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^
14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8
+ 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^
16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^
6 + a^8*e^8))*log(-625*c^6*d^8*x - 3250*a*c^5*d^6*x*e^2 - 4944*a^2*c^4*d^4*
x*e^4 - 686*a^3*c^3*d^2*x*e^6 + 2401*a^4*c^2*x*e^8 + (75*a^3*c^5*d^8*e + 41
8*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e
^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^
2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a
*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*
d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10
*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d
^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 +
a^17*e^16)))*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 +
(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^
8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3
868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a
^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 5
6*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*
c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^
6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))) + (a^2*c^3*d^5*x^5
 + a^3*c^2*d^5*x + (a^4*c*d*x^5 + a^5*d*x)*e^4 + 2*(a^3*c^2*d^3*x^5 + a^4*c
```

```
*d^3*x)*e^2)*sqrt(-(30*c^4*d^5*e + 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 +
(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^
8*e^8)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3
868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a
^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 5
6*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*
c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^
6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8))*log(-625*c^6*d^8*x
- 3250*a*c^5*d^6*x*e^2 - 4944*a^2*c^4*d^4*x*e^4 - 686*a^3*c^3*d^2*x*e^6 + 2
401*a^4*c^2*x*e^8 - (75*a^3*c^5*d^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d
^4*e^5 + 126*a^6*c^2*d^2*e^7 - 343*a^7*c*e^9 - (5*a^7*c^5*d^11 + 29*a^8*c^4
*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 74*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9
*a^12*d*e^10)*sqrt(-(625*c^9*d^12 + 4050*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*
e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 +
 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*
e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 2
8*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14 + a^17*e^16)))*sqrt(-(30*c^4*d^5*e
+ 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 + (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050
*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^
5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^
10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4
*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
+ a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7
*c*d^2*e^6 + a^8*e^8))) - (a^2*c^3*d^5*x^5 + a^3*c^2*d^5*x + (a^4*c*d*x^5 +
 a^5*d*x)*e^4 + 2*(a^3*c^2*d^3*x^5 + a^4*c*d^3*x)*e^2)*sqrt(-(30*c^4*d^5*e
+ 124*a*c^3*d^3*e^3 + 126*a^2*c^2*d*e^5 - (a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2
+ 6*a^6*c^2*d^4*e^4 + 4*a^7*c*d^2*e^6 + a^8*e^8)*sqrt(-(625*c^9*d^12 + 4050
*a*c^8*d^10*e^2 + 8511*a^2*c^7*d^8*e^4 + 3868*a^3*c^6*d^6*e^6 - 6417*a^4*c^
5*d^4*e^8 - 3822*a^5*c^4*d^2*e^10 + 2401*a^6*c^3*e^12)/(a^9*c^8*d^16 + 8*a^
10*c^7*d^14*e^2 + 28*a^11*c^6*d^12*e^4 + 56*a^12*c^5*d^10*e^6 + 70*a^13*c^4
*d^8*e^8 + 56*a^14*c^3*d^6*e^10 + 28*a^15*c^2*d^4*e^12 + 8*a^16*c*d^2*e^14
+ a^17*e^16)))/(a^4*c^4*d^8 + 4*a^5*c^3*d^6*e^2 + 6*a^6*c^2*d^4*e^4 + 4*a^7
*c*d^2*e^6 + a^8*e^8))*log(-625*c^6*d^8*x - 3250*a*c^5*d^6*x*e^2 - 4944*a^2
*c^4*d^4*x*e^4 - 686*a^3*c^3*d^2*x*e^6 + 2401*a^4*c^2*x*e^8 + (75*a^3*c^5*d
^8*e + 418*a^4*c^4*d^6*e^3 + 684*a^5*c^3*d^4*e^5 + 126*a^6*c^2*d^2*e^7 - 34
3*a^7*c*e^9 + (5*a^7*c^5*d^11 + 29*a^8*c^4*d^9*e^2 + 66*a^9*c^3*d^7*e^4 + 7
4*a^10*c^2*d^5*e^6 + 41*a^11*c*d^3*e^8 + 9*a^12...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.20, size = 639, normalized size = 0.86

$$-\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+5\left(ac^3\right)^{\frac{3}{4}}cd^3+7\left(ac^3\right)^{\frac{1}{4}}a^2ce^3+9\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4+2\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)}-\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e+5\left(ac^3\right)^{\frac{3}{4}}cd^3+7\left(ac^3\right)^{\frac{1}{4}}a^2ce^3+9\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}a^3c^3d^4+2\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)}-\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e-5\left(ac^3\right)^{\frac{3}{4}}cd^3+7\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-9\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\log\left(x^2+\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4+2\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)}+\frac{\left(3\left(ac^3\right)^{\frac{1}{4}}ac^2d^2e-5\left(ac^3\right)^{\frac{3}{4}}cd^3+7\left(ac^3\right)^{\frac{1}{4}}a^2ce^3-9\left(ac^3\right)^{\frac{3}{4}}ade^2\right)\log\left(x^2-\sqrt{2}x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}a^3c^3d^4+2\sqrt{2}a^4c^2d^2e^2+\sqrt{2}a^5ce^4\right)}-\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{9}{2}}}{\left(c^2d^5+2acd^3e^2+a^2de^4\right)\sqrt{d}}-\frac{5c^2d^2x^4+4acx^4e^2+acdx^2e+4acd^2+4a^2e^2}{4\left(a^2cd^3+a^3de^2\right)\left(cx^5+ax\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)
*a^2*c*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/
c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + s
qrt(2)*a^5*c*e^4) - 1/8*(3*(a*c^3)^(1/4)*a*c^2*d^2*e + 5*(a*c^3)^(3/4)*c*d^
3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 + 9*(a*c^3)^(3/4)*a*d*e^2)*arctan(1/2*sqrt(2)
*(2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*
a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2*e -
5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3)^(3/4)*a*d*e^2
)*log(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqr
t(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(3*(a*c^3)^(1/4)*a*c^2*d^2
*e - 5*(a*c^3)^(3/4)*c*d^3 + 7*(a*c^3)^(1/4)*a^2*c*e^3 - 9*(a*c^3)^(3/4)*a*
d*e^2)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 +
2*sqrt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - arctan(x*e^(1/2)/sqrt(d))*
e^(9/2)/((c^2*d^5 + 2*a*c*d^3*e^2 + a^2*d*e^4)*sqrt(d)) - 1/4*(5*c^2*d^2*x^
4 + 4*a*c*x^4*e^2 + a*c*d*x^2*e + 4*a*c*d^2 + 4*a^2*e^2)/((a^2*c*d^3 + a^3*
d*e^2)*(c*x^5 + a*x))
```

Mupad [B]
time = 5.16, size = 2500, normalized size = 3.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] - (1/(a*d) + (c*e*x^2)/(4*a*(a*e^2 + c*d^2)) + (c*x^4*(4*a*e^2 + 5*c*d^2))/
(4*a^2*d*(a*e^2 + c*d^2)))/(a*x + c*x^5) - atan(((11875*a^5*c^10*d^15*e - a
^9*c^3*(72128*a^3*d*e^15 + 265655*c^3*d^7*e^9 - 76440*a*c^2*d^5*e^11 - 1785
85*a^2*c*d^3*e^13) + 68800*a^6*c^9*d^13*e^3 + 89403*a^7*c^8*d^11*e^5 - 1264
88*a^8*c^7*d^9*e^7)*(a^25*d^2*e^19*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^
3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3
*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^
(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*
a^11*c^2*d^4*e^4))^(5/2)*2i - a^15*c^2*e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2
```

) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*3136i - a^11*c^10*d^19*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*25i - a^16*c^9*d^20*e*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*2i + a^24*c*d^4*e^17*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(5/2)*14i + a^8*c^9*d^14*e^3*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*1250i + a^9*c^8*d^12*e^5*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*9900i + a^10*c^7*d^10*e^7*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*31902i + a^11*c^6*d^8*e^9*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*52008i + a^12*c^5*d^6*e^11*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*42238i + a^13*c^4*d^4*e^13*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(1/2)*10924i - a^14*c^3*d^2*e^15*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^

```
2 + 6*a^11*c^2*d^4*e^4))^(1/2)*5694i - a^12*c^9*d^17*e^2*x*(-(49*a^3*e^6*(-
a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^5*e + 126*a^7*c
^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)^(1/2) - 39*a^2
*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^12*c*d^2*e^6 + 4
*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*216i - a^13*c^8*d^15*e^4*x*(
-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2) + 30*a^5*c^4*d^
5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4*e^2*(-a^9*c^3)
^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*c^4*d^8 + 4*a^1
2*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2 + 6*a^11*c^2*d^4*e^4))^(3/2)*700i - a^14*c
^7*d^13*e^6*x*(-(49*a^3*e^6*(-a^9*c^3)^(1/2) - 25*c^3*d^6*(-a^9*c^3)^(1/2)
+ 30*a^5*c^4*d^5*e + 126*a^7*c^2*d*e^5 + 124*a^6*c^3*d^3*e^3 - 81*a*c^2*d^4
*e^2*(-a^9*c^3)^(1/2) - 39*a^2*c*d^2*e^4*(-a^9*c^3)^(1/2))/(a^13*e^8 + a^9*
c^4*d^8 + 4*a^12*c*d^2*e^6 + 4*a^10*c^3*d^6*e^2...
```

3.258 $\int \frac{1}{x^4(d+ex^2)(a+cx^4)^2}\,dx$

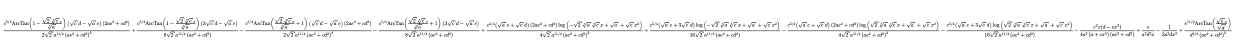
Optimal. Leaf size=751

$$-\frac{1}{3a^2dx^3}+\frac{e}{a^2d^2x}-\frac{c^2x(d-ex^2)}{4a^2(cd^2+ae^2)(a+cx^4)}+\frac{e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)^2}+\frac{c^{5/4}(3\sqrt{c}\,d-\sqrt{a}\,e)\tan^{-1}\left(1-\frac{\sqrt{2}\,\sqrt[4]{}}{\sqrt[4]{a}}\right.}{8\sqrt{2}\,a^{11/4}(cd^2+ae^2)}$$

```
[Out] -1/3/a^2/d/x^3+e/a^2/d^2/x-1/4*c^2*x*(-e*x^2+d)/a^2/(a*e^2+c*d^2)/(c*x^4+a)
+e^(11/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/(a*e^2+c*d^2)^2-1/4*c^(5/4)*(2*
a*e^2+c*d^2)*arctan(-1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^
(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/4*c^(5/4)*(2*a*e^2+c*d^2)*arctan(1+c^(1/4)
*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)
+1/8*c^(5/4)*(2*a*e^2+c*d^2)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1
/2))*(e*a^(1/2)+d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/8*c^(5/4)*(2*
a*e^2+c*d^2)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+d
*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)^2*2^(1/2)-1/16*c^(5/4)*arctan(-1+c^(1/4)*x
*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)-1
/16*c^(5/4)*arctan(1+c^(1/4)*x*2^(1/2)/a^(1/4))*(-e*a^(1/2)+3*d*c^(1/2))/a^
(11/4)/(a*e^2+c*d^2)*2^(1/2)+1/32*c^(5/4)*ln(-a^(1/4)*c^(1/4)*x*2^(1/2)+a^(
1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)-1/
32*c^(5/4)*ln(a^(1/4)*c^(1/4)*x*2^(1/2)+a^(1/2)+x^2*c^(1/2))*(e*a^(1/2)+3*d
*c^(1/2))/a^(11/4)/(a*e^2+c*d^2)*2^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 751, normalized size of antiderivative = 1.00, number of steps used = 22, number of rules used = 9, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.409$, Rules used = {1350, 211, 1193, 1182, 1176, 631, 210, 1179, 642}



Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] -1/3*1/(a^2*d*x^3) + e/(a^2*d^2*x) - (c^2*x*(d - e*x^2))/(4*a^2*(c*d^2 + a*
e^2)*(a + c*x^4)) + (e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2
+ a*e^2)^2) + (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*ArcTan[1 - (Sqrt[2]*c^(1/4
)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d -
 Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(2*S
qrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d - Sqrt[a]*e)*Arc
Tan[1 + (Sqrt[2]*c^(1/4)*x)/a^(1/4)])/(8*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2))
- (c^(5/4)*(Sqrt[c]*d - Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(2*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) + (c^(5/4)*(3*Sqr
t[c]*d + Sqrt[a]*e)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])
```

```
/(16*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)) + (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(
c*d^2 + 2*a*e^2)*Log[Sqrt[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4
*Sqrt[2]*a^(11/4)*(c*d^2 + a*e^2)^2) - (c^(5/4)*(3*Sqrt[c]*d + Sqrt[a]*e)*L
og[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(16*Sqrt[2]*a^(11/4)
*(c*d^2 + a*e^2)) - (c^(5/4)*(Sqrt[c]*d + Sqrt[a]*e)*(c*d^2 + 2*a*e^2)*Log[
Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(4*Sqrt[2]*a^(11/4)*(c*
d^2 + a*e^2)^2)
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1182

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[(-a)
*c]
```

Rule 1193

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(-x
)*(d + e*x^2)*((a + c*x^4)^(p + 1)/(4*a*(p + 1))), x] + Dist[1/(4*a*(p + 1)
), Int[Simp[d*(4*p + 5) + e*(4*p + 7)*x^2, x]*(a + c*x^4)^(p + 1), x], x] /
; FreeQ[{a, c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && LtQ[p, -1] && IntegerQ
[2*p]
```

Rule 1350

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*(a + c*x^4)^p,
x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && (IGtQ[p, 0] || IGtQ[q, 0] |
| IntegersQ[m, q])
```

Rubi steps

$$\int \frac{1}{x^4 (d+ex^2)(a+cx^4)^2} dx = \int \left(\frac{1}{a^2 d x^4} - \frac{e}{a^2 d^2 x^2} + \frac{e^6}{d^2 (cd^2+ae^2)^2 (d+ex^2)} - \frac{c^2(d-ex^2)}{a(cd^2+ae^2)(a+cx^4)^2} \right.$$

$$= -\frac{1}{3a^2 d x^3} + \frac{e}{a^2 d^2 x} + \frac{e^6 \int \frac{1}{d+ex^2} dx}{d^2 (cd^2+ae^2)^2} - \frac{c^2 \int \frac{d-ex^2}{(a+cx^4)^2} dx}{a(cd^2+ae^2)} - \frac{(c^2(cd^2+2ae^2)) \int}{a^2 (cd^2+ae^2}$$

$$= -\frac{1}{3a^2 d x^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x(d-ex^2)}{4a^2 (cd^2+ae^2)(a+cx^4)} + \frac{e^{11/2} \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{d^{5/2} (cd^2+ae^2)^2} + \frac{c^2 \int}{4a^2}$$

$$= -\frac{1}{3a^2 d x^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x(d-ex^2)}{4a^2 (cd^2+ae^2)(a+cx^4)} + \frac{e^{11/2} \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{d^{5/2} (cd^2+ae^2)^2} - \frac{(c(}{}$$

$$= -\frac{1}{3a^2 d x^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x(d-ex^2)}{4a^2 (cd^2+ae^2)(a+cx^4)} + \frac{e^{11/2} \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{d^{5/2} (cd^2+ae^2)^2} + \frac{c^{5/4}}{}$$

$$= -\frac{1}{3a^2 d x^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x(d-ex^2)}{4a^2 (cd^2+ae^2)(a+cx^4)} + \frac{e^{11/2} \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{d^{5/2} (cd^2+ae^2)^2} + \frac{c^{5/4}}{}$$

$$= -\frac{1}{3a^2 d x^3} + \frac{e}{a^2 d^2 x} - \frac{c^2 x(d-ex^2)}{4a^2 (cd^2+ae^2)(a+cx^4)} + \frac{e^{11/2} \tan^{-1}\left(\frac{\sqrt{e}\, x}{\sqrt{d}}\right)}{d^{5/2} (cd^2+ae^2)^2} + \frac{c^{5/4}}{}$$

Mathematica [A]
time = 0.26, size = 513, normalized size = 0.68

$$\frac{1}{96}\left(-\frac{32}{a^2 d x^3} + \frac{96e}{a^2 d^2 x} - \frac{24c^2 x(d-ex^2)}{a^2(cd^2+ae^2)(a+cx^4)} + \frac{96e^{11/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}(cd^2+ae^2)^2} + \frac{6\sqrt{2}\,c^{5/4}(7c^{3/2}d^3 - 5\sqrt{a}\,cd^2e + 11a\sqrt{c}\,de^2 - 9a^{3/2}e^3)\tan^{-1}\left(1-\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{11/4}(cd^2+ae^2)^2} + \frac{6\sqrt{2}\,c^{5/4}(-7c^{3/2}d^3 + 5\sqrt{a}\,cd^2e - 11a\sqrt{c}\,de^2 + 9a^{3/2}e^3)\tan^{-1}\left(1+\frac{\sqrt{2}\sqrt[4]{c}\,x}{\sqrt[4]{a}}\right)}{a^{11/4}(cd^2+ae^2)^2} + \frac{3\sqrt{2}\,c^{5/4}(7c^{3/2}d^3 + 5\sqrt{a}\,cd^2e + 11a\sqrt{c}\,de^2 + 9a^{3/2}e^3)\log\left(\sqrt{a} - \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{a^{11/4}(cd^2+ae^2)^2} - \frac{3\sqrt{2}\,c^{5/4}(7c^{3/2}d^3 + 5\sqrt{a}\,cd^2e + 11a\sqrt{c}\,de^2 + 9a^{3/2}e^3)\log\left(\sqrt{a} + \sqrt{2}\sqrt[4]{a}\sqrt[4]{c}\,x + \sqrt{c}\,x^2\right)}{a^{11/4}(cd^2+ae^2)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + c*x^4)^2),x]
```

```
[Out] (-32/(a^2*d*x^3) + (96*e)/(a^2*d^2*x) - (24*c^2*x*(d - e*x^2))/(a^2*(c*d^2
+ a*e^2)*(a + c*x^4)) + (96*e^(11/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*
(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 - 5*Sqrt[a]*c*d^2*e
+ 11*a*Sqrt[c]*d*e^2 - 9*a^(3/2)*e^3)*ArcTan[1 - (Sqrt[2]*c^(1/4)*x)/a^(1/4
)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (6*Sqrt[2]*c^(5/4)*(-7*c^(3/2)*d^3 + 5*S
qrt[a]*c*d^2*e - 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*ArcTan[1 + (Sqrt[2]*c^
(1/4)*x)/a^(1/4)])/(a^(11/4)*(c*d^2 + a*e^2)^2) + (3*Sqrt[2]*c^(5/4)*(7*c^(
```

```
3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*e^2 + 9*a^(3/2)*e^3)*Log[Sqrt
[a] - Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2])/(a^(11/4)*(c*d^2 + a*e^2)^2
) - (3*Sqrt[2]*c^(5/4)*(7*c^(3/2)*d^3 + 5*Sqrt[a]*c*d^2*e + 11*a*Sqrt[c]*d*
e^2 + 9*a^(3/2)*e^3)*Log[Sqrt[a] + Sqrt[2]*a^(1/4)*c^(1/4)*x + Sqrt[c]*x^2]
)/(a^(11/4)*(c*d^2 + a*e^2)^2))/96
```

Maple [A]
time = 0.23, size = 355, normalized size = 0.47

method	result
default	$\frac{e^6 \arctan\left(\frac{ex}{\sqrt{de}}\right)}{d^2(a e^2+c d^2)^2 \sqrt{de}} - \frac{c^2\left(\frac{\left(-\frac{1}{4} a e^3-\frac{1}{4} c d^2 e\right) x^3+\left(\frac{1}{4} d e^2 a+\frac{1}{4} c d^3\right) x}{c x^4+a}+\frac{\left(11 d e^2 a+7 c d^3\right)\left(\frac{a}{c}\right)^{\frac{1}{4}} \sqrt{2}\left(\ln\left(\frac{x^2+\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}{x^2-\left(\frac{a}{c}\right)^{\frac{1}{4}} x \sqrt{2}+\sqrt{\frac{a}{c}}}\right)+\cdots\right.}{32a}\cdots\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^2*e^6/(a*e^2+c*d^2)^2/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))-c^2/(a*e^2+c*
d^2)^2/a^2*(((-1/4*a*e^3-1/4*c*d^2*e)*x^3+(1/4*d*e^2*a+1/4*c*d^3)*x)/(c*x^4
+a)+1/32*(11*a*d*e^2+7*c*d^3)*(a/c)^(1/4)/a*2^(1/2)*(ln((x^2+(a/c)^(1/4)*x*
2^(1/2)+(a/c)^(1/2))/(x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1
/2)/(a/c)^(1/4)*x+1)+2*arctan(2^(1/2)/(a/c)^(1/4)*x-1))+1/32*(-9*a*e^3-5*c*
d^2*e)/c/(a/c)^(1/4)*2^(1/2)*(ln((x^2-(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2))/(x
^2+(a/c)^(1/4)*x*2^(1/2)+(a/c)^(1/2)))+2*arctan(2^(1/2)/(a/c)^(1/4)*x+1)+2*
arctan(2^(1/2)/(a/c)^(1/4)*x-1)))-1/3/a^2/d/x^3+e/a^2/d^2/x
```

Maxima [A]
time = 0.54, size = 527, normalized size = 0.70

$$-\frac{c^2\left(\frac{2\sqrt{2}\left(7 c^{\frac{3}{2}} d^3-5 \sqrt{a} c d^2 e+11 a \sqrt{c} d e^2-9 a^{\frac{3}{2}} e^3\right) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c} x+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{2\sqrt{2}\left(7 c^{\frac{3}{2}} d^3-5 \sqrt{a} c d^2 e+11 a \sqrt{c} d e^2-9 a^{\frac{3}{2}} e^3\right) \arctan\left(\frac{\sqrt{2}\left(2 \sqrt{c} x-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}}\right)}{2 \sqrt{\sqrt{a} \sqrt{c}}}\right)}{\sqrt{a} \sqrt{\sqrt{a} \sqrt{c}} \sqrt{c}}+\frac{\sqrt{2}\left(7 c^{\frac{3}{2}} d^3+5 \sqrt{a} c d^2 e+11 a \sqrt{c} d e^2+9 a^{\frac{3}{2}} e^3\right) \log\left(\sqrt{c} x^2+\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}-\frac{\sqrt{2}\left(7 c^{\frac{3}{2}} d^3+5 \sqrt{a} c d^2 e+11 a \sqrt{c} d e^2+9 a^{\frac{3}{2}} e^3\right) \log\left(\sqrt{c} x^2-\sqrt{2} a^{\frac{1}{4}} c^{\frac{1}{4}} x+\sqrt{a}\right)}{a^{\frac{3}{4}} c^{\frac{3}{4}}}\right)}{32\left(a^2 c^2 d^4+2 a^3 c d^2 e^2+a^4 e^4\right)}+\frac{\arctan\left(\frac{x e^{\frac{1}{2}}}{\sqrt{d}}\right) e^{\frac{11}{2}}}{\left(c^2 d^6+2 a c d^4 e^2+a^2 d^2 e^4\right) \sqrt{d}}+\frac{3\left(5 c^2 d^2 e+4 a c e^3\right) x^6-4 a c d^3-\left(7 c^2 d^3+4 a c d e^2\right) x^4-4 a^2 d e^2+12\left(a c d^2 e+a^2 e^3\right) x^2}{12\left(\left(a^2 c^2 d^4+a^3 c d^2 e^2\right) x^7+\left(a^3 c d^4+a^4 d^2 e^2\right) x^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="maxima")
```

```
[Out] -1/32*c^2*(2*sqrt(2)*(7*c^(3/2)*d^3 - 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^
2 - 9*a^(3/2)*e^3)*arctan(1/2*sqrt(2)*(2*sqrt(c)*x + sqrt(2)*a^(1/4)*c^(1/4
))/sqrt(sqrt(a)*sqrt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + 2*sqrt(
2)*(7*c^(3/2)*d^3 - 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^2 - 9*a^(3/2)*e^3)
*arctan(1/2*sqrt(2)*(2*sqrt(c)*x - sqrt(2)*a^(1/4)*c^(1/4))/sqrt(sqrt(a)*sq
rt(c)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(c))*sqrt(c)) + sqrt(2)*(7*c^(3/2)*d^3 +
5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^2 + 9*a^(3/2)*e^3)*log(sqrt(c)*x^2 + s
```

```
qrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)) - sqrt(2)*(7*c^(3/2)*
d^3 + 5*sqrt(a)*c*d^2*e + 11*a*sqrt(c)*d*e^2 + 9*a^(3/2)*e^3)*log(sqrt(c)*x
^2 - sqrt(2)*a^(1/4)*c^(1/4)*x + sqrt(a))/(a^(3/4)*c^(3/4)))/(a^2*c^2*d^4 +
 2*a^3*c*d^2*e^2 + a^4*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(11/2)/((c^2*d^6
+ 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d)) + 1/12*(3*(5*c^2*d^2*e + 4*a*c*e^3)
*x^6 - 4*a*c*d^3 - (7*c^2*d^3 + 4*a*c*d*e^2)*x^4 - 4*a^2*d*e^2 + 12*(a*c*d^
2*e + a^2*e^3)*x^2)/((a^2*c^2*d^4 + a^3*c*d^2*e^2)*x^7 + (a^3*c*d^4 + a^4*d
^2*e^2)*x^3)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4891 vs. $2(572) = 1144$.
time = 63.54, size = 9816, normalized size = 13.07

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="fricas")
```

```
[Out] [-1/48*(28*c^3*d^5*x^4 + 16*a*c^2*d^5 - 24*(a^2*c*x^7 + a^3*x^3)*sqrt(-e/d)
*e^5*log((x^2*e + 2*d*x*sqrt(-e/d) - d)/(x^2*e + d)) - 3*(a^2*c^3*d^6*x^7 +
 a^3*c^2*d^6*x^3 + (a^4*c*d^2*x^7 + a^5*d^2*x^3)*e^4 + 2*(a^3*c^2*d^4*x^7 +
 a^4*c*d^4*x^3)*e^2)*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d
*e^5 + (a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e
^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9
*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^
10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*
d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^1
0 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 +
4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))*log(-24
01*c^8*d^8*x - 10290*a*c^7*d^6*x*e^2 - 11968*a^2*c^6*d^4*x*e^4 + 1458*a^3*c
^5*d^2*x*e^6 + 6561*a^4*c^4*x*e^8 + (343*a^3*c^7*d^9 + 1442*a^4*c^6*d^7*e^2
 + 1636*a^5*c^5*d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7*c^3*d*e^8 + (5*a^9*
c^5*d^10*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c^3*d^6*e^5 + 74*a^12*c^2*d^4*e^
7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^11)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d
^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^
8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*
d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^
8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*
e^16)))*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 + (a^5*c
^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8)
*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60
*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6
*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56
*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c
^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*d^6
```

```
*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))) + 3*(a^2*c^3*d^6*x^
7 + a^3*c^2*d^6*x^3 + (a^4*c*d^2*x^7 + a^5*d^2*x^3)*e^4 + 2*(a^3*c^2*d^4*x^
7 + a^4*c*d^4*x^3)*e^2)*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^
3*d*e^5 + (a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^
2*e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*
c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2
*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c
^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*
e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8
 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))*log(
-2401*c^8*d^8*x - 10290*a*c^7*d^6*x*e^2 - 11968*a^2*c^6*d^4*x*e^4 + 1458*a^
3*c^5*d^2*x*e^6 + 6561*a^4*c^4*x*e^8 - (343*a^3*c^7*d^9 + 1442*a^4*c^6*d^7*
e^2 + 1636*a^5*c^5*d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7*c^3*d*e^8 + (5*a
^9*c^5*d^10*e + 29*a^10*c^4*d^8*e^3 + 66*a^11*c^3*d^6*e^5 + 74*a^12*c^2*d^4
*e^7 + 41*a^13*c*d^2*e^9 + 9*a^14*e^11)*sqrt(-(2401*c^11*d^12 + 12642*a*c^1
0*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4
*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c
^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8
*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^
19*e^16)))*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2*c^3*d*e^5 + (a^
5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e
^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a^2*c^9*d^8*e^4 +
 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*d^2*e^10 + 6561*
a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^13*c^6*d^12*e^4 +
 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d^6*e^10 + 28*a^1
7*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*d^8 + 4*a^6*c^3*
d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))) - 3*(a^2*c^3*d^6
*x^7 + a^3*c^2*d^6*x^3 + (a^4*c*d^2*x^7 + a^5*d^2*x^3)*e^4 + 2*(a^3*c^2*d^4
*x^7 + a^4*c*d^4*x^3)*e^2)*sqrt((70*c^5*d^5*e + 236*a*c^4*d^3*e^3 + 198*a^2
*c^3*d*e^5 - (a^5*c^4*d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c
*d^2*e^6 + a^9*e^8)*sqrt(-(2401*c^11*d^12 + 12642*a*c^10*d^10*e^2 + 19679*a
^2*c^9*d^8*e^4 + 60*a^3*c^8*d^6*e^6 - 19937*a^4*c^7*d^4*e^8 - 5022*a^5*c^6*
d^2*e^10 + 6561*a^6*c^5*e^12)/(a^11*c^8*d^16 + 8*a^12*c^7*d^14*e^2 + 28*a^1
3*c^6*d^12*e^4 + 56*a^14*c^5*d^10*e^6 + 70*a^15*c^4*d^8*e^8 + 56*a^16*c^3*d
^6*e^10 + 28*a^17*c^2*d^4*e^12 + 8*a^18*c*d^2*e^14 + a^19*e^16)))/(a^5*c^4*
d^8 + 4*a^6*c^3*d^6*e^2 + 6*a^7*c^2*d^4*e^4 + 4*a^8*c*d^2*e^6 + a^9*e^8))*l
og(-2401*c^8*d^8*x - 10290*a*c^7*d^6*x*e^2 - 11968*a^2*c^6*d^4*x*e^4 + 1458
*a^3*c^5*d^2*x*e^6 + 6561*a^4*c^4*x*e^8 + (343*a^3*c^7*d^9 + 1442*a^4*c^6*d
^7*e^2 + 1636*a^5*c^5*d^5*e^4 - 226*a^6*c^4*d^3*e^6 - 891*a^7*c^3*d*e^8 - (
5*a^9*c^5*d^10*e + 29*a^10*c^4*d^8*e^3 + 66*a^1...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+a)**2,x)
```

```
[Out] Timed out
```

Giac [A]
time = 5.36, size = 628, normalized size = 0.84

$$-\frac{\left(7\,(ac^3)^{\frac{1}{4}}c^3d^3+11\,(ac^3)^{\frac{1}{4}}ac^2de^2-5\,(ac^3)^{\frac{3}{4}}cd^2e-9\,(ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x+\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^3c^3d^4+2\sqrt{2}\,a^4c^2d^2e^2+\sqrt{2}\,a^5ce^4\right)}-\frac{\left(7\,(ac^3)^{\frac{1}{4}}c^3d^3+11\,(ac^3)^{\frac{1}{4}}ac^2de^2-5\,(ac^3)^{\frac{3}{4}}cd^2e-9\,(ac^3)^{\frac{3}{4}}ae^3\right)\arctan\left(\frac{\sqrt{2}\left(2x-\sqrt{2}\left(\frac{a}{c}\right)^{\frac{1}{4}}\right)}{2\left(\frac{a}{c}\right)^{\frac{1}{4}}}\right)}{8\left(\sqrt{2}\,a^3c^3d^4+2\sqrt{2}\,a^4c^2d^2e^2+\sqrt{2}\,a^5ce^4\right)}-\frac{\left(7\,(ac^3)^{\frac{1}{4}}c^3d^3+11\,(ac^3)^{\frac{1}{4}}ac^2de^2+5\,(ac^3)^{\frac{3}{4}}cd^2e+9\,(ac^3)^{\frac{3}{4}}ae^3\right)\log\left(x^2+\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,a^3c^3d^4+2\sqrt{2}\,a^4c^2d^2e^2+\sqrt{2}\,a^5ce^4\right)}+\frac{\left(7\,(ac^3)^{\frac{1}{4}}c^3d^3+11\,(ac^3)^{\frac{1}{4}}ac^2de^2+5\,(ac^3)^{\frac{3}{4}}cd^2e+9\,(ac^3)^{\frac{3}{4}}ae^3\right)\log\left(x^2-\sqrt{2}\,x\left(\frac{a}{c}\right)^{\frac{1}{4}}+\sqrt{\frac{a}{c}}\right)}{16\left(\sqrt{2}\,a^3c^3d^4+2\sqrt{2}\,a^4c^2d^2e^2+\sqrt{2}\,a^5ce^4\right)}+\frac{\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{11}{2}}}{\left(c^2d^6+2\,acd^4e^2+a^2d^2e^4\right)\sqrt{d}}+\frac{c^2x^3e-c^2dx}{4\left(a^2cd^2+a^3e^2\right)\left(cx^4+a\right)}+\frac{3\,x^2e-d}{3\,a^2d^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+a)^2,x, algorithm="giac")
```

```
[Out] -1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*e^2 - 5*(a*c^3)^(3
/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(2*x + sqrt(2)*(a/c
)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^4*c^2*d^2*e^2 + sq
rt(2)*a^5*c*e^4) - 1/8*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*c^3)^(1/4)*a*c^2*d*
e^2 - 5*(a*c^3)^(3/4)*c*d^2*e - 9*(a*c^3)^(3/4)*a*e^3)*arctan(1/2*sqrt(2)*(
2*x - sqrt(2)*(a/c)^(1/4))/(a/c)^(1/4))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2)*a^
4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) - 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11*(a*
c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^3)*l
og(x^2 + sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sqrt(2
)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + 1/16*(7*(a*c^3)^(1/4)*c^3*d^3 + 11
*(a*c^3)^(1/4)*a*c^2*d*e^2 + 5*(a*c^3)^(3/4)*c*d^2*e + 9*(a*c^3)^(3/4)*a*e^
3)*log(x^2 - sqrt(2)*x*(a/c)^(1/4) + sqrt(a/c))/(sqrt(2)*a^3*c^3*d^4 + 2*sq
rt(2)*a^4*c^2*d^2*e^2 + sqrt(2)*a^5*c*e^4) + arctan(x*e^(1/2)/sqrt(d))*e^(1
1/2)/((c^2*d^6 + 2*a*c*d^4*e^2 + a^2*d^2*e^4)*sqrt(d)) + 1/4*(c^2*x^3*e - c
^2*d*x)/((a^2*c*d^2 + a^3*e^2)*(c*x^4 + a)) + 1/3*(3*x^2*e - d)/(a^2*d^2*x^
3)
```

Mupad [B]
time = 5.22, size = 2500, normalized size = 3.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(a + c*x^4)^2*(d + e*x^2)),x)
```

```
[Out] atan(((x*(4917248*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498
304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^
14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^
17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709
312*a^19*c^9*d^18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^1
4*e^27) - ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 7
0*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e
```

^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4 - 698089472*a^16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^33*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*d^29*e^14 + 11879841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^18 + 6274940928*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d^15*e^28) + ((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(x*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*(134217728*a^20*c^16*d^42*e^3 + 1342177280*a^21*c^15*d^40*e^5 + 5905580032*a^22*c^14*d^38*e^7 + 14763950080*a^23*c^13*d^36*e^9 + 22145925120*a^24*c^12*d^34*e^11 + 17716740096*a^25*c^11*d^32*e^13 - 17716740096*a^27*c^9*d^28*e^17 - 22145925120*a^28*c^8*d^26*e^19 - 14763950080*a^29*c^7*d^24*e^21 - 5905580032*a^30*c^6*d^22*e^23 - 1342177280*a^31*c^5*d^20*e^25 - 134217728*a^32*c^4*d^18*e^27) + 29360128*a^17*c^17*d^42*e^2 + 239075328*a^18*c^16*d^40*e^4 + 708837376*a^19*c^15*d^38*e^6 + 465567744*a^20*c^14*d^36*e^8 - 2726297600*a^21*c^13*d^34*e^10 - 9084862464*a^22*c^12*d^32*e^12 - 13614710784*a^23*c^11*d^30*e^14 - 10745806848*a^24*c^10*d^28*e^16 - 2403336192*a^25*c^9*d^26*e^18 + 3879731200*a^26*c^8*d^24*e^20 + 4517265408*a^27*c^7*d^22*e^22 + 2294284288*a^28*c^6*d^20*e^24 + 603979776*a^29*c^5*d^18*e^26 + 67108864*a^30*c^4*d^16*e^28))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2) + 7225344*a^12*c^18*d^39*e^3 + 76972032*a^13*c^17*d^37*e^5 + 367607808*a^14*c^16*d^35*e^7 + 1036910592*a^15*c^15*d^33*e^9 + 1876983808*a^16*c^14*d^31*e^11 + 2115436544*a^17*c^13*d^29*e^13 + 1052803072*a^18*c^12*d^27*e^15 - 848429056*a^19*c^11*d^25*e^17 - 2105458688*a^20*c^10*d^23*e^19 - 1909030912*a^21*c^9*d^21*e^21 - 959037440*a^22*c^8*d^19*e^23 - 262144000*a^23*c^7*d^17*e^25 - 30408704*a^24*c^6*d^15*e^27))*((81*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*1i + (x*(4917248*a^10*c^18*d^36*e^5 + 50677760*a^11*c^17*d^34*e^7 + 230498304*a^12*c^16*d^32*e^9 + 607559680*a^13*c^15*d^30*e^11 + 1026486272*a^14*c^14*d^28*e^13 + 1166602240*a^15*c^13*d^26*e^15 + 923508736*a^16*c^12*d^24*e^17 + 539500544*a^17*c^11*d^22*e^19 + 259409920*a^18*c^10*d^20*e^21 + 109709312*a^19*c^9*d^

```
18*e^23 + 34537472*a^20*c^8*d^16*e^25 + 5308416*a^21*c^7*d^14*e^27) - ((81*
a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5*e
 + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5)^
(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8 +
 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^13*c^2*d^4*e^4)))^(1/2)*((x*(1
787297792*a^19*c^13*d^31*e^12 - 147587072*a^15*c^17*d^39*e^4 - 698089472*a^
16*c^16*d^37*e^6 - 1660157952*a^17*c^15*d^35*e^8 - 1588068352*a^18*c^14*d^3
3*e^10 - 12845056*a^14*c^18*d^41*e^2 + 7839678464*a^20*c^12*d^29*e^14 + 118
79841792*a^21*c^11*d^27*e^16 + 10631249920*a^22*c^10*d^25*e^18 + 6274940928
*a^23*c^9*d^23*e^20 + 2652110848*a^24*c^8*d^21*e^22 + 891027456*a^25*c^7*d^
19*e^24 + 234881024*a^26*c^6*d^17*e^26 + 33554432*a^27*c^5*d^15*e^28) - ((8
1*a^3*e^6*(-a^11*c^5)^(1/2) - 49*c^3*d^6*(-a^11*c^5)^(1/2) + 70*a^6*c^5*d^5
*e + 198*a^8*c^3*d*e^5 + 236*a^7*c^4*d^3*e^3 - 129*a*c^2*d^4*e^2*(-a^11*c^5
)^(1/2) - 31*a^2*c*d^2*e^4*(-a^11*c^5)^(1/2))/(256*(a^15*e^8 + a^11*c^4*d^8
 + 4*a^14*c*d^2*e^6 + 4*a^12*c^3*d^6*e^2 + 6*a^...
```

3.259 $\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=70

$$-\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

```
[Out] -1/4*arctan(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {1332, 226, 1713, 209}

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{4\sqrt{x^4+1}}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] -1/2*ArcTan[(Sqrt[2]*x)/Sqrt[1 + x^4]]/Sqrt[2] + ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1332

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
```

```
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1+x^2)\sqrt{1+x^4}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx - \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{1+x^4}}\,dx \\ &= \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right) \\ &= -\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 40, normalized size = 0.57

$$\sqrt[4]{-1}\left(-F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)+\Pi\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + EllipticPi[-I, I*ArcS
inh[(-1)^(1/4)*x], -1])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 110, normalized size = 1.57

method	result
default	$\frac{\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}} + \frac{(-1)^{\frac{3}{4}}\sqrt{-i x^2+1}\,\sqrt{i x^2+1}\,\text{EllipticPi}\left((-1)^{\frac{1}{4}}x, i, -\sqrt{}\right.}{\sqrt{x^4+1}}$

elliptic	$\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}+\frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticPi}\left((-1)^{\frac{1}{4}}x,i,-\right.}{\sqrt{x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2)
*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)+(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+I
*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,I,(-I)^(1/2)/(-1)^(1/4))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Fricas [**C**] Result contains complex when optimal does not.
time = 0.13, size = 31, normalized size = 0.44

$$-\frac{1}{4}\sqrt{2}\arctan\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)-\frac{1}{2}i\sqrt{i}\,\text{ellipticF}\left(\sqrt{i}\,x,-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(2)*arctan(sqrt(2)*x/sqrt(x^4 + 1)) - 1/2*I*sqrt(I)*ellipticF(sqrt
(I)*x, -1)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{(x^2+1)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] Integral(x**2/((x**2 + 1)*sqrt(x**4 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(x^4 + 1)*(x^2 + 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(x^2+1\right)\,\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)*(x^4 + 1)^(1/2)),x)
[Out] int(x^2/((x^2 + 1)*(x^4 + 1)^(1/2)), x)
```

3.260 $\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}}$$

```
[Out] 1/4*arctanh(x*2^(1/2)/(x^4+1)^(1/2))*2^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(x^4+1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1332, 226, 1713, 212}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{x^4+1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,\middle|\,\frac{1}{2}\right)}{4\sqrt{x^4+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] ArcTanh[(Sqrt[2]*x)/Sqrt[1 + x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[1 + x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1332

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
```

```
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(1-x^2)\sqrt{1+x^4}}\,dx &= -\left(\frac{1}{2}\int \frac{1}{\sqrt{1+x^4}}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{(1-x^2)\sqrt{1+x^4}}\,dx \\
&= -\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}}+\frac{1}{2}\mathrm{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{1+x^4}}\right. \\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{1+x^4}}\right)}{2\sqrt{2}}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)\middle|\frac{1}{2}\right)}{4\sqrt{1+x^4}}
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.09, size = 36, normalized size = 0.51

$$\sqrt[4]{-1}\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)-\Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[1 + x^4]),x]
```

```
[Out] (-1)^(1/4)*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - EllipticPi[I, ArcSin[(
-1)^(3/4)*x], -1])
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 112, normalized size = 1.60

method	result
default	$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right), i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\mathrm{EllipticPi}\left((-1)^{\frac{1}{4}}x, -\right.}{\sqrt{x^4+1}}$

elliptic	$-\frac{\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticF}\left(x\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right),i\right)}{\left(\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\sqrt{x^4+1}}-\frac{(-1)^{\frac{3}{4}}\sqrt{-ix^2+1}\,\sqrt{ix^2+1}\,\text{EllipticPi}\left((-1)^{\frac{1}{4}}x,\right.}{\sqrt{x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1/2*2^(1/2)+1/2*I*2^(1/2))*(1-I*x^2)^(1/2)*(1+I*x^2)^(1/2)/(x^4+1)^(1/2
)*EllipticF(x*(1/2*2^(1/2)+1/2*I*2^(1/2)),I)-(-1)^(3/4)*(1-I*x^2)^(1/2)*(1+
I*x^2)^(1/2)/(x^4+1)^(1/2)*EllipticPi((-1)^(1/4)*x,-I,(-I)^(1/2)/(-1)^(1/4)
)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.12, size = 55, normalized size = 0.79

$$\frac{1}{2}i\sqrt{i}\,\text{ellipticF}\left(\sqrt{i}\,x,-1\right)+\frac{1}{8}\sqrt{2}\,\log\left(\frac{x^4+2\sqrt{2}\,\sqrt{x^4+1}\,x+2x^2+1}{x^4-2x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*I*sqrt(I)*ellipticF(sqrt(I)*x, -1) + 1/8*sqrt(2)*log((x^4 + 2*sqrt(2)*s
qrt(x^4 + 1)*x + 2*x^2 + 1)/(x^4 - 2*x^2 + 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{x^2\sqrt{x^4+1}-\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(x**4+1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(x**4 + 1) - sqrt(x**4 + 1)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4+1)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(x^4 + 1)*(x^2 - 1)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{\left(x^2-1\right)\sqrt{x^4+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(x^4 + 1)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(x^4 + 1)^(1/2)), x)
```

3.261 $\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=99

$$-\frac{x(1-x^2)}{2\sqrt{1-x^4}}-\frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E(\sin^{-1}(x)|-1)}{2\sqrt{1-x^4}}+\frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,F(\sin^{-1}(x)|-1)}{\sqrt{1-x^4}}$$

```
[Out] -1/2*x*(-x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1
/2)/(-x^4+1)^(1/2)+EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/
2)
```

Rubi [A]
time = 0.03, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {1270, 482, 434, 435, 254, 227}

$$\frac{\sqrt{x^2+1}\,\sqrt{1-x^2}\,F(\text{ArcSin}(x)|-1)}{\sqrt{1-x^4}}-\frac{\sqrt{x^2+1}\,\sqrt{1-x^2}\,E(\text{ArcSin}(x)|-1)}{2\sqrt{1-x^4}}-\frac{x(1-x^2)}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] -1/2*(x*(1 - x^2))/Sqrt[1 - x^4] - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[1 - x^4]) + (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticF[A
rcSin[x], -1])/Sqrt[1 - x^4]
```

Rule 227

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> Simp[EllipticF[ArcSin[Rt[-b,
4]*(x/Rt[a, 4])], -1]/(Rt[a, 4]*Rt[-b, 4]), x] /; FreeQ[{a, b}, x] && NegQ[
b/a] && GtQ[a, 0]
```

Rule 254

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_.)*((a2_.) + (b2_.)*(x_)^(n_))^(p_.), x_
Symbol] :> Int[(a1*a2 + b1*b2*x^(2*n))^p, x] /; FreeQ[{a1, b1, a2, b2, n, p
}, x] && EqQ[a2*b1 + a1*b2, 0] && (IntegerQ[p] || (GtQ[a1, 0] && GtQ[a2, 0]
))
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1270

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c/e)*x^2
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{(1+x^2)\sqrt{1-x^4}}\,dx = \frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{x^2}{\sqrt{1-x^2}\,(1+x^2)^{3/2}}\,dx}{\sqrt{1-x^4}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} + \frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{\sqrt{1-x^2}}{\sqrt{1+x^2}}\,dx}{2\sqrt{1-x^4}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1-x^4}} + \frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \ldots}{\sqrt{1-\ldots}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E(\sin^{-1}(x)|-1)}{2\sqrt{1-x^4}} + \frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \ldots}{\sqrt{1-x^4}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E(\sin^{-1}(x)|-1)}{2\sqrt{1-x^4}} + \frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,F(\sin^{-}\ldots}{\sqrt{1-x^4}}$$

Mathematica [A]

time = 10.10, size = 46, normalized size = 0.46

$$\frac{1}{2}\left(-\frac{x}{\sqrt{1-x^4}}+\frac{x^3}{\sqrt{1-x^4}}-E\left(\sin^{-1}(x)\middle|-1\right)+2F\left(\sin^{-1}(x)\middle|-1\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (-(x/Sqrt[1 - x^4]) + x^3/Sqrt[1 - x^4] - EllipticE[ArcSin[x], -1] + 2*Elli
pticF[ArcSin[x], -1])/2
```

Maple [A]
time = 0.14, size = 96, normalized size = 0.97

method	result
risch	$\frac{x(x^2-1)}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{2\sqrt{-x^4+1}}+\frac{\text{EllipticF}(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}$
default	$\frac{\text{EllipticF}(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}-\frac{(-x^2+1)x}{2\sqrt{(-x^2+1)(x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{Elli}}{2\sqrt{-x^4+1}}$
elliptic	$\frac{\text{EllipticF}(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}-\frac{(-x^2+1)x}{2\sqrt{(-x^2+1)(x^2+1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{Elli}}{2\sqrt{-x^4+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)-1/2*(-x^2+1)
*x/((-x^2+1)*(x^2+1))^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)
*(EllipticF(x,I)-EllipticE(x,I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly
1 arguments (2 given)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{-(x-1)(x+1)(x^2+1)}\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(-x**4+1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt(-(x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(-x^4 + 1)*(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(x^2+1)\,\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)),x)
```

```
[Out] int(x^2/((x^2 + 1)*(1 - x^4)^(1/2)), x)
```

3.262 $\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx$

Optimal. Leaf size=61

$$\frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E(\sin^{-1}(x)|-1)}{2\sqrt{1-x^4}}$$

```
[Out] 1/2*x*(x^2+1)/(-x^4+1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2
)/(-x^4+1)^(1/2)
```

Rubi [A]
time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1270, 482, 435}

$$\frac{x(x^2+1)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\,\sqrt{x^2+1}\,E(\text{ArcSin}(x)|-1)}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (x*(1 + x^2))/(2*Sqrt[1 - x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[Ar
cSin[x], -1])/(2*Sqrt[1 - x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1270

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c/e)*x^2
```

```
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^2}{(1-x^2)\sqrt{1-x^4}}\,dx &= \frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{x^2}{(1-x^2)^{3/2}\sqrt{1+x^2}}\,dx}{\sqrt{1-x^4}}\\
&= \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\left(\sqrt{1-x^2}\,\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1-x^4}}\\
&= \frac{x(1+x^2)}{2\sqrt{1-x^4}} - \frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E(\sin^{-1}(x)|-1)}{2\sqrt{1-x^4}}
\end{aligned}
$$

Mathematica [A]
time = 10.07, size = 37, normalized size = 0.61

$$\frac{x+x^3-\sqrt{1-x^4}\,E(\sin^{-1}(x)|-1)}{2\sqrt{1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[1 - x^4]),x]
```

```
[Out] (x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[1 - x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 142 vs. 2(49) = 98.
time = 0.16, size = 143, normalized size = 2.34

method	result
risch	$\frac{x(x^2+1)}{2\sqrt{-x^4+1}}-\frac{\text{EllipticF}(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{EllipticE}(x,i))}{2\sqrt{-x^4+1}}$
elliptic	$-\frac{(-x^2-1)x}{2\sqrt{(x^2-1)(-x^2-1)}}-\frac{\text{EllipticF}(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{EllipticF}(x,i)-\text{Ell}}{2\sqrt{-x^4+1}}$
default	$-\frac{\text{EllipticF}(x,i)\sqrt{-x^2+1}\,\sqrt{x^2+1}}{2\sqrt{-x^4+1}}-\frac{-x^3-x^2-x-1}{4\sqrt{(-1+x)(-x^3-x^2-x-1)}}+\frac{\sqrt{-x^2+1}\,\sqrt{x^2+1}\,(\text{Elli}}{2\sqrt{-x^4+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(-x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

[Out] -1/2*EllipticF(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)-1/4*(-x^3-x^2-x-1)/((-1+x)*(-x^3-x^2-x-1))^(1/2)+1/2*(-x^2+1)^(1/2)*(x^2+1)^(1/2)/(-x^4+1)^(1/2)*(EllipticF(x,I)-EllipticE(x,I))-1/4*(-x^3+x^2-x+1)/((1+x)*(-x^3+x^2-x+1))^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="maxima")

[Out] -integrate(x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)

Fricas [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="fricas")

[Out] Exception raised: TypeError >> Symbolic function elliptic_ec takes exactly 1 arguments (2 given)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2}{x^2\sqrt{1-x^4} - \sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x**2/(-x**2+1)/(-x**4+1)**(1/2),x)

[Out] -Integral(x**2/(x**2*sqrt(1 - x**4) - sqrt(1 - x**4)), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^2/(-x^2+1)/(-x^4+1)^(1/2),x, algorithm="giac")

[Out] integrate(-x^2/(sqrt(-x^4 + 1)*(x^2 - 1)), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2}{(x^2-1)\sqrt{1-x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(1 - x^4)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(1 - x^4)^(1/2)), x)
```

3.263 $\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=113

$$-\frac{x(1-x^2)}{2\sqrt{-1+x^4}}-\frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E(\sin^{-1}(x)|-1)}{2\sqrt{-1+x^4}}+\frac{\sqrt{-1+x^2}\,\sqrt{1+x^2}\,F\left(\sin^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1+x^2}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{-1+x^4}}$$

```
[Out] -1/2*x*(-x^2+1)/(x^4-1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/
2)/(x^4-1)^(1/2)+1/2*EllipticF(x*2^(1/2)/(x^2-1)^(1/2),1/2*2^(1/2))*(x^2-1)
^(1/2)*(x^2+1)^(1/2)*2^(1/2)/(x^4-1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {1270, 482, 434, 438, 435, 259, 228}

$$\frac{\sqrt{x^2-1}\,\sqrt{x^2+1}\,F\left(\text{ArcSin}\left(\frac{\sqrt{2}\,x}{\sqrt{x^2-1}}\right)|\frac{1}{2}\right)}{\sqrt{2}\,\sqrt{x^4-1}}-\frac{\sqrt{x^2+1}\,\sqrt{1-x^2}\,E(\text{ArcSin}(x)|-1)}{2\sqrt{x^4-1}}-\frac{x(1-x^2)}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] -1/2*(x*(1 - x^2))/Sqrt[-1 + x^4] - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[
ArcSin[x], -1])/(2*Sqrt[-1 + x^4]) + (Sqrt[-1 + x^2]*Sqrt[1 + x^2]*Elliptic
F[ArcSin[(Sqrt[2]*x)/Sqrt[-1 + x^2]], 1/2])/(Sqrt[2]*Sqrt[-1 + x^4])
```

Rule 228

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[(-a)*b, 2]}, Sim
p[Sqrt[-a + q*x^2]*(Sqrt[(a + q*x^2)/q]/(Sqrt[2]*Sqrt[-a]*Sqrt[a + b*x^4]))
*EllipticF[ArcSin[x/Sqrt[(a + q*x^2)/(2*q)]], 1/2], x] /; IntegerQ[q]] /; F
reeQ[{a, b}, x] && LtQ[a, 0] && GtQ[b, 0]
```

Rule 259

```
Int[((a1_.) + (b1_.)*(x_)^(n_))^(p_)*((a2_.) + (b2_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Dist[(a1 + b1*x^n)^FracPart[p]*((a2 + b2*x^n)^FracPart[p]/(a1*a2 +
 b1*b2*x^(2*n))^FracPart[p]), Int[(a1*a2 + b1*b2*x^(2*n))^p, x], x] /; Free
Q[{a1, b1, a2, b2, n, p}, x] && EqQ[a2*b1 + a1*b2, 0] &&  !IntegerQ[p]
```

Rule 434

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
```

```
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 438

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[1 + (d/c)*x^2]/Sqrt[c + d*x^2], Int[Sqrt[a + b*x^2]/Sqrt[1 + (d/c)*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] &&  !GtQ[c, 0]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1270

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c/e)*x^2
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{(1+x^2)\sqrt{-1+x^4}}\,dx = \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{x^2}{\sqrt{-1+x^2}\,(1+x^2)^{3/2}}\,dx}{\sqrt{-1+x^4}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{-1+x^2}}{\sqrt{1+x^2}}\,dx}{2\sqrt{-1+x^4}}$$

$$= -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\left(\sqrt{-1+x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{-1+x^2}}\,dx}{2\sqrt{-1+x^4}} + \frac{\left(\sqrt{-1+x^2}\sqrt{1\right.}{}$$

$$= -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\left(\sqrt{1-x^2}\sqrt{1+x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{-1+x^4}} + \int \frac{1}{\sqrt{-1+x^4}}\,dx$$

$$= -\frac{x(1-x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\sqrt{1+x^2}\,E(\sin^{-1}(x)|-1)}{2\sqrt{-1+x^4}} + \frac{\sqrt{-1+x^2}\sqrt{1+x^2}}{\sqrt{2}}$$

Mathematica [**A**]
time = 10.07, size = 54, normalized size = 0.48

$$\frac{-x+x^3-\sqrt{1-x^4}\,E(\sin^{-1}(x)|-1)+2\sqrt{1-x^4}\,F(\sin^{-1}(x)|-1)}{2\sqrt{-1+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[-1 + x^4]),x]
[Out] (-x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1] + 2*Sqrt[1 - x^4]*Ellipt
icF[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Maple [**A**]
time = 0.16, size = 99, normalized size = 0.88

method	result
risch	$\frac{x(x^2-1)}{2\sqrt{x^4-1}} + \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{2\sqrt{x^4-1}} - \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}$
default	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}} + \frac{(x^2-1)x}{2\sqrt{(x^2+1)(x^2-1)}} + \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-}{2\sqrt{x^4-1}}$
elliptic	$-\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}} + \frac{(x^2-1)x}{2\sqrt{(x^2+1)(x^2-1)}} + \frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-}{2\sqrt{x^4-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)+1/2*(x^2
-1)*x/((x^2+1)*(x^2-1))^(1/2)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1
/2)*(EllipticF(I*x,I)-EllipticE(I*x,I))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

Fricas [A]
time = 0.10, size = 17, normalized size = 0.15

$$\frac{\sqrt{x^4-1}\,x}{2\,(x^2+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^4 - 1)*x/(x^2 + 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{(x-1)\,(x+1)\,(x^2+1)}\,\,(x^2+1)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/(sqrt((x - 1)*(x + 1)*(x**2 + 1))*(x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/(sqrt(x^4 - 1)*(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{(x^2+1)\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)*(x^4 - 1)^(1/2)),x)
[Out] int(x^2/((x^2 + 1)*(x^4 - 1)^(1/2)), x)
```

3.264 $\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx$

Optimal. Leaf size=57

$$\frac{x(1+x^2)}{2\sqrt{-1+x^4}}-\frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E(\sin^{-1}(x)|-1)}{2\sqrt{-1+x^4}}$$

```
[Out] 1/2*x*(x^2+1)/(x^4-1)^(1/2)-1/2*EllipticE(x,I)*(-x^2+1)^(1/2)*(x^2+1)^(1/2)
/(x^4-1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1270, 482, 437, 435}

$$\frac{x(x^2+1)}{2\sqrt{x^4-1}}-\frac{\sqrt{1-x^2}\,\sqrt{x^2+1}\,E(\text{ArcSin}(x)|-1)}{2\sqrt{x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] (x*(1 + x^2))/(2*Sqrt[-1 + x^4]) - (Sqrt[1 - x^2]*Sqrt[1 + x^2]*EllipticE[A
rcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Rule 435

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]/(Sqrt[c]*Rt[-d/c, 2]))*EllipticE[ArcSin[Rt[-d/c, 2]*x], b*(c/(a*d)
)], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 437

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b/a)*x^2], Int[Sqrt[1 + (b/a)*x^2]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 1270

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_
), x_Symbol] :> Dist[(a + c*x^4)^FracPart[p]/((d + e*x^2)^FracPart[p]*(a/d
+ (c*x^2)/e)^FracPart[p]), Int[(f*x)^m*(d + e*x^2)^(q + p)*(a/d + (c/e)*x^2
)^p, x], x] /; FreeQ[{a, c, d, e, f, m, p, q}, x] && EqQ[c*d^2 + a*e^2, 0]
&&  !IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2}{(1-x^2)\sqrt{-1+x^4}}\,dx &= \frac{\left(\sqrt{-1-x^2}\,\sqrt{1-x^2}\right)\int \frac{x^2}{\sqrt{-1-x^2}\,(1-x^2)^{3/2}}\,dx}{\sqrt{-1+x^4}} \\
&= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} + \frac{\left(\sqrt{-1-x^2}\,\sqrt{1-x^2}\right)\int \frac{\sqrt{-1-x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{-1+x^4}} \\
&= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} + \frac{\left((-1-x^2)\sqrt{1-x^2}\right)\int \frac{\sqrt{1+x^2}}{\sqrt{1-x^2}}\,dx}{2\sqrt{1+x^2}\,\sqrt{-1+x^4}} \\
&= \frac{x(1+x^2)}{2\sqrt{-1+x^4}} - \frac{\sqrt{1-x^2}\,\sqrt{1+x^2}\,E(\sin^{-1}(x)|-1)}{2\sqrt{-1+x^4}}
\end{aligned}$$

Mathematica [A]
time = 10.07, size = 35, normalized size = 0.61

$$\frac{x+x^3-\sqrt{1-x^4}\,E(\sin^{-1}(x)|-1)}{2\sqrt{-1+x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[-1 + x^4]),x]
```

```
[Out] (x + x^3 - Sqrt[1 - x^4]*EllipticE[ArcSin[x], -1])/(2*Sqrt[-1 + x^4])
```

Maple [B] Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 133 vs. 2(45) = 90.
time = 0.15, size = 134, normalized size = 2.35

method	result

risch	$\frac{x(x^2+1)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{EllipticE}(ix,i))}{2\sqrt{x^4-1}}$
elliptic	$\frac{(x^2+1)x}{2\sqrt{(x^2+1)(x^2-1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{EllipticF}(ix,i)-\mathrm{Elli}}{2\sqrt{x^4-1}}$
default	$\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,\mathrm{EllipticF}(ix,i)}{2\sqrt{x^4-1}}+\frac{x^3+x^2+x+1}{4\sqrt{(-1+x)(x^3+x^2+x+1)}}+\frac{i\sqrt{x^2+1}\,\sqrt{-x^2+1}\,(\mathrm{Ellipt}}{2\sqrt{x^4-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x^4-1)^(1/2)*EllipticF(I*x,I)+1/4*(x^3+
x^2+x+1)/((-1+x)*(x^3+x^2+x+1))^(1/2)+1/2*I*(x^2+1)^(1/2)*(-x^2+1)^(1/2)/(x
^4-1)^(1/2)*(EllipticF(I*x,I)-EllipticE(I*x,I))+1/4*(x^3-x^2+x-1)/((1+x)*(x
^3-x^2+x-1))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

Fricas [A]
time = 0.08, size = 17, normalized size = 0.30

$$\frac{\sqrt{x^4-1}\,x}{2\,(x^2-1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(x^4 - 1)*x/(x^2 - 1)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^2}{x^2\sqrt{x^4-1}-\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(x**4-1)**(1/2),x)
[Out] -Integral(x**2/(x**2*sqrt(x**4 - 1) - sqrt(x**4 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(x^4 - 1)*(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$-\int \frac{x^2}{\left(x^2-1\right)\,\sqrt{x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(x^4 - 1)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(x^4 - 1)^(1/2)), x)
```

3.265 $\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx$

Optimal. Leaf size=74

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}}+\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

```
[Out] -1/4*arctanh(x*2^(1/2)/(-x^4-1)^(1/2))*2^(1/2)+1/4*(x^2+1)*(cos(2*arctan(x)
)^2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1
)/(x^2+1)^2)^(1/2)/(-x^4-1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {1332, 226, 1713, 212}

$$\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{4\sqrt{-x^4-1}}-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] -1/2*ArcTanh[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/Sqrt[2] + ((1 + x^2)*Sqrt[(1 + x^4
)/(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1332

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
```

```
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx &= \frac{1}{2}\int \frac{1}{\sqrt{-1-x^4}}\,dx - \frac{1}{2}\int \frac{1-x^2}{(1+x^2)\sqrt{-1-x^4}}\,dx\\ &= \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}} - \frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \frac{x}{\sqrt{-1-x^4}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}} + \frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 60, normalized size = 0.81

$$\frac{\sqrt[4]{-1}\sqrt{1+x^4}\left(-F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)+\Pi\left(-i;i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)\right)}{\sqrt{-1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 + x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] ((-1)^(1/4)*Sqrt[1 + x^4]*(-EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] + Ellipt
icPi[-I, I*ArcSinh[(-1)^(1/4)*x], -1]))/Sqrt[-1 - x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.19, size = 168, normalized size = 2.27

method	result
default	$\frac{\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}} - \frac{i\sqrt{-i}\,\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\,\text{EllipticPi}\left(\sqrt{-i}\,x\right.}{2\sqrt{-x^4-1}}$

elliptic	$\frac{\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}} - \frac{i\sqrt{-i}\,\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\,\text{EllipticPi}\left(\sqrt{-i}\,x,-\right.}{2\sqrt{-x^4-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2
)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)-1/2*I*(-I)^(1/2)*(1+I*x^2)^(1/
2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I
)^(1/2))-1/2/(-I)^(1/2)*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*Elli
pticPi((-I)^(1/2)*x,-I,(-1)^(1/4)/(-I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.10, size = 74, normalized size = 1.00

$$-\frac{1}{2}\sqrt{i}\,\text{ellipticF}\left(\sqrt{i}\,x,-1\right)-\frac{1}{8}\sqrt{2}\,\log\left(\frac{\sqrt{2}\,x+\sqrt{-x^4-1}}{x^2+1}\right)+\frac{1}{8}\sqrt{2}\,\log\left(-\frac{\sqrt{2}\,x-\sqrt{-x^4-1}}{x^2+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(I)*ellipticF(sqrt(I)*x, -1) - 1/8*sqrt(2)*log((sqrt(2)*x + sqrt(-
x^4 - 1))/(x^2 + 1)) + 1/8*sqrt(2)*log(-(sqrt(2)*x - sqrt(-x^4 - 1))/(x^2 +
 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^2}{(x^2+1)\sqrt{-x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(x**2+1)/(-x**4-1)**(1/2),x)
```

```
[Out] Integral(x**2/((x**2 + 1)*sqrt(-x**4 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(-x^4 - 1)*(x^2 + 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2}{\left(x^2+1\right)\,\sqrt{-x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((x^2 + 1)*(- x^4 - 1)^(1/2)),x)
[Out] int(x^2/((x^2 + 1)*(- x^4 - 1)^(1/2)), x)
```

3.266 $\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx$

Optimal. Leaf size=74

$$\frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

```
[Out] 1/4*arctan(x*2^(1/2)/(-x^4-1)^(1/2))*2^(1/2)-1/4*(x^2+1)*(cos(2*arctan(x))^
2)^(1/2)/cos(2*arctan(x))*EllipticF(sin(2*arctan(x)),1/2*2^(1/2))*((x^4+1)/
(x^2+1)^2)^(1/2)/(-x^4-1)^(1/2)
```

Rubi [A]
time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {1332, 226, 1713, 209}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\,x}{\sqrt{-x^4-1}}\right)}{2\sqrt{2}}-\frac{(x^2+1)\sqrt{\frac{x^4+1}{(x^2+1)^2}}\,F\left(2\text{ArcTan}(x)\,|\,\frac{1}{2}\right)}{4\sqrt{-x^4-1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
```

```
[Out] ArcTan[(Sqrt[2]*x)/Sqrt[-1 - x^4]]/(2*Sqrt[2]) - ((1 + x^2)*Sqrt[(1 + x^4)/
(1 + x^2)^2]*EllipticF[2*ArcTan[x], 1/2])/(4*Sqrt[-1 - x^4])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 226

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^4], x_Symbol] :> With[{q = Rt[b/a, 4]}, Simp[(
1 + q^2*x^2)*(Sqrt[(a + b*x^4)/(a*(1 + q^2*x^2)^2)]/(2*q*Sqrt[a + b*x^4]))*
EllipticF[2*ArcTan[q*x], 1/2], x]] /; FreeQ[{a, b}, x] && PosQ[b/a]
```

Rule 1332

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4]), x_Symbol] :>
Dist[d/(2*d*e), Int[1/Sqrt[a + c*x^4], x], x] - Dist[d/(2*d*e), Int[(d - e*
x^2)/((d + e*x^2)*Sqrt[a + c*x^4]), x], x] /; FreeQ[{a, c, d, e}, x] && NeQ
```

```
[c*d^2 + a*e^2, 0] && PosQ[c/a] && EqQ[c*d^2 - a*e^2, 0]
```

Rule 1713

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (c_.)*(x_)^4])
, x_Symbol] :> Dist[A, Subst[Int[1/(d + 2*a*e*x^2), x], x, x/Sqrt[a + c*x^4
]], x] /; FreeQ[{a, c, d, e, A, B}, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[c*d^
2 - a*e^2, 0] && EqQ[B*d + A*e, 0]
```

Rubi steps

$$\int \frac{x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx = -\left(\frac{1}{2}\int \frac{1}{\sqrt{-1-x^4}}\,dx\right)+\frac{1}{2}\int \frac{1+x^2}{(1-x^2)\sqrt{-1-x^4}}\,dx$$

$$= -\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx, x, \frac{x}{\sqrt{-1-\dots}}\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{2}\,x}{\sqrt{-1-x^4}}\right)}{2\sqrt{2}}-\frac{(1+x^2)\sqrt{\frac{1+x^4}{(1+x^2)^2}}\,F\left(2\tan^{-1}(x)|\frac{1}{2}\right)}{4\sqrt{-1-x^4}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.08, size = 56, normalized size = 0.76

$$\frac{\sqrt[4]{-1}\,\sqrt{1+x^4}\,\left(F\left(i\sinh^{-1}\left(\sqrt[4]{-1}\,x\right)\middle|-1\right)-\Pi\left(i;\sin^{-1}\left((-1)^{3/4}x\right)\middle|-1\right)\right)}{\sqrt{-1-x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((1 - x^2)*Sqrt[-1 - x^4]),x]
[Out] ((-1)^(1/4)*Sqrt[1 + x^4]*(EllipticF[I*ArcSinh[(-1)^(1/4)*x], -1] - Ellipti
cPi[I, ArcSin[(-1)^(3/4)*x], -1]))/Sqrt[-1 - x^4]
```

Maple [C] Result contains complex when optimal does not.
time = 0.20, size = 115, normalized size = 1.55

method	result
default	$-\frac{\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\,\text{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x,i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^4-1}}+\frac{\sqrt{ix^2+1}\,\sqrt{-ix^2+1}\,\text{EllipticPi}\left(\sqrt{-i}\,x,i,\dots\right.}{\sqrt{-i}\,\sqrt{-x^4-1}}$

elliptic	$-\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\mathrm{EllipticF}\left(\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)x, i\right)}{\left(\frac{\sqrt{2}}{2}-\frac{i\sqrt{2}}{2}\right)\sqrt{-x^{4}-1}}+\frac{\sqrt{i x^{2}+1}\,\sqrt{-i x^{2}+1}\,\mathrm{EllipticPi}\left(\sqrt{-i}\,x, i, (-1)\right.}{\sqrt{-i}\,\sqrt{-x^{4}-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(-x^2+1)/(-x^4-1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(1/2*2^(1/2)-1/2*I*2^(1/2))*(1+I*x^2)^(1/2)*(1-I*x^2)^(1/2)/(-x^4-1)^(1/
2)*EllipticF((1/2*2^(1/2)-1/2*I*2^(1/2))*x,I)+1/(-I)^(1/2)*(1+I*x^2)^(1/2)*
(1-I*x^2)^(1/2)/(-x^4-1)^(1/2)*EllipticPi((-I)^(1/2)*x,I,(-1)^(1/4)/(-I)^(1
/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="maxima")
```

```
[Out] -integrate(x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

Fricas [C] Result contains complex when optimal does not.
time = 0.11, size = 73, normalized size = 0.99

$$\frac{1}{2}\sqrt{i}\,\mathrm{ellipticF}\left(\sqrt{i}\,x,-1\right)-\frac{1}{8}i\sqrt{2}\,\log\left(\frac{i\sqrt{2}\,x+\sqrt{-x^{4}-1}}{x^{2}-1}\right)+\frac{1}{8}i\sqrt{2}\,\log\left(\frac{-i\sqrt{2}\,x+\sqrt{-x^{4}-1}}{x^{2}-1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(I)*ellipticF(sqrt(I)*x, -1) - 1/8*I*sqrt(2)*log((I*sqrt(2)*x + sqr
t(-x^4 - 1))/(x^2 - 1)) + 1/8*I*sqrt(2)*log((-I*sqrt(2)*x + sqrt(-x^4 - 1))
/(x^2 - 1))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{x^{2}}{x^{2}\sqrt{-x^{4}-1}-\sqrt{-x^{4}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(-x**2+1)/(-x**4-1)**(1/2),x)
```

```
[Out] -Integral(x**2/(x**2*sqrt(-x**4 - 1) - sqrt(-x**4 - 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(-x^2+1)/(-x^4-1)^(1/2),x, algorithm="giac")
[Out] integrate(-x^2/(sqrt(-x^4 - 1)*(x^2 - 1)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$-\int \frac{x^2}{(x^2-1)\sqrt{-x^4-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-x^2/((x^2 - 1)*(- x^4 - 1)^(1/2)),x)
[Out] -int(x^2/((x^2 - 1)*(- x^4 - 1)^(1/2)), x)
```

3.267 $\int x^2\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=243

$$-\frac{c(bc-2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\,(a+bx^2)}-\frac{(bc-2ad)x^3\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)}+\frac{bx^3(c+dx^2)^{3/}}{6}$$

```
[Out] 1/6*b*x^3*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)+1/16*c^2*(-2*a*d+
b*c)*arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/d^(5/2)/(b*x^2+
a)-1/16*c*(-2*a*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)-
1/8*(-2*a*d+b*c)*x^3*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A]
time = 0.09, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.162$, Rules used = {1264, 470, 285, 327, 223, 212}

$$\frac{c^2\sqrt{a^2+2abx^2+b^2x^4}\,(bc-2ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{16d^{5/2}\,(a+bx^2)}-\frac{cx\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(bc-2ad)}{16d^2\,(a+bx^2)}+\frac{bx^3\sqrt{a^2+2abx^2+b^2x^4}\,(c+dx^2)^{3/2}}{6d\,(a+bx^2)}-\frac{x^3\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(bc-2ad)}{8d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/16*(c*(b*c - 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(
d^2*(a + b*x^2)) - ((b*c - 2*a*d)*x^3*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4])/(8*d*(a + b*x^2)) + (b*x^3*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x
^2 + b^2*x^4])/(6*d*(a + b*x^2)) + (c^2*(b*c - 2*a*d)*Sqrt[a^2 + 2*a*b*x^2
+ b^2*x^4]*ArcTanh[(Sqrt[d]*x)/Sqrt[c + d*x^2]])/(16*d^(5/2)*(a + b*x^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 285

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Dist[a*n*(p/(m + n*p + 1
)), Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m}, x] && IG
tQ[n, 0] && GtQ[p, 0] && NeQ[m + n*p + 1, 0] && IntBinomialQ[a, b, c, n, m,
```

```
p, x]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(b*e*(m + n*(p
+ 1) + 1))), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(b*(m + n*(p
 + 1) + 1)), Int[(e*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, m,
n, p}, x] && NeQ[b*c - a*d, 0] && NeQ[m + n*(p + 1) + 1, 0]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int x^2\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\int x^2(ab+b^2x^2)\,\sqrt{c+dx^2}\,dx}{ab+b^2x^2}$$

$$= \frac{bx^3(c+dx^2)^{3/2}\,\sqrt{a^2+2abx^2+b^2x^4}}{6d\,(a+bx^2)} - \frac{\Big(b(bc-2ad)\sqrt{a^2+2abx^2}}{2d\,(ab}$$

$$= -\frac{(bc-2ad)x^3\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)} + \frac{bx^3(c+dx^2)^{3/2}}{6d\,(a}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\,(a+bx^2)} - \frac{(bc-2ad)x^3\sqrt{c}}{8}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\,(a+bx^2)} - \frac{(bc-2ad)x^3\sqrt{c}}{8}$$

$$= -\frac{c(bc-2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{16d^2\,(a+bx^2)} - \frac{(bc-2ad)x^3\sqrt{c}}{8}$$

Mathematica [A]
time = 0.12, size = 120, normalized size = 0.49

$$\frac{\sqrt{(a+bx^2)^2}\,\left(\sqrt{d}\,x\sqrt{c+dx^2}\,(6ad(c+2dx^2)+b(-3c^2+2cdx^2+8d^2x^4))-3c^2(bc-2ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)\right)}{48d^{5/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[d]*x*Sqrt[c + d*x^2]*(6*a*d*(c + 2*d*x^2) + b*(-
3*c^2 + 2*c*d*x^2 + 8*d^2*x^4)) - 3*c^2*(b*c - 2*a*d)*Log[-(Sqrt[d]*x) + Sq
rt[c + d*x^2]]))/(48*d^(5/2)*(a + b*x^2))
```

Maple [A]
time = 0.12, size = 159, normalized size = 0.65

method	result
risch	$\frac{x(8bx^4d^2+12ad^2x^2+2bcdx^2+6acd-3bc^2)\sqrt{dx^2+c}\,\sqrt{(bx^2+a)^2}}{48d^2(bx^2+a)} + \frac{\left(-\frac{c^2a\ln\left(\sqrt{d}\,x+\sqrt{dx^2+c}\right)}{8d^{\frac{3}{2}}}+\frac{c^3\ln\left(\sqrt{d}\,x\right.}{}\right.}{bx^2+a}$

default	$\frac{\sqrt{(bx^2+a)^2}\left(8(dx^2+c)^{\frac{3}{2}}d^{\frac{3}{2}}bx^3+12(dx^2+c)^{\frac{3}{2}}d^{\frac{3}{2}}ax-6(dx^2+c)^{\frac{3}{2}}\sqrt{d}\,bcx-6\sqrt{dx^2+c}\,d^{\frac{3}{2}}acx+3\sqrt{dx^2+c}\,\sqrt{d}\cdots\right.}{48(bx^2+a)d^{\frac{5}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/48*((b*x^2+a)^2)^(1/2)*(8*(d*x^2+c)^(3/2)*d^(3/2)*b*x^3+12*(d*x^2+c)^(3/2
)*d^(3/2)*a*x-6*(d*x^2+c)^(3/2)*d^(1/2)*b*c*x-6*(d*x^2+c)^(1/2)*d^(3/2)*a*c
*x+3*(d*x^2+c)^(1/2)*d^(1/2)*b*c^2*x-6*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c^2*
d+3*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^3)/(b*x^2+a)/d^(5/2)
```

Maxima [A]
time = 0.29, size = 124, normalized size = 0.51

$$\frac{(dx^2+c)^{\frac{3}{2}}bx^3}{6\,d}-\frac{(dx^2+c)^{\frac{3}{2}}bcx}{8\,d^2}+\frac{\sqrt{dx^2+c}\,bc^2x}{16\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}ax}{4\,d}-\frac{\sqrt{dx^2+c}\,acx}{8\,d}+\frac{bc^3\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{16\,d^{\frac{5}{2}}}-\frac{ac^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/6*(d*x^2 + c)^(3/2)*b*x^3/d - 1/8*(d*x^2 + c)^(3/2)*b*c*x/d^2 + 1/16*sqrt
(d*x^2 + c)*b*c^2*x/d^2 + 1/4*(d*x^2 + c)^(3/2)*a*x/d - 1/8*sqrt(d*x^2 + c)
*a*c*x/d + 1/16*b*c^3*arcsinh(d*x/sqrt(c*d))/d^(5/2) - 1/8*a*c^2*arcsinh(d*
x/sqrt(c*d))/d^(3/2)
```

Fricas [A]
time = 0.38, size = 206, normalized size = 0.85

$$\left[-\frac{3\,(bc^3-2\,ac^2d)\sqrt{d}\,\log\left(-2\,dx^2+2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\,(8\,bd^3x^5+2\,(bcd^2+6\,ad^3)x^3-3\,(bc^2d-2\,acd^2)x)\sqrt{dx^2+c}}{96\,d^3},\,-\frac{3\,(bc^3-2\,ac^2d)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(8\,bd^3x^5+2\,(bcd^2+6\,ad^3)x^3-3\,(bc^2d-2\,acd^2)x)\sqrt{dx^2+c}}{48\,d^3}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*(b*c^3 - 2*a*c^2*d)*sqrt(d)*log(-2*d*x^2 + 2*sqrt(d*x^2 + c)*sqrt
(d)*x - c) - 2*(8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 - 3*(b*c^2*d - 2*a*
c*d^2)*x)*sqrt(d*x^2 + c))/d^3, -1/48*(3*(b*c^3 - 2*a*c^2*d)*sqrt(-d)*arcta
n(sqrt(-d)*x/sqrt(d*x^2 + c)) - (8*b*d^3*x^5 + 2*(b*c*d^2 + 6*a*d^3)*x^3 -
3*(b*c^2*d - 2*a*c*d^2)*x)*sqrt(d*x^2 + c))/d^3]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x^2\sqrt{c+dx^2}\,\sqrt{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x**2*sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

Giac [A]
time = 4.42, size = 156, normalized size = 0.64

$$\frac{1}{48}\left(2\left(4\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd^3\mathrm{sgn}\left(bx^2+a\right)+6\,ad^4\mathrm{sgn}\left(bx^2+a\right)}{d^4}\right)x^2-\frac{3\left(bc^2d^2\mathrm{sgn}\left(bx^2+a\right)-2\,acd^3\mathrm{sgn}\left(bx^2+a\right)\right)}{d^4}\right)\sqrt{dx^2+c}\,x-\frac{\left(bc^3\mathrm{sgn}\left(bx^2+a\right)-2\,ac^2d\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{16\,d^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/48*(2*(4*b*x^2*sgn(b*x^2 + a) + (b*c*d^3*sgn(b*x^2 + a) + 6*a*d^4*sgn(b*x
^2 + a))/d^4)*x^2 - 3*(b*c^2*d^2*sgn(b*x^2 + a) - 2*a*c*d^3*sgn(b*x^2 + a))
/d^4)*sqrt(d*x^2 + c)*x - 1/16*(b*c^3*sgn(b*x^2 + a) - 2*a*c^2*d*sgn(b*x^2
+ a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(5/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int x^2\,\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x^2*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.268 $\int x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=108

$$-\frac{(bc-ad)\,(c+dx^2)^{3/2}\,\sqrt{a^2+2abx^2+b^2x^4}}{3d^2\,(a+bx^2)}+\frac{b(c+dx^2)^{5/2}\,\sqrt{a^2+2abx^2+b^2x^4}}{5d^2\,(a+bx^2)}$$

```
[Out] -1/3*(-a*d+b*c)*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)+1/5*b*(d*
x^2+c)^(5/2)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2+a)
```

Rubi [A]
time = 0.07, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {1261, 660, 45}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,(c+dx^2)^{5/2}}{5d^2\,(a+bx^2)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(c+dx^2)^{3/2}\,(bc-ad)}{3d^2\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/3*((b*c - a*d)*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d^2*(
a + b*x^2)) + (b*(c + d*x^2)^(5/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(5*d^2*
(a + b*x^2))
```

Rule 45

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 660

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_Sy
mbol] :> Dist[(a + b*x + c*x^2)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x)^(2*Fr
acPart[p])), Int[(d + e*x)^m*(b/2 + c*x)^(2*p), x], x] /; FreeQ[{a, b, c, d
, e, m, p}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p] && NeQ[2*c*d - b*e,
0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{1}{2}\text{Subst}\left(\int\sqrt{c+dx}\,\sqrt{a^2+2abx+b^2x^2}\,dx,x,x^2\right)$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(ab+b^2x\right)\sqrt{c+dx}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\left(-\frac{b(bc-ad)\sqrt{c+dx}}{d}+\frac{b^2(c+dx)^{3/2}}{d}\right)\,d\right.}{2\left(ab+b^2x^2\right)}$$

$$= -\frac{(bc-ad)\left(c+dx^2\right)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d^2\left(a+bx^2\right)}+\frac{b\left(c+dx^2\right)^{5/2}\sqrt{a^2}}{5d^2\left(a+\right.}$$

Mathematica [A]
time = 0.03, size = 56, normalized size = 0.52

$$\frac{\sqrt{\left(a+bx^2\right)^2}\left(c+dx^2\right)^{3/2}\left(-2bc+5ad+3bdx^2\right)}{15d^2\left(a+bx^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(c + d*x^2)^(3/2)*(-2*b*c + 5*a*d + 3*b*d*x^2))/(15*d^
2*(a + b*x^2))
```

Maple [A]
time = 0.12, size = 51, normalized size = 0.47

method	result	size
gosper	$\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}\left(3b\,x^{2}d+5ad-2bc\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{15d^{2}\left(b\,x^{2}+a\right)}$	51
default	$\frac{\left(d\,x^{2}+c\right)^{\frac{3}{2}}\left(3b\,x^{2}d+5ad-2bc\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{15d^{2}\left(b\,x^{2}+a\right)}$	51
risch	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\left(3b\,x^{4}d^{2}+5a\,d^{2}x^{2}+bcd\,x^{2}+5acd-2b\,c^{2}\right)\sqrt{d\,x^{2}+c}}{15\left(b\,x^{2}+a\right)d^{2}}$	72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/15*(d*x^2+c)^(3/2)*(3*b*d*x^2+5*a*d-2*b*c)*((b*x^2+a)^2)^(1/2)/d^2/(b*x^2
+a)
```

Maxima [A]
time = 0.27, size = 50, normalized size = 0.46

$$\frac{(dx^2+c)^{\frac{3}{2}}bx^2}{5\,d}-\frac{2\,(dx^2+c)^{\frac{3}{2}}bc}{15\,d^2}+\frac{(dx^2+c)^{\frac{3}{2}}a}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/5*(d*x^2 + c)^(3/2)*b*x^2/d - 2/15*(d*x^2 + c)^(3/2)*b*c/d^2 + 1/3*(d*x^2
 + c)^(3/2)*a/d
```

Fricas [A]
time = 0.34, size = 50, normalized size = 0.46

$$\frac{(3\,bd^2x^4-2\,bc^2+5\,acd+(bcd+5\,ad^2)x^2)\sqrt{dx^2+c}}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*b*d^2*x^4 - 2*b*c^2 + 5*a*c*d + (b*c*d + 5*a*d^2)*x^2)*sqrt(d*x^2 +
 c)/d^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int x\sqrt{c+dx^2}\,\sqrt{(a+bx^2)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(x*sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

Giac [A]
time = 3.98, size = 68, normalized size = 0.63

$$\frac{3\,(dx^2+c)^{\frac{5}{2}}b\mathrm{sgn}(bx^2+a)-5\,(dx^2+c)^{\frac{3}{2}}bc\mathrm{sgn}(bx^2+a)+5\,(dx^2+c)^{\frac{3}{2}}ad\mathrm{sgn}(bx^2+a)}{15\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/15*(3*(d*x^2 + c)^(5/2)*b*sgn(b*x^2 + a) - 5*(d*x^2 + c)^(3/2)*b*c*sgn(b*
x^2 + a) + 5*(d*x^2 + c)^(3/2)*a*d*sgn(b*x^2 + a))/d^2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int x\,\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int(x*(c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.269 $\int \sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx$

Optimal. Leaf size=178

$$-\frac{(bc-4ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)}+\frac{bx(c+dx^2)^{3/2}\,\sqrt{a^2+2abx^2+b^2x^4}}{4d\,(a+bx^2)}-\frac{c(bc-4ad)\sqrt{a^2+2ab}}{8c}$$

```
[Out] 1/4*b*x*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-1/8*c*(-4*a*d+b*c)*
arctanh(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/d^(3/2)/(b*x^2+a)-1/
8*(-4*a*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)
```

Rubi [A]

time = 0.05, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {1162, 396, 201, 223, 212}

$$-\frac{c\sqrt{a^2+2abx^2+b^2x^4}\,(bc-4ad)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{8d^{3/2}\,(a+bx^2)}+\frac{bx\sqrt{a^2+2abx^2+b^2x^4}\,(c+dx^2)^{3/2}}{4d\,(a+bx^2)}-\frac{x\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(bc-4ad)}{8d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] -1/8*((b*c - 4*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(d*(
a + b*x^2)) + (b*x*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(4*d*
(a + b*x^2)) - (c*(b*c - 4*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sq
rt[d]*x)/Sqrt[c + d*x^2]])/(8*d^(3/2)*(a + b*x^2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_),
x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^IntPart[p]*(b/2 + c*x^
2)^(2*FracPart[p])), Int[(d + e*x^2)^q*(b/2 + c*x^2)^(2*p), x], x] /; FreeQ
[{a, b, c, d, e, p, q}, x] && EqQ[b^2 - 4*a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int (ab+b^2x^2)\sqrt{c+dx^2}\,dx}{ab+b^2x^2}$$

$$= \frac{bx(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{4d\,(a+bx^2)} - \frac{\left(b(bc-4ad)\sqrt{a^2+2abx^2+b\right.}{4d\,(ab+b^2}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}\sqrt{a^2}}{4d\,(a+}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}\sqrt{a^2}}{4d\,(a+}$$

$$= -\frac{(bc-4ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{8d\,(a+bx^2)} + \frac{bx(c+dx^2)^{3/2}\sqrt{a^2}}{4d\,(a+}$$

Mathematica [A]
time = 0.09, size = 96, normalized size = 0.54

$$\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}\,x\sqrt{c+dx^2}\,(4ad+b(c+2dx^2))+c(bc-4ad)\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)\right)}{8d^{3/2}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4],x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[d]*x*Sqrt[c + d*x^2]*(4*a*d + b*(c + 2*d*x^2)) +
 c*(b*c - 4*a*d)*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(8*d^(3/2)*(a + b*x^
2))
```

Maple [A]
time = 0.13, size = 119, normalized size = 0.67

method	result
default	$\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(2\sqrt{d}\,\left(d\,x^{2}+c\right)^{\frac{3}{2}} b x+4 d^{\frac{3}{2}} \sqrt{d\,x^{2}+c}\,a x-\sqrt{d}\,\sqrt{d\,x^{2}+c}\,b c x+4\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right) a c d-\ln}{8\left(b\,x^{2}+a\right) d^{\frac{3}{2}}}$
risch	$\frac{x\left(2 b\,x^{2} d+4 a d+b c\right)\sqrt{d\,x^{2}+c}\,\sqrt{\left(b\,x^{2}+a\right)^{2}}}{8 d\left(b\,x^{2}+a\right)}+\frac{\left(\frac{a c\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)}{2\sqrt{d}}-\frac{c^{2}\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right) b}{8 d^{\frac{3}{2}}}\right)\sqrt{(}}{b\,x^{2}+a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*((b*x^2+a)^2)^(1/2)*(2*d^(1/2)*(d*x^2+c)^(3/2)*b*x+4*d^(3/2)*(d*x^2+c)^
(1/2)*a*x-d^(1/2)*(d*x^2+c)^(1/2)*b*c*x+4*ln(d^(1/2)*x+(d*x^2+c)^(1/2))*a*c
*d-ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2)/(b*x^2+a)/d^(3/2)
```

Maxima [A]
time = 0.27, size = 81, normalized size = 0.46

$$\frac{1}{2}\sqrt{dx^2+c}\,ax+\frac{(dx^2+c)^{\frac{3}{2}}bx}{4\,d}-\frac{\sqrt{dx^2+c}\,bcx}{8\,d}-\frac{bc^2\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{8\,d^{\frac{3}{2}}}+\frac{ac\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\,\sqrt{d}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*a*x + 1/4*(d*x^2 + c)^(3/2)*b*x/d - 1/8*sqrt(d*x^2 + c)
*b*c*x/d - 1/8*b*c^2*arcsinh(d*x/sqrt(c*d))/d^(3/2) + 1/2*a*c*arcsinh(d*x/s
qrt(c*d))/sqrt(d)
```

Fricas [A]
time = 0.38, size = 155, normalized size = 0.87

$$\left[-\frac{(bc^2-4\,acd)\sqrt{d}\,\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)-2\,(2\,bd^2x^3+(bcd+4\,ad^2)x)\sqrt{dx^2+c}}{16\,d^2},\ \frac{(bc^2-4\,acd)\sqrt{-d}\,\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)+(2\,bd^2x^3+(bcd+4\,ad^2)x)\sqrt{dx^2+c}}{8\,d^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*((b*c^2 - 4*a*c*d)*sqrt(d)*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*
x - c) - 2*(2*b*d^2*x^3 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2, 1/8*((
b*c^2 - 4*a*c*d)*sqrt(-d)*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) + (2*b*d^2*x^3
 + (b*c*d + 4*a*d^2)*x)*sqrt(d*x^2 + c))/d^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c+dx^2}\,\sqrt{\left(a+bx^2\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2), x)
```

Giac [A]
time = 2.78, size = 109, normalized size = 0.61

$$\frac{1}{8}\left(2\,bx^2\mathrm{sgn}\left(bx^2+a\right)+\frac{bcd\mathrm{sgn}(bx^2+a)+4\,ad^2\mathrm{sgn}(bx^2+a)}{d^2}\right)\sqrt{dx^2+c}\,x+\frac{\left(bc^2\mathrm{sgn}(bx^2+a)-4\,acd\mathrm{sgn}(bx^2+a)\right)\log\left(\left|-\sqrt{d}\,x+\sqrt{dx^2+c}\right|\right)}{8\,d^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*(2*b*x^2*sgn(b*x^2 + a) + (b*c*d*sgn(b*x^2 + a) + 4*a*d^2*sgn(b*x^2 + a
))/d^2)*sqrt(d*x^2 + c)*x + 1/8*(b*c^2*sgn(b*x^2 + a) - 4*a*c*d*sgn(b*x^2 +
 a))*log(abs(-sqrt(d)*x + sqrt(d*x^2 + c)))/d^(3/2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2),x)
```

```
[Out] int((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2), x)
```

3.270 $\int \frac{\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx$

Optimal. Leaf size=152

$$\frac{a\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2}+\frac{b(c+dx^2)^{3/2}\,\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)}-\frac{a\sqrt{c}\,\sqrt{a^2+2abx^2+b^2x^4}\,\tanh^{-1}\left(\right.}{a+bx^2}$$

```
[Out] 1/3*b*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/d/(b*x^2+a)-a*arctanh((d*x^2+c)^(
1/2)/c^(1/2))*c^(1/2)*((b*x^2+a)^2)^(1/2)/(b*x^2+a)+a*(d*x^2+c)^(1/2)*((b*x
^2+a)^2)^(1/2)/(b*x^2+a)
```

Rubi [A]
time = 0.06, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1264, 457, 81, 52, 65, 214}

$$\frac{b\sqrt{a^2+2abx^2+b^2x^4}\,(c+dx^2)^{3/2}}{3d\,(a+bx^2)}+\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}}{a+bx^2}-\frac{a\sqrt{c}\,\sqrt{a^2+2abx^2+b^2x^4}\,\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{a+bx^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (a*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(a + b*x^2) + (b*(c + d
*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(3*d*(a + b*x^2)) - (a*Sqrt[c]
*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]])/(a + b*x
^2)
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 81

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p +
 2))), x] + Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(
n + p + 2)), Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f
, n, p}, x] && NeQ[n + p + 2, 0]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{x}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int\frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x}\,dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int\frac{(ab+b^2x)\sqrt{c+dx}}{x}\,dx,x,x^2\right)}{2\left(ab+b^2x^2\right)}$$

$$= \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{3d\,(a+bx^2)} + \frac{\left(ab\sqrt{a^2+2abx^2+b^2x^4}\right)\text{Su}}{2\,(ab+}$$

$$= \frac{a\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b}}{3d\,(a+bx^2)}$$

$$= \frac{a\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b}}{3d\,(a+bx^2)}$$

$$= \frac{a\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{a+bx^2} + \frac{b(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b}}{3d\,(a+bx^2)}$$

Mathematica [A]
time = 0.06, size = 83, normalized size = 0.55

$$\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c+dx^2}\,(3ad+b(c+dx^2))-3a\sqrt{c}\,d\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{3d\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x,x]
```

```
[Out] (Sqrt[(a + b*x^2)^2]*(Sqrt[c + d*x^2]*(3*a*d + b*(c + d*x^2)) - 3*a*Sqrt[c]
*d*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(3*d*(a + b*x^2))
```

Maple [A]
time = 0.12, size = 80, normalized size = 0.53

method	result	size
default	$-\frac{\sqrt{(b\,x^2+a)^2}\left(3\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)ad-b(d\,x^2+c)^{\frac{3}{2}}-3\sqrt{d\,x^2+c}\,ad\right)}{3(b\,x^2+a)d}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*((b*x^2+a)^2)^(1/2)*(3*c^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a*d
-b*(d*x^2+c)^(3/2)-3*(d*x^2+c)^(1/2)*a*d)/(b*x^2+a)/d
```

Maxima [A]
time = 0.28, size = 45, normalized size = 0.30

$$-a\sqrt{c}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)+\sqrt{dx^2+c}\,a+\frac{(dx^2+c)^{\frac{3}{2}}b}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="maxima")
```

```
[Out] -a*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) + sqrt(d*x^2 + c)*a + 1/3*(d*x^2 +
 c)^(3/2)*b/d
```

Fricas [A]
time = 0.38, size = 123, normalized size = 0.81

$$\left[\frac{3\,a\sqrt{c}\,d\log\left(-\frac{dx^2-2\sqrt{dx^2+c}\,\sqrt{c}+2c}{x^2}\right)+2\,(bdx^2+bc+3\,ad)\sqrt{dx^2+c}}{6\,d},\ \frac{3\,a\sqrt{-c}\,d\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(bdx^2+bc+3\,ad)\sqrt{dx^2+c}}{3\,d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="fricas")
```

```
[Out] [1/6*(3*a*sqrt(c)*d*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2*c)/x^2) + 2
*(b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d, 1/3*(3*a*sqrt(-c)*d*arctan(sqr
t(-c)/sqrt(d*x^2 + c)) + (b*d*x^2 + b*c + 3*a*d)*sqrt(d*x^2 + c))/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{c+dx^2}\,\sqrt{(a+bx^2)^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x, x)
```

Giac [A]
time = 3.48, size = 84, normalized size = 0.55

$$\frac{ac\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)\operatorname{sgn}(bx^2+a)}{\sqrt{-c}}+\frac{(dx^2+c)^{\frac{3}{2}}bd^2\operatorname{sgn}(bx^2+a)+3\sqrt{dx^2+c}\,ad^3\operatorname{sgn}(bx^2+a)}{3\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x,x, algorithm="giac")
```

```
[Out] a*c*arctan(sqrt(d*x^2 + c)/sqrt(-c))*sgn(b*x^2 + a)/sqrt(-c) + 1/3*((d*x^2
+ c)^(3/2)*b*d^2*sgn(b*x^2 + a) + 3*sqrt(d*x^2 + c)*a*d^3*sgn(b*x^2 + a))/d
^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x, x)
```

3.271 $\int \frac{\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx$

Optimal. Leaf size=177

$$\frac{(bc+2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)}-\frac{a(c+dx^2)^{3/2}\,\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)}+\frac{(bc+2ad)\sqrt{a^2+2abx^2+\cdots}}{2\sqrt{d}\,(\cdots}$$

```
[Out] -a*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x/(b*x^2+a)+1/2*(2*a*d+b*c)*arctan
h(x*d^(1/2)/(d*x^2+c)^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/d^(1/2)+1/2*(2*a
*d+b*c)*x*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/(b*x^2+a)
```

Rubi [A]
time = 0.06, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}} = 0.135$, Rules used = {1264, 464, 201, 223, 212}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(c+dx^2)^{3/2}}{cx\,(a+bx^2)}+\frac{x\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(2ad+bc)}{2c\,(a+bx^2)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(2ad+bc)\tanh^{-1}\left(\frac{\sqrt{d}\,x}{\sqrt{c+dx^2}}\right)}{2\sqrt{d}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] ((b*c + 2*a*d)*x*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a +
 b*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(c*x*(a +
b*x^2)) + ((b*c + 2*a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[(Sqrt[d]*x
)/Sqrt[c + d*x^2]])/(2*Sqrt[d]*(a + b*x^2))
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{x^2}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^2}\,dx}{ab+b^2x^2}$$

$$= -\frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{cx\,(a+bx^2)} + -\frac{\Big((-b^2c-2abd)\sqrt{a^2+2}}{c\,(a}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+}}{cx\,(a+b}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+}}{cx\,(a+b}$$

$$= \frac{(bc+2ad)x\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+}}{cx\,(a+b}$$

Mathematica [A]
time = 0.11, size = 93, normalized size = 0.53

$$-\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{d}\,(2a-bx^2)\sqrt{c+dx^2}+(bc+2ad)x\log\left(-\sqrt{d}\,x+\sqrt{c+dx^2}\right)\right)}{2\sqrt{d}\,x\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^2,x]
```

```
[Out] -1/2*(Sqrt[(a + b*x^2)^2]*(Sqrt[d]*(2*a - b*x^2)*Sqrt[c + d*x^2] + (b*c + 2
*a*d)*x*Log[-(Sqrt[d]*x) + Sqrt[c + d*x^2]]))/(Sqrt[d]*x*(a + b*x^2))
```

Maple [A]
time = 0.12, size = 130, normalized size = 0.73

method	result
risch	$-\frac{\sqrt{d\,x^{2}+c}\,\left(-b\,x^{2}+2 a\right)\sqrt{\left(b\,x^{2}+a\right)^{2}}}{2 x \left(b\,x^{2}+a\right)}+\frac{\left(a\sqrt{d}\,\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)+\frac{\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)bc}{2\sqrt{d}}\right)\sqrt{(b}}{b\,x^{2}+a}$
default	$-\frac{\sqrt{\left(b\,x^{2}+a\right)^{2}}\,\left(-2\sqrt{d\,x^{2}+c}\,d^{\frac{3}{2}}a\,x^{2}-\sqrt{d\,x^{2}+c}\,\sqrt{d}\,bc\,x^{2}+2\left(d\,x^{2}+c\right)^{\frac{3}{2}}\sqrt{d}\,a-2\ln\left(\sqrt{d}\,x+\sqrt{d\,x^{2}+c}\right)ac}{2\left(b\,x^{2}+a\right)cx\sqrt{d}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((b*x^2+a)^2)^(1/2)*(-2*(d*x^2+c)^(1/2)*d^(3/2)*a*x^2-(d*x^2+c)^(1/2)*
d^(1/2)*b*c*x^2+2*(d*x^2+c)^(3/2)*d^(1/2)*a-2*ln(d^(1/2)*x+(d*x^2+c)^(1/2))
*a*c*d*x-ln(d^(1/2)*x+(d*x^2+c)^(1/2))*b*c^2*x)/(b*x^2+a)/c/x/d^(1/2)
```

Maxima [A]
time = 0.27, size = 59, normalized size = 0.33

$$\frac{1}{2}\sqrt{dx^2+c}\,bx+\frac{bc\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)}{2\sqrt{d}}+a\sqrt{d}\,\mathrm{arsinh}\left(\frac{dx}{\sqrt{cd}}\right)-\frac{\sqrt{dx^2+c}\,a}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x + 1/2*b*c*arcsinh(d*x/sqrt(c*d))/sqrt(d) + a*sqrt(d
)*arcsinh(d*x/sqrt(c*d)) - sqrt(d*x^2 + c)*a/x
```

Fricas [A]
time = 0.37, size = 134, normalized size = 0.76

$$\left[\frac{(bc+2\,ad)\sqrt{d}\,x\log\left(-2\,dx^2-2\,\sqrt{dx^2+c}\,\sqrt{d}\,x-c\right)+2\,(bdx^2-2\,ad)\sqrt{dx^2+c}}{4\,dx},-\frac{(bc+2\,ad)\sqrt{-d}\,x\arctan\left(\frac{\sqrt{-d}\,x}{\sqrt{dx^2+c}}\right)-(bdx^2-2\,ad)\sqrt{dx^2+c}}{2\,dx}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*c + 2*a*d)*sqrt(d)*x*log(-2*d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(d)*x -
c) + 2*(b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c))/(d*x), -1/2*((b*c + 2*a*d)*sqrt(-
d)*x*arctan(sqrt(-d)*x/sqrt(d*x^2 + c)) - (b*d*x^2 - 2*a*d)*sqrt(d*x^2 + c)
)/(d*x)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c+dx^2}\,\sqrt{(a+bx^2)^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**2,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x**2, x)
```

Giac [A]
time = 3.62, size = 116, normalized size = 0.66

$$\frac{1}{2}\sqrt{dx^2+c}\,bx\mathrm{sgn}\left(bx^2+a\right)+\frac{2\,ac\sqrt{d}\,\mathrm{sgn}\left(bx^2+a\right)}{\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2-c}-\frac{\left(bc\sqrt{d}\,\mathrm{sgn}\left(bx^2+a\right)+2\,ad^{\frac{3}{2}}\mathrm{sgn}\left(bx^2+a\right)\right)\log\left(\left(\sqrt{d}\,x-\sqrt{dx^2+c}\right)^2\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^2,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(d*x^2 + c)*b*x*sgn(b*x^2 + a) + 2*a*c*sqrt(d)*sgn(b*x^2 + a)/((sqr
t(d)*x - sqrt(d*x^2 + c))^2 - c) - 1/4*(b*c*sqrt(d)*sgn(b*x^2 + a) + 2*a*d^
(3/2)*sgn(b*x^2 + a))*log((sqrt(d)*x - sqrt(d*x^2 + c))^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2+c}\,\sqrt{{\left(b\,x^2+a\right)}^2}}{x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^2, x)
```

3.272 $\int \frac{\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx$

Optimal. Leaf size=177

$$\frac{(2bc+ad)\sqrt{c+dx^2}\,\sqrt{a^2+2abx^2+b^2x^4}}{2c\,(a+bx^2)}-\frac{a(c+dx^2)^{3/2}\,\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2\,(a+bx^2)}-\frac{(2bc+ad)\sqrt{a^2+2abx^2+b^2}}{2\sqrt{c}\,(a}$$

```
[Out] -1/2*a*(d*x^2+c)^(3/2)*((b*x^2+a)^2)^(1/2)/c/x^2/(b*x^2+a)-1/2*(a*d+2*b*c)*
arctanh((d*x^2+c)^(1/2)/c^(1/2))*((b*x^2+a)^2)^(1/2)/(b*x^2+a)/c^(1/2)+1/2*
(a*d+2*b*c)*(d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/c/(b*x^2+a)
```

Rubi [A]

time = 0.08, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.162, Rules used = {1264, 457, 79, 52, 65, 214}

$$-\frac{a\sqrt{a^2+2abx^2+b^2x^4}\,(c+dx^2)^{3/2}}{2cx^2\,(a+bx^2)}+\frac{\sqrt{a^2+2abx^2+b^2x^4}\,\sqrt{c+dx^2}\,(ad+2bc)}{2c\,(a+bx^2)}-\frac{\sqrt{a^2+2abx^2+b^2x^4}\,(ad+2bc)\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)}{2\sqrt{c}\,(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] ((2*b*c + a*d)*Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*(a + b
*x^2)) - (a*(c + d*x^2)^(3/2)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/(2*c*x^2*(a
+ b*x^2)) - ((2*b*c + a*d)*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4]*ArcTanh[Sqrt[c +
 d*x^2]/Sqrt[c]])/(2*Sqrt[c]*(a + b*x^2))
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 79

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/
(f*(p + 1)*(c*f - d*e))), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c
*f*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || I
ntegerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 457

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p
*(c + d*x)^q, x], x, x^n], x] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[
b*c - a*d, 0] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 1264

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_), x_Symbol] :> Dist[(a + b*x^2 + c*x^4)^FracPart[p]/(c^Int
Part[p]*(b/2 + c*x^2)^(2*FracPart[p])), Int[(f*x)^m*(d + e*x^2)^q*(b/2 + c*
x^2)^(2*p), x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && EqQ[b^2 - 4*
a*c, 0] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{x^3}\,dx = \frac{\sqrt{a^2+2abx^2+b^2x^4}\int \frac{(ab+b^2x^2)\sqrt{c+dx^2}}{x^3}\,dx}{ab+b^2x^2}$$

$$= \frac{\sqrt{a^2+2abx^2+b^2x^4}\,\text{Subst}\left(\int \frac{(ab+b^2x)\sqrt{c+dx}}{x^2}\,dx, x, x^2\right)}{2(ab+b^2x^2)}$$

$$= -\frac{a(c+dx^2)^{3/2}\sqrt{a^2+2abx^2+b^2x^4}}{2cx^2(a+bx^2)} + \frac{\left(\left(b^2c+\frac{abd}{2}\right)\sqrt{a^2+2abx^2+\right.}{2c}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+2a}}{2cx^2(a+bx}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+2a}}{2cx^2(a+bx}$$

$$= \frac{(2bc+ad)\sqrt{c+dx^2}\sqrt{a^2+2abx^2+b^2x^4}}{2c(a+bx^2)} - \frac{a(c+dx^2)^{3/2}\sqrt{a^2+2a}}{2cx^2(a+bx}$$

Mathematica [A]
time = 0.10, size = 90, normalized size = 0.51

$$-\frac{\sqrt{(a+bx^2)^2}\left(\sqrt{c}\,(a-2bx^2)\sqrt{c+dx^2}+(2bc+ad)x^2\tanh^{-1}\left(\frac{\sqrt{c+dx^2}}{\sqrt{c}}\right)\right)}{2\sqrt{c}\,x^2(a+bx^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[c + d*x^2]*Sqrt[a^2 + 2*a*b*x^2 + b^2*x^4])/x^3,x]
```

```
[Out] -1/2*(Sqrt[(a + b*x^2)^2]*(Sqrt[c]*(a - 2*b*x^2)*Sqrt[c + d*x^2] + (2*b*c +
 a*d)*x^2*ArcTanh[Sqrt[c + d*x^2]/Sqrt[c]]))/(Sqrt[c]*x^2*(a + b*x^2))
```

Maple [A]
time = 0.14, size = 133, normalized size = 0.75

method	result

risch	$-\frac{a\sqrt{d\,x^2+c}\,\sqrt{(b\,x^2+a)^2}}{2x^2(b\,x^2+a)}+\frac{\left(b\sqrt{d\,x^2+c}-\frac{da\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)}{2\sqrt{c}}-b\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+}\right.\right.}{b\,x^2+a}$
default	$-\frac{\sqrt{(b\,x^2+a)^2}\left(\sqrt{c}\,\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)a\,d\,x^2+2c^{\frac{3}{2}}\ln\left(\frac{2c+2\sqrt{c}\,\sqrt{d\,x^2+c}}{x}\right)b\,x^2-\sqrt{d\,x^2+c}\,a\,d\,x^2\right.}{2(b\,x^2+a)c\,x^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*((b*x^2+a)^2)^(1/2)*(c^(1/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*a*d*x
^2+2*c^(3/2)*ln(2*(c^(1/2)*(d*x^2+c)^(1/2)+c)/x)*b*x^2-(d*x^2+c)^(1/2)*a*d*
x^2-2*(d*x^2+c)^(1/2)*b*c*x^2+(d*x^2+c)^(3/2)*a)/(b*x^2+a)/c/x^2
```

Maxima [A]
time = 0.28, size = 83, normalized size = 0.47

$$-b\sqrt{c}\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)-\frac{ad\,\operatorname{arsinh}\left(\frac{c}{\sqrt{cd}\,|x|}\right)}{2\,\sqrt{c}}+\sqrt{dx^2+c}\,b+\frac{\sqrt{dx^2+c}\,ad}{2\,c}-\frac{(dx^2+c)^{\frac{3}{2}}a}{2\,cx^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="maxima")
```

```
[Out] -b*sqrt(c)*arcsinh(c/(sqrt(c*d)*abs(x))) - 1/2*a*d*arcsinh(c/(sqrt(c*d)*abs
(x)))/sqrt(c) + sqrt(d*x^2 + c)*b + 1/2*sqrt(d*x^2 + c)*a*d/c - 1/2*(d*x^2
+ c)^(3/2)*a/(c*x^2)
```

Fricas [A]
time = 0.37, size = 141, normalized size = 0.80

$$\left[\frac{(2\,bc+ad)\sqrt{c}\,x^2\log\left(-\frac{dx^2-2\,\sqrt{dx^2+c}\,\sqrt{c}+2\,c}{x^2}\right)+2\,(2\,bcx^2-ac)\sqrt{dx^2+c}}{4\,cx^2},\frac{(2\,bc+ad)\sqrt{-c}\,x^2\arctan\left(\frac{\sqrt{-c}}{\sqrt{dx^2+c}}\right)+(2\,bcx^2-ac)\sqrt{dx^2+c}}{2\,cx^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*b*c + a*d)*sqrt(c)*x^2*log(-(d*x^2 - 2*sqrt(d*x^2 + c)*sqrt(c) + 2
*c)/x^2) + 2*(2*b*c*x^2 - a*c)*sqrt(d*x^2 + c))/(c*x^2), 1/2*((2*b*c + a*d)
*sqrt(-c)*x^2*arctan(sqrt(-c)/sqrt(d*x^2 + c)) + (2*b*c*x^2 - a*c)*sqrt(d*x
^2 + c))/(c*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{c + dx^2}\,\sqrt{(a + bx^2)^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x**2+c)**(1/2)*((b*x**2+a)**2)**(1/2)/x**3,x)
```

```
[Out] Integral(sqrt(c + d*x**2)*sqrt((a + b*x**2)**2)/x**3, x)
```

Giac [A]
time = 4.55, size = 100, normalized size = 0.56

$$\frac{2\,\sqrt{dx^2 + c}\,bd\mathrm{sgn}(bx^2 + a) + \frac{(2\,bcd\mathrm{sgn}(bx^2+a)+ad^2\mathrm{sgn}(bx^2+a))\,\arctan\left(\frac{\sqrt{dx^2+c}}{\sqrt{-c}}\right)}{\sqrt{-c}} - \frac{\sqrt{dx^2+c}\,ad\mathrm{sgn}(bx^2+a)}{x^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*x^2+c)^(1/2)*((b*x^2+a)^2)^(1/2)/x^3,x, algorithm="giac")
```

```
[Out] 1/2*(2*sqrt(d*x^2 + c)*b*d*sgn(b*x^2 + a) + (2*b*c*d*sgn(b*x^2 + a) + a*d^2
*sgn(b*x^2 + a))*arctan(sqrt(d*x^2 + c)/sqrt(-c))/sqrt(-c) - sqrt(d*x^2 + c
)*a*d*sgn(b*x^2 + a)/x^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{d\,x^2 + c}\,\sqrt{{\left(b\,x^2 + a\right)}^2}}{x^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3,x)
```

```
[Out] int(((c + d*x^2)^(1/2)*((a + b*x^2)^2)^(1/2))/x^3, x)
```

3.273 $\int x^3(d+ex^2)^2(a+bx^2+cx^4)\,dx$

Optimal. Leaf size=78

$$\frac{1}{4}ad^2x^4+\frac{1}{6}d(bd+2ae)x^6+\frac{1}{8}\left(cd^2+e(2bd+ae)\right)x^8+\frac{1}{10}e(2cd+be)x^{10}+\frac{1}{12}ce^2x^{12}$$

```
[Out] 1/4*a*d^2*x^4+1/6*d*(2*a*e+b*d)*x^6+1/8*(c*d^2+e*(a*e+2*b*d))*x^8+1/10*e*(b
*e+2*c*d)*x^10+1/12*c*e^2*x^12
```

Rubi [A]
time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.080$, Rules used = {1265, 785}

$$\frac{1}{8}x^8\left(e(ae+2bd)+cd^2\right)+\frac{1}{6}dx^6(2ae+bd)+\frac{1}{4}ad^2x^4+\frac{1}{10}ex^{10}(be+2cd)+\frac{1}{12}ce^2x^{12}$$

Antiderivative was successfully verified.

```
[In] Int[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^4)/4 + (d*(b*d + 2*a*e)*x^6)/6 + ((c*d^2 + e*(2*b*d + a*e))*x^8)/8
 + (e*(2*c*d + b*e)*x^10)/10 + (c*e^2*x^12)/12
```

Rule 785

```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (
c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g*x)*
(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^
2 - 4*a*c, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0] && IntegerQ[m]))
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int x^3\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx&=\frac{1}{2}\text{Subst}\left(\int x(d+ex)^2\left(a+bx+cx^2\right)\,dx,x,x^2\right)\\&=\frac{1}{2}\text{Subst}\left(\int\left(ad^2x+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^3+e(2cd\right.\right.\\&=\frac{1}{4}ad^2x^4+\frac{1}{6}d(bd+2ae)x^6+\frac{1}{8}\left(cd^2+e(2bd+ae)\right)x^8+\frac{1}{10}e(2cd+be\end{aligned}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 0.92

$$\frac{1}{120}x^4\left(30ad^2+20d(bd+2ae)x^2+15\left(cd^2+e(2bd+ae)\right)x^4+12e(2cd+be)x^6+10ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^4*(30*a*d^2 + 20*d*(b*d + 2*a*e)*x^2 + 15*(c*d^2 + e*(2*b*d + a*e))*x^4
+ 12*e*(2*c*d + b*e)*x^6 + 10*c*e^2*x^8))/120
```

Maple [A]
time = 0.17, size = 73, normalized size = 0.94

method	result	size
default	$\frac{c\,e^2x^{12}}{12}+\frac{(e^2b+2cde)x^{10}}{10}+\frac{(a\,e^2+2deb+c\,d^2)x^8}{8}+\frac{(2ade+d^2b)x^6}{6}+\frac{a\,d^2x^4}{4}$	73
norman	$\frac{c\,e^2x^{12}}{12}+\left(\frac{1}{10}e^2b+\frac{1}{5}cde\right)x^{10}+\left(\frac{1}{8}a\,e^2+\frac{1}{4}deb+\frac{1}{8}c\,d^2\right)x^8+\left(\frac{1}{3}ade+\frac{1}{6}d^2b\right)x^6+\frac{a\,d^2x^4}{4}$	74
gosper	$\frac{1}{12}c\,e^2x^{12}+\frac{1}{10}x^{10}e^2b+\frac{1}{5}x^{10}cde+\frac{1}{8}x^8a\,e^2+\frac{1}{4}x^8deb+\frac{1}{8}x^8c\,d^2+\frac{1}{3}x^6ade+\frac{1}{6}x^6d^2b+\frac{1}{4}a\,d^2x^4$	80
risch	$\frac{1}{12}c\,e^2x^{12}+\frac{1}{10}x^{10}e^2b+\frac{1}{5}x^{10}cde+\frac{1}{8}x^8a\,e^2+\frac{1}{4}x^8deb+\frac{1}{8}x^8c\,d^2+\frac{1}{3}x^6ade+\frac{1}{6}x^6d^2b+\frac{1}{4}a\,d^2x^4$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/12*c*e^2*x^12+1/10*(b*e^2+2*c*d*e)*x^10+1/8*(a*e^2+2*b*d*e+c*d^2)*x^8+1/6
*(2*a*d*e+b*d^2)*x^6+1/4*a*d^2*x^4
```

Maxima [A]
time = 0.27, size = 72, normalized size = 0.92

$$\frac{1}{12}\,cx^{12}e^2+\frac{1}{10}\,(2\,cde+be^2)x^{10}+\frac{1}{8}\,(cd^2+2\,bde+ae^2)x^8+\frac{1}{4}\,ad^2x^4+\frac{1}{6}\,(bd^2+2\,ade)x^6$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/12*c*x^12*e^2 + 1/10*(2*c*d*e + b*e^2)*x^10 + 1/8*(c*d^2 + 2*b*d*e + a*e^
2)*x^8 + 1/4*a*d^2*x^4 + 1/6*(b*d^2 + 2*a*d*e)*x^6
```

Fricas [A]
time = 0.34, size = 77, normalized size = 0.99

$$\frac{1}{8}\,cd^2x^8+\frac{1}{6}\,bd^2x^6+\frac{1}{4}\,ad^2x^4+\frac{1}{120}\,(10\,cx^{12}+12\,bx^{10}+15\,ax^8)e^2+\frac{1}{60}\,(12\,cdx^{10}+15\,bdx^8+20\,adx^6)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/8*c*d^2*x^8 + 1/6*b*d^2*x^6 + 1/4*a*d^2*x^4 + 1/120*(10*c*x^12 + 12*b*x^1
0 + 15*a*x^8)*e^2 + 1/60*(12*c*d*x^10 + 15*b*d*x^8 + 20*a*d*x^6)*e
```

Sympy [A]
time = 0.01, size = 76, normalized size = 0.97

$$\frac{ad^2x^4}{4}+\frac{ce^2x^{12}}{12}+x^{10}\left(\frac{be^2}{10}+\frac{cde}{5}\right)+x^8\left(\frac{ae^2}{8}+\frac{bde}{4}+\frac{cd^2}{8}\right)+x^6\left(\frac{ade}{3}+\frac{bd^2}{6}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**4/4 + c*e**2*x**12/12 + x**10*(b*e**2/10 + c*d*e/5) + x**8*(a*e**
2/8 + b*d*e/4 + c*d**2/8) + x**6*(a*d*e/3 + b*d**2/6)
```

Giac [A]
time = 3.85, size = 79, normalized size = 1.01

$$\frac{1}{12}\,cx^{12}e^2+\frac{1}{5}\,cdx^{10}e+\frac{1}{10}\,bx^{10}e^2+\frac{1}{8}\,cd^2x^8+\frac{1}{4}\,bdx^8e+\frac{1}{8}\,ax^8e^2+\frac{1}{6}\,bd^2x^6+\frac{1}{3}\,adx^6e+\frac{1}{4}\,ad^2x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/12*c*x^12*e^2 + 1/5*c*d*x^10*e + 1/10*b*x^10*e^2 + 1/8*c*d^2*x^8 + 1/4*b*
d*x^8*e + 1/8*a*x^8*e^2 + 1/6*b*d^2*x^6 + 1/3*a*d*x^6*e + 1/4*a*d^2*x^4
```

Mupad [B]
time = 0.04, size = 73, normalized size = 0.94

$$x^8\left(\frac{c\,d^2}{8}+\frac{b\,d\,e}{4}+\frac{a\,e^2}{8}\right)+x^6\left(\frac{b\,d^2}{6}+\frac{a\,e\,d}{3}\right)+x^{10}\left(\frac{b\,e^2}{10}+\frac{c\,d\,e}{5}\right)+\frac{a\,d^2\,x^4}{4}+\frac{c\,e^2\,x^{12}}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^8*((a*e^2)/8 + (c*d^2)/8 + (b*d*e)/4) + x^6*((b*d^2)/6 + (a*d*e)/3) + x^1
0*((b*e^2)/10 + (c*d*e)/5) + (a*d^2*x^4)/4 + (c*e^2*x^12)/12
```

3.274 $\int x^2(d+ex^2)^2\left(a+bx^2+cx^4\right)\,dx$

Optimal. Leaf size=78

$$\frac{1}{3}ad^2x^3+\frac{1}{5}d(bd+2ae)x^5+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{9}e(2cd+be)x^9+\frac{1}{11}ce^2x^{11}$$

```
[Out] 1/3*a*d^2*x^3+1/5*d*(2*a*e+b*d)*x^5+1/7*(c*d^2+e*(a*e+2*b*d))*x^7+1/9*e*(b*
e+2*c*d)*x^9+1/11*c*e^2*x^11
```

Rubi [A]
time = 0.04, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.040$, Rules used = {1275}

$$\frac{1}{7}x^7\left(e(ae+2bd)+cd^2\right)+\frac{1}{5}dx^5(2ae+bd)+\frac{1}{3}ad^2x^3+\frac{1}{9}ex^9(be+2cd)+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Int[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + e*(2*b*d + a*e))*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int x^2\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)\,dx&=\int\left(ad^2x^2+d(bd+2ae)x^4+\left(cd^2+e(2bd+ae)\right)x^6+e(2cd+be)x^8+\right.\\&=\frac{1}{3}ad^2x^3+\frac{1}{5}d(bd+2ae)x^5+\frac{1}{7}\left(cd^2+e(2bd+ae)\right)x^7+\frac{1}{9}e(2cd+be)x\end{aligned}$$

Mathematica [A]
time = 0.01, size = 78, normalized size = 1.00

$$\frac{1}{3}ad^2x^3+\frac{1}{5}d(bd+2ae)x^5+\frac{1}{7}\left(cd^2+2bde+ae^2\right)x^7+\frac{1}{9}e(2cd+be)x^9+\frac{1}{11}ce^2x^{11}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (a*d^2*x^3)/3 + (d*(b*d + 2*a*e)*x^5)/5 + ((c*d^2 + 2*b*d*e + a*e^2)*x^7)/7
 + (e*(2*c*d + b*e)*x^9)/9 + (c*e^2*x^11)/11
```

Maple [A]
time = 0.14, size = 73, normalized size = 0.94

method	result	size
default	$\frac{c\,e^{2} x^{11}}{11}+\frac{\left(e^{2} b+2 c d e\right) x^{9}}{9}+\frac{\left(a\,e^{2}+2 d e b+c\,d^{2}\right) x^{7}}{7}+\frac{\left(2 a d e+d^{2} b\right) x^{5}}{5}+\frac{a\,d^{2} x^{3}}{3}$	73
norman	$\frac{c\,e^{2} x^{11}}{11}+\left(\frac{1}{9} e^{2} b+\frac{2}{9} c d e\right) x^{9}+\left(\frac{1}{7} a\,e^{2}+\frac{2}{7} d e b+\frac{1}{7} c\,d^{2}\right) x^{7}+\left(\frac{2}{5} a d e+\frac{1}{5} d^{2} b\right) x^{5}+\frac{a\,d^{2} x^{3}}{3}$	74
gosper	$\frac{1}{11} c\,e^{2} x^{11}+\frac{1}{9} x^{9} e^{2} b+\frac{2}{9} x^{9} c d e+\frac{1}{7} x^{7} a\,e^{2}+\frac{2}{7} x^{7} d e b+\frac{1}{7} x^{7} c\,d^{2}+\frac{2}{5} x^{5} a d e+\frac{1}{5} x^{5} d^{2} b+\frac{1}{3} a\,d^{2} x^{3}$	80
risch	$\frac{1}{11} c\,e^{2} x^{11}+\frac{1}{9} x^{9} e^{2} b+\frac{2}{9} x^{9} c d e+\frac{1}{7} x^{7} a\,e^{2}+\frac{2}{7} x^{7} d e b+\frac{1}{7} x^{7} c\,d^{2}+\frac{2}{5} x^{5} a d e+\frac{1}{5} x^{5} d^{2} b+\frac{1}{3} a\,d^{2} x^{3}$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/11*c*e^2*x^11+1/9*(b*e^2+2*c*d*e)*x^9+1/7*(a*e^2+2*b*d*e+c*d^2)*x^7+1/5*(
2*a*d*e+b*d^2)*x^5+1/3*a*d^2*x^3
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.92

$$\frac{1}{11}\,cx^{11}e^{2}+\frac{1}{9}\,\left(2\,cde+be^{2}\right)x^{9}+\frac{1}{7}\,\left(cd^{2}+2\,bde+ae^{2}\right)x^{7}+\frac{1}{3}\,ad^{2}x^{3}+\frac{1}{5}\,\left(bd^{2}+2\,ade\right)x^{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/11*c*x^11*e^2 + 1/9*(2*c*d*e + b*e^2)*x^9 + 1/7*(c*d^2 + 2*b*d*e + a*e^2)
*x^7 + 1/3*a*d^2*x^3 + 1/5*(b*d^2 + 2*a*d*e)*x^5
```

Fricas [A]
time = 0.35, size = 77, normalized size = 0.99

$$\frac{1}{7}\,cd^{2}x^{7}+\frac{1}{5}\,bd^{2}x^{5}+\frac{1}{3}\,ad^{2}x^{3}+\frac{1}{693}\,\left(63\,cx^{11}+77\,bx^{9}+99\,ax^{7}\right)e^{2}+\frac{2}{315}\,\left(35\,cdx^{9}+45\,bdx^{7}+63\,adx^{5}\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/7*c*d^2*x^7 + 1/5*b*d^2*x^5 + 1/3*a*d^2*x^3 + 1/693*(63*c*x^11 + 77*b*x^9
 + 99*a*x^7)*e^2 + 2/315*(35*c*d*x^9 + 45*b*d*x^7 + 63*a*d*x^5)*e
```

Sympy [A]
time = 0.01, size = 82, normalized size = 1.05

$$\frac{ad^{2}x^{3}}{3}+\frac{ce^{2}x^{11}}{11}+x^{9}\left(\frac{be^{2}}{9}+\frac{2cde}{9}\right)+x^{7}\left(\frac{ae^{2}}{7}+\frac{2bde}{7}+\frac{cd^{2}}{7}\right)+x^{5}\cdot\left(\frac{2ade}{5}+\frac{bd^{2}}{5}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**3/3 + c*e**2*x**11/11 + x**9*(b*e**2/9 + 2*c*d*e/9) + x**7*(a*e**
2/7 + 2*b*d*e/7 + c*d**2/7) + x**5*(2*a*d*e/5 + b*d**2/5)
```

Giac [A]
time = 3.52, size = 79, normalized size = 1.01

$$\frac{1}{11}cx^{11}e^2 + \frac{2}{9}cdx^9e + \frac{1}{9}bx^9e^2 + \frac{1}{7}cd^2x^7 + \frac{2}{7}bdx^7e + \frac{1}{7}ax^7e^2 + \frac{1}{5}bd^2x^5 + \frac{2}{5}adx^5e + \frac{1}{3}ad^2x^3$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/11*c*x^11*e^2 + 2/9*c*d*x^9*e + 1/9*b*x^9*e^2 + 1/7*c*d^2*x^7 + 2/7*b*d*x
^7*e + 1/7*a*x^7*e^2 + 1/5*b*d^2*x^5 + 2/5*a*d*x^5*e + 1/3*a*d^2*x^3
```

Mupad [B]
time = 0.03, size = 73, normalized size = 0.94

$$x^7\left(\frac{c\,d^2}{7}+\frac{2\,b\,d\,e}{7}+\frac{a\,e^2}{7}\right)+x^5\left(\frac{b\,d^2}{5}+\frac{2\,a\,e\,d}{5}\right)+x^9\left(\frac{b\,e^2}{9}+\frac{2\,c\,d\,e}{9}\right)+\frac{a\,d^2\,x^3}{3}+\frac{c\,e^2\,x^{11}}{11}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^7*((a*e^2)/7 + (c*d^2)/7 + (2*b*d*e)/7) + x^5*((b*d^2)/5 + (2*a*d*e)/5) +
 x^9*((b*e^2)/9 + (2*c*d*e)/9) + (a*d^2*x^3)/3 + (c*e^2*x^11)/11
```

3.275 $\int x(d+ex^2)^2 \left(a+bx^2+cx^4\right) dx$

Optimal. Leaf size=75

$$\frac{(cd^2-bde+ae^2)\left(d+ex^2\right)^3}{6e^3}-\frac{(2cd-be)\left(d+ex^2\right)^4}{8e^3}+\frac{c(d+ex^2)^5}{10e^3}$$

```
[Out] 1/6*(a*e^2-b*d*e+c*d^2)*(e*x^2+d)^3/e^3-1/8*(-b*e+2*c*d)*(e*x^2+d)^4/e^3+1/
10*c*(e*x^2+d)^5/e^3
```

Rubi [A]
time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {1261, 712}

$$\frac{\left(d+ex^2\right)^3\left(ae^2-bde+cd^2\right)}{6e^3}-\frac{\left(d+ex^2\right)^4(2cd-be)}{8e^3}+\frac{c(d+ex^2)^5}{10e^3}$$

Antiderivative was successfully verified.

```
[In] Int[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*(d + e*x^2)^3)/(6*e^3) - ((2*c*d - b*e)*(d + e*x^2
)^4)/(8*e^3) + (c*(d + e*x^2)^5)/(10*e^3)
```

Rule 712

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /; F
reeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a
*e^2, 0] && NeQ[2*c*d - b*e, 0] && IntegerQ[p] && (GtQ[p, 0] || (EqQ[a, 0]
&& IntegerQ[m]))
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int x\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)dx &= \frac{1}{2}\text{Subst}\left(\int (d+ex)^2\left(a+bx+cx^2\right)dx, x, x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(cd^2-bde+ae^2)(d+ex)^2}{e^2}+\frac{(-2cd+be)(d+ex)^3}{e^2}+\right.\right.\\ &= \frac{(cd^2-bde+ae^2)\left(d+ex^2\right)^3}{6e^3}-\frac{(2cd-be)\left(d+ex^2\right)^4}{8e^3}+\frac{c(d+ex^2)^5}{10e^3}\end{aligned}$$

Mathematica [A]
time = 0.02, size = 72, normalized size = 0.96

$$\frac{1}{120}x^2\left(60ad^2+30d(bd+2ae)x^2+20\left(cd^2+e(2bd+ae)\right)x^4+15e(2cd+be)x^6+12ce^2x^8\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] (x^2*(60*a*d^2 + 30*d*(b*d + 2*a*e)*x^2 + 20*(c*d^2 + e*(2*b*d + a*e))*x^4
+ 15*e*(2*c*d + b*e)*x^6 + 12*c*e^2*x^8))/120
```

Maple [A]
time = 0.14, size = 73, normalized size = 0.97

method	result	size
default	$\frac{c\,e^2x^{10}}{10}+\frac{(e^2b+2cde)x^8}{8}+\frac{(a\,e^2+2deb+c\,d^2)x^6}{6}+\frac{(2ade+d^2b)x^4}{4}+\frac{a\,d^2x^2}{2}$	73
norman	$\frac{c\,e^2x^{10}}{10}+\left(\frac{1}{8}e^2b+\frac{1}{4}cde\right)x^8+\left(\frac{1}{6}a\,e^2+\frac{1}{3}deb+\frac{1}{6}c\,d^2\right)x^6+\left(\frac{1}{2}ade+\frac{1}{4}d^2b\right)x^4+\frac{a\,d^2x^2}{2}$	74
gosper	$\frac{1}{10}c\,e^2x^{10}+\frac{1}{8}x^8e^2b+\frac{1}{4}x^8cde+\frac{1}{6}x^6a\,e^2+\frac{1}{3}x^6deb+\frac{1}{6}x^6c\,d^2+\frac{1}{2}x^4ade+\frac{1}{4}b\,x^4d^2+\frac{1}{2}a\,d^2x^2$	80
risch	$\frac{1}{10}c\,e^2x^{10}+\frac{1}{8}x^8e^2b+\frac{1}{4}x^8cde+\frac{1}{6}x^6a\,e^2+\frac{1}{3}x^6deb+\frac{1}{6}x^6c\,d^2+\frac{1}{2}x^4ade+\frac{1}{4}b\,x^4d^2+\frac{1}{2}a\,d^2x^2$	80

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*c*e^2*x^10+1/8*(b*e^2+2*c*d*e)*x^8+1/6*(a*e^2+2*b*d*e+c*d^2)*x^6+1/4*(
2*a*d*e+b*d^2)*x^4+1/2*a*d^2*x^2
```

Maxima [A]
time = 0.29, size = 72, normalized size = 0.96

$$\frac{1}{10}\,cx^{10}e^2+\frac{1}{8}\left(2\,cde+be^2\right)x^8+\frac{1}{6}\left(cd^2+2\,bde+ae^2\right)x^6+\frac{1}{2}\,ad^2x^2+\frac{1}{4}\left(bd^2+2\,ade\right)x^4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/10*c*x^10*e^2 + 1/8*(2*c*d*e + b*e^2)*x^8 + 1/6*(c*d^2 + 2*b*d*e + a*e^2)
*x^6 + 1/2*a*d^2*x^2 + 1/4*(b*d^2 + 2*a*d*e)*x^4
```

Fricas [A]
time = 0.33, size = 77, normalized size = 1.03

$$\frac{1}{6}\,cd^2x^6+\frac{1}{4}\,bd^2x^4+\frac{1}{2}\,ad^2x^2+\frac{1}{120}\left(12\,cx^{10}+15\,bx^8+20\,ax^6\right)e^2+\frac{1}{12}\left(3\,cdx^8+4\,bdx^6+6\,adx^4\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/6*c*d^2*x^6 + 1/4*b*d^2*x^4 + 1/2*a*d^2*x^2 + 1/120*(12*c*x^10 + 15*b*x^8
 + 20*a*x^6)*e^2 + 1/12*(3*c*d*x^8 + 4*b*d*x^6 + 6*a*d*x^4)*e
```

Sympy [A]
time = 0.01, size = 76, normalized size = 1.01

$$\frac{ad^2x^2}{2} + \frac{ce^2x^{10}}{10} + x^8\left(\frac{be^2}{8} + \frac{cde}{4}\right) + x^6\left(\frac{ae^2}{6} + \frac{bde}{3} + \frac{cd^2}{6}\right) + x^4\left(\frac{ade}{2} + \frac{bd^2}{4}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x**2/2 + c*e**2*x**10/10 + x**8*(b*e**2/8 + c*d*e/4) + x**6*(a*e**2/
6 + b*d*e/3 + c*d**2/6) + x**4*(a*d*e/2 + b*d**2/4)
```

Giac [A]
time = 3.03, size = 79, normalized size = 1.05

$$\frac{1}{10}\,cx^{10}e^2 + \frac{1}{4}\,cdx^8e + \frac{1}{8}\,bx^8e^2 + \frac{1}{6}\,cd^2x^6 + \frac{1}{3}\,bdx^6e + \frac{1}{6}\,ax^6e^2 + \frac{1}{4}\,bd^2x^4 + \frac{1}{2}\,adx^4e + \frac{1}{2}\,ad^2x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/10*c*x^10*e^2 + 1/4*c*d*x^8*e + 1/8*b*x^8*e^2 + 1/6*c*d^2*x^6 + 1/3*b*d*x
^6*e + 1/6*a*x^6*e^2 + 1/4*b*d^2*x^4 + 1/2*a*d*x^4*e + 1/2*a*d^2*x^2
```

Mupad [B]
time = 0.03, size = 73, normalized size = 0.97

$$x^6\left(\frac{c\,d^2}{6} + \frac{b\,d\,e}{3} + \frac{a\,e^2}{6}\right) + x^4\left(\frac{b\,d^2}{4} + \frac{a\,e\,d}{2}\right) + x^8\left(\frac{b\,e^2}{8} + \frac{c\,d\,e}{4}\right) + \frac{a\,d^2\,x^2}{2} + \frac{c\,e^2\,x^{10}}{10}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^6*((a*e^2)/6 + (c*d^2)/6 + (b*d*e)/3) + x^4*((b*d^2)/4 + (a*d*e)/2) + x^8
*((b*e^2)/8 + (c*d*e)/4) + (a*d^2*x^2)/2 + (c*e^2*x^10)/10
```

3.276 $\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx$

Optimal. Leaf size=73

$$ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9$$

```
[Out] a*d^2*x+1/3*d*(2*a*e+b*d)*x^3+1/5*(c*d^2+e*(a*e+2*b*d))*x^5+1/7*e*(b*e+2*c*
d)*x^7+1/9*c*e^2*x^9
```

Rubi [A]
time = 0.03, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.045$, Rules used = {1167}

$$\frac{1}{5}x^5\left(e(ae+2bd)+cd^2\right)+\frac{1}{3}dx^3(2ae+bd)+ad^2x+\frac{1}{7}ex^7(be+2cd)+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + e*(2*b*d + a*e))*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \left(d+ex^2\right)^2 \left(a+bx^2+cx^4\right) \, dx &= \int \left(ad^2+d(bd+2ae)x^2+\left(cd^2+e(2bd+ae)\right)x^4+e(2cd+be)x^6+ce^2\right. \\ &= ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+e(2bd+ae)\right)x^5+\frac{1}{7}e(2cd+be)x^7+\end{aligned}$$

Mathematica [A]
time = 0.01, size = 73, normalized size = 1.00

$$ad^2x+\frac{1}{3}d(bd+2ae)x^3+\frac{1}{5}\left(cd^2+2bde+ae^2\right)x^5+\frac{1}{7}e(2cd+be)x^7+\frac{1}{9}ce^2x^9$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^2*(a + b*x^2 + c*x^4),x]
```

```
[Out] a*d^2*x + (d*(b*d + 2*a*e)*x^3)/3 + ((c*d^2 + 2*b*d*e + a*e^2)*x^5)/5 + (e*
(2*c*d + b*e)*x^7)/7 + (c*e^2*x^9)/9
```

Maple [A]
time = 0.11, size = 70, normalized size = 0.96

method	result	size
default	$\frac{c\,e^{2} x^{9}}{9}+\frac{\left(e^{2} b+2 c d e\right) x^{7}}{7}+\frac{\left(a\,e^{2}+2 d e b+c\,d^{2}\right) x^{5}}{5}+\frac{\left(2 a d e+d^{2} b\right) x^{3}}{3}+a\,d^{2} x$	70
norman	$\frac{c\,e^{2} x^{9}}{9}+\left(\frac{1}{7} e^{2} b+\frac{2}{7} c d e\right) x^{7}+\left(\frac{1}{5} a\,e^{2}+\frac{2}{5} d e b+\frac{1}{5} c\,d^{2}\right) x^{5}+\left(\frac{2}{3} a d e+\frac{1}{3} d^{2} b\right) x^{3}+a\,d^{2} x$	71
gosper	$\frac{1}{9} c\,e^{2} x^{9}+\frac{1}{7} x^{7} e^{2} b+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{2}{5} x^{5} d e b+\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+\frac{1}{3} x^{3} d^{2} b+a\,d^{2} x$	77
risch	$\frac{1}{9} c\,e^{2} x^{9}+\frac{1}{7} x^{7} e^{2} b+\frac{2}{7} c d e\,x^{7}+\frac{1}{5} x^{5} a\,e^{2}+\frac{2}{5} x^{5} d e b+\frac{1}{5} x^{5} c\,d^{2}+\frac{2}{3} a d e\,x^{3}+\frac{1}{3} x^{3} d^{2} b+a\,d^{2} x$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/9*c*e^2*x^9+1/7*(b*e^2+2*c*d*e)*x^7+1/5*(a*e^2+2*b*d*e+c*d^2)*x^5+1/3*(2*
a*d*e+b*d^2)*x^3+a*d^2*x
```

Maxima [A]
time = 0.30, size = 69, normalized size = 0.95

$$\frac{1}{9}\,cx^{9}e^{2}+\frac{1}{7}\,{\left(2\,cde+be^{2}\right)}x^{7}+\frac{1}{5}\,{\left(cd^{2}+2\,bde+ae^{2}\right)}x^{5}+ad^{2}x+\frac{1}{3}\,{\left(bd^{2}+2\,ade\right)}x^{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] 1/9*c*x^9*e^2 + 1/7*(2*c*d*e + b*e^2)*x^7 + 1/5*(c*d^2 + 2*b*d*e + a*e^2)*x
^5 + a*d^2*x + 1/3*(b*d^2 + 2*a*d*e)*x^3
```

Fricas [A]
time = 0.37, size = 74, normalized size = 1.01

$$\frac{1}{5}\,cd^{2}x^{5}+\frac{1}{3}\,bd^{2}x^{3}+ad^{2}x+\frac{1}{315}\,{\left(35\,cx^{9}+45\,bx^{7}+63\,ax^{5}\right)}e^{2}+\frac{2}{105}\,{\left(15\,cdx^{7}+21\,bdx^{5}+35\,adx^{3}\right)}e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/5*c*d^2*x^5 + 1/3*b*d^2*x^3 + a*d^2*x + 1/315*(35*c*x^9 + 45*b*x^7 + 63*a
*x^5)*e^2 + 2/105*(15*c*d*x^7 + 21*b*d*x^5 + 35*a*d*x^3)*e
```

Sympy [A]
time = 0.01, size = 78, normalized size = 1.07

$$ad^{2}x+\frac{ce^{2}x^{9}}{9}+x^{7}\left(\frac{be^{2}}{7}+\frac{2cde}{7}\right)+x^{5}\left(\frac{ae^{2}}{5}+\frac{2bde}{5}+\frac{cd^{2}}{5}\right)+x^{3}\cdot\left(\frac{2ade}{3}+\frac{bd^{2}}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a),x)
```

```
[Out] a*d**2*x + c*e**2*x**9/9 + x**7*(b*e**2/7 + 2*c*d*e/7) + x**5*(a*e**2/5 + 2
*b*d*e/5 + c*d**2/5) + x**3*(2*a*d*e/3 + b*d**2/3)
```

Giac [A]
time = 2.91, size = 76, normalized size = 1.04

$$\frac{1}{9}\,cx^9e^2 + \frac{2}{7}\,cdx^7e + \frac{1}{7}\,bx^7e^2 + \frac{1}{5}\,cd^2x^5 + \frac{2}{5}\,bdx^5e + \frac{1}{5}\,ax^5e^2 + \frac{1}{3}\,bd^2x^3 + \frac{2}{3}\,adx^3e + ad^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/9*c*x^9*e^2 + 2/7*c*d*x^7*e + 1/7*b*x^7*e^2 + 1/5*c*d^2*x^5 + 2/5*b*d*x^5
*e + 1/5*a*x^5*e^2 + 1/3*b*d^2*x^3 + 2/3*a*d*x^3*e + a*d^2*x
```

Mupad [B]
time = 0.03, size = 70, normalized size = 0.96

$$x^5\left(\frac{c\,d^2}{5}+\frac{2\,b\,d\,e}{5}+\frac{a\,e^2}{5}\right)+x^3\left(\frac{b\,d^2}{3}+\frac{2\,a\,e\,d}{3}\right)+x^7\left(\frac{b\,e^2}{7}+\frac{2\,c\,d\,e}{7}\right)+\frac{c\,e^2\,x^9}{9}+a\,d^2\,x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^2*(a + b*x^2 + c*x^4),x)
```

```
[Out] x^5*((a*e^2)/5 + (c*d^2)/5 + (2*b*d*e)/5) + x^3*((b*d^2)/3 + (2*a*d*e)/3) +
 x^7*((b*e^2)/7 + (2*c*d*e)/7) + (c*e^2*x^9)/9 + a*d^2*x
```

3.277 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x} \, dx$

Optimal. Leaf size=74

$$\frac{1}{2}d(bd+2ae)x^2+\frac{1}{4}\left(cd^2+e(2bd+ae)\right)x^4+\frac{1}{6}e(2cd+be)x^6+\frac{1}{8}ce^2x^8+ad^2\log(x)$$

```
[Out] 1/2*d*(2*a*e+b*d)*x^2+1/4*(c*d^2+e*(a*e+2*b*d))*x^4+1/6*e*(b*e+2*c*d)*x^6+1
/8*c*e^2*x^8+a*d^2*ln(x)
```

Rubi [A]
time = 0.07, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.080$, Rules used = {1265, 907}

$$\frac{1}{4}x^4\left(e(ae+2bd)+cd^2\right)+\frac{1}{2}dx^2(2ae+bd)+ad^2\log(x)+\frac{1}{6}ex^6(be+2cd)+\frac{1}{8}ce^2x^8$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + e*(2*b*d + a*e))*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(d+ex^2)^2 (a+bx^2+cx^4)}{x}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^2 (a+bx+cx^2)}{x}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(d(bd+2ae) + \frac{ad^2}{x} + \left(cd^2+e(2bd+ae)\right)x + e(2cd+be)x^2\right.\right.$$
$$= \frac{1}{2}d(bd+2ae)x^2 + \frac{1}{4}\left(cd^2+e(2bd+ae)\right)x^4 + \frac{1}{6}e(2cd+be)x^6 + \frac{1}{8}ce^2x^8 +$$

Mathematica [A]
time = 0.01, size = 74, normalized size = 1.00

$$\frac{1}{2}d(bd+2ae)x^2 + \frac{1}{4}\left(cd^2+2bde+ae^2\right)x^4 + \frac{1}{6}e(2cd+be)x^6 + \frac{1}{8}ce^2x^8 + ad^2\log(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x]
```

```
[Out] (d*(b*d + 2*a*e)*x^2)/2 + ((c*d^2 + 2*b*d*e + a*e^2)*x^4)/4 + (e*(2*c*d + b
*e)*x^6)/6 + (c*e^2*x^8)/8 + a*d^2*Log[x]
```

Maple [A]
time = 0.11, size = 77, normalized size = 1.04

method	result	size
norman	$\left(\frac{1}{6}e^2b+\frac{1}{3}cde\right)x^6+\left(ade+\frac{1}{2}d^2b\right)x^2+\left(\frac{1}{4}a\,e^2+\frac{1}{2}deb+\frac{1}{4}c\,d^2\right)x^4+\frac{c\,e^2x^8}{8}+a\,d^2\ln(x)$	71
default	$\frac{c\,e^2x^8}{8}+\frac{b\,e^2x^6}{6}+\frac{cde\,x^6}{3}+\frac{a\,e^2x^4}{4}+\frac{bde\,x^4}{2}+\frac{c\,d^2x^4}{4}+ade\,x^2+\frac{b\,d^2x^2}{2}+a\,d^2\ln(x)$	77
risch	$\frac{c\,e^2x^8}{8}+\frac{b\,e^2x^6}{6}+\frac{cde\,x^6}{3}+\frac{a\,e^2x^4}{4}+\frac{bde\,x^4}{2}+\frac{c\,d^2x^4}{4}+ade\,x^2+\frac{b\,d^2x^2}{2}+a\,d^2\ln(x)$	77

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*c*e^2*x^8+1/6*b*e^2*x^6+1/3*c*d*e*x^6+1/4*a*e^2*x^4+1/2*b*d*e*x^4+1/4*c
*d^2*x^4+a*d*e*x^2+1/2*b*d^2*x^2+a*d^2*ln(x)
```

Maxima [A]
time = 0.28, size = 73, normalized size = 0.99

$$\frac{1}{8}\,cx^8e^2 + \frac{1}{6}\,(2\,cde+be^2)x^6 + \frac{1}{4}\,(cd^2+2\,bde+ae^2)x^4 + \frac{1}{2}\,ad^2\log\left(x^2\right) + \frac{1}{2}\,(bd^2+2\,ade)x^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="maxima")
```

```
[Out] 1/8*c*x^8*e^2 + 1/6*(2*c*d*e + b*e^2)*x^6 + 1/4*(c*d^2 + 2*b*d*e + a*e^2)*x
^4 + 1/2*a*d^2*log(x^2) + 1/2*(b*d^2 + 2*a*d*e)*x^2
```

Fricas [A]
time = 0.33, size = 75, normalized size = 1.01

$$\frac{1}{4}cd^2x^4 + \frac{1}{2}bd^2x^2 + ad^2\log(x) + \frac{1}{24}\left(3\,cx^8 + 4\,bx^6 + 6\,ax^4\right)e^2 + \frac{1}{6}\left(2\,cdx^6 + 3\,bdx^4 + 6\,adx^2\right)e$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="fricas")
```

```
[Out] 1/4*c*d^2*x^4 + 1/2*b*d^2*x^2 + a*d^2*log(x) + 1/24*(3*c*x^8 + 4*b*x^6 + 6*
a*x^4)*e^2 + 1/6*(2*c*d*x^6 + 3*b*d*x^4 + 6*a*d*x^2)*e
```

Sympy [A]
time = 0.06, size = 73, normalized size = 0.99

$$ad^2\log(x) + \frac{ce^2x^8}{8} + x^6\left(\frac{be^2}{6} + \frac{cde}{3}\right) + x^4\left(\frac{ae^2}{4} + \frac{bde}{2} + \frac{cd^2}{4}\right) + x^2\left(ade + \frac{bd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x,x)
```

```
[Out] a*d**2*log(x) + c*e**2*x**8/8 + x**6*(b*e**2/6 + c*d*e/3) + x**4*(a*e**2/4
+ b*d*e/2 + c*d**2/4) + x**2*(a*d*e + b*d**2/2)
```

Giac [A]
time = 3.11, size = 79, normalized size = 1.07

$$\frac{1}{8}cx^8e^2 + \frac{1}{3}cdx^6e + \frac{1}{6}bx^6e^2 + \frac{1}{4}cd^2x^4 + \frac{1}{2}bdx^4e + \frac{1}{4}ax^4e^2 + \frac{1}{2}bd^2x^2 + adx^2e + \frac{1}{2}ad^2\log\left(x^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x,x, algorithm="giac")
```

```
[Out] 1/8*c*x^8*e^2 + 1/3*c*d*x^6*e + 1/6*b*x^6*e^2 + 1/4*c*d^2*x^4 + 1/2*b*d*x^4
*e + 1/4*a*x^4*e^2 + 1/2*b*d^2*x^2 + a*d*x^2*e + 1/2*a*d^2*log(x^2)
```

Mupad [B]
time = 0.03, size = 70, normalized size = 0.95

$$x^4\left(\frac{c\,d^2}{4} + \frac{b\,d\,e}{2} + \frac{a\,e^2}{4}\right) + x^2\left(\frac{b\,d^2}{2} + a\,e\,d\right) + x^6\left(\frac{b\,e^2}{6} + \frac{c\,d\,e}{3}\right) + \frac{c\,e^2\,x^8}{8} + a\,d^2\,\ln(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x,x)
```

```
[Out] x^4*((a*e^2)/4 + (c*d^2)/4 + (b*d*e)/2) + x^2*((b*d^2)/2 + a*d*e) + x^6*((b
*e^2)/6 + (c*d*e)/3) + (c*e^2*x^8)/8 + a*d^2*log(x)
```

3.278 $\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx$

Optimal. Leaf size=71

$$-\frac{ad^2}{x}+d(bd+2ae)x+\frac{1}{3}\left(cd^2+e(2bd+ae)\right)x^3+\frac{1}{5}e(2cd+be)x^5+\frac{1}{7}ce^2x^7$$

```
[Out] -a*d^2/x+d*(2*a*e+b*d)*x+1/3*(c*d^2+e*(a*e+2*b*d))*x^3+1/5*e*(b*e+2*c*d)*x^
5+1/7*c*e^2*x^7
```

Rubi [A]
time = 0.03, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.040, Rules used = {1275}

$$\frac{1}{3}x^3\left(e(ae+2bd)+cd^2\right)+dx(2ae+bd)-\frac{ad^2}{x}+\frac{1}{5}ex^5(be+2cd)+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + e*(2*b*d + a*e))*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^2}\,dx &= \int \left(d(bd+2ae)+\frac{ad^2}{x^2}+\left(cd^2+e(2bd+ae)\right)x^2+e(2cd+be)x^4+ce^2x^6\right.\\ &= -\frac{ad^2}{x}+d(bd+2ae)x+\frac{1}{3}\left(cd^2+e(2bd+ae)\right)x^3+\frac{1}{5}e(2cd+be)x^5+\frac{1}{7}ce\end{aligned}$$

Mathematica [A]
time = 0.02, size = 71, normalized size = 1.00

$$-\frac{ad^2}{x}+d(bd+2ae)x+\frac{1}{3}\left(cd^2+2bde+ae^2\right)x^3+\frac{1}{5}e(2cd+be)x^5+\frac{1}{7}ce^2x^7$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x]
```

```
[Out] -((a*d^2)/x) + d*(b*d + 2*a*e)*x + ((c*d^2 + 2*b*d*e + a*e^2)*x^3)/3 + (e*(
2*c*d + b*e)*x^5)/5 + (c*e^2*x^7)/7
```

Maple [A]
time = 0.12, size = 75, normalized size = 1.06

method	result	size
norman	$\frac{\frac{c\,e^{2} x^{8}}{7}+\left(\frac{1}{5} e^{2} b+\frac{2}{5} c d e\right) x^{6}+\left(\frac{1}{3} a\,e^{2}+\frac{2}{3} d e b+\frac{1}{3} c\,d^{2}\right) x^{4}+\left(2 a d e+d^{2} b\right) x^{2}-a\,d^{2}}{x}$	74
default	$\frac{c\,e^{2} x^{7}}{7}+\frac{b\,e^{2} x^{5}}{5}+\frac{2 c d e\,x^{5}}{5}+\frac{a\,e^{2} x^{3}}{3}+\frac{2 b d e\,x^{3}}{3}+\frac{c\,d^{2} x^{3}}{3}+2 a d e x+d^{2} b x-\frac{a\,d^{2}}{x}$	75
risch	$\frac{c\,e^{2} x^{7}}{7}+\frac{b\,e^{2} x^{5}}{5}+\frac{2 c d e\,x^{5}}{5}+\frac{a\,e^{2} x^{3}}{3}+\frac{2 b d e\,x^{3}}{3}+\frac{c\,d^{2} x^{3}}{3}+2 a d e x+d^{2} b x-\frac{a\,d^{2}}{x}$	75
gosper	$-\frac{-15 c\,e^{2} x^{8}-21 b\,e^{2} x^{6}-42 c d e\,x^{6}-35 a\,e^{2} x^{4}-70 b d e\,x^{4}-35 c\,d^{2} x^{4}-210 a d e\,x^{2}-105 b\,d^{2} x^{2}+105 a\,d^{2}}{105 x}$	82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/7*c*e^2*x^7+1/5*b*e^2*x^5+2/5*c*d*e*x^5+1/3*a*e^2*x^3+2/3*b*d*e*x^3+1/3*c
*d^2*x^3+2*a*d*e*x+d^2*b*x-a*d^2/x
```

Maxima [A]
time = 0.30, size = 69, normalized size = 0.97

$$\frac{1}{7}\,cx^{7}e^{2}+\frac{1}{5}\,\left(2\,cde+be^{2}\right)x^{5}+\frac{1}{3}\,\left(cd^{2}+2\,bde+ae^{2}\right)x^{3}-\frac{ad^{2}}{x}+\left(bd^{2}+2\,ade\right)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="maxima")
```

```
[Out] 1/7*c*x^7*e^2 + 1/5*(2*c*d*e + b*e^2)*x^5 + 1/3*(c*d^2 + 2*b*d*e + a*e^2)*x
^3 - a*d^2/x + (b*d^2 + 2*a*d*e)*x
```

Fricas [A]
time = 0.35, size = 78, normalized size = 1.10

$$\frac{35\,cd^{2}x^{4}+105\,bd^{2}x^{2}-105\,ad^{2}+\left(15\,cx^{8}+21\,bx^{6}+35\,ax^{4}\right)e^{2}+14\,\left(3\,cdx^{6}+5\,bdx^{4}+15\,adx^{2}\right)e}{105\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="fricas")
```

```
[Out] 1/105*(35*c*d^2*x^4 + 105*b*d^2*x^2 - 105*a*d^2 + (15*c*x^8 + 21*b*x^6 + 35
*a*x^4)*e^2 + 14*(3*c*d*x^6 + 5*b*d*x^4 + 15*a*d*x^2)*e)/x
```

Sympy [A]
time = 0.06, size = 73, normalized size = 1.03

$$-\frac{ad^2}{x}+\frac{ce^2x^7}{7}+x^5\left(\frac{be^2}{5}+\frac{2cde}{5}\right)+x^3\left(\frac{ae^2}{3}+\frac{2bde}{3}+\frac{cd^2}{3}\right)+x\left(2ade+bd^2\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**2,x)
```

```
[Out] -a*d**2/x + c*e**2*x**7/7 + x**5*(b*e**2/5 + 2*c*d*e/5) + x**3*(a*e**2/3 +
2*b*d*e/3 + c*d**2/3) + x*(2*a*d*e + b*d**2)
```

Giac [A]
time = 5.14, size = 74, normalized size = 1.04

$$\frac{1}{7}\,cx^7e^2+\frac{2}{5}\,cdx^5e+\frac{1}{5}\,bx^5e^2+\frac{1}{3}\,cd^2x^3+\frac{2}{3}\,bdx^3e+\frac{1}{3}\,ax^3e^2+bd^2x+2\,adxe-\frac{ad^2}{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^2,x, algorithm="giac")
```

```
[Out] 1/7*c*x^7*e^2 + 2/5*c*d*x^5*e + 1/5*b*x^5*e^2 + 1/3*c*d^2*x^3 + 2/3*b*d*x^3
*e + 1/3*a*x^3*e^2 + b*d^2*x + 2*a*d*x*e - a*d^2/x
```

Mupad [B]
time = 0.03, size = 70, normalized size = 0.99

$$x^3\,\left(\frac{c\,d^2}{3}+\frac{2\,b\,d\,e}{3}+\frac{a\,e^2}{3}\right)+x\,\left(b\,d^2+2\,a\,e\,d\right)+x^5\,\left(\frac{b\,e^2}{5}+\frac{2\,c\,d\,e}{5}\right)-\frac{a\,d^2}{x}+\frac{c\,e^2\,x^7}{7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^2,x)
```

```
[Out] x^3*((a*e^2)/3 + (c*d^2)/3 + (2*b*d*e)/3) + x*(b*d^2 + 2*a*d*e) + x^5*((b*e
^2)/5 + (2*c*d*e)/5) - (a*d^2)/x + (c*e^2*x^7)/7
```

3.279 $\int \frac{\left(d+ex^2\right)^2\left(a+bx^2+cx^4\right)}{x^3}\,dx$

Optimal. Leaf size=74

$$-\frac{ad^2}{2x^2}+\frac{1}{2}\left(cd^2+e(2bd+ae)\right)x^2+\frac{1}{4}e(2cd+be)x^4+\frac{1}{6}ce^2x^6+d(bd+2ae)\log(x)$$

```
[Out] -1/2*a*d^2/x^2+1/2*(c*d^2+e*(a*e+2*b*d))*x^2+1/4*e*(b*e+2*c*d)*x^4+1/6*c*e^
2*x^6+d*(2*a*e+b*d)*ln(x)
```

Rubi [A]
time = 0.06, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.080$, Rules used = {1265, 907}

$$\frac{1}{2}x^2\left(e(ae+2bd)+cd^2\right)+d\log(x)(2ae+bd)-\frac{ad^2}{2x^2}+\frac{1}{4}ex^4(be+2cd)+\frac{1}{6}ce^2x^6$$

Antiderivative was successfully verified.

```
[In] Int[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] -1/2*(a*d^2)/x^2 + ((c*d^2 + e*(2*b*d + a*e))*x^2)/2 + (e*(2*c*d + b*e)*x^4
)/4 + (c*e^2*x^6)/6 + d*(b*d + 2*a*e)*Log[x]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(d+ex^2)^2(a+bx^2+cx^4)}{x^3}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^2(a+bx+cx^2)}{x^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(cd^2\left(1+\frac{e(2bd+ae)}{cd^2}\right)+\frac{ad^2}{x^2}+\frac{d(bd+2ae)}{x}+e(2cd+be)x\right.\right.$$
$$= -\frac{ad^2}{2x^2}+\frac{1}{2}\left(cd^2+e(2bd+ae)\right)x^2+\frac{1}{4}e(2cd+be)x^4+\frac{1}{6}ce^2x^6+d(bd+2a$$

Mathematica [A]
time = 0.03, size = 71, normalized size = 0.96

$$\frac{1}{12}\left(-\frac{6ad^2}{x^2}+6\left(cd^2+e(2bd+ae)\right)x^2+3e(2cd+be)x^4+2ce^2x^6+12d(bd+2ae)\log(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x]
```

```
[Out] ((-6*a*d^2)/x^2 + 6*(c*d^2 + e*(2*b*d + a*e))*x^2 + 3*e*(2*c*d + b*e)*x^4 +
 2*c*e^2*x^6 + 12*d*(b*d + 2*a*e)*Log[x])/12
```

Maple [A]
time = 0.12, size = 74, normalized size = 1.00

method	result	size
norman	$\frac{\left(\frac{1}{4}e^2b+\frac{1}{2}cde\right)x^6+\left(\frac{1}{2}a\,e^2+deb+\frac{1}{2}c\,d^2\right)x^4-\frac{a\,d^2}{2}+\frac{c\,e^2x^8}{6}}{x^2}+(2ade+d^2b)\ln(x)$	73
default	$\frac{c\,e^2x^6}{6}+\frac{b\,e^2x^4}{4}+\frac{cde\,x^4}{2}+\frac{a\,e^2x^2}{2}+bde\,x^2+\frac{c\,d^2x^2}{2}-\frac{a\,d^2}{2x^2}+d(2ae+bd)\ln(x)$	74
risch	$\frac{c\,e^2x^6}{6}+\frac{b\,e^2x^4}{4}+\frac{cde\,x^4}{2}+\frac{a\,e^2x^2}{2}+bde\,x^2+\frac{c\,d^2x^2}{2}-\frac{a\,d^2}{2x^2}+2\ln(x)\,ade+\ln(x)\,b\,d^2$	76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*c*e^2*x^6+1/4*b*e^2*x^4+1/2*c*d*e*x^4+1/2*a*e^2*x^2+b*d*e*x^2+1/2*c*d^2
*x^2-1/2*a*d^2/x^2+d*(2*a*e+b*d)*ln(x)
```

Maxima [A]
time = 0.29, size = 73, normalized size = 0.99

$$\frac{1}{6}cx^6e^2+\frac{1}{4}\left(2\,cde+be^2\right)x^4+\frac{1}{2}\left(cd^2+2\,bde+ae^2\right)x^2+\frac{1}{2}\left(bd^2+2\,ade\right)\log\left(x^2\right)-\frac{ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="maxima")
```

```
[Out] 1/6*c*x^6*e^2 + 1/4*(2*c*d*e + b*e^2)*x^4 + 1/2*(c*d^2 + 2*b*d*e + a*e^2)*x
^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*a*d^2/x^2
```

Fricas [A]

time = 0.37, size = 83, normalized size = 1.12

$$\frac{6\,cd^2x^4 - 6\,ad^2 + (2\,cx^8 + 3\,bx^6 + 6\,ax^4)e^2 + 6\,(cdx^6 + 2\,bdx^4)e + 12\,(bd^2x^2 + 2\,adx^2e)\log(x)}{12\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="fricas")
```

```
[Out] 1/12*(6*c*d^2*x^4 - 6*a*d^2 + (2*c*x^8 + 3*b*x^6 + 6*a*x^4)*e^2 + 6*(c*d*x^
6 + 2*b*d*x^4)*e + 12*(b*d^2*x^2 + 2*a*d*x^2*e)*log(x))/x^2
```

Sympy [A]

time = 0.12, size = 71, normalized size = 0.96

$$-\frac{ad^2}{2x^2} + \frac{ce^2x^6}{6} + d(2ae + bd)\log(x) + x^4\left(\frac{be^2}{4} + \frac{cde}{2}\right) + x^2\left(\frac{ae^2}{2} + bde + \frac{cd^2}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**2*(c*x**4+b*x**2+a)/x**3,x)
```

```
[Out] -a*d**2/(2*x**2) + c*e**2*x**6/6 + d*(2*a*e + b*d)*log(x) + x**4*(b*e**2/4
+ c*d*e/2) + x**2*(a*e**2/2 + b*d*e + c*d**2/2)
```

Giac [A]

time = 6.22, size = 97, normalized size = 1.31

$$\frac{1}{6}\,cx^6e^2 + \frac{1}{2}\,cdx^4e + \frac{1}{4}\,bx^4e^2 + \frac{1}{2}\,cd^2x^2 + bdx^2e + \frac{1}{2}\,ax^2e^2 + \frac{1}{2}\,(bd^2 + 2\,ade)\log(x^2) - \frac{bd^2x^2 + 2\,adx^2e + ad^2}{2\,x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^2*(c*x^4+b*x^2+a)/x^3,x, algorithm="giac")
```

```
[Out] 1/6*c*x^6*e^2 + 1/2*c*d*x^4*e + 1/4*b*x^4*e^2 + 1/2*c*d^2*x^2 + b*d*x^2*e +
 1/2*a*x^2*e^2 + 1/2*(b*d^2 + 2*a*d*e)*log(x^2) - 1/2*(b*d^2*x^2 + 2*a*d*x^
2*e + a*d^2)/x^2
```

Mupad [B]

time = 0.04, size = 70, normalized size = 0.95

$$x^2\left(\frac{c\,d^2}{2} + b\,d\,e + \frac{a\,e^2}{2}\right) + x^4\left(\frac{b\,e^2}{4} + \frac{c\,d\,e}{2}\right) + \ln(x)\,\left(b\,d^2 + 2\,a\,e\,d\right) - \frac{a\,d^2}{2\,x^2} + \frac{c\,e^2\,x^6}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((d + e*x^2)^2*(a + b*x^2 + c*x^4))/x^3,x)
```

```
[Out] x^2*((a*e^2)/2 + (c*d^2)/2 + b*d*e) + x^4*((b*e^2)/4 + (c*d*e)/2) + log(x)*
(b*d^2 + 2*a*d*e) - (a*d^2)/(2*x^2) + (c*e^2*x^6)/6
```

3.280 $\int \frac{x^6\left(a+bx^2+cx^4\right)}{\left(d+ex^2\right)^2}\,dx$

Optimal. Leaf size=168

$$-\frac{d(4cd^2-e(3bd-2ae))\,x}{e^5}+\frac{(3cd^2-e(2bd-ae))\,x^3}{3e^4}-\frac{(2cd-be)x^5}{5e^3}+\frac{cx^7}{7e^2}-\frac{d^2(cd^2-bde+ae^2)\,x}{2e^5\,(d+ex^2)}+\frac{d^{3/2}(9cd^2}{}$$

```
[Out] -d*(4*c*d^2-e*(-2*a*e+3*b*d))*x/e^5+1/3*(3*c*d^2-e*(-a*e+2*b*d))*x^3/e^4-1/
5*(-b*e+2*c*d)*x^5/e^3+1/7*c*x^7/e^2-1/2*d^2*(a*e^2-b*d*e+c*d^2)*x/e^5/(e*x
^2+d)+1/2*d^(3/2)*(9*c*d^2-e*(-5*a*e+7*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(1
1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1271, 1824, 211}

$$\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(9cd^2-e(7bd-5ae))}{2e^{11/2}}-\frac{dx(4cd^2-e(3bd-2ae))}{e^5}+\frac{x^3(3cd^2-e(2bd-ae))}{3e^4}-\frac{d^2x(ae^2-bde+cd^2)}{2e^5\,(d+ex^2)}-\frac{x^5(2cd-be)}{5e^3}+\frac{cx^7}{7e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -((d*(4*c*d^2 - e*(3*b*d - 2*a*e))*x)/e^5) + ((3*c*d^2 - e*(2*b*d - a*e))*x
^3)/(3*e^4) - ((2*c*d - b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - (d^2*(c*d^2 -
 b*d*e + a*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - e*(7*b*d - 5*a
*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = -\frac{d^2(cd^2-bde+ae^2)\,x}{2e^5\,(d+ex^2)} - \frac{\int \frac{-d^2(cd^2-bde+ae^2)+2de(cd^2-bde+ae^2)x^2-2e^2(cd^2-bde+ae^2)x^4+2}{d+ex^2}}{2e^5}$$

$$= -\frac{d^2(cd^2-bde+ae^2)\,x}{2e^5\,(d+ex^2)} - \frac{\int \Big(2d(4cd^2-e(3bd-2ae)) - 2e(3cd^2-e(2bd-ae)\Big)}{}$$

$$= -\frac{d(4cd^2-e(3bd-2ae))\,x}{e^5} + \frac{(3cd^2-e(2bd-ae))\,x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} -$$

$$= -\frac{d(4cd^2-e(3bd-2ae))\,x}{e^5} + \frac{(3cd^2-e(2bd-ae))\,x^3}{3e^4} - \frac{(2cd-be)x^5}{5e^3} + \frac{cx^7}{7e^2} -$$

Mathematica [A]
time = 0.09, size = 165, normalized size = 0.98

$$-\frac{d(4cd^2-3bde+2ae^2)\,x}{e^5} + \frac{(3cd^2-2bde+ae^2)\,x^3}{3e^4} + \frac{(-2cd+be)x^5}{5e^3} + \frac{cx^7}{7e^2} - \frac{(cd^4-bd^3e+ad^2e^2)\,x}{2e^5\,(d+ex^2)} + \frac{d^{3/2}(9cd^2-7bde+5ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
[Out] -((d*(4*c*d^2 - 3*b*d*e + 2*a*e^2)*x)/e^5) + ((3*c*d^2 - 2*b*d*e + a*e^2)*x
^3)/(3*e^4) + ((-2*c*d + b*e)*x^5)/(5*e^3) + (c*x^7)/(7*e^2) - ((c*d^4 - b*
d^3*e + a*d^2*e^2)*x)/(2*e^5*(d + e*x^2)) + (d^(3/2)*(9*c*d^2 - 7*b*d*e + 5
*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(11/2))
```

Maple [A]
time = 0.13, size = 159, normalized size = 0.95

method	result
default	$-\frac{-\frac{1}{7}c\,x^7e^3-\frac{1}{5}b\,e^3x^5+\frac{2}{5}cd\,x^5e^2-\frac{1}{3}a\,e^3x^3+\frac{2}{3}bd\,e^2x^3-c\,d^2e\,x^3+2d\,e^2ax-3d^2ebx+4c\,d^3x}{e^5} + \frac{d^2\left(\frac{\left(-\frac{1}{2}a\,e^2+\frac{1}{2}deb-\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{(5a\,e^2-}{}\right.}{e^5}$
risch	$\frac{c\,x^7}{7e^2}+\frac{b\,x^5}{5e^2}-\frac{2cd\,x^5}{5e^3}+\frac{a\,x^3}{3e^2}-\frac{2bd\,x^3}{3e^3}+\frac{c\,d^2x^3}{e^4}-\frac{2dax}{e^3}+\frac{3d^2bx}{e^4}-\frac{4c\,d^3x}{e^5}+\frac{\left(-\frac{1}{2}d^2e^2a+\frac{1}{2}d^3eb-\frac{1}{2}d^4c\right)x}{e^5(e\,x^2+d)}+\frac{5\sqrt{-de}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/e^5*(-1/7*c*x^7*e^3-1/5*b*e^3*x^5+2/5*c*d*x^5*e^2-1/3*a*e^3*x^3+2/3*b*d*
e^2*x^3-c*d^2*e*x^3+2*d*e^2*a*x-3*d^2*e*b*x+4*c*d^3*x)+d^2/e^5*((-1/2*a*e^2
+1/2*d*e*b-1/2*c*d^2)*x/(e*x^2+d)+1/2*(5*a*e^2-7*b*d*e+9*c*d^2)/(d*e)^(1/2)
*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.52, size = 154, normalized size = 0.92

$$\frac{\left(9\,cd^4-7\,bd^3e+5\,ad^2e^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{105}\left(15\,cx^7e^3-21\left(2\,cde^2-be^3\right)x^5+35\left(3\,cd^2e-2\,bde^2+ae^3\right)x^3-105\left(4\,cd^3-3\,bd^2e+2\,ade^2\right)x\right)e^{(-5)}-\frac{\left(cd^4-bd^3e+ad^2e^2\right)x}{2\left(x^2e^6+de^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/2)
/sqrt(d) + 1/105*(15*c*x^7*e^3 - 21*(2*c*d*e^2 - b*e^3)*x^5 + 35*(3*c*d^2*e
 - 2*b*d*e^2 + a*e^3)*x^3 - 105*(4*c*d^3 - 3*b*d^2*e + 2*a*d*e^2)*x)*e^(-5)
 - 1/2*(c*d^4 - b*d^3*e + a*d^2*e^2)*x/(x^2*e^6 + d*e^5)
```

Fricas [A]
time = 0.37, size = 415, normalized size = 2.47

$$\left[-\frac{1890\,cd^4x-105\left(9\,cd^4+5\,adx^2e^3-\left(7\,bd^2x^2-5\,ad^2\right)e^2+\left(9\,cd^3x^2-7\,bd^3\right)e\right)\sqrt{-de^{(-1)}}\log\left(\frac{x^2e+2\,\sqrt{-de^{(-1)}}\,xe-d}{x^2e+d}\right)-4\left(15\,cx^9+21\,bx^7+35\,ax^5\right)e^4+4\left(27\,cdx^7+49\,bdx^5+175\,adx^3\right)e^3-14\left(18\,cd^2x^5+70\,bd^2x^3-75\,ad^2x\right)e^2+210\left(6\,cd^3x^3-7\,bd^3x\right)e}{420\left(x^2e^6+de^5\right)},-\frac{945\,cd^4x-105\left(9\,cd^4+5\,adx^2e^3-\left(7\,bd^2x^2-5\,ad^2\right)e^2+\left(9\,cd^3x^2-7\,bd^3\right)e\right)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}-2\left(15\,cx^9+21\,bx^7+35\,ax^5\right)e^4+2\left(27\,cdx^7+49\,bdx^5+175\,adx^3\right)e^3-7\left(18\,cd^2x^5+70\,bd^2x^3-75\,ad^2x\right)e^2+105\left(6\,cd^3x^3-7\,bd^3x\right)e}{210\left(x^2e^6+de^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/420*(1890*c*d^4*x - 105*(9*c*d^4 + 5*a*d*x^2*e^3 - (7*b*d^2*x^2 - 5*a*d
^2)*e^2 + (9*c*d^3*x^2 - 7*b*d^3)*e)*sqrt(-d*e^(-1))*log((x^2*e + 2*sqrt(-d
*e^(-1))*x*e - d)/(x^2*e + d)) - 4*(15*c*x^9 + 21*b*x^7 + 35*a*x^5)*e^4 + 4
*(27*c*d*x^7 + 49*b*d*x^5 + 175*a*d*x^3)*e^3 - 14*(18*c*d^2*x^5 + 70*b*d^2*
x^3 - 75*a*d^2*x)*e^2 + 210*(6*c*d^3*x^3 - 7*b*d^3*x)*e)/(x^2*e^6 + d*e^5),
 -1/210*(945*c*d^4*x - 105*(9*c*d^4 + 5*a*d*x^2*e^3 - (7*b*d^2*x^2 - 5*a*d^
2)*e^2 + (9*c*d^3*x^2 - 7*b*d^3)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1
/2) - 2*(15*c*x^9 + 21*b*x^7 + 35*a*x^5)*e^4 + 2*(27*c*d*x^7 + 49*b*d*x^5 +
 175*a*d*x^3)*e^3 - 7*(18*c*d^2*x^5 + 70*b*d^2*x^3 - 75*a*d^2*x)*e^2 + 105*
(6*c*d^3*x^3 - 7*b*d^3*x)*e)/(x^2*e^6 + d*e^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(153) = 306$.
time = 0.67, size = 320, normalized size = 1.90

$$\frac{cx^7}{7e^2}+x^5\left(\frac{b}{5e^2}-\frac{2cd}{5e^3}\right)+x^3\left(\frac{a}{3e^2}-\frac{2bd}{3e^3}+\frac{cd^2}{e^4}\right)+x\left(-\frac{2ad}{e^3}+\frac{3bd^2}{e^4}-\frac{4cd^3}{e^5}\right)+\frac{x\left(-ad^2e^2+bd^3e-cd^4\right)}{2de^5+2e^6x^2}-\frac{\sqrt{-\frac{d^3}{e^{11}}}\cdot\left(5ae^2-7bde+9cd^2\right)\log\left(-\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}\cdot\left(5ae^2-7bde+9cd^2\right)}{5ade^2-7bd^2e+9cd^3}+x\right)}{4}+\frac{\sqrt{-\frac{d^3}{e^{11}}}\cdot\left(5ae^2-7bde+9cd^2\right)\log\left(\frac{e^5\sqrt{-\frac{d^3}{e^{11}}}\cdot\left(5ae^2-7bde+9cd^2\right)}{5ade^2-7bd^2e+9cd^3}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**7/(7*e**2) + x**5*(b/(5*e**2) - 2*c*d/(5*e**3)) + x**3*(a/(3*e**2) - 2
*b*d/(3*e**3) + c*d**2/e**4) + x*(-2*a*d/e**3 + 3*b*d**2/e**4 - 4*c*d**3/e*
*5) + x*(-a*d**2*e**2 + b*d**3*e - c*d**4)/(2*d*e**5 + 2*e**6*x**2) - sqrt(
-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(-e**5*sqrt(-d**3/e**11)*(5
*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x)/4 +
 sqrt(-d**3/e**11)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)*log(e**5*sqrt(-d**3/e**1
1)*(5*a*e**2 - 7*b*d*e + 9*c*d**2)/(5*a*d*e**2 - 7*b*d**2*e + 9*c*d**3) + x
)/4
```

Giac [A]
time = 4.39, size = 160, normalized size = 0.95

$$\frac{(9\,cd^4-7\,bd^3e+5\,ad^2e^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{105}\,(15\,cx^7e^{12}-42\,cdx^5e^{11}+21\,bx^5e^{12}+105\,cd^2x^3e^{10}-70\,bdx^3e^{11}-420\,cd^3xe^9+35\,ax^3e^{12}+315\,bd^2xe^{10}-210\,adxe^{11})e^{(-14)}-\frac{(cd^4x-bd^3xe+ad^2xe^2)e^{(-5)}}{2\,(x^2e+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(9*c*d^4 - 7*b*d^3*e + 5*a*d^2*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/2)
/sqrt(d) + 1/105*(15*c*x^7*e^12 - 42*c*d*x^5*e^11 + 21*b*x^5*e^12 + 105*c*d
^2*x^3*e^10 - 70*b*d*x^3*e^11 - 420*c*d^3*x*e^9 + 35*a*x^3*e^12 + 315*b*d^2
*x*e^10 - 210*a*d*x*e^11)*e^(-14) - 1/2*(c*d^4*x - b*d^3*x*e + a*d^2*x*e^2)
*e^(-5)/(x^2*e + d)
```

Mupad [B]
time = 0.33, size = 251, normalized size = 1.49

$$x^5\left(\frac{b}{5\,e^2}-\frac{2\,c\,d}{5\,e^3}\right)-x^3\left(\frac{c\,d^2}{3\,e^4}-\frac{a}{3\,e^2}+\frac{2\,d\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{3\,e}\right)+x\left(\frac{2\,d\,\left(\frac{c\,d^2}{e^4}-\frac{a}{e^2}+\frac{2\,d\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e}\right)}{e}-\frac{d^2\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e^2}\right)-\frac{x\,\left(\frac{c\,d^4}{2}-\frac{b\,d^3\,e}{2}+\frac{a\,d^2\,e^2}{2}\right)}{e^6\,x^2+d\,e^5}+\frac{c\,x^7}{7\,e^2}+\frac{d^{3/2}\,\mathrm{atan}\left(\frac{d^{3/2}\,\sqrt{e}\,x\,\left(9\,c\,d^2-7\,b\,d\,e+5\,a\,e^2\right)}{9\,c\,d^4-7\,b\,d^3\,e+5\,a\,d^2\,e^2}\right)\,\left(9\,c\,d^2-7\,b\,d\,e+5\,a\,e^2\right)}{2\,e^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x^5*(b/(5*e^2) - (2*c*d)/(5*e^3)) - x^3*((c*d^2)/(3*e^4) - a/(3*e^2) + (2*d
*(b/e^2 - (2*c*d)/e^3))/(3*e)) + x*((2*d*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2
 - (2*c*d)/e^3))/e))/e - (d^2*(b/e^2 - (2*c*d)/e^3))/e^2) - (x*((c*d^4)/2 +
 (a*d^2*e^2)/2 - (b*d^3*e)/2))/(d*e^5 + e^6*x^2) + (c*x^7)/(7*e^2) + (d^(3/
2)*atan((d^(3/2)*e^(1/2)*x*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(9*c*d^4 + 5*a*d^
2*e^2 - 7*b*d^3*e))*(5*a*e^2 + 9*c*d^2 - 7*b*d*e))/(2*e^(11/2))
```

3.281 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2} dx$

Optimal. Leaf size=135

$$\frac{(3cd^2-e(2bd-ae))x}{e^4}-\frac{(2cd-be)x^3}{3e^3}+\frac{cx^5}{5e^2}+\frac{d(cd^2-bde+ae^2)x}{2e^4(d+ex^2)}-\frac{\sqrt{d}(7cd^2-e(5bd-3ae))\tan^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)}{2e^{9/2}}$$

```
[Out] (3*c*d^2-e*(-a*e+2*b*d))*x/e^4-1/3*(-b*e+2*c*d)*x^3/e^3+1/5*c*x^5/e^2+1/2*d
*(a*e^2-b*d*e+c*d^2)*x/e^4/(e*x^2+d)-1/2*(7*c*d^2-e*(-3*a*e+5*b*d))*arctan(
x*e^(1/2)/d^(1/2))*d^(1/2)/e^(9/2)
```

Rubi [A]
time = 0.11, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1271, 1824, 211}

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{e}x}{\sqrt{d}}\right)(7cd^2-e(5bd-3ae))}{2e^{9/2}}+\frac{x(3cd^2-e(2bd-ae))}{e^4}+\frac{dx(ae^2-bde+cd^2)}{2e^4(d+ex^2)}-\frac{x^3(2cd-be)}{3e^3}+\frac{cx^5}{5e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - e*(2*b*d - a*e))*x)/e^4 - ((2*c*d - b*e)*x^3)/(3*e^3) + (c*x^5)
/(5*e^2) + (d*(c*d^2 - b*d*e + a*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*(7*
c*d^2 - e*(5*b*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx = \frac{d(cd^2-bde+ae^2)\,x}{2e^4\,(d+ex^2)} - \frac{\int \frac{d(cd^2-bde+ae^2)-2e(cd^2-bde+ae^2)x^2+2e^2(cd-be)x^4-2ce^3x^6}{d+ex^2}\,dx}{2e^4}$$

$$= \frac{d(cd^2-bde+ae^2)\,x}{2e^4\,(d+ex^2)} - \frac{\int \Big(-2(3cd^2-2bde+ae^2)+2e(2cd-be)x^2-2ce^2x^4+$$

$$= \frac{(3cd^2-e(2bd-ae))\,x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d(cd^2-bde+ae^2)\,x}{2e^4\,(d+ex^2)} - \frac{(d(7cd^2$$

$$= \frac{(3cd^2-e(2bd-ae))\,x}{e^4} - \frac{(2cd-be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{d(cd^2-bde+ae^2)\,x}{2e^4\,(d+ex^2)} - \frac{\sqrt{d}\,(7c$$

Mathematica [A]
time = 0.05, size = 133, normalized size = 0.99

$$\frac{(3cd^2-2bde+ae^2)\,x}{e^4} + \frac{(-2cd+be)x^3}{3e^3} + \frac{cx^5}{5e^2} + \frac{(cd^3-bd^2e+ade^2)\,x}{2e^4\,(d+ex^2)} - \frac{\sqrt{d}\,(7cd^2-5bde+3ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] ((3*c*d^2 - 2*b*d*e + a*e^2)*x)/e^4 + ((-2*c*d + b*e)*x^3)/(3*e^3) + (c*x^5
)/(5*e^2) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/(2*e^4*(d + e*x^2)) - (Sqrt[d]*
(7*c*d^2 - 5*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*e^(9/2))
```

Maple [A]
time = 0.14, size = 122, normalized size = 0.90

method	result
default	$\frac{\frac{1}{5}c\,x^5e^2+\frac{1}{3}b\,e^2x^3-\frac{2}{3}cde\,x^3+a\,e^2x-2debx+3c\,d^2x}{e^4} - \frac{d\left(\frac{\left(-\frac{1}{2}a\,e^2+\frac{1}{2}deb-\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(3a\,e^2-5deb+7c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}\right)}{e^4}$
risch	$\frac{cx^5}{5e^2}+\frac{bx^3}{3e^2}-\frac{2cd\,x^3}{3e^3}+\frac{ax}{e^2}-\frac{2dbx}{e^3}+\frac{3c\,d^2x}{e^4}+\frac{\left(\frac{1}{2}d\,e^2a-\frac{1}{2}d^2eb+\frac{1}{2}c\,d^3\right)x}{e^4(e\,x^2+d)}+\frac{3\sqrt{-de}\,\ln\left(-\sqrt{-de}\,x-d\right)a}{4e^3}-\frac{5\sqrt{-}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/5*c*x^5*e^2+1/3*b*e^2*x^3-2/3*c*d*e*x^3+a*e^2*x-2*d*e*b*x+3*c*d^2*
x)-d/e^4*((-1/2*a*e^2+1/2*d*e*b-1/2*c*d^2)*x/(e*x^2+d)+1/2*(3*a*e^2-5*b*d*e
+7*c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.50, size = 122, normalized size = 0.90

$$-\frac{\left(7\,cd^3-5\,bd^2e+3\,ade^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{15}\left(3\,cx^5e^2-5\left(2\,cde-be^2\right)x^3+15\left(3\,cd^2-2\,bde+ae^2\right)x\right)e^{(-4)}+\frac{\left(cd^3-bd^2e+ade^2\right)x}{2\left(x^2e^5+de^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/s
qrt(d) + 1/15*(3*c*x^5*e^2 - 5*(2*c*d*e - b*e^2)*x^3 + 15*(3*c*d^2 - 2*b*d*
e + a*e^2)*x)*e^(-4) + 1/2*(c*d^3 - b*d^2*e + a*d*e^2)*x/(x^2*e^5 + d*e^4)
```

Fricas [A]
time = 0.39, size = 341, normalized size = 2.53

$$\left[\frac{210\,cd^3x+15\left(7\,cd^3+3\,ax^2e^3-\left(5\,bdx^2-3\,ad\right)e^2+\left(7\,cd^2x^2-5\,bd^2\right)e\right)\sqrt{-de^{(-1)}}\log\left(\frac{x^2e-2\,\sqrt{-de^{(-1)}}\,xe-d}{x^2e+d}\right)+4\left(3\,cx^7+5\,bx^5+15\,ax^3\right)e^3-2\left(14\,cdx^5+50\,bdx^3-45\,adx\right)e^2+10\left(14\,cd^2x^3-15\,bd^2x\right)e}{60\left(x^2e^5+de^4\right)},\ \frac{105\,cd^3x-15\left(7\,cd^3+3\,ax^2e^3-\left(5\,bdx^2-3\,ad\right)e^2+\left(7\,cd^2x^2-5\,bd^2\right)e\right)\sqrt{d}\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}+2\left(3\,cx^7+5\,bx^5+15\,ax^3\right)e^3-\left(14\,cdx^5+50\,bdx^3-45\,adx\right)e^2+5\left(14\,cd^2x^3-15\,bd^2x\right)e}{30\left(x^2e^5+de^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/60*(210*c*d^3*x + 15*(7*c*d^3 + 3*a*x^2*e^3 - (5*b*d*x^2 - 3*a*d)*e^2 +
(7*c*d^2*x^2 - 5*b*d^2)*e)*sqrt(-d*e^(-1))*log((x^2*e - 2*sqrt(-d*e^(-1))*x
*e - d)/(x^2*e + d)) + 4*(3*c*x^7 + 5*b*x^5 + 15*a*x^3)*e^3 - 2*(14*c*d*x^5
 + 50*b*d*x^3 - 45*a*d*x)*e^2 + 10*(14*c*d^2*x^3 - 15*b*d^2*x)*e)/(x^2*e^5
+ d*e^4), 1/30*(105*c*d^3*x - 15*(7*c*d^3 + 3*a*x^2*e^3 - (5*b*d*x^2 - 3*a*
d)*e^2 + (7*c*d^2*x^2 - 5*b*d^2)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1
/2) + 2*(3*c*x^7 + 5*b*x^5 + 15*a*x^3)*e^3 - (14*c*d*x^5 + 50*b*d*x^3 - 45*
a*d*x)*e^2 + 5*(14*c*d^2*x^3 - 15*b*d^2*x)*e)/(x^2*e^5 + d*e^4)]
```

Sympy [A]
time = 0.71, size = 189, normalized size = 1.40

$$\frac{cx^5}{5e^2}+x^3\left(\frac{b}{3e^2}-\frac{2cd}{3e^3}\right)+x\left(\frac{a}{e^2}-\frac{2bd}{e^3}+\frac{3cd^2}{e^4}\right)+\frac{x\left(ade^2-bd^2e+cd^3\right)}{2de^4+2e^5x^2}+\frac{\sqrt{-\frac{d}{e^9}}\cdot\left(3ae^2-5bde+7cd^2\right)\log\left(-e^4\sqrt{-\frac{d}{e^9}}+x\right)}{4}-\frac{\sqrt{-\frac{d}{e^9}}\cdot\left(3ae^2-5bde+7cd^2\right)\log\left(e^4\sqrt{-\frac{d}{e^9}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**5/(5*e**2) + x**3*(b/(3*e**2) - 2*c*d/(3*e**3)) + x*(a/e**2 - 2*b*d/e*
*3 + 3*c*d**2/e**4) + x*(a*d*e**2 - b*d**2*e + c*d**3)/(2*d*e**4 + 2*e**5*x
**2) + sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(-e**4*sqrt(-d/e**9
) + x)/4 - sqrt(-d/e**9)*(3*a*e**2 - 5*b*d*e + 7*c*d**2)*log(e**4*sqrt(-d/e
**9) + x)/4
```

Giac [A]
time = 4.16, size = 125, normalized size = 0.93

$$-\frac{(7\,cd^3-5\,bd^2e+3\,ade^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{15}\,(3\,cx^5e^8-10\,cdx^3e^7+5\,bx^3e^8+45\,cd^2xe^6-30\,bdxe^7+15\,axe^8)e^{(-10)}+\frac{(cd^3x-bd^2xe+adxe^2)e^{(-4)}}{2\,(x^2e+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(7*c*d^3 - 5*b*d^2*e + 3*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/s
qrt(d) + 1/15*(3*c*x^5*e^8 - 10*c*d*x^3*e^7 + 5*b*x^3*e^8 + 45*c*d^2*x*e^6
- 30*b*d*x*e^7 + 15*a*x*e^8)*e^(-10) + 1/2*(c*d^3*x - b*d^2*x*e + a*d*x*e^2
)*e^(-4)/(x^2*e + d)
```

Mupad [B]
time = 0.32, size = 179, normalized size = 1.33

$$x^3\left(\frac{b}{3\,e^2}-\frac{2\,c\,d}{3\,e^3}\right)-x\left(\frac{c\,d^2}{e^4}-\frac{a}{e^2}+\frac{2\,d\,\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)}{e}\right)+\frac{c\,x^5}{5\,e^2}+\frac{x\,\left(\frac{c\,d^3}{2}-\frac{b\,d^2\,e}{2}+\frac{a\,d\,e^2}{2}\right)}{e^5\,x^2+d\,e^4}-\frac{\sqrt{d}\ \mathrm{atan}\left(\frac{\sqrt{d}\ \sqrt{e}\ x\,(7\,c\,d^2-5\,b\,d\,e+3\,a\,e^2)}{7\,c\,d^3-5\,b\,d^2\,e+3\,a\,d\,e^2}\right)\,(7\,c\,d^2-5\,b\,d\,e+3\,a\,e^2)}{2\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x^3*(b/(3*e^2) - (2*c*d)/(3*e^3)) - x*((c*d^2)/e^4 - a/e^2 + (2*d*(b/e^2 -
(2*c*d)/e^3))/e) + (c*x^5)/(5*e^2) + (x*((c*d^3)/2 + (a*d*e^2)/2 - (b*d^2*e
)/2))/(d*e^4 + e^5*x^2) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(3*a*e^2 + 7*c*d
^2 - 5*b*d*e))/(7*c*d^3 + 3*a*d*e^2 - 5*b*d^2*e))*(3*a*e^2 + 7*c*d^2 - 5*b*
d*e))/(2*e^(9/2))
```

3.282 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx$

Optimal. Leaf size=106

$$-\frac{(2cd-be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}+\frac{(5cd^2-e(3bd-ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{7/2}}$$

```
[Out] -(-b*e+2*c*d)*x/e^3+1/3*c*x^3/e^2-1/2*(a*e^2-b*d*e+c*d^2)*x/e^3/(e*x^2+d)+1
/2*(5*c*d^2-e*(-a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(7/2)/d^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {1271, 1167, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(5cd^2-e(3bd-ae))}{2\sqrt{d}\,e^{7/2}}-\frac{x(ae^2-bde+cd^2)}{2e^3\,(d+ex^2)}-\frac{x(2cd-be)}{e^3}+\frac{cx^3}{3e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
```

```
[Out] -(((2*c*d - b*e)*x)/e^3) + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2
*e^3*(d + e*x^2)) + ((5*c*d^2 - e*(3*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]
])/(2*Sqrt[d]*e^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
```

```
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^2}\,dx &= -\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}-\frac{\int \frac{-cd^2+bde-ae^2+2e(cd-be)x^2-2ce^2x^4}{d+ex^2}\,dx}{2e^3}\\ &= -\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}-\frac{\int \left(2(2cd-be)-2cex^2+\frac{-5cd^2+3bde-ae^2}{d+ex^2}\right)dx}{2e^3}\\ &= -\frac{(2cd-be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}-\frac{(-5cd^2+e(3bd-ae))\int \frac{1}{d+ex^2}\,dx}{2e^3}\\ &= -\frac{(2cd-be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}+\frac{(5cd^2-e(3bd-ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{7/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 102, normalized size = 0.96

$$\frac{(-2cd+be)x}{e^3}+\frac{cx^3}{3e^2}-\frac{(cd^2-bde+ae^2)\,x}{2e^3\,(d+ex^2)}+\frac{(5cd^2-3bde+ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2\sqrt{d}\,e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x]
[Out] ((-2*c*d + b*e)*x)/e^3 + (c*x^3)/(3*e^2) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*e
^3*(d + e*x^2)) + ((5*c*d^2 - 3*b*d*e + a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*Sqrt[d]*e^(7/2))
```

Maple [A]
time = 0.13, size = 90, normalized size = 0.85

method	result
default	$\frac{\frac{1}{3}ce\,x^3+ebx-2cdx}{e^3}+\frac{\frac{\left(-\frac{1}{2}a\,e^2+\frac{1}{2}deb-\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(a\,e^2-3deb+5c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}}{e^3}$

risch	$\frac{c\,x^3}{3e^2}+\frac{bx}{e^2}-\frac{2cdx}{e^3}+\frac{\left(-\frac{1}{2}a\,e^2+\frac{1}{2}deb-\frac{1}{2}c\,d^2\right)x}{e^3(e\,x^2+d)}-\frac{\ln\left(ex+\sqrt{-de}\right)a}{4e\sqrt{-de}}+\frac{3\ln\left(ex+\sqrt{-de}\right)db}{4e^2\sqrt{-de}}-\frac{5\ln\left(ex+\sqrt{-de}\right)c\,d^2}{4e^3\sqrt{-de}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^3*(1/3*c*e*x^3+e*b*x-2*c*d*x)+1/e^3*((-1/2*a*e^2+1/2*d*e*b-1/2*c*d^2)*x
/(e*x^2+d)+1/2*(a*e^2-3*b*d*e+5*c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.52, size = 90, normalized size = 0.85

$$\frac{(5\,cd^2-3\,bde+ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{3}\left(cx^3e-3\,(2\,cd-be)x\right)e^{(-3)}-\frac{(cd^2-bde+ae^2)x}{2\,(x^2e^4+de^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d)
+ 1/3*(c*x^3*e - 3*(2*c*d - b*e)*x)*e^(-3) - 1/2*(c*d^2 - b*d*e + a*e^2)*x/
(x^2*e^4 + d*e^3)
```

Fricas [A]
time = 0.35, size = 300, normalized size = 2.83

$$\left[-\frac{30\,cd^3xe+3\,(5\,cd^3+ax^2e^3-(3\,bdx^2-ad)e^2+(5\,cd^2x^2-3\,bd^2)e)\sqrt{-de}\,\log\left(\frac{x^2e-2\,\sqrt{-de}\,x-d}{x^2e+d}\right)-2\,(2\,cdx^5+6\,bdx^3-3\,adx)e^3+2\,(10\,cd^2x^3-9\,bd^2x)e^2}{12\,(dx^2e^5+d^2e^4)},-\frac{15\,cd^3xe-3\,(5\,cd^3+ax^2e^3-(3\,bdx^2-ad)e^2+(5\,cd^2x^2-3\,bd^2)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-(2\,cdx^5+6\,bdx^3-3\,adx)e^3+(10\,cd^2x^3-9\,bd^2x)e^2}{6\,(dx^2e^5+d^2e^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/12*(30*c*d^3*x*e + 3*(5*c*d^3 + a*x^2*e^3 - (3*b*d*x^2 - a*d)*e^2 + (5*
c*d^2*x^2 - 3*b*d^2)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e
+ d)) - 2*(2*c*d*x^5 + 6*b*d*x^3 - 3*a*d*x)*e^3 + 2*(10*c*d^2*x^3 - 9*b*d^2
*x)*e^2)/(d*x^2*e^5 + d^2*e^4), -1/6*(15*c*d^3*x*e - 3*(5*c*d^3 + a*x^2*e^3
 - (3*b*d*x^2 - a*d)*e^2 + (5*c*d^2*x^2 - 3*b*d^2)*e)*sqrt(d)*arctan(x*e^(1
/2)/sqrt(d))*e^(1/2) - (2*c*d*x^5 + 6*b*d*x^3 - 3*a*d*x)*e^3 + (10*c*d^2*x^
3 - 9*b*d^2*x)*e^2)/(d*x^2*e^5 + d^2*e^4)]
```

Sympy [A]
time = 0.61, size = 162, normalized size = 1.53

$$\frac{cx^3}{3e^2}+x\left(\frac{b}{e^2}-\frac{2cd}{e^3}\right)+\frac{x(-ae^2+bde-cd^2)}{2de^3+2e^4x^2}-\frac{\sqrt{-\frac{1}{de^7}}\,(ae^2-3bde+5cd^2)\log\left(-de^3\sqrt{-\frac{1}{de^7}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{de^7}}\,(ae^2-3bde+5cd^2)\log\left(de^3\sqrt{-\frac{1}{de^7}}+x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x**3/(3*e**2) + x*(b/e**2 - 2*c*d/e**3) + x*(-a*e**2 + b*d*e - c*d**2)/(2
*d*e**3 + 2*e**4*x**2) - sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e + 5*c*d**2)*lo
g(-d*e**3*sqrt(-1/(d*e**7)) + x)/4 + sqrt(-1/(d*e**7))*(a*e**2 - 3*b*d*e +
5*c*d**2)*log(d*e**3*sqrt(-1/(d*e**7)) + x)/4
```

Giac [**A**]
time = 5.89, size = 91, normalized size = 0.86

$$\frac{(5\,cd^2-3\,bde+ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{2\,\sqrt{d}}+\frac{1}{3}\,(cx^3e^4-6\,cdxe^3+3\,bxe^4)e^{(-6)}-\frac{(cd^2x-bdxe+axe^2)e^{(-3)}}{2\,(x^2e+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2 - 3*b*d*e + a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-7/2)/sqrt(d)
+ 1/3*(c*x^3*e^4 - 6*c*d*x*e^3 + 3*b*x*e^4)*e^(-6) - 1/2*(c*d^2*x - b*d*x*e
 + a*x*e^2)*e^(-3)/(x^2*e + d)
```

Mupad [**B**]
time = 0.34, size = 95, normalized size = 0.90

$$x\left(\frac{b}{e^2}-\frac{2\,c\,d}{e^3}\right)-\frac{x\left(\frac{c\,d^2}{2}-\frac{b\,d\,e}{2}+\frac{a\,e^2}{2}\right)}{e^4\,x^2+d\,e^3}+\frac{c\,x^3}{3\,e^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(5\,c\,d^2-3\,b\,d\,e+a\,e^2\right)}{2\,\sqrt{d}\,e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^2,x)
```

```
[Out] x*(b/e^2 - (2*c*d)/e^3) - (x*((a*e^2)/2 + (c*d^2)/2 - (b*d*e)/2))/(d*e^3 +
e^4*x^2) + (c*x^3)/(3*e^2) + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 + 5*c*d^2 -
3*b*d*e))/(2*d^(1/2)*e^(7/2))
```

3.283 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^2} dx$

Optimal. Leaf size=83

$$\frac{cx}{e^2} + \frac{\left(a + \frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{(3cd^2 - e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

```
[Out] c*x/e^2+1/2*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)-1/2*(3*c*d^2-e*(a*e+b*d))*ar
ctan(x*e^(1/2)/d^(1/2))/d^(3/2)/e^(5/2)
```

Rubi [A]
time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1171, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(3cd^2 - e(ae+bd))}{2d^{3/2}e^{5/2}} + \frac{x(ae^2 - bde + cd^2)}{2de^2\,(d+ex^2)} + \frac{cx}{e^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
```

```
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 e*(b*d + a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
```

```
- b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{(d+ex^2)^2}\,dx &= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{\int \frac{\frac{cd^2-e(bd+ae)}{e^2}-\frac{2cdx^2}{e}}{d+ex^2}\,dx}{2d}\\ &= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\int \frac{1}{d+ex^2}\,dx}{2de^2}\\ &= \frac{cx}{e^2} + \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{2d\,(d+ex^2)} - \frac{(3cd^2-e(bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 88, normalized size = 1.06

$$\frac{cx}{e^2} + \frac{(cd^2-bde+ae^2)\,x}{2de^2\,(d+ex^2)} - \frac{(3cd^2-bde-ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{3/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^2,x]
[Out] (c*x)/e^2 + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d*e^2*(d + e*x^2)) - ((3*c*d^2 -
 b*d*e - a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(3/2)*e^(5/2))
```

Maple [A]
time = 0.13, size = 79, normalized size = 0.95

method	result
default	$\frac{cx}{e^2} + \frac{\frac{(a e^2-deb+c d^2)x}{2d(e x^2+d)} + \frac{(a e^2+deb-3c d^2)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2d\sqrt{de}}}{e^2}$
risch	$\frac{cx}{e^2} + \frac{(a e^2-deb+c d^2)x}{2d e^2(e x^2+d)} - \frac{\ln\left(ex+\sqrt{-de}\right)a}{4\sqrt{-de}\,d} - \frac{\ln\left(ex+\sqrt{-de}\right)b}{4e\sqrt{-de}} + \frac{3d\ln\left(ex+\sqrt{-de}\right)c}{4e^2\sqrt{-de}} + \frac{\ln\left(-ex+\sqrt{-de}\right)a}{4\sqrt{-de}\,d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^2+1/e^2*(1/2*(a*e^2-b*d*e+c*d^2)/d*x/(e*x^2+d)+1/2*(a*e^2+b*d*e-3*c*d
^2)/d/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.50, size = 74, normalized size = 0.89

$$cxe^{(-2)} - \frac{(3\,cd^2 - bde - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(cd^2 - bde + ae^2)x}{2\,(dx^2e^3 + d^2e^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2 - b*d*e + a*e^2)*x/(d*x^2*e^3 + d^2*e^2)
```

Fricas [A]
time = 0.35, size = 266, normalized size = 3.20

$$\left[\frac{6\,cd^3xe + 2\,adxe^3 + (3\,cd^3 - ax^2e^3 - (bdx^2 + ad)e^2 + (3\,cd^2x^2 - bd^2)e)\sqrt{-de}\,\log\left(\frac{x^2e - 2\,\sqrt{-de}\,x - d}{x^2e + d}\right) + 2\,(2\,cd^2x^3 - bd^2x)e^2}{4\,(d^2x^2e^4 + d^3e^3)}, \frac{3\,cd^3xe + adxe^3 - (3\,cd^3 - ax^2e^3 - (bdx^2 + ad)e^2 + (3\,cd^2x^2 - bd^2)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}} + (2\,cd^2x^3 - bd^2x)e^2}{2\,(d^2x^2e^4 + d^3e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(6*c*d^3*x*e + 2*a*d*x*e^3 + (3*c*d^3 - a*x^2*e^3 - (b*d*x^2 + a*d)*e^
2 + (3*c*d^2*x^2 - b*d^2)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x
^2*e + d)) + 2*(2*c*d^2*x^3 - b*d^2*x)*e^2)/(d^2*x^2*e^4 + d^3*e^3), 1/2*(3
*c*d^3*x*e + a*d*x*e^3 - (3*c*d^3 - a*x^2*e^3 - (b*d*x^2 + a*d)*e^2 + (3*c*
d^2*x^2 - b*d^2)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) + (2*c*d^2*x^
3 - b*d^2*x)*e^2)/(d^2*x^2*e^4 + d^3*e^3)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(75) = 150$.
time = 0.43, size = 153, normalized size = 1.84

$$\frac{cx}{e^2} + \frac{x(ae^2 - bde + cd^2)}{2d^2e^2 + 2de^3x^2} - \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 + bde - 3cd^2)\log\left(-d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4} + \frac{\sqrt{-\frac{1}{d^3e^5}}\,(ae^2 + bde - 3cd^2)\log\left(d^2e^2\sqrt{-\frac{1}{d^3e^5}} + x\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**2,x)
```

```
[Out] c*x/e**2 + x*(a*e**2 - b*d*e + c*d**2)/(2*d**2*e**2 + 2*d*e**3*x**2) - sqrt
(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(-d**2*e**2*sqrt(-1/(d**3*e
**5)) + x)/4 + sqrt(-1/(d**3*e**5))*(a*e**2 + b*d*e - 3*c*d**2)*log(d**2*e*
*2*sqrt(-1/(d**3*e**5)) + x)/4
```

Giac [A]
time = 4.11, size = 75, normalized size = 0.90

$$cxe^{(-2)} - \frac{(3\,cd^2 - bde - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{2\,d^{\frac{3}{2}}} + \frac{(cd^2x - bdxe + axe^2)e^{(-2)}}{2\,(x^2e + d)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] c*x*e^(-2) - 1/2*(3*c*d^2 - b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/
2)/d^(3/2) + 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)*e^(-2)/((x^2*e + d)*d)
```

Mupad [B]
time = 0.36, size = 77, normalized size = 0.93

$$\frac{c\,x}{e^2} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(-3\,c\,d^2 + b\,d\,e + a\,e^2\right)}{2\,d^{3/2}\,e^{5/2}} + \frac{x\,\left(c\,d^2 - b\,d\,e + a\,e^2\right)}{2\,d\,\left(e^3\,x^2 + d\,e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^2,x)
```

```
[Out] (c*x)/e^2 + (atan((e^(1/2)*x)/d^(1/2))*(a*e^2 - 3*c*d^2 + b*d*e))/(2*d^(3/2
)*e^(5/2)) + (x*(a*e^2 + c*d^2 - b*d*e))/(2*d*(d*e^2 + e^3*x^2))
```

3.284 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^2} \, dx$

Optimal. Leaf size=89

$$-\frac{a}{d^2x}-\frac{(cd^2-bde+ae^2)\,x}{2d^2e\,(d+ex^2)}+\frac{(cd^2+e(bd-3ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}$$

```
[Out] -a/d^2/x-1/2*(a*e^2-b*d*e+c*d^2)*x/d^2/e/(e*x^2+d)+1/2*(c*d^2+e*(-3*a*e+b*d
))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/e^(3/2)
```

Rubi [A]

time = 0.08, antiderivative size = 86, normalized size of antiderivative = 0.97, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1273, 464, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(e(bd-3ae)+cd^2)}{2d^{5/2}e^{3/2}}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{2\,(d+ex^2)}-\frac{a}{d^2x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(2*(d + e*x^2)) + ((c*d^2 + e*(b
*d - 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
```

```
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^2}\,dx &= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)}-\frac{\int \frac{-2ade^2-e\left(cd^2+e(bd-ae)\right)x^2}{x^2\left(d+ex^2\right)}\,dx}{2d^2e^2}\\
&= -\frac{a}{d^2x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)}+\frac{1}{2}\left(\frac{c}{e}+\frac{bd-3ae}{d^2}\right)\int \frac{1}{d+ex^2}\,dx\\
&= -\frac{a}{d^2x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{2\left(d+ex^2\right)}+\frac{\left(cd^2+e(bd-3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}
\end{aligned}
$$

Mathematica [A]
time = 0.04, size = 89, normalized size = 1.00

$$
-\frac{a}{d^2x}-\frac{\left(cd^2-bde+ae^2\right)x}{2d^2e\left(d+ex^2\right)}+\frac{\left(cd^2+bde-3ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{5/2}e^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x]
```

```
[Out] -(a/(d^2*x)) - ((c*d^2 - b*d*e + a*e^2)*x)/(2*d^2*e*(d + e*x^2)) + ((c*d^2
+ b*d*e - 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(5/2)*e^(3/2))
```

Maple [A]
time = 0.14, size = 85, normalized size = 0.96

method	result
default	$-\frac{\frac{\left(a\,e^2-deb+c\,d^2\right)x}{2e\left(e\,x^2+d\right)}+\frac{\left(3a\,e^2-deb-c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2e\sqrt{de}}}{d^2}-\frac{a}{d^2x}$
risch	$\frac{-\frac{\left(3a\,e^2-deb+c\,d^2\right)x^2}{2d^2e}-\frac{a}{d}}{x(e\,x^2+d)}-\frac{3e\ln\left(-\sqrt{-de}\,x-d\right)a}{4\sqrt{-de}\,d^2}+\frac{\ln\left(-\sqrt{-de}\,x-d\right)b}{4\sqrt{-de}\,d}+\frac{\ln\left(-\sqrt{-de}\,x-d\right)c}{4\sqrt{-de}\,e}+\frac{3e\ln\left(-\sqrt{-de}\right.}{4\sqrt{-de}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^2*(1/2*(a*e^2-b*d*e+c*d^2)/e*x/(e*x^2+d)+1/2*(3*a*e^2-b*d*e-c*d^2)/e/(
d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-a/d^2/x
```

Maxima [A]
time = 0.48, size = 81, normalized size = 0.91

$$\frac{(cd^2 + bde - 3\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{2\,d^{\frac{5}{2}}} - \frac{(cd^2 - bde + 3\,ae^2)x^2 + 2\,ade}{2\,(d^2x^3e^2 + d^3xe)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(5/2) -
1/2*((c*d^2 - b*d*e + 3*a*e^2)*x^2 + 2*a*d*e)/(d^2*x^3*e^2 + d^3*x*e)
```

Fricas [A]
time = 0.38, size = 274, normalized size = 3.08

$$\left[-\frac{2\,cd^3x^2e + 6\,adx^2e^3 - (cd^3x - 3\,ax^3e^3 + (bdx^3 - 3\,adx)e^2 + (cd^2x^3 + bd^2x)e)\sqrt{-de}\,\log\left(\frac{x^2e + 2\,\sqrt{-de}\,x - d}{x^2e + d}\right) - 2\,(bd^2x^2 - 2\,ad^2)e^2}{4\,(d^3x^3e^3 + d^4xe^2)}, -\frac{cd^3x^2e + 3\,adx^2e^3 - (cd^3x - 3\,ax^3e^3 + (bdx^3 - 3\,adx)e^2 + (cd^2x^3 + bd^2x)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}} - (bd^2x^2 - 2\,ad^2)e^2}{2\,(d^3x^3e^3 + d^4xe^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*c*d^3*x^2*e + 6*a*d*x^2*e^3 - (c*d^3*x - 3*a*x^3*e^3 + (b*d*x^3 -
3*a*d*x)*e^2 + (c*d^2*x^3 + b*d^2*x)*e)*sqrt(-d*e)*log((x^2*e + 2*sqrt(-d*e
)*x - d)/(x^2*e + d)) - 2*(b*d^2*x^2 - 2*a*d^2)*e^2)/(d^3*x^3*e^3 + d^4*x*e
^2), -1/2*(c*d^3*x^2*e + 3*a*d*x^2*e^3 - (c*d^3*x - 3*a*x^3*e^3 + (b*d*x^3
- 3*a*d*x)*e^2 + (c*d^2*x^3 + b*d^2*x)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))
*e^(1/2) - (b*d^2*x^2 - 2*a*d^2)*e^2)/(d^3*x^3*e^3 + d^4*x*e^2)]
```

Sympy [A]
time = 0.61, size = 155, normalized size = 1.74

$$\frac{\sqrt{-\frac{1}{d^5e^3}}\cdot(3ae^2 - bde - cd^2)\log\left(-d^3e\sqrt{-\frac{1}{d^5e^3}} + x\right)}{4} - \frac{\sqrt{-\frac{1}{d^5e^3}}\cdot(3ae^2 - bde - cd^2)\log\left(d^3e\sqrt{-\frac{1}{d^5e^3}} + x\right)}{4} + \frac{-2ade + x^2(-3ae^2 + bde - cd^2)}{2d^3ex + 2d^2e^2x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(-d**3*e*sqrt(-1/(d**5*
e**3)) + x)/4 - sqrt(-1/(d**5*e**3))*(3*a*e**2 - b*d*e - c*d**2)*log(d**3*e
*sqrt(-1/(d**5*e**3)) + x)/4 + (-2*a*d*e + x**2*(-3*a*e**2 + b*d*e - c*d**2
))/(2*d**3*e*x + 2*d**2*e**2*x**3)
```

Giac [A]
time = 4.41, size = 83, normalized size = 0.93

$$\frac{(cd^2 + bde - 3\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{2\,d^{\frac{5}{2}}} - \frac{(cd^2x^2 - bdx^2e + 3\,ax^2e^2 + 2\,ade)e^{(-1)}}{2\,(x^3e + dx)d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 + b*d*e - 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(5/2) -
1/2*(c*d^2*x^2 - b*d*x^2*e + 3*a*x^2*e^2 + 2*a*d*e)*e^(-1)/((x^3*e + d*x)*d
^2)
```

Mupad [B]
time = 0.37, size = 81, normalized size = 0.91

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2 + b\,d\,e - 3\,a\,e^2\right)}{2\,d^{5/2}\,e^{3/2}} - \frac{\frac{a}{d} + \frac{x^2\,\left(c\,d^2 - b\,d\,e + 3\,a\,e^2\right)}{2\,d^2\,e}}{e\,x^3 + d\,x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^2),x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 3*a*e^2 + b*d*e))/(2*d^(5/2)*e^(3/2)) -
 (a/d + (x^2*(3*a*e^2 + c*d^2 - b*d*e))/(2*d^2*e))/(d*x + e*x^3)
```

3.285 $\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^2}\,dx$

Optimal. Leaf size=106

$$-\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{(cd^2-bde+ae^2)\,x}{2d^3\,(d+ex^2)}+\frac{(cd^2-e(3bd-5ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}$$

```
[Out] -1/3*a/d^2/x^3+(2*a*e-b*d)/d^3/x+1/2*(a*e^2-b*d*e+c*d^2)*x/d^3/(e*x^2+d)+1/
2*(c*d^2-e*(-5*a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(7/2)/e^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1273, 1275, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(cd^2-e(3bd-5ae))}{2d^{7/2}\sqrt{e}}+\frac{x(ae^2-bde+cd^2)}{2d^3\,(d+ex^2)}-\frac{bd-2ae}{d^3x}-\frac{a}{3d^2x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -1/3*a/(d^2*x^3) - (b*d - 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(2*d
^3*(d + e*x^2)) + ((c*d^2 - e*(3*b*d - 5*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])
/(2*d^(7/2)*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^2}\,dx &= \frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\int \frac{2ad^2e^2+2de^2(bd-ae)x^2+e^2\left(cd^2-bde+ae^2\right)x^4}{x^4\left(d+ex^2\right)}\,dx}{2d^3e^2}\\ &= \frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\int \left(\frac{2ade^2}{x^4}-\frac{2e^2(-bd+2ae)}{x^2}+\frac{e^2\left(cd^2-e(3bd-5ae)\right)}{d+ex^2}\right)dx}{2d^3e^2}\\ &= -\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\left(cd^2-e(3bd-5ae)\right)\int \frac{1}{d+ex^2}\,dx}{2d^3}\\ &= -\frac{a}{3d^2x^3}-\frac{bd-2ae}{d^3x}+\frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\left(cd^2-e(3bd-5ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 105, normalized size = 0.99

$$-\frac{a}{3d^2x^3}+\frac{-bd+2ae}{d^3x}+\frac{\left(cd^2-bde+ae^2\right)x}{2d^3\left(d+ex^2\right)}+\frac{\left(cd^2-3bde+5ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{7/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x]
```

```
[Out] -1/3*a/(d^2*x^3) + (-(b*d) + 2*a*e)/(d^3*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(
2*d^3*(d + e*x^2)) + ((c*d^2 - 3*b*d*e + 5*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d
]])/(2*d^(7/2)*Sqrt[e])
```

Maple [A]
time = 0.13, size = 94, normalized size = 0.89

method	result
default	$\frac{\frac{\left(\frac{1}{2}a\,e^2-\frac{1}{2}deb+\frac{1}{2}c\,d^2\right)x}{e\,x^2+d}+\frac{\left(5a\,e^2-3deb+c\,d^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}}{d^3}-\frac{a}{3x^3d^2}-\frac{-2ae+bd}{d^3x}$

risch	$\frac{\frac{\left(5a\,e^{2}-3deb+c\,d^{2}\right)x^{4}}{2d^{3}}+\frac{(5ae-3bd)x^{2}}{3d^{2}}-\frac{a}{3d}}{x^{3}(e\,x^{2}+d)}-\frac{5\ln\left(-\sqrt{-de}\,x+d\right)a\,e^{2}}{4\sqrt{-de}\,d^{3}}+\frac{3\ln\left(-\sqrt{-de}\,x+d\right)eb}{4\sqrt{-de}\,d^{2}}-\frac{\ln\left(-\sqrt{-de}\,x+d\right)c}{4\sqrt{-de}\,d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d^3*((1/2*a*e^2-1/2*d*e*b+1/2*c*d^2)*x/(e*x^2+d)+1/2*(5*a*e^2-3*b*d*e+c*d
^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/3*a/x^3/d^2-(-2*a*e+b*d)/d^3/x
```

Maxima [A]
time = 0.49, size = 101, normalized size = 0.95

$$\frac{3\,(cd^{2}-3\,bde+5\,ae^{2})x^{4}-2\,ad^{2}-2\,(3\,bd^{2}-5\,ade)x^{2}}{6\,(d^{3}x^{5}e+d^{4}x^{3})}+\frac{(cd^{2}-3\,bde+5\,ae^{2})\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(c*d^2 - 3*b*d*e + 5*a*e^2)*x^4 - 2*a*d^2 - 2*(3*b*d^2 - 5*a*d*e)*x^
2)/(d^3*x^5*e + d^4*x^3) + 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(x*e^(1/2)
/sqrt(d))*e^(-1/2)/d^(7/2)
```

Fricas [A]
time = 0.35, size = 340, normalized size = 3.21

$$\left[\frac{30\,adx^{4}e^{3}-3\,(cd^{3}x^{3}+5\,ax^{5}e^{3}-(3\,bdx^{5}-5\,adx^{3})e^{2}+(cd^{2}x^{5}-3\,bd^{2}x^{3})e)\sqrt{-de}\,\log\left(\frac{x^{2}e-2\,\sqrt{-de}\,x-d}{x^{2}e+d}\right)-2\,(9\,bd^{2}x^{4}-10\,ad^{2}x^{2})e^{2}+2\,(3\,cd^{3}x^{4}-6\,bd^{3}x^{2}-2\,ad^{3})e}{12\,(d^{4}x^{5}e^{2}+d^{5}x^{3}e)},\frac{15\,adx^{4}e^{3}+3\,(cd^{3}x^{3}+5\,ax^{5}e^{3}-(3\,bdx^{5}-5\,adx^{3})e^{2}+(cd^{2}x^{5}-3\,bd^{2}x^{3})e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-(9\,bd^{2}x^{4}-10\,ad^{2}x^{2})e^{2}+(3\,cd^{3}x^{4}-6\,bd^{3}x^{2}-2\,ad^{3})e}{6\,(d^{4}x^{5}e^{2}+d^{5}x^{3}e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(30*a*d*x^4*e^3 - 3*(c*d^3*x^3 + 5*a*x^5*e^3 - (3*b*d*x^5 - 5*a*d*x^3
)*e^2 + (c*d^2*x^5 - 3*b*d^2*x^3)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x
 - d)/(x^2*e + d)) - 2*(9*b*d^2*x^4 - 10*a*d^2*x^2)*e^2 + 2*(3*c*d^3*x^4 -
6*b*d^3*x^2 - 2*a*d^3)*e)/(d^4*x^5*e^2 + d^5*x^3*e), 1/6*(15*a*d*x^4*e^3 +
3*(c*d^3*x^3 + 5*a*x^5*e^3 - (3*b*d*x^5 - 5*a*d*x^3)*e^2 + (c*d^2*x^5 - 3*b
*d^2*x^3)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - (9*b*d^2*x^4 - 10*
a*d^2*x^2)*e^2 + (3*c*d^3*x^4 - 6*b*d^3*x^2 - 2*a*d^3)*e)/(d^4*x^5*e^2 + d^
5*x^3*e)]
```

Sympy [A]
time = 0.81, size = 167, normalized size = 1.58

$$-\frac{\sqrt{-\frac{1}{d^{7}e}}\,(5ae^{2}-3bde+cd^{2})\log\left(-d^{4}\sqrt{-\frac{1}{d^{7}e}}+x\right)}{4}+\frac{\sqrt{-\frac{1}{d^{7}e}}\,(5ae^{2}-3bde+cd^{2})\log\left(d^{4}\sqrt{-\frac{1}{d^{7}e}}+x\right)}{4}+\frac{-2ad^{2}+x^{4}\cdot(15ae^{2}-9bde+3cd^{2})+x^{2}\cdot(10ade-6bd^{2})}{6d^{4}x^{3}+6d^{3}ex^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(-d**4*sqrt(-1/(d**7*e)
) + x)/4 + sqrt(-1/(d**7*e))*(5*a*e**2 - 3*b*d*e + c*d**2)*log(d**4*sqrt(-1
/(d**7*e)) + x)/4 + (-2*a*d**2 + x**4*(15*a*e**2 - 9*b*d*e + 3*c*d**2) + x*
*2*(10*a*d*e - 6*b*d**2))/(6*d**4*x**3 + 6*d**3*e*x**5)
```

Giac [A]
time = 3.66, size = 94, normalized size = 0.89

$$\frac{(cd^2 - 3\,bde + 5\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{7}{2}}} + \frac{cd^2x - bdxe + axe^2}{2\,(x^2e + d)d^3} - \frac{3\,bdx^2 - 6\,ax^2e + ad}{3\,d^3x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(c*d^2 - 3*b*d*e + 5*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(7/2)
+ 1/2*(c*d^2*x - b*d*x*e + a*x*e^2)/((x^2*e + d)*d^3) - 1/3*(3*b*d*x^2 - 6*
a*x^2*e + a*d)/(d^3*x^3)
```

Mupad [B]
time = 0.36, size = 98, normalized size = 0.92

$$\frac{\frac{x^2\,(5\,a\,e-3\,b\,d)}{3\,d^2} - \frac{a}{3\,d} + \frac{x^4\,(c\,d^2-3\,b\,d\,e+5\,a\,e^2)}{2\,d^3}}{e\,x^5 + d\,x^3} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(c\,d^2 - 3\,b\,d\,e + 5\,a\,e^2)}{2\,d^{7/2}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^2),x)
```

```
[Out] ((x^2*(5*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (x^4*(5*a*e^2 + c*d^2 - 3*b*d*e)
)/(2*d^3))/(d*x^3 + e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(5*a*e^2 + c*d^2 -
3*b*d*e))/(2*d^(7/2)*e^(1/2))
```

3.286 $\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2}\,dx$

Optimal. Leaf size=136

$$-\frac{a}{5d^2x^5}-\frac{bd-2ae}{3d^3x^3}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{e(cd^2-bde+ae^2)\,x}{2d^4\,(d+ex^2)}-\frac{\sqrt{e}\,(3cd^2-e(5bd-7ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{9/2}}$$

```
[Out] -1/5*a/d^2/x^5+1/3*(2*a*e-b*d)/d^3/x^3+(-c*d^2+e*(-3*a*e+2*b*d))/d^4/x-1/2*
e*(a*e^2-b*d*e+c*d^2)*x/d^4/(e*x^2+d)-1/2*(3*c*d^2-e*(-7*a*e+5*b*d))*arctan
(x*e^(1/2)/d^(1/2))*e^(1/2)/d^(9/2)
```

Rubi [A]
time = 0.17, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1273, 1816, 211}

$$-\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(3cd^2-e(5bd-7ae))}{2d^{9/2}}-\frac{ex(ae^2-bde+cd^2)}{2d^4\,(d+ex^2)}-\frac{cd^2-e(2bd-3ae)}{d^4x}-\frac{bd-2ae}{3d^3x^3}-\frac{a}{5d^2x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
```

```
[Out] -1/5*a/(d^2*x^5) - (b*d - 2*a*e)/(3*d^3*x^3) - (c*d^2 - e*(2*b*d - 3*a*e))/
(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e]*(3*c
*d^2 - e*(5*b*d - 7*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^2}\,dx = -\frac{e(cd^2-bde+ae^2)\,x}{2d^4(d+ex^2)} - \frac{\int \frac{-2ad^3e^2-2d^2e^2(bd-ae)x^2-2de^2(cd^2-bde+ae^2)x^4+e^3(cd^2-bde+ae^2)x^6}{x^6(d+ex^2)}\,d}{2d^4e^2}$$

$$= -\frac{e(cd^2-bde+ae^2)\,x}{2d^4(d+ex^2)} - \frac{\int \left(-\frac{2ad^2e^2}{x^6} - \frac{2de^2(bd-2ae)}{x^4} + \frac{2e^2(-cd^2+e(2bd-3ae))}{x^2} + \frac{e^3(3cd^2-e(5b}{d+ex}\right.}{2d^4e^2}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e(cd^2-bde+ae^2)\,x}{2d^4(d+ex^2)} - \frac{(e(3cd^2-e(5bc}{}$$

$$= -\frac{a}{5d^2x^5} - \frac{bd-2ae}{3d^3x^3} - \frac{cd^2-e(2bd-3ae)}{d^4x} - \frac{e(cd^2-bde+ae^2)\,x}{2d^4(d+ex^2)} - \frac{\sqrt{e}\,(3cd^2-e(5}{}$$

Mathematica [A]
time = 0.06, size = 135, normalized size = 0.99

$$-\frac{a}{5d^2x^5} + \frac{-bd+2ae}{3d^3x^3} + \frac{-cd^2+2bde-3ae^2}{d^4x} - \frac{e(cd^2-bde+ae^2)\,x}{2d^4(d+ex^2)} - \frac{\sqrt{e}\,(3cd^2-5bde+7ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x]
[Out] -1/5*a/(d^2*x^5) + (-(b*d) + 2*a*e)/(3*d^3*x^3) + (-(c*d^2) + 2*b*d*e - 3*a
*e^2)/(d^4*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(2*d^4*(d + e*x^2)) - (Sqrt[e
]*(3*c*d^2 - 5*b*d*e + 7*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(9/2))
```

Maple [A]
time = 0.13, size = 122, normalized size = 0.90

method	result
default	$-\frac{e\left(\frac{\left(\frac{1}{2}ae^2-\frac{1}{2}deb+\frac{1}{2}cd^2\right)x}{ex^2+d}+\frac{\left(7ae^2-5deb+3cd^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}\right)}{d^4} - \frac{a}{5x^5d^2} - \frac{-2ae+bd}{3d^3x^3} - \frac{3ae^2-2deb+cd^2}{d^4x}$

risch	$\frac{-\frac{e\left(7a\,e^2-5deb+3c\,d^2\right)x^6}{2d^4}-\frac{\left(7a\,e^2-5deb+3c\,d^2\right)x^4}{3d^3}+\frac{(7ae-5bd)x^2}{15d^2}-\frac{a}{5d}}{x^5(e\,x^2+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(d^9\_Z^2+49a^2e^5-70abd\,e^4+42ac\,d^2e^3+25b^2d^2e^3-30\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x,method=_RETURNVERBOSE)
```

```
[Out] -e/d^4*((1/2*a*e^2-1/2*d*e*b+1/2*c*d^2)*x/(e*x^2+d)+1/2*(7*a*e^2-5*b*d*e+3*
c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/5*a/x^5/d^2-1/3*(-2*a*e+b*d)/
d^3/x^3-(3*a*e^2-2*b*d*e+c*d^2)/d^4/x
```

Maxima [A]
time = 0.49, size = 135, normalized size = 0.99

$$-\frac{15\,(3\,cd^2e-5\,bde^2+7\,ae^3)x^6+10\,(3\,cd^3-5\,bd^2e+7\,ade^2)x^4+6\,ad^3+2\,(5\,bd^3-7\,ad^2e)x^2}{30\,(d^4x^7e+d^5x^5)}-\frac{(3\,cd^2e-5\,bde^2+7\,ae^3)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="maxima")
```

```
[Out] -1/30*(15*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*x^6 + 10*(3*c*d^3 - 5*b*d^2*e +
 7*a*d*e^2)*x^4 + 6*a*d^3 + 2*(5*b*d^3 - 7*a*d^2*e)*x^2)/(d^4*x^7*e + d^5*x
^5) - 1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1
/2)/d^(9/2)
```

Fricas [A]
time = 0.37, size = 389, normalized size = 2.86

$$\left[-\frac{60\,cd^3x^4+210\,ax^6e^3+20\,bd^3x^2+12\,ad^3-15\,(3\,cd^3x^5+7\,ax^7e^3-(5\,bdx^7-7\,adx^5)e^2+(3\,cd^2x^7-5\,bd^2x^5)e)\sqrt{-\frac{e}{d}}\log\left(\frac{x^2e-2\,dx\sqrt{-\frac{e}{d}}-d}{x^2e+d}\right)-10\,(15\,bdx^6-14\,adx^4)e^2+2\,(45\,cd^2x^6-50\,bd^2x^4-14\,ad^2x^2)e}{60\,(d^4x^7e+d^5x^5)},\ -\frac{30\,cd^3x^4+105\,ax^6e^3+10\,bd^3x^2+6\,ad^3+\frac{15\,(3\,cd^3x^5+7\,ax^7e^3-(5\,bdx^7-7\,adx^5)e^2+(3\,cd^2x^7-5\,bd^2x^5)e)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{\sqrt{d}}-5\,(15\,bdx^6-14\,adx^4)e^2+(45\,cd^2x^6-50\,bd^2x^4-14\,ad^2x^2)e}{30\,(d^4x^7e+d^5x^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="fricas")
```

```
[Out] [-1/60*(60*c*d^3*x^4 + 210*a*x^6*e^3 + 20*b*d^3*x^2 + 12*a*d^3 - 15*(3*c*d^
3*x^5 + 7*a*x^7*e^3 - (5*b*d*x^7 - 7*a*d*x^5)*e^2 + (3*c*d^2*x^7 - 5*b*d^2*
x^5)*e)*sqrt(-e/d)*log((x^2*e - 2*d*x*sqrt(-e/d) - d)/(x^2*e + d)) - 10*(15
*b*d*x^6 - 14*a*d*x^4)*e^2 + 2*(45*c*d^2*x^6 - 50*b*d^2*x^4 - 14*a*d^2*x^2)
*e)/(d^4*x^7*e + d^5*x^5), -1/30*(30*c*d^3*x^4 + 105*a*x^6*e^3 + 10*b*d^3*x
^2 + 6*a*d^3 + 15*(3*c*d^3*x^5 + 7*a*x^7*e^3 - (5*b*d*x^7 - 7*a*d*x^5)*e^2
+ (3*c*d^2*x^7 - 5*b*d^2*x^5)*e)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/sqrt(d)
- 5*(15*b*d*x^6 - 14*a*d*x^4)*e^2 + (45*c*d^2*x^6 - 50*b*d^2*x^4 - 14*a*d^2
*x^2)*e)/(d^4*x^7*e + d^5*x^5)]
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(126) = 252$.
time = 1.09, size = 284, normalized size = 2.09

$$\frac{\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)\log\left(-\frac{d^5\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4}-\frac{\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)\log\left(\frac{d^5\sqrt{-\frac{e}{d^9}}\cdot(7ae^2-5bde+3cd^2)}{7ae^3-5bde^2+3cd^2e}+x\right)}{4}+\frac{-6ad^3+x^6(-105ae^3+75bde^2-45cd^2e)+x^4(-70ade^2+50bd^2e-30cd^3)+x^2\cdot(14ad^2e-10bd^3)}{30d^5x^5+30d^4ex^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**2,x)
```

```
[Out] sqrt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-e/d**9)*(7*a*
e**2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 - sq
rt(-e/d**9)*(7*a*e**2 - 5*b*d*e + 3*c*d**2)*log(d**5*sqrt(-e/d**9)*(7*a*e**
2 - 5*b*d*e + 3*c*d**2)/(7*a*e**3 - 5*b*d*e**2 + 3*c*d**2*e) + x)/4 + (-6*a
*d**3 + x**6*(-105*a*e**3 + 75*b*d*e**2 - 45*c*d**2*e) + x**4*(-70*a*d*e**2
 + 50*b*d**2*e - 30*c*d**3) + x**2*(14*a*d**2*e - 10*b*d**3))/(30*d**5*x**5
 + 30*d**4*e*x**7)
```

Giac [A]
time = 3.51, size = 131, normalized size = 0.96

$$-\frac{(3\,cd^2e-5\,bde^2+7\,ae^3)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{9}{2}}}-\frac{cd^2xe-bdxe^2+axe^3}{2\,(x^2e+d)d^4}-\frac{15\,cd^2x^4-30\,bdx^4e+45\,ax^4e^2+5\,bd^2x^2-10\,adx^2e+3\,ad^2}{15\,d^4x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] -1/2*(3*c*d^2*e - 5*b*d*e^2 + 7*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d
^(9/2) - 1/2*(c*d^2*x*e - b*d*x*e^2 + a*x*e^3)/((x^2*e + d)*d^4) - 1/15*(15
*c*d^2*x^4 - 30*b*d*x^4*e + 45*a*x^4*e^2 + 5*b*d^2*x^2 - 10*a*d*x^2*e + 3*a
*d^2)/(d^4*x^5)
```

Mupad [B]
time = 0.38, size = 128, normalized size = 0.94

$$-\frac{\frac{a}{5\,d}-\frac{x^2\,(7\,a\,e-5\,b\,d)}{15\,d^2}+\frac{x^4\,(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2)}{3\,d^3}+\frac{e\,x^6\,(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2)}{2\,d^4}}{e\,x^7+d\,x^5}-\frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(3\,c\,d^2-5\,b\,d\,e+7\,a\,e^2)}{2\,d^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^2),x)
```

```
[Out] - (a/(5*d) - (x^2*(7*a*e - 5*b*d))/(15*d^2) + (x^4*(7*a*e^2 + 3*c*d^2 - 5*b
*d*e))/(3*d^3) + (e*x^6*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))/(2*d^4))/(d*x^5 + e*
x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(7*a*e^2 + 3*c*d^2 - 5*b*d*e))/(2
*d^(9/2))
```

3.287 $\int \frac{a+bx^2+cx^4}{x^8(d+ex^2)^2}\,dx$

Optimal. Leaf size=167

$$-\frac{a}{7d^2x^7}-\frac{bd-2ae}{5d^3x^5}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}+\frac{e(2cd^2-e(3bd-4ae))}{d^5x}+\frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\,(d+ex^2)}+\frac{e^{3/2}(5cd^2-e(7bd$$

```
[Out] -1/7*a/d^2/x^7+1/5*(2*a*e-b*d)/d^3/x^5+1/3*(-c*d^2+e*(-3*a*e+2*b*d))/d^4/x^
3+e*(2*c*d^2-e*(-4*a*e+3*b*d))/d^5/x+1/2*e^2*(a*e^2-b*d*e+c*d^2)*x/d^5/(e*x
^2+d)+1/2*e^(3/2)*(5*c*d^2-e*(-9*a*e+7*b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(1
1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1273, 1816, 211}

$$\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(5cd^2-e(7bd-9ae))}{2d^{11/2}}+\frac{e^2x(ae^2-bde+cd^2)}{2d^5\,(d+ex^2)}+\frac{e(2cd^2-e(3bd-4ae))}{d^5x}-\frac{cd^2-e(2bd-3ae)}{3d^4x^3}-\frac{bd-2ae}{5d^3x^5}-\frac{a}{7d^2x^7}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
```

```
[Out] -1/7*a/(d^2*x^7) - (b*d - 2*a*e)/(5*d^3*x^5) - (c*d^2 - e*(2*b*d - 3*a*e))/
(3*d^4*x^3) + (e*(2*c*d^2 - e*(3*b*d - 4*a*e)))/(d^5*x) + (e^2*(c*d^2 - b*d
*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - e*(7*b*d - 9*a*e))
*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^8\left(d+ex^2\right)^2}\,dx = \frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\left(d+ex^2\right)} + \frac{\int \frac{2ad^4e^2+2d^3e^2(bd-ae)x^2+2d^2e^2\left(cd^2-bde+ae^2\right)x^4-2de^3\left(cd^2-bde+ae^2\right)x^6+e}{x^8(d+ex^2)}}{2d^5e^2}$$

$$= \frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\left(d+ex^2\right)} + \frac{\int \left(\frac{2ad^3e^2}{x^8} + \frac{2d^2e^2(bd-2ae)}{x^6} + \frac{2de^2\left(cd^2-e(2bd-3ae)\right)}{x^4} + \frac{2e^3\left(-2cd^2+e(3b\right.}{x^2}\right.}{2d^5e^2}$$

$$= -\frac{a}{7d^2x^7} - \frac{bd-2ae}{5d^3x^5} - \frac{cd^2-e(2bd-3ae)}{3d^4x^3} + \frac{e(2cd^2-e(3bd-4ae))}{d^5x} + \frac{e^2(cd^2-bde}{2d^5\,(d+}$$

$$= -\frac{a}{7d^2x^7} - \frac{bd-2ae}{5d^3x^5} - \frac{cd^2-e(2bd-3ae)}{3d^4x^3} + \frac{e(2cd^2-e(3bd-4ae))}{d^5x} + \frac{e^2(cd^2-bde}{2d^5\,(d+}$$

Mathematica [A]
time = 0.06, size = 166, normalized size = 0.99

$$-\frac{a}{7d^2x^7}+\frac{-bd+2ae}{5d^3x^5}+\frac{-cd^2+2bde-3ae^2}{3d^4x^3}+\frac{e(2cd^2-3bde+4ae^2)}{d^5x}+\frac{e^2(cd^2-bde+ae^2)\,x}{2d^5\,(d+ex^2)}+\frac{e^{3/2}(5cd^2-7bde+9ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{2d^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x]
[Out] -1/7*a/(d^2*x^7) + (-(b*d) + 2*a*e)/(5*d^3*x^5) + (-(c*d^2) + 2*b*d*e - 3*a
*e^2)/(3*d^4*x^3) + (e*(2*c*d^2 - 3*b*d*e + 4*a*e^2))/(d^5*x) + (e^2*(c*d^2
 - b*d*e + a*e^2)*x)/(2*d^5*(d + e*x^2)) + (e^(3/2)*(5*c*d^2 - 7*b*d*e + 9*
a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(2*d^(11/2))
```

Maple [A]
time = 0.13, size = 149, normalized size = 0.89

method	result
default	$\frac{e^2\left(\frac{\left(\frac{1}{2}ae^2-\frac{1}{2}deb+\frac{1}{2}cd^2\right)x}{ex^2+d}+\frac{\left(9ae^2-7deb+5cd^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{2\sqrt{de}}\right)}{d^5}-\frac{a}{7x^7d^2}-\frac{-2ae+bd}{5d^3x^5}-\frac{3ae^2-2deb+cd^2}{3d^4x^3}+\frac{e(4ae^2-3deb}{d^5x}$

risch	$\frac{\frac{e^{2}\left(9a\,e^{2}-7deb+5c\,d^{2}\right)x^{8}}{2d^{5}}+\frac{e\left(9a\,e^{2}-7deb+5c\,d^{2}\right)x^{6}}{3d^{4}}-\frac{\left(9a\,e^{2}-7deb+5c\,d^{2}\right)x^{4}}{15d^{3}}+\frac{(9ae-7bd)x^{2}}{35d^{2}}-\frac{a}{7d}}{x^{7}(e\,x^{2}+d)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(d^{11}\_Z^{2}+81a^{2}e^{7}-126abd\,e\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x,method=_RETURNVERBOSE)

[Out] e^2/d^5*((1/2*a*e^2-1/2*d*e*b+1/2*c*d^2)*x/(e*x^2+d)+1/2*(9*a*e^2-7*b*d*e+5*c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/7*a/x^7/d^2-1/5*(-2*a*e+b*d)/d^3/x^5-1/3*(3*a*e^2-2*b*d*e+c*d^2)/d^4/x^3+e*(4*a*e^2-3*b*d*e+2*c*d^2)/d^5/x

Maxima [A]
time = 0.52, size = 165, normalized size = 0.99

$$\frac{105\,(5\,cd^2e^2-7\,bde^3+9\,ae^4)x^8+70\,(5\,cd^3e-7\,bd^2e^2+9\,ade^3)x^6-30\,ad^4-14\,(5\,cd^4-7\,bd^3e+9\,ad^2e^2)x^4-6\,(7\,bd^4-9\,ad^3e)x^2}{210\,(d^5x^9e+d^6x^7)}+\frac{(5\,cd^2e^2-7\,bde^3+9\,ae^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="maxima")

[Out] 1/210*(105*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*x^8 + 70*(5*c*d^3*e - 7*b*d^2*e^2 + 9*a*d*e^3)*x^6 - 30*a*d^4 - 14*(5*c*d^4 - 7*b*d^3*e + 9*a*d^2*e^2)*x^4 - 6*(7*b*d^4 - 9*a*d^3*e)*x^2)/(d^5*x^9*e + d^6*x^7) + 1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(11/2)

Fricas [A]
time = 0.37, size = 458, normalized size = 2.74

$$\left[\frac{1890\,ax^8e^4-140\,cd^4x^4-84\,bd^4x^2-60\,ad^4+105\,(5\,cd^3x^7e+9\,ax^9e^4-(7\,bdx^9-9\,adx^7)e^3+(5\,cd^2x^9-7\,bd^2x^7)e^2)\sqrt{-\frac{e}{d}}\log\left(\frac{x^2e+2\,dx\sqrt{-\frac{e}{d}}-d}{x^2e+d}\right)-210\,(7\,bdx^8-6\,adx^6)e^3+14\,(75\,cd^2x^8-70\,bd^2x^6-18\,ad^2x^4)e^2+4\,(175\,cd^3x^6+49\,bd^3x^4+27\,ad^3x^2)e}{420\,(d^5x^9e+d^6x^7)},\frac{945\,ax^8e^4-70\,cd^4x^4-42\,bd^4x^2-30\,ad^4+\frac{105\,(5\,cd^3x^7e+9\,ax^9e^4-(7\,bdx^9-9\,adx^7)e^3+(5\,cd^2x^9-7\,bd^2x^7)e^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{\sqrt{d}}-105\,(7\,bdx^8-6\,adx^6)e^3+7\,(75\,cd^2x^8-70\,bd^2x^6-18\,ad^2x^4)e^2+2\,(175\,cd^3x^6+49\,bd^3x^4+27\,ad^3x^2)e}{210\,(d^5x^9e+d^6x^7)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="fricas")

[Out] [1/420*(1890*a*x^8*e^4 - 140*c*d^4*x^4 - 84*b*d^4*x^2 - 60*a*d^4 + 105*(5*c*d^3*x^7*e + 9*a*x^9*e^4 - (7*b*d*x^9 - 9*a*d*x^7)*e^3 + (5*c*d^2*x^9 - 7*b*d^2*x^7)*e^2)*sqrt(-e/d)*log((x^2*e + 2*d*x*sqrt(-e/d) - d)/(x^2*e + d)) - 210*(7*b*d*x^8 - 6*a*d*x^6)*e^3 + 14*(75*c*d^2*x^8 - 70*b*d^2*x^6 - 18*a*d^2*x^4)*e^2 + 4*(175*c*d^3*x^6 + 49*b*d^3*x^4 + 27*a*d^3*x^2)*e)/(d^5*x^9*e + d^6*x^7), 1/210*(945*a*x^8*e^4 - 70*c*d^4*x^4 - 42*b*d^4*x^2 - 30*a*d^4 + 105*(5*c*d^3*x^7*e + 9*a*x^9*e^4 - (7*b*d*x^9 - 9*a*d*x^7)*e^3 + (5*c*d^2*x^9 - 7*b*d^2*x^7)*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/sqrt(d) - 105*(7*b*d*x^8 - 6*a*d*x^6)*e^3 + 7*(75*c*d^2*x^8 - 70*b*d^2*x^6 - 18*a*d^2*x^4)*e^2 + 2*(175*c*d^3*x^6 + 49*b*d^3*x^4 + 27*a*d^3*x^2)*e)/(d^5*x^9*e + d^6*x^7)]

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 328 vs. $2(156) = 312$.
time = 1.35, size = 328, normalized size = 1.96

$$-\frac{\sqrt{-\frac{e^3}{d^{11}}}\cdot(9ae^2-7bde+5cd^2)\log\left(-\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\cdot(9ae^2-7bde+5cd^2)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4}+\frac{\sqrt{-\frac{e^3}{d^{11}}}\cdot(9ae^2-7bde+5cd^2)\log\left(\frac{d^6\sqrt{-\frac{e^3}{d^{11}}}\cdot(9ae^2-7bde+5cd^2)}{9ae^4-7bde^3+5cd^2e^2}+x\right)}{4}+\frac{-30ad^4+x^8\cdot(945ae^4-735bde^3+525cd^2e^2)+x^6\cdot(630ade^3-490bd^2e^2+350cd^3e)+x^4(-126ad^2e^2+98bd^3e-70cd^4)+x^2\cdot(54ad^3e-42bd^4)}{210d^6x^7+210d^5ex^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**8/(e*x**2+d)**2,x)
```

```
[Out] -sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(-d**6*sqrt(-e**3/d**
11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d**2*e**2)
 + x)/4 + sqrt(-e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)*log(d**6*sqrt(-
e**3/d**11)*(9*a*e**2 - 7*b*d*e + 5*c*d**2)/(9*a*e**4 - 7*b*d*e**3 + 5*c*d*
*2*e**2) + x)/4 + (-30*a*d**4 + x**8*(945*a*e**4 - 735*b*d*e**3 + 525*c*d**
2*e**2) + x**6*(630*a*d*e**3 - 490*b*d**2*e**2 + 350*c*d**3*e) + x**4*(-126
*a*d**2*e**2 + 98*b*d**3*e - 70*c*d**4) + x**2*(54*a*d**3*e - 42*b*d**4))/(
210*d**6*x**7 + 210*d**5*e*x**9)
```

Giac [A]
time = 4.41, size = 164, normalized size = 0.98

$$\frac{(5\,cd^2e^2-7\,bde^3+9\,ae^4)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{2\,d^{\frac{11}{2}}}+\frac{cd^2xe^2-bdxe^3+axe^4}{2\,(x^2e+d)d^5}+\frac{210\,cd^2x^6e-315\,bdx^6e^2-35\,cd^3x^4+420\,ax^6e^3+70\,bd^2x^4e-105\,adx^4e^2-21\,bd^3x^2+42\,ad^2x^2e-15\,ad^3}{105\,d^5x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^8/(e*x^2+d)^2,x, algorithm="giac")
```

```
[Out] 1/2*(5*c*d^2*e^2 - 7*b*d*e^3 + 9*a*e^4)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/
d^(11/2) + 1/2*(c*d^2*x*e^2 - b*d*x*e^3 + a*x*e^4)/((x^2*e + d)*d^5) + 1/10
5*(210*c*d^2*x^6*e - 315*b*d*x^6*e^2 - 35*c*d^3*x^4 + 420*a*x^6*e^3 + 70*b*
d^2*x^4*e - 105*a*d*x^4*e^2 - 21*b*d^3*x^2 + 42*a*d^2*x^2*e - 15*a*d^3)/(d^
5*x^7)
```

Mupad [B]
time = 0.40, size = 156, normalized size = 0.93

$$\frac{\frac{x^2\,(9\,a\,e-7\,b\,d)}{35\,d^2}-\frac{a}{7\,d}-\frac{x^4\,(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2)}{15\,d^3}+\frac{e\,x^6\,(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2)}{3\,d^4}+\frac{e^2\,x^8\,(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2)}{2\,d^5}}{e\,x^9+d\,x^7}+\frac{e^{3/2}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(5\,c\,d^2-7\,b\,d\,e+9\,a\,e^2)}{2\,d^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^8*(d + e*x^2)^2),x)
```

```
[Out] ((x^2*(9*a*e - 7*b*d))/(35*d^2) - a/(7*d) - (x^4*(9*a*e^2 + 5*c*d^2 - 7*b*d
*e))/(15*d^3) + (e*x^6*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(3*d^4) + (e^2*x^8*(9
*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^5))/(d*x^7 + e*x^9) + (e^(3/2)*atan((e^(1
/2)*x)/d^(1/2))*(9*a*e^2 + 5*c*d^2 - 7*b*d*e))/(2*d^(11/2))
```

3.288 $\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=173

$$\frac{(6cd^2-e(3bd-ae))\,x}{e^5}-\frac{(3cd-be)x^3}{3e^4}+\frac{cx^5}{5e^3}-\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2}+\frac{d(17cd^2-e(13bd-9ae))\,x}{8e^5\,(d+ex^2)}-\frac{\sqrt{d}\,(63cd$$

```
[Out] (6*c*d^2-e*(-a*e+3*b*d))*x/e^5-1/3*(-b*e+3*c*d)*x^3/e^4+1/5*c*x^5/e^3-1/4*d
^2*(a*e^2-b*d*e+c*d^2)*x/e^5/(e*x^2+d)^2+1/8*d*(17*c*d^2-e*(-9*a*e+13*b*d))
*x/e^5/(e*x^2+d)-1/8*(15*a*e^2-35*b*d*e+63*c*d^2)*arctan(x*e^(1/2)/d^(1/2))
*d^(1/2)/e^(11/2)
```

Rubi [A]
time = 0.21, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1271, 1828, 1824, 211}

$$-\frac{\sqrt{d}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(15ae^2-35bde+63cd^2)}{8e^{11/2}}+\frac{dx(17cd^2-e(13bd-9ae))}{8e^5\,(d+ex^2)}+\frac{x(6cd^2-e(3bd-ae))}{e^5}-\frac{d^2x(ae^2-bde+cd^2)}{4e^5\,(d+ex^2)^2}-\frac{x^3(3cd-be)}{3e^4}+\frac{cx^5}{5e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 - e*(3*b*d - a*e))*x)/e^5 - ((3*c*d - b*e)*x^3)/(3*e^4) + (c*x^5)
/(5*e^3) - (d^2*(c*d^2 - b*d*e + a*e^2)*x)/(4*e^5*(d + e*x^2)^2) + (d*(17*c
*d^2 - e*(13*b*d - 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^2 - 35
*b*d*e + 15*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1824

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^6(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = -\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} - \frac{\int \frac{-d^2(cd^2-bde+ae^2)+4de(cd^2-bde+ae^2)x^2-4e^2(cd^2-bde+ae^2)x^4+4}{(d+ex^2)^2}}{4e^5}$$

$$= -\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} + \frac{d(17cd^2-e(13bd-9ae))\,x}{8e^5\,(d+ex^2)} + \int \frac{-d^2(15cd^2-e(11bd-7ae))+}{}$$

$$= -\frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} + \frac{d(17cd^2-e(13bd-9ae))\,x}{8e^5\,(d+ex^2)} + \int \Big(8d(6cd^2-e(3bd-$$

$$= \frac{(6cd^2-e(3bd-ae))\,x}{e^5} - \frac{(3cd-be)x^3}{3e^4} + \frac{cx^5}{5e^3} - \frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} + \frac{d(17cd}{}$$

$$= \frac{(6cd^2-e(3bd-ae))\,x}{e^5} - \frac{(3cd-be)x^3}{3e^4} + \frac{cx^5}{5e^3} - \frac{d^2(cd^2-bde+ae^2)\,x}{4e^5\,(d+ex^2)^2} + \frac{d(17cd}{}$$

Mathematica [A]
time = 0.07, size = 170, normalized size = 0.98

$$\frac{(6cd^2+e(-3bd+ae))\,x}{e^5} + \frac{(-3cd+be)x^3}{3e^4} + \frac{cx^5}{5e^3} - \frac{(cd^4+d^2e(-bd+ae))\,x}{4e^5\,(d+ex^2)^2} + \frac{(17cd^3+de(-13bd+9ae))\,x}{8e^5\,(d+ex^2)} - \frac{\sqrt{d}\,(63cd^2+5e(-7bd+3ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8e^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] ((6*c*d^2 + e*(-3*b*d + a*e))*x)/e^5 + ((-3*c*d + b*e)*x^3)/(3*e^4) + (c*x^
5)/(5*e^3) - ((c*d^4 + d^2*e*(-(b*d) + a*e))*x)/(4*e^5*(d + e*x^2)^2) + ((1
7*c*d^3 + d*e*(-13*b*d + 9*a*e))*x)/(8*e^5*(d + e*x^2)) - (Sqrt[d]*(63*c*d^
2 + 5*e*(-7*b*d + 3*a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*e^(11/2))
```

Maple [A]
time = 0.14, size = 151, normalized size = 0.87

method	result
default	$\frac{\frac{1}{5}c\,x^5e^2+\frac{1}{3}b\,e^2x^3-cde\,x^3+a\,e^2x-3debx+6c\,d^2x}{e^5}-\frac{d\left(\frac{\left(-\frac{9}{8}a\,e^3+\frac{13}{8}d\,e^2b-\frac{17}{8}c\,d^2e\right)x^3-\frac{d\left(7a\,e^2-11deb+15c\,d^2\right)x}{8}}{\left(e\,x^2+d\right)^2}+\frac{\left(15a\,e^2-35deb+63c\,d^2\right.}{8\sqrt{d}}\right.}{e^5}$
risch	$\frac{c\,x^5}{5e^3}+\frac{b\,x^3}{3e^3}-\frac{cd\,x^3}{e^4}+\frac{ax}{e^3}-\frac{3dbx}{e^4}+\frac{6c\,d^2x}{e^5}+\frac{\left(\frac{9}{8}d\,e^3a-\frac{13}{8}d^2e^2b+\frac{17}{8}d^3ec\right)x^3+\frac{d^2\left(7a\,e^2-11deb+15c\,d^2\right)x}{8}}{e^5\left(e\,x^2+d\right)^2}+\frac{15\sqrt{-de}\,\ln\left(-\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^5*(1/5*c*x^5*e^2+1/3*b*e^2*x^3-c*d*e*x^3+a*e^2*x-3*d*e*b*x+6*c*d^2*x)-d
/e^5*(((-9/8*a*e^3+13/8*d*e^2*b-17/8*c*d^2*e)*x^3-1/8*d*(7*a*e^2-11*b*d*e+1
5*c*d^2)*x)/(e*x^2+d)^2+1/8*(15*a*e^2-35*b*d*e+63*c*d^2)/(d*e)^(1/2)*arctan
(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.51, size = 165, normalized size = 0.95

$$-\frac{\left(63\,cd^3-35\,bd^2e+15\,ade^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{8\sqrt{d}}+\frac{1}{15}\left(3\,cx^5e^2-5\left(3\,cde-be^2\right)x^3+15\left(6\,cd^2-3\,bde+ae^2\right)x\right)e^{(-5)}+\frac{\left(17\,cd^3e-13\,bd^2e^2+9\,ade^3\right)x^3+\left(15\,cd^4-11\,bd^3e+7\,ad^2e^2\right)x}{8\left(x^4e^7+2\,dx^2e^6+d^2e^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/
2)/sqrt(d) + 1/15*(3*c*x^5*e^2 - 5*(3*c*d*e - b*e^2)*x^3 + 15*(6*c*d^2 - 3*
b*d*e + a*e^2)*x)*e^(-5) + 1/8*((17*c*d^3*e - 13*b*d^2*e^2 + 9*a*d*e^3)*x^3
 + (15*c*d^4 - 11*b*d^3*e + 7*a*d^2*e^2)*x)/(x^4*e^7 + 2*d*x^2*e^6 + d^2*e^
5)
```

Fricas [A]
time = 0.37, size = 488, normalized size = 2.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/240*(1890*c*d^4*x + 15*(15*a*x^4*e^4 + 63*c*d^4 - 5*(7*b*d*x^4 - 6*a*d*x
^2)*e^3 + (63*c*d^2*x^4 - 70*b*d^2*x^2 + 15*a*d^2)*e^2 + 7*(18*c*d^3*x^2 -
5*b*d^3)*e)*sqrt(-d*e^(-1))*log((x^2*e - 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e
```

```
+ d)) + 16*(3*c*x^9 + 5*b*x^7 + 15*a*x^5)*e^4 - 2*(72*c*d*x^7 + 280*b*d*x^5
 - 375*a*d*x^3)*e^3 + 2*(504*c*d^2*x^5 - 875*b*d^2*x^3 + 225*a*d^2*x)*e^2 +
 1050*(3*c*d^3*x^3 - b*d^3*x)*e)/(x^4*e^7 + 2*d*x^2*e^6 + d^2*e^5), 1/120*(
945*c*d^4*x - 15*(15*a*x^4*e^4 + 63*c*d^4 - 5*(7*b*d*x^4 - 6*a*d*x^2)*e^3 +
 (63*c*d^2*x^4 - 70*b*d^2*x^2 + 15*a*d^2)*e^2 + 7*(18*c*d^3*x^2 - 5*b*d^3)*
e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2) + 8*(3*c*x^9 + 5*b*x^7 + 15*a
*x^5)*e^4 - (72*c*d*x^7 + 280*b*d*x^5 - 375*a*d*x^3)*e^3 + (504*c*d^2*x^5 -
 875*b*d^2*x^3 + 225*a*d^2*x)*e^2 + 525*(3*c*d^3*x^3 - b*d^3*x)*e)/(x^4*e^7
 + 2*d*x^2*e^6 + d^2*e^5)]
```

Sympy [A]
time = 2.02, size = 235, normalized size = 1.36

$$\frac{cx^5}{5e^3}+x^3\left(\frac{b}{3e^3}-\frac{cd}{e^4}\right)+x\left(\frac{a}{e^3}-\frac{3bd}{e^4}+\frac{6cd^2}{e^5}\right)+\frac{\sqrt{-\frac{d}{e^{11}}}\cdot(15ae^2-35bde+63cd^2)\log\left(-e^5\sqrt{-\frac{d}{e^{11}}}+x\right)}{16}-\frac{\sqrt{-\frac{d}{e^{11}}}\cdot(15ae^2-35bde+63cd^2)\log\left(e^5\sqrt{-\frac{d}{e^{11}}}+x\right)}{16}+\frac{x^3\cdot(9ade^3-13bd^2e^2+17cd^3e)+x(7ad^2e^2-11bd^3e+15cd^4)}{8d^2e^5+16de^6x^2+8e^7x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**5/(5*e**3) + x**3*(b/(3*e**3) - c*d/e**4) + x*(a/e**3 - 3*b*d/e**4 + 6
*c*d**2/e**5) + sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)*log(-e**5
*sqrt(-d/e**11) + x)/16 - sqrt(-d/e**11)*(15*a*e**2 - 35*b*d*e + 63*c*d**2)
*log(e**5*sqrt(-d/e**11) + x)/16 + (x**3*(9*a*d*e**3 - 13*b*d**2*e**2 + 17*
c*d**3*e) + x*(7*a*d**2*e**2 - 11*b*d**3*e + 15*c*d**4))/(8*d**2*e**5 + 16*
d*e**6*x**2 + 8*e**7*x**4)
```

Giac [A]
time = 4.47, size = 160, normalized size = 0.92

$$-\frac{(63\,cd^3-35\,bd^2e+15\,ade^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{11}{2}\right)}}{8\sqrt{d}}+\frac{1}{15}\left(3\,cx^5e^{12}-15\,cdx^3e^{11}+5\,bx^3e^{12}+90\,cd^2xe^{10}-45\,bdxe^{11}+15\,axe^{12}\right)e^{(-15)}+\frac{(17\,cd^3x^3e-13\,bd^2x^3e^2+15\,cd^4x+9\,adx^3e^3-11\,bd^3xe+7\,ad^2xe^2)e^{(-5)}}{8\,(x^2e+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(63*c*d^3 - 35*b*d^2*e + 15*a*d*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-11/
2)/sqrt(d) + 1/15*(3*c*x^5*e^12 - 15*c*d*x^3*e^11 + 5*b*x^3*e^12 + 90*c*d^2
*x*e^10 - 45*b*d*x*e^11 + 15*a*x*e^12)*e^(-15) + 1/8*(17*c*d^3*x^3*e - 13*b
*d^2*x^3*e^2 + 15*c*d^4*x + 9*a*d*x^3*e^3 - 11*b*d^3*x*e + 7*a*d^2*x*e^2)*e
^(-5)/(x^2*e + d)^2
```

Mupad [B]
time = 0.35, size = 223, normalized size = 1.29

$$x^3\left(\frac{b}{3\,e^3}-\frac{c\,d}{e^4}\right)-x\left(\frac{3\,c\,d^2}{e^5}-\frac{a}{e^3}+\frac{3\,d\,\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right)}{e}\right)+\frac{\left(\frac{17\,c\,d^3\,e}{8}-\frac{13\,b\,d^2\,e^2}{8}+\frac{9\,a\,d\,e^3}{8}\right)\,x^3+\left(\frac{15\,c\,d^4}{8}-\frac{11\,b\,d^3\,e}{8}+\frac{7\,a\,d^2\,e^2}{8}\right)\,x}{d^2\,e^5+2\,d\,e^6\,x^2+e^7\,x^4}+\frac{c\,x^5}{5\,e^3}-\frac{\sqrt{d}\,\mathrm{atan}\left(\frac{\sqrt{d}\,\sqrt{e}\,x\,(63\,c\,d^2-35\,b\,d\,e+15\,a\,e^2)}{63\,c\,d^3-35\,b\,d^2\,e+15\,a\,d\,e^2}\right)\,(63\,c\,d^2-35\,b\,d\,e+15\,a\,e^2)}{8\,e^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] x^3*(b/(3*e^3) - (c*d)/e^4) - x*((3*c*d^2)/e^5 - a/e^3 + (3*d*(b/e^3 - (3*c
*d)/e^4))/e) + (x^3*((9*a*d*e^3)/8 - (13*b*d^2*e^2)/8 + (17*c*d^3*e)/8) + x
*((15*c*d^4)/8 + (7*a*d^2*e^2)/8 - (11*b*d^3*e)/8))/(d^2*e^5 + e^7*x^4 + 2*
d*e^6*x^2) + (c*x^5)/(5*e^3) - (d^(1/2)*atan((d^(1/2)*e^(1/2)*x*(15*a*e^2 +
 63*c*d^2 - 35*b*d*e))/(63*c*d^3 + 15*a*d*e^2 - 35*b*d^2*e))*(15*a*e^2 + 63
*c*d^2 - 35*b*d*e))/(8*e^(11/2))
```

3.289 $\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=143

$$-\frac{(3cd-be)x}{e^4}+\frac{cx^3}{3e^3}+\frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2}-\frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)}+\frac{(35cd^2-3e(5bd-ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8\sqrt{d}\,e^{9/2}}$$

```
[Out] -(-b*e+3*c*d)*x/e^4+1/3*c*x^3/e^3+1/4*d*(a*e^2-b*d*e+c*d^2)*x/e^4/(e*x^2+d)
^2-1/8*(13*c*d^2-e*(-5*a*e+9*b*d))*x/e^4/(e*x^2+d)+1/8*(35*c*d^2-3*e*(-a*e+
5*b*d))*arctan(x*e^(1/2)/d^(1/2))/e^(9/2)/d^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1271, 1828, 1167, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(35cd^2-3e(5bd-ae))}{8\sqrt{d}\,e^{9/2}}-\frac{x(13cd^2-e(9bd-5ae))}{8e^4\,(d+ex^2)}+\frac{dx(ae^2-bde+cd^2)}{4e^4\,(d+ex^2)^2}-\frac{x(3cd-be)}{e^4}+\frac{cx^3}{3e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] -(((3*c*d - b*e)*x)/e^4) + (c*x^3)/(3*e^3) + (d*(c*d^2 - b*d*e + a*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - e*(9*b*d - 5*a*e))*x)/(8*e^4*(d + e*x^
2)) + ((35*c*d^2 - 3*e*(5*b*d - a*e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt[
d]*e^(9/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
```

```
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1828

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b
*f*x)*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{x^4(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx = \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{\int \frac{d(cd^2-bde+ae^2)-4e(cd^2-bde+ae^2)x^2+4e^2(cd-be)x^4-4ce^3x^6}{(d+ex^2)^2}\,dx}{4e^4}$$

$$= \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)} + \frac{\int \frac{d(11cd^2-e(7bd-3ae))-8de(2cd-be)\ldots}{d+ex^2}}{8de^4}$$

$$= \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)} + \frac{\int \Big(-8d(3cd-be)+8cdex^2+\ldots}{8de^4}$$

$$= -\frac{(3cd-be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)} + \frac{(35cd\ldots}{}$$

$$= -\frac{(3cd-be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{d(cd^2-bde+ae^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{(13cd^2-e(9bd-5ae))\,x}{8e^4\,(d+ex^2)} + \frac{(35cd\ldots}{}$$

Mathematica [A]
time = 0.06, size = 141, normalized size = 0.99

$$\frac{(-3cd+be)x}{e^4} + \frac{cx^3}{3e^3} + \frac{(cd^3-bd^2e+ade^2)\,x}{4e^4\,(d+ex^2)^2} - \frac{(13cd^2-9bde+5ae^2)\,x}{8e^4\,(d+ex^2)} + \frac{(35cd^2-15bde+3ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8\sqrt{d}\,e^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
[Out] ((-3*c*d + b*e)*x)/e^4 + (c*x^3)/(3*e^3) + ((c*d^3 - b*d^2*e + a*d*e^2)*x)/
(4*e^4*(d + e*x^2)^2) - ((13*c*d^2 - 9*b*d*e + 5*a*e^2)*x)/(8*e^4*(d + e*x^
```

```
2)) + ((35*c*d^2 - 15*b*d*e + 3*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*Sqrt
[d]*e^(9/2))
```

Maple [A]
time = 0.14, size = 120, normalized size = 0.84

method	result
default	$\frac{\frac{1}{3} c e\,x^{3}+e b x-3 c d x}{e^{4}}+\frac{\frac{\left(-\frac{5}{8} a\,e^{3}+\frac{9}{8} d\,e^{2} b-\frac{13}{8} c\,d^{2} e\right) x^{3}-\frac{d\left(3 a\,e^{2}-7 d e b+11 c\,d^{2}\right) x}{8}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(3 a\,e^{2}-15 d e b+35 c\,d^{2}\right) \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{8 \sqrt{d e}}}{e^{4}}$
risch	$\frac{c\,x^{3}}{3 e^{3}}+\frac{b x}{e^{3}}-\frac{3 c d x}{e^{4}}+\frac{\left(-\frac{5}{8} a\,e^{3}+\frac{9}{8} d\,e^{2} b-\frac{13}{8} c\,d^{2} e\right) x^{3}-\frac{d\left(3 a\,e^{2}-7 d e b+11 c\,d^{2}\right) x}{8}}{e^{4}\left(e\,x^{2}+d\right)^{2}}-\frac{3 \ln\left(e x+\sqrt{-d e}\right) a}{16 e^{2} \sqrt{-d e}}+\frac{15 \ln\left(e x+\sqrt{-d e}\right)}{16 e^{3} \sqrt{-d e}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/e^4*(1/3*c*e*x^3+e*b*x-3*c*d*x)+1/e^4*(((-5/8*a*e^3+9/8*d*e^2*b-13/8*c*d^
2*e)*x^3-1/8*d*(3*a*e^2-7*b*d*e+11*c*d^2)*x)/(e*x^2+d)^2+1/8*(3*a*e^2-15*b*
d*e+35*c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))
```

Maxima [A]
time = 0.49, size = 132, normalized size = 0.92

$$\frac{(35\,cd^2-15\,bde+3\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,\sqrt{d}}+\frac{1}{3}\left(cx^3e-3\,(3\,cd-be)x\right)e^{(-4)}-\frac{(13\,cd^2e-9\,bde^2+5\,ae^3)x^3+(11\,cd^3-7\,bd^2e+3\,ade^2)x}{8\,(x^4e^6+2\,dx^2e^5+d^2e^4)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(35*c*d^2 - 15*b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt
(d) + 1/3*(c*x^3*e - 3*(3*c*d - b*e)*x)*e^(-4) - 1/8*((13*c*d^2*e - 9*b*d*e
^2 + 5*a*e^3)*x^3 + (11*c*d^3 - 7*b*d^2*e + 3*a*d*e^2)*x)/(x^4*e^6 + 2*d*x^
2*e^5 + d^2*e^4)
```

Fricas [A]
time = 0.37, size = 450, normalized size = 3.15

$$\left[-\frac{210\,cd^4xe+3\,(3\,ax^4e^4+35\,cd^4-3\,(5\,bdx^4-2\,adx^2)e^3+(35\,cd^2x^4-30\,bd^2x^2+3\,ad^2)e^2+5\,(14\,cd^3x^2-3\,bd^3)e)\sqrt{-de}\,\log\left(\frac{x^2e-2\,\sqrt{-de}\,x-d}{x^2e+d}\right)-2\,(8\,cdx^7+24\,bdx^5-15\,adx^3)e^4+2\,(56\,cd^2x^5-75\,bd^2x^3+9\,ad^2x)e^3+10\,(35\,cd^3x^3-9\,bd^3x)e^2}{48\,(dx^4e^7+2\,d^2x^2e^6+d^3e^5)},-\frac{105\,cd^4xe-3\,(3\,ax^4e^4+35\,cd^4-3\,(5\,bdx^4-2\,adx^2)e^3+(35\,cd^2x^4-30\,bd^2x^2+3\,ad^2)e^2+5\,(14\,cd^3x^2-3\,bd^3)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-(8\,cdx^7+24\,bdx^5-15\,adx^3)e^4+(56\,cd^2x^5-75\,bd^2x^3+9\,ad^2x)e^3+5\,(35\,cd^3x^3-9\,bd^3x)e^2}{24\,(dx^4e^7+2\,d^2x^2e^6+d^3e^5)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/48*(210*c*d^4*x*e + 3*(3*a*x^4*e^4 + 35*c*d^4 - 3*(5*b*d*x^4 - 2*a*d*x^
2)*e^3 + (35*c*d^2*x^4 - 30*b*d^2*x^2 + 3*a*d^2)*e^2 + 5*(14*c*d^3*x^2 - 3*
```

```
b*d^3)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) - 2*(8*c
*d*x^7 + 24*b*d*x^5 - 15*a*d*x^3)*e^4 + 2*(56*c*d^2*x^5 - 75*b*d^2*x^3 + 9*
a*d^2*x)*e^3 + 10*(35*c*d^3*x^3 - 9*b*d^3*x)*e^2)/(d*x^4*e^7 + 2*d^2*x^2*e^
6 + d^3*e^5), -1/24*(105*c*d^4*x*e - 3*(3*a*x^4*e^4 + 35*c*d^4 - 3*(5*b*d*x
^4 - 2*a*d*x^2)*e^3 + (35*c*d^2*x^4 - 30*b*d^2*x^2 + 3*a*d^2)*e^2 + 5*(14*c
*d^3*x^2 - 3*b*d^3)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - (8*c*d*x
^7 + 24*b*d*x^5 - 15*a*d*x^3)*e^4 + (56*c*d^2*x^5 - 75*b*d^2*x^3 + 9*a*d^2*
x)*e^3 + 5*(35*c*d^3*x^3 - 9*b*d^3*x)*e^2)/(d*x^4*e^7 + 2*d^2*x^2*e^6 + d^3
*e^5)]
```

Sympy [A]
time = 1.87, size = 212, normalized size = 1.48

$$\frac{cx^3}{3e^3}+x\left(\frac{b}{e^3}-\frac{3cd}{e^4}\right)-\frac{\sqrt{-\frac{1}{de^9}}\cdot(3ae^2-15bde+35cd^2)\log\left(-de^4\sqrt{-\frac{1}{de^9}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{de^9}}\cdot(3ae^2-15bde+35cd^2)\log\left(de^4\sqrt{-\frac{1}{de^9}}+x\right)}{16}+\frac{x^3(-5ae^3+9bde^2-13cd^2e)+x(-3ade^2+7bd^2e-11cd^3)}{8d^2e^4+16de^5x^2+8e^6x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x**3/(3*e**3) + x*(b/e**3 - 3*c*d/e**4) - sqrt(-1/(d*e**9))*(3*a*e**2 - 1
5*b*d*e + 35*c*d**2)*log(-d*e**4*sqrt(-1/(d*e**9)) + x)/16 + sqrt(-1/(d*e**
9))*(3*a*e**2 - 15*b*d*e + 35*c*d**2)*log(d*e**4*sqrt(-1/(d*e**9)) + x)/16
+ (x**3*(-5*a*e**3 + 9*b*d*e**2 - 13*c*d**2*e) + x*(-3*a*d*e**2 + 7*b*d**2*
e - 11*c*d**3))/(8*d**2*e**4 + 16*d*e**5*x**2 + 8*e**6*x**4)
```

Giac [A]
time = 4.67, size = 125, normalized size = 0.87

$$\frac{(35\,cd^2-15\,bde+3\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{9}{2}\right)}}{8\,\sqrt{d}}+\frac{1}{3}\left(cx^3e^6-9\,cdxe^5+3\,bxe^6\right)e^{(-9)}-\frac{(13\,cd^2x^3e-9\,bdx^3e^2+11\,cd^3x+5\,ax^3e^3-7\,bd^2xe+3\,adxe^2)e^{(-4)}}{8\,(x^2e+d)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(35*c*d^2 - 15*b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-9/2)/sqrt
(d) + 1/3*(c*x^3*e^6 - 9*c*d*x*e^5 + 3*b*x*e^6)*e^(-9) - 1/8*(13*c*d^2*x^3*
e - 9*b*d*x^3*e^2 + 11*c*d^3*x + 5*a*x^3*e^3 - 7*b*d^2*x*e + 3*a*d*x*e^2)*e
^(-4)/(x^2*e + d)^2
```

Mupad [B]
time = 0.34, size = 137, normalized size = 0.96

$$x\left(\frac{b}{e^3}-\frac{3\,c\,d}{e^4}\right)-\frac{\left(\frac{13\,c\,d^2\,e}{8}-\frac{9\,b\,d\,e^2}{8}+\frac{5\,a\,e^3}{8}\right)\,x^3+\left(\frac{11\,c\,d^3}{8}-\frac{7\,b\,d^2\,e}{8}+\frac{3\,a\,d\,e^2}{8}\right)\,x}{d^2\,e^4+2\,d\,e^5\,x^2+e^6\,x^4}+\frac{c\,x^3}{3\,e^3}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(35\,c\,d^2-15\,b\,d\,e+3\,a\,e^2\right)}{8\,\sqrt{d}\,e^{9/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] x*(b/e^3 - (3*c*d)/e^4) - (x*((11*c*d^3)/8 + (3*a*d*e^2)/8 - (7*b*d^2*e)/8)
 + x^3*((5*a*e^3)/8 - (9*b*d*e^2)/8 + (13*c*d^2*e)/8))/(d^2*e^4 + e^6*x^4 +
 2*d*e^5*x^2) + (c*x^3)/(3*e^3) + (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 35*
c*d^2 - 15*b*d*e))/(8*d^(1/2)*e^(9/2))
```

3.290 $\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx$

Optimal. Leaf size=124

$$\frac{cx}{e^3}-\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}+\frac{(9cd^2-e(5bd-ae))\,x}{8de^3\,(d+ex^2)}-\frac{(15cd^2-e(3bd+ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}}$$

```
[Out] c*x/e^3-1/4*(a*e^2-b*d*e+c*d^2)*x/e^3/(e*x^2+d)^2+1/8*(9*c*d^2-e*(-a*e+5*b*
d))*x/d/e^3/(e*x^2+d)-1/8*(15*c*d^2-e*(a*e+3*b*d))*arctan(x*e^(1/2)/d^(1/2)
)/d^(3/2)/e^(7/2)
```

Rubi [A]
time = 0.09, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}=0.160$,
Rules used = {1271, 1171, 396, 211}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(15cd^2-e(ae+3bd))}{8d^{3/2}e^{7/2}}+\frac{x(9cd^2-e(5bd-ae))}{8de^3\,(d+ex^2)}-\frac{x(ae^2-bde+cd^2)}{4e^3\,(d+ex^2)^2}+\frac{cx}{e^3}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 e*(5*b*d - a*e))*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - e*(3*b*d + a*e))*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
```

```
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1271

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
 + e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*x^2))*(2*e
^(2*p + m/2)*(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*
d*e + a*e^2)^p*(d + e*(2*q + 3)*x^2))], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2(a+bx^2+cx^4)}{(d+ex^2)^3}\,dx &= -\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}-\frac{\int \frac{-cd^2+bde-ae^2+4e(cd-be)x^2-4ce^2x^4}{(d+ex^2)^2}\,dx}{4e^3}\\
&= -\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}+\frac{(9cd^2-e(5bd-ae))\,x}{8de^3\,(d+ex^2)}+\frac{\int \frac{-7cd^2+e(3bd+ae)+8cdex^2}{d+ex^2}\,dx}{8de^3}\\
&= \frac{cx}{e^3}-\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}+\frac{(9cd^2-e(5bd-ae))\,x}{8de^3\,(d+ex^2)}-\frac{(15cd^2-e(3bd+ae))\int \frac{}{d}}{8de^3}\\
&= \frac{cx}{e^3}-\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}+\frac{(9cd^2-e(5bd-ae))\,x}{8de^3\,(d+ex^2)}-\frac{(15cd^2-e(3bd+ae))\tan}{8d^{3/2}e^{7/2}}
\end{aligned}$$

Mathematica [A]
time = 0.07, size = 122, normalized size = 0.98

$$\frac{cx}{e^3}-\frac{(cd^2-bde+ae^2)\,x}{4e^3\,(d+ex^2)^2}+\frac{(9cd^2-5bde+ae^2)\,x}{8de^3\,(d+ex^2)}-\frac{(15cd^2-3bde-ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{3/2}e^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x]
```

```
[Out] (c*x)/e^3 - ((c*d^2 - b*d*e + a*e^2)*x)/(4*e^3*(d + e*x^2)^2) + ((9*c*d^2 -
 5*b*d*e + a*e^2)*x)/(8*d*e^3*(d + e*x^2)) - ((15*c*d^2 - 3*b*d*e - a*e^2)*
ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(3/2)*e^(7/2))
```

Maple [A]

time = 0.16, size = 106, normalized size = 0.85

method	result
default	$\frac{cx}{e^{3}}+\frac{\frac{\frac{e \left(a \,e^{2}-5 d e b +9 c \,d^{2}\right) x^{3}}{8 d}+\left(-\frac{1}{8} a \,e^{2}-\frac{3}{8} d e b +\frac{7}{8} c \,d^{2}\right) x}{\left(e \,x^{2}+d \right)^{2}}+\frac{\left(a \,e^{2}+3 d e b -15 c \,d^{2}\right) \arctan \left(\frac{e x}{\sqrt{d e}}\right)}{8 d \sqrt{d e}}}{e^{3}}$
risch	$\frac{cx}{e^{3}}+\frac{\frac{e \left(a \,e^{2}-5 d e b +9 c \,d^{2}\right) x^{3}}{8 d}+\left(-\frac{1}{8} a \,e^{2}-\frac{3}{8} d e b +\frac{7}{8} c \,d^{2}\right) x}{e^{3} \left(e \,x^{2}+d \right)^{2}}-\frac{\ln \left(e x +\sqrt{-d e}\right) a}{16 e \sqrt{-d e}\, d}-\frac{3 \ln \left(e x +\sqrt{-d e}\right) b}{16 e^{2} \sqrt{-d e}}+\frac{15 d \ln \left(e x +\sqrt{-d e}\right.}{16 e^{3} \sqrt{-d e}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] c*x/e^3+1/e^3*((1/8*e*(a*e^2-5*b*d*e+9*c*d^2)/d*x^3+(-1/8*a*e^2-3/8*d*e*b+7
/8*c*d^2)*x)/(e*x^2+d)^2+1/8*(a*e^2+3*b*d*e-15*c*d^2)/d/(d*e)^(1/2)*arctan(
e*x/(d*e)^(1/2)))
```

Maxima [A]

time = 0.52, size = 114, normalized size = 0.92

$$cxe^{(-3)} - \frac{(15\,cd^2 - 3\,bde - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{8\,d^{\frac{3}{2}}} + \frac{(9\,cd^2e - 5\,bde^2 + ae^3)x^3 + (7\,cd^3 - 3\,bd^2e - ade^2)x}{8\,(dx^4e^5 + 2\,d^2x^2e^4 + d^3e^3)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] c*x*e^(-3) - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(
-7/2)/d^(3/2) + 1/8*((9*c*d^2*e - 5*b*d*e^2 + a*e^3)*x^3 + (7*c*d^3 - 3*b*d
^2*e - a*d*e^2)*x)/(d*x^4*e^5 + 2*d^2*x^2*e^4 + d^3*e^3)
```

Fricas [A]

time = 0.36, size = 415, normalized size = 3.35

$$\left[\frac{30\,cd^4xe + 2\,adx^3e^4 - (ax^4e^4 - 15\,cd^4 + (3\,bdx^4 + 2\,adx^2)e^3 - (15\,cd^2x^4 - 6\,bd^2x^2 - ad^2)e^2 - 3\,(10\,cd^3x^2 - bd^3)e)\sqrt{-de}\,\log\left(\frac{x^2e - 2\,\sqrt{-de}\,x - d}{x^2e + d}\right) + 2\,(8\,cd^2x^5 - 5\,bd^2x^3 - ad^2x)e^3 + 2\,(25\,cd^3x^3 - 3\,bd^3x)e^2}{16\,(d^2x^4e^6 + 2\,d^3x^2e^5 + d^4e^4)}, \frac{15\,cd^4xe + adx^3e^4 + (ax^4e^4 - 15\,cd^4 + (3\,bdx^4 + 2\,adx^2)e^3 - (15\,cd^2x^4 - 6\,bd^2x^2 - ad^2)e^2 - 3\,(10\,cd^3x^2 - bd^3)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}} + (8\,cd^2x^5 - 5\,bd^2x^3 - ad^2x)e^3 + (25\,cd^3x^3 - 3\,bd^3x)e^2}{8\,(d^2x^4e^6 + 2\,d^3x^2e^5 + d^4e^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(30*c*d^4*x*e + 2*a*d*x^3*e^4 - (a*x^4*e^4 - 15*c*d^4 + (3*b*d*x^4 +
2*a*d*x^2)*e^3 - (15*c*d^2*x^4 - 6*b*d^2*x^2 - a*d^2)*e^2 - 3*(10*c*d^3*x^2
 - b*d^3)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) + 2*(
8*c*d^2*x^5 - 5*b*d^2*x^3 - a*d^2*x)*e^3 + 2*(25*c*d^3*x^3 - 3*b*d^3*x)*e^2
)/(d^2*x^4*e^6 + 2*d^3*x^2*e^5 + d^4*e^4), 1/8*(15*c*d^4*x*e + a*d*x^3*e^4
```

```
+ (a*x^4*e^4 - 15*c*d^4 + (3*b*d*x^4 + 2*a*d*x^2)*e^3 - (15*c*d^2*x^4 - 6*b
*d^2*x^2 - a*d^2)*e^2 - 3*(10*c*d^3*x^2 - b*d^3)*e)*sqrt(d)*arctan(x*e^(1/2
)/sqrt(d))*e^(1/2) + (8*c*d^2*x^5 - 5*b*d^2*x^3 - a*d^2*x)*e^3 + (25*c*d^3*
x^3 - 3*b*d^3*x)*e^2)/(d^2*x^4*e^6 + 2*d^3*x^2*e^5 + d^4*e^4)]
```

Sympy [A]
time = 1.37, size = 201, normalized size = 1.62

$$\frac{cx}{e^3} - \frac{\sqrt{-\frac{1}{d^3e^7}}\,(ae^2+3bde-15cd^2)\log\left(-d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{16} + \frac{\sqrt{-\frac{1}{d^3e^7}}\,(ae^2+3bde-15cd^2)\log\left(d^2e^3\sqrt{-\frac{1}{d^3e^7}}+x\right)}{16} + \frac{x^3(ae^3-5bde^2+9cd^2e)+x(-ade^2-3bd^2e+7cd^3)}{8d^3e^3+16d^2e^4x^2+8de^5x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] c*x/e**3 - sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e - 15*c*d**2)*log(-d**2*e*
*3*sqrt(-1/(d**3*e**7)) + x)/16 + sqrt(-1/(d**3*e**7))*(a*e**2 + 3*b*d*e -
15*c*d**2)*log(d**2*e**3*sqrt(-1/(d**3*e**7)) + x)/16 + (x**3*(a*e**3 - 5*b
*d*e**2 + 9*c*d**2*e) + x*(-a*d*e**2 - 3*b*d**2*e + 7*c*d**3))/(8*d**3*e**3
 + 16*d**2*e**4*x**2 + 8*d*e**5*x**4)
```

Giac [A]
time = 4.89, size = 107, normalized size = 0.86

$$cxe^{(-3)} - \frac{(15\,cd^2 - 3\,bde - ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{7}{2}\right)}}{8\,d^{\frac{3}{2}}} + \frac{(9\,cd^2x^3e - 5\,bdx^3e^2 + 7\,cd^3x + ax^3e^3 - 3\,bd^2xe - adxe^2)e^{(-3)}}{8\,(x^2e+d)^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] c*x*e^(-3) - 1/8*(15*c*d^2 - 3*b*d*e - a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(
-7/2)/d^(3/2) + 1/8*(9*c*d^2*x^3*e - 5*b*d*x^3*e^2 + 7*c*d^3*x + a*x^3*e^3
- 3*b*d^2*x*e - a*d*x*e^2)*e^(-3)/((x^2*e + d)^2*d)
```

Mupad [B]
time = 0.39, size = 118, normalized size = 0.95

$$\frac{c\,x}{e^3} - \frac{x\left(-\frac{7\,c\,d^2}{8} + \frac{3\,b\,d\,e}{8} + \frac{a\,e^2}{8}\right) - \frac{x^3\,(9\,c\,d^2\,e - 5\,b\,d\,e^2 + a\,e^3)}{8\,d}}{d^2\,e^3 + 2\,d\,e^4\,x^2 + e^5\,x^4} + \frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(-15\,c\,d^2 + 3\,b\,d\,e + a\,e^2)}{8\,d^{3/2}\,e^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(a + b*x^2 + c*x^4))/(d + e*x^2)^3,x)
```

```
[Out] (c*x)/e^3 - (x*((a*e^2)/8 - (7*c*d^2)/8 + (3*b*d*e)/8) - (x^3*(a*e^3 - 5*b*
d*e^2 + 9*c*d^2*e))/(8*d))/(d^2*e^3 + e^5*x^4 + 2*d*e^4*x^2) + (atan((e^(1/
2)*x)/d^(1/2))*(a*e^2 - 15*c*d^2 + 3*b*d*e))/(8*d^(3/2)*e^(7/2))
```

3.291 $\int \frac{a+bx^2+cx^4}{(d+ex^2)^3} dx$

Optimal. Leaf size=115

$$\frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2}-\frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)}+\frac{\left(3cd^2+e(bd+3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

```
[Out] 1/4*(a+d*(-b*e+c*d)/e^2)*x/d/(e*x^2+d)^2-1/8*(5*c*d^2-e*(3*a*e+b*d))*x/d^2/
e^2/(e*x^2+d)+1/8*(3*c*d^2+e*(3*a*e+b*d))*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)
/e^(5/2)
```

Rubi [A]
time = 0.08, antiderivative size = 119, normalized size of antiderivative = 1.03, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {1171, 393, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\left(e(3ae+bd)+3cd^2\right)}{8d^{5/2}e^{5/2}}-\frac{x\left(5cd^2-e(3ae+bd)\right)}{8d^2e^2\left(d+ex^2\right)}+\frac{x\left(ae^2-bde+cd^2\right)}{4de^2\left(d+ex^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
```

```
[Out] ((c*d^2 - b*d*e + a*e^2)*x)/(4*d*e^2*(d + e*x^2)^2) - ((5*c*d^2 - e*(b*d +
3*a*e))*x)/(8*d^2*e^2*(d + e*x^2)) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(S
qrt[e]*x)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 393

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(-(b*c - a*d))*x*((a + b*x^n)^(p + 1)/(a*b*n*(p + 1))), x] - Dist[(a*d -
 b*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; F
reeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n
 + p, 0])
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Dist[1/(2*d*(q
 + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x
], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{\left(d+ex^2\right)^3}\,dx = \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\int \frac{-3a+\frac{d(cd-be)}{e^2}-\frac{4cdx^2}{e}}{\left(d+ex^2\right)^2}\,dx}{4d}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)} - \frac{\left(-\frac{4cd^2}{e}+e\left(-3a+\frac{d(cd-be)}{e^2}\right)\right)\int \frac{1}{d+ex^2}\,d}{8d^2e}$$

$$= \frac{\left(a+\frac{d(cd-be)}{e^2}\right)x}{4d\left(d+ex^2\right)^2} - \frac{\left(5cd^2-e(bd+3ae)\right)x}{8d^2e^2\left(d+ex^2\right)} + \frac{\left(3cd^2+e(bd+3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Mathematica [A]
time = 0.06, size = 110, normalized size = 0.96

$$\frac{x\left(-cd^2\left(3d+5ex^2\right)+e\left(bd\left(-d+ex^2\right)+ae\left(5d+3ex^2\right)\right)\right)}{8d^2e^2\left(d+ex^2\right)^2}+\frac{\left(3cd^2+e(bd+3ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{5/2}e^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(d + e*x^2)^3,x]
[Out] (x*(-(c*d^2*(3*d + 5*e*x^2)) + e*(b*d*(-d + e*x^2) + a*e*(5*d + 3*e*x^2))))
/(8*d^2*e^2*(d + e*x^2)^2) + ((3*c*d^2 + e*(b*d + 3*a*e))*ArcTan[(Sqrt[e]*x
)/Sqrt[d]])/(8*d^(5/2)*e^(5/2))
```

Maple [A]
time = 0.13, size = 107, normalized size = 0.93

method	result
default	$\frac{\frac{\left(3a e^2+deb-5c d^2\right)x^3}{8d^2e}+\frac{\left(5a e^2-deb-3c d^2\right)x}{8d e^2}}{\left(e x^2+d\right)^2}+\frac{\left(3a e^2+deb+3c d^2\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8d^2e^2\sqrt{de}}$
risch	$\frac{\frac{\left(3a e^2+deb-5c d^2\right)x^3}{8d^2e}+\frac{\left(5a e^2-deb-3c d^2\right)x}{8d e^2}}{\left(e x^2+d\right)^2}-\frac{3\ln\left(ex+\sqrt{-de}\right)a}{16\sqrt{-de}\,d^2}-\frac{\ln\left(ex+\sqrt{-de}\right)b}{16\sqrt{-de}\,ed}-\frac{3\ln\left(ex+\sqrt{-de}\right)c}{16\sqrt{-de}\,e^2}+\frac{3\ln\left(-\right.}{16}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] (1/8*(3*a*e^2+b*d*e-5*c*d^2)/d^2/e*x^3+1/8*(5*a*e^2-b*d*e-3*c*d^2)/d/e^2*x)
/(e*x^2+d)^2+1/8*(3*a*e^2+b*d*e+3*c*d^2)/d^2/e^2/(d*e)^(1/2)*arctan(e*x/(d*
e)^(1/2))
```

Maxima [A]
time = 0.51, size = 110, normalized size = 0.96

$$\frac{\left(3\,cd^2+bde+3\,ae^2\right)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{\left(5\,cd^2e-bde^2-3\,ae^3\right)x^3+\left(3\,cd^3+bd^2e-5\,ade^2\right)x}{8\left(d^2x^4e^4+2\,d^3x^2e^3+d^4e^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2)
- 1/8*((5*c*d^2*e - b*d*e^2 - 3*a*e^3)*x^3 + (3*c*d^3 + b*d^2*e - 5*a*d*e^2
)*x)/(d^2*x^4*e^4 + 2*d^3*x^2*e^3 + d^4*e^2)
```

Fricas [A]
time = 0.36, size = 389, normalized size = 3.38

$$\left[-\frac{6\,cd^4xe-6\,adx^3e^4+\left(3\,ax^4e^4+3\,cd^4+\left(bdx^4+6\,adx^2\right)e^3+\left(3\,cd^2x^4+2\,bd^2x^2+3\,ad^2\right)e^2+\left(6\,cd^3x^2+bd^3\right)e\right)\sqrt{-de}\,\log\left(\frac{x^2e-2\,\sqrt{-de}\,x-d}{x^2e+d}\right)-2\left(bd^2x^3+5\,ad^2x\right)e^3+2\left(5\,cd^3x^3+bd^3x\right)e^2}{16\left(d^3x^4e^5+2\,d^4x^2e^4+d^5e^3\right)},-\frac{3\,cd^4xe-3\,adx^3e^4-\left(3\,ax^4e^4+3\,cd^4+\left(bdx^4+6\,adx^2\right)e^3+\left(3\,cd^2x^4+2\,bd^2x^2+3\,ad^2\right)e^2+\left(6\,cd^3x^2+bd^3\right)e\right)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-\left(bd^2x^3+5\,ad^2x\right)e^3+\left(5\,cd^3x^3+bd^3x\right)e^2}{8\left(d^3x^4e^5+2\,d^4x^2e^4+d^5e^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(6*c*d^4*x*e - 6*a*d*x^3*e^4 + (3*a*x^4*e^4 + 3*c*d^4 + (b*d*x^4 + 6
*a*d*x^2)*e^3 + (3*c*d^2*x^4 + 2*b*d^2*x^2 + 3*a*d^2)*e^2 + (6*c*d^3*x^2 +
b*d^3)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) - 2*(b*d
^2*x^3 + 5*a*d^2*x)*e^3 + 2*(5*c*d^3*x^3 + b*d^3*x)*e^2)/(d^3*x^4*e^5 + 2*d
^4*x^2*e^4 + d^5*e^3), -1/8*(3*c*d^4*x*e - 3*a*d*x^3*e^4 - (3*a*x^4*e^4 + 3
*c*d^4 + (b*d*x^4 + 6*a*d*x^2)*e^3 + (3*c*d^2*x^4 + 2*b*d^2*x^2 + 3*a*d^2)*
e^2 + (6*c*d^3*x^2 + b*d^3)*e)*sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) -
(b*d^2*x^3 + 5*a*d^2*x)*e^3 + (5*c*d^3*x^3 + b*d^3*x)*e^2)/(d^3*x^4*e^5 + 2
*d^4*x^2*e^4 + d^5*e^3)]
```

Sympy [A]
time = 0.78, size = 196, normalized size = 1.70

$$-\frac{\sqrt{-\frac{1}{d^5e^5}}\cdot\left(3ae^2+bde+3cd^2\right)\log\left(-d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^5e^5}}\cdot\left(3ae^2+bde+3cd^2\right)\log\left(d^3e^2\sqrt{-\frac{1}{d^5e^5}}+x\right)}{16}+\frac{x^3\cdot\left(3ae^3+bde^2-5cd^2e\right)+x\left(5ade^2-bd^2e-3cd^3\right)}{8d^4e^2+16d^3e^3x^2+8d^2e^4x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*log(-d**3*e**2*sqrt(-1/
(d**5*e**5)) + x)/16 + sqrt(-1/(d**5*e**5))*(3*a*e**2 + b*d*e + 3*c*d**2)*l
og(d**3*e**2*sqrt(-1/(d**5*e**5)) + x)/16 + (x**3*(3*a*e**3 + b*d*e**2 - 5*
c*d**2*e) + x*(5*a*d*e**2 - b*d**2*e - 3*c*d**3))/(8*d**4*e**2 + 16*d**3*e*
*3*x**2 + 8*d**2*e**4*x**4)
```

Giac [A]
time = 3.23, size = 101, normalized size = 0.88

$$\frac{(3\,cd^2+bde+3\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{5}{2}\right)}}{8\,d^{\frac{5}{2}}}-\frac{(5\,cd^2x^3e-bdx^3e^2+3\,cd^3x-3\,ax^3e^3+bd^2xe-5\,adxe^2)e^{(-2)}}{8\,(x^2e+d)^2d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 + b*d*e + 3*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-5/2)/d^(5/2)
- 1/8*(5*c*d^2*x^3*e - b*d*x^3*e^2 + 3*c*d^3*x - 3*a*x^3*e^3 + b*d^2*x*e -
5*a*d*x*e^2)*e^(-2)/((x^2*e + d)^2*d^2)
```

Mupad [B]
time = 0.38, size = 112, normalized size = 0.97

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(3\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^{5/2}\,e^{5/2}}-\frac{\frac{x\,\left(3\,c\,d^2+b\,d\,e-5\,a\,e^2\right)}{8\,d\,e^2}-\frac{x^3\,\left(-5\,c\,d^2+b\,d\,e+3\,a\,e^2\right)}{8\,d^2\,e}}{d^2+2\,d\,e\,x^2+e^2\,x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(d + e*x^2)^3,x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(3*a*e^2 + 3*c*d^2 + b*d*e))/(8*d^(5/2)*e^(5/2))
 - ((x*(3*c*d^2 - 5*a*e^2 + b*d*e))/(8*d*e^2) - (x^3*(3*a*e^2 - 5*c*d^2 + b
*d*e))/(8*d^2*e))/(d^2 + e^2*x^4 + 2*d*e*x^2)
```

3.292 $\int \frac{a+bx^2+cx^4}{x^2(d+ex^2)^3} dx$

Optimal. Leaf size=127

$$-\frac{a}{d^3x}-\frac{(cd^2-bde+ae^2)\,x}{4d^2e\,(d+ex^2)^2}+\frac{(cd^2+e(3bd-7ae))\,x}{8d^3e\,(d+ex^2)}+\frac{(cd^2+3e(bd-5ae))\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{7/2}e^{3/2}}$$

```
[Out] -a/d^3/x-1/4*(a*e^2-b*d*e+c*d^2)*x/d^2/e/(e*x^2+d)^2+1/8*(c*d^2+e*(-7*a*e+3
*b*d))*x/d^3/e/(e*x^2+d)+1/8*(c*d^2+3*e*(-5*a*e+b*d))*arctan(x*e^(1/2)/d^(1
/2))/d^(7/2)/e^(3/2)
```

Rubi [A]

time = 0.13, antiderivative size = 124, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1273, 467, 464, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(3e(bd-5ae)+cd^2)}{8d^{7/2}e^{3/2}}-\frac{x\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)}{4\,(d+ex^2)^2}+\frac{x(e(3bd-7ae)+cd^2)}{8d^3e\,(d+ex^2)}-\frac{a}{d^3x}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] -(a/(d^3*x)) - ((c/e - (b*d - a*e)/d^2)*x)/(4*(d + e*x^2)^2) + ((c*d^2 + e*
(3*b*d - 7*a*e))*x)/(8*d^3*e*(d + e*x^2)) + ((c*d^2 + 3*e*(b*d - 5*a*e))*Ar
cTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(7/2)*e^(3/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 464

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*e*(m + 1))),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 467

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
```

```
+ 1))), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[x^m*(a + b*x^2)^(p + 1)*Ex
pandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c -
 a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/2
, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{a+bx^2+cx^4}{x^2\left(d+ex^2\right)^3}\,dx &= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}-\frac{\int \frac{-4ade^2-e\left(cd^2+3e(bd-ae)\right)x^2}{x^2\left(d+ex^2\right)^2}\,dx}{4d^2e^2}\\
&= -\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\int \frac{8ae^2+e\left(cd+e\left(3b-\frac{7ae}{d}\right)\right)x^2}{x^2\left(d+ex^2\right)}\,dx}{8d^2e^2}\\
&= -\frac{a}{d^3x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\left(cd^2+3e(bd-5ae)\right)\int \frac{1}{d+ex^2}\,dx}{8d^3e}\\
&= -\frac{a}{d^3x}-\frac{\left(\frac{c}{e}-\frac{bd-ae}{d^2}\right)x}{4\left(d+ex^2\right)^2}+\frac{\left(cd^2+e(3bd-7ae)\right)x}{8d^3e\left(d+ex^2\right)}+\frac{\left(cd^2+3e(bd-5ae)\right)\tan^{-1}\left(\frac{\sqrt{e}}{\sqrt{d}}\right.}{8d^{7/2}e^{3/2}}
\end{aligned}$$

Mathematica [A]
time = 0.09, size = 124, normalized size = 0.98

$$\frac{\frac{\sqrt{d}\left(-ae\left(8d^2+25dex^2+15e^2x^4\right)+dx^2\left(cd\left(-d+ex^2\right)+be\left(5d+3ex^2\right)\right)\right)}{ex\left(d+ex^2\right)^2}+\frac{\left(cd^2+3e(bd-5ae)\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}}}{8d^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x]
```

```
[Out] ((Sqrt[d]*(-(a*e*(8*d^2 + 25*d*e*x^2 + 15*e^2*x^4)) + d*x^2*(c*d*(-d + e*x^
2) + b*e*(5*d + 3*e*x^2))))/(e*x*(d + e*x^2)^2) + ((c*d^2 + 3*e*(b*d - 5*a*
e))*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/e^(3/2))/(8*d^(7/2))
```

Maple [A]
time = 0.16, size = 111, normalized size = 0.87

method	result
default	$-\frac{\frac{\left(\frac{7}{8} a\,e^{2}-\frac{3}{8} d e b-\frac{1}{8} c\,d^{2}\right) x^{3}+\frac{d \left(9 a\,e^{2}-5 d e b+c\,d^{2}\right) x}{8 e}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(15 a\,e^{2}-3 d e b-c\,d^{2}\right) \arctan\left(\frac{e x}{\sqrt{d e}}\right)}{8 e \sqrt{d e}}}{d^{3}}-\frac{a}{d^{3} x}$
risch	$\frac{-\frac{\left(15 a\,e^{2}-3 d e b-c\,d^{2}\right) x^{4}}{8 d^{3}}-\frac{\left(25 a\,e^{2}-5 d e b+c\,d^{2}\right) x^{2}}{8 d^{2} e}-\frac{a}{d}}{x \left(e\,x^{2}+d\right)^{2}}-\frac{15 e \ln\left(-\sqrt{-d e}\, x-d\right) a}{16 \sqrt{-d e}\, d^{3}}+\frac{3 \ln\left(-\sqrt{-d e}\, x-d\right) b}{16 \sqrt{-d e}\, d^{2}}+\frac{\ln\left(-\sqrt{-d e}\, x-\right.}{16 \sqrt{-d e}\, ec}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -1/d^3*(((7/8*a*e^2-3/8*d*e*b-1/8*c*d^2)*x^3+1/8*d*(9*a*e^2-5*b*d*e+c*d^2)/
e*x)/(e*x^2+d)^2+1/8*(15*a*e^2-3*b*d*e-c*d^2)/e/(d*e)^(1/2)*arctan(e*x/(d*e
)^(1/2)))-a/d^3/x
```

Maxima [A]
time = 0.52, size = 121, normalized size = 0.95

$$\frac{(cd^2e + 3\,bde^2 - 15\,ae^3)x^4 - 8\,ad^2e - (cd^3 - 5\,bd^2e + 25\,ade^2)x^2}{8\,(d^3x^5e^3 + 2\,d^4x^3e^2 + d^5xe)} + \frac{(cd^2 + 3\,bde - 15\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{8\,d^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((c*d^2*e + 3*b*d*e^2 - 15*a*e^3)*x^4 - 8*a*d^2*e - (c*d^3 - 5*b*d^2*e
+ 25*a*d*e^2)*x^2)/(d^3*x^5*e^3 + 2*d^4*x^3*e^2 + d^5*x*e) + 1/8*(c*d^2 + 3
*b*d*e - 15*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(7/2)
```

Fricas [A]
time = 0.38, size = 428, normalized size = 3.37

$$\left[-\frac{2\,cd^4x^2e + 30\,adx^4e^4 + (15\,ax^5e^4 - cd^4x - 3\,(bdx^5 - 10\,adx^3)e^3 - (cd^2x^5 + 6\,bd^2x^3 - 15\,ad^2x)e^2 - (2\,cd^3x^3 + 3\,bd^3x)e)\sqrt{-de}\,\log\left(\frac{x^2e + 2\,\sqrt{-de}\,x - d}{x^2e + d}\right) - 2\,(3\,bd^2x^4 - 25\,ad^2x^2)e^3 - 2\,(cd^3x^4 + 5\,bd^3x^2 - 8\,ad^3)e^2}{16\,(d^4x^5e^4 + 2\,d^5x^3e^3 + d^6xe^2)}, -\frac{cd^4x^2e + 15\,adx^4e^4 + (15\,ax^5e^4 - cd^4x - 3\,(bdx^5 - 10\,adx^3)e^3 - (cd^2x^5 + 6\,bd^2x^3 - 15\,ad^2x)e^2 - (2\,cd^3x^3 + 3\,bd^3x)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}} - (3\,bd^2x^4 - 25\,ad^2x^2)e^3 - (cd^3x^4 + 5\,bd^3x^2 - 8\,ad^3)e^2}{8\,(d^4x^5e^4 + 2\,d^5x^3e^3 + d^6xe^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(2*c*d^4*x^2*e + 30*a*d*x^4*e^4 + (15*a*x^5*e^4 - c*d^4*x - 3*(b*d*x
^5 - 10*a*d*x^3)*e^3 - (c*d^2*x^5 + 6*b*d^2*x^3 - 15*a*d^2*x)*e^2 - (2*c*d^
```

```
3*x^3 + 3*b*d^3*x)*e)*sqrt(-d*e)*log((x^2*e + 2*sqrt(-d*e)*x - d)/(x^2*e +
d)) - 2*(3*b*d^2*x^4 - 25*a*d^2*x^2)*e^3 - 2*(c*d^3*x^4 + 5*b*d^3*x^2 - 8*a
*d^3)*e^2)/(d^4*x^5*e^4 + 2*d^5*x^3*e^3 + d^6*x*e^2), -1/8*(c*d^4*x^2*e + 1
5*a*d*x^4*e^4 + (15*a*x^5*e^4 - c*d^4*x - 3*(b*d*x^5 - 10*a*d*x^3)*e^3 - (c
*d^2*x^5 + 6*b*d^2*x^3 - 15*a*d^2*x)*e^2 - (2*c*d^3*x^3 + 3*b*d^3*x)*e)*sqr
t(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2) - (3*b*d^2*x^4 - 25*a*d^2*x^2)*e^3 -
 (c*d^3*x^4 + 5*b*d^3*x^2 - 8*a*d^3)*e^2)/(d^4*x^5*e^4 + 2*d^5*x^3*e^3 + d^
6*x*e^2)]
```

Sympy [A]

time = 1.10, size = 202, normalized size = 1.59

$$\frac{\sqrt{-\frac{1}{d^7e^3}}\cdot(15ae^2-3bde-cd^2)\log\left(-d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16}-\frac{\sqrt{-\frac{1}{d^7e^3}}\cdot(15ae^2-3bde-cd^2)\log\left(d^4e\sqrt{-\frac{1}{d^7e^3}}+x\right)}{16}+\frac{-8ad^2e+x^4(-15ae^3+3bde^2+cd^2e)+x^2(-25ade^2+5bd^2e-cd^3)}{8d^5ex+16d^4e^2x^3+8d^3e^3x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**2/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log(-d**4*e*sqrt(-1/(d*
*7*e**3)) + x)/16 - sqrt(-1/(d**7*e**3))*(15*a*e**2 - 3*b*d*e - c*d**2)*log
(d**4*e*sqrt(-1/(d**7*e**3)) + x)/16 + (-8*a*d**2*e + x**4*(-15*a*e**3 + 3*
b*d*e**2 + c*d**2*e) + x**2*(-25*a*d*e**2 + 5*b*d**2*e - c*d**3))/(8*d**5*e
*x + 16*d**4*e**2*x**3 + 8*d**3*e**3*x**5)
```

Giac [A]

time = 3.30, size = 110, normalized size = 0.87

$$\frac{(cd^2+3\,bde-15\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{3}{2}\right)}}{8\,d^{\frac{7}{2}}}-\frac{a}{d^3x}+\frac{(cd^2x^3e+3\,bdx^3e^2-cd^3x-7\,ax^3e^3+5\,bd^2xe-9\,adxe^2)e^{(-1)}}{8\,(x^2e+d)^2d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^2/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(c*d^2 + 3*b*d*e - 15*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-3/2)/d^(7/2)
 - a/(d^3*x) + 1/8*(c*d^2*x^3*e + 3*b*d*x^3*e^2 - c*d^3*x - 7*a*x^3*e^3 + 5
*b*d^2*x*e - 9*a*d*x*e^2)*e^(-1)/((x^2*e + d)^2*d^3)
```

Mupad [B]

time = 0.39, size = 118, normalized size = 0.93

$$\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(c\,d^2+3\,b\,d\,e-15\,a\,e^2\right)}{8\,d^{7/2}\,e^{3/2}}-\frac{\frac{a}{d}-\frac{x^4\,\left(c\,d^2+3\,b\,d\,e-15\,a\,e^2\right)}{8\,d^3}+\frac{x^2\,\left(c\,d^2-5\,b\,d\,e+25\,a\,e^2\right)}{8\,d^2\,e}}{d^2\,x+2\,d\,e\,x^3+e^2\,x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^2*(d + e*x^2)^3),x)
```

```
[Out] (atan((e^(1/2)*x)/d^(1/2))*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^(7/2)*e^(3/2)
) - (a/d - (x^4*(c*d^2 - 15*a*e^2 + 3*b*d*e))/(8*d^3) + (x^2*(25*a*e^2 + c*
d^2 - 5*b*d*e))/(8*d^2*e))/(d^2*x + e^2*x^5 + 2*d*e*x^3)
```

3.293 $\int \frac{a+bx^2+cx^4}{x^4\left(d+ex^2\right)^3}\,dx$

Optimal. Leaf size=142

$$-\frac{a}{3d^3x^3}-\frac{bd-3ae}{d^4x}+\frac{(cd^2-bde+ae^2)\,x}{4d^3\,(d+ex^2)^2}+\frac{(3cd^2-e(7bd-11ae))\,x}{8d^4\,(d+ex^2)}+\frac{(3cd^2-15bde+35ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{9/2}\sqrt{e}}$$

```
[Out] -1/3*a/d^3/x^3+(3*a*e-b*d)/d^4/x+1/4*(a*e^2-b*d*e+c*d^2)*x/d^3/(e*x^2+d)^2+
1/8*(3*c*d^2-e*(-11*a*e+7*b*d))*x/d^4/(e*x^2+d)+1/8*(35*a*e^2-15*b*d*e+3*c*
d^2)*arctan(x*e^(1/2)/d^(1/2))/d^(9/2)/e^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1273, 1275, 211}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(35ae^2-15bde+3cd^2)}{8d^{9/2}\sqrt{e}}+\frac{x(3cd^2-e(7bd-11ae))}{8d^4\,(d+ex^2)}+\frac{x(ae^2-bde+cd^2)}{4d^3\,(d+ex^2)^2}-\frac{bd-3ae}{d^4x}-\frac{a}{3d^3x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -1/3*a/(d^3*x^3) - (b*d - 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(4*d
^3*(d + e*x^2)^2) + ((3*c*d^2 - e*(7*b*d - 11*a*e))*x)/(8*d^4*(d + e*x^2))
+ ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/2)*
Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^4(d+ex^2)^3}\,dx = \frac{(cd^2-bde+ae^2)\,x}{4d^3(d+ex^2)^2} + \frac{\int \frac{4ad^2e^2+4de^2(bd-ae)x^2+3e^2(cd^2-bde+ae^2)x^4}{x^4(d+ex^2)^2}\,dx}{4d^3e^2}$$

$$= \frac{(cd^2-bde+ae^2)\,x}{4d^3(d+ex^2)^2} + \frac{(3cd^2-e(7bd-11ae))\,x}{8d^4(d+ex^2)} + \frac{\int \frac{8ad^4e^4+8d^3e^4(bd-2ae)x^2+d^2e^4(3cd^2-e(}{x^4(d+ex^2)}}{8d^6e^4}$$

$$= \frac{(cd^2-bde+ae^2)\,x}{4d^3(d+ex^2)^2} + \frac{(3cd^2-e(7bd-11ae))\,x}{8d^4(d+ex^2)} + \frac{\int \left(\frac{8ad^3e^4}{x^4} + \frac{8d^2e^4(bd-3ae)}{x^2} + \frac{d^2e^4(3cd}{}\right.}{8d^6e^4}$$

$$= -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{(cd^2-bde+ae^2)\,x}{4d^3(d+ex^2)^2} + \frac{(3cd^2-e(7bd-11ae))\,x}{8d^4(d+ex^2)} + \frac{(3cd^2-15}{}$$

$$= -\frac{a}{3d^3x^3} - \frac{bd-3ae}{d^4x} + \frac{(cd^2-bde+ae^2)\,x}{4d^3(d+ex^2)^2} + \frac{(3cd^2-e(7bd-11ae))\,x}{8d^4(d+ex^2)} + \frac{(3cd^2-15}{}$$

Mathematica [A]
time = 0.06, size = 141, normalized size = 0.99

$$-\frac{a}{3d^3x^3} + \frac{-bd+3ae}{d^4x} + \frac{(cd^2-bde+ae^2)\,x}{4d^3(d+ex^2)^2} + \frac{(3cd^2-7bde+11ae^2)\,x}{8d^4(d+ex^2)} + \frac{(3cd^2-15bde+35ae^2)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{9/2}\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x]
```

```
[Out] -1/3*a/(d^3*x^3) + (-(b*d) + 3*a*e)/(d^4*x) + ((c*d^2 - b*d*e + a*e^2)*x)/(
4*d^3*(d + e*x^2)^2) + ((3*c*d^2 - 7*b*d*e + 11*a*e^2)*x)/(8*d^4*(d + e*x^2
)) + ((3*c*d^2 - 15*b*d*e + 35*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(9/
2)*Sqrt[e])
```

Maple [A]
time = 0.14, size = 124, normalized size = 0.87

method	result

default	$\frac{\frac{\left(\frac{11}{8}a\,e^{3}-\frac{7}{8}d\,e^{2}b+\frac{3}{8}c\,d^{2}e\right)x^{3}+\frac{d\left(13a\,e^{2}-9deb+5c\,d^{2}\right)x}{8}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(35a\,e^{2}-15deb+3c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}}}{d^{4}}-\frac{a}{3d^{3}x^{3}}-\frac{-3ae+bd}{d^{4}x}$
risch	$\frac{\frac{e\left(35a\,e^{2}-15deb+3c\,d^{2}\right)x^{6}}{8d^{4}}+\frac{5\left(35a\,e^{2}-15deb+3c\,d^{2}\right)x^{4}}{24d^{3}}+\frac{(7ae-3bd)x^{2}}{3d^{2}}-\frac{a}{3d}}{x^{3}\left(e\,x^{2}+d\right)^{2}}+\frac{\left(\sum_{\_R=\text{RootOf}\left(d^{9}e\_Z^{2}+1225a^{2}e^{4}-1050abd\,e^{3}+210ac\,d^{2}e^{2}+225\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x,method=_RETURNVERBOSE)

[Out] 1/d^4*(((11/8*a*e^3-7/8*d*e^2*b+3/8*c*d^2*e)*x^3+1/8*d*(13*a*e^2-9*b*d*e+5*c*d^2)*x)/(e*x^2+d)^2+1/8*(35*a*e^2-15*b*d*e+3*c*d^2)/(d*e)^(1/2)*arctan(e*x/(d*e)^(1/2)))-1/3*a/d^3/x^3-(-3*a*e+b*d)/d^4/x

Maxima [A]
time = 0.50, size = 143, normalized size = 1.01

$$\frac{3\,(3\,cd^2e-15\,bde^2+35\,ae^3)x^6+5\,(3\,cd^3-15\,bd^2e+35\,ade^2)x^4-8\,ad^3-8\,(3\,bd^3-7\,ad^2e)x^2}{24\,(d^4x^7e^2+2\,d^5x^5e+d^6x^3)}+\frac{(3\,cd^2-15\,bde+35\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{9}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="maxima")

[Out] 1/24*(3*(3*c*d^2*e - 15*b*d*e^2 + 35*a*e^3)*x^6 + 5*(3*c*d^3 - 15*b*d^2*e + 35*a*d*e^2)*x^4 - 8*a*d^3 - 8*(3*b*d^3 - 7*a*d^2*e)*x^2)/(d^4*x^7*e^2 + 2*d^5*x^5*e + d^6*x^3) + 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(9/2)

Fricas [A]
time = 0.37, size = 491, normalized size = 3.46

$$\left[\frac{210\,adx^6e^4-3\,(35\,ax^7e^4+3\,cd^4x^3-5\,(3\,bdx^7-14\,adx^5)e^3+(3\,cd^2x^7-30\,bd^2x^5+35\,ad^2x^3)e^2+3\,(2\,cd^3x^5-5\,bd^3x^3)e)\sqrt{-de}\,\log\left(\frac{x^2e-2\sqrt{-de}\,x-d}{x^2e+d}\right)-10\,(9\,bd^2x^6-35\,ad^2x^4)e^3+2\,(9\,cd^3x^6-75\,bd^3x^4+56\,ad^3x^2)e^2+2\,(15\,cd^4x^4-24\,bd^4x^2-8\,ad^4)e}{48\,(d^5x^7e^3+2\,d^6x^5e^2+d^7x^3e)},\ \frac{105\,adx^6e^4+3\,(35\,ax^7e^4+3\,cd^4x^3-5\,(3\,bdx^7-14\,adx^5)e^3+(3\,cd^2x^7-30\,bd^2x^5+35\,ad^2x^3)e^2+3\,(2\,cd^3x^5-5\,bd^3x^3)e)\sqrt{d}\,\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}-5\,(9\,bd^2x^6-35\,ad^2x^4)e^3+(9\,cd^3x^6-75\,bd^3x^4+56\,ad^3x^2)e^2+(15\,cd^4x^4-24\,bd^4x^2-8\,ad^4)e}{24\,(d^5x^7e^3+2\,d^6x^5e^2+d^7x^3e)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="fricas")

[Out] [1/48*(210*a*d*x^6*e^4 - 3*(35*a*x^7*e^4 + 3*c*d^4*x^3 - 5*(3*b*d*x^7 - 14*a*d*x^5)*e^3 + (3*c*d^2*x^7 - 30*b*d^2*x^5 + 35*a*d^2*x^3)*e^2 + 3*(2*c*d^3*x^5 - 5*b*d^3*x^3)*e)*sqrt(-d*e)*log((x^2*e - 2*sqrt(-d*e)*x - d)/(x^2*e + d)) - 10*(9*b*d^2*x^6 - 35*a*d^2*x^4)*e^3 + 2*(9*c*d^3*x^6 - 75*b*d^3*x^4 + 56*a*d^3*x^2)*e^2 + 2*(15*c*d^4*x^4 - 24*b*d^4*x^2 - 8*a*d^4)*e)/(d^5*x^7*e^3 + 2*d^6*x^5*e^2 + d^7*x^3*e), 1/24*(105*a*d*x^6*e^4 + 3*(35*a*x^7*e^4 + 3*c*d^4*x^3 - 5*(3*b*d*x^7 - 14*a*d*x^5)*e^3 + (3*c*d^2*x^7 - 30*b*d^2*x^

```
5 + 35*a*d^2*x^3)*e^2 + 3*(2*c*d^3*x^5 - 5*b*d^3*x^3)*e)*sqrt(d)*arctan(x*e
^(1/2)/sqrt(d))*e^(1/2) - 5*(9*b*d^2*x^6 - 35*a*d^2*x^4)*e^3 + (9*c*d^3*x^6
 - 75*b*d^3*x^4 + 56*a*d^3*x^2)*e^2 + (15*c*d^4*x^4 - 24*b*d^4*x^2 - 8*a*d^
4)*e)/(d^5*x^7*e^3 + 2*d^6*x^5*e^2 + d^7*x^3*e)]
```

Sympy [A]
time = 1.42, size = 214, normalized size = 1.51

$$-\frac{\sqrt{-\frac{1}{d^9e}}\,(35ae^2-15bde+3cd^2)\log\left(-d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}+\frac{\sqrt{-\frac{1}{d^9e}}\,(35ae^2-15bde+3cd^2)\log\left(d^5\sqrt{-\frac{1}{d^9e}}+x\right)}{16}+\frac{-8ad^3+x^6\cdot(105ae^3-45bde^2+9cd^2e)+x^4\cdot(175ade^2-75bd^2e+15cd^3)+x^2\cdot(56ad^2e-24bd^3)}{24d^6x^3+48d^5ex^5+24d^4e^2x^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**4/(e*x**2+d)**3,x)
```

```
[Out] -sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(-d**5*sqrt(-1/(d**
9*e)) + x)/16 + sqrt(-1/(d**9*e))*(35*a*e**2 - 15*b*d*e + 3*c*d**2)*log(d**
5*sqrt(-1/(d**9*e)) + x)/16 + (-8*a*d**3 + x**6*(105*a*e**3 - 45*b*d*e**2 +
 9*c*d**2*e) + x**4*(175*a*d*e**2 - 75*b*d**2*e + 15*c*d**3) + x**2*(56*a*d
**2*e - 24*b*d**3))/(24*d**6*x**3 + 48*d**5*e*x**5 + 24*d**4*e**2*x**7)
```

Giac [A]
time = 4.46, size = 128, normalized size = 0.90

$$\frac{(3\,cd^2-15\,bde+35\,ae^2)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{9}{2}}}+\frac{3\,cd^2x^3e-7\,bdx^3e^2+5\,cd^3x+11\,ax^3e^3-9\,bd^2xe+13\,adxe^2}{8\,(x^2e+d)^2d^4}-\frac{3\,bdx^2-9\,ax^2e+ad}{3\,d^4x^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^4/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*c*d^2 - 15*b*d*e + 35*a*e^2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/d^(9
/2) + 1/8*(3*c*d^2*x^3*e - 7*b*d*x^3*e^2 + 5*c*d^3*x + 11*a*x^3*e^3 - 9*b*d
^2*x*e + 13*a*d*x*e^2)/((x^2*e + d)^2*d^4) - 1/3*(3*b*d*x^2 - 9*a*x^2*e + a
*d)/(d^4*x^3)
```

Mupad [B]
time = 0.40, size = 138, normalized size = 0.97

$$\frac{\frac{x^2\,(7\,a\,e-3\,b\,d)}{3\,d^2}-\frac{a}{3\,d}+\frac{5\,x^4\,(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2)}{24\,d^3}+\frac{e\,x^6\,(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2)}{8\,d^4}}{d^2\,x^3+2\,d\,e\,x^5+e^2\,x^7}+\frac{\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,(3\,c\,d^2-15\,b\,d\,e+35\,a\,e^2)}{8\,d^{9/2}\,\sqrt{e}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^4*(d + e*x^2)^3),x)
```

```
[Out] ((x^2*(7*a*e - 3*b*d))/(3*d^2) - a/(3*d) + (5*x^4*(35*a*e^2 + 3*c*d^2 - 15*
b*d*e))/(24*d^3) + (e*x^6*(35*a*e^2 + 3*c*d^2 - 15*b*d*e))/(8*d^4))/(d^2*x^
3 + e^2*x^7 + 2*d*e*x^5) + (atan((e^(1/2)*x)/d^(1/2))*(35*a*e^2 + 3*c*d^2 -
 15*b*d*e))/(8*d^(9/2)*e^(1/2))
```

3.294 $\int \frac{a+bx^2+cx^4}{x^6(d+ex^2)^3}\,dx$

Optimal. Leaf size=171

$$-\frac{a}{5d^3x^5}-\frac{bd-3ae}{3d^4x^3}-\frac{cd^2-3bde+6ae^2}{d^5x}-\frac{e(cd^2-bde+ae^2)\,x}{4d^4\,(d+ex^2)^2}-\frac{e(7cd^2-e(11bd-15ae))\,x}{8d^5\,(d+ex^2)}-\frac{\sqrt{e}\,(15cd^2-3\ldots$$

```
[Out] -1/5*a/d^3/x^5+1/3*(3*a*e-b*d)/d^4/x^3+(-6*a*e^2+3*b*d*e-c*d^2)/d^5/x-1/4*e
*(a*e^2-b*d*e+c*d^2)*x/d^4/(e*x^2+d)^2-1/8*e*(7*c*d^2-e*(-15*a*e+11*b*d))*x
/d^5/(e*x^2+d)-1/8*(63*a*e^2-35*b*d*e+15*c*d^2)*arctan(x*e^(1/2)/d^(1/2))*e
^(1/2)/d^(11/2)
```

Rubi [A]
time = 0.24, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {1273, 1819, 1816, 211}

$$-\frac{\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)(63ae^2-35bde+15cd^2)}{8d^{11/2}}-\frac{6ae^2-3bde+cd^2}{d^5x}-\frac{ex(7cd^2-e(11bd-15ae))}{8d^5\,(d+ex^2)}-\frac{ex(ae^2-bde+cd^2)}{4d^4\,(d+ex^2)^2}-\frac{bd-3ae}{3d^4x^3}-\frac{a}{5d^3x^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -1/5*a/(d^3*x^5) - (b*d - 3*a*e)/(3*d^4*x^3) - (c*d^2 - 3*b*d*e + 6*a*e^2)/
(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - (e*(7*c*d^2
 - e*(11*b*d - 15*a*e))*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2 - 35*b*
d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1273

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e*
x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] &
& ILtQ[m/2, 0]
```

Rule 1816

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 1819

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[(a*g - b*f*x)*((a + b*x^2)^(p + 1)/(2*a*
b*(p + 1))), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{a+bx^2+cx^4}{x^6\left(d+ex^2\right)^3}\,dx = -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{\int \frac{-4ad^3e^2-4d^2e^2(bd-ae)x^2-4de^2\left(cd^2-bde+ae^2\right)x^4+3e^3\left(cd^2-bde+ae^2\right)x^6}{x^6\left(d+ex^2\right)^2}\,d}{4d^4e^2}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} + \frac{\int \frac{8ad^3e^2+8d^2e^2(bd-2ae)x^2+8de^2(c}{}}{}$$

$$= -\frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd-15ae)\right)x}{8d^5\left(d+ex^2\right)} + \frac{\int \left(\frac{8ad^2e^2}{x^6} + \frac{8de^2(bd-3ae)}{x^4} + \frac{8e^2}{}\right.}{}$$

$$= -\frac{a}{5d^3x^5} - \frac{bd-3ae}{3d^4x^3} - \frac{cd^2-3bde+6ae^2}{d^5x} - \frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd\right.}{8d^5\left(d+\right.}$$

$$= -\frac{a}{5d^3x^5} - \frac{bd-3ae}{3d^4x^3} - \frac{cd^2-3bde+6ae^2}{d^5x} - \frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{e\left(7cd^2-e(11bd\right.}{8d^5\left(d+\right.}$$

Mathematica [A]
time = 0.08, size = 173, normalized size = 1.01

$$-\frac{a}{5d^3x^5} + \frac{-bd+3ae}{3d^4x^3} + \frac{-cd^2+3bde-6ae^2}{d^5x} - \frac{e\left(cd^2-bde+ae^2\right)x}{4d^4\left(d+ex^2\right)^2} - \frac{\left(7cd^2e-11bde^2+15ae^3\right)x}{8d^5\left(d+ex^2\right)} - \frac{\sqrt{e}\left(15cd^2-35bde+63ae^2\right)\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{8d^{11/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x]
```

```
[Out] -1/5*a/(d^3*x^5) + (-(b*d) + 3*a*e)/(3*d^4*x^3) + (-(c*d^2) + 3*b*d*e - 6*a
*e^2)/(d^5*x) - (e*(c*d^2 - b*d*e + a*e^2)*x)/(4*d^4*(d + e*x^2)^2) - ((7*c
*d^2*e - 11*b*d*e^2 + 15*a*e^3)*x)/(8*d^5*(d + e*x^2)) - (Sqrt[e]*(15*c*d^2
 - 35*b*d*e + 63*a*e^2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(8*d^(11/2))
```

Maple [A]
time = 0.14, size = 151, normalized size = 0.88

method	result
default	$-\frac{e\left(\frac{\left(\frac{15}{8}a\,e^{3}-\frac{11}{8}d\,e^{2}b+\frac{7}{8}c\,d^{2}e\right)x^{3}+\frac{d\left(17a\,e^{2}-13deb+9c\,d^{2}\right)x}{8}}{\left(e\,x^{2}+d\right)^{2}}+\frac{\left(63a\,e^{2}-35deb+15c\,d^{2}\right)\arctan\left(\frac{ex}{\sqrt{de}}\right)}{8\sqrt{de}}\right)}{d^{5}}-\frac{a}{5x^{5}d^{3}}-\frac{-3ae+bd}{3d^{4}x^{3}}-\frac{6a}{}$
risch	$\frac{-\frac{e^{2}\left(63a\,e^{2}-35deb+15c\,d^{2}\right)x^{8}}{8d^{5}}-\frac{5e\left(63a\,e^{2}-35deb+15c\,d^{2}\right)x^{6}}{24d^{4}}-\frac{\left(63a\,e^{2}-35deb+15c\,d^{2}\right)x^{4}}{15d^{3}}+\frac{(9ae-5bd)x^{2}}{15d^{2}}-\frac{a}{5d}}{x^{5}\left(e\,x^{2}+d\right)^{2}}+\left(\sum_{\_R=\mathrm{RootOf}\left(d^{11}\_Z^{2}+396\right.}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x,method=_RETURNVERBOSE)
```

```
[Out] -e/d^5*(((15/8*a*e^3-11/8*d*e^2*b+7/8*c*d^2*e)*x^3+1/8*d*(17*a*e^2-13*b*d*e
+9*c*d^2)*x)/(e*x^2+d)^2+1/8*(63*a*e^2-35*b*d*e+15*c*d^2)/(d*e)^(1/2)*arcta
n(e*x/(d*e)^(1/2)))-1/5*a/x^5/d^3-1/3*(-3*a*e+b*d)/d^4/x^3-(6*a*e^2-3*b*d*e
+c*d^2)/d^5/x
```

Maxima [A]
time = 0.51, size = 175, normalized size = 1.02

$$-\frac{15\,(15\,cd^{2}e^{2}-35\,bde^{3}+63\,ae^{4})x^{8}+25\,(15\,cd^{3}e-35\,bd^{2}e^{2}+63\,ade^{3})x^{6}+24\,ad^{4}+8\,(15\,cd^{4}-35\,bd^{3}e+63\,ad^{2}e^{2})x^{4}+8\,(5\,bd^{4}-9\,ad^{3}e)x^{2}}{120\,(d^{5}x^{9}e^{2}+2\,d^{6}x^{7}e+d^{7}x^{5})}-\frac{(15\,cd^{2}e-35\,bde^{2}+63\,ae^{3})\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{11}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="maxima")
```

```
[Out] -1/120*(15*(15*c*d^2*e^2 - 35*b*d*e^3 + 63*a*e^4)*x^8 + 25*(15*c*d^3*e - 35
*b*d^2*e^2 + 63*a*d*e^3)*x^6 + 24*a*d^4 + 8*(15*c*d^4 - 35*b*d^3*e + 63*a*d
^2*e^2)*x^4 + 8*(5*b*d^4 - 9*a*d^3*e)*x^2)/(d^5*x^9*e^2 + 2*d^6*x^7*e + d^7
*x^5) - 1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*
e^(-1/2)/d^(11/2)
```

Fricas [A]
time = 0.36, size = 534, normalized size = 3.12

$$\left[-\frac{1890\,ax^{8}e^{4}+240\,cd^{4}x^{4}+80\,bd^{4}x^{2}+48\,ad^{4}-15\,(63\,ax^{9}e^{4}+15\,cd^{4}x^{5}-7\,(5\,bdx^{9}-18\,adx^{7})e^{3}+(15\,cd^{2}x^{9}-70\,bd^{2}x^{7}+63\,ad^{2}x^{5})e^{2}+5\,(6\,cd^{3}x^{7}-7\,bd^{3}x^{5})e)\sqrt{-\frac{e}{d}}\log\left(\frac{x^{2}e-2\,dx\sqrt{-\frac{e}{d}}-d}{x^{2}e+d}\right)-1050\,(bdx^{8}-3\,adx^{6})e^{3}+2\,(225\,cd^{2}x^{8}-875\,bd^{2}x^{6}+504\,ad^{2}x^{4})e^{2}+2\,(375\,cd^{3}x^{6}-280\,bd^{3}x^{4}-72\,ad^{3}x^{2})e}{240\,(d^{5}x^{9}e^{2}+2\,d^{6}x^{7}e+d^{7}x^{5})},\ -\frac{945\,ax^{8}e^{4}+120\,cd^{4}x^{4}+40\,bd^{4}x^{2}+24\,ad^{4}+\frac{15\,(63\,ax^{9}e^{4}+15\,cd^{4}x^{5}-7\,(5\,bdx^{9}-18\,adx^{7})e^{3}+(15\,cd^{2}x^{9}-70\,bd^{2}x^{7}+63\,ad^{2}x^{5})e^{2}+5\,(6\,cd^{3}x^{7}-7\,bd^{3}x^{5})e)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\frac{1}{2}}}{\sqrt{d}}-525\,(bdx^{8}-3\,adx^{6})e^{3}+(225\,cd^{2}x^{8}-875\,bd^{2}x^{6}+504\,ad^{2}x^{4})e^{2}+(375\,cd^{3}x^{6}-280\,bd^{3}x^{4}-72\,ad^{3}x^{2})e}{120\,(d^{5}x^{9}e^{2}+2\,d^{6}x^{7}e+d^{7}x^{5})}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="fricas")
```

```
[Out] [-1/240*(1890*a*x^8*e^4 + 240*c*d^4*x^4 + 80*b*d^4*x^2 + 48*a*d^4 - 15*(63*
a*x^9*e^4 + 15*c*d^4*x^5 - 7*(5*b*d*x^9 - 18*a*d*x^7)*e^3 + (15*c*d^2*x^9 -
 70*b*d^2*x^7 + 63*a*d^2*x^5)*e^2 + 5*(6*c*d^3*x^7 - 7*b*d^3*x^5)*e)*sqrt(-
e/d)*log((x^2*e - 2*d*x*sqrt(-e/d) - d)/(x^2*e + d)) - 1050*(b*d*x^8 - 3*a*
d*x^6)*e^3 + 2*(225*c*d^2*x^8 - 875*b*d^2*x^6 + 504*a*d^2*x^4)*e^2 + 2*(375
*c*d^3*x^6 - 280*b*d^3*x^4 - 72*a*d^3*x^2)*e)/(d^5*x^9*e^2 + 2*d^6*x^7*e +
d^7*x^5), -1/120*(945*a*x^8*e^4 + 120*c*d^4*x^4 + 40*b*d^4*x^2 + 24*a*d^4 +
 15*(63*a*x^9*e^4 + 15*c*d^4*x^5 - 7*(5*b*d*x^9 - 18*a*d*x^7)*e^3 + (15*c*d
^2*x^9 - 70*b*d^2*x^7 + 63*a*d^2*x^5)*e^2 + 5*(6*c*d^3*x^7 - 7*b*d^3*x^5)*e
)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/sqrt(d) - 525*(b*d*x^8 - 3*a*d*x^6)*e^3
 + (225*c*d^2*x^8 - 875*b*d^2*x^6 + 504*a*d^2*x^4)*e^2 + (375*c*d^3*x^6 - 2
80*b*d^3*x^4 - 72*a*d^3*x^2)*e)/(d^5*x^9*e^2 + 2*d^6*x^7*e + d^7*x^5)]
```

Sympy [**B**] Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(163) = 326$.
time = 1.95, size = 330, normalized size = 1.93

$$\frac{\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)\log\left(-\frac{d^6\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}-\frac{\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)\log\left(\frac{d^6\sqrt{-\frac{e}{d^{11}}}\cdot(63ae^2-35bde+15cd^2)}{63ae^3-35bde^2+15cd^2e}+x\right)}{16}+\frac{-24ad^4+x^8(-945ae^4+525bde^3-225cd^2e^2)+x^6(-1575ade^3+875bd^2e^2-375cd^3e)+x^4(-504ad^2e^2+280bd^3e-120cd^4)+x^2\cdot(72ad^3e-40bd^4)}{120d^7x^5+240d^6ex^7+120d^5e^2x^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)/x**6/(e*x**2+d)**3,x)
```

```
[Out] sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(-d**6*sqrt(-e/d**11)*
(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c*d**2*e)
+ x)/16 - sqrt(-e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)*log(d**6*sqrt(-
e/d**11)*(63*a*e**2 - 35*b*d*e + 15*c*d**2)/(63*a*e**3 - 35*b*d*e**2 + 15*c
*d**2*e) + x)/16 + (-24*a*d**4 + x**8*(-945*a*e**4 + 525*b*d*e**3 - 225*c*d
**2*e**2) + x**6*(-1575*a*d*e**3 + 875*b*d**2*e**2 - 375*c*d**3*e) + x**4*(
-504*a*d**2*e**2 + 280*b*d**3*e - 120*c*d**4) + x**2*(72*a*d**3*e - 40*b*d*
*4))/(120*d**7*x**5 + 240*d**6*e*x**7 + 120*d**5*e**2*x**9)
```

Giac [**A**]
time = 4.01, size = 164, normalized size = 0.96

$$-\frac{(15\,cd^2e-35\,bde^2+63\,ae^3)\arctan\left(\frac{xe^{\frac{1}{2}}}{\sqrt{d}}\right)e^{\left(-\frac{1}{2}\right)}}{8\,d^{\frac{11}{2}}}-\frac{7\,cd^2x^3e^2-11\,bdx^3e^3+9\,cd^3xe+15\,ax^3e^4-13\,bd^2xe^2+17\,adxe^3}{8\,(x^2e+d)^2d^5}-\frac{15\,cd^2x^4-45\,bdx^4e+90\,ax^4e^2+5\,bd^2x^2-15\,adx^2e+3\,ad^2}{15\,d^5x^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)/x^6/(e*x^2+d)^3,x, algorithm="giac")
```

```
[Out] -1/8*(15*c*d^2*e - 35*b*d*e^2 + 63*a*e^3)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2
)/d^(11/2) - 1/8*(7*c*d^2*x^3*e^2 - 11*b*d*x^3*e^3 + 9*c*d^3*x*e + 15*a*x^3
*e^4 - 13*b*d^2*x*e^2 + 17*a*d*x*e^3)/((x^2*e + d)^2*d^5) - 1/15*(15*c*d^2*
x^4 - 45*b*d*x^4*e + 90*a*x^4*e^2 + 5*b*d^2*x^2 - 15*a*d*x^2*e + 3*a*d^2)/(
d^5*x^5)
```

Mupad [B]
time = 0.41, size = 168, normalized size = 0.98

$$-\frac{\frac{a}{5\,d}-\frac{x^2\,\left(9\,a\,e-5\,b\,d\right)}{15\,d^2}+\frac{x^4\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{15\,d^3}+\frac{5\,e\,x^6\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{24\,d^4}+\frac{e^2\,x^8\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{8\,d^5}}{d^2\,x^5+2\,d\,e\,x^7+e^2\,x^9}-\frac{\sqrt{e}\,\mathrm{atan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)\,\left(15\,c\,d^2-35\,b\,d\,e+63\,a\,e^2\right)}{8\,d^{11/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)/(x^6*(d + e*x^2)^3),x)
```

```
[Out] - (a/(5*d) - (x^2*(9*a*e - 5*b*d))/(15*d^2) + (x^4*(63*a*e^2 + 15*c*d^2 - 3
5*b*d*e))/(15*d^3) + (5*e*x^6*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(24*d^4) +
(e^2*x^8*(63*a*e^2 + 15*c*d^2 - 35*b*d*e))/(8*d^5))/(d^2*x^5 + e^2*x^9 + 2*
d*e*x^7) - (e^(1/2)*atan((e^(1/2)*x)/d^(1/2))*(63*a*e^2 + 15*c*d^2 - 35*b*d
*e))/(8*d^(11/2))
```

3.295 $\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=230

$$-\frac{(cd+be)x^2}{2c^2e^2}+\frac{x^4}{4ce}-\frac{(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}+\frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)}$$

```
[Out] -1/2*(b*e+c*d)*x^2/c^2/e^2+1/4*x^4/c/e+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e
+c*d^2)-1/4*(a^2*c*e-a*b^2*e-2*a*b*c*d+b^3*d)*ln(c*x^4+b*x^2+a)/c^3/(a*e^2-
b*d*e+c*d^2)-1/2*(3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-4*a*b^2*c*d+b^4*d)*arctan
h((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^3/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2
)
```

Rubi [A]
time = 0.33, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1265, 1642, 648, 632, 212, 642}

$$-\frac{(a^2ce-ab^2e-2abcd+b^3d)\log(a+bx^2+cx^4)}{4c^3(ae^2-bde+cd^2)}-\frac{(3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^3\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^4\log(d+ex^2)}{2e^3(ae^2-bde+cd^2)}-\frac{x^2(be+cd)}{2c^2e^2}+\frac{x^4}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((c*d + b*e)*x^2)/(c^2*e^2) + x^4/(4*c*e) - ((b^4*d - 4*a*b^2*c*d + 2*
a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(2*c^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^4*Log[d + e*x^2])/
(2*e^3*(c*d^2 - b*d*e + a*e^2)) - ((b^3*d - 2*a*b*c*d - a*b^2*e + a^2*c*e)*
Log[a + b*x^2 + c*x^4])/(4*c^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^9}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^4}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{-cd-be}{c^2e^2} + \frac{x}{ce} + \frac{d^4}{e^2(cd^2-bde+ae^2)(d+ex)} + \frac{-a(b^2d-\cdots}{\cdots}\right.\right. \\
&= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{-a(b^2d-acd-abe)-(b\cdots}{a+b\cdots}\right.}{2c^2(cd^2-\cdots} \\
&= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} - \frac{(b^3d-2abcd-ab^2e+a^2c\cdots}{4c^3(cd^2-\cdots} \\
&= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} + \frac{d^4\log(d+ex^2)}{2e^3(cd^2-bde+ae^2)} - \frac{(b^3d-2abcd-ab^2e+a^2c\cdots}{4c^3(cd^2-bd\cdots} \\
&= -\frac{(cd+be)x^2}{2c^2e^2} + \frac{x^4}{4ce} - \frac{(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\tanh^{-1}\left(\cdots\right.}{2c^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}
\end{aligned}$$

Mathematica [A]

time = 0.16, size = 228, normalized size = 0.99

$$\frac{1}{4}\left(-\frac{2(cd+be)x^2}{c^2e^2}+\frac{x^4}{ce}-\frac{2(b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)}{c^3\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}+\frac{2d^4\log(d+ex^2)}{e^3(cd^2+e(-bd+ae))}+\frac{(-b^3d+2abcd+ab^2e-a^2ce)\log(a+bx^2+cx^4)}{c^3(cd^2+e(-bd+ae))}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-2*(c*d + b*e)*x^2)/(c^2*e^2) + x^4/(c*e) - (2*(b^4*d - 4*a*b^2*c*d + 2*a
^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]])
/(c^3*Sqrt[-b^2 + 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) + (2*d^4*Log[d + e*x^2
])/(e^3*(c*d^2 + e*(-(b*d) + a*e))) + ((-(b^3*d) + 2*a*b*c*d + a*b^2*e - a^
2*c*e)*Log[a + b*x^2 + c*x^4])/(c^3*(c*d^2 + e*(-(b*d) + a*e))))/4
```

Maple [A]
time = 0.26, size = 216, normalized size = 0.94

method	result
default	$\frac{(-ce\,x^2+eb+cd)^2}{4c^3e^3}+\frac{\frac{(-a^2ce+a\,b^2e+2abcd-b^3d)\ln(c\,x^4+b\,x^2+a)}{2c}+\frac{2\left(a^2be+a^2cd-a\,b^2d-\frac{(-a^2ce+a\,b^2e+2abcd-b^3d)b}{2c}\right)\arctan\left(\frac{2c\,x}{\sqrt{4ac}}\right.}{\sqrt{4ac-b^2}}}{2(a\,e^2-deb+c\,d^2)c^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4*(-c*e*x^2+b*e+c*d)^2/c^3/e^3+1/2/(a*e^2-b*d*e+c*d^2)/c^2*(1/2*(-a^2*c*e
+a*b^2*e+2*a*b*c*d-b^3*d)/c*ln(c*x^4+b*x^2+a)+2*(a^2*b*e+a^2*c*d-a*b^2*d-1/
2*(-a^2*c*e+a*b^2*e+2*a*b*c*d-b^3*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2
+b)/(4*a*c-b^2)^(1/2)))+1/2*d^4*ln(e*x^2+d)/e^3/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**9/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.17, size = 236, normalized size = 1.03

$$\frac{d^4 \log\left(|x^2 e + d|\right)}{2\,(cd^2e^3 - bde^4 + ae^5)} - \frac{(b^3d - 2\,abcd - ab^2e + a^2ce)\log\left(cx^4 + bx^2 + a\right)}{4\,(c^4d^2 - bc^3de + ac^3e^2)} + \frac{(b^4d - 4\,ab^2cd + 2\,a^2c^2d - ab^3e + 3\,a^2bce)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(c^4d^2 - bc^3de + ac^3e^2)\sqrt{-b^2+4\,ac}} + \frac{(cx^4e - 2\,cdx^2 - 2\,bx^2e)e^{(-2)}}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^9/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^4*log(abs(x^2*e + d))/(c*d^2*e^3 - b*d*e^4 + a*e^5) - 1/4*(b^3*d - 2*
a*b*c*d - a*b^2*e + a^2*c*e)*log(c*x^4 + b*x^2 + a)/(c^4*d^2 - b*c^3*d*e +
a*c^3*e^2) + 1/2*(b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e
)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^4*d^2 - b*c^3*d*e + a*c^3*e^
2)*sqrt(-b^2 + 4*a*c)) + 1/4*(c*x^4*e - 2*c*d*x^2 - 2*b*x^2*e)*e^(-2)/c^2
```

Mupad [B]
time = 69.94, size = 2500, normalized size = 10.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^9/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^4*log(d + e*x^2))/(2*a*e^5 + 2*c*d^2*e^3 - 2*b*d*e^4) + (log((x^2*(a^7*e
^7 + b^7*d^7 - 2*a^3*b*c^3*d^7 - a^4*c^3*d^6*e - 2*a^6*c*d^2*e^5 + 7*a^2*b^
3*c^2*d^7 + 3*a^2*b^5*d^5*e^2 + 4*a^3*b^4*d^4*e^3 + 4*a^4*b^3*d^3*e^4 + 3*a
```

^5*b^2*d^2*e^5 + 2*a^5*c^2*d^4*e^3 - 5*a*b^5*c*d^7 + 2*a*b^6*d^6*e + 2*a^6*
b*d*e^6 - 8*a^2*b^4*c*d^6*e - 6*a^5*b*c*d^3*e^4 + 8*a^3*b^2*c^2*d^6*e - 9*a
^3*b^3*c*d^5*e^2 + 5*a^4*b*c^2*d^5*e^2 - 9*a^4*b^2*c*d^4*e^3))/(c^4*e^4) +
(a*d*(a^3*e^3 + b^3*d^3 - 2*a*b*c*d^3 + a*b^2*d^2*e + a^2*b*d*e^2 - a^2*c*d
^2*e)^2)/(c^4*e^4) + (((x^2*(4*a^2*c^6*d^8 + 6*a^4*b^4*e^8 + 18*a^6*c^2*e^8
 + 6*b^4*c^4*d^8 + 6*b^8*d^4*e^4 - 16*a*b^2*c^5*d^8 - 26*a^5*b^2*c*e^8 + 8*
a*b^7*d^3*e^5 + 8*a^3*b^5*d*e^7 - 2*b^5*c^3*d^7*e - 2*b^7*c*d^5*e^3 + 8*a^2
*b^6*d^2*e^6 - 20*a^3*c^5*d^6*e^2 + 40*a^4*c^4*d^4*e^4 - 36*a^5*c^3*d^2*e^6
 + 2*b^6*c^2*d^6*e^2 + 42*a^2*b^2*c^4*d^6*e^2 - 28*a^2*b^3*c^3*d^5*e^3 + 80
*a^2*b^4*c^2*d^4*e^4 - 64*a^3*b^2*c^3*d^4*e^4 + 80*a^3*b^3*c^2*d^3*e^5 + 48
*a^4*b^2*c^2*d^2*e^6 + 18*a*b^3*c^4*d^7*e - 40*a*b^6*c*d^4*e^4 - 26*a^2*b*c
^5*d^7*e - 32*a^4*b^3*c*d*e^7 + 12*a^5*b*c^2*d*e^7 - 16*a*b^4*c^3*d^6*e^2 +
 10*a*b^5*c^2*d^5*e^3 - 48*a^2*b^5*c*d^3*e^5 + 46*a^3*b*c^4*d^5*e^3 - 40*a^
3*b^4*c*d^2*e^6 - 48*a^4*b*c^3*d^3*e^5))/(c^4*e^4) + (((x^2*(8*a*b^8*e^9 +
8*b*c^8*d^9 + 8*b^9*d*e^8 + 120*a^5*c^4*e^9 - 72*a^2*b^6*c*e^9 - 8*b^2*c^7*
d^8*e - 8*b^8*c*d^2*e^7 + 212*a^3*b^4*c^2*e^9 - 240*a^4*b^2*c^3*e^9 - 112*a
^2*c^7*d^6*e^3 + 240*a^3*c^6*d^4*e^5 - 228*a^4*c^5*d^2*e^7 + 4*b^3*c^6*d^7*
e^2 - 24*b^4*c^5*d^6*e^3 + 32*b^5*c^4*d^5*e^4 - 24*b^6*c^3*d^4*e^5 + 4*b^7*
c^2*d^3*e^6 + 32*a*c^8*d^8*e - 56*a*b^7*c*d*e^8 - 428*a^2*b^2*c^5*d^4*e^5 +
 108*a^2*b^3*c^4*d^3*e^6 - 216*a^2*b^4*c^3*d^2*e^7 + 424*a^3*b^2*c^4*d^2*e^
7 - 16*a*b*c^7*d^7*e^2 + 8*a^4*b*c^4*d*e^8 + 88*a*b^2*c^6*d^6*e^3 - 116*a*b
^3*c^5*d^5*e^4 + 188*a*b^4*c^4*d^4*e^5 - 36*a*b^5*c^3*d^3*e^6 + 60*a*b^6*c^
2*d^2*e^7 + 40*a^2*b*c^6*d^5*e^4 + 100*a^2*b^5*c^2*d*e^8 - 72*a^3*b*c^5*d^3
*e^6 - 4*a^3*b^3*c^3*d*e^8))/(c^4*e^4) - (((x^2*(32*a*b^6*c^3*e^10 - 352*a^
4*c^6*e^10 + 128*a*c^9*d^6*e^4 + 32*b*c^9*d^7*e^3 + 32*b^7*c^3*d*e^9 - 256*
a^2*b^4*c^4*e^10 + 600*a^3*b^2*c^5*e^10 - 464*a^2*c^8*d^4*e^6 + 592*a^3*c^7
*d^2*e^8 - 64*b^2*c^8*d^6*e^4 + 56*b^3*c^7*d^5*e^5 - 48*b^4*c^6*d^4*e^6 + 5
6*b^5*c^5*d^3*e^7 - 64*b^6*c^4*d^2*e^8 - 688*a^2*b^2*c^6*d^2*e^8 - 192*a*b*
c^8*d^5*e^5 - 224*a*b^5*c^4*d*e^9 - 72*a^3*b*c^6*d*e^9 + 272*a*b^2*c^7*d^4*
e^6 - 200*a*b^3*c^6*d^3*e^7 + 360*a*b^4*c^5*d^2*e^8 + 136*a^2*b*c^7*d^3*e^7
 + 424*a^2*b^3*c^5*d*e^9))/(c^4*e^4) + (32*a*d*(2*b^6*e^6 + 2*c^6*d^6 - 15*
a^3*c^3*e^6 - 10*a*c^5*d^4*e^2 + 29*a^2*b^2*c^2*e^6 + 17*a^2*c^4*d^2*e^4 +
3*b^2*c^4*d^4*e^2 - b^3*c^3*d^3*e^3 + 3*b^4*c^2*d^2*e^4 - 14*a*b^4*c*e^6 -
2*b*c^5*d^5*e - 2*b^5*c*d*e^5 + 2*a*b*c^4*d^3*e^3 + 6*a*b^3*c^2*d*e^5 + a^2
*b*c^3*d*e^5 - 13*a*b^2*c^3*d^2*e^4))/(c*e) - (8*e^2*(b^2*e^2 + c^2*d^2 - 3
*a*c*e^2 - b*c*d*e)*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^
4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^2
*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2 - 4*a*c)^(1/2) +
3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2))*(2*a*c^2*d^3 + a*b^2*e^3*x^2 + b*c^2*d^3*x
^2 - 4*a^2*c*e^3*x^2 + b^3*d*e^2*x^2 + 2*a*b^2*d*e^2 - 6*a^2*c*d*e^2 + 4*a*
c^2*d^2*e*x^2 - 2*b^2*c*d^2*e*x^2 - 2*a*b*c*d^2*e - 3*a*b*c*d*e^2*x^2))/(c*
(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2)
- 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2
*b*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b
^2 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*

```
(a*e^2 + c*d^2 - b*d*e)) + (4*a*d*(4*b^8*e^8 + 4*c^8*d^8 + 37*a^4*c^4*e^8 -
 16*a*c^7*d^6*e^2 + 84*a^2*b^4*c^2*e^8 - 84*a^3*b^2*c^3*e^8 + 40*a^2*c^6*d^
4*e^4 - 56*a^3*c^5*d^2*e^6 + 4*b^2*c^6*d^6*e^2 - 4*b^3*c^5*d^5*e^3 + 13*b^4
*c^4*d^4*e^4 - 4*b^5*c^3*d^3*e^5 + 4*b^6*c^2*d^2*e^6 - 32*a*b^6*c*e^8 + 98*
a^2*b^2*c^4*d^2*e^6 - 8*a*b^5*c^2*d*e^7 - 4*a^3*b*c^4*d*e^7 - 52*a*b^2*c^5*
d^4*e^4 + 20*a*b^3*c^4*d^3*e^5 - 36*a*b^4*c^3*d^2*e^6 - 16*a^2*b*c^5*d^3*e^
5 + 28*a^2*b^3*c^3*d*e^7))/(c^4*e^4))*(b^5*d + b^4*d*(b^2 - 4*a*c)^(1/2) -
4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e*(b^2 - 4*a*c)^(1/2) + 8*a^2*b
*c^2*d + 5*a^2*b^2*c*e + 2*a^2*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*d*(b^2
 - 4*a*c)^(1/2) + 3*a^2*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*(a
*e^2 + c*d^2 - b*d*e)) + (4*a*d*(2*a^3*b^4*e^7 + 5*a^5*c^2*e^7 + 2*b^3*c^4*
d^7 + 2*b^7*d^3*e^4 - 8*a^4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 -
 2*a^2*c^5*d^6*e + 6*a^3*c^4*d^4*e^3 - 9*a^4*c^3*d^2*e^5 + b^5*c^2*d^5*e^2
- 4*a*b*c^5*d^7 - a^2*b^2*c^3*d^4*e^3 + 20*a^2*b^3*c^2*d^3*e^4 + 12*a^3*b^2
*c^2*d^2*e^5 + 2*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 8*a^3*b^3*c*d*e^6 +
 3*a^4*b*c^2*d*e^6 - 6*a*b^3*c^3*d^5*e^2 - a*b^4*c^2*d^4*e^3 + 10*a^2*b*c^4
*d^5*e^2 - 10*a^2*b^4*c*d^2*e^5 - 12*a^3*b*c^3*d^3*e^4))/(c^4*e^4))*(b^5*d
+ b^4*d*(b^2 - 4*a*c)^(1/2) - 4*a^3*c^2*e - a*b^4*e - 6*a*b^3*c*d - a*b^3*e
*(b^2 - 4*a*c)^(1/2) + 8*a^2*b*c^2*d + 5*a^2*b^...
```

3.296 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=189

$$\frac{x^2}{2ce}+\frac{(b^3d-3abcd-ab^2e+2a^2ce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}-\frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)}+\frac{(b^2d-acd-abe)\log(a}{4c^2(cd^2-bde+}$$

```
[Out] 1/2*x^2/c/e-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^2)+1/4*(-a*b*e-a*c*d+b
^2*d)*ln(c*x^4+b*x^2+a)/c^2/(a*e^2-b*d*e+c*d^2)+1/2*(2*a^2*c*e-a*b^2*e-3*a*
b*c*d+b^3*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/c^2/(a*e^2-b*d*e+c*d^2
)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1265, 1642, 648, 632, 212, 642}

$$\frac{(2a^2ce-ab^2e-3abcd+b^3d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{(-abe-acd+b^2d)\log(a+bx^2+cx^4)}{4c^2(ae^2-bde+cd^2)}-\frac{d^3\log(d+ex^2)}{2e^2(ae^2-bde+cd^2)}+\frac{x^2}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x^2/(2*c*e) + ((b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTanh[(b + 2*c*x
^2)/Sqrt[b^2 - 4*a*c]])/(2*c^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) -
 (d^3*Log[d + e*x^2])/(2*e^2*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a
*b*e)*Log[a + b*x^2 + c*x^4])/(4*c^2*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2-bde+ae^2)(d+ex)} + \frac{a(bd-ae)+(b^2d-ac\\
&= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{a(bd-ae)+(b^2d-acd-abe)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2c(cd^2-bde+ae^2)}\\
&= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\,\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x\\
&= \frac{x^2}{2ce} - \frac{d^3\log(d+ex^2)}{2e^2(cd^2-bde+ae^2)} + \frac{(b^2d-acd-abe)\log(a+bx^2+cx^4)}{4c^2(cd^2-bde+ae^2)} + \frac{(b^3\\
&= \frac{x^2}{2ce} + \frac{(b^3d-3abcd-ab^2e+2a^2ce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c^2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d^3\log(d\\
\end{aligned}$$

Mathematica [A]

time = 0.12, size = 186, normalized size = 0.98

$$\frac{2e^2(b^3d-3abcd-ab^2e+2a^2ce)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,(2c^2d^3\log(d+ex^2)+e(-2c(cd^2-bde+ae^2)x^2+e(-b^2d+acd+abe)\log(a+bx^2+cx^4)))}{4c^2\sqrt{-b^2+4ac}\,e^2(-cd^2+e(bd-ae))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*e^2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*ArcTan[(b + 2*c*x^2)/Sqrt[
-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a*c]*(2*c^2*d^3*Log[d + e*x^2] + e*(-2*c*(c*
d^2 - b*d*e + a*e^2)*x^2 + e*(-(b^2*d) + a*c*d + a*b*e)*Log[a + b*x^2 + c*x
^4])))/(4*c^2*Sqrt[-b^2 + 4*a*c]*e^2*(-(c*d^2) + e*(b*d - a*e)))
```

Maple [A]
time = 0.18, size = 172, normalized size = 0.91

method	result
default	$\frac{x^2}{2ce}-\frac{\frac{\left(abe+acd-b^2d\right)\ln\left(cx^4+bx^2+a\right)}{2c}+\frac{2\left(ea^2-abd-\frac{\left(abe+acd-b^2d\right)b}{2c}\right)\arctan\left(\frac{2cx^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(ae^2-deb+cd^2)c}-\frac{d^3\ln(ex^2+d)}{2e^2(ae^2-deb+cd^2)}$
risch	$\frac{x^2}{2ce}-\frac{d^3\ln(ex^2+d)}{2e^2(ae^2-deb+cd^2)}+\frac{\sum\_R=\text{RootOf}\left(\left(4c^2e^2a^2-ce^2ab^2-4abc^2de+4ac^3d^2+b^3cde-b^2c^2d^2\right)\_Z^2+\left(4ce^2a^2b+4a^2c^2de-e^2ab^3-5ab\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2*x^2/c/e-1/2/(a*e^2-b*d*e+c*d^2)/c*(1/2*(a*b*e+a*c*d-b^2*d)/c*ln(c*x^4+b
*x^2+a)+2*(e*a^2-a*b*d-1/2*(a*b*e+a*c*d-b^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arcta
n((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*d^3*ln(e*x^2+d)/e^2/(a*e^2-b*d*e+c*d^
2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**A**]
time = 117.40, size = 609, normalized size = 3.22

$$\left[\frac{2\,(b^2c^2-4\,ac^3)d^2x^2e-2\,(b^3c-4\,abc^2)dx^2e^2-2\,(b^2c^2-4\,ac^3)d^3\log\left(x^2e+d\right)+2\,(ab^2c-4\,a^2c^2)x^2e^3+((b^3-3\,abc)de^2-(ab^2-2\,a^2c)e^3)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)+((b^4-5\,ab^2c+4\,a^2c^2)de^2-(ab^3-4\,a^2bc)e^3)\log\left(cx^4+bx^2+a\right)}{4\,((b^2c^3-4\,ac^4)d^2e^2-(b^3c^2-4\,abc^3)de^3+(ab^2c^2-4\,a^2c^3)e^4)},\ \frac{2\,(b^2c^2-4\,ac^3)d^2x^2e-2\,(b^3c-4\,abc^2)dx^2e^2-2\,(b^2c^2-4\,ac^3)d^3\log\left(x^2e+d\right)+2\,(ab^2c-4\,a^2c^2)x^2e^3+2\,((b^3-3\,abc)de^2-(ab^2-2\,a^2c)e^3)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)+((b^4-5\,ab^2c+4\,a^2c^2)de^2-(ab^3-4\,a^2bc)e^3)\log\left(cx^4+bx^2+a\right)}{4\,((b^2c^3-4\,ac^4)d^2e^2-(b^3c^2-4\,abc^3)de^3+(ab^2c^2-4\,a^2c^3)e^4)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c^2 - 4*a*c^3)*d^2*x^2*e - 2*(b^3*c - 4*a*b*c^2)*d*x^2*e^2 - 2
*(b^2*c^2 - 4*a*c^3)*d^3*log(x^2*e + d) + 2*(a*b^2*c - 4*a^2*c^2)*x^2*e^3 +
 ((b^3 - 3*a*b*c)*d*e^2 - (a*b^2 - 2*a^2*c)*e^3)*sqrt(b^2 - 4*a*c)*log((2*c
^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4
+ b*x^2 + a)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2*b*c)*
e^3)*log(c*x^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4)*d^2*e^2 - (b^3*c^2 - 4*a*
b*c^3)*d*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*e^4), 1/4*(2*(b^2*c^2 - 4*a*c^3)*d^2
*x^2*e - 2*(b^3*c - 4*a*b*c^2)*d*x^2*e^2 - 2*(b^2*c^2 - 4*a*c^3)*d^3*log(x^
2*e + d) + 2*(a*b^2*c - 4*a^2*c^2)*x^2*e^3 + 2*((b^3 - 3*a*b*c)*d*e^2 - (a*
b^2 - 2*a^2*c)*e^3)*sqrt(-b^2 + 4*a*c)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*
a*c)/(b^2 - 4*a*c)) + ((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e^2 - (a*b^3 - 4*a^2
*b*c)*e^3)*log(c*x^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4)*d^2*e^2 - (b^3*c^2
- 4*a*b*c^3)*d*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*e^4)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.48, size = 194, normalized size = 1.03

$$-\frac{d^3\log\left(|x^2e+d|\right)}{2\,(cd^2e^2-bde^3+ae^4)}+\frac{x^2e^{(-1)}}{2\,c}+\frac{(b^2d-acd-abe)\log\left(cx^4+bx^2+a\right)}{4\,(c^3d^2-bc^2de+ac^2e^2)}-\frac{(b^3d-3\,abcd-ab^2e+2\,a^2ce)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(c^3d^2-bc^2de+ac^2e^2)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d^3*log(abs(x^2*e + d))/(c*d^2*e^2 - b*d*e^3 + a*e^4) + 1/2*x^2*e^(-1)
/c + 1/4*(b^2*d - a*c*d - a*b*e)*log(c*x^4 + b*x^2 + a)/(c^3*d^2 - b*c^2*d*
e + a*c^2*e^2) - 1/2*(b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*arctan((2*c*
x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-b^2 +
 4*a*c))
```

Mupad [B]
time = 15.21, size = 2304, normalized size = 12.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
[Out] x^2/(2*c*e) - (d^3*log(d + e*x^2))/(2*(a*e^4 + c*d^2*e^2 - b*d*e^3)) - (log
(a*e^5*x^2*(b^2 - 4*a*c)^(7/2) - 128*a^5*c^3*e^5 - 8*c^3*d^5*(b^2 - 4*a*c)^
(5/2) - 512*a^3*c^5*d^4*e + 8*b^2*c^3*d^5*(b^2 - 4*a*c)^(3/2) + 6*b^3*d^2*e
^3*(b^2 - 4*a*c)^(5/2) - 3*b^5*d^2*e^3*(b^2 - 4*a*c)^(3/2) + 32*a^4*b^2*c^2
*e^5 + 384*a^4*c^4*d^2*e^3 + 256*a^2*c^6*d^5*x^2 + 16*b^4*c^4*d^5*x^2 + 3*a
*d*e^4*(b^2 - 4*a*c)^(7/2) - 3*b*d^2*e^3*(b^2 - 4*a*c)^(7/2) - 3*c*d^3*e^2*
(b^2 - 4*a*c)^(7/2) - 16*a^2*b^3*c^3*d^3*e^2 + 48*a^2*b^4*c^2*d^2*e^3 - 288
*a^3*b^2*c^3*d^2*e^3 + 16*a^3*b^3*c^2*e^5*x^2 - 384*a^3*c^5*d^3*e^2*x^2 + 1
6*b^6*c^2*d^3*e^2*x^2 - 6*a*b^2*d*e^4*(b^2 - 4*a*c)^(5/2) + 3*a*b^4*d*e^4*(
b^2 - 4*a*c)^(3/2) + 8*b*c^2*d^4*e*(b^2 - 4*a*c)^(5/2) - 32*a*b^4*c^3*d^4*e
 + 192*a^4*b*c^3*d*e^4 - 2*b^2*c*d^3*e^2*(b^2 - 4*a*c)^(5/2) - 8*b^3*c^2*d^
4*e*(b^2 - 4*a*c)^(3/2) + 5*b^4*c*d^3*e^2*(b^2 - 4*a*c)^(3/2) - 2*a*b^2*e^5
*x^2*(b^2 - 4*a*c)^(5/2) + a*b^4*e^5*x^2*(b^2 - 4*a*c)^(3/2) + 16*b*c^4*d^5
*x^2*(b^2 - 4*a*c)^(3/2) - 3*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 16*c^3*d^4
*e*x^2*(b^2 - 4*a*c)^(5/2) + 256*a^2*b^2*c^4*d^4*e + 64*a^3*b*c^4*d^3*e^2 -
 48*a^3*b^3*c^2*d*e^4 - 128*a*b^2*c^5*d^5*x^2 - 64*a^4*b*c^3*e^5*x^2 + 384*
a^4*c^4*d*e^4*x^2 - 32*b^5*c^3*d^4*e*x^2 + 480*a^2*b^2*c^4*d^3*e^2*x^2 + 48
*a^2*b^3*c^3*d^2*e^3*x^2 + 256*a*b^3*c^4*d^4*e*x^2 - 512*a^2*b*c^5*d^4*e*x^
2 + 8*b*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(5/2) + 6*b^2*c*d^2*e^3*x^2*(b^2 - 4*
a*c)^(5/2) - 16*b^2*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(3/2) - 3*b^4*c*d^2*e^3*x^2
*(b^2 - 4*a*c)^(3/2) - 160*a*b^4*c^3*d^3*e^2*x^2 - 192*a^3*b*c^4*d^2*e^3*x^
2 - 96*a^3*b^2*c^3*d*e^4*x^2 + 8*b^3*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(
b^4*d - b^3*d*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*d - a*b^3*e - 5*a*b^2*c*d + 4
*a^2*b*c*e + a*b^2*e*(b^2 - 4*a*c)^(1/2) - 2*a^2*c*e*(b^2 - 4*a*c)^(1/2) +
3*a*b*c*d*(b^2 - 4*a*c)^(1/2)))/(4*(4*a*c^4*d^2 + 4*a^2*c^3*e^2 - b^2*c^3*d
^2 - a*b^2*c^2*e^2 + b^3*c^2*d*e - 4*a*b*c^3*d*e)) - (log(8*c^3*d^5*(b^2 -
4*a*c)^(5/2) - 128*a^5*c^3*e^5 - a*e^5*x^2*(b^2 - 4*a*c)^(7/2) - 512*a^3*c^
5*d^4*e - 8*b^2*c^3*d^5*(b^2 - 4*a*c)^(3/2) - 6*b^3*d^2*e^3*(b^2 - 4*a*c)^(
5/2) + 3*b^5*d^2*e^3*(b^2 - 4*a*c)^(3/2) + 32*a^4*b^2*c^2*e^5 + 384*a^4*c^4
*d^2*e^3 + 256*a^2*c^6*d^5*x^2 + 16*b^4*c^4*d^5*x^2 - 3*a*d*e^4*(b^2 - 4*a*
c)^(7/2) + 3*b*d^2*e^3*(b^2 - 4*a*c)^(7/2) + 3*c*d^3*e^2*(b^2 - 4*a*c)^(7/2
) - 16*a^2*b^3*c^3*d^3*e^2 + 48*a^2*b^4*c^2*d^2*e^3 - 288*a^3*b^2*c^3*d^2*e
^3 + 16*a^3*b^3*c^2*e^5*x^2 - 384*a^3*c^5*d^3*e^2*x^2 + 16*b^6*c^2*d^3*e^2*
x^2 + 6*a*b^2*d*e^4*(b^2 - 4*a*c)^(5/2) - 3*a*b^4*d*e^4*(b^2 - 4*a*c)^(3/2)
 - 8*b*c^2*d^4*e*(b^2 - 4*a*c)^(5/2) - 32*a*b^4*c^3*d^4*e + 192*a^4*b*c^3*d
*e^4 + 2*b^2*c*d^3*e^2*(b^2 - 4*a*c)^(5/2) + 8*b^3*c^2*d^4*e*(b^2 - 4*a*c)^
(3/2) - 5*b^4*c*d^3*e^2*(b^2 - 4*a*c)^(3/2) + 2*a*b^2*e^5*x^2*(b^2 - 4*a*c)
```

```
^(5/2) - a*b^4*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^4*d^5*x^2*(b^2 - 4*a*c)
^(3/2) + 3*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 16*c^3*d^4*e*x^2*(b^2 - 4*a*
c)^(5/2) + 256*a^2*b^2*c^4*d^4*e + 64*a^3*b*c^4*d^3*e^2 - 48*a^3*b^3*c^2*d*
e^4 - 128*a*b^2*c^5*d^5*x^2 - 64*a^4*b*c^3*e^5*x^2 + 384*a^4*c^4*d*e^4*x^2
- 32*b^5*c^3*d^4*e*x^2 + 480*a^2*b^2*c^4*d^3*e^2*x^2 + 48*a^2*b^3*c^3*d^2*e
^3*x^2 + 256*a*b^3*c^4*d^4*e*x^2 - 512*a^2*b*c^5*d^4*e*x^2 - 8*b*c^2*d^3*e^
2*x^2*(b^2 - 4*a*c)^(5/2) - 6*b^2*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 16*b^
2*c^3*d^4*e*x^2*(b^2 - 4*a*c)^(3/2) + 3*b^4*c*d^2*e^3*x^2*(b^2 - 4*a*c)^(3/
2) - 160*a*b^4*c^3*d^3*e^2*x^2 - 192*a^3*b*c^4*d^2*e^3*x^2 - 96*a^3*b^2*c^3
*d*e^4*x^2 - 8*b^3*c^2*d^3*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(b^4*d + b^3*d*(b^2
 - 4*a*c)^(1/2) + 4*a^2*c^2*d - a*b^3*e - 5*a*b^2*c*d + 4*a^2*b*c*e - a*b^2
*e*(b^2 - 4*a*c)^(1/2) + 2*a^2*c*e*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*d*(b^2 - 4
*a*c)^(1/2)))/(4*(4*a*c^4*d^2 + 4*a^2*c^3*e^2 - b^2*c^3*d^2 - a*b^2*c^2*e^2
 + b^3*c^2*d*e - 4*a*b*c^3*d*e))
```

3.297 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)} dx$

Optimal. Leaf size=158

$$-\frac{(b^2d-2acd-abe)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}+\frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)}-\frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(cd^2-bde+ae^2)}$$

```
[Out] 1/2*d^2*ln(e*x^2+d)/e/(a*e^2-b*d*e+c*d^2)-1/4*(-a*e+b*d)*ln(c*x^4+b*x^2+a)/
c/(a*e^2-b*d*e+c*d^2)-1/2*(-a*b*e-2*a*c*d+b^2*d)*arctanh((2*c*x^2+b)/(-4*a*
c+b^2)^(1/2))/c/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1265, 1642, 648, 632, 212, 642}

$$-\frac{(-abe-2acd+b^2d)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{d^2\log(d+ex^2)}{2e(ae^2-bde+cd^2)}-\frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((b^2*d - 2*a*c*d - a*b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(
c*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + (d^2*Log[d + e*x^2])/(2*e*(c
*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*Log[a + b*x^2 + c*x^4])/(4*c*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex)} + \frac{-ad-(bd-ae)x}{(cd^2-bde+ae^2)(a+bx+cx\right.\right.\\
&= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{-ad-(bd-ae)x}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)}\\
&= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4c(cd^2-bde+ae^2)} + \frac{(b^2d-2ac\cdots}{}\\
&= \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd-ae)\log(a+bx^2+cx^4)}{4c(cd^2-bde+ae^2)} - \frac{(b^2d-2acd-abe\cdots}{2c\cdots}\\
&= -\frac{(b^2d-2acd-abe)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2c\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} + \frac{d^2\log(d+ex^2)}{2e(cd^2-bde+ae^2)} - \frac{(bd\cdots}{}
\end{aligned}$$

Mathematica [A]

time = 0.07, size = 139, normalized size = 0.88

$$-\frac{2e(-b^2d+2acd+abe)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,(-2cd^2\log(d+ex^2)+e(bd-ae)\log(a+bx^2+cx^4))}{4c\sqrt{-b^2+4ac}\,e\,(cd^2+e(-bd+ae))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*(2*e*(-(b^2*d) + 2*a*c*d + a*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a
*c]] + Sqrt[-b^2 + 4*a*c]*(-2*c*d^2*Log[d + e*x^2] + e*(b*d - a*e)*Log[a + 
b*x^2 + c*x^4]))/(c*Sqrt[-b^2 + 4*a*c]*e*(c*d^2 + e*(-(b*d) + a*e)))
```

Maple [A]
time = 0.23, size = 137, normalized size = 0.87

method	result	size
default	$-\frac{\frac{(-ae+bd)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(ad-\frac{(-ae+bd)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-deb+c\,d^2)}+\frac{d^2\ln\left(e\,x^2+d\right)}{2e(a\,e^2-deb+c\,d^2)}$	137
risch	Expression too large to display	9388

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*(1/2*(-a*e+b*d)/c*ln(c*x^4+b*x^2+a)+2*(a*d-1/2*(-a
*e+b*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))+1/2*d
^2*ln(e*x^2+d)/e/(a*e^2-b*d*e+c*d^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 33.10, size = 421, normalized size = 2.66

$$\left[\frac{2\,(b^2c-4\,ac^2)d^2\log(x^2e+d)+(abe^2-(b^2-2\,ac)de)\sqrt{b^2-4\,ac}\,\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)-((b^3-4\,abc)de-(ab^2-4\,a^2c)e^2)\log(cx^4+bx^2+a)}{4\,((b^2c^2-4\,ac^3)d^2e-(b^3c-4\,abc^2)de^2+(ab^2c-4\,a^2c^2)e^3)},\ \frac{2\,(b^2c-4\,ac^2)d^2\log(x^2e+d)+2\,(abe^2-(b^2-2\,ac)de)\sqrt{-b^2+4\,ac}\,\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)-((b^3-4\,abc)de-(ab^2-4\,a^2c)e^2)\log(cx^4+bx^2+a)}{4\,((b^2c^2-4\,ac^3)d^2e-(b^3c-4\,abc^2)de^2+(ab^2c-4\,a^2c^2)e^3)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b^2*c - 4*a*c^2)*d^2*log(x^2*e + d) + (a*b*e^2 - (b^2 - 2*a*c)*d*e
)*sqrt(b^2 - 4*a*c)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2*a*c + (2*c*x^2 + b
)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)) - ((b^3 - 4*a*b*c)*d*e - (a*b^2 -
 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e - (b^3*c
- 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3), 1/4*(2*(b^2*c - 4*a*c^2)*d
^2*log(x^2*e + d) + 2*(a*b*e^2 - (b^2 - 2*a*c)*d*e)*sqrt(-b^2 + 4*a*c)*arct
an(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a*c)) - ((b^3 - 4*a*b*c)*d*e
- (a*b^2 - 4*a^2*c)*e^2)*log(c*x^4 + b*x^2 + a))/((b^2*c^2 - 4*a*c^3)*d^2*e
 - (b^3*c - 4*a*b*c^2)*d*e^2 + (a*b^2*c - 4*a^2*c^2)*e^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.52, size = 157, normalized size = 0.99

$$\frac{d^2\log\left(|x^2e+d|\right)}{2\left(cd^2e-bde^2+ae^3\right)}-\frac{(bd-ae)\log\left(cx^4+bx^2+a\right)}{4\left(c^2d^2-bcde+ace^2\right)}+\frac{(b^2d-2\,acd-abe)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(c^2d^2-bcde+ace^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*d^2*log(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) - 1/4*(b*d - a*e)*l
og(c*x^4 + b*x^2 + a)/(c^2*d^2 - b*c*d*e + a*c*e^2) + 1/2*(b^2*d - 2*a*c*d
- a*b*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((c^2*d^2 - b*c*d*e + a*c
*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [B]
time = 11.05, size = 1853, normalized size = 11.73

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^2*log(d + e*x^2))/(2*a*e^3 - 2*b*d*e^2 + 2*c*d^2*e) + (log(4*a^2*e^4*(b^
2 - 4*a*c)^(5/2) + 8*c^2*d^4*(b^2 - 4*a*c)^(5/2) + 5*d^2*e^2*(b^2 - 4*a*c)^
(7/2) + 3*d*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 16*a^3*b^3*c*e^4 + 64*a^4*b*c^2*e
^4 + 640*a^3*c^4*d^3*e - 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*(b^2 - 4*a*c)^(3
/2) - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^(3/2) - 6*b^2*d^2*e^2*(b^2 - 4*a*c)^(5/2)
 + b^4*d^2*e^2*(b^2 - 4*a*c)^(3/2) - 256*a^2*c^5*d^4*x^2 - 128*a^4*c^3*e^4*
x^2 - 16*b^4*c^3*d^4*x^2 + 80*a^2*b^3*c^2*d^2*e^2 + 96*a^3*b^2*c^2*e^4*x^2
+ 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^(3/2) + 4*a*b*e^4*x
^2*(b^2 - 4*a*c)^(5/2) + 48*a*b^4*c^2*d^3*e - 16*a*b^5*c*d^2*e^2 - 4*a*b^3*
e^4*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c)^(3/2) - 6*b^2*
d*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 3*b^4*d*e^3*x^2*(b^2 - 4*a*c)^(3/2) + 20*c^
2*d^3*e*x^2*(b^2 - 4*a*c)^(5/2) - 352*a^2*b^2*c^3*d^3*e - 64*a^3*b*c^3*d^2*
e^2 + 96*a^3*b^2*c^2*d*e^3 + 128*a*b^2*c^4*d^4*x^2 - 16*a^2*b^4*c*e^4*x^2 +
 32*b^5*c^2*d^3*e*x^2 - 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*(b^2 - 4*a*c)^(5
/2) - 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2 - 4*a*c)^(5/2)
- 240*a*b^3*c^3*d^3*e*x^2 + 448*a^2*b*c^4*d^3*e*x^2 - 192*a^3*b*c^3*d*e^3*x
^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(3/2) - 4*b^3*c*d^2*e^2*x^2*(b^2 -
4*a*c)^(3/2) + 144*a*b^4*c^2*d^2*e^2*x^2 + 48*a^2*b^3*c^2*d*e^3*x^2)*((b^3*
d)/4 + e*(a^2*c - (a*b^2)/4 + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^2*d*(b^2 -
4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 - a*b*c*d))/(4*a*c^3*d^2 +
4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a*b*c^2*d*e) - (l
og(4*a^2*e^4*(b^2 - 4*a*c)^(5/2) + 8*c^2*d^4*(b^2 - 4*a*c)^(5/2) + 5*d^2*e^
2*(b^2 - 4*a*c)^(7/2) + 3*d*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 16*a^3*b^3*c*e^4
- 64*a^4*b*c^2*e^4 - 640*a^3*c^4*d^3*e + 384*a^4*c^3*d*e^3 - 4*a^2*b^2*e^4*
(b^2 - 4*a*c)^(3/2) - 8*b^2*c^2*d^4*(b^2 - 4*a*c)^(3/2) - 6*b^2*d^2*e^2*(b^
2 - 4*a*c)^(5/2) + b^4*d^2*e^2*(b^2 - 4*a*c)^(3/2) + 256*a^2*c^5*d^4*x^2 +
128*a^4*c^3*e^4*x^2 + 16*b^4*c^3*d^4*x^2 - 80*a^2*b^3*c^2*d^2*e^2 - 96*a^3*
b^2*c^2*e^4*x^2 - 640*a^3*c^4*d^2*e^2*x^2 + 4*b^3*c*d^3*e*(b^2 - 4*a*c)^(3/
2) + 4*a*b*e^4*x^2*(b^2 - 4*a*c)^(5/2) - 48*a*b^4*c^2*d^3*e + 16*a*b^5*c*d^
2*e^2 - 4*a*b^3*e^4*x^2*(b^2 - 4*a*c)^(3/2) - 16*b*c^3*d^4*x^2*(b^2 - 4*a*c
)^(3/2) - 6*b^2*d*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 3*b^4*d*e^3*x^2*(b^2 - 4*a*
c)^(3/2) + 20*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(5/2) + 352*a^2*b^2*c^3*d^3*e + 6
4*a^3*b*c^3*d^2*e^2 - 96*a^3*b^2*c^2*d*e^3 - 128*a*b^2*c^4*d^4*x^2 + 16*a^2
*b^4*c*e^4*x^2 - 32*b^5*c^2*d^3*e*x^2 + 16*b^6*c*d^2*e^2*x^2 - 4*b*c*d^3*e*
(b^2 - 4*a*c)^(5/2) + 480*a^2*b^2*c^3*d^2*e^2*x^2 - 12*b*c*d^2*e^2*x^2*(b^2
 - 4*a*c)^(5/2) + 240*a*b^3*c^3*d^3*e*x^2 - 448*a^2*b*c^4*d^3*e*x^2 + 192*a
^3*b*c^3*d*e^3*x^2 + 12*b^2*c^2*d^3*e*x^2*(b^2 - 4*a*c)^(3/2) - 4*b^3*c*d^2
*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 144*a*b^4*c^2*d^2*e^2*x^2 - 48*a^2*b^3*c^2*d
*e^3*x^2)*(e*((a*b^2)/4 - a^2*c + (a*b*(b^2 - 4*a*c)^(1/2))/4) - (b^3*d)/4
- (b^2*d*(b^2 - 4*a*c)^(1/2))/4 + (a*c*d*(b^2 - 4*a*c)^(1/2))/2 + a*b*c*d))
/(4*a*c^3*d^2 + 4*a^2*c^2*e^2 - b^2*c^2*d^2 + b^3*c*d*e - a*b^2*c*e^2 - 4*a
*b*c^2*d*e)
```

3.298 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=132

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}-\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)}+\frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}$$

```
[Out] -1/2*d*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)+1/4*d*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e
+c*d^2)+1/2*(-2*a*e+b*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d
*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {1265, 814, 648, 632, 212, 642}

$$\frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{d\log(d+ex^2)}{2(ae^2-bde+cd^2)}+\frac{d\log(a+bx^2+cx^4)}{4(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - 2*a*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*Sqrt[b^2 - 4*a*
c]*(c*d^2 - b*d*e + a*e^2)) - (d*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e^2)
) + (d*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 814

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)/(a +
 b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*
c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)(a+bx+cx^2)}\,dx, x, x^2\right) \\
&= \frac{1}{2}\text{Subst}\left(\int \left(-\frac{de}{(cd^2-bde+ae^2)(d+ex)} + \frac{ae+cdx}{(cd^2-bde+ae^2)(a+bx+\right.\right. \\
&= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{\text{Subst}\left(\int \frac{ae+cdx}{a+bx+cx^2}\,dx, x, x^2\right)}{2(cd^2-bde+ae^2)} \\
&= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4(cd^2-bde+ae^2)} - \frac{(bd-2ae)\text{Subst}}{4(cd^2-} \\
&= -\frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)} + \frac{(bd-2ae)\text{Subst}\left(\int \frac{1}{b^2-4a}\right.}{2(cd^2-bde} \\
&= \frac{(bd-2ae)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)} - \frac{d\log(d+ex^2)}{2(cd^2-bde+ae^2)} + \frac{d\log(a+bx^2}{4(cd^2-bde}
\end{aligned}$$

Mathematica [A]
time = 0.05, size = 114, normalized size = 0.86

$$\frac{2(bd-2ae)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,d(2\log(d+ex^2)-\log(a+bx^2+cx^4))}{4\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*(b*d - 2*a*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*a
*c]*d*(2*Log[d + e*x^2] - Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*(-
(c*d^2) + e*(b*d - a*e)))
```

Maple [A]
time = 0.16, size = 112, normalized size = 0.85

method	result
default	$\frac{\frac{d\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2}+\frac{2\left(ae-\frac{bd}{2}\right)\arctan\left(\frac{2c\,x^{2}+b}{\sqrt{4ac-b^{2}}}\right)}{\sqrt{4ac-b^{2}}}}{2a\,e^{2}-2deb+2c\,d^{2}}-\frac{d\ln\left(e\,x^{2}+d\right)}{2\left(a\,e^{2}-deb+c\,d^{2}\right)}$
risch	$-\frac{d\ln\left(e\,x^{2}+d\right)}{2\left(a\,e^{2}-deb+c\,d^{2}\right)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{2}c\,e^{2}-a\,b^{2}e^{2}-4abcde+4a\,c^{2}d^{2}+b^{3}de-b^{2}c\,d^{2}\right)\_Z^{2}+\left(-4acd+b^{2}d\right)\_Z+a\right)}\_R\ln\left(\left(\left(4e^{3}c\,a^{2}-a\,b\right.\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)*(1/2*d*ln(c*x^4+b*x^2+a)+2*(a*e-1/2*b*d)/(4*a*c-b^2
)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*d*ln(e*x^2+d)/(a*e^2-b*d
*e+c*d^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 11.05, size = 325, normalized size = 2.46

$$\left[\frac{(b^2-4\,ac)d\log\left(cx^4+bx^2+a\right)-2\,(b^2-4\,ac)d\log\left(x^2e+d\right)-\sqrt{b^2-4\,ac}\,(bd-2\,ae)\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac-(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{4\left((b^2c-4\,ac^2)d^2-(b^3-4\,abc)de+(ab^2-4\,a^2c)e^2\right)}, \frac{(b^2-4\,ac)d\log\left(cx^4+bx^2+a\right)-2\,(b^2-4\,ac)d\log\left(x^2e+d\right)+2\,\sqrt{-b^2+4\,ac}\,(bd-2\,ae)\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{4\left((b^2c-4\,ac^2)d^2-(b^3-4\,abc)de+(ab^2-4\,a^2c)e^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/4*((b^2 - 4*a*c)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(x^2*e
+ d) - sqrt(b^2 - 4*a*c)*(b*d - 2*a*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 - 2
*a*c - (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*a
*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), 1/4*((b^2 - 4*a*c
)*d*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*d*log(x^2*e + d) + 2*sqrt(-b^2
 + 4*a*c)*(b*d - 2*a*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4*a
*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 5.53, size = 133, normalized size = 1.01

$$-\frac{de\log\left(|x^2e+d|\right)}{2\left(cd^2e-bde^2+ae^3\right)}+\frac{d\log\left(cx^4+bx^2+a\right)}{4\left(cd^2-bde+ae^2\right)}-\frac{(bd-2\,ae)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(cd^2-bde+ae^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*d*e*log(abs(x^2*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/4*d*log(c*x^4
+ b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) - 1/2*(b*d - 2*a*e)*arctan((2*c*x^2 +
b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [**B**]
time = 9.75, size = 2500, normalized size = 18.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(76*d^3*e^3*(b^2 - 4*a*c)^(9/2) - 64*a^3*b^6*e^6 - 4608*a^3*c^6*d^6 + 5
12*a^6*c^3*e^6 - 320*a*b^4*c^4*d^6 + 512*a^4*b^4*c*e^6 - 64*a*b^8*d^2*e^4 -
 128*a^2*b^7*d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^(3/2) - 48*b^3*c^3*d^6*(b
```

^2 - 4*a*c)^(3/2) - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^(7/2) - 28*b^4*d^3*e^3*(b^
2 - 4*a*c)^(5/2) + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 4*a^2*e^6*x^2*(b^2
- 4*a*c)^(7/2) + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^(5/2) + 39*d^2*e^4*x^2*(b^2
- 4*a*c)^(9/2) + 2432*a^2*b^2*c^5*d^6 - 1152*a^5*b^2*c^2*e^6 + 40448*a^4*c^
5*d^4*e^2 - 19968*a^5*c^4*d^2*e^4 - 64*a^2*b^7*e^6*x^2 - 64*b^5*c^4*d^6*x^2
 - 64*b^9*d^2*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^(5/2) + 48*b*c^3*d^6*(b^
2 - 4*a*c)^(5/2) + 40*a^2*d*e^5*(b^2 - 4*a*c)^(7/2) + 168*c^2*d^5*e*(b^2 -
4*a*c)^(7/2) + 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) + 20*a^2*b^4*e^6*x^2*
(b^2 - 4*a*c)^(3/2) - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^(3/2) + 155*b^2*d^2*
e^4*x^2*(b^2 - 4*a*c)^(7/2) - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 25*
b^6*d^2*e^4*x^2*(b^2 - 4*a*c)^(3/2) + 316*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(7/
2) + 5120*a^2*b^4*c^3*d^4*e^2 - 4096*a^2*b^5*c^2*d^3*e^3 - 24448*a^3*b^2*c^
4*d^4*e^2 + 21760*a^3*b^3*c^3*d^3*e^3 - 9920*a^3*b^4*c^2*d^2*e^4 + 26240*a^
4*b^2*c^3*d^2*e^4 - 1600*a^4*b^3*c^2*e^6*x^2 + 38912*a^4*c^5*d^3*e^3*x^2 -
384*b^7*c^2*d^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^(7/2) - 176*b*c*d^4
e^2(b^2 - 4*a*c)^(7/2) + 256*a*b^5*c^3*d^5*e + 256*a*b^7*c*d^3*e^3 + 2560
*a^3*b*c^5*d^5*e + 1664*a^3*b^5*c*d*e^5 + 8704*a^5*b*c^3*d*e^5 - 128*a*b^8*
d*e^5*x^2 - 168*a*b^3*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*a*b^5*d^2*e^4*(b^2 -
 4*a*c)^(3/2) + 144*a^2*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 56*a^2*b^4*d*e^5*(b
^2 - 4*a*c)^(3/2) - 272*b^2*c^2*d^5*e*(b^2 - 4*a*c)^(5/2) + 256*b^3*c*d^4*e
^2*(b^2 - 4*a*c)^(5/2) + 104*b^4*c^2*d^5*e*(b^2 - 4*a*c)^(3/2) - 80*b^5*c*d
^4*e^2*(b^2 - 4*a*c)^(3/2) - 384*a*b^6*c^2*d^4*e^2 - 1664*a^2*b^3*c^4*d^5*e
 + 1408*a^2*b^6*c*d^2*e^4 - 37888*a^4*b*c^4*d^3*e^3 - 6784*a^4*b^3*c^2*d*e^
5 + 448*a*b^3*c^5*d^6*x^2 - 768*a^2*b*c^6*d^6*x^2 + 576*a^3*b^5*c*e^6*x^2 +
 1280*a^5*b*c^3*e^6*x^2 - 21504*a^3*c^6*d^5*e*x^2 - 5120*a^5*c^4*d*e^5*x^2
+ 256*b^6*c^3*d^5*e*x^2 + 256*b^8*c*d^3*e^3*x^2 - 26560*a^2*b^3*c^4*d^4*e^2
*x^2 + 25600*a^2*b^4*c^3*d^3*e^3*x^2 - 11264*a^2*b^5*c^2*d^2*e^4*x^2 - 5888
0*a^3*b^2*c^4*d^3*e^3*x^2 + 34880*a^3*b^3*c^3*d^2*e^4*x^2 + 80*a*b^3*d*e^5*
x^2*(b^2 - 4*a*c)^(5/2) - 40*a*b^5*d*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 448*b*c*
d^3*e^3*x^2*(b^2 - 4*a*c)^(7/2) - 416*b*c^3*d^5*e*x^2*(b^2 - 4*a*c)^(5/2) -
 3200*a*b^4*c^4*d^5*e*x^2 + 1472*a*b^7*c*d^2*e^4*x^2 + 1792*a^2*b^6*c*d*e^5
*x^2 + 192*b^3*c*d^3*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 160*b^3*c^3*d^5*e*x^2*(b
^2 - 4*a*c)^(3/2) + 5504*a*b^5*c^3*d^4*e^2*x^2 - 4352*a*b^6*c^2*d^3*e^3*x^2
 + 14080*a^2*b^2*c^5*d^5*e*x^2 + 42752*a^3*b*c^5*d^4*e^2*x^2 - 8320*a^3*b^4
*c^2*d*e^5*x^2 - 37120*a^4*b*c^4*d^2*e^4*x^2 + 14080*a^4*b^2*c^3*d*e^5*x^2
+ 88*a*b*d*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 168*b^2*c^2*d^4*e^2*x^2*(b^2 - 4*a
*c)^(5/2) - 100*b^4*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(3/2))*(d*((b*(b^2 - 4*a*
c)^(1/2))/4 - a*c + b^2/4) - (a*e*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a
*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) - (log(76*d^3*e
^3*(b^2 - 4*a*c)^(9/2) + 64*a^3*b^6*e^6 + 4608*a^3*c^6*d^6 - 512*a^6*c^3*e^
6 + 320*a*b^4*c^4*d^6 - 512*a^4*b^4*c*e^6 + 64*a*b^8*d^2*e^4 + 128*a^2*b^7*
d*e^5 + 32*a^3*b^3*e^6*(b^2 - 4*a*c)^(3/2) - 48*b^3*c^3*d^6*(b^2 - 4*a*c)^(
3/2) - 68*b^2*d^3*e^3*(b^2 - 4*a*c)^(7/2) - 28*b^4*d^3*e^3*(b^2 - 4*a*c)^(5
/2) + 20*b^6*d^3*e^3*(b^2 - 4*a*c)^(3/2) + 4*a^2*e^6*x^2*(b^2 - 4*a*c)^(7/2
) + 144*c^4*d^6*x^2*(b^2 - 4*a*c)^(5/2) + 39*d^2*e^4*x^2*(b^2 - 4*a*c)^(9/2

) - 2432*a^2*b^2*c^5*d^6 + 1152*a^5*b^2*c^2*e^6 - 40448*a^4*c^5*d^4*e^2 + 19968*a^5*c^4*d^2*e^4 + 64*a^2*b^7*e^6*x^2 + 64*b^5*c^4*d^6*x^2 + 64*b^9*d^2*e^4*x^2 + 32*a^3*b*e^6*(b^2 - 4*a*c)^(5/2) + 48*b*c^3*d^6*(b^2 - 4*a*c)^(5/2) + 40*a^2*d*e^5*(b^2 - 4*a*c)^(7/2) + 168*c^2*d^5*e*(b^2 - 4*a*c)^(7/2) + 40*a^2*b^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) + 20*a^2*b^4*e^6*x^2*(b^2 - 4*a*c)^(3/2) - 80*b^2*c^4*d^6*x^2*(b^2 - 4*a*c)^(3/2) + 155*b^2*d^2*e^4*x^2*(b^2 - 4*a*c)^(7/2) - 155*b^4*d^2*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 25*b^6*d^2*e^4*x^2*(b^2 - 4*a*c)^(3/2) + 316*c^2*d^4*e^2*x^2*(b^2 - 4*a*c)^(7/2) - 5120*a^2*b^4*c^3*d^4*e^2 + 4096*a^2*b^5*c^2*d^3*e^3 + 24448*a^3*b^2*c^4*d^4*e^2 - 21760*a^3*b^3*c^3*d^3*e^3 + 9920*a^3*b^4*c^2*d^2*e^4 - 26240*a^4*b^2*c^3*d^2*e^4 + 1600*a^4*b^3*c^2*e^6*x^2 - 38912*a^4*c^5*d^3*e^3*x^2 + 384*b^7*c^2*d^4*e^2*x^2 + 212*a*b*d^2*e^4*(b^2 - 4*a*c)^(7/2) - 176*b*c*d^4*e^2*(b^2 - 4*a*c)^(7/2) - 256*a*b^5*c^3*d^5*e - 256*a*b^7*c*d^3*e^3 - 2560*a^3*b*c^5*d^5*e - 1664*a^3*b^5*c*d*e^5 - 8704*a^5*b*c^3*d*e^5 + 128*a*b^8*d*e^5*x^2 - 168*a*b^3*d^2*e^4*(b^2 - 4*a*c)^(5/2) + 20*a*b^5*d^2*e^4*(b^2 - 4*a*c)^(3/2) + 144*a^2*b^2*d*e^5*(b^2 - 4*a*c)^(5/2) - 56*a^2*b^4*d*e^5*(b^2 - 4*a*c)^(3/2) - 272*b^2*c^2*d^5*e*(b^2 - 4*a*c)^(5/2) + 256*b^3*c*d^4*e^2*(b^2 - 4*a*c)^(5/2) + 104*b^4*c^2*d^5*e*(b^2 - 4*a*c)^(3/2) - 80*b^5*c*d^4*e^2*(b^2 - 4*a*c)^(3/2) + 384*a*b^6*c^2*d^4*e^2 + 1664*a^...

3.299 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=133

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}+\frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)}-\frac{e\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}$$

```
[Out] 1/2*e*ln(e*x^2+d)/(a*e^2-b*d*e+c*d^2)-1/4*e*ln(c*x^4+b*x^2+a)/(a*e^2-b*d*e+
c*d^2)-1/2*(-b*e+2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*
e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {1261, 719, 31, 648, 632, 212, 642}

$$-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}+\frac{e\log(d+ex^2)}{2(ae^2-bde+cd^2)}-\frac{e\log(a+bx^2+cx^4)}{4(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(Sqrt[b^2 - 4
*a*c]*(c*d^2 - b*d*e + a*e^2)) + (e*Log[d + e*x^2])/(2*(c*d^2 - b*d*e + a*e
^2)) - (e*Log[a + b*x^2 + c*x^4])/(4*(c*d^2 - b*d*e + a*e^2))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 719

```
Int[1/(((d_.) + (e_.)*(x_))*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)), x_Symbol]
 :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[1/(d + e*x), x], x] + Dist[1/(c*d
^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x)/(a + b*x + c*x^2), x], x] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^
2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)(a+bx+cx^2)}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{cd-be-cex}{a+bx+cx^2}\,dx,x,x^2\right)}{2(cd^2-bde+ae^2)}+\frac{e^2\text{Subst}\left(\int \frac{1}{d+ex}\,dx,x,x^2\right)}{2(cd^2-bde+ae^2)}\\
&= \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)}-\frac{e\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx,x,x^2\right)}{4(cd^2-bde+ae^2)}+\frac{(2cd-be)\text{Subst}\left(\int\right.}{4(cd^2-b}\\
&= \frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)}-\frac{e\log(a+bx^2+cx^4)}{4(cd^2-bde+ae^2)}-\frac{(2cd-be)\text{Subst}\left(\int \frac{1}{b^2-4ac-}\right.}{2(cd^2-bde+}\\
&= -\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}+\frac{e\log(d+ex^2)}{2(cd^2-bde+ae^2)}-\frac{e\log(a+b}{4(cd^2-bd}
\end{aligned}$$

Mathematica [A]

time = 0.05, size = 112, normalized size = 0.84

$$\frac{(-4cd+2be)\tan^{-1}\left(\frac{b+2cx^2}{\sqrt{-b^2+4ac}}\right)+\sqrt{-b^2+4ac}\,e(-2\log(d+ex^2)+\log(a+bx^2+cx^4))}{4\sqrt{-b^2+4ac}\,(-cd^2+e(bd-ae))}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*c*d + 2*b*e)*ArcTan[(b + 2*c*x^2)/Sqrt[-b^2 + 4*a*c]] + Sqrt[-b^2 + 4*
a*c]*e*(-2*Log[d + e*x^2] + Log[a + b*x^2 + c*x^4]))/(4*Sqrt[-b^2 + 4*a*c]*
(-(c*d^2) + e*(b*d - a*e)))
```

Maple [A]
time = 0.20, size = 113, normalized size = 0.85

method	result	size
default	$-\frac{\frac{e\ln\left(c\,x^4+b\,x^2+a\right)}{2}+\frac{2\left(\frac{eb}{2}-cd\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-deb+c\,d^2)}+\frac{e\ln\left(e\,x^2+d\right)}{2a\,e^2-2deb+2c\,d^2}$	113
risch	Expression too large to display	3744

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)*(1/2*e*ln(c*x^4+b*x^2+a)+2*(1/2*e*b-c*d)/(4*a*c-b^
2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))+1/2*e*ln(e*x^2+d)/(a*e^2-b*
d*e+c*d^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [A]
time = 7.89, size = 329, normalized size = 2.47

$$\left[-\frac{(b^2-4\,ac)e\log(cx^4+bx^2+a)-2\,(b^2-4\,ac)e\log(x^2e+d)+\sqrt{b^2-4\,ac}\,(2\,cd-be)\log\left(\frac{2\,c^2x^4+2\,bcx^2+b^2-2\,ac+(2\,cx^2+b)\sqrt{b^2-4\,ac}}{cx^4+bx^2+a}\right)}{4\,((b^2c-4\,ac^2)d^2-(b^3-4\,abc)de+(ab^2-4\,a^2c)e^2)},\ -\frac{(b^2-4\,ac)e\log(cx^4+bx^2+a)-2\,(b^2-4\,ac)e\log(x^2e+d)+2\,\sqrt{-b^2+4\,ac}\,(2\,cd-be)\arctan\left(-\frac{(2\,cx^2+b)\sqrt{-b^2+4\,ac}}{b^2-4\,ac}\right)}{4\,((b^2c-4\,ac^2)d^2-(b^3-4\,abc)de+(ab^2-4\,a^2c)e^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*((b^2 - 4*a*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(x^2*e
 + d) + sqrt(b^2 - 4*a*c)*(2*c*d - b*e)*log((2*c^2*x^4 + 2*b*c*x^2 + b^2 -
2*a*c + (2*c*x^2 + b)*sqrt(b^2 - 4*a*c))/(c*x^4 + b*x^2 + a)))/((b^2*c - 4*
a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2), -1/4*((b^2 - 4*a
*c)*e*log(c*x^4 + b*x^2 + a) - 2*(b^2 - 4*a*c)*e*log(x^2*e + d) + 2*sqrt(-b
^2 + 4*a*c)*(2*c*d - b*e)*arctan(-(2*c*x^2 + b)*sqrt(-b^2 + 4*a*c)/(b^2 - 4
*a*c)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^
2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 6.40, size = 134, normalized size = 1.01

$$-\frac{e\log\left(cx^4+bx^2+a\right)}{4\left(cd^2-bde+ae^2\right)}+\frac{e^2\log\left(\left|x^2e+d\right|\right)}{2\left(cd^2e-bde^2+ae^3\right)}+\frac{\left(2\,cd-be\right)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(cd^2-bde+ae^2\right)\sqrt{-b^2+4\,ac}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*e*log(c*x^4 + b*x^2 + a)/(c*d^2 - b*d*e + a*e^2) + 1/2*e^2*log(abs(x^2
*e + d))/(c*d^2*e - b*d*e^2 + a*e^3) + 1/2*(2*c*d - b*e)*arctan((2*c*x^2 +
b)/sqrt(-b^2 + 4*a*c))/((c*d^2 - b*d*e + a*e^2)*sqrt(-b^2 + 4*a*c))
```

Mupad [**B**]
time = 8.71, size = 2434, normalized size = 18.30

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (e*log(d + e*x^2))/(2*a*e^2 + 2*c*d^2 - 2*b*d*e) - (log(36*a^4*c^3*e^5 - 4*
a*b^6*e^5 - 4*b^7*e^5*x^2 + 32*a^2*b^4*c*e^5 + 36*a^2*c^5*d^4*e - 4*a*c^6*d
```

^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4*a*c)^(1/2) - 73*a^3*b^2*c^2*e^5 - 184*a^3*c^4*d^2*e^3 + b^2*c^5*d^5*x^2 - 4*a*b^5*e^5*(b^2 - 4*a*c)^(1/2) + 2*a*c^5*d^5*(b^2 - 4*a*c)^(1/2) + 16*a*b^5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 146*a^2*b^2*c^3*d^2*e^3 - 101*a^2*b^3*c^2*e^5*x^2 + 120*a^2*c^5*d^3*e^2*x^2 + 19*b^4*c^3*d^3*e^2*x^2 - 25*b^5*c^2*d^2*e^3*x^2 - 9*a*b^2*c^4*d^4*e + 184*a^3*b*c^3*d*e^4 + 36*a*b^5*c*e^5*x^2 + 16*b^6*c*d*e^4*x^2 + 24*a^2*b^3*c*e^5*(b^2 - 4*a*c)^(1/2) - 33*a^3*b*c^2*e^5*(b^2 - 4*a*c)^(1/2) + 66*a^3*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) + b*c^5*d^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*b^3*c^3*d^3*e^2 - 25*a*b^4*c^2*d^2*e^3 - 72*a^2*b*c^4*d^3*e^2 - 110*a^2*b^3*c^2*d*e^4 + 84*a^3*b*c^3*e^5*x^2 - 132*a^3*c^4*d*e^4*x^2 - 7*b^3*c^4*d^4*e*x^2 + 28*a*b^4*c*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*c^5*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) + 16*b^5*c*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) - 126*a*b^4*c^2*d*e^4*x^2 + 20*a*b^2*c^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 25*a*b^3*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 90*a^2*b*c^3*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 78*a^2*b^2*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) - 7*b^2*c^4*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) - 106*a*b^2*c^4*d^3*e^2*x^2 + 168*a*b^3*c^3*d^2*e^3*x^2 - 272*a^2*b*c^4*d^2*e^3*x^2 + 281*a^2*b^2*c^3*d*e^4*x^2 - 5*a*b*c^4*d^4*e*(b^2 - 4*a*c)^(1/2) + 16*a*b^4*c*d*e^4*(b^2 - 4*a*c)^(1/2) - 53*a^2*b^2*c^2*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 28*a*b*c^5*d^4*e*x^2 - 92*a^2*c^4*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 19*b^3*c^3*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 25*b^4*c^2*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) + 118*a*b^2*c^3*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) - 66*a*b*c^4*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 94*a*b^3*c^2*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) + 125*a^2*b*c^3*d*e^4*x^2*(b^2 - 4*a*c)^(1/2))*(e*((b*(b^2 - 4*a*c)^(1/2))/4 - a*c + b^2/4) - (c*d*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e) + (log(4*a*b^6*e^5 - 36*a^4*c^3*e^5 + 4*b^7*e^5*x^2 - 32*a^2*b^4*c*e^5 - 36*a^2*c^5*d^4*e + 4*a*c^6*d^5*x^2 - 4*b^6*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 73*a^3*b^2*c^2*e^5 + 184*a^3*c^4*d^2*e^3 - b^2*c^5*d^5*x^2 - 4*a*b^5*e^5*(b^2 - 4*a*c)^(1/2) + 2*a*c^5*d^5*(b^2 - 4*a*c)^(1/2) - 16*a*b^5*c*d*e^4 - 60*a^2*c^4*d^3*e^2*(b^2 - 4*a*c)^(1/2) + 18*a^3*c^3*e^5*x^2*(b^2 - 4*a*c)^(1/2) - 146*a^2*b^2*c^3*d^2*e^3 + 101*a^2*b^3*c^2*e^5*x^2 - 120*a^2*c^5*d^3*e^2*x^2 - 19*b^4*c^3*d^3*e^2*x^2 + 25*b^5*c^2*d^2*e^3*x^2 + 9*a*b^2*c^4*d^4*e - 184*a^3*b*c^3*d*e^4 - 36*a*b^5*c*e^5*x^2 - 16*b^6*c*d*e^4*x^2 + 24*a^2*b^3*c*e^5*(b^2 - 4*a*c)^(1/2) - 33*a^3*b*c^2*e^5*(b^2 - 4*a*c)^(1/2) + 66*a^3*c^3*d*e^4*(b^2 - 4*a*c)^(1/2) + b*c^5*d^5*x^2*(b^2 - 4*a*c)^(1/2) - 18*a*b^3*c^3*d^3*e^2 + 25*a*b^4*c^2*d^2*e^3 + 72*a^2*b*c^4*d^3*e^2 + 110*a^2*b^3*c^2*d*e^4 - 84*a^3*b*c^3*e^5*x^2 + 132*a^3*c^4*d*e^4*x^2 + 7*b^3*c^4*d^4*e*x^2 + 28*a*b^4*c*e^5*x^2*(b^2 - 4*a*c)^(1/2) + 18*a*c^5*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) + 16*b^5*c*d*e^4*x^2*(b^2 - 4*a*c)^(1/2) + 126*a*b^4*c^2*d*e^4*x^2 + 20*a*b^2*c^3*d^3*e^2*(b^2 - 4*a*c)^(1/2) - 25*a*b^3*c^2*d^2*e^3*(b^2 - 4*a*c)^(1/2) + 90*a^2*b*c^3*d^2*e^3*(b^2 - 4*a*c)^(1/2) - 78*a^2*b^2*c^2*d*e^4*(b^2 - 4*a*c)^(1/2) - 7*b^2*c^4*d^4*e*x^2*(b^2 - 4*a*c)^(1/2) + 106*a*b^2*c^4*d^3*e^2*x^2 - 168*a*b^3*c^3*d^2*e^3*x^2 + 272*a^2*b*c^4*d^2*e^3*x^2 - 281*a^2*b^2*c^3*d*e^4*x^2 - 5*a*b*c^4*d^4*e*(b^2 - 4*a*c)^(1/2) + 16*a*b^4*c*d*e^4*(b^2 - 4*a*c)^(1/2) - 53*a^2*b^2*c^2*e^5*x^2*(b^2

```
 - 4*a*c)^(1/2) - 28*a*b*c^5*d^4*e*x^2 - 92*a^2*c^4*d^2*e^3*x^2*(b^2 - 4*a*
c)^(1/2) + 19*b^3*c^3*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 25*b^4*c^2*d^2*e^3*
x^2*(b^2 - 4*a*c)^(1/2) + 118*a*b^2*c^3*d^2*e^3*x^2*(b^2 - 4*a*c)^(1/2) - 6
6*a*b*c^4*d^3*e^2*x^2*(b^2 - 4*a*c)^(1/2) - 94*a*b^3*c^2*d*e^4*x^2*(b^2 - 4
*a*c)^(1/2) + 125*a^2*b*c^3*d*e^4*x^2*(b^2 - 4*a*c)^(1/2))*(e*(a*c + (b*(b^
2 - 4*a*c)^(1/2))/4 - b^2/4) - (c*d*(b^2 - 4*a*c)^(1/2))/2))/(a*b^2*e^2 - 4
*a*c^2*d^2 - 4*a^2*c*e^2 + b^2*c*d^2 - b^3*d*e + 4*a*b*c*d*e)
```

3.300 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=167

$$\frac{(bcd-b^2e+2ace)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}+\frac{\log(x)}{ad}-\frac{e^2\log(d+ex^2)}{2d(cd^2-bde+ae^2)}-\frac{(cd-be)\log(a+bx^2+cx^4)}{4a(cd^2-bde+ae^2)}$$

```
[Out] ln(x)/a/d-1/2*e^2*ln(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)-1/4*(-b*e+c*d)*ln(c*x^4
+b*x^2+a)/a/(a*e^2-b*d*e+c*d^2)+1/2*(2*a*c*e-b^2*e+b*c*d)*arctanh((2*c*x^2+
b)/(-4*a*c+b^2)^(1/2))/a/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1265, 907, 648, 632, 212, 642}

$$\frac{(2ace+b^2(-e)+bcd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{e^2\log(d+ex^2)}{2d(ae^2-bde+cd^2)}-\frac{(cd-be)\log(a+bx^2+cx^4)}{4a(ae^2-bde+cd^2)}+\frac{\log(x)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*c*d - b^2*e + 2*a*c*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a*S
qrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + Log[x]/(a*d) - (e^2*Log[d + e*x
^2])/(2*d*(c*d^2 - b*d*e + a*e^2)) - ((c*d - b*e)*Log[a + b*x^2 + c*x^4])/(
4*a*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)}\,dx, x, x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx}-\frac{e^3}{d\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{-bcd+b^2e-ace\ldots}{a\left(cd^2-bde+ae^2\right)\ldots}\right.\right.\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}+\frac{\text{Subst}\left(\int \frac{-bcd+b^2e-ace-c(cd-be)x}{a+bx+cx^2}\,dx, x, x\ldots\right.}{2a\left(cd^2-bde+ae^2\right)}\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\text{Subst}\left(\int \frac{b+2cx}{a+bx+cx^2}\,dx, x, x^2\right)}{4a\left(cd^2-bde+ae^2\right)}\ldots\\
&= \frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex^2\right)}{2d\left(cd^2-bde+ae^2\right)}-\frac{(cd-be)\log\left(a+bx^2+cx^4\right)}{4a\left(cd^2-bde+ae^2\right)}+\frac{(bcd-\ldots}{\ldots}\\
&= \frac{\left(bcd-b^2e+2ace\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}+\frac{\log(x)}{ad}-\frac{e^2\log\left(d+ex\ldots\right.}{2d\left(cd^2-bde+\ldots\right.}
\end{aligned}$$

Mathematica [A]

time = 0.20, size = 242, normalized size = 1.45

$$\frac{4\sqrt{b^2-4ac}\,(cd^2+e(-bd+ae))\log(x)-d\left(bcd+c\sqrt{b^2-4ac}\,d-b^2e+2ace-b\sqrt{b^2-4ac}\,e\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)+d\left(bcd-c\sqrt{b^2-4ac}\,d-b^2e+2ace+b\sqrt{b^2-4ac}\,e\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)-2a\sqrt{b^2-4ac}\,e^2\log(d+ex^2)}{4a\sqrt{b^2-4ac}\,d\,(cd^2+e(-bd+ae))}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (4*Sqrt[b^2 - 4*a*c]*(c*d^2 + e*(-(b*d) + a*e))*Log[x] - d*(b*c*d + c*Sqrt[
b^2 - 4*a*c]*d - b^2*e + 2*a*c*e - b*Sqrt[b^2 - 4*a*c]*e)*Log[b - Sqrt[b^2
- 4*a*c] + 2*c*x^2] + d*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e +
b*Sqrt[b^2 - 4*a*c]*e)*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2] - 2*a*Sqrt[b^2
- 4*a*c]*e^2*Log[d + e*x^2])/(4*a*Sqrt[b^2 - 4*a*c]*d*(c*d^2 + e*(-(b*d) +
a*e)))
```

Maple [A]
time = 0.22, size = 166, normalized size = 0.99

method	result
default	$-\frac{\frac{\left(-bce+c^2d\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(ace-b^2e+bcd-\frac{\left(-bce+c^2d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-deb+c\,d^2)a}-\frac{e^2\ln\left(e\,x^2+d\right)}{2d(a\,e^2-deb+c\,d^2)}+\frac{\ln(x)}{ad}$
risch	$\frac{\ln(x)}{ad}-\frac{e^2\ln\left(-e\,x^2-d\right)}{2d(a\,e^2-deb+c\,d^2)}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^3c\,e^2-a^2b^2e^2-4a^2bcde+4a^2c^2d^2+a\,b^3de-a\,b^2c\,d^2\right)\_Z^2+\left(-4abce+4a\,c^2d+b^3e-b^2cd\right)\_Z\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/(a*e^2-b*d*e+c*d^2)/a*(1/2*(-b*c*e+c^2*d)/c*ln(c*x^4+b*x^2+a)+2*(a*c*e
-b^2*e+b*c*d-1/2*(-b*c*e+c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(
4*a*c-b^2)^(1/2)))-1/2*e^2*ln(e*x^2+d)/d/(a*e^2-b*d*e+c*d^2)+ln(x)/a/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 7.26, size = 172, normalized size = 1.03

$$-\frac{(cd-be)\log\left(cx^4+bx^2+a\right)}{4\left(acd^2-abde+a^2e^2\right)}-\frac{e^3\log\left(|x^2e+d|\right)}{2\left(cd^3e-bd^2e^2+ade^3\right)}-\frac{(bcd-b^2e+2\,ace)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(acd^2-abde+a^2e^2\right)\sqrt{-b^2+4\,ac}}+\frac{\log\left(x^2\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(c*d - b*e)*log(c*x^4 + b*x^2 + a)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/2
*e^3*log(abs(x^2*e + d))/(c*d^3*e - b*d^2*e^2 + a*d*e^3) - 1/2*(b*c*d - b^2
*e + 2*a*c*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a*c*d^2 - a*b*d*e
+ a^2*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*log(x^2)/(a*d)
```

Mupad [**B**]
time = 17.20, size = 2500, normalized size = 14.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(256*a^4*e^8*(4*a*c - b^2)^4 - 80*c^4*d^8*(4*a*c - b^2)^4 - 61*d^4*e^4*
(4*a*c - b^2)^6 + 160*b^3*c^4*d^8*(b^2 - 4*a*c)^(5/2) + 16*b^5*c^4*d^8*(b^2
 - 4*a*c)^(3/2) - 184*b^3*d^4*e^4*(b^2 - 4*a*c)^(9/2) + 370*b^5*d^4*e^4*(b^
```

2 - 4*a*c)^(7/2) + 128*b^7*d^4*e^4*(b^2 - 4*a*c)^(5/2) + 5*b^9*d^4*e^4*(b^2 - 4*a*c)^(3/2) + 128*a^3*e^8*x^2*(b^2 - 4*a*c)^(9/2) + 160*c^5*d^8*x^2*(b^2 - 4*a*c)^(7/2) - 256*a^4*b^2*e^8*(4*a*c - b^2)^3 + 32*b^2*c^4*d^8*(4*a*c - b^2)^3 + 112*b^4*c^4*d^8*(4*a*c - b^2)^2 - 144*a^2*d^2*e^6*(4*a*c - b^2)^5 + 544*b^2*d^4*e^4*(4*a*c - b^2)^5 + 382*b^4*d^4*e^4*(4*a*c - b^2)^4 - 152*b^6*d^4*e^4*(4*a*c - b^2)^3 + 71*b^8*d^4*e^4*(4*a*c - b^2)^2 + 200*c^2*d^6*e^2*(4*a*c - b^2)^5 - 13*d^3*e^5*x^2*(4*a*c - b^2)^6 + 512*a^4*b*e^8*(b^2 - 4*a*c)^(7/2) - 176*b*c^4*d^8*(b^2 - 4*a*c)^(7/2) - 26*a*d^3*e^5*(b^2 - 4*a*c)^(11/2) + 352*a^3*d*e^7*(b^2 - 4*a*c)^(9/2) - 319*b*d^4*e^4*(b^2 - 4*a*c)^(11/2) + 148*c*d^5*e^3*(b^2 - 4*a*c)^(11/2) + 168*c^3*d^7*e*(b^2 - 4*a*c)^(9/2) - 768*a*b^3*d^3*e^5*(4*a*c - b^2)^4 - 368*a*b^5*d^3*e^5*(4*a*c - b^2)^3 + 128*a^3*b^3*d*e^7*(4*a*c - b^2)^3 - 32*a*b^7*d^3*e^5*(4*a*c - b^2)^2 - 672*a^2*b^3*d^2*e^6*(b^2 - 4*a*c)^(7/2) - 272*a^2*b^5*d^2*e^6*(b^2 - 4*a*c)^(5/2) + 408*b^3*c*d^5*e^3*(4*a*c - b^2)^4 + 256*b^3*c^3*d^7*e*(4*a*c - b^2)^3 + 792*b^5*c*d^5*e^3*(4*a*c - b^2)^3 - 352*b^5*c^3*d^7*e*(4*a*c - b^2)^2 - 248*b^7*c*d^5*e^3*(4*a*c - b^2)^2 - 328*b^3*c^2*d^6*e^2*(b^2 - 4*a*c)^(7/2) + 1064*b^5*c^2*d^6*e^2*(b^2 - 4*a*c)^(5/2) + 40*b^7*c^2*d^6*e^2*(b^2 - 4*a*c)^(3/2) + 384*a^3*b*e^8*x^2*(4*a*c - b^2)^4 + 384*a^3*b^2*e^8*x^2*(b^2 - 4*a*c)^(7/2) - 512*b*c^5*d^8*x^2*(4*a*c - b^2)^3 + 576*b^2*c^5*d^8*x^2*(b^2 - 4*a*c)^(5/2) + 32*b^4*c^5*d^8*x^2*(b^2 - 4*a*c)^(3/2) - 176*a^2*d*e^7*x^2*(4*a*c - b^2)^5 - 800*b^3*d^3*e^5*x^2*(b^2 - 4*a*c)^(9/2) + 158*b^5*d^3*e^5*x^2*(b^2 - 4*a*c)^(7/2) + 56*b^7*d^3*e^5*x^2*(b^2 - 4*a*c)^(5/2) - b^9*d^3*e^5*x^2*(b^2 - 4*a*c)^(3/2) - 336*c^4*d^7*e*x^2*(4*a*c - b^2)^4 + 400*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(9/2) - 608*a^2*b^2*d^2*e^6*(4*a*c - b^2)^4 + 560*a^2*b^4*d^2*e^6*(4*a*c - b^2)^3 - 1096*b^2*c^2*d^6*e^2*(4*a*c - b^2)^4 - 872*b^4*c^2*d^6*e^2*(4*a*c - b^2)^3 + 424*b^6*c^2*d^6*e^2*(4*a*c - b^2)^2 - 128*a^3*b^3*e^8*x^2*(4*a*c - b^2)^3 + 256*b^3*c^5*d^8*x^2*(4*a*c - b^2)^2 + 584*b^2*d^3*e^5*x^2*(4*a*c - b^2)^5 - 410*b^4*d^3*e^5*x^2*(4*a*c - b^2)^4 - 256*b^6*d^3*e^5*x^2*(4*a*c - b^2)^3 - 17*b^8*d^3*e^5*x^2*(4*a*c - b^2)^2 + 296*c^2*d^5*e^3*x^2*(4*a*c - b^2)^5 + 336*a*b*d^3*e^5*(4*a*c - b^2)^5 + 384*a^3*b*d*e^7*(4*a*c - b^2)^4 - 832*a*b^2*d^3*e^5*(b^2 - 4*a*c)^(9/2) - 52*a*b^4*d^3*e^5*(b^2 - 4*a*c)^(7/2) + 144*a*b^6*d^3*e^5*(b^2 - 4*a*c)^(5/2) - 2*a*b^8*d^3*e^5*(b^2 - 4*a*c)^(3/2) - 80*a^2*b*d^2*e^6*(b^2 - 4*a*c)^(9/2) - 192*a^3*b^2*d*e^7*(b^2 - 4*a*c)^(7/2) + 96*a^3*b^4*d*e^7*(b^2 - 4*a*c)^(5/2) - 632*b*c*d^5*e^3*(4*a*c - b^2)^5 + 608*b*c^3*d^7*e*(4*a*c - b^2)^4 - 776*b*c^2*d^6*e^2*(b^2 - 4*a*c)^(9/2) + 920*b^2*c*d^5*e^3*(b^2 - 4*a*c)^(9/2) + 584*b^2*c^3*d^7*e*(b^2 - 4*a*c)^(7/2) - 384*b^4*c*d^5*e^3*(b^2 - 4*a*c)^(7/2) - 712*b^4*c^3*d^7*e*(b^2 - 4*a*c)^(5/2) - 664*b^6*c*d^5*e^3*(b^2 - 4*a*c)^(5/2) - 40*b^6*c^3*d^7*e*(b^2 - 4*a*c)^(3/2) - 20*b^8*c*d^5*e^3*(b^2 - 4*a*c)^(3/2) + 72*a*d^2*e^6*x^2*(b^2 - 4*a*c)^(11/2) - 181*b*d^3*e^5*x^2*(b^2 - 4*a*c)^(11/2) + 122*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(11/2) + 368*a^2*b*d*e^7*x^2*(b^2 - 4*a*c)^(9/2) - 1552*b*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(7/2) - 3400*b^2*c^2*d^5*e^3*x^2*(4*a*c - b^2)^4 - 4800*b^3*c^3*d^6*e^2*x^2*(4*a*c - b^2)^3 + 3448*b^4*c^2*d^5*e^3*x^2*(4*a*c - b^2)^3 + 928*b^5*c^3*d^6*e^2*x^2*(4*a*c - b^2)^2 - 536*b^6*c^2*d^5*e^3*x^2*(4*a*c - b^2)^2 - 32*

a*b*d^2*e^6*x^2*(4*a*c - b^2)^5 - 344*a*b^2*d^2*e^6*x^2*(b^2 - 4*a*c)^(9/2) - 616*a*b^4*d^2*e^6*x^2*(b^2 - 4*a*c)^(7/2) - 136*a*b^6*d^2*e^6*x^2*(b^2 - 4*a*c)^(5/2) - 160*a^2*b^3*d*e^7*x^2*(b^2 - 4*a*c)^(7/2) + 48*a^2*b^5*d*e^7*x^2*(b^2 - 4*a*c)^(5/2) - 760*b*c*d^4*e^4*x^2*(4*a*c - b^2)^5 - 1560*b*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(9/2) + 1848*b^2*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(9/2) - 2208*b^3*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(5/2) + 1452*b^4*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(7/2) - 80*b^5*c^4*d^7*e*x^2*(b^2 - 4*a*c)^(3/2) + 408*b^6*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(5/2) + 10*b^8*c*d^4*e^4*x^2*(b^2 - 4*a*c)^(3/2) - 640*a*b^3*d^2*e^6*x^2*(4*a*c - b^2)^4 + 96*a^2*b^2*d*e^7*x^2*(4*a*c - b^2)^4 + 416*a*b^5*d^2*e^6*x^2*(4*a*c - b^2)^3 + 16*a^2*b^4*d*e^7*x^2*(4*a*c - b^2)^3 + 1952*b*c^3*d^6*e^2*x^2*(4*a*c - b^2)^4 + 2216*b^3*c*d^4*e^4*x^2*(4*a*c - b^2)^4 + 2720*b^2*c^4*d^7*e*x^2*(4*a*c - b^2)^3 - 712*b^5*c*d^4*e^4*x^2*(4*a*c - b^2)^3 - 784*b^4*c^4*d^7*e*x^2*(4*a*c - b^2)^2 + 152*b^7*c*d^4*e^4*x^2*(4*a*c - b^2)^2 + 4144*b^2*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(7/2) - 4216*b^3*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(7/2) + 3056*b^4*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(5/2) - 1864*b^5*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(5/2) + 80*b^6*c^3*d^6*e^2*x^2*(b^2 - 4*a*c)^(3/2) - 40*b^7*c^2*d^5*e^3*x^2*(b^2 - 4*a*c)^(3/2))*(d*((b^2*c)/4 - a*c^2 + (b*c*(b^2 - 4*a*c)^(1/2))/4) - (b^3*e)/4 - (b^2*e*(b^2 - 4*a*c)^(1/2))/4 + (a*c*e*(b^2 - 4*a*c...

3.301 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=205

$$-\frac{1}{2adx^2}-\frac{(b^2cd-2ac^2d-b^3e+3abce)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log(d+ex^2)}{2d^2(cd^2-bde+ae^2)}+\frac{(bc$$

```
[Out] -1/2/a/d/x^2-(a*e+b*d)*ln(x)/a^2/d^2+1/2*e^3*ln(e*x^2+d)/d^2/(a*e^2-b*d*e+c
*d^2)+1/4*(a*c*e-b^2*e+b*c*d)*ln(c*x^4+b*x^2+a)/a^2/(a*e^2-b*d*e+c*d^2)-1/2
*(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2)^(1/2)
)/a^2/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1265, 907, 648, 632, 212, 642}

$$\frac{(ace+b^2(-e)+bcd)\log(a+bx^2+cx^4)}{4a^2(ae^2-bde+cd^2)}-\frac{(3abce-2ac^2d+b^3(-e)+b^2cd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,(ae^2-bde+cd^2)}-\frac{\log(x)(ae+bd)}{a^2d^2}+\frac{e^3\log(d+ex^2)}{2d^2(ae^2-bde+cd^2)}-\frac{1}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*1/(a*d*x^2) - ((b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*ArcTanh[(b +
2*c*x^2)/Sqrt[b^2 - 4*a*c]])/(2*a^2*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^
2)) - ((b*d + a*e)*Log[x])/(a^2*d^2) + (e^3*Log[d + e*x^2])/(2*d^2*(c*d^2 -
 b*d*e + a*e^2)) + ((b*c*d - b^2*e + a*c*e)*Log[a + b*x^2 + c*x^4])/(4*a^2*
(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^2}+\frac{-bd-ae}{a^2d^2x}+\frac{e^4}{d^2\left(cd^2-bde+ae^2\right)(d+ex)}+\frac{b^2c\ldots}{\ldots}\right.\right.$$

$$= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\text{Subst}\left(\int \frac{b^2cd-\ldots}{\ldots}\right.}{\ldots}$$

$$= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+a\ldots\right)}{4a^2\ldots}$$

$$= -\frac{1}{2adx^2}-\frac{(bd+ae)\log(x)}{a^2d^2}+\frac{e^3\log\left(d+ex^2\right)}{2d^2\left(cd^2-bde+ae^2\right)}+\frac{\left(bcd-b^2e+a\ldots\right)}{4a^2\left(cd\ldots\right)}$$

$$= -\frac{1}{2adx^2}-\frac{\left(b^2cd-2ac^2d-b^3e+3abce\right)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^2\sqrt{b^2-4ac}\,\left(cd^2-bde+ae^2\right)}-\frac{(bd\ldots}{\ldots}$$

Mathematica [A]

time = 0.22, size = 331, normalized size = 1.61

$$\frac{1}{4}\left(-\frac{2}{adx^2}-\frac{4(bd+ae)\log(x)}{a^2d^2}+\frac{\left(b^3e-bc\left(\sqrt{b^2-4ac}\,d+3ae\right)+ac\left(2cd-\sqrt{b^2-4ac}\,e\right)+b^2\left(-cd+\sqrt{b^2-4ac}\,e\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{a^2\sqrt{b^2-4ac}\,(-cd^2+e(bd-ae))}+\frac{\left(-b^3e+bc\left(-\sqrt{b^2-4ac}\,d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ac}\,e\right)-ac\left(2cd+\sqrt{b^2-4ac}\,e\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{a^2\sqrt{b^2-4ac}\,(-cd^2+e(bd-ae))}+\frac{2e^3\log(d+ex^2)}{cd^4+d^2e(-bd+ae)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-2/(a*d*x^2) - (4*(b*d + a*e)*Log[x])/(a^2*d^2) + ((b^3*e - b*c*(Sqrt[b^2
- 4*a*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqr
t[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*
a*c]*(-(c*d^2) + e*(b*d - a*e))) + ((-(b^3*e) + b*c*(-(Sqrt[b^2 - 4*a*c]*d)
 + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c
]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^2*Sqrt[b^2 - 4*a*c]*(-(c*d^2
) + e*(b*d - a*e))) + (2*e^3*Log[d + e*x^2])/(c*d^4 + d^2*e*(-(b*d) + a*e))
)/4
```

Maple [A]

time = 0.20, size = 215, normalized size = 1.05

method	result
default	$\frac{\frac{\left(a\,c^2e-b^2ce+b\,c^2d\right)\ln\left(c\,x^4+b\,x^2+a\right)}{2c}+\frac{2\left(2abce-a\,c^2d-b^3e+b^2cd-\frac{\left(a\,c^2e-b^2ce+b\,c^2d\right)b}{2c}\right)\arctan\left(\frac{2c\,x^2+b}{\sqrt{4ac-b^2}}\right)}{\sqrt{4ac-b^2}}}{2(a\,e^2-deb+c\,d^2)a^2}+\frac{e^3\ln\left(e\,x^2+d\right)}{2d^2(a\,e^2-deb+c\,d^2}$
risch	$-\frac{1}{2ad\,x^2}-\frac{e\ln(x)}{a\,d^2}-\frac{\ln(x)b}{a^2d}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^4c\,e^2-a^3b^2e^2-4a^3bcde+4a^3c^2d^2+a^2b^3de-a^2b^2c\,d^2\right)\_Z^2+\left(-4a^2c^2e+5a\,b^2ce-4ab\,c^2\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(a*e^2-b*d*e+c*d^2)/a^2*(1/2*(a*c^2*e-b^2*c*e+b*c^2*d)/c*ln(c*x^4+b*x^2
+a)+2*(2*a*b*c*e-a*c^2*d-b^3*e+b^2*c*d-1/2*(a*c^2*e-b^2*c*e+b*c^2*d)*b/c)/(
4*a*c-b^2)^(1/2)*arctan((2*c*x^2+b)/(4*a*c-b^2)^(1/2)))+1/2*e^3*ln(e*x^2+d)
/d^2/(a*e^2-b*d*e+c*d^2)-1/2/a/d/x^2+1/a^2/d^2*(-a*e-b*d)*ln(x)
```

Maxima [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 4.37, size = 237, normalized size = 1.16

$$\frac{(bcd-b^2e+ace)\log\left(cx^4+bx^2+a\right)}{4\left(a^2cd^2-a^2bde+a^3e^2\right)}+\frac{e^4\log\left(|x^2e+d|\right)}{2\left(cd^4e-bd^3e^2+ad^2e^3\right)}+\frac{(b^2cd-2\,ac^2d-b^3e+3\,abce)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\left(a^2cd^2-a^2bde+a^3e^2\right)\sqrt{-b^2+4\,ac}}-\frac{(bd+ae)\log\left(x^2\right)}{2\,a^2d^2}+\frac{bdx^2+ax^2e-ad}{2\,a^2d^2x^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(b*c*d - b^2*e + a*c*e)*log(c*x^4 + b*x^2 + a)/(a^2*c*d^2 - a^2*b*d*e +
 a^3*e^2) + 1/2*e^4*log(abs(x^2*e + d))/(c*d^4*e - b*d^3*e^2 + a*d^2*e^3) +
 1/2*(b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)*arctan((2*c*x^2 + b)/sqrt(-b
^2 + 4*a*c))/((a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*sqrt(-b^2 + 4*a*c)) - 1/2*(
b*d + a*e)*log(x^2)/(a^2*d^2) + 1/2*(b*d*x^2 + a*x^2*e - a*d)/(a^2*d^2*x^2)
```

Mupad [**B**]
time = 62.95, size = 2500, normalized size = 12.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4*b^7*d^2*
e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e
^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 - 4*b^5*c^
2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3*e^4 - 76*
a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20*a*b^2*c^
4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b*c^4*d^4*e
^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6))/(a^2*d^2) + (((16*c^2*e^
2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^4*b^2*c*e
^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c^3*d^6*e
- 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b^5*c^2*d^5
*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3*e^4 + 48*a
^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*e - 12*a*b^5*c*d^3*e^4 - 15*a^3*b^3*c
*d*e^6 + 28*a^4*b*c^2*d*e^6 - 27*a*b^3*c^3*d^5*e^2 + 27*a*b^4*c^2*d^4*e^3 +
 27*a^2*b*c^4*d^5*e^2 - 18*a^2*b^4*c*d^2*e^5 - 52*a^3*b*c^3*d^3*e^4))/(a*d)
 + (8*c^2*e^2*x^2*(10*a*c^6*d^7 + a^2*b^5*e^7 + b^2*c^5*d^7 + b^7*d^2*e^5 -
 11*a^3*b^3*c*e^7 + 28*a^4*b*c^2*e^7 - 88*a^4*c^3*d*e^6 - 6*b^3*c^4*d^6*e -
 6*b^6*c*d^3*e^4 + 26*a^2*c^5*d^5*e^2 + 88*a^3*c^4*d^3*e^4 + 5*b^4*c^3*d^5*
e^2 + 5*b^5*c^2*d^4*e^3 + 12*a*b^6*d*e^6 - 3*a*b*c^5*d^6*e - 110*a^2*b^2*c^
3*d^3*e^4 + 155*a^2*b^3*c^2*d^2*e^5 - 28*a*b^5*c*d^2*e^5 - 93*a^2*b^4*c*d*e
^6 - 10*a*b^2*c^4*d^5*e^2 - 27*a*b^3*c^3*d^4*e^3 + 46*a*b^4*c^2*d^3*e^4 + 1
5*a^2*b*c^4*d^4*e^3 - 236*a^3*b*c^3*d^2*e^5 + 202*a^3*b^2*c^2*d*e^6))/(a*d)
 + (4*c^2*e^2*(a*b^2*e^3 + b*c^2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^
2*e - 2*b^2*c*d^2*e - 3*a*b*c*d*e^2)*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4
*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^
(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2))*(a*b^3
*d^2*e^2 + a^2*b^2*d*e^3 + 4*a^2*c^2*d^3*e - 10*a*c^3*d^4*x^2 - 12*a^3*c*e^
4*x^2 + 3*a^2*b^2*e^4*x^2 + 3*b^2*c^2*d^4*x^2 + 3*b^4*d^2*e^2*x^2 + a*b*c^2
*d^4 - 4*a^3*c*d*e^3 - 2*a*b^2*c*d^3*e - 14*a^2*c^2*d^2*e^2*x^2 - 3*a^2*b*c
*d^2*e^2 - 4*a*b^3*d*e^3*x^2 - 6*b^3*c*d^3*e*x^2 - 8*a*b^2*c*d^2*e^2*x^2 +
22*a*b*c^2*d^3*e*x^2 + 16*a^2*b*c*d*e^3*x^2))/(a^2*(4*a*c - b^2)*(a*e^2 + c
*d^2 - b*d*e)))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2*e - b^3*c*d
+ 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) - b^2*c*d*(b^2
- 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*(a*e^
2 + c*d^2 - b*d*e)) - (4*c^2*e^2*x^2*(6*a*b^6*e^7 + 6*b*c^6*d^7 + 6*b^7*d*e
^6 - 16*a^4*c^3*e^7 - 44*a^2*b^4*c*e^7 - 8*b^2*c^5*d^6*e - 8*b^6*c*d^2*e^5
+ 84*a^3*b^2*c^2*e^7 + 30*a^2*c^5*d^4*e^3 - 2*b^3*c^4*d^5*e^2 + 8*b^4*c^3*d
^4*e^3 - 2*b^5*c^2*d^3*e^4 + 11*a*c^6*d^6*e - 47*a*b^5*c*d*e^6 - 96*a^2*b^2
*c^3*d^2*e^5 + 14*a*b*c^5*d^5*e^2 - 94*a^3*b*c^3*d*e^6 - 35*a*b^2*c^4*d^4*e
^3 + 7*a*b^3*c^3*d^3*e^4 + 56*a*b^4*c^2*d^2*e^5 - 17*a^2*b*c^4*d^3*e^4 + 11
7*a^2*b^3*c^2*d*e^6))/(a^2*d^2))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2
*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2
) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4
*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) + (4*c^2*e^2*x^2*(b^7*e^7 + c^7*d^7 -
```

```
6*a^3*b*c^3*e^7 + 2*a*c^6*d^5*e^2 - 4*a^3*c^4*d*e^6 + 14*a^2*b^3*c^2*e^7 +
6*a^2*c^5*d^3*e^4 + b^3*c^4*d^4*e^3 + b^4*c^3*d^3*e^4 - 7*a*b^5*c*e^7 + 2*a
*b^4*c^2*d*e^6 - 6*a*b^2*c^4*d^3*e^4 + 3*a*b^3*c^3*d^2*e^5 - 9*a^2*b*c^4*d^
2*e^5 - 5*a^2*b^2*c^3*d*e^6))/(a^3*d^3) + (4*c^2*e^2*(a*e + b*d)*(b^3*e^3 +
 c^3*d^3 - 3*a*b*c*e^3)^2)/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) +
4*a^2*c^2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)
^(1/2) - b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*a
^2*(4*a*c - b^2)*(a*e^2 + c*d^2 - b*d*e)) - (2*c^5*e^5*x^2*(b^3*e^3 + c^3*d
^3 - 3*a*b*c*e^3))/(a^3*d^3))*(b^4*e + b^3*e*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^
2*e - b^3*c*d + 4*a*b*c^2*d - 5*a*b^2*c*e + 2*a*c^2*d*(b^2 - 4*a*c)^(1/2) -
 b^2*c*d*(b^2 - 4*a*c)^(1/2) - 3*a*b*c*e*(b^2 - 4*a*c)^(1/2)))/(4*(4*a^4*c*
e^2 - a^3*b^2*e^2 + 4*a^3*c^2*d^2 - a^2*b^2*c*d^2 + a^2*b^3*d*e - 4*a^3*b*c
*d*e)) + (log((((((4*c^2*e^2*(a*c^6*d^7 - 4*a^2*b^5*e^7 - 4*b^2*c^5*d^7 - 4
*b^7*d^2*e^5 + 28*a^3*b^3*c*e^7 - 48*a^4*b*c^2*e^7 + 8*b^3*c^4*d^6*e + 8*b^
6*c*d^3*e^4 - 16*a^2*c^5*d^5*e^2 + 16*a^3*c^4*d^3*e^4 - 4*b^4*c^3*d^5*e^2 -
 4*b^5*c^2*d^4*e^3 - 7*a*b^6*d*e^6 - 20*a*b*c^5*d^6*e + 56*a^2*b^2*c^3*d^3*
e^4 - 76*a^2*b^3*c^2*d^2*e^5 + 32*a*b^5*c*d^2*e^5 + 46*a^2*b^4*c*d*e^6 + 20
*a*b^2*c^4*d^5*e^2 + 6*a*b^3*c^3*d^4*e^3 - 44*a*b^4*c^2*d^3*e^4 + 22*a^2*b*
c^4*d^4*e^3 + 48*a^3*b*c^3*d^2*e^5 - 75*a^3*b^2*c^2*d*e^6))/(a^2*d^2) + (((
16*c^2*e^2*(a^3*b^4*e^7 + 16*a^5*c^2*e^7 + b^3*c^4*d^7 + b^7*d^3*e^4 - 8*a^
4*b^2*c*e^7 + 2*a*b^6*d^2*e^5 + 2*a^2*b^5*d*e^6 - 4*a^2*c^5*d^6*e - 4*b^4*c
^3*d^6*e - 4*b^6*c*d^4*e^3 + 20*a^3*c^4*d^4*e^3 - 32*a^4*c^3*d^2*e^5 + 6*b^
5*c^2*d^5*e^2 - a*b*c^5*d^7 - 52*a^2*b^2*c^3*d^4*e^3 + 45*a^2*b^3*c^2*d^3*e
^4 + 48*a^3*b^2*c^2*d^2*e^5 + 11*a*b^2*c^4*d^6*...
```

3.302 $\int \frac{1}{x^5(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=268

$$-\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\,(cd^2-bde+ae^2)}+\frac{(b^2d^2+abde-a(cd^2-}{a^3d^3}$$

```
[Out] -1/4/a/d/x^4+1/2*(a*e+b*d)/a^2/d^2/x^2+(b^2*d^2+a*b*d*e-a*(-a*e^2+c*d^2))*l
n(x)/a^3/d^3-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)-1/4*(2*a*b*c*e-a*c
^2*d-b^3*e+b^2*c*d)*ln(c*x^4+b*x^2+a)/a^3/(a*e^2-b*d*e+c*d^2)+1/2*(-2*a^2*c
^2*e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)*arctanh((2*c*x^2+b)/(-4*a*c+b^2
)^(1/2))/a^3/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.37, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1265, 907, 648, 632, 212, 642}

$$\frac{\log(x)\left(abde-a(cd^2-ae^2)+b^2d^2\right)}{a^3d^3}-\frac{(2abce-ac^2d+b^3(-e)+b^2cd)\log\left(a+bx^2+cx^4\right)}{4a^3\left(ae^2-bde+cd^2\right)}+\frac{ae+bd}{2a^2d^2x^2}+\frac{(-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd)\tanh^{-1}\left(\frac{b+2cx^2}{\sqrt{b^2-4ac}}\right)}{2a^3\sqrt{b^2-4ac}\,\left(ae^2-bde+cd^2\right)}-\frac{e^4\log\left(d+ex^2\right)}{2d^3\left(ae^2-bde+cd^2\right)}-\frac{1}{4adx^4}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*1/(a*d*x^4) + (b*d + a*e)/(2*a^2*d^2*x^2) + ((b^3*c*d - 3*a*b*c^2*d -
b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)*ArcTanh[(b + 2*c*x^2)/Sqrt[b^2 - 4*a*c]]
)/(2*a^3*Sqrt[b^2 - 4*a*c]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d^2 + a*b*d*e -
 a*(c*d^2 - a*e^2))*Log[x])/(a^3*d^3) - (e^4*Log[d + e*x^2])/(2*d^3*(c*d^2
- b*d*e + a*e^2)) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*Log[a + b*x^2
+ c*x^4])/(4*a^3*(c*d^2 - b*d*e + a*e^2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 632

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 648

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 907

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && I
ntegerQ[p] && ((EqQ[p, 1] && IntegersQ[m, n]) || (ILtQ[m, 0] && ILtQ[n, 0])
)
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^5\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx=\frac{1}{2}\text{Subst}\left(\int \frac{1}{x^3(d+ex)\left(a+bx+cx^2\right)}\,dx,x,x^2\right)$$

$$=\frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{adx^3}+\frac{-bd-ae}{a^2d^2x^2}+\frac{b^2d^2+abde-a(cd^2-ae^2)}{a^3d^3x}-\frac{}{d^3\left(cd\right.}\right.\right.$$

$$=-\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{\left(b^2d^2+abde-a(cd^2-ae^2)\right)\log(x)}{a^3d^3}-\frac{e^4\log\left(\right.}{2d^3\left(cd^2-\right.}$$

$$=-\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{\left(b^2d^2+abde-a(cd^2-ae^2)\right)\log(x)}{a^3d^3}-\frac{e^4\log\left(\right.}{2d^3\left(cd^2-\right.}$$

$$=-\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{\left(b^2d^2+abde-a(cd^2-ae^2)\right)\log(x)}{a^3d^3}-\frac{e^4\log\left(\right.}{2d^3\left(cd^2-\right.}$$

$$=-\frac{1}{4adx^4}+\frac{bd+ae}{2a^2d^2x^2}+\frac{\left(b^3cd-3abc^2d-b^4e+4ab^2ce-2a^2c^2e\right)\tanh^{-1}\left(\right.}{2a^3\sqrt{b^2-4ac}\left(cd^2-bde+ae^2\right)}$$

Mathematica [A]
time = 0.27, size = 426, normalized size = 1.59

$$\frac{1}{4}\left(-\frac{1}{adx^4}+\frac{2(bd+ae)}{a^2d^2x^2}+\frac{4\left(b^2d^2+abde+a\left(-cd^2+ae^2\right)\right)\log(x)}{a^3d^3}-\frac{\left(b^4e+ac^2\left(\sqrt{b^2-4ac}\,d+2ae\right)-b^2c\left(\sqrt{b^2-4ac}\,d+4ae\right)+abc\left(3cd-2\sqrt{b^2-4ac}\,e\right)+b^3\left(-cd+\sqrt{b^2-4ac}\,e\right)\right)\log\left(b-\sqrt{b^2-4ac}+2cx^2\right)}{a^3\sqrt{b^2-4ac}\left(-cd^2+e(bd-ae)\right)}-\frac{\left(-b^4e+ac^2\left(\sqrt{b^2-4ac}\,d-2ae\right)+b^2c\left(-\sqrt{b^2-4ac}\,d+4ae\right)+b^3\left(cd+\sqrt{b^2-4ac}\,e\right)-abc\left(3cd+2\sqrt{b^2-4ac}\,e\right)\right)\log\left(b+\sqrt{b^2-4ac}+2cx^2\right)}{a^3\sqrt{b^2-4ac}\left(-cd^2+e(bd-ae)\right)}-\frac{2e^4\log\left(d+ex^2\right)}{cd^5+d^3e(-bd+ae)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(1/(a*d*x^4)) + (2*(b*d + a*e))/(a^2*d^2*x^2) + (4*(b^2*d^2 + a*b*d*e + a
*(-(c*d^2) + a*e^2))*Log[x])/(a^3*d^3) - ((b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]
*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b
^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2 - 4*a*c]*e))*Log[b - Sqrt[b^2 - 4*a
*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*d^2) + e*(b*d - a*e))) - ((-(b^
4*e) + a*c^2*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d)
+ 4*a*e) + b^3*(c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*
a*c]*e))*Log[b + Sqrt[b^2 - 4*a*c] + 2*c*x^2])/(a^3*Sqrt[b^2 - 4*a*c]*(-(c*
d^2) + e*(b*d - a*e))) - (2*e^4*Log[d + e*x^2])/(c*d^5 + d^3*e*(-(b*d) + a*
e)))/4
```

Maple [A]
time = 0.23, size = 284, normalized size = 1.06

method	result

default	$\frac{\frac{\left(-2ab\,c^{2}e+a\,c^{3}d+ce\,b^{3}-b^{2}c^{2}d\right)\ln\left(c\,x^{4}+b\,x^{2}+a\right)}{2c}+\frac{2\left(a^{2}c^{2}e-3a\,b^{2}ce+2ab\,c^{2}d+b^{4}e-b^{3}cd-\frac{\left(-2ab\,c^{2}e+a\,c^{3}d+ce\,b^{3}-b^{2}c^{2}d\right)b}{2c}\right)\arctan\left(\frac{2c\,x}{\sqrt{4ac}}\right.}{\sqrt{4ac-b^{2}}}}{2(a\,e^{2}-deb+c\,d^{2})a^{3}}$
risch	$\frac{\frac{(ae+bd)x^{2}}{2a^{2}d^{2}}-\frac{1}{4da}}{x^{4}}+\frac{\ln(x)e^{2}}{d^{3}a}+\frac{\ln(x)be}{d^{2}a^{2}}-\frac{\ln(x)c}{d\,a^{2}}+\frac{\ln(x)b^{2}}{d\,a^{3}}-\frac{e^{4}\ln(e\,x^{2}+d)}{2d^{3}(a\,e^{2}-deb+c\,d^{2})}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\left(4a^{5}c\,e^{2}-a^{4}b^{2}e^{2}-4a^{4}bcde\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/2/(a*e^2-b*d*e+c*d^2)/a^3*(1/2*(-2*a*b*c^2*e+a*c^3*d+b^3*c*e-b^2*c^2*d)/c
*ln(c*x^4+b*x^2+a)+2*(a^2*c^2*e-3*a*b^2*c*e+2*a*b*c^2*d+b^4*e-b^3*c*d-1/2*(
-2*a*b*c^2*e+a*c^3*d+b^3*c*e-b^2*c^2*d)*b/c)/(4*a*c-b^2)^(1/2)*arctan((2*c*
x^2+b)/(4*a*c-b^2)^(1/2)))-1/2*e^4*ln(e*x^2+d)/d^3/(a*e^2-b*d*e+c*d^2)-1/4/
a/d/x^4-1/2*(-a*e-b*d)/a^2/d^2/x^2+(a^2*e^2+a*b*d*e-a*c*d^2+b^2*d^2)/d^3/a^
3*ln(x)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a*c-b^2>0)', see `assume?` for mo
re deta

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**5/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [A]
time = 4.45, size = 332, normalized size = 1.24

$$-\frac{(b^2cd - ac^2d - b^3e + 2\,abce)\log\left(cx^4 + bx^2 + a\right)}{4\,(a^3cd^2 - a^3bde + a^4e^2)} - \frac{e^5\log\left(|x^2e + d|\right)}{2\,(cd^5e - bd^4e^2 + ad^3e^3)} - \frac{(b^3cd - 3\,abc^2d - b^4e + 4\,ab^2ce - 2\,a^2c^2e)\arctan\left(\frac{2\,cx^2+b}{\sqrt{-b^2+4\,ac}}\right)}{2\,(a^3cd^2 - a^3bde + a^4e^2)\sqrt{-b^2+4\,ac}} + \frac{(b^2d^2 - acd^2 + abde + a^2e^2)\log\left(x^2\right)}{2\,a^3d^3} - \frac{3\,b^2d^2x^4 - 3\,acd^2x^4 + 3\,abdx^4e + 3\,a^2x^4e^2 - 2\,abd^2x^2 - 2\,a^2dx^2e + a^2d^2}{4\,a^3d^3x^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^5/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e)*log(c*x^4 + b*x^2 + a)/(a^3*c*
d^2 - a^3*b*d*e + a^4*e^2) - 1/2*e^5*log(abs(x^2*e + d))/(c*d^5*e - b*d^4*e
^2 + a*d^3*e^3) - 1/2*(b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*
c^2*e)*arctan((2*c*x^2 + b)/sqrt(-b^2 + 4*a*c))/((a^3*c*d^2 - a^3*b*d*e + a
^4*e^2)*sqrt(-b^2 + 4*a*c)) + 1/2*(b^2*d^2 - a*c*d^2 + a*b*d*e + a^2*e^2)*l
og(x^2)/(a^3*d^3) - 1/4*(3*b^2*d^2*x^4 - 3*a*c*d^2*x^4 + 3*a*b*d*x^4*e + 3*
a^2*x^4*e^2 - 2*a*b*d^2*x^2 - 2*a^2*d*x^2*e + a^2*d^2)/(a^3*d^3*x^4)
```

Mupad [B]
time = 144.76, size = 2500, normalized size = 9.33

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^5*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((c^8*e^8*(a^2*e^2 + b^2*d^2 - a*c*d^2 + a*b*d*e))/(a^6*d^6) - (c^9*e^9
*x^2)/(a^5*d^5) - (((c^5*e^5*(4*a^3*b^3*e^6 + 4*b^3*c^3*d^6 + 4*b^6*d^3*e^3
 + 8*a*b^5*d^2*e^4 + 8*a^2*b^4*d*e^5 + 4*a^2*c^4*d^5*e + 16*a^4*c^2*d*e^5 -
 19*a^3*c^3*d^3*e^3 - 4*a*b*c^4*d^6 - 12*a^4*b*c*e^6 + 36*a^2*b^2*c^2*d^3*e
^3 - 24*a*b^4*c*d^3*e^3 - 32*a^3*b^2*c*d*e^5 - 36*a^2*b^3*c*d^2*e^4 + 28*a^
3*b*c^2*d^2*e^4))/(a^6*d^6) - (((4*a^4*b^6*c^2*e^12 - 24*a^5*b^4*c^3*e^12 +
 36*a^6*b^2*c^4*e^12 - 4*a^3*c^9*d^8*e^4 + 64*a^4*c^8*d^6*e^6 - 144*a^5*c^7
*d^4*e^8 + 96*a^6*c^6*d^2*e^10 + 4*b^4*c^8*d^10*e^2 + 8*b^7*c^5*d^7*e^5 + 4
*b^10*c^2*d^4*e^8 + 64*a^2*b^3*c^7*d^7*e^5 - 8*a^2*b^4*c^6*d^6*e^6 - 8*a^2*
b^5*c^5*d^5*e^7 + 172*a^2*b^6*c^4*d^4*e^8 - 112*a^2*b^7*c^3*d^3*e^9 + 16*a^
2*b^8*c^2*d^2*e^10 - 72*a^3*b^2*c^7*d^6*e^6 + 56*a^3*b^3*c^6*d^5*e^7 - 312*
a^3*b^4*c^5*d^4*e^8 + 348*a^3*b^5*c^4*d^3*e^9 - 132*a^3*b^6*c^3*d^2*e^10 +
324*a^4*b^2*c^6*d^4*e^8 - 428*a^4*b^3*c^5*d^3*e^9 + 344*a^4*b^4*c^4*d^2*e^1
0 - 300*a^5*b^2*c^5*d^2*e^10 - 96*a^6*b*c^5*d*e^11 - 4*a*b^2*c^9*d^10*e^2 -
 4*a*b^3*c^8*d^9*e^3 - 48*a*b^5*c^6*d^7*e^5 + 8*a*b^6*c^5*d^6*e^6 - 44*a*b^
8*c^3*d^4*e^8 + 12*a*b^9*c^2*d^3*e^9 + 8*a^2*b*c^9*d^9*e^3 - 24*a^3*b*c^8*d
```

^7*e^5 + 12*a^3*b^7*c^2*d*e^11 - 88*a^4*b*c^7*d^5*e^7 - 88*a^4*b^5*c^3*d*e^11 + 228*a^5*b*c^6*d^3*e^9 + 188*a^5*b^3*c^4*d*e^11)/(a^6*d^6) + (x^2*(32*a^6*c^6*d*e^11 - 24*a^6*b*c^5*e^12 + 4*a^3*b^7*c^2*e^12 - 28*a^4*b^5*c^3*e^12 + 56*a^5*b^3*c^4*e^12 + 2*a^3*c^9*d^7*e^5 + 104*a^4*c^8*d^5*e^7 - 156*a^5*c^7*d^3*e^9 + 4*b^3*c^9*d^10*e^2 + 4*b^6*c^6*d^7*e^5 + 4*b^7*c^5*d^6*e^6 + 4*b^10*c^2*d^3*e^9 + 8*a^2*b^2*c^8*d^7*e^5 + 40*a^2*b^3*c^7*d^6*e^6 - 12*a^2*b^5*c^5*d^4*e^8 + 180*a^2*b^6*c^4*d^3*e^9 - 116*a^2*b^7*c^3*d^2*e^10 - 92*a^3*b^2*c^7*d^5*e^7 + 84*a^3*b^3*c^6*d^4*e^8 - 350*a^3*b^4*c^5*d^3*e^9 + 388*a^3*b^5*c^4*d^2*e^10 + 348*a^4*b^2*c^6*d^3*e^9 - 524*a^4*b^3*c^5*d^2*e^10 - 4*a*b^2*c^9*d^9*e^3 - 20*a*b^4*c^7*d^7*e^5 - 20*a*b^5*c^6*d^6*e^6 + 4*a*b^6*c^5*d^5*e^7 - 44*a*b^8*c^3*d^3*e^9 + 12*a*b^9*c^2*d^2*e^10 + 8*a^2*b*c^9*d^8*e^4 + 12*a^2*b^8*c^2*d*e^11 - 36*a^3*b*c^8*d^6*e^6 - 100*a^3*b^6*c^3*d*e^11 - 132*a^4*b*c^7*d^4*e^8 + 264*a^4*b^4*c^4*d*e^11 + 264*a^5*b*c^6*d^2*e^10 - 224*a^5*b^2*c^5*d*e^11))/(a^6*d^6) + (((192*a^6*b*c^4*e^11 - 256*a^6*c^5*d*e^10 + 16*a^4*b^5*c^2*e^11 - 112*a^5*b^3*c^3*e^11 + 60*a^3*c^8*d^7*e^4 - 320*a^4*c^7*d^5*e^6 + 480*a^5*c^6*d^3*e^8 + 16*b^4*c^7*d^9*e^2 - 32*b^5*c^6*d^8*e^3 + 16*b^6*c^5*d^7*e^4 + 16*b^7*c^4*d^6*e^5 - 32*b^8*c^3*d^5*e^6 + 16*b^9*c^2*d^4*e^7 + 16*a^2*b^2*c^7*d^7*e^4 + 120*a^2*b^3*c^6*d^6*e^5 - 816*a^2*b^4*c^5*d^5*e^6 + 880*a^2*b^5*c^4*d^4*e^7 - 424*a^2*b^6*c^3*d^3*e^8 + 56*a^2*b^7*c^2*d^2*e^9 + 832*a^3*b^2*c^6*d^5*e^6 - 1424*a^3*b^3*c^5*d^4*e^7 + 1340*a^3*b^4*c^4*d^3*e^8 - 464*a^3*b^5*c^3*d^2*e^9 - 1512*a^4*b^2*c^5*d^3*e^8 + 1144*a^4*b^3*c^4*d^2*e^9 - 20*a*b^2*c^8*d^9*e^2 + 96*a*b^3*c^7*d^8*e^3 - 64*a*b^4*c^6*d^7*e^4 - 88*a*b^5*c^5*d^6*e^5 + 288*a*b^6*c^4*d^5*e^6 - 208*a*b^7*c^3*d^4*e^7 + 44*a*b^8*c^2*d^3*e^8 - 40*a^2*b*c^8*d^8*e^3 - 88*a^3*b*c^7*d^6*e^5 + 44*a^3*b^6*c^2*d*e^10 + 704*a^4*b*c^6*d^4*e^7 - 328*a^4*b^4*c^3*d*e^10 - 736*a^5*b*c^5*d^2*e^9 + 684*a^5*b^2*c^4*d*e^10)/(a^4*d^4) + (((256*a^6*c^4*e^10 + 16*a^4*b^4*c^2*e^10 - 128*a^5*b^2*c^3*e^10 - 192*a^3*c^7*d^6*e^4 + 448*a^4*c^6*d^4*e^6 - 512*a^5*c^5*d^2*e^8 + 16*b^4*c^6*d^8*e^2 - 64*b^5*c^5*d^7*e^3 + 96*b^6*c^4*d^6*e^4 - 64*b^7*c^3*d^5*e^5 + 16*b^8*c^2*d^4*e^6 + 768*a^2*b^2*c^6*d^6*e^4 - 1200*a^2*b^3*c^5*d^5*e^5 + 896*a^2*b^4*c^4*d^4*e^6 - 320*a^2*b^5*c^3*d^3*e^7 + 32*a^2*b^6*c^2*d^2*e^8 - 1392*a^3*b^2*c^5*d^4*e^6 + 1024*a^3*b^3*c^4*d^3*e^7 - 288*a^3*b^4*c^3*d^2*e^8 + 768*a^4*b^2*c^4*d^2*e^8 + 448*a^5*b*c^4*d*e^9 - 32*a*b^2*c^7*d^8*e^2 + 240*a*b^3*c^6*d^7*e^3 - 528*a*b^4*c^5*d^6*e^4 + 496*a*b^5*c^4*d^5*e^5 - 208*a*b^6*c^3*d^4*e^6 + 32*a*b^7*c^2*d^3*e^7 - 176*a^2*b*c^7*d^7*e^3 + 848*a^3*b*c^6*d^5*e^5 + 32*a^3*b^5*c^2*d*e^9 - 1024*a^4*b*c^5*d^3*e^7 - 240*a^4*b^3*c^3*d*e^9)/(a^2*d^2) + (8*c^2*e^2*x^2*(a^3*b^5*e^8 + b^3*c^5*d^8 + b^8*d^3*e^5 - 11*a^4*b^3*c*e^8 + 28*a^5*b*c^2*e^8 + 8*a*b^7*d^2*e^6 + 8*a^2*b^6*d*e^7 - 30*a^2*c^6*d^7*e - 24*a^5*c^3*d*e^7 - 6*b^4*c^4*d^7*e - 6*b^7*c*d^4*e^4 - 18*a^3*c^5*d^5*e^3 + 180*a^4*c^4*d^3*e^5 + 5*b^5*c^3*d^6*e^2 + 5*b^6*c^2*d^5*e^3 + 5*a*b*c^6*d^8 + 13*a^2*b^2*c^4*d^5*e^3 - 82*a^2*b^3*c^3*d^4*e^4 + 110*a^2*b^4*c^2*d^3*e^5 - 277*a^3*b^2*c^3*d^3*e^5 + 328*a^3*b^3*c^2*d^2*e^6 + 15*a*b^2*c^5*d^7*e - 17*a*b^6*c*d^3*e^5 - 57*a^3*b^4*c*d*e^7 - 27*a*b^3*c^4*d^6*e^2 - 24*a*b^4*c^3*d^5*e^3 + 40*a*b^5*c^2*d^4*e^4 + 67*a^2*b*c^5*d^6*e^2 - 92*a^2*b^5*c*d^2*e^6 + 72*a^3*b*c^4*d^4*e^4 - 352*a^4*b*c^3

```
*d^2*e^6 + 106*a^4*b^2*c^2*d*e^7))/(a^2*d^2) - (4*c^2*e^2*(a*b^2*e^3 + b*c^
2*d^3 - 4*a^2*c*e^3 + b^3*d*e^2 + 4*a*c^2*d^2*e - 2*b^2*c*d^2*e - 3*a*b*c*d
*e^2)*(b^4*e*(b^2 - 4*a*c)^(1/2) - b^5*e + 4*a^2*c^3*d + b^4*c*d + 6*a*b^3*
c*e - b^3*c*d*(b^2 - 4*a*c)^(1/2) - 5*a*b^2*c^2*d - 8*a^2*b*c^2*e + 2*a^2*c
^2*e*(b^2 - 4*a*c)^(1/2) + 3*a*b*c^2*d*(b^2 - 4*a*c)^(1/2) - 4*a*b^2*c*e*(b
^2 - 4*a*c)^(1/2))*(a*b^3*d^2*e^2 + a^2*b^2*d*e...
```

3.303 $\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=387

$$-\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}-\frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+3a^2bce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$

```
[Out] -(b*e+c*d)*x/c^2/e^2+1/3*x^3/c/e+d^(7/2)*arctan(x*e^(1/2)/d^(1/2))/e^(5/2)/
(a*e^2-b*d*e+c*d^2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*(b^3*d-2*a*b*c*d-a*b^2*e+a^2*c*e+(-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*
b^2*c*d-b^4*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(
-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2)
)^(1/2))*(b^3*d-2*a*b*c*d-a*b^2*e+a^2*c*e+(3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-
4*a*b^2*c*d+b^4*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 2.77, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1301, 211, 1180}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{3a^2bce+2a^2c^2d-ab^3e-4ab^2cd+b^4d}{\sqrt{b^2-4ac}}+a^2ce-ab^2e-2abcd+b^3d\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{d^{7/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}(ae^2-bde+cd^2)}-\frac{x(be+cd)}{c^2e^2}+\frac{x^3}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) - ((b^3*d - 2*a*b*c*d - a*b^2*e
+ a^2*c*e - (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a*b^3*e + 3*a^2*b*c*e)/Sqr
t[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(S
qrt[2]*c^(5/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - ((b^3
*d - 2*a*b*c*d - a*b^2*e + a^2*c*e + (b^4*d - 4*a*b^2*c*d + 2*a^2*c^2*d - a
*b^3*e + 3*a^2*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b
+ Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) + (d^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^8}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{-cd-be}{c^2e^2}+\frac{x^2}{ce}+\frac{d^4}{e^2(cd^2-bde+ae^2)(d+ex^2)}+\frac{-a(b^2d-acd-a\cdots}{c^2(cd^2-\cdots}\right.$$

$$= -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}+\frac{\int \frac{-a(b^2d-acd-abe)+(-b^3d+2abcd+ab^2e-a^2ce)x^2}{a+bx^2+cx^4}\,dx}{c^2(cd^2-bde+ae^2)}+\frac{d^4\cdots}{e^2(cd\cdots}$$

$$= -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}(cd^2-bde+ae^2)}-\frac{\left(b^3d-2abcd-ab^2e+a^2c\cdots\right.}{\cdots}$$

$$= -\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}-\frac{\left(b^3d-2abcd-ab^2e+a^2ce-\frac{b^4d-4ab^2cd+2a^2c^2d-ab^3e+\cdots}{\sqrt{b^2-4ac}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b-\sqrt{b^2-4ac}}\,(c\cdots}$$

Mathematica [A]
time = 0.40, size = 463, normalized size = 1.20

$$-\frac{(cd+be)x}{c^2e^2}+\frac{x^3}{3ce}+\frac{\left(-b^4d+b^3\left(\sqrt{b^2-4ac}\,d+ae\right)-abc\left(2\sqrt{b^2-4ac}\,d+3ae\right)+ab^2\left(4cd-\sqrt{b^2-4ac}\,e\right)+a^2c\left(-2cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}+\frac{\left(b^4d+b^3\left(\sqrt{b^2-4ac}\,d-ae\right)+abc\left(-2\sqrt{b^2-4ac}\,d+3ae\right)+a^2c\left(2cd+\sqrt{b^2-4ac}\,e\right)-ab^2\left(4cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))}+\frac{d^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{5/2}(cd^2-bde+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((c*d + b*e)*x)/(c^2*e^2)) + x^3/(3*c*e) + ((-(b^4*d) + b^3*(Sqrt[b^2 - 4
*a*c]*d + a*e) - a*b*c*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a*b^2*(4*c*d - Sqr
t[b^2 - 4*a*c]*e) + a^2*c*(-2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*S
qrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^4*d + b^3*(Sq
```

```
rt[b^2 - 4*a*c]*d - a*e) + a*b*c*(-2*Sqrt[b^2 - 4*a*c]*d + 3*a*e) + a^2*c*(
2*c*d + Sqrt[b^2 - 4*a*c]*e) - a*b^2*(4*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2
 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + (d^(7/2
)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Maple [A]
time = 0.24, size = 413, normalized size = 1.07

method	result
default	$-\frac{-\frac{1}{3}ce\,x^3+ebx+cdx}{c^2e^2}+\frac{-\frac{\left(-a^2ce\sqrt{-4ac+b^2}+a\,b^2e\sqrt{-4ac+b^2}+2\sqrt{-4ac+b^2}\,abcd-\sqrt{-4ac+b^2}\,b^3d+3a^2b\cdots\right.}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-\cdots\right.}}\cdots}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^2/e^2*(-1/3*c*e*x^3+e*b*x+c*d*x)+4/(a*e^2-b*d*e+c*d^2)/c*(-1/8*(-a^2*c
*e*(-4*a*c+b^2)^(1/2)+a*b^2*e*(-4*a*c+b^2)^(1/2)+2*(-4*a*c+b^2)^(1/2)*a*b*c
*d-(-4*a*c+b^2)^(1/2)*b^3*d+3*a^2*b*c*e+2*a^2*c^2*d-a*b^3*e-4*a*b^2*c*d+b^4
*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(
c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-a^2*c*e*(-4*a*c+b^2)^(
1/2)+a*b^2*e*(-4*a*c+b^2)^(1/2)+2*(-4*a*c+b^2)^(1/2)*a*b*c*d-(-4*a*c+b^2)^(
1/2)*b^3*d-3*a^2*b*c*e-2*a^2*c^2*d+a*b^3*e+4*a*b^2*c*d-b^4*d)/c/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4
*a*c+b^2)^(1/2))*c)^(1/2)))+1/e^2*d^4/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arcta
n(e*x/(d*e)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 - b*d*e^3 + a*e^4) +
1/3*(c*x^3*e - 3*(c*d + b*e)*x)*e^(-2)/c^2 + integrate((a^2*b*e + (a*b^2*e
- a^2*c*e - (b^3 - 2*a*b*c)*d)*x^2 - (a*b^2 - a^2*c)*d)/(c*x^4 + b*x^2 + a)
, x)/(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 12087 vs. $2(346) = 692$.

time = 162.32, size = 24206, normalized size = 62.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/6*(2*c^2*d^2*x^3*e + 3*sqrt(-d*e^(-1))*c^2*d^3*log((x^2*e + 2*sqrt(-d*e^
(-1))*x*e - d)/(x^2*e + d)) - 6*c^2*d^3*x + 3*sqrt(1/2)*(c^3*d^2*e^2 - b*c^
2*d*e^3 + a*c^2*e^4)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5
- 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + ((b^2*c^7 - 4*a*c^8)*d^4 - 2*(b^3*c^6 -
4*a*b*c^7)*d^3*e + (b^4*c^5 - 2*a*b^2*c^6 - 8*a^2*c^7)*d^2*e^2 - 2*(a*b^3*c
^5 - 4*a^2*b*c^6)*d*e^3 + (a^2*b^2*c^5 - 4*a^3*c^6)*e^4)*sqrt(((b^12 - 10*a
*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5
 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3
 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*
a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b
^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (
a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4)/((b^
2*c^14 - 4*a*c^15)*d^8 - 4*(b^3*c^13 - 4*a*b*c^14)*d^7*e + 2*(3*b^4*c^12 -
10*a*b^2*c^13 - 8*a^2*c^14)*d^6*e^2 - 4*(b^5*c^11 - a*b^3*c^12 - 12*a^2*b*c
^13)*d^5*e^3 + (b^6*c^10 + 8*a*b^4*c^11 - 42*a^2*b^2*c^12 - 24*a^3*c^13)*d^
4*e^4 - 4*(a*b^5*c^10 - a^2*b^3*c^11 - 12*a^3*b*c^12)*d^3*e^5 + 2*(3*a^2*b^
4*c^10 - 10*a^3*b^2*c^11 - 8*a^4*c^12)*d^2*e^6 - 4*(a^3*b^3*c^10 - 4*a^4*b*
c^11)*d*e^7 + (a^4*b^2*c^10 - 4*a^5*c^11)*e^8)))/((b^2*c^7 - 4*a*c^8)*d^4 -
 2*(b^3*c^6 - 4*a*b*c^7)*d^3*e + (b^4*c^5 - 2*a*b^2*c^6 - 8*a^2*c^7)*d^2*e^
2 - 2*(a*b^3*c^5 - 4*a^2*b*c^6)*d*e^3 + (a^2*b^2*c^5 - 4*a^3*c^6)*e^4))*log
(2*(a^3*b^6 - 5*a^4*b^4*c + 6*a^5*b^2*c^2 - a^6*c^3)*d^2*x - 4*(a^4*b^5 - 4
*a^5*b^3*c + 3*a^6*b*c^2)*d*x*e + 2*(a^5*b^4 - 3*a^6*b^2*c + a^7*c^2)*x*e^2
 + sqrt(1/2)*((b^10 - 10*a*b^8*c + 35*a^2*b^6*c^2 - 51*a^3*b^4*c^3 + 29*a^4
*b^2*c^4 - 4*a^5*c^5)*d^3 - (3*a*b^9 - 27*a^2*b^7*c + 80*a^3*b^5*c^2 - 87*a
^4*b^3*c^3 + 28*a^5*b*c^4)*d^2*e + (3*a^2*b^8 - 24*a^3*b^6*c + 58*a^4*b^4*c
^2 - 41*a^5*b^2*c^3 + 4*a^6*c^4)*d*e^2 - (a^3*b^7 - 7*a^4*b^5*c + 13*a^5*b^
3*c^2 - 4*a^6*b*c^3)*e^3 - ((b^5*c^7 - 7*a*b^3*c^8 + 12*a^2*b*c^9)*d^5 - (2
*b^6*c^6 - 13*a*b^4*c^7 + 18*a^2*b^2*c^8 + 8*a^3*c^9)*d^4*e + (b^7*c^5 - 3*
a*b^5*c^6 - 14*a^2*b^3*c^7 + 40*a^3*b*c^8)*d^3*e^2 - (3*a*b^6*c^5 - 18*a^2*
b^4*c^6 + 20*a^3*b^2*c^7 + 16*a^4*c^8)*d^2*e^3 + (3*a^2*b^5*c^5 - 19*a^3*b^
3*c^6 + 28*a^4*b*c^7)*d*e^4 - (a^3*b^4*c^5 - 6*a^4*b^2*c^6 + 8*a^5*c^7)*e^5
)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*
c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d^3*e + 2*(3*a^2*b^10
- 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^3 + 27*a^6*b^2*c^4 - a^7*c^5
```

```
)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b^5*c^2 - 13*a^6*b^3*c^3 + 3*
a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*a^6*b^4*c^2 - 6*a^7*b^2*c^3
+ a^8*c^4)*e^4)/((b^2*c^14 - 4*a*c^15)*d^8 - 4*(b^3*c^13 - 4*a*b*c^14)*d^7*
e + 2*(3*b^4*c^12 - 10*a*b^2*c^13 - 8*a^2*c^14)*d^6*e^2 - 4*(b^5*c^11 - a*b
^3*c^12 - 12*a^2*b*c^13)*d^5*e^3 + (b^6*c^10 + 8*a*b^4*c^11 - 42*a^2*b^2*c^
12 - 24*a^3*c^13)*d^4*e^4 - 4*(a*b^5*c^10 - a^2*b^3*c^11 - 12*a^3*b*c^12)*d
^3*e^5 + 2*(3*a^2*b^4*c^10 - 10*a^3*b^2*c^11 - 8*a^4*c^12)*d^2*e^6 - 4*(a^3
*b^3*c^10 - 4*a^4*b*c^11)*d*e^7 + (a^4*b^2*c^10 - 4*a^5*c^11)*e^8)))*sqrt(-
((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^
4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2
)*e^2 + ((b^2*c^7 - 4*a*c^8)*d^4 - 2*(b^3*c^6 - 4*a*b*c^7)*d^3*e + (b^4*c^5
 - 2*a*b^2*c^6 - 8*a^2*c^7)*d^2*e^2 - 2*(a*b^3*c^5 - 4*a^2*b*c^6)*d*e^3 + (
a^2*b^2*c^5 - 4*a^3*c^6)*e^4)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 -
62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^4 - 4*(a*b^11
 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b
*c^5)*d^3*e + 2*(3*a^2*b^10 - 24*a^3*b^8*c + 66*a^4*b^6*c^2 - 72*a^5*b^4*c^
3 + 27*a^6*b^2*c^4 - a^7*c^5)*d^2*e^2 - 4*(a^3*b^9 - 7*a^4*b^7*c + 16*a^5*b
^5*c^2 - 13*a^6*b^3*c^3 + 3*a^7*b*c^4)*d*e^3 + (a^4*b^8 - 6*a^5*b^6*c + 11*
a^6*b^4*c^2 - 6*a^7*b^2*c^3 + a^8*c^4)*e^4)/((b^2*c^14 - 4*a*c^15)*d^8 - 4*
(b^3*c^13 - 4*a*b*c^14)*d^7*e + 2*(3*b^4*c^12 - 10*a*b^2*c^13 - 8*a^2*c^14)
*d^6*e^2 - 4*(b^5*c^11 - a*b^3*c^12 - 12*a^2*b*c^13)*d^5*e^3 + (b^6*c^10 +
8*a*b^4*c^11 - 42*a^2*b^2*c^12 - 24*a^3*c^13)*d^4*e^4 - 4*(a*b^5*c^10 - a^2
*b^3*c^11 - 12*a^3*b*c^12)*d^3*e^5 + 2*(3*a^2*b^4*c^10 - 10*a^3*b^2*c^11 -
8*a^4*c^12)*d^2*e^6 - 4*(a^3*b^3*c^10 - 4*a^4*b*c^11)*d*e^7 + (a^4*b^2*c^10
 - 4*a^5*c^11)*e^8)))/((b^2*c^7 - 4*a*c^8)*d^4 - 2*(b^3*c^6 - 4*a*b*c^7)*d^
3*e + (b^4*c^5 - 2*a*b^2*c^6 - 8*a^2*c^7)*d^2*e^2 - 2*(a*b^3*c^5 - 4*a^2*b*
c^6)*d*e^3 + (a^2*b^2*c^5 - 4*a^3*c^6)*e^4))) - 3*sqrt(1/2)*(c^3*d^2*e^2 -
b*c^2*d*e^3 + a*c^2*e^4)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b
*c^3)*d^2 - 2*(a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*d*e + (a^2*
b^5 - 5*a^3*b^3*c + 5*a^4*b*c^2)*e^2 + ((b^2*c^7 - 4*a*c^8)*d^4 - 2*(b^3*c^
6 - 4*a*b*c^7)*d^3*e + (b^4*c^5 - 2*a*b^2*c^6 -...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 12506 vs. $2(346) = 692$.
time = 13.67, size = 12506, normalized size = 32.32

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^8/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")

[Out] d^(7/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e^2 - b*d*e^3 + a*e^4) + 1/8*((2*b^7*c^8 - 16*a*b^5*c^9 + 36*a^2*b^3*c^10 - 16*a^3*b*c^11 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 - 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^8 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^9 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^10 - 2*(b^2 - 4*a*c)*b^5*c^8 + 8*(b^2 - 4*a*c)*a*b^3*c^9 - 4*(b^2 - 4*a*c)*a^2*b*c^10)*d^5 - (4*b^8*c^7 - 30*a*b^6*c^8 + 58*a^2*b^4*c^9 - 8*a^3*b^2*c^10 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 15*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^6 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 29*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 - 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^7 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^7 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^9 - 4*(b^2 - 4*a*c)*b^6*c^7 + 14*(b^2 - 4*a*c)*a*b^4*c^8 - 2*(b^2 - 4*a*c)*a^2*b^2*c^9)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^4 - 9*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 + 2*a*b^6*c^5 + 24*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^6 + 10*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^6 - 18*a^2*b^4*c^6 - 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^7 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^7 - 5*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^7 + 48*a^3*b^2*c^7 + 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*c^8 - 32*a^4*c^8 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 10*(b^2 - 4*a*c)*a^2*b^2*c^6 - 8*(b^2 - 4*a*c)*a^3*c^7)*d^3*abs(-c^3*d^2 + b*c^2*d*e - a*c^2*e^2) + (2*b^9*c^6 - 8*a*b^7*c^7 - 24*a^2*b^5*c^8 + 104*a^3*b^3*c^9 - 32*a^4*b*c^10 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^9*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^8*c^5 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^6 - 52*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^7 - 24*sqrt(2)*

```
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^7 + 16*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*b*c^8 + 8*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^8 + 12*sqrt(2)*sqr
t(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^8 - 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^9 - 2*(b^2 - 4*a*c)*b
^7*c^6 + 24*(b^2 - 4*a*c)*a^2*b^3*c^8 - 8*(b^2 - 4*a*c)*a^3*b*c^9)*d^3*e^2
+ 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^3 - 8*sqrt(2)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^6*c^4 + 2*a*b^7*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a
^3*b^3*c^5 + 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^5 + sqrt(2
)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^5 - 16*a^2*b^5*c^5 - 4*sqrt(2)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^6 + 32*a^3*b^3*c^6 - 2*(b^2 - 4*a*c
)*a*b^5*c^4 + 8*(b^2 - 4*a*c)*a^2*b^3*c^5)*d^2*abs(-c^3*d^2 + b*c^2*d*e - a
*c^2*e^2)*e - (2*b^7*c^2 - 20*a*b^5*c^3 + 64*a^2*b^3*c^4 - 64*a^3*b*c^5 - s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7 + 10*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c - 32*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 12*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
- sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 + 12*(b^2 - 4*a*
c)*a*b^3*c^3 - 16*(b^2 - 4*a*c)*a^2*b*c^4)*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2
)^2*d - (6*a*b^8*c^6 - 42*a^2*b^6*c^7 + 68*a^3*...
```

Mupad [B]
time = 7.13, size = 2500, normalized size = 6.46

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((((((192*a^3*c^8*d^6*e^5 + 384*a^4*c^7*d^4*e^7 + 192*a^5*c^6*d^2*e^9
- 48*a^2*b^2*c^7*d^6*e^5 + 96*a^2*b^3*c^6*d^5*e^6 - 48*a^2*b^4*c^5*d^4*e^7
+ 96*a^3*b^2*c^6*d^4*e^7 + 96*a^3*b^3*c^5*d^3*e^8 - 48*a^4*b^2*c^5*d^2*e^9
- 384*a^3*b*c^7*d^5*e^6 - 384*a^4*b*c^6*d^3*e^8)/(c^3*e^3) - (2*x*(-(b^9*d^
2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a
^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3
*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c
- b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2)
 - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) +
 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*
```

d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2)*(128*a^4*b^2*c^6*e^12 - 16*a^3*b^4*c^5*e^12 - 256*a^5*c^7*e^12 + 256*a^2*c^10*d^6*e^6 + 256*a^3*c^9*d^4*e^8 - 256*a^4*c^8*d^2*e^10 - 16*b^3*c^9*d^7*e^5 + 64*b^4*c^8*d^6*e^6 - 96*b^5*c^7*d^5*e^7 + 64*b^6*c^6*d^4*e^8 - 16*b^7*c^5*d^3*e^9 + 256*a^2*b^2*c^8*d^4*e^8 + 144*a^2*b^3*c^7*d^3*e^9 - 96*a^2*b^4*c^6*d^2*e^10 + 192*a^3*b^2*c^7*d^2*e^10 + 64*a*b*c^10*d^7*e^5 + 320*a^4*b*c^7*d*e^11 - 320*a*b^2*c^9*d^6*e^6 + 528*a*b^3*c^8*d^5*e^7 - 336*a*b^4*c^7*d^4*e^8 + 48*a*b^5*c^6*d^3*e^9 + 16*a*b^6*c^5*d^2*e^10 - 576*a^2*b*c^9*d^5*e^7 + 16*a^2*b^5*c^5*d*e^11 - 320*a^3*b*c^8*d^3*e^9 - 144*a^3*b^3*c^6*d*e^11))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e - 3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 16*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 - 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) + (2*x*(4*a^3*b^7*e^10 + 4*b^3*c^7*d^10 + 4*b^10*d^3*e^7 - 36*a^4*b^5*c*e^10 - 80*a^6*b*c^3*e^10 - 4*a*b^9*d^2*e^8 - 4*a^2*b^8*d*e^9 - 64*a^2*c^8*d^9*e - 56*a^6*c^4*d*e^9 - 8*b^4*c^6*d^9*e - 8*b^9*c*d^4*e^6 + 100*a^5*b^3*c^2*e^10 + 8*a^4*c^6*d^5*e^5 + 16*a^5*c^5*d^3*e^7 + 4*b^5*c^5*d^8*e^2 + 4*b^8*c^2*d^5*e^5 - 16*a*b*c^8*d^10 + 80*a^2*b^4*c^4*d^5*e^5 - 160*a^2*b^5*c^3*d^4*e^6 + 16*a^2*b^6*c^2*d^3*e^7 - 64*a^3*b^2*c^5*d^5*e^5 + 128*a^3*b^3*c^4*d^4*e^6 + 96*a^3*b^4*c^3*d^3*e^7 + 8*a^3*b^5*c^2*d^2*e^8 - 120*a^4*b^2*c^4*d^3*e^7 - 124*a^4*b^3*c^3*d^2*e^8 + 48*a*b^2*c^7*d^9*e - 24*a*b^8*c*d^3*e^7 + 48*a^3*b^6*c*d*e^9 - 28*a*b^3*c^6*d^8*e^2 - 32*a*b^6*c^3*d^5*e^5 + 64*a*b^7*c^2*d^4*e^6 + 48*a^2*b*c^7*d^8*e^2 + 20*a^2*b^7*c*d^2*e^8 - 16*a^4*b*c^5*d^4*e^6 - 184*a^4*b^4*c^2*d*e^9 + 96*a^5*b*c^4*d^2*e^8 + 240*a^5*b^2*c^3*d*e^9))/(c^3*e^3))*(-(b^9*d^2 + a^2*b^7*e^2 + b^6*d^2*(-(4*a*c - b^2)^3)^(1/2) + 28*a^4*b*c^4*d^2 - 9*a^3*b^5*c*e^2 - 20*a^5*b*c^3*e^2 - 2*a*b^8*d*e + 42*a^2*b^5*c^2*d^2 - 63*a^3*b^3*c^3*d^2 + a^2*b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^3*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) + 25*a^4*b^3*c^2*e^2 + a^4*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*d^2 - 16*a^5*c^4*d*e - 2*a*b^5*d*e*(-(4*a*c - b^2)^3)^(1/2) + 20*a^2*b^6*c*d*e + 6*a^2*b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 5*a*b^4*

```
c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^3*b^4*c^2*d*e + 76*a^4*b^2*c^3*d*e -
3*a^3*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) + 8*a^2*b^3*c*d*e*(-(4*a*c - b^2)^
3)^(1/2) - 6*a^3*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9*d^4 + 1
6*a^4*c^7*e^4 + b^4*c^7*d^4 - 8*a*b^2*c^8*d^4 - 2*b^5*c^6*d^3*e + a^2*b^4*c
^5*e^4 - 8*a^3*b^2*c^6*e^4 + 32*a^3*c^8*d^2*e^2 + b^6*c^5*d^2*e^2 + 16*a*b^
3*c^7*d^3*e - 2*a*b^5*c^5*d*e^3 - 32*a^2*b*c^8*d^3*e - 32*a^3*b*c^7*d*e^3 -
 6*a*b^4*c^6*d^2*e^2 + 16*a^2*b^3*c^6*d*e^3)))^(1/2) - (16*a^3*c^6*d^9 + 4*
a*b^4*c^4*d^9 + 4*a*b^8*d^5*e^4 + 4*a^5*b^4*d*e^8 + 4*a^7*c^2*d*e^8 - 20*a^
2*b^2*c^5*d^9 - 4*a^2*b^7*d^4*e^5 - 4*a^4*b^5*d^2*e^7 - 64*a^4*c^5*d^7*e^2
+ 64*a^5*c^4*d^5*e^4 + 4*a^6*c^3*d^3*e^6 - 36*a^2*b^4*c^3*d^7*e^2 - 40*a^2*
b^5*c^2*d^6*e^3 + 96*a^3*b^2*c^4*d^7*e^2 + 128*a^3*b^3*c^3*d^6*e^3 + 164*a^
3*b^4*c^2*d^5*e^4 - 224*a^4*b^2*c^3*d^5*e^4 - 1...
```

3.304 $\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=323

$$\frac{x}{ce}+\frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}+\frac{\left(b^2d-acd-abe+\frac{b^3d-3abcd-ab}{\sqrt{b^2-}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+\sqrt{b^2}}}$$

[Out] x/c/e-d^(5/2)*arctan(x*e^(1/2)/d^(1/2))/e^(3/2)/(a*e^2-b*d*e+c*d^2)+1/2*arc tan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(-2* a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(a*e^2-b*d*e+c *d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b+ (-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+ b^3*d)/(-4*a*c+b^2)^(1/2))/c^(3/2)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b ^2)^(1/2))^(1/2)

Rubi [A]
time = 0.94, antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1301, 211, 1180}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{d^{5/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(ae^2-bde+cd^2)}+\frac{x}{ce}$$

Antiderivative was successfully verified.

[In] Int[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] x/(c*e) + ((b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c* e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b ^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt [2]*c^(3/2)*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (d^(5/2) *ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 - b*d*e + a*e^2))

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^6}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{1}{ce} - \frac{d^3}{e(cd^2-bde+ae^2)(d+ex^2)} + \frac{a(bd-ae)+(b^2d-acd-abe}{c(cd^2-bde+ae^2)(a+bx^2+}\right.$$

$$= \frac{x}{ce} + \frac{\int \frac{a(bd-ae)+(b^2d-acd-abe)x^2}{a+bx^2+cx^4}\,dx}{c(cd^2-bde+ae^2)} - \frac{d^3\int \frac{1}{d+ex^2}\,dx}{e(cd^2-bde+ae^2)}$$

$$= \frac{x}{ce} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(cd^2-bde+ae^2)} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\int}{2c(cd^2-bde+ae^2}$$

$$= \frac{x}{ce} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+2a^2ce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$

Mathematica [A]
time = 0.34, size = 385, normalized size = 1.19

$$\frac{x}{ce} + \frac{\left(b^3d - b^2\left(\sqrt{b^2-4ac}\,d+ae\right)+ac\left(\sqrt{b^2-4ac}\,d+2ae\right)+ab\left(-3cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+e(bd-ae))} + \frac{\left(b^3d + b^2\left(\sqrt{b^2-4ac}\,d-ae\right)+ac\left(-\sqrt{b^2-4ac}\,d+2ae\right)-ab\left(3cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2+e(-bd+ae))} - \frac{d^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{e^{3/2}(cd^2-bde+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*e) + ((b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c
]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*
x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + e*(b*d - a*e))) + ((b^3*d + b^2*(Sqrt[b^2
- 4*a*c]*d - a*e) + a*c*(-(Sqrt[b^2 - 4*a*c]*d) + 2*a*e) - a*b*(3*c*d + Sqr
t[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])
```

```
/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (d^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(e^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

Maple [A]
time = 0.21, size = 331, normalized size = 1.02

method	result
default	$\frac{x}{ce}+\frac{-\frac{\left(-abe\sqrt{-4ac+b^2}-\sqrt{-4ac+b^2}\,acd+\sqrt{-4ac+b^2}\,b^2d-2a^2ce+a\,b^2e+3abcd-b^3d\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{\cdots}{\sqrt{(-b+\cdots}}\right)}{2c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\cdots}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] x/c/e+4/(a*e^2-b*d*e+c*d^2)*(-1/8*(-a*b*e*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(
1/2)*a*c*d+(-4*a*c+b^2)^(1/2)*b^2*d-2*a^2*c*e+a*b^2*e+3*a*b*c*d-b^3*d)/c/(-
4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1
/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(-a*b*e*(-4*a*c+b^2)^(1/2)-(-4*a
*c+b^2)^(1/2)*a*c*d+(-4*a*c+b^2)^(1/2)*b^2*d+2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^
3*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c
*x*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))-1/e*d^3/(a*e^2-b*d*e+c*d^2)/(
d*e)^(1/2)*arctan(e*x/(d*e)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e - b*d*e^2 + a*e^3) + x
*e^(-1)/c - integrate(-(a*b*d - (a*b*e - (b^2 - a*c)*d)*x^2 - a^2*e)/(c*x^4
 + b*x^2 + a), x)/(c^2*d^2 - b*c*d*e + a*c*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9909 vs. 2(284) = 568.
time = 25.98, size = 19849, normalized size = 61.45

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-d*e^(-1))*c*d^2*log((x^2*e - 2*sqrt(-d*e^(-1))*x*e - d)/(x^2*e
+ d)) + 2*c*d^2*x - 2*b*d*x*e + 2*a*x*e^2 + sqrt(1/2)*(c^2*d^2*e - b*c*d*e^
2 + a*c*e^3)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^4 - 4*a^2*
b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + ((b^2*c^5 - 4*a*c^6)*d
^4 - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^
2*e^2 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^3 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^4)*
sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*
(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 -
12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c
 + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4)/((b^2*c^10 -
 4*a*c^11)*d^8 - 4*(b^3*c^9 - 4*a*b*c^10)*d^7*e + 2*(3*b^4*c^8 - 10*a*b^2*c
^9 - 8*a^2*c^10)*d^6*e^2 - 4*(b^5*c^7 - a*b^3*c^8 - 12*a^2*b*c^9)*d^5*e^3 +
 (b^6*c^6 + 8*a*b^4*c^7 - 42*a^2*b^2*c^8 - 24*a^3*c^9)*d^4*e^4 - 4*(a*b^5*c
^6 - a^2*b^3*c^7 - 12*a^3*b*c^8)*d^3*e^5 + 2*(3*a^2*b^4*c^6 - 10*a^3*b^2*c^
7 - 8*a^4*c^8)*d^2*e^6 - 4*(a^3*b^3*c^6 - 4*a^4*b*c^7)*d*e^7 + (a^4*b^2*c^6
 - 4*a^5*c^7)*e^8)))/((b^2*c^5 - 4*a*c^6)*d^4 - 2*(b^3*c^4 - 4*a*b*c^5)*d^3
*e + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^2 - 2*(a*b^3*c^3 - 4*a^2*b*c
^4)*d*e^3 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^4))*log(-2*(a^2*b^4 - 3*a^3*b^2*c +
 a^4*c^2)*d^2*x + 4*(a^3*b^3 - 2*a^4*b*c)*d*x*e - 2*(a^4*b^2 - a^5*c)*x*e^2
 + sqrt(1/2)*((b^7 - 7*a*b^5*c + 13*a^2*b^3*c^2 - 4*a^3*b*c^3)*d^3 - (3*a*b
^6 - 19*a^2*b^4*c + 29*a^3*b^2*c^2 - 4*a^4*c^3)*d^2*e + (3*a^2*b^5 - 17*a^3
*b^3*c + 20*a^4*b*c^2)*d*e^2 - (a^3*b^4 - 5*a^4*b^2*c + 4*a^5*c^2)*e^3 - ((
b^4*c^5 - 6*a*b^2*c^6 + 8*a^2*c^7)*d^5 - (2*b^5*c^4 - 11*a*b^3*c^5 + 12*a^2
*b*c^6)*d^4*e + (b^6*c^3 - 2*a*b^4*c^4 - 12*a^2*b^2*c^5 + 16*a^3*c^6)*d^3*e
^2 - (3*a*b^5*c^3 - 14*a^2*b^3*c^4 + 8*a^3*b*c^5)*d^2*e^3 + (3*a^2*b^4*c^3
- 14*a^3*b^2*c^4 + 8*a^4*c^5)*d*e^4 - (a^3*b^3*c^3 - 4*a^4*b*c^4)*e^5)*sqrt
(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4 - 4*(a*b
^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^6 - 12*a
^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b^3*c + 2
*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4)/((b^2*c^10 - 4*a
*c^11)*d^8 - 4*(b^3*c^9 - 4*a*b*c^10)*d^7*e + 2*(3*b^4*c^8 - 10*a*b^2*c^9 -
 8*a^2*c^10)*d^6*e^2 - 4*(b^5*c^7 - a*b^3*c^8 - 12*a^2*b*c^9)*d^5*e^3 + (b^
6*c^6 + 8*a*b^4*c^7 - 42*a^2*b^2*c^8 - 24*a^3*c^9)*d^4*e^4 - 4*(a*b^5*c^6 -
 a^2*b^3*c^7 - 12*a^3*b*c^8)*d^3*e^5 + 2*(3*a^2*b^4*c^6 - 10*a^3*b^2*c^7 -
8*a^4*c^8)*d^2*e^6 - 4*(a^3*b^3*c^6 - 4*a^4*b*c^7)*d*e^7 + (a^4*b^2*c^6 - 4
*a^5*c^7)*e^8)))*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^4 - 4*
a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + ((b^2*c^5 - 4*a*c^
6)*d^4 - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5
)*d^2*e^2 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^3 + (a^2*b^2*c^3 - 4*a^3*c^4)*e
^4)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^4
- 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2*b^
```

```
6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^4*b
^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4)/((b^2*c^
10 - 4*a*c^11)*d^8 - 4*(b^3*c^9 - 4*a*b*c^10)*d^7*e + 2*(3*b^4*c^8 - 10*a*b
^2*c^9 - 8*a^2*c^10)*d^6*e^2 - 4*(b^5*c^7 - a*b^3*c^8 - 12*a^2*b*c^9)*d^5*e
^3 + (b^6*c^6 + 8*a*b^4*c^7 - 42*a^2*b^2*c^8 - 24*a^3*c^9)*d^4*e^4 - 4*(a*b
^5*c^6 - a^2*b^3*c^7 - 12*a^3*b*c^8)*d^3*e^5 + 2*(3*a^2*b^4*c^6 - 10*a^3*b^
2*c^7 - 8*a^4*c^8)*d^2*e^6 - 4*(a^3*b^3*c^6 - 4*a^4*b*c^7)*d*e^7 + (a^4*b^2
*c^6 - 4*a^5*c^7)*e^8)))/((b^2*c^5 - 4*a*c^6)*d^4 - 2*(b^3*c^4 - 4*a*b*c^5)
*d^3*e + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^2 - 2*(a*b^3*c^3 - 4*a^2
*b*c^4)*d*e^3 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^4))) - sqrt(1/2)*(c^2*d^2*e - b
*c*d*e^2 + a*c*e^3)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^2 - 2*(a*b^4 -
 4*a^2*b^2*c + 2*a^3*c^2)*d*e + (a^2*b^3 - 3*a^3*b*c)*e^2 + ((b^2*c^5 - 4*a
*c^6)*d^4 - 2*(b^3*c^4 - 4*a*b*c^5)*d^3*e + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*
c^5)*d^2*e^2 - 2*(a*b^3*c^3 - 4*a^2*b*c^4)*d*e^3 + (a^2*b^2*c^3 - 4*a^3*c^4
)*e^4)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d
^4 - 4*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^3*e + 2*(3*a^2
*b^6 - 12*a^3*b^4*c + 12*a^4*b^2*c^2 - a^5*c^3)*d^2*e^2 - 4*(a^3*b^5 - 3*a^
4*b^3*c + 2*a^5*b*c^2)*d*e^3 + (a^4*b^4 - 2*a^5*b^2*c + a^6*c^2)*e^4)/((b^2
*c^10 - 4*a*c^11)*d^8 - 4*(b^3*c^9 - 4*a*b*c^10)*d^7*e + 2*(3*b^4*c^8 - 10*
a*b^2*c^9 - 8*a^2*c^10)*d^6*e^2 - 4*(b^5*c^7 - a*b^3*c^8 - 12*a^2*b*c^9)*d^
5*e^3 + (b^6*c^6 + 8*a*b^4*c^7 - 42*a^2*b^2*c^8 - 24*a^3*c^9)*d^4*e^4 - 4*(
a*b^5*c^6 - a^2*b^3*c^7 - 12*a^3*b*c^8)*d^3*e^5 + 2*(3*a^2*b^4*c^6 - 10*a^3
*b^2*c^7 - 8*a^4*c^8)*d^2*e^6 - 4*(a^3*b^3*c^6 - 4*a^4*b*c^7)*d*e^7 + (a^4*
b^2*c^6 - 4*a^5*c^7)*e^8)))/((b^2*c^5 - 4*a*c^6)*d^4 - 2*(b^3*c^4 - 4*a*b*c
^5)*d^3*e + (b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*e^2 - 2*(a*b^3*c^3 - 4*
a^2*b*c^4)*d*e^3 + (a^2*b^2*c^3 - 4*a^3*c^4)*e^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 11030 vs. 2(284) = 568.
time = 9.92, size = 11030, normalized size = 34.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

[Out] -d^(5/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2*e - b*d*e^2 + a*e^3) - 1
/8*((2*b^6*c^6 - 14*a*b^4*c^7 + 24*a^2*b^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^3*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^
2 - 4*a*c)*c)*b^4*c^6 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4
*a*c)*c)*a*b^2*c^7 - 2*(b^2 - 4*a*c)*b^4*c^6 + 6*(b^2 - 4*a*c)*a*b^2*c^7)*d
^5 - (4*b^7*c^5 - 26*a*b^5*c^6 + 36*a^2*b^3*c^7 + 16*a^3*b*c^8 - 2*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^7*c^3 + 13*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^4 + 4*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6*c^4 - 18*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^5 - 10*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c^5 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^6 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^6 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^6 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^7 - 4*(b^2 - 4*a*c)*b^5*c^5 + 10*(b^2 - 4*a
*c)*a*b^3*c^6 + 4*(b^2 - 4*a*c)*a^2*b*c^7)*d^4*e - 2*(sqrt(2)*sqrt(b*c - sq
rt(b^2 - 4*a*c)*c)*a*b^5*c^3 - 8*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^
2*b^3*c^4 - 2*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 + 2*a*b^5*c
^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 + 8*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^5 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a*b^3*c^5 - 16*a^2*b^3*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c
)*a^2*b*c^6 + 32*a^3*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^4 + 8*(b^2 - 4*a*c)*a^
2*b*c^5)*d^3*abs(-c^2*d^2 + b*c*d*e - a*c*e^2) + (2*b^8*c^4 - 6*a*b^6*c^5 -
 28*a^2*b^4*c^6 + 80*a^3*b^2*c^7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqr
t(b^2 - 4*a*c)*c)*b^8*c^2 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2
 - 4*a*c)*c)*a*b^6*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
4*a*c)*c)*b^7*c^3 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a*b^5*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*
b^6*c^4 - 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*
b^2*c^5 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*
b^3*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c
^5 + 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c
^6 - 2*(b^2 - 4*a*c)*b^6*c^4 - 2*(b^2 - 4*a*c)*a*b^4*c^5 + 20*(b^2 - 4*a*c)
*a^2*b^2*c^6)*d^3*e^2 + 2*(sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^
2 - 7*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 2*sqrt(2)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^3 + 2*a*b^6*c^3 + 8*sqrt(2)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 6*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)
*a^2*b^3*c^4 + sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^4 - 14*a^2*b
^4*c^4 + 16*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a^4*c^5 + 8*sqrt(2)*sqr

```
t(b*c - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 3*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a^2*b^2*c^5 + 16*a^3*b^2*c^5 - 4*sqrt(2)*sqrt(b*c - sqrt(b^2 - 4*a*
c)*c)*a^3*c^6 + 32*a^4*c^6 - 2*(b^2 - 4*a*c)*a*b^4*c^3 + 6*(b^2 - 4*a*c)*a^
2*b^2*c^4 + 8*(b^2 - 4*a*c)*a^3*c^5)*d^2*abs(-c^2*d^2 + b*c*d*e - a*c*e^2)*
e - (2*b^6*c^2 - 18*a*b^4*c^3 + 48*a^2*b^2*c^4 - 32*a^3*c^5 - sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - s
qrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(
b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 -
 4*a*c)*c)*a^2*b*c^3 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*
a*c)*c)*a*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c - sqrt(b^2 - 4*a*c
)*c)*a^2*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^
2 - 4*a*c)*a^2*c^4)*(c^2*d^2 - b*c*d*e + a*c*e^2)^2*d - (6*a*b^7*c^4 - 36*a
^2*b^5*c^5 + 40*a^3*b^3*c^6 + 32*a^4*b*c^7 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c - sqrt(b^2 - 4*a*c)*c)*a*b^7*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c - sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c - sqrt(b^2 - 4*a*c)*c)*a*b^6*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c - sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 - 3*sqrt(2...
```

Mupad [B]
time = 6.45, size = 2500, normalized size = 7.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] x/(c*e) - atan(((((((64*a^5*c^4*d*e^8 + 64*a^3*c^6*d^5*e^4 + 128*a^4*c^5*d^
3*e^6 - 144*a^2*b^2*c^5*d^5*e^4 + 64*a^2*b^3*c^4*d^4*e^5 + 16*a^2*b^4*c^3*d
^3*e^6 - 96*a^3*b^2*c^4*d^3*e^6 + 16*a^3*b^3*c^3*d^2*e^7 - 16*a*b^3*c^5*d^6
*e^3 + 32*a*b^4*c^4*d^5*e^4 - 16*a*b^5*c^3*d^4*e^5 + 64*a^2*b*c^6*d^6*e^3 -
 64*a^4*b*c^4*d^2*e^7 - 16*a^4*b^2*c^3*d*e^8)/(c*e) - (2*x*(-(b^7*d^2 + a^2
*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2 - 7*a^3*b^3*
c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 2*a*b^6*d*e
 + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) - a^2*c^2*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*a*b^3*d*e*(-(
4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(1
6*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*
d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e^2 + b^6*c^3*
d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^6*d^3*e - 32*
```

```
a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3)))^(1/2)*(128*
a^4*b^2*c^4*e^10 - 16*a^3*b^4*c^3*e^10 - 256*a^5*c^5*e^10 + 256*a^2*c^8*d^6
*e^4 + 256*a^3*c^7*d^4*e^6 - 256*a^4*c^6*d^2*e^8 - 16*b^3*c^7*d^7*e^3 + 64*
b^4*c^6*d^6*e^4 - 96*b^5*c^5*d^5*e^5 + 64*b^6*c^4*d^4*e^6 - 16*b^7*c^3*d^3*
e^7 + 256*a^2*b^2*c^6*d^4*e^6 + 144*a^2*b^3*c^5*d^3*e^7 - 96*a^2*b^4*c^4*d^
2*e^8 + 192*a^3*b^2*c^5*d^2*e^8 + 64*a*b*c^8*d^7*e^3 + 320*a^4*b*c^5*d*e^9
- 320*a*b^2*c^7*d^6*e^4 + 528*a*b^3*c^6*d^5*e^5 - 336*a*b^4*c^5*d^4*e^6 + 4
8*a*b^5*c^4*d^3*e^7 + 16*a*b^6*c^3*d^2*e^8 - 576*a^2*b*c^7*d^5*e^5 + 16*a^2
*b^5*c^3*d*e^9 - 320*a^3*b*c^6*d^3*e^7 - 144*a^3*b^3*c^4*d*e^9))/(c*e))*(-(
b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*d^2
 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) -
 a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*e + 2*
a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b^2)^3)
^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c^6*d^4
 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6*d^2*e
^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d*e^3 - 32*a^2*b*c^
6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a^2*b^3*c^4*d*e^3))
)^(1/2) + (2*x*(4*a^3*b^5*e^8 + 4*b^3*c^5*d^8 + 4*b^8*d^3*e^5 - 28*a^4*b^3*
c*e^8 + 48*a^5*b*c^2*e^8 - 4*a*b^7*d^2*e^6 - 4*a^2*b^6*d*e^7 - 64*a^2*c^6*d
^7*e + 56*a^5*c^3*d*e^7 - 8*b^4*c^4*d^7*e - 8*b^7*c*d^4*e^4 - 8*a^3*c^5*d^5
*e^3 - 16*a^4*c^4*d^3*e^5 + 4*b^5*c^3*d^6*e^2 + 4*b^6*c^2*d^5*e^3 - 16*a*b*
c^6*d^8 + 36*a^2*b^2*c^4*d^5*e^3 - 72*a^2*b^3*c^3*d^4*e^4 - 12*a^2*b^4*c^2*
d^3*e^5 + 64*a^3*b^2*c^3*d^3*e^5 + 28*a^3*b^3*c^2*d^2*e^6 + 48*a*b^2*c^5*d^
7*e - 16*a*b^6*c*d^3*e^5 + 40*a^3*b^4*c*d*e^7 - 28*a*b^3*c^4*d^6*e^2 - 24*a
*b^4*c^3*d^5*e^3 + 48*a*b^5*c^2*d^4*e^4 + 48*a^2*b*c^5*d^6*e^2 + 12*a^2*b^5
*c*d^2*e^6 + 16*a^3*b*c^4*d^4*e^4 - 64*a^4*b*c^3*d^2*e^6 - 108*a^4*b^2*c^2*
d*e^7))/(c*e))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2)
- 20*a^3*b*c^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*
c - b^2)^3)^(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c
 - b^2)^3)^(1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 1
6*a^4*c^3*d*e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3
*a*b^2*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*
(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4
 - 8*a*b^2*c^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4
+ 32*a^3*c^6*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e - 2*a*b^5*c^3*d
*e^3 - 32*a^2*b*c^6*d^3*e - 32*a^3*b*c^5*d*e^3 - 6*a*b^4*c^4*d^2*e^2 + 16*a
^2*b^3*c^4*d*e^3)))^(1/2) - (4*a*b^3*c^3*d^7 - 16*a^2*b*c^4*d^7 + 4*a*b^6*d
^4*e^3 + 4*a^4*b^3*d*e^6 + 48*a^3*c^4*d^6*e - 4*a^2*b^5*d^3*e^4 - 4*a^3*b^4
*d^2*e^5 - 60*a^4*c^3*d^4*e^3 + 4*a^5*c^2*d^2*e^5 - 8*a^5*b*c*d*e^6 - 32*a^
2*b^3*c^2*d^5*e^2 + 92*a^3*b^2*c^2*d^4*e^3 + 4*a*b^4*c^2*d^6*e + 4*a*b^5*c*
d^5*e^2 - 28*a^2*b^2*c^3*d^6*e - 36*a^2*b^4*c*d^4*e^3 + 64*a^3*b*c^3*d^5*e^
2 + 36*a^3*b^3*c*d^3*e^4 - 60*a^4*b*c^2*d^3*e^4 + 4*a^4*b^2*c*d^2*e^5)/(c*e
))*(-(b^7*d^2 + a^2*b^5*e^2 - b^4*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c
```

```
^3*d^2 - 7*a^3*b^3*c*e^2 + 12*a^4*b*c^2*e^2 + a^3*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 2*a*b^6*d*e + 25*a^2*b^3*c^2*d^2 - a^2*b^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) - a^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*d^2 + 16*a^4*c^3*d*
e + 2*a*b^3*d*e*(-(4*a*c - b^2)^3)^(1/2) + 16*a^2*b^4*c*d*e + 3*a*b^2*c*d^2
*(-(4*a*c - b^2)^3)^(1/2) - 36*a^3*b^2*c^2*d*e - 4*a^2*b*c*d*e*(-(4*a*c - b
^2)^3)^(1/2))/(8*(16*a^2*c^7*d^4 + 16*a^4*c^5*e^4 + b^4*c^5*d^4 - 8*a*b^2*c
^6*d^4 - 2*b^5*c^4*d^3*e + a^2*b^4*c^3*e^4 - 8*a^3*b^2*c^4*e^4 + 32*a^3*c^6
*d^2*e^2 + b^6*c^3*d^2*e^2 + 16*a*b^3*c^5*d^3*e...
```

3.305 $\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=280

$$-\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}-\frac{\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$

```
[Out] d^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/e^(1/2)-1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d
)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/c^(1/2)/(b-(-4*a*c+b^2)^(
1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d
-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)
/c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1301, 211, 1180}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{\sqrt{2}\sqrt{c}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{d^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2
]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2))) - ((b*d - a*e + (b^2*d - 2*a*c*d - a
*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)
) + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^4}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{d^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{-ad-(bd-ae)x^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{-ad+(-bd+ae)x^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,d}{2(cd^2-bde+ae^2)}$$

$$= -\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\left(bd-ae+\right.}{\sqrt{2}\sqrt{c}}$$

Mathematica [A]
time = 0.22, size = 323, normalized size = 1.15

$$\frac{\left(-b^2d+2acd+b\sqrt{b^2-4ac}\,d+abe-a\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{\left(b^2d-2acd+b\sqrt{b^2-4ac}\,d-abe-a\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{d^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{e}\,(cd^2-bde+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-(b^2*d) + 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + ((b^2*d - 2*a*c*d + b*Sqrt[b^2 - 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*
e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c
]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))
 + (d^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Maple [A]
time = 0.18, size = 264, normalized size = 0.94

method	result
default	$\frac{4c\left(-\frac{\left(\sqrt{-4ac+b^2}\,ae-bd\sqrt{-4ac+b^2}-abe-2acd+b^2d\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8c\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(\sqrt{-4a\right.}{}}{}\right.}{a\,e^2-deb+c\,d^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)*c*(-1/8*((-4*a*c+b^2)^(1/2)*a*e-b*d*(-4*a*c+b^2)^(1/2
)-a*b*e-2*a*c*d+b^2*d)/c/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*((-4*
a*c+b^2)^(1/2)*a*e-b*d*(-4*a*c+b^2)^(1/2)+a*b*e+2*a*c*d-b^2*d)/c/(-4*a*c+b^
2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-
4*a*c+b^2)^(1/2))*c)^(1/2)))+d^2/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e*x
/(d*e)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 - b*d*e + a*e^2) - integr
ate(((b*d - a*e)*x^2 + a*d)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7614 vs. 2(238) = 476.
time = 2.60, size = 15257, normalized size = 54.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-4*a^2*b*d*x*e + 2*a^3*x*e^2 + 2*(a*b^2 - a^2*c)*d^2*x + sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2)*e^5)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 + 4*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*

```
c^5)*d^5*e^3 + (b^6*c^2 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^
4 - 4*(a*b^5*c^2 - a^2*b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 -
 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7
+ (a^4*b^2*c^2 - 4*a^5*c^3)*e^8)))/((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 -
4*a*b*c^3)*d^3*e + (b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c -
 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*c - 4*a^3*c^2)*e^4))*log(-4*a^2*b*d*x*e + 2*
a^3*x*e^2 + 2*(a*b^2 - a^2*c)*d^2*x - sqrt(1/2)*((b^4 - 5*a*b^2*c + 4*a^2*c
^2)*d^3 - 2*(a*b^3 - 4*a^2*b*c)*d^2*e + (a^2*b^2 - 4*a^3*c)*d*e^2 - ((b^3*c
^3 - 4*a*b*c^4)*d^5 - 2*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^4*e + (b^5*c
+ 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^3*e^2 - 4*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^3*
c^3)*d^2*e^3 + 5*(a^2*b^3*c - 4*a^3*b*c^2)*d*e^4 - 2*(a^3*b^2*c - 4*a^4*c^2
)*e^5)*sqrt(-(4*a^3*b*d*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^4 - a^4*e^4 + 4
*(a*b^3 - a^2*b*c)*d^3*e - 2*(3*a^2*b^2 - a^3*c)*d^2*e^2)/((b^2*c^6 - 4*a*c
^7)*d^8 - 4*(b^3*c^5 - 4*a*b*c^6)*d^7*e + 2*(3*b^4*c^4 - 10*a*b^2*c^5 - 8*a
^2*c^6)*d^6*e^2 - 4*(b^5*c^3 - a*b^3*c^4 - 12*a^2*b*c^5)*d^5*e^3 + (b^6*c^2
 + 8*a*b^4*c^3 - 42*a^2*b^2*c^4 - 24*a^3*c^5)*d^4*e^4 - 4*(a*b^5*c^2 - a^2*
b^3*c^3 - 12*a^3*b*c^4)*d^3*e^5 + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4
*c^4)*d^2*e^6 - 4*(a^3*b^3*c^2 - 4*a^4*b*c^3)*d*e^7 + (a^4*b^2*c^2 - 4*a^5*
c^3)*e^8)))*sqrt(-(a^2*b*e^2 + (b^3 - 3*a*b*c)*d^2 - 2*(a*b^2 - 2*a^2*c)*d*
e + ((b^2*c^3 - 4*a*c^4)*d^4 - 2*(b^3*c^2 - 4*a*b*c^3)*d^3*e + (b^4*c - 2*a
*b^2*c^2 - 8*a^2*c^3)*d^2*e^2 - 2*(a*b^3*c - 4*a^2*b*c^2)*d*e^3 + (a^2*b^2*
c - 4*a^3*c^2)*e^4)*sqrt(-(4*a^3*b*d*e^3 - (b^4...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8658 vs. $2(238) = 476$.
time = 6.55, size = 8658, normalized size = 30.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^(3/2)*arctan(x*e^(1/2)/sqrt(d))*e^(-1/2)/(c*d^2 - b*d*e + a*e^2) - 1/8*((
2*b^5*c^4 - 12*a*b^3*c^5 + 16*a^2*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
```

(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5)*d^5 - (4*b^6*c^3 - 22*a*b^4*c^4 + 24*a^2*b^2*c^5 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 11*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 4*(b^2 - 4*a*c)*b^4*c^3 + 6*(b^2 - 4*a*c)*a*b^2*c^4)*d^4*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 2*a*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 + 16*a^2*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 - 32*a^3*c^5 + 2*(b^2 - 4*a*c)*a*b^2*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + (2*b^7*c^2 - 4*a*b^5*c^3 - 24*a^2*b^3*c^4 + 32*a^3*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 2*(b^2 - 4*a*c)*b^5*c^2 - 4*(b^2 - 4*a*c)*a*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*e^2 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 - 2*a*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 16*a^2*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 - 32*a^3*b*c^4 + 2*(b^2 - 4*a*c)*a*b^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c

```
^3 - 2*(b^2 - 4*a*c)*b^3*c^2 + 8*(b^2 - 4*a*c)*a*b*c^3)*(c*d^2 - b*d*e + a*
e^2)^2*d - (6*a*b^6*c^2 - 28*a^2*b^4*c^3 + 16*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6 + 14*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*(b^2 - 4*a*c)*a*b^4*c^2 + 4*(b^2
- 4*a*c)*a^2*b^2*c^3)*d^2*e^3 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^4*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^2 - 2*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^2 - 2*a^2*b^4*c^2 + 16*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -
4*a*c)*c)*a^3*b*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 +
 16*a^3*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^4 - 32*a^
4*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3)*d*abs(c*d^2
- b*d*e + a*e^2)*e^2 + (2*a*b^4*c^2 - 16*a^2*b^2*c^3 + 32*a^3*c^4 - sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)...
```

Mupad [B]
time = 5.80, size = 2500, normalized size = 8.93

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2
 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a
^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*
a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3
*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e
^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4
*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*((x*(8*a^3*b^3*c*e^7 - 32*a^4*
b*c^2*e^7 - 112*a^4*c^3*d*e^6 + 8*b^3*c^4*d^6*e + 8*b^6*c*d^3*e^4 - 112*a^2
*c^5*d^5*e^2 + 32*a^3*c^4*d^3*e^4 - 8*b^4*c^3*d^5*e^2 - 8*b^5*c^2*d^4*e^3 -
 32*a*b*c^5*d^6*e - 48*a^2*b^2*c^3*d^3*e^4 + 8*a^2*b^3*c^2*d^2*e^5 - 8*a*b^
5*c*d^2*e^5 - 8*a^2*b^4*c*d*e^6 + 64*a*b^2*c^4*d^5*e^2 + 8*a*b^3*c^3*d^4*e^
3 - 16*a*b^4*c^2*d^3*e^4 + 64*a^2*b*c^4*d^4*e^3 + 64*a^3*b*c^3*d^2*e^5 + 64
*a^3*b^2*c^2*d*e^6) + (-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)
^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e
- 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3
*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^
2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4
```

- 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2
- 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*
d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(64*a^2*c^6*d^6
e^2 - x(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*
d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d
^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12
*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 1
6*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d
^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d
*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b
^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*(256*a^4*b^2*c^3*e^9 - 32*a^
3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5
 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5
*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d
^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^
4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e
^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6
+ 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 64
0*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) + 128*a^3*c^5*d^4*e^4 + 64*a^4
*c^4*d^2*e^6 - 96*a^2*b^2*c^4*d^4*e^4 + 64*a^2*b^3*c^3*d^3*e^5 + 32*a^2*b^4
*c^2*d^2*e^6 - 144*a^3*b^2*c^3*d^2*e^6 + 64*a^4*b*c^3*d*e^7 - 16*a*b^2*c^5*
d^6*e^2 + 16*a*b^3*c^4*d^5*e^3 + 16*a*b^4*c^3*d^4*e^4 - 16*a*b^5*c^2*d^3*e^
5 - 64*a^2*b*c^5*d^5*e^3 - 16*a^3*b^3*c^2*d*e^7))*(-(b^5*d^2 + a^2*b^3*e^2
+ a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*
a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*
c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*
b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c
^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2
*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*
e^3)))^(1/2) + 16*a^2*c^4*d^5*e + 4*a^4*c^2*d*e^5 - 60*a^3*c^3*d^3*e^3 + 24
*a^2*b^2*c^2*d^3*e^3 - 4*a*b^2*c^3*d^5*e - 4*a*b^4*c*d^3*e^3 - 4*a^3*b^2*c*
d*e^5 - 4*a*b^3*c^2*d^4*e^2 + 20*a^2*b*c^3*d^4*e^2 + 8*a^2*b^3*c*d^2*e^4 -
16*a^3*b*c^2*d^2*e^4) + x*(2*a^4*c*e^5 + 4*a^2*c^3*d^4*e + 2*b^4*c*d^4*e -
8*a*b^2*c^2*d^4*e))*(-(b^5*d^2 + a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(
1/2) + b^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e -
7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c
^2*d*e + 12*a^2*b^2*c*d*e - 2*a*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*
c^5*d^4 + 16*a^4*c^3*e^4 + b^4*c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 -
2*b^5*c^2*d^3*e + b^6*c*d^2*e^2 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 -
2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*
e^3 - 6*a*b^4*c^2*d^2*e^2 + 16*a^2*b^3*c^2*d*e^3)))^(1/2)*1i + ((-(b^5*d^2
+ a^2*b^3*e^2 + a^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) + 12*a^2*b*c^2*d^2 - 2*a*b^4*d*e - 7*a*b^3*c*d^2 - a*c*d^2*(-(4*a*
c - b^2)^3)^(1/2) - 4*a^3*b*c*e^2 - 16*a^3*c^2*d*e + 12*a^2*b^2*c*d*e - 2*a

```
*b*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^5*d^4 + 16*a^4*c^3*e^4 + b^4*
c^3*d^4 - 8*a*b^2*c^4*d^4 + a^2*b^4*c*e^4 - 2*b^5*c^2*d^3*e + b^6*c*d^2*e^2
 - 8*a^3*b^2*c^2*e^4 + 32*a^3*c^4*d^2*e^2 - 2*a*b^5*c*d*e^3 + 16*a*b^3*c^3*
d^3*e - 32*a^2*b*c^4*d^3*e - 32*a^3*b*c^3*d*e^3...
```

3.306 $\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=251

$$\frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}+\frac{\sqrt{c}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$

```
[Out] -arctan(x*e^(1/2)/d^(1/2))*d^(1/2)*e^(1/2)/(a*e^2-b*d*e+c*d^2)+1/2*arctan(x
*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(d+(2*a*e-b*d)/(-4*a
*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2
*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(d+(-2*a*e+
b*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))
^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 251, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1301, 211, 1180}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{\sqrt{d}\sqrt{e}\,\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{ae^2-bde+cd^2}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 -
b*d*e + a*e^2)) + (Sqrt[c]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqr
t[d]])/(c*d^2 - b*d*e + a*e^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(-\frac{de}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{ae+cdx^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right)$$

$$= \frac{\int \frac{ae+cdx^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} - \frac{(de)\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2-bde+ae^2} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+c}}{2(cd^2-bde+ae^2)}$$

$$= \frac{\sqrt{c}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} + \frac{\sqrt{c}\left(d+\frac{bd-}{\sqrt{b^2-}}\right.}{\sqrt{2}\sqrt{b+}}$$

Mathematica [A]
time = 0.32, size = 277, normalized size = 1.10

$$-\frac{\sqrt{c}\left(-bd+\sqrt{b^2-4ac}\,d+2ae\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} - \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} - \frac{\sqrt{d}\sqrt{e}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{cd^2-bde+ae^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x
)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^
2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2))) - (Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c
]*d - 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2
)) - (Sqrt[d]*Sqrt[e]*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(c*d^2 - b*d*e + a*e^2)
```

Maple [A]
time = 0.20, size = 213, normalized size = 0.85

method	result
default	$\frac{4c\left(-\frac{\left(d\sqrt{-4ac+b^2}+2ae-bd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(bd-2ae+d\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arctan}\left(\dots\right)}{8\sqrt{-4ac+b^2}\sqrt{\left(b+\dots\right.}}\right)}{a\,e^2-deb+c\,d^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)*c*(-1/8*(d*(-4*a*c+b^2)^(1/2)+2*a*e-b*d)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(
-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(b*d-2*a*e+d*(-4*a*c+b^2)^(1/2))/(-4*a*c+b
^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(
-4*a*c+b^2)^(1/2))*c)^(1/2)))-d*e/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e*
x/(d*e)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 - b*d*e + a*e^2) + integr
ate((c*d*x^2 + a*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5974 vs. $2(209) = 418$.
time = 1.29, size = 11974, normalized size = 47.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^
2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2
```

```
*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)
*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4
*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6
*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42
*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d
^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*
a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b
^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3
- 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))*log(-2*c^2*d^2*x + 2*a*c*x*e
^2 + sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3 - (2*(b^2*c
^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^4*e + 4*(b^4*c - 3*a*b^2*c^2
- 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^2*e^3 + 2*(a*b^4
- 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 - 4*a^3*b*c)*e^5)*sqrt((c^2*d^4
 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c
^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a
*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a
^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b
^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a
^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 -
 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)
*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2
*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a
*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c
 - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 -
24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a
^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7
+ (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^
2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*
e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) - sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-
(b*c*d^2 - 4*a*c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*
b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c
)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)
/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 -
10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^
5*e^3 + (b^6 + 8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5
- a^2*b^3*c - 12*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c
^2)*d^2*e^6 - 4*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((
b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8
*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))
*log(-2*c^2*d^2*x + 2*a*c*x*e^2 - sqrt(1/2)*((b^2*c - 4*a*c^2)*d^2*e - (a*b
^2 - 4*a^2*c)*e^3 - (2*(b^2*c^3 - 4*a*c^4)*d^5 - 5*(b^3*c^2 - 4*a*b*c^3)*d^
4*e + 4*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^3*e^2 - (b^5 + 2*a*b^3*c - 24*a
^2*b*c^2)*d^2*e^3 + 2*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d*e^4 - (a^2*b^3 -
4*a^3*b*c)*e^5)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^
5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^
```

```
2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a*
b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 12*a^
3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4*(a^
3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))*sqrt(-(b*c*d^2 - 4*a*
c*d*e + a*b*e^2 + ((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e +
(b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*
b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*
a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 -
8*a^2*c^4)*d^6*e^2 - 4*(b^5*c - a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 +
8*a*b^4*c - 42*a^2*b^2*c^2 - 24*a^3*c^3)*d^4*e^4 - 4*(a*b^5 - a^2*b^3*c - 1
2*a^3*b*c^2)*d^3*e^5 + 2*(3*a^2*b^4 - 10*a^3*b^2*c - 8*a^4*c^2)*d^2*e^6 - 4
*(a^3*b^3 - 4*a^4*b*c)*d*e^7 + (a^4*b^2 - 4*a^5*c)*e^8)))/((b^2*c^2 - 4*a*c
^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e
^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4))) + sqrt(1/2)*(
c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c*d^2 - 4*a*c*d*e + a*b*e^2 - ((b^2*c^2 - 4
*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d
^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)*sqrt((c^2*d
^4 - 2*a*c*d^2*e^2 + a^2*e^4)/((b^2*c^4 - 4*a*c^5)*d^8 - 4*(b^3*c^3 - 4*a*b
*c^4)*d^7*e + 2*(3*b^4*c^2 - 10*a*b^2*c^3 - 8*a^2*c^4)*d^6*e^2 - 4*(b^5*c -
 a*b^3*c^2 - 12*a^2*b*c^3)*d^5*e^3 + (b^6 + 8*a...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 6921 vs. $2(209) = 418$.
time = 6.35, size = 6921, normalized size = 27.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -sqrt(d)*arctan(x*e^(1/2)/sqrt(d))*e^(1/2)/(c*d^2 - b*d*e + a*e^2) + 1/8*((
2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^
4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^5 - 2*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5
```

```
 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)
*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 -
 4*a*c)*a*b*c^4)*d^4*e + (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*
sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c
)*a*b^2*c^3)*d^3*e^2 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 2*sqrt(2)*sqrt(b*c
 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 -
 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 -
4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*
e - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sq
rt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*
c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^3 - 2
*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e + a*e^2)^2*d
 - 4*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c
+ sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^2*b*c^3)*d
^2*e^3 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*
b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*a^2*b^3*c^2 - 4*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 2*(b^2 - 4*a*c)*a*b^3*c
 - 8*(b^2 - 4*a*c)*a^2*b*c^2)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 + 5*(2*a^2*b
```

```
^4*c^2 - 8*a^3*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*
c)*a^3*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*
a^2*b^3*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b
^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2)*d*e^4 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c
 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*a^2*b^4*c + 16*s
qrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2
*c^2 + 16*a^3*b^2*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 -
 32*a^4*c^3 + 2*(b^2 - 4*a*c)*a^2*b^2*c - 8*(b^2 - 4*a*c)*a^3*c^2)*abs(c*d^
2 - b*d*e + a*e^2)*e^3 - 2*(2*a^3*b^3*c^2 - 8*a^4*b*c^3 - sqrt(2)*sqrt(b^2
- 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3 + 4*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b
*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 2*(b^2 - ...
```

Mupad [B]
time = 4.96, size = 2500, normalized size = 9.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(b^4*d^3*e^5 - a*b^3*d^2*e^6 + a*c^3*d^5*e^3 - b^3*c*d^4*e^4 + 2*a^2*c^
2*d^3*e^5 + a^3*c*d*e^7 + b^4*e^3*x*(-d*e)^(5/2) + a*b^3*e^5*x*(-d*e)^(3/2)
 + a^3*c*e^7*x*(-d*e)^(1/2) + 2*a*b*c^2*d^4*e^4 - 3*a*b^2*c*d^3*e^5 + 2*a^2
*b*c*d^2*e^6 + 2*a^2*c^2*e^3*x*(-d*e)^(5/2) - a*c^3*d*x*(-d*e)^(7/2) + b^3*
c*e*x*(-d*e)^(7/2) - 2*a*b*c^2*e*x*(-d*e)^(7/2) - 3*a*b^2*c*e^3*x*(-d*e)^(5
/2) - 2*a^2*b*c*e^5*x*(-d*e)^(3/2))*(-d*e)^(1/2))/(2*a*e^2 + 2*c*d^2 - 2*b*
d*e) - atan(((x*(2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(
a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b
^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d
*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d
^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d
*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^
3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^
3*e^7 + 16*a*c^6*d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*
c^5*d^3*e^4 - 8*b^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8
*b^5*c^2*d^2*e^5 - 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c
^4*d^2*e^5 + 24*a^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3
)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^
2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^
4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^
```

2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2
*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a
^3*b*c^2*d*e^3)))^(1/2)*(x*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) +
b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2
+ 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4
*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 +
32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6
*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d
e^3)))^(1/2)(256*a^4*b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 +
512*a^2*c^7*d^6*e^3 + 512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c
^6*d^7*e^2 + 128*b^4*c^5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^
5 - 32*b^7*c^2*d^3*e^6 + 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6
- 192*a^2*b^4*c^3*d^2*e^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 +
640*a^4*b*c^4*d*e^8 - 640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672
*a*b^4*c^4*d^4*e^5 + 96*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2
*b*c^6*d^5*e^4 + 32*a^2*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3
*c^3*d*e^8) - 192*a^4*c^4*d*e^7 - 192*a^2*c^6*d^5*e^3 - 384*a^3*c^5*d^3*e^5
- 96*a^2*b^2*c^4*d^3*e^5 - 96*a^2*b^3*c^3*d^2*e^6 + 48*a*b^2*c^5*d^5*e^3 -
96*a*b^3*c^4*d^4*e^4 + 48*a*b^4*c^3*d^3*e^5 + 384*a^2*b*c^5*d^4*e^4 + 384*
a^3*b*c^4*d^2*e^6 + 48*a^3*b^2*c^3*d*e^7))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c -
b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2
- 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*
c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*
a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b
^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3
- 32*a^3*b*c^2*d*e^3)))^(1/2) - 4*a*c^5*d^4*e^2 - 52*a^2*c^4*d^2*e^4 + 8*a*
b*c^4*d^3*e^3 - 4*a*b^3*c^2*d*e^5 + 20*a^2*b*c^3*d*e^5 + 8*a*b^2*c^3*d^2*e^
4))*(-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*
a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*
b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4
+ b^6*d^2*e^2 - 8*a*b^2*c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*
a*b^5*d*e^3 - 2*b^5*c*d^3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a
^2*b*c^3*d^3*e + 16*a^2*b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*1i + (x*(
2*a^2*c^3*e^5 - 4*a*c^4*d^2*e^3 + 2*b^2*c^3*d^2*e^3) - (-(a*b^3*e^2 - a*e^2
*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*
a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e
^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*
c^3*d^4 - 8*a^3*b^2*c*e^4 + 32*a^3*c^3*d^2*e^2 - 2*a*b^5*d*e^3 - 2*b^5*c*d^
3*e + 16*a*b^3*c^2*d^3*e - 6*a*b^4*c*d^2*e^2 - 32*a^2*b*c^3*d^3*e + 16*a^2*
b^3*c*d*e^3 - 32*a^3*b*c^2*d*e^3)))^(1/2)*((x*(32*a^3*b*c^3*e^7 + 16*a*c^6*
d^5*e^2 - 112*a^3*c^4*d*e^6 - 8*a^2*b^3*c^2*e^7 + 160*a^2*c^5*d^3*e^4 - 8*b
^2*c^5*d^5*e^2 + 8*b^3*c^4*d^4*e^3 + 8*b^4*c^3*d^3*e^4 - 8*b^5*c^2*d^2*e^5
- 96*a*b^2*c^4*d^3*e^4 + 64*a*b^3*c^3*d^2*e^5 - 96*a^2*b*c^4*d^2*e^5 + 24*a
^2*b^2*c^3*d*e^6) + (-(a*b^3*e^2 - a*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^3*c*d
^2 + c*d^2*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b*c^2*d^2 - 4*a^2*b*c*e^2 + 16*a^

```
2*c^2*d*e - 4*a*b^2*c*d*e)/(8*(a^2*b^4*e^4 + 16*a^2*c^4*d^4 + 16*a^4*c^2*e^
4 + b^4*c^2*d^4 + b^6*d^2*e^2 - 8*a*b^2*c^3*d^4...
```

3.307 $\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=254

$$-\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}-\frac{\sqrt{c}\left(e+\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}$$

```
[Out] e^(3/2)*arctan(x*e^(1/2)/d^(1/2))/(a*e^2-b*d*e+c*d^2)/d^(1/2)-1/2*arctan(x*
2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(b*e-2*c*d)/(-4*a*
c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(e+(-b*e+2*c
*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2)
```

Rubi [A]

time = 0.36, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1184, 211, 1180}

$$-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)}{\sqrt{2}\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{e^{3/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[c]*(e - (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)
/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2))) - (Sqrt[c]*(e + (2*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b + Sqrt[b^
2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]]
)/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1184

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{e^2}{(cd^2-bde+ae^2)(d+ex^2)} + \frac{cd-be-cex^2}{(cd^2-bde+ae^2)(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{cd-be-cex^2}{a+bx^2+cx^4}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{d+ex^2}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\frac{b}{2}-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx}{2(cd^2-bde+ae^2)}$$

$$= -\frac{\sqrt{c}\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \frac{\sqrt{c}\left(e+\frac{2cd}{\sqrt{b^2-}}\right.}{\sqrt{2}\sqrt{b+}}$$

Mathematica [A]
time = 0.16, size = 274, normalized size = 1.08

$$\frac{\sqrt{c}\left(-2cd+be+\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{\sqrt{c}\left(2cd-be+\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(-cd^2+bde-ae^2)} + \frac{e^{3/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[c]*(-2*c*d + b*e + Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sq
rt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 -
4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) + (Sqrt[c]*(2*c*d - b*e + Sqrt[b^2 - 4*
a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(-(c*d^2) + b*d*e - a*e^2)) +
(e^(3/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(Sqrt[d]*(c*d^2 - b*d*e + a*e^2))
```

Maple [A]
time = 0.11, size = 215, normalized size = 0.85

method	result
default	$\frac{4c\left(-\frac{\left(-e\sqrt{-4ac+b^2}-eb+2cd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(eb-2cd-e\sqrt{-4ac+b^2}\right)\sqrt{2}\,\operatorname{arc}}{8\sqrt{-4ac+b^2}\,\sqrt{(b}}\right.}{a\,e^2-deb+c\,d^2}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)*c*(-1/8*(-e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/(-4*a*c+b^2
)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+
(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(e*b-2*c*d-e*(-4*a*c+b^2)^(1/2))/(-4*a*c+
b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+
(-4*a*c+b^2)^(1/2))*c)^(1/2)))+e^2/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(x*e^(1/2)/sqrt(d))*e^(3/2)/((c*d^2 - b*d*e + a*e^2)*sqrt(d)) - integ
rate((c*x^2*e - c*d + b*e)/(c*x^4 + b*x^2 + a), x)/(c*d^2 - b*d*e + a*e^2)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7850 vs. $2(213) = 426$.
time = 13.06, size = 15733, normalized size = 61.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(1/2)*(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c^2*d^2 - 2*(b^2*c - 2*a*c
^2)*d*e + (b^3 - 3*a*b*c)*e^2 + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c -
```

4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^
3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4)*sqrt((c^4*d^4 - 4*b*c^3*d^3
e + 2(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*a
*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2*b^3*c^3 - 4*
a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^6*e^2 -
4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6 + 8*a^3*b^4*
c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^3*c - 12*a^5*
b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*e^6 - 4*(a^5*
b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))/((a*b^2*c^2 - 4*a^2*c^3
)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)
*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4))*log(-2
*c^4*d^2*x + 4*b*c^3*d*x*e - 2*(b^2*c^2 - a*c^3)*x*e^2 + sqrt(1/2)*((b^2*c^
3 - 4*a*c^4)*d^3 - 3*(b^3*c^2 - 4*a*b*c^3)*d^2*e + (3*b^4*c - 13*a*b^2*c^2
+ 4*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^3 - ((a*b^3*c^3 - 4*
a^2*b*c^4)*d^5 - (3*a*b^4*c^2 - 14*a^2*b^2*c^3 + 8*a^3*c^4)*d^4*e + (3*a*b^
5*c - 14*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^3*e^2 - (a*b^6 - 2*a^2*b^4*c - 12*a^3
*b^2*c^2 + 16*a^4*c^3)*d^2*e^3 + (2*a^2*b^5 - 11*a^3*b^3*c + 12*a^4*b*c^2)*
d*e^4 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^5)*sqrt((c^4*d^4 - 4*b*c^3*d^
3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 - 2*
a*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2*b^3*c^3 - 4
*a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^6*e^2
- 4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6 + 8*a^3*b^4
*c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^3*c - 12*a^5
*b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*e^6 - 4*(a^5
*b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))*sqrt(-(b*c^2*d^2 - 2*(
b^2*c - 2*a*c^2)*d*e + (b^3 - 3*a*b*c)*e^2 + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 -
2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^
2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4)*sqrt((c^4*d^4
- 4*b*c^3*d^3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3
+ (b^4 - 2*a*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2
*b^3*c^3 - 4*a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c
^4)*d^6*e^2 - 4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6
+ 8*a^3*b^4*c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^
3*c - 12*a^5*b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*
e^6 - 4*(a^5*b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))/((a*b^2*c^
2 - 4*a^2*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c
- 8*a^3*c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)
e^4))) - sqrt(1/2)(c*d^2 - b*d*e + a*e^2)*sqrt(-(b*c^2*d^2 - 2*(b^2*c - 2
*a*c^2)*d*e + (b^3 - 3*a*b*c)*e^2 + ((a*b^2*c^2 - 4*a^2*c^3)*d^4 - 2*(a*b^3
*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2*e^2 - 2*(a^
2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4)*sqrt((c^4*d^4 - 4*b*c^3
*d^3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4 -
2*a*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2*b^3*c^3
- 4*a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^6*e
^2 - 4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6 + 8*a^3*

```
b^4*c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^3*c - 12*
a^5*b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*e^6 - 4*(
a^5*b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))/((a*b^2*c^2 - 4*a^2
*c^3)*d^4 - 2*(a*b^3*c - 4*a^2*b*c^2)*d^3*e + (a*b^4 - 2*a^2*b^2*c - 8*a^3*
c^2)*d^2*e^2 - 2*(a^2*b^3 - 4*a^3*b*c)*d*e^3 + (a^3*b^2 - 4*a^4*c)*e^4))*lo
g(-2*c^4*d^2*x + 4*b*c^3*d*x*e - 2*(b^2*c^2 - a*c^3)*x*e^2 - sqrt(1/2)*((b^
2*c^3 - 4*a*c^4)*d^3 - 3*(b^3*c^2 - 4*a*b*c^3)*d^2*e + (3*b^4*c - 13*a*b^2*
c^2 + 4*a^2*c^3)*d*e^2 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^3 - ((a*b^3*c^3
- 4*a^2*b*c^4)*d^5 - (3*a*b^4*c^2 - 14*a^2*b^2*c^3 + 8*a^3*c^4)*d^4*e + (3*
a*b^5*c - 14*a^2*b^3*c^2 + 8*a^3*b*c^3)*d^3*e^2 - (a*b^6 - 2*a^2*b^4*c - 12
*a^3*b^2*c^2 + 16*a^4*c^3)*d^2*e^3 + (2*a^2*b^5 - 11*a^3*b^3*c + 12*a^4*b*c
^2)*d*e^4 - (a^3*b^4 - 6*a^4*b^2*c + 8*a^5*c^2)*e^5)*sqrt((c^4*d^4 - 4*b*c^
3*d^3*e + 2*(3*b^2*c^2 - a*c^3)*d^2*e^2 - 4*(b^3*c - a*b*c^2)*d*e^3 + (b^4
- 2*a*b^2*c + a^2*c^2)*e^4)/((a^2*b^2*c^4 - 4*a^3*c^5)*d^8 - 4*(a^2*b^3*c^3
 - 4*a^3*b*c^4)*d^7*e + 2*(3*a^2*b^4*c^2 - 10*a^3*b^2*c^3 - 8*a^4*c^4)*d^6*
e^2 - 4*(a^2*b^5*c - a^3*b^3*c^2 - 12*a^4*b*c^3)*d^5*e^3 + (a^2*b^6 + 8*a^3
*b^4*c - 42*a^4*b^2*c^2 - 24*a^5*c^3)*d^4*e^4 - 4*(a^3*b^5 - a^4*b^3*c - 12
*a^5*b*c^2)*d^3*e^5 + 2*(3*a^4*b^4 - 10*a^5*b^2*c - 8*a^6*c^2)*d^2*e^6 - 4*
(a^5*b^3 - 4*a^6*b*c)*d*e^7 + (a^6*b^2 - 4*a^7*c)*e^8)))*sqrt(-(b*c^2*d^2 -
 2*(b^2*c - 2*a*c^2)*d*e + (b^3 - 3*a*b*c)*e^2 ...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 7650 vs. $2(213) = 426$.
time = 5.89, size = 7650, normalized size = 30.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*(2*b^3*c^5 - 8*a*b*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b
*c^5 - 2*(b^2 - 4*a*c)*b*c^5)*d^5 - 5*(2*b^4*c^4 - 8*a*b^2*c^5 - sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^
```

2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 - 2*(b^2 - 4*a*c)*b^2*c^4)*d^4*e - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 2*b^4*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^4 + 16*a*b^2*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^5 - 32*a^2*c^5 + 2*(b^2 - 4*a*c)*b^2*c^3 - 8*(b^2 - 4*a*c)*a*c^4)*d^3*abs(c*d^2 - b*d*e + a*e^2) + 4*(2*b^5*c^3 - 6*a*b^3*c^4 - 8*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 - 2*(b^2 - 4*a*c)*a*b*c^4)*d^3*e^2 + 4*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 2*b^5*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^3 + 16*a*b^3*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 32*a^2*b*c^4 + 2*(b^2 - 4*a*c)*b^3*c^2 - 8*(b^2 - 4*a*c)*a*b*c^3)*d^2*abs(c*d^2 - b*d*e + a*e^2)*e - (2*b^6*c^2 + 4*a*b^4*c^3 - 48*a^2*b^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^4*c^2 - 12*(b^2 - 4*a*c)*a*b^2*c^3)*d^2*e^3 - 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^6 - 7*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c - 2*b^6*c + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 6*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 14*a*b^4*c^2 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 3*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 16*a^2*b^2*c^3 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*b^4*c - 6*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*d*abs(c*d^2 - b*d*e + a*e^2)*e^2 - (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr

```
t(b^2 - 4*a*c)*c)*a^2*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a*b*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^2*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*(c*d^2 - b*d*e +
a*e^2)^2*e + 2*(2*a*b^5*c^2 - 6*a^2*b^3*c^3 - 8*a^3*b*c^4 - sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 + 3*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
(b^2 - 4*a*c)*c)*a^2*b*c^3 - 2*(b^2 - 4*a*c)*a*b^3*c^2 - 2*(b^2 - 4*a*c)*a^
2*b*c^3)*d*e^4 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c - 2*sqrt(2)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^3*b*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 -...
```

Mupad [B]
time = 5.61, size = 2500, normalized size = 9.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^
2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2
 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a
*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3
*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2
 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3
*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*
b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16*b^5*c^2*e^7 + 16*c^7*d
^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*a*c^6*d^3*e^4 - 240*a^2
*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 16*b^2*c^5*d^3*e^4 + 16*
b^3*c^4*d^2*e^5 - 96*a*b*c^5*d^2*e^5 + 192*a*b^2*c^4*d*e^6) - (-(b^5*e^2 +
b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a
*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^
2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^
3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^
2*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b
^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*
```

```
c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2) -
16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*
(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*
c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^
2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*
b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*(256*a^4*
b^2*c^3*e^9 - 32*a^3*b^4*c^2*e^9 - 512*a^5*c^4*e^9 + 512*a^2*c^7*d^6*e^3 +
512*a^3*c^6*d^4*e^5 - 512*a^4*c^5*d^2*e^7 - 32*b^3*c^6*d^7*e^2 + 128*b^4*c^
5*d^6*e^3 - 192*b^5*c^4*d^5*e^4 + 128*b^6*c^3*d^4*e^5 - 32*b^7*c^2*d^3*e^6
+ 512*a^2*b^2*c^5*d^4*e^5 + 288*a^2*b^3*c^4*d^3*e^6 - 192*a^2*b^4*c^3*d^2*e
^7 + 384*a^3*b^2*c^4*d^2*e^7 + 128*a*b*c^7*d^7*e^2 + 640*a^4*b*c^4*d*e^8 -
640*a*b^2*c^6*d^6*e^3 + 1056*a*b^3*c^5*d^5*e^4 - 672*a*b^4*c^4*d^4*e^5 + 96
*a*b^5*c^3*d^3*e^6 + 32*a*b^6*c^2*d^2*e^7 - 1152*a^2*b*c^6*d^5*e^4 + 32*a^2
*b^5*c^2*d*e^8 - 640*a^3*b*c^5*d^3*e^6 - 288*a^3*b^3*c^3*d*e^8) - 256*a^4*c
^4*e^8 + 64*a*c^7*d^6*e^2 - 16*a^2*b^4*c^2*e^8 + 128*a^3*b^2*c^3*e^8 - 128*
a^2*c^6*d^4*e^4 - 448*a^3*c^5*d^2*e^6 - 16*b^2*c^6*d^6*e^2 + 64*b^3*c^5*d^5
*e^3 - 96*b^4*c^4*d^4*e^4 + 64*b^5*c^3*d^3*e^5 - 16*b^6*c^2*d^2*e^6 + 240*a
^2*b^2*c^4*d^2*e^6 - 256*a*b*c^6*d^5*e^3 + 32*a*b^5*c^2*d*e^7 + 384*a^3*b*c
^4*d*e^7 + 416*a*b^2*c^5*d^4*e^4 - 288*a*b^3*c^4*d^3*e^5 + 32*a*b^4*c^3*d^2
*e^6 + 128*a^2*b*c^5*d^3*e^5 - 224*a^2*b^3*c^3*d*e^7))*(-(b^5*e^2 + b^3*c^2
*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2)
+ 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*
(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-
(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 +
 a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*
b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 1
6*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c
*d^2*e^2)))^(1/2) - 4*b^3*c^3*e^6 - 4*c^6*d^3*e^3 + 4*b*c^5*d^2*e^4 + 4*b^2
*c^4*d*e^5 + 16*a*b*c^4*e^6 - 20*a*c^5*d*e^5) + 6*c^5*e^5*x)*(-(b^5*e^2 + b
^3*c^2*d^2 + b^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^
(1/2) + 12*a^2*b*c^2*e^2 - 2*b^4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*
c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*
d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2
*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 -
8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3
*e + 16*a^3*b^3*c*d*e^3 - 32*a^4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2
*b^4*c*d^2*e^2)))^(1/2)*1i + ((-(b^5*e^2 + b^3*c^2*d^2 + b^2*e^2*(-(4*a*c -
 b^2)^3)^(1/2) + c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) + 12*a^2*b*c^2*e^2 - 2*b^
4*c*d*e - 4*a*b*c^3*d^2 - 7*a*b^3*c*e^2 - a*c*e^2*(-(4*a*c - b^2)^3)^(1/2)
- 16*a^2*c^3*d*e + 12*a*b^2*c^2*d*e - 2*b*c*d*e*(-(4*a*c - b^2)^3)^(1/2))/(
8*(a^3*b^4*e^4 + 16*a^3*c^4*d^4 + 16*a^5*c^2*e^4 + a*b^4*c^2*d^4 - 8*a^4*b^
2*c*e^4 + a*b^6*d^2*e^2 - 2*a^2*b^5*d*e^3 - 8*a^2*b^2*c^3*d^4 + 32*a^4*c^3*
d^2*e^2 - 2*a*b^5*c*d^3*e - 32*a^3*b*c^3*d^3*e + 16*a^3*b^3*c*d*e^3 - 32*a^
4*b*c^2*d*e^3 + 16*a^2*b^3*c^2*d^3*e - 6*a^2*b^4*c*d^2*e^2)))^(1/2)*((x*(16
*b^5*c^2*e^7 + 16*c^7*d^5*e^2 - 112*a*b^3*c^3*e^7 + 192*a^2*b*c^4*e^7 + 32*
```

```
a*c^6*d^3*e^4 - 240*a^2*c^5*d*e^6 - 32*b*c^6*d^4*e^3 - 32*b^4*c^3*d*e^6 + 1
6*b^2*c^5*d^3*e^4 + 16*b^3*c^4*d^2*e^5 - 96*a*b...
```

3.308 $\int \frac{1}{x^2(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=298

$$-\frac{1}{adx}-\frac{\sqrt{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}-\frac{\sqrt{c}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\cdots\right.}{\sqrt{2}\,a\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2-\cdots}$$

```
[Out] -1/a/d/x-e^(5/2)*arctan(x*e^(1/2)/d^(1/2))/d^(3/2)/(a*e^2-b*d*e+c*d^2)-1/2*
arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(c*d-b*e+(2*
a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))/a/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4
*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^
(1/2))*c^(1/2)*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))/a/(a*e^2
-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.62, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1301, 211, 1180}

$$-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}-\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{\sqrt{2}\,a\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}-\frac{e^{5/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}(ae^2-bde+cd^2)}-\frac{1}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) - (Sqrt[c]*(c*d - b*e
- (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*x)/S
qrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2
- b*d*e + a*e^2)) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
 b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^2 (d+ex^2)(a+bx^2+cx^4)}\,dx = \int \left(\frac{1}{adx^2} - \frac{e^3}{d(cd^2-bde+ae^2)(d+ex^2)} + \frac{-bcd+b^2e-ace-c(c\ldots}{a(cd^2-bde+ae^2)(a+\ldots}\right.$$

$$= -\frac{1}{adx} + \frac{\int \frac{-bcd+b^2e-ace-c(cd-be)x^2}{a+bx^2+cx^4}\,dx}{a(cd^2-bde+ae^2)} - \frac{e^3\int \frac{1}{d+ex^2}\,dx}{d(cd^2-bde+ae^2)}$$

$$= -\frac{1}{adx} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}(cd^2-bde+ae^2)} - \frac{\left(c\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\ldots}{\frac{b}{2}+\ldots}}{2a(cd^2-bde+ae\ldots}$$

$$= -\frac{1}{adx} - \frac{\sqrt{c}\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)} - \ldots$$

Mathematica [A]

time = 0.26, size = 340, normalized size = 1.14

$$-\frac{1}{adx} - \frac{\sqrt{c}\left(bcd+c\sqrt{b^2-4ac}\,d-b^2e+2ace-b\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2+e(-bd+ae))} + \frac{\sqrt{c}\left(bcd-c\sqrt{b^2-4ac}\,d-b^2e+2ace+b\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{b+\sqrt{b^2-4ac}}\,(cd^2+e(-bd+ae))} - \frac{e^{5/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{3/2}(cd^2-bde+ae^2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(1/(a*d*x)) - (Sqrt[c]*(b*c*d + c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e -
b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c
]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-
(b*d) + a*e))) + (Sqrt[c]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e
+ b*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a
*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*
(-(b*d) + a*e))) - (e^(5/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d^(3/2)*(c*d^2 -
b*d*e + a*e^2))
```

Maple [A]
time = 0.21, size = 276, normalized size = 0.93

method	result
default	$\frac{4c\left(-\frac{\left(eb\sqrt{-4ac+b^2}-cd\sqrt{-4ac+b^2}-2ace+b^2e-bcd\right)\sqrt{2}\,\operatorname{arctanh}\left(\frac{cx\sqrt{2}}{\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}\right)}{8\sqrt{-4ac+b^2}\,\sqrt{\left(-b+\sqrt{-4ac+b^2}\right)c}}+\frac{\left(eb\sqrt{-4ac\right.}}{}\right.}{(a\,e^2-deb+c\,d^2)a}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)/a*c*(-1/8*(e*b*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*c+b^2)^(1
/2)-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2)
)*c)^(1/2)*arctanh(c*x*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(e*b*
(-4*a*c+b^2)^(1/2)-c*d*(-4*a*c+b^2)^(1/2)+2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)
^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(1/2)/((b+(-4*
a*c+b^2)^(1/2))*c)^(1/2)))-1/d*e^3/(a*e^2-b*d*e+c*d^2)/(d*e)^(1/2)*arctan(e
*x/(d*e)^(1/2))-1/a/d/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] -arctan(x*e^(1/2)/sqrt(d))*e^(5/2)/((c*d^3 - b*d^2*e + a*d*e^2)*sqrt(d)) -
integrate((b*c*d + (c^2*d - b*c*e)*x^2 - b^2*e + a*c*e)/(c*x^4 + b*x^2 + a)
, x)/(a*c*d^2 - a*b*d*e + a^2*e^2) - 1/(a*d*x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10147 vs. $2(258) = 516$.
time = 163.30, size = 20327, normalized size = 68.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [1/2*(a*x*sqrt(-e/d)*e^2*log((x^2*e - 2*d*x*sqrt(-e/d) - d)/(x^2*e + d)) -
2*c*d^2 + 2*b*d*e + sqrt(1/2)*(a*c*d^3*x - a*b*d^2*x*e + a^2*d*x*e^2)*sqrt(
-((b^3*c^2 - 3*a*b*c^3)*d^2 - 2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d*e + (b^
5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^2 + ((a^3*b^2*c^2 - 4*a^4*c^3)*d^4 - 2*(a^3*
b^3*c - 4*a^4*b*c^2)*d^3*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^2*e^2 -
2*(a^4*b^3 - 4*a^5*b*c)*d*e^3 + (a^5*b^2 - 4*a^6*c)*e^4)*sqrt(((b^4*c^4 - 2
*a*b^2*c^5 + a^2*c^6)*d^4 - 4*(b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*d^3*e +
 2*(3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*d^2*e^2 - 4*(b^7*c
 - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^3 + (b^8 - 6*a*b^6*c + 11
*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^4)/((a^6*b^2*c^4 - 4*a^7*c^5)*d^8
 - 4*(a^6*b^3*c^3 - 4*a^7*b*c^4)*d^7*e + 2*(3*a^6*b^4*c^2 - 10*a^7*b^2*c^3
- 8*a^8*c^4)*d^6*e^2 - 4*(a^6*b^5*c - a^7*b^3*c^2 - 12*a^8*b*c^3)*d^5*e^3 +
 (a^6*b^6 + 8*a^7*b^4*c - 42*a^8*b^2*c^2 - 24*a^9*c^3)*d^4*e^4 - 4*(a^7*b^5
 - a^8*b^3*c - 12*a^9*b*c^2)*d^3*e^5 + 2*(3*a^8*b^4 - 10*a^9*b^2*c - 8*a^10
*c^2)*d^2*e^6 - 4*(a^9*b^3 - 4*a^10*b*c)*d*e^7 + (a^10*b^2 - 4*a^11*c)*e^8)
))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^4 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^3*e + (a^
3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^2*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d*e^3 +
(a^5*b^2 - 4*a^6*c)*e^4))*log(2*(b^2*c^5 - a*c^6)*d^2*x - 4*(b^3*c^4 - 2*a*
b*c^5)*d*x*e + 2*(b^4*c^3 - 3*a*b^2*c^4 + a^2*c^5)*x*e^2 + sqrt(1/2)*((b^5*
c^3 - 5*a*b^3*c^4 + 4*a^2*b*c^5)*d^3 - (3*b^6*c^2 - 18*a*b^4*c^3 + 25*a^2*b
^2*c^4 - 4*a^3*c^5)*d^2*e + (3*b^7*c - 21*a*b^5*c^2 + 41*a^2*b^3*c^3 - 20*a
^3*b*c^4)*d*e^2 - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^
4*c^4)*e^3 - ((a^3*b^4*c^3 - 6*a^4*b^2*c^4 + 8*a^5*c^5)*d^5 - (3*a^3*b^5*c^
2 - 19*a^4*b^3*c^3 + 28*a^5*b*c^4)*d^4*e + (3*a^3*b^6*c - 18*a^4*b^4*c^2 +
20*a^5*b^2*c^3 + 16*a^6*c^4)*d^3*e^2 - (a^3*b^7 - 3*a^4*b^5*c - 14*a^5*b^3*
c^2 + 40*a^6*b*c^3)*d^2*e^3 + (2*a^4*b^6 - 13*a^5*b^4*c + 18*a^6*b^2*c^2 +
8*a^7*c^3)*d*e^4 - (a^5*b^5 - 7*a^6*b^3*c + 12*a^7*b*c^2)*e^5)*sqrt(((b^4*c
^4 - 2*a*b^2*c^5 + a^2*c^6)*d^4 - 4*(b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*b*c^5)*d
^3*e + 2*(3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*d^2*e^2 - 4*
(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^3 + (b^8 - 6*a*b^6*
c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^4)/((a^6*b^2*c^4 - 4*a^7*c^
5)*d^8 - 4*(a^6*b^3*c^3 - 4*a^7*b*c^4)*d^7*e + 2*(3*a^6*b^4*c^2 - 10*a^7*b^
2*c^3 - 8*a^8*c^4)*d^6*e^2 - 4*(a^6*b^5*c - a^7*b^3*c^2 - 12*a^8*b*c^3)*d^5
*e^3 + (a^6*b^6 + 8*a^7*b^4*c - 42*a^8*b^2*c^2 - 24*a^9*c^3)*d^4*e^4 - 4*(a
^7*b^5 - a^8*b^3*c - 12*a^9*b*c^2)*d^3*e^5 + 2*(3*a^8*b^4 - 10*a^9*b^2*c -
8*a^10*c^2)*d^2*e^6 - 4*(a^9*b^3 - 4*a^10*b*c)*d*e^7 + (a^10*b^2 - 4*a^11*c
)*e^8)))*sqrt(-((b^3*c^2 - 3*a*b*c^3)*d^2 - 2*(b^4*c - 4*a*b^2*c^2 + 2*a^2*
c^3)*d*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^2 + ((a^3*b^2*c^2 - 4*a^4*c^3)
*d^4 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^3*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c
^2)*d^2*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d*e^3 + (a^5*b^2 - 4*a^6*c)*e^4)*sqrt
(((b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*d^4 - 4*(b^5*c^3 - 3*a*b^3*c^4 + 2*a^2*
b*c^5)*d^3*e + 2*(3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*c^5)*d^2*
e^2 - 4*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^3 + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^4)/((a^6*b^2*c^4 -
4*a^7*c^5)*d^8 - 4*(a^6*b^3*c^3 - 4*a^7*b*c^4)*d^7*e + 2*(3*a^6*b^4*c^2 - 1
```

```
0*a^7*b^2*c^3 - 8*a^8*c^4)*d^6*e^2 - 4*(a^6*b^5*c - a^7*b^3*c^2 - 12*a^8*b*
c^3)*d^5*e^3 + (a^6*b^6 + 8*a^7*b^4*c - 42*a^8*b^2*c^2 - 24*a^9*c^3)*d^4*e^
4 - 4*(a^7*b^5 - a^8*b^3*c - 12*a^9*b*c^2)*d^3*e^5 + 2*(3*a^8*b^4 - 10*a^9*
b^2*c - 8*a^10*c^2)*d^2*e^6 - 4*(a^9*b^3 - 4*a^10*b*c)*d*e^7 + (a^10*b^2 -
4*a^11*c)*e^8)))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^4 - 2*(a^3*b^3*c - 4*a^4*b*c^
2)*d^3*e + (a^3*b^4 - 2*a^4*b^2*c - 8*a^5*c^2)*d^2*e^2 - 2*(a^4*b^3 - 4*a^5
*b*c)*d*e^3 + (a^5*b^2 - 4*a^6*c)*e^4))) - sqrt(1/2)*(a*c*d^3*x - a*b*d^2*x
*e + a^2*d*x*e^2)*sqrt(-((b^3*c^2 - 3*a*b*c^3)*d^2 - 2*(b^4*c - 4*a*b^2*c^2
 + 2*a^2*c^3)*d*e + (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e^2 + ((a^3*b^2*c^2 - 4
*a^4*c^3)*d^4 - 2*(a^3*b^3*c - 4*a^4*b*c^2)*d^3*e + (a^3*b^4 - 2*a^4*b^2*c
- 8*a^5*c^2)*d^2*e^2 - 2*(a^4*b^3 - 4*a^5*b*c)*d*e^3 + (a^5*b^2 - 4*a^6*c)*
e^4)*sqrt(((b^4*c^4 - 2*a*b^2*c^5 + a^2*c^6)*d^4 - 4*(b^5*c^3 - 3*a*b^3*c^4
 + 2*a^2*b*c^5)*d^3*e + 2*(3*b^6*c^2 - 12*a*b^4*c^3 + 12*a^2*b^2*c^4 - a^3*
c^5)*d^2*e^2 - 4*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e^3
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^4)/((a^6*b
^2*c^4 - 4*a^7*c^5)*d^8 - 4*(a^6*b^3*c^3 - 4*a^7*b*c^4)*d^7*e + 2*(3*a^6*b^
4*c^2 - 10*a^7*b^2*c^3 - 8*a^8*c^4)*d^6*e^2 - 4*(a^6*b^5*c - a^7*b^3*c^2 -
12*a^8*b*c^3)*d^5*e^3 + (a^6*b^6 + 8*a^7*b^4*c - 42*a^8*b^2*c^2 - 24*a^9*c^
3)*d^4*e^4 - 4*(a^7*b^5 - a^8*b^3*c - 12*a^9*b*c^2)*d^3*e^5 + 2*(3*a^8*b^4
- 10*a^9*b^2*c - 8*a^10*c^2)*d^2*e^6 - 4*(a^9*b^3 - 4*a^10*b*c)*d*e^7 + (a^
10*b^2 - 4*a^11*c)*e^8)))/((a^3*b^2*c^2 - 4*a^4*c^3)*d^4 - 2*(a^3*b^3*c - 4
*a^4*b*c^2)*d^3*e + (a^3*b^4 - 2*a^4*b^2*c - 8*...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10058 vs. 2(258) = 516.
time = 7.54, size = 10058, normalized size = 33.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*((2*a^2*b^4*c^5 - 8*a^3*b^2*c^6 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt
```

(b^2 - 4*a*c)*c)*a^2*b^2*c^5 - 2*(b^2 - 4*a*c)*a^2*b^2*c^5)*d^5 - (6*a^2*b^5*c^4 - 28*a^3*b^3*c^5 + 16*a^4*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^4 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^5 - 6*(b^2 - 4*a*c)*a^2*b^3*c^4 + 4*(b^2 - 4*a*c)*a^3*b*c^5)*d^4*e + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 2*a*b^5*c^3 + 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 + 16*a^2*b^3*c^4 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 - 32*a^3*b*c^5 + 2*(b^2 - 4*a*c)*a*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*b*c^4)*d^3*abs(a*c*d^2 - a*b*d*e + a^2*e^2) + (6*a^2*b^6*c^3 - 28*a^3*b^4*c^4 + 16*a^4*b^2*c^5 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^2 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^2*c^3 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^3 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^4 - 6*(b^2 - 4*a*c)*a^2*b^4*c^3 + 4*(b^2 - 4*a*c)*a^3*b^2*c^4)*d^3*e^2 - 2*(2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 17*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 - 4*a*b^6*c^2 + 40*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^3 + 18*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 + 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 34*a^2*b^4*c^3 - 16*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^4 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 - 9*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 80*a^3*b^2*c^4 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*c^5 + 32*a^4*c^5 + 4*(b^2 - 4*a*c)*a*b^4*c^2 - 18*(b^2 - 4*a*c)*a^2*b^2*c^3 + 8*(b^2 - 4*a*c)*a^3*c^4)*d^2*abs(a*c*d^2 - a*b*d*e + a^2*e^2)*e + (2*b^4*c^3 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^4*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a*c^4)*(a*c*d^2 - a*b*d*e + a^2*e^2)^2*d - (2*a^2*b^7*c^2 - 4*a^3*b^5*c^3 - 24*a^4*b^3*c^4 + 32*a^5*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr

```
t(b^2 - 4*a*c)*c)*a^3*b^5*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b
^2 - 4*a*c)*c)*a^2*b^6*c + 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^4*b^3*c^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2
- 4*a*c)*c)*a^3*b^4*c^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^2*b^5*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^5*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^4*b^2*c^3 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c
)*c)*a^3*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)
*c)*a^4*b*c^4 - 2*(b^2 - 4*a*c)*a^2*b^5*c^2 - 4*(b^2 - 4*a*c)*a^3*b^3*c^3 +
 8*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*e^3 + 2*(sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a*b^7 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c - 2*sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^6*c - 2*a*b^7*c + 16*sqrt(2)*sqrt(b*
c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^2 + 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*a^2*b^4*c^2 + sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + 16*
a^2*b^5*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 32*a^
3*b^3*c^3 + 2*(b^2 - 4*a*c)*a*b^5*c - 8*(b^2 - 4*a*c)*a^2*b^3*c^2)*d*abs(a*
c*d^2 - a*b*d*e + a^2*e^2)*e^2 - (2*b^5*c^2 - 16*a*b^3*c^3 + 32*a^2*b*c^4 -
 sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 ...
```

Mupad [B]
time = 5.89, size = 2500, normalized size = 8.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^
3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*
b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^
3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^
4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c
*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c
 - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6
*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^
6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 1
6*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c
*d^2*e^2)))^(1/2)*(((-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(
1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1
/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(
4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^
2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2
) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2
*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^
2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3
```

*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*
c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e
- 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(192*a^10*c^7*d^14*e^3 - x*(-(b^7*e^2 + b^5*
c^2*d^2 + b^4*e^2*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4
*d^2 - a*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e
+ 25*a^2*b^3*c^2*e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(
-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e
 - 2*b^3*c*d*e*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^
(1/2) - 36*a^2*b^2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^
5*b^4*e^4 + 16*a^5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d
*e^3 + a^3*b^4*c^2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d
^2*e^2 - 2*a^3*b^5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a
^6*b*c^2*d*e^3 + 16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(1/2)*(512*a
^11*c^7*d^15*e^3 + 512*a^12*c^6*d^13*e^5 - 512*a^13*c^5*d^11*e^7 - 512*a^14
*c^4*d^9*e^9 - 32*a^9*b^3*c^6*d^16*e^2 + 128*a^9*b^4*c^5*d^15*e^3 - 192*a^9
*b^5*c^4*d^14*e^4 + 128*a^9*b^6*c^3*d^13*e^5 - 32*a^9*b^7*c^2*d^12*e^6 - 64
0*a^10*b^2*c^6*d^15*e^3 + 1056*a^10*b^3*c^5*d^14*e^4 - 672*a^10*b^4*c^4*d^1
3*e^5 + 96*a^10*b^5*c^3*d^12*e^6 + 32*a^10*b^6*c^2*d^11*e^7 + 512*a^11*b^2*
c^5*d^13*e^5 + 288*a^11*b^3*c^4*d^12*e^6 - 192*a^11*b^4*c^3*d^11*e^7 + 32*a
^11*b^5*c^2*d^10*e^8 + 384*a^12*b^2*c^4*d^11*e^7 - 288*a^12*b^3*c^3*d^10*e^
8 - 32*a^12*b^4*c^2*d^9*e^9 + 256*a^13*b^2*c^3*d^9*e^9 + 128*a^10*b*c^7*d^1
6*e^2 - 1152*a^11*b*c^6*d^14*e^4 - 640*a^12*b*c^5*d^12*e^6 + 640*a^13*b*c^4
*d^10*e^8) + 128*a^11*c^6*d^12*e^5 - 320*a^12*c^5*d^10*e^7 - 256*a^13*c^4*d
^8*e^9 - 16*a^8*b^3*c^6*d^15*e^2 + 64*a^8*b^4*c^5*d^14*e^3 - 96*a^8*b^5*c^4
*d^13*e^4 + 64*a^8*b^6*c^3*d^12*e^5 - 16*a^8*b^7*c^2*d^11*e^6 - 304*a^9*b^2
*c^6*d^14*e^3 + 512*a^9*b^3*c^5*d^13*e^4 - 352*a^9*b^4*c^4*d^12*e^5 + 64*a^
9*b^5*c^3*d^11*e^6 + 16*a^9*b^6*c^2*d^10*e^7 + 352*a^10*b^2*c^5*d^12*e^5 +
80*a^10*b^3*c^4*d^11*e^6 - 128*a^10*b^4*c^3*d^10*e^7 + 16*a^10*b^5*c^2*d^9*
e^8 + 336*a^11*b^2*c^4*d^10*e^7 - 128*a^11*b^3*c^3*d^9*e^8 - 16*a^11*b^4*c^
2*d^8*e^9 + 128*a^12*b^2*c^3*d^8*e^9 + 64*a^9*b*c^7*d^15*e^2 - 512*a^10*b*c
^6*d^13*e^4 - 320*a^11*b*c^5*d^11*e^6 + 256*a^12*b*c^4*d^9*e^8) + x*(112*a^
10*c^6*d^10*e^6 - 32*a^9*c^7*d^12*e^4 - 16*a^8*c^8*d^14*e^2 - 128*a^11*c^5*
d^8*e^8 + 8*a^7*b^2*c^7*d^14*e^2 - 16*a^7*b^3*c^6*d^13*e^3 + 8*a^7*b^4*c^5*
d^12*e^4 + 8*a^7*b^5*c^4*d^11*e^5 - 16*a^7*b^6*c^3*d^10*e^6 + 8*a^7*b^7*c^2
*d^9*e^7 - 72*a^8*b^3*c^5*d^11*e^5 + 128*a^8*b^4*c^4*d^10*e^6 - 72*a^8*b^5*
c^3*d^9*e^7 - 280*a^9*b^2*c^5*d^10*e^6 + 208*a^9*b^3*c^4*d^9*e^7 - 16*a^9*b
^4*c^3*d^8*e^8 + 8*a^9*b^5*c^2*d^7*e^9 + 96*a^10*b^2*c^4*d^8*e^8 - 56*a^10*
b^3*c^3*d^7*e^9 + 32*a^8*b*c^7*d^13*e^3 + 128*a^9*b*c^6*d^11*e^5 - 192*a^10
*b*c^5*d^9*e^7 + 96*a^11*b*c^4*d^7*e^9))*(-(b^7*e^2 + b^5*c^2*d^2 + b^4*e^2
*(-(4*a*c - b^2)^3)^(1/2) - 7*a*b^3*c^3*d^2 + 12*a^2*b*c^4*d^2 - a*c^3*d^2*
(-(4*a*c - b^2)^3)^(1/2) - 20*a^3*b*c^3*e^2 - 2*b^6*c*d*e + 25*a^2*b^3*c^2*
e^2 + a^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + b^2*c^2*d^2*(-(4*a*c - b^2)^3)
^(1/2) - 9*a*b^5*c*e^2 + 16*a^3*c^4*d*e + 16*a*b^4*c^2*d*e - 2*b^3*c*d*e*(-
(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 36*a^2*b^
2*c^3*d*e + 4*a*b*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^5*b^4*e^4 + 16*a^

```
5*c^4*d^4 + 16*a^7*c^2*e^4 - 8*a^6*b^2*c*e^4 - 2*a^4*b^5*d*e^3 + a^3*b^4*c^
2*d^4 - 8*a^4*b^2*c^3*d^4 + a^3*b^6*d^2*e^2 + 32*a^6*c^3*d^2*e^2 - 2*a^3*b^
5*c*d^3*e - 32*a^5*b*c^3*d^3*e + 16*a^5*b^3*c*d*e^3 - 32*a^6*b*c^2*d*e^3 +
16*a^4*b^3*c^2*d^3*e - 6*a^4*b^4*c*d^2*e^2)))^(...
```

3.309 $\int \frac{1}{x^4(d+ex^2)(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=348

$$-\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\sqrt{c}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}\,(cd^2-bde+ae^2)}+\frac{\sqrt{c}\left(bcd-\right.}{}$$

```
[Out] -1/3/a/d/x^3+(a*e+b*d)/a^2/d^2/x+e^(7/2)*arctan(x*e^(1/2)/d^(1/2))/d^(5/2)/
(a*e^2-b*d*e+c*d^2)+1/2*arctan(x*2^(1/2)*c^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/
2))*c^(1/2)*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+
b^2)^(1/2))/a^2/(a*e^2-b*d*e+c*d^2)*2^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/
2*arctan(x*2^(1/2)*c^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(b*c*d-b^2
*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/a^2/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.06, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1301, 211, 1180}

$$\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}\,(ae^2-bde+cd^2)}+\frac{\sqrt{c}\,\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{\sqrt{b^2-4ac}+b}}\right)\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b}\,(ae^2-bde+cd^2)}+\frac{ae+bd}{a^2d^2x}+\frac{e^{7/2}\text{ArcTan}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}(ae^2-bde+cd^2)}-\frac{1}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*1/(a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(b*c*d - b^2*e + a*c*
e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b - Sqrt[b
^2 - 4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (Sqrt[c]*(b*c*d - b^2*e + a*c*e - (
b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]
*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + Sqrt[b^2 -
4*a*c]]*(c*d^2 - b*d*e + a*e^2)) + (e^(7/2)*ArcTan[(Sqrt[e]*x)/Sqrt[d]])/(d
^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{1}{x^4\left(d+ex^2\right)\left(a+bx^2+cx^4\right)}\,dx = \int \left(\frac{1}{adx^4}+\frac{-bd-ae}{a^2d^2x^2}+\frac{e^4}{d^2\left(cd^2-bde+ae^2\right)\left(d+ex^2\right)}+\frac{b^2cd-ac^2d\ldots}{a^2\left(cd\ldots\right.}\right.$$

$$= -\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\int \frac{b^2cd-ac^2d-b^3e+2abce+c\left(bcd-b^2e+ace\right)x^2}{a+bx^2+cx^4}\,dx}{a^2\left(cd^2-bde+ae^2\right)}+\frac{e^4\int \ldots}{d^2\left(cd^2-\ldots\right.}$$

$$= -\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2-bde+ae^2\right)}+\frac{\left(c\left(bcd-b^2e+ace-\ldots\right.\right.}{\ldots}$$

$$= -\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\sqrt{c}\left(bcd-b^2e+ace+\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-}\ldots}{\sqrt{2}\,a^2\sqrt{b-\sqrt{b^2-4ac}}\,\left(cd^2-b\ldots\right.}$$

Mathematica [A]
time = 0.36, size = 410, normalized size = 1.18

$$-\frac{1}{3adx^3}+\frac{bd+ae}{a^2d^2x}+\frac{\sqrt{c}\left(-b^3e+bc\left(\sqrt{b^2-4ac}\,d+3ae\right)+b^2\left(cd-\sqrt{b^2-4ac}\,e\right)+ac\left(-2cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\left(cd^2+e(-bd+ae)\right)}+\frac{\sqrt{c}\left(b^3e+bc\left(\sqrt{b^2-4ac}\,d-3ae\right)-b^2\left(cd+\sqrt{b^2-4ac}\,e\right)+ac\left(2cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\,x}{\sqrt{b+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}\,\left(cd^2+e(-bd+ae)\right)}+\frac{e^{7/2}\tan^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d}}\right)}{d^{5/2}\left(cd^2-bde+ae^2\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*1/(a*d*x^3) + (b*d + a*e)/(a^2*d^2*x) + (Sqrt[c]*(-(b^3*e) + b*c*(Sqrt
[b^2 - 4*a*c]*d + 3*a*e) + b^2*(c*d - Sqrt[b^2 - 4*a*c]*e) + a*c*(-2*c*d +
Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*x)/Sqrt[b - Sqrt[b^2 - 4*a*c]
]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(
-(b*d) + a*e))) + (Sqrt[c]*(b^3*e + b*c*(Sqrt[b^2 - 4*a*c]*d - 3*a*e) - b^2
*(c*d + Sqrt[b^2 - 4*a*c]*e) + a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(S
```

```
qrt[2]*Sqrt[c]*x)/Sqrt[b + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a
*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*(c*d^2 + e*(-(b*d) + a*e))) + (e^(7/2)*ArcT
an[(Sqrt[e]*x)/Sqrt[d]])/(d^(5/2)*(c*d^2 - b*d*e + a*e^2))
```

Maple [A]
time = 0.22, size = 349, normalized size = 1.00

method	result
default	$\frac{4c\left(-\frac{\left(ace\sqrt{-4ac+b^2}-b^2e\sqrt{-4ac+b^2}+bcd\sqrt{-4ac+b^2}+3abce-2a\,c^2d-b^3e+b^2cd\right)\sqrt{2}\ \mathrm{arctanh}\left(\frac{cx\sqrt{\cdots}}{\sqrt{\left(-b+\sqrt{-\cdots}\right.}}\right.}{8\sqrt{-4ac+b^2}\ \sqrt{\left(-b+\sqrt{-4ac+b^2}\,\right)c}}\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 4/(a*e^2-b*d*e+c*d^2)/a^2*c*(-1/8*(a*c*e*(-4*a*c+b^2)^(1/2)-b^2*e*(-4*a*c+b
^2)^(1/2)+b*c*d*(-4*a*c+b^2)^(1/2)+3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a
*c+b^2)^(1/2)*2^(1/2)/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctanh(c*x*2^(1/2)
/((-b+(-4*a*c+b^2)^(1/2))*c)^(1/2))+1/8*(a*c*e*(-4*a*c+b^2)^(1/2)-b^2*e*(-4
*a*c+b^2)^(1/2)+b*c*d*(-4*a*c+b^2)^(1/2)-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)
/(-4*a*c+b^2)^(1/2)*2^(1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)*arctan(c*x*2^(
1/2)/((b+(-4*a*c+b^2)^(1/2))*c)^(1/2)))+1/d^2*e^4/(a*e^2-b*d*e+c*d^2)/(d*e)
^(1/2)*arctan(e*x/(d*e)^(1/2))-1/3/a/d/x^3-1/a^2/d^2*(-a*e-b*d)/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] arctan(x*e^(1/2)/sqrt(d))*e^(7/2)/((c*d^4 - b*d^3*e + a*d^2*e^2)*sqrt(d)) +
 integrate(-(b^3*e - 2*a*b*c*e - (b*c^2*d - b^2*c*e + a*c^2*e)*x^2 - (b^2*c
 - a*c^2)*d)/(c*x^4 + b*x^2 + a), x)/(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + 1/
3*(3*(b*d + a*e)*x^2 - a*d)/(a^2*d^2*x^3)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 12268 vs. $2(311) = 622$.
time = 7.60, size = 12268, normalized size = 35.25

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*((2*a^4*b^5*c^5 - 12*a^5*b^3*c^6 + 16*a^6*b*c^7 - sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 + 6*sqrt(2)*sqrt(b^2 - 4*a
*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 2*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*c^5 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^6 - 2*(b^2 - 4*a*c)*a^4*b^3*c^5 + 4*(b^2
 - 4*a*c)*a^5*b*c^6)*d^5 - (6*a^4*b^6*c^4 - 38*a^5*b^4*c^5 + 56*a^6*b^2*c^6
 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2
+ 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3
+ 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 -
 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 -
 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 -
 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^4*c^4 +
7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^2*c^5 - 6
*(b^2 - 4*a*c)*a^4*b^4*c^4 + 14*(b^2 - 4*a*c)*a^5*b^2*c^5)*d^4*e + 2*(sqrt(
2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^6*c^2 - 9*sqrt(2)*sqrt(b*c + sqrt(
b^2 - 4*a*c)*c)*a^3*b^4*c^3 - 2*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2
*b^5*c^3 - 2*a^2*b^6*c^3 + 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b
^2*c^4 + 10*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c^4 + sqrt(2)*s
```

qrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^4*c^4 + 18*a^3*b^4*c^4 - 16*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*c^5 - 8*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a
*c)*c)*a^4*b*c^5 - 5*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^2*c^5 -
48*a^4*b^2*c^5 + 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*c^6 + 32*a^5
c^6 + 2(b^2 - 4*a*c)*a^2*b^4*c^3 - 10*(b^2 - 4*a*c)*a^3*b^2*c^4 + 8*(b^2
- 4*a*c)*a^4*c^5)*d^3*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2) + (6*a^4*b^7*c^3
 - 36*a^5*b^5*c^4 + 40*a^6*b^3*c^5 + 32*a^7*b*c^6 - 3*sqrt(2)*sqrt(b^2 - 4*
a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^7*c + 18*sqrt(2)*sqrt(b^2 - 4*a*
c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2 - 20*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^4*c^3 - 3*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^5*c^3 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b*c^4 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^2*c^4 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^3*c^4 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b*c^5 - 6*(b^2 - 4*a*c)*a^4*b^5*c^3 + 12*(b
^2 - 4*a*c)*a^5*b^3*c^4 + 8*(b^2 - 4*a*c)*a^6*b*c^5)*d^3*e^2 - 2*(2*sqrt(2)
*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^7*c - 19*sqrt(2)*sqrt(b*c + sqrt(b^2
 - 4*a*c)*c)*a^3*b^5*c^2 - 4*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^
6*c^2 - 4*a^2*b^7*c^2 + 56*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^3*
c^3 + 22*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^4*c^3 + 2*sqrt(2)*sq
rt(b*c + sqrt(b^2 - 4*a*c)*c)*a^2*b^5*c^3 + 38*a^3*b^5*c^3 - 48*sqrt(2)*sqr
t(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b*c^4 - 24*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4
*a*c)*c)*a^4*b^2*c^4 - 11*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^3*b^3*c
^4 - 112*a^4*b^3*c^4 + 12*sqrt(2)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b*c^5
 + 96*a^5*b*c^5 + 4*(b^2 - 4*a*c)*a^2*b^5*c^2 - 22*(b^2 - 4*a*c)*a^3*b^3*c^
3 + 24*(b^2 - 4*a*c)*a^4*b*c^4)*d^2*abs(a^2*c*d^2 - a^2*b*d*e + a^3*e^2)*e
+ (2*b^5*c^3 - 16*a*b^3*c^4 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt
(b*c + sqrt(b^2 - 4*a*c)*c)*b^5*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c +
sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqr
t(b^2 - 4*a*c)*c)*b^4*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^
2 - 4*a*c)*c)*a^2*b*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 -
 4*a*c)*c)*a*b^2*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*
c)*c)*b^3*c^3 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)
*a*b*c^4 - 2*(b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a*b*c^4)*(a^2*c*d^2 -
a^2*b*d*e + a^3*e^2)^2*d - (2*a^4*b^8*c^2 - 6*a^5*b^6*c^3 - 28*a^6*b^4*c^4
+ 80*a^7*b^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c
)*a^4*b^8 + 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5
*b^6*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^
7*c + 14*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^4*
c^2 + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^5*b^5*c
^2 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^4*b^6*c^2
- 40*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^7*b^2*c^3
- 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2 - 4*a*c)*c)*a^6*b^3*c^3

```
- sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(b*c + sqrt(b^2...
```

Mupad [B]
time = 6.73, size = 2500, normalized size = 7.18

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log(c^9*d^27*e^6 - b^9*d^18*e^15 + 2*a*c^8*d^25*e^8 - 2*b*c^8*d^26*e^7 + 2
*b^8*c*d^19*e^14 + a^5*b^4*d^13*e^20 + a^2*c^7*d^23*e^10 + 16*a^4*c^5*d^19*
e^14 + 16*a^7*c^2*d^13*e^20 + b^2*c^7*d^25*e^8 - b^7*c^2*d^20*e^13 - 25*a^2
*b^3*c^4*d^20*e^13 + 66*a^2*b^4*c^3*d^19*e^14 - 42*a^2*b^5*c^2*d^18*e^15 -
76*a^3*b^2*c^4*d^19*e^14 + 63*a^3*b^3*c^3*d^18*e^15 - a^5*b^4*e^3*x*(-d^5*e
^7)^(5/2) + a^2*c^7*d^15*x*(-d^5*e^7)^(3/2) - 16*a^7*c^2*e^3*x*(-d^5*e^7)^(
5/2) - b^9*d^10*e^5*x*(-d^5*e^7)^(3/2) - c^9*d^24*e^3*x*(-d^5*e^7)^(1/2) -
2*a*b*c^7*d^24*e^9 + 11*a*b^7*c*d^18*e^15 + 9*a*b^5*c^3*d^20*e^13 - 20*a*b^
6*c^2*d^19*e^14 + 20*a^3*b*c^5*d^20*e^13 - 28*a^4*b*c^4*d^18*e^15 - 8*a^6*b
^2*c*d^13*e^20 + 16*a^4*c^5*d^11*e^4*x*(-d^5*e^7)^(3/2) - b^7*c^2*d^12*e^3*
x*(-d^5*e^7)^(3/2) - b^2*c^7*d^22*e^5*x*(-d^5*e^7)^(1/2) + 8*a^6*b^2*c*e^3*
x*(-d^5*e^7)^(5/2) - 2*a*c^8*d^22*e^5*x*(-d^5*e^7)^(1/2) + 2*b^8*c*d^11*e^4
*x*(-d^5*e^7)^(3/2) + 2*b*c^8*d^23*e^4*x*(-d^5*e^7)^(1/2) + 11*a*b^7*c*d^10
*e^5*x*(-d^5*e^7)^(3/2) + 2*a*b*c^7*d^21*e^6*x*(-d^5*e^7)^(1/2) + 9*a*b^5*c
^3*d^12*e^3*x*(-d^5*e^7)^(3/2) - 20*a*b^6*c^2*d^11*e^4*x*(-d^5*e^7)^(3/2) +
 20*a^3*b*c^5*d^12*e^3*x*(-d^5*e^7)^(3/2) - 28*a^4*b*c^4*d^10*e^5*x*(-d^5*e
^7)^(3/2) - 25*a^2*b^3*c^4*d^12*e^3*x*(-d^5*e^7)^(3/2) + 66*a^2*b^4*c^3*d^1
1*e^4*x*(-d^5*e^7)^(3/2) - 42*a^2*b^5*c^2*d^10*e^5*x*(-d^5*e^7)^(3/2) - 76*
a^3*b^2*c^4*d^11*e^4*x*(-d^5*e^7)^(3/2) + 63*a^3*b^3*c^3*d^10*e^5*x*(-d^5*e
^7)^(3/2))*(-d^5*e^7)^(1/2))/(2*c*d^7 + 2*a*d^5*e^2 - 2*b*d^6*e) - atan((((
-(b^9*e^2 + b^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^
2 - 20*a^3*b*c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2
- a^2*c^4*d^2*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^
3*e^2 + a^3*c^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^
3)^(1/2) - 11*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e
*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*
a*b^4*c*e^2*(-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*
d*e + 3*a*b^2*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c -
 b^2)^3)^(1/2) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4
+ 16*a^7*c^4*d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5
*b^4*c^2*d^4 - 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2
*a^5*b^5*c*d^3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d
*e^3 + 16*a^6*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(((-(b^9*e^2 + b
^7*c^2*d^2 - b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*
c^5*d^2 + 28*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2
```

*(-(4*a*c - b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c
^3*e^2*(-(4*a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11
*a*b^7*c*e^2 - 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b
^2)^3)^(1/2) - 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(
-(4*a*c - b^2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2
*c^3*d^2*(-(4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2
) + 6*a^2*b*c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*
d^4 + 16*a^9*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4
- 8*a^6*b^2*c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^
3*e - 32*a^7*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6
*b^3*c^2*d^3*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(x*(-(b^9*e^2 + b^7*c^2*d^2 -
b^6*e^2*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c^3*d^2 - 20*a^3*b*c^5*d^2 + 28
*a^4*b*c^4*e^2 - 2*b^8*c*d*e + 25*a^2*b^3*c^4*d^2 - a^2*c^4*d^2*(-(4*a*c -
b^2)^3)^(1/2) + 42*a^2*b^5*c^2*e^2 - 63*a^3*b^3*c^3*e^2 + a^3*c^3*e^2*(-(4*
a*c - b^2)^3)^(1/2) - b^4*c^2*d^2*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e^2
- 16*a^4*c^5*d*e + 20*a*b^6*c^2*d*e + 2*b^5*c*d*e*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^2*b^2*c^2*e^2*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e^2*(-(4*a*c - b^
2)^3)^(1/2) - 66*a^2*b^4*c^3*d*e + 76*a^3*b^2*c^4*d*e + 3*a*b^2*c^3*d^2*(-(
4*a*c - b^2)^3)^(1/2) - 8*a*b^3*c^2*d*e*(-(4*a*c - b^2)^3)^(1/2) + 6*a^2*b*
c^3*d*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^7*b^4*e^4 + 16*a^7*c^4*d^4 + 16*a^9
*c^2*e^4 - 8*a^8*b^2*c*e^4 - 2*a^6*b^5*d*e^3 + a^5*b^4*c^2*d^4 - 8*a^6*b^2*
c^3*d^4 + a^5*b^6*d^2*e^2 + 32*a^8*c^3*d^2*e^2 - 2*a^5*b^5*c*d^3*e - 32*a^7
*b*c^3*d^3*e + 16*a^7*b^3*c*d*e^3 - 32*a^8*b*c^2*d*e^3 + 16*a^6*b^3*c^2*d^3
*e - 6*a^6*b^4*c*d^2*e^2)))^(1/2)*(512*a^20*c^7*d^24*e^3 + 512*a^21*c^6*d^2
2*e^5 - 512*a^22*c^5*d^20*e^7 - 512*a^23*c^4*d^18*e^9 - 32*a^18*b^3*c^6*d^2
5*e^2 + 128*a^18*b^4*c^5*d^24*e^3 - 192*a^18*b^5*c^4*d^23*e^4 + 128*a^18*b^
6*c^3*d^22*e^5 - 32*a^18*b^7*c^2*d^21*e^6 - 640*a^19*b^2*c^6*d^24*e^3 + 105
6*a^19*b^3*c^5*d^23*e^4 - 672*a^19*b^4*c^4*d^22*e^5 + 96*a^19*b^5*c^3*d^21*
e^6 + 32*a^19*b^6*c^2*d^20*e^7 + 512*a^20*b^2*c^5*d^22*e^5 + 288*a^20*b^3*c
^4*d^21*e^6 - 192*a^20*b^4*c^3*d^20*e^7 + 32*a^20*b^5*c^2*d^19*e^8 + 384*a^
21*b^2*c^4*d^20*e^7 - 288*a^21*b^3*c^3*d^19*e^8 - 32*a^21*b^4*c^2*d^18*e^9
+ 256*a^22*b^2*c^3*d^18*e^9 + 128*a^19*b*c^7*d^...

3.310 $\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=866

$$\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\sqrt{f}}\right)}{\sqrt[4]{2}\,\sqrt{b^2-4ac}\,\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f}}-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tan^{-}}{\sqrt[4]{2}\,\sqrt{b^2-4ac}\,\left(-b+\sqrt{b^2-4ac}\right.}$$

```
[Out] -1/2*e^(7/4)*arctan(1-e^(1/4)*2^(1/2)*(f*x)^(1/2)/d^(1/4)/f^(1/2))/d^(3/4)/
(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/2*e^(7/4)*arctan(1+e^(1/4)*2^(1/2)*(f
*x)^(1/2)/d^(1/4)/f^(1/2))/d^(3/4)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)-1/4*
e^(7/4)*ln(d^(1/2)*f^(1/2)+x*e^(1/2)*f^(1/2)-d^(1/4)*e^(1/4)*2^(1/2)*(f*x)^
(1/2))/d^(3/4)/(a*e^2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/4*e^(7/4)*ln(d^(1/2)*f
^(1/2)+x*e^(1/2)*f^(1/2)+d^(1/4)*e^(1/4)*2^(1/2)*(f*x)^(1/2))/d^(3/4)/(a*e^
2-b*d*e+c*d^2)*2^(1/2)/f^(1/2)+1/2*c^(3/4)*arctan(2^(1/4)*c^(1/4)*(f*x)^(1/
2)/(-b-(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*
2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2
)/f^(1/2)+1/2*c^(3/4)*arctanh(2^(1/4)*c^(1/4)*(f*x)^(1/2)/(-b-(-4*a*c+b^2)^
(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e
+c*d^2)/(-b-(-4*a*c+b^2)^(1/2))^(3/4)/(-4*a*c+b^2)^(1/2)/f^(1/2)-1/2*c^(3/4
)*arctan(2^(1/4)*c^(1/4)*(f*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))
*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^
(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(3/4)/f^(1/2)-1/2*c^(3/4)*arctanh(2^(1/4)*c^(
1/4)*(f*x)^(1/2)/(-b+(-4*a*c+b^2)^(1/2))^(1/4)/f^(1/2))*(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))*2^(3/4)/(a*e^2-b*d*e+c*d^2)/(-4*a*c+b^2)^(1/2)/(-b+(-4*a*c+b^
2)^(1/2))^(3/4)/f^(1/2)
```

Rubi [A]
time = 1.77, antiderivative size = 866, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 12, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.387$, Rules used = {1283, 1438, 217, 1179, 642, 1176, 631, 210, 1436, 218, 214, 211}

$$-\frac{\mathrm{ArcTan}\left(1-\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{fx}}{\sqrt[4]{d}\,\sqrt{f}}\right)e^{7/4}}{\sqrt{2}\,d^{3/4}(cd^2-bed+ae^2)\sqrt{f}}+\frac{\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt[4]{e}\,\sqrt{fx}}{\sqrt[4]{d}\,\sqrt{f}}+1\right)e^{7/4}}{\sqrt{2}\,d^{3/4}(cd^2-bed+ae^2)\sqrt{f}}-\frac{\log\left(\sqrt{e}\,\sqrt{f}\,x+\sqrt{d}\,\sqrt{f}-\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}\,d^{3/4}(cd^2-bed+ae^2)\sqrt{f}}+\frac{\log\left(\sqrt{e}\,\sqrt{f}\,x+\sqrt{d}\,\sqrt{f}+\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{fx}\right)e^{7/4}}{2\sqrt{2}\,d^{3/4}(cd^2-bed+ae^2)\sqrt{f}}+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\mathrm{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\sqrt{f}}\right)}{\sqrt[4]{2}\,\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bed+ae^2)\sqrt{f}}-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\mathrm{ArcTan}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\sqrt{f}}\right)}{\sqrt[4]{2}\,\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}(cd^2-bed+ae^2)\sqrt{f}}+\frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\sqrt{f}}\right)}{\sqrt[4]{2}\,\sqrt{b^2-4ac}\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bed+ae^2)\sqrt{f}}-\frac{c^{3/4}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\tanh^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{fx}}{\sqrt[4]{\sqrt{b^2-4ac}-b}\,\sqrt{f}}\right)}{\sqrt[4]{2}\,\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}-b\right)^{3/4}(cd^2-bed+ae^2)\sqrt{f}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c^(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f
*x])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*
(-b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*
(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTan[(2^(1/4)*c^(1/4)*Sqrt[f*x]])/((-b
 + Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqr
```

```
t[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*ArcTan[1
- (Sqrt[2]*e^(1/4)*Sqrt[f*x])/(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*ArcTan[1 + (Sqrt[2]*e^(1/4)*Sqrt[f*x])/
(d^(1/4)*Sqrt[f])])/(Sqrt[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (c^
(3/4)*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x
])/((-b - Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-
b - Sqrt[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (c^(3/4)*(2
*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(2^(1/4)*c^(1/4)*Sqrt[f*x])/((-b
+ Sqrt[b^2 - 4*a*c])^(1/4)*Sqrt[f])])/(2^(1/4)*Sqrt[b^2 - 4*a*c]*(-b + Sqrt
[b^2 - 4*a*c])^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) - (e^(7/4)*Log[Sqrt[d
]*Sqrt[f] + Sqrt[e]*Sqrt[f]*x - Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt
[2]*d^(3/4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f]) + (e^(7/4)*Log[Sqrt[d]*Sqrt[f]
 + Sqrt[e]*Sqrt[f]*x + Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[f*x]])/(2*Sqrt[2]*d^(3/
4)*(c*d^2 - b*d*e + a*e^2)*Sqrt[f])
```

Rule 210

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 217

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Dist[1/(2*r), Int[(r - s*x^2)/(a + b*x^4),
 x], x] + Dist[1/(2*r), Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a, b
}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &&
AtomQ[SplitProduct[SumBaseQ, b]]))
```

Rule 218

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x], x]
+ Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a/b
, 0]
```

Rule 631

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b)
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 642

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1176

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 1179

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 1283

```
Int[((f_.)*(x_))^(m_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c
_.)*(x_)^4)^(p_), x_Symbol] :> With[{k = Denominator[m]}, Dist[k/f, Subst[I
nt[x^(k*(m + 1) - 1)*(d + e*(x^(2*k)/f^2))^q*(a + b*(x^(2*k)/f^k) + c*(x^(4
*k)/f^4))^p, x], x, (f*x)^(1/k)], x]] /; FreeQ[{a, b, c, d, e, f, p, q}, x]
 && NeQ[b^2 - 4*a*c, 0] && FractionQ[m] && IntegerQ[p]
```

Rule 1436

```
Int[((d_) + (e_.)*(x_)^(n_))/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_)), x
_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q),
Int[1/(b/2 - q/2 + c*x^n), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(
b/2 + q/2 + c*x^n), x], x]] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (PosQ[b^2 - 4*a
*c] ||  !IGtQ[n/2, 0])
```

Rule 1438

```
Int[((d_) + (e_.)*(x_)^(n_))^(q_)/((a_) + (b_.)*(x_)^(n_) + (c_.)*(x_)^(n2_
)), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^n)^q/(a + b*x^n + c*x^(2*n)),
```

```
x], x] /; FreeQ[{a, b, c, d, e, n}, x] && EqQ[n2, 2*n] && NeQ[b^2 - 4*a*c,
0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)(a+bx^2+cx^4)}\,dx = \frac{2\text{Subst}\left(\int \frac{1}{\left(d+\frac{ex^4}{f^2}\right)\left(a+\frac{bx^4}{f^2}+\frac{cx^8}{f^4}\right)}\,dx, x, \sqrt{fx}\right)}{f}$$

$$= \frac{2\text{Subst}\left(\int \left(\frac{e^2f^2}{(cd^2-bde+ae^2)(df^2+ex^4)} + \frac{cdf^4-bef^4-cef^2x^4}{(cd^2-bde+ae^2)(af^4+bf^2x^4+cx^8)}\right)\,dx, \ldots\right.}{f}$$

$$= \frac{2\text{Subst}\left(\int \frac{cdf^4-bef^4-cef^2x^4}{af^4+bf^2x^4+cx^8}\,dx, x, \sqrt{fx}\right)}{(cd^2-bde+ae^2)\,f} + \frac{(2e^2f)\,\text{Subst}\left(\int \frac{1}{df^2+ex^4} \ldots\right.}{cd^2-bde+\ldots}$$

$$= \frac{e^2\text{Subst}\left(\int \frac{\sqrt{d}\,f-\sqrt{e}\,x^2}{df^2+ex^4}\,dx, x, \sqrt{fx}\right)}{\sqrt{d}\,(cd^2-bde+ae^2)} + \frac{e^2\text{Subst}\left(\int \frac{\sqrt{d}\,f+\sqrt{e}\,x^2}{df^2+ex^4} \ldots\right.}{\sqrt{d}\,(cd^2-bde \ldots}$$

$$= \frac{e^{3/2}\text{Subst}\left(\int \frac{1}{\frac{\sqrt{d}\,f}{\sqrt{e}}-\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt{f}\,x}{\sqrt[4]{e}}+x^2}\,dx, x, \sqrt{fx}\right)}{2\sqrt{d}\,(cd^2-bde+ae^2)} + \frac{e^{3/2}\text{Subst} \ldots}{\ldots}$$

$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\sqrt{f} \ldots}\right.}{\sqrt[4]{2}\,\sqrt{b^2-4ac}\,\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f} \ldots}$$

$$= \frac{c^{3/4}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt[4]{2}\,\sqrt[4]{c}\,\sqrt{fx}}{\sqrt[4]{-b-\sqrt{b^2-4ac}}\,\sqrt{f} \ldots}\right.}{\sqrt[4]{2}\,\sqrt{b^2-4ac}\,\left(-b-\sqrt{b^2-4ac}\right)^{3/4}(cd^2-bde+ae^2)\sqrt{f} \ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.25, size = 215, normalized size = 0.25

$$-\frac{\sqrt{x}\left(\sqrt{2}\,e^{7/4}\left(\tan^{-1}\left(\frac{\sqrt{d}-\sqrt{e}\,x}{\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}}\right)-\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt[4]{d}\,\sqrt[4]{e}\,\sqrt{x}}{\sqrt{d}+\sqrt{e}\,x}\right)\right)+d^{3/4}\text{RootSum}\left[a+b\#1^4+c\#1^8\&, \frac{-cd\log\left(\sqrt{x}-\#1\right)+be\log\left(\sqrt{x}-\#1\right)+ce\log\left(\sqrt{x}-\#1\right)\#1^4}{b\#1^3+2c\#1^7}\&\right]\right)}{2d^{3/4}\left(cd^2+e(-bd+ae)\right)\sqrt{fx}}$$

Antiderivative was successfully verified.

[In] Integrate[1/(Sqrt[f*x]*(d + e*x^2)*(a + b*x^2 + c*x^4)),x]

[Out] -1/2*(Sqrt[x]*(Sqrt[2]*e^(7/4)*(ArcTan[(Sqrt[d] - Sqrt[e]*x)/(Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x])] - ArcTanh[(Sqrt[2]*d^(1/4)*e^(1/4)*Sqrt[x])/(Sqrt[d] + Sqrt[e]*x)]) + d^(3/4)*RootSum[a + b*#1^4 + c*#1^8 & , (-(c*d*Log[Sqrt[x] - #1]) + b*e*Log[Sqrt[x] - #1] + c*e*Log[Sqrt[x] - #1]*#1^4)/(b*#1^3 + 2*c*#1^7) &]))/(d^(3/4)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[f*x])

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 264, normalized size = 0.30

method	result
derivativedivides	$2f^5\left(\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+bf^2\_Z^4+af^4\right)}\frac{\left(-\_R^4ce-bef^2+cdf^2\right)\ln\left(\sqrt{fx}-\_R\right)}{2\_R^7c+\_R^3bf^2}}{4f^4(ae^2-deb+cd^2)}+\frac{e^2\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fx+}{fx-}\right.\right.}{}\right.$
default	$2f^5\left(\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+bf^2\_Z^4+af^4\right)}\frac{\left(-\_R^4ce-bef^2+cdf^2\right)\ln\left(\sqrt{fx}-\_R\right)}{2\_R^7c+\_R^3bf^2}}{4f^4(ae^2-deb+cd^2)}+\frac{e^2\left(\frac{df^2}{e}\right)^{\frac{1}{4}}\sqrt{2}\left(\ln\left(\frac{fx+}{fx-}\right.\right.}{}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2*f^5*(1/4/f^4/(a*e^2-b*d*e+c*d^2)*sum((-_R^4*c*e-b*e*f^2+c*d*f^2)/(2*_R^7*c+_R^3*b*f^2)*ln((f*x)^(1/2)-_R),_R=RootOf(_Z^8*c+_Z^4*b*f^2+a*f^4))+1/8*e^2/f^6/(a*e^2-b*d*e+c*d^2)*(d*f^2/e)^(1/4)/d*2^(1/2)*(ln((f*x+(d*f^2/e)^(1/4)*(f*x)^(1/2)*2^(1/2)+(d*f^2/e)^(1/2))/(f*x-(d*f^2/e)^(1/4)*(f*x)^(1/2)*2^(1/2)+(d*f^2/e)^(1/2)))+2*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)+1)+2*arctan(2^(1/2)/(d*f^2/e)^(1/4)*(f*x)^(1/2)-1)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="maxima")

[Out] 1/4*(2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2)*d^(1/4)*e^(1/4) + 2*e^(1/2*log(x) + 1/2))*e^(-1/4)/d^(1/4))*e^(7/4)/d^(3/4) + 2*sqrt(2)*arctan(-1/2*sqrt(2)

```
*(sqrt(2)*d^(1/4)*e^(1/4) - 2*e^(1/2*log(x) + 1/2))*e^(-1/4)/d^(1/4))*e^(7/
4)/d^(3/4) + sqrt(2)*e^(7/4)*log(sqrt(2)*d^(1/4)*e^(1/2*log(x) + 1/4) + sqr
t(d) + e^(log(x) + 1/2))/d^(3/4) - sqrt(2)*e^(7/4)*log(-sqrt(2)*d^(1/4)*e^(
1/2*log(x) + 1/4) + sqrt(d) + e^(log(x) + 1/2))/d^(3/4))/(c*d^2*sqrt(f) - b
*d*sqrt(f)*e + a*sqrt(f)*e^2) - 2*e^(1/2*log(x) + 2)/(c*d^3*sqrt(f) - b*d^2
*sqrt(f)*e + a*d*sqrt(f)*e^2) + 2*sqrt(x)/(a*d*sqrt(f)) + integrate(-((c^2*
d - b*c*e)*x^(7/2) + (b*c*d - (b^2 - a*c)*e)*x^(3/2))/(a^2*c*d^2*sqrt(f) -
a^2*b*d*sqrt(f)*e + (a*c^2*d^2*sqrt(f) - a*b*c*d*sqrt(f)*e + a^2*c*sqrt(f)*
e^2)*x^4 + a^3*sqrt(f)*e^2 + (a*b*c*d^2*sqrt(f) - a*b^2*d*sqrt(f)*e + a^2*b
*sqrt(f)*e^2)*x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{fx}\,(d+ex^2)\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)/(c*x**4+b*x**2+a)/(f*x)**(1/2),x)
[Out] Integral(1/(sqrt(f*x)*(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)/(c*x^4+b*x^2+a)/(f*x)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [B]
time = 6.84, size = 2500, normalized size = 2.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((f*x)^(1/2)*(d + e*x^2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] symsum(log(-root(8388608*a^7*b*c^11*d^18*e*f^6*h^12 - 513802240*a^10*b^2*c^
7*d^11*e^8*f^6*h^12 - 381681664*a^11*b^2*c^6*d^9*e^10*f^6*h^12 - 381681664*
a^9*b^2*c^8*d^13*e^6*f^6*h^12 - 300941312*a^9*b^5*c^5*d^10*e^9*f^6*h^12 - 3
00941312*a^8*b^5*c^6*d^12*e^7*f^6*h^12 + 293601280*a^10*b^3*c^6*d^10*e^9*f^
6*h^12 + 293601280*a^9*b^3*c^7*d^12*e^7*f^6*h^12 - 168820736*a^10*b^5*c^4*d
^8*e^11*f^6*h^12 - 168820736*a^7*b^5*c^7*d^14*e^5*f^6*h^12 + 166068224*a^8*
b^6*c^5*d^11*e^8*f^6*h^12 - 146800640*a^12*b^2*c^5*d^7*e^12*f^6*h^12 - 1468
00640*a^8*b^2*c^9*d^15*e^4*f^6*h^12 + 124780544*a^10*b^4*c^5*d^9*e^10*f^6*h
^12 + 124780544*a^8*b^4*c^7*d^13*e^6*f^6*h^12 + 119275520*a^9*b^4*c^6*d^11*
e^8*f^6*h^12 + 117440512*a^11*b^3*c^5*d^8*e^11*f^6*h^12 + 117440512*a^8*b^3
*c^8*d^14*e^5*f^6*h^12 + 102760448*a^9*b^6*c^4*d^9*e^10*f^6*h^12 + 10276044
8*a^7*b^6*c^6*d^13*e^6*f^6*h^12 + 91750400*a^11*b^4*c^4*d^7*e^12*f^6*h^12 +
 91750400*a^7*b^4*c^8*d^15*e^4*f^6*h^12 - 71065600*a^7*b^8*c^4*d^11*e^8*f^6
*h^12 - 53444608*a^8*b^8*c^3*d^9*e^10*f^6*h^12 - 53444608*a^6*b^8*c^5*d^13*
e^6*f^6*h^12 + 40370176*a^9*b^7*c^3*d^8*e^11*f^6*h^12 + 40370176*a^6*b^7*c^
6*d^14*e^5*f^6*h^12 - 36700160*a^11*b^5*c^3*d^6*e^13*f^6*h^12 - 36700160*a^
6*b^5*c^8*d^16*e^3*f^6*h^12 + 34078720*a^8*b^7*c^4*d^10*e^9*f^6*h^12 + 3407
8720*a^7*b^7*c^5*d^12*e^7*f^6*h^12 + 26214400*a^12*b^4*c^3*d^5*e^14*f^6*h^1
2 + 26214400*a^6*b^4*c^9*d^17*e^2*f^6*h^12 + 22118400*a^7*b^9*c^3*d^10*e^9*
f^6*h^12 + 22118400*a^6*b^9*c^4*d^12*e^7*f^6*h^12 - 20971520*a^13*b^2*c^4*d
^5*e^14*f^6*h^12 - 20971520*a^7*b^2*c^10*d^17*e^2*f^6*h^12 + 18350080*a^10*
b^7*c^2*d^6*e^13*f^6*h^12 + 18350080*a^5*b^7*c^7*d^16*e^3*f^6*h^12 - 166297
60*a^9*b^8*c^2*d^7*e^12*f^6*h^12 - 16629760*a^5*b^8*c^6*d^15*e^4*f^6*h^12 -
 10485760*a^11*b^6*c^2*d^5*e^14*f^6*h^12 - 10485760*a^5*b^6*c^8*d^17*e^2*f^
6*h^12 + 9175040*a^10*b^6*c^3*d^7*e^12*f^6*h^12 + 9175040*a^6*b^6*c^7*d^15*
e^4*f^6*h^12 - 8388608*a^13*b^3*c^3*d^4*e^15*f^6*h^12 + 5619712*a^7*b^10*c^
2*d^9*e^10*f^6*h^12 + 5619712*a^5*b^10*c^4*d^13*e^6*f^6*h^12 - 5570560*a^6*
b^11*c^2*d^10*e^9*f^6*h^12 - 5570560*a^5*b^11*c^3*d^12*e^7*f^6*h^12 + 43581
44*a^8*b^9*c^2*d^8*e^11*f^6*h^12 + 4358144*a^5*b^9*c^5*d^14*e^5*f^6*h^12 +
4259840*a^6*b^10*c^3*d^11*e^8*f^6*h^12 + 3899392*a^4*b^10*c^5*d^15*e^4*f^6*
h^12 - 3440640*a^4*b^9*c^6*d^16*e^3*f^6*h^12 + 3145728*a^12*b^5*c^2*d^4*e^1
5*f^6*h^12 - 2523136*a^4*b^11*c^4*d^14*e^5*f^6*h^12 + 1802240*a^4*b^8*c^7*d
^17*e^2*f^6*h^12 + 1556480*a^5*b^12*c^2*d^11*e^8*f^6*h^12 + 1048576*a^14*b^
2*c^3*d^3*e^16*f^6*h^12 + 688128*a^4*b^12*c^3*d^13*e^6*f^6*h^12 - 393216*a^
13*b^4*c^2*d^3*e^16*f^6*h^12 - 286720*a^3*b^12*c^4*d^15*e^4*f^6*h^12 + 2293
76*a^3*b^13*c^3*d^14*e^5*f^6*h^12 + 229376*a^3*b^11*c^5*d^16*e^3*f^6*h^12 +
 163840*a^4*b^13*c^2*d^12*e^7*f^6*h^12 - 114688*a^3*b^14*c^2*d^13*e^6*f^6*h
^12 - 114688*a^3*b^10*c^6*d^17*e^2*f^6*h^12 + 293601280*a^11*b*c^7*d^10*e^9
*f^6*h^12 + 293601280*a^10*b*c^8*d^12*e^7*f^6*h^12 + 176160768*a^12*b*c^6*d
^8*e^11*f^6*h^12 + 176160768*a^9*b*c^9*d^14*e^5*f^6*h^12 + 58720256*a^13*b*
c^5*d^6*e^13*f^6*h^12 + 58720256*a^8*b*c^10*d^16*e^3*f^6*h^12 + 8388608*a^1
```

4*b*c^4*d^4*e^15*f^6*h^12 - 8388608*a^6*b^3*c^10*d^18*e*f^6*h^12 + 3899392*
a^8*b^10*c*d^7*e^12*f^6*h^12 - 3440640*a^9*b^9*c*d^6*e^13*f^6*h^12 + 314572
8*a^5*b^5*c^9*d^18*e*f^6*h^12 - 2523136*a^7*b^11*c*d^8*e^11*f^6*h^12 + 1802
240*a^10*b^8*c*d^5*e^14*f^6*h^12 + 688128*a^6*b^12*c*d^9*e^10*f^6*h^12 - 52
4288*a^11*b^7*c*d^4*e^15*f^6*h^12 - 524288*a^4*b^7*c^8*d^18*e*f^6*h^12 + 16
3840*a^5*b^13*c*d^10*e^9*f^6*h^12 - 163840*a^4*b^14*c*d^11*e^8*f^6*h^12 + 6
5536*a^12*b^6*c*d^3*e^16*f^6*h^12 + 32768*a^3*b^15*c*d^12*e^7*f^6*h^12 + 32
768*a^3*b^9*c^7*d^18*e*f^6*h^12 - 73400320*a^11*c^8*d^11*e^8*f^6*h^12 - 587
20256*a^12*c^7*d^9*e^10*f^6*h^12 - 58720256*a^10*c^9*d^13*e^6*f^6*h^12 - 29
360128*a^13*c^6*d^7*e^12*f^6*h^12 - 29360128*a^9*c^10*d^15*e^4*f^6*h^12 - 8
388608*a^14*c^5*d^5*e^14*f^6*h^12 - 8388608*a^8*c^11*d^17*e^2*f^6*h^12 - 10
48576*a^15*c^4*d^3*e^16*f^6*h^12 - 286720*a^7*b^12*d^7*e^12*f^6*h^12 + 2293
76*a^8*b^11*d^6*e^13*f^6*h^12 + 229376*a^6*b^13*d^8*e^11*f^6*h^12 - 114688*
a^9*b^10*d^5*e^14*f^6*h^12 - 114688*a^5*b^14*d^9*e^10*f^6*h^12 + 32768*a^10
*b^9*d^4*e^15*f^6*h^12 + 32768*a^4*b^15*d^10*e^9*f^6*h^12 - 4096*a^11*b^8*d
^3*e^16*f^6*h^12 - 4096*a^3*b^16*d^11*e^8*f^6*h^12 + 1048576*a^6*b^2*c^11*d
^19*f^6*h^12 - 393216*a^5*b^4*c^10*d^19*f^6*h^12 + 65536*a^4*b^6*c^9*d^19*f
^6*h^12 - 4096*a^3*b^8*c^8*d^19*f^6*h^12 - 1048576*a^7*c^12*d^19*f^6*h^12 +
 262144*a^10*b*c^4*d*e^14*f^4*h^8 - 23552*a*b^6*c^8*d^14*e*f^4*h^8 - 16384*
a^7*b^7*c*d*e^14*f^4*h^8 - 3328*a*b^13*c*d^7*e^8*f^4*h^8 + 2429952*a^4*b^5*
c^6*d^9*e^6*f^4*h^8 - 1865728*a^6*b^3*c^6*d^7*e^8*f^4*h^8 - 1716224*a^4*b^4
*c^7*d^10*e^5*f^4*h^8 + 1605632*a^6*b^2*c^7*d^8*e^7*f^4*h^8 + 1584384*a^5*b
^5*c^5*d^7*e^8*f^4*h^8 + 1572864*a^5*b^2*c^8*d^10*e^5*f^4*h^8 - 1433600*a^5
*b^3*c^7*d^9*e^6*f^4*h^8 - 1261568*a^4*b^6*c^5*d^8*e^7*f^4*h^8 - 1124352*a^
3*b^4*c^8*d^12*e^3*f^4*h^8 - 1110016*a^7*b^3*c^5*d^5*e^10*f^4*h^8 + 1106176
*a^3*b^5*c^7*d^11*e^4*f^4*h^8 - 936960*a^5*b^6*...

3.311 $\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=272

$$\frac{((2cd-be)(4cd+be)-2ce(2cd+be)x^2)\sqrt{a+bx^2+cx^4}}{16c^2e^3}+\frac{(a+bx^2+cx^4)^{3/2}}{6ce}-\frac{(16c^3d^3-b^3e^3-2bce^2(bd-$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/c/e-1/32*(16*c^3*d^3-b^3*e^3-2*b*c*e^2*(-2*a*e+b*
d)-8*c^2*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1
/2))/c^(5/2)/e^4+1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*
d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^4+1/16*
((-b*e+2*c*d)*(b*e+4*c*d)-2*c*e*(b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2/
e^3
```

Rubi [A]
time = 0.38, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1265, 1667, 828, 857, 635, 212, 738}

$$-\frac{(-8c^2de(bd-ae)-2bce^2(bd-2ae)-b^3e^3+16c^3d^3)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{5/2}e^4}+\frac{\sqrt{a+bx^2+cx^4}\,((2cd-be)(be+4cd)-2cex^2(be+2cd))}{16c^2e^3}+\frac{d^2\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}+\frac{(a+bx^2+cx^4)^{3/2}}{6ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (((2*c*d - b*e)*(4*c*d + b*e) - 2*c*e*(2*c*d + b*e)*x^2)*Sqrt[a + b*x^2 + c
*x^4])/(16*c^2*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*c*e) - ((16*c^3*d^3 - b^
3*e^3 - 2*b*c*e^2*(b*d - 2*a*e) - 8*c^2*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x
^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(5/2)*e^4) + (d^2*Sqrt[c*d^
2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2
- b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
```

```
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right) \\
&= \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{3}{2}bde-\frac{3}{2}e(2cd+be)x\right)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{6ce^2} \\
&= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} \\
&= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} \\
&= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce} \\
&= \frac{\left((2cd-be)(4cd+be)-2ce(2cd+be)x^2\right)\sqrt{a+bx^2+cx^4}}{16c^2e^3} + \frac{\left(a+bx^2+cx^4\right)^{3/2}}{6ce}
\end{aligned}
$$

Mathematica [A]
time = 0.96, size = 259, normalized size = 0.95

$$
\frac{2\sqrt{c}\,e\sqrt{a+bx^2+cx^4}\left(-3b^2e^2+2ce(-3bd+4ae+bex^2)+4c^2(6d^2-3dex^2+2e^2x^4)\right)+96c^{5/2}d^2\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)+3(16c^3d^3-b^3e^3-2bce^2(bd-2ae)+8c^2de(-bd+ae))\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{96c^{5/2}e^4}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[a + b*x^2 + c*x^4]*(-3*b^2*e^2 + 2*c*e*(-3*b*d + 4*a*e +
b*e*x^2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)) + 96*c^(5/2)*d^2*Sqrt[-(c
*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x
^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] + 3*(16*c^3*d^3 - b^3*e^3 - 2*b*c*e^2*
(b*d - 2*a*e) + 8*c^2*d*e*(-(b*d) + a*e))*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[
a + b*x^2 + c*x^4]])/(96*c^(5/2)*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1012 vs. 2(244) = 488.
time = 0.17, size = 1013, normalized size = 3.72 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/6*(c*x^4+b*x^2+a)^(3/2)/c-1/8*b/c*x^2*(c*x^4+b*x^2+a)^(1/2)-1/16*b^2
/c^2*(c*x^4+b*x^2+a)^(1/2)-1/8*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*
x^2+a)^(1/2))*a+1/32*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2)))-d/e^2*(1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/4/c^(1/2)*ln((1/2*b
+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/c^(3/2)*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))*b^2)+1/e^2*d^2*(1/2/e*(c*(x^2+d/e)^2+(b*e-2*c*d)
/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/4/e*ln((1/2*(b*e-2*c*d)/e+c*(
x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/c^(1/2)*b-1/2/e^2*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+
(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1
/2)*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(
b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*
e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2/e^2/(
(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*
(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x
^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d*b-1/2/e^3/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c*d^2)
```

<u>**Maxima**</u> **[F(-2)]**
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

<u>**Fricas**</u> **[A]**
time = 98.96, size = 1501, normalized size = 5.52

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/192*(48*sqrt(c*d^2 - b*d*e + a*e^2)*c^3*d^2*log(-(8*c^2*d^2*x^4 + 8*b*c*
d^2*x^2 + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d -
(b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x
```

```
^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4
*e^2 + 2*d*x^2*e + d^2)) + 3*(16*c^3*d^3 - 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c
^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 +
 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(24*c^3*d^2*e
 + (8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c + 8*a*c^2)*e^3 - 6*(2*c^3*d*x^2 + b*c
^2*d)*e^2)*sqrt(c*x^4 + b*x^2 + a))*e^(-4)/c^3, 1/192*(96*sqrt(-c*d^2 + b*d
*e - a*e^2)*c^3*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d -
(b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 +
a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e
)) + 3*(16*c^3*d^3 - 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a
*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*(24*c^3*d^2*e + (8*c^3*x^4 + 2*b*c^
2*x^2 - 3*b^2*c + 8*a*c^2)*e^3 - 6*(2*c^3*d*x^2 + b*c^2*d)*e^2)*sqrt(c*x^4
+ b*x^2 + a))*e^(-4)/c^3, 1/96*(24*sqrt(c*d^2 - b*d*e + a*e^2)*c^3*d^2*log(
-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2
+ a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^
2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*
d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2)) + 3*(16*c^3*d^3 - 8*b*c^2*
d^2*e - 2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c)*e^3)*sqrt(-c)*arctan(1/
2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c))
 + 2*(24*c^3*d^2*e + (8*c^3*x^4 + 2*b*c^2*x^2 - 3*b^2*c + 8*a*c^2)*e^3 - 6*
(2*c^3*d*x^2 + b*c^2*d)*e^2)*sqrt(c*x^4 + b*x^2 + a))*e^(-4)/c^3, 1/96*(48*
sqrt(-c*d^2 + b*d*e - a*e^2)*c^3*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2
*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4
 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2
*d*x^2 + a*b*d)*e)) + 3*(16*c^3*d^3 - 8*b*c^2*d^2*e - 2*(b^2*c - 4*a*c^2)*d
*e^2 - (b^3 - 4*a*b*c)*e^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*(24*c^3*d^2*e + (8*c^3*x
^4 + 2*b*c^2*x^2 - 3*b^2*c + 8*a*c^2)*e^3 - 6*(2*c^3*d*x^2 + b*c^2*d)*e^2)*
sqrt(c*x^4 + b*x^2 + a))*e^(-4)/c^3]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**5*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x^5*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

3.312 $\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=208

$$-\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2}+\frac{(8c^2d^2-b^2e^2-4ce(bd-ae))\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{}}{}$$

```
[Out] 1/16*(8*c^2*d^2-b^2*e^2-4*c*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(
c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e^3-1/2*d*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)
*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^
(1/2)/e^3-1/8*(-2*c*e*x^2-b*e+4*c*d)*(c*x^4+b*x^2+a)^(1/2)/c/e^2
```

Rubi [A]
time = 0.21, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 828, 857, 635, 212, 738}

$$\frac{(-4ce(bd-ae)-b^2e^2+8c^2d^2)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16c^{3/2}e^3}-\frac{d\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^3}-\frac{\sqrt{a+bx^2+cx^4}\,(-be+4cd-2cex^2)}{8ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] -1/8*((4*c*d - b*e - 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(c*e^2) + ((8*c^2*
d^2 - b^2*e^2 - 4*c*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
+ b*x^2 + c*x^4])])/(16*c^(3/2)*e^3) - (d*Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTa
nh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a
+ b*x^2 + c*x^4])])/(2*e^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)$$

$$= -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2} - \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}d(4bcd-b^2e-4ace)-\frac{1}{2}(8c^2d^2-b^2e^2-}{(d+ex)\sqrt{a+bx+cx^2}}\right.}{8ce^2}$$

$$= -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2} - \frac{(d(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{}{(d+ex)\sqrt{a}}\right.}{2e^3}$$

$$= -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{(d(cd^2-bde+ae^2))\,\text{Subst}\left(\int \frac{1}{4cd^2-4bde-}\right.}{e^3}$$

$$= -\frac{(4cd-be-2cex^2)\sqrt{a+bx^2+cx^4}}{8ce^2} + \frac{(8c^2d^2-b^2e^2-4ce(bd-ae))\tanh^{-1}\left(\right.}{16c^{3/2}e^3}$$

Mathematica [A]
time = 0.60, size = 200, normalized size = 0.96

$$\frac{2\sqrt{c}\,e(-4cd+be+2cex^2)\sqrt{a+bx^2+cx^4}-16c^{3/2}d\sqrt{-cd^2+e(bd-ae)}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)+(-8c^2d^2+b^2e^2+4ce(bd-ae))\log\left(c\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)}{16c^{3/2}e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (2*Sqrt[c]*e*(-4*c*d + b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4] - 16*c^(3/2
)*d*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a +
 b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] + (-8*c^2*d^2 + b^2*e^2 +
4*c*e*(b*d - a*e))*Log[c*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]
)/(16*c^(3/2)*e^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 867 vs. 2(184) = 368.
time = 0.16, size = 868, normalized size = 4.17 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/8*(2*c*x^2+b)*(c*x^4+b*x^2+a)^(1/2)/c+1/4/c^(1/2)*ln((1/2*b+c*x^2)/c
^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a-1/16/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))*b^2)-d/e*(1/2/e*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)+1/4/e*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/
2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c
```

```
^(1/2)*b-1/2/e^2*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+
(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d-1/2/e/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2/e^2/((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d*b-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(
1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+
c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2
)/e^2)^(1/2))/(x^2+d/e))*c*d^2)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

Fricas [A]
time = 11.53, size = 1247, normalized size = 6.00

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/32*(8*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*
x^2 + (b^2 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x
^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 +
 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2
 + 2*d*x^2*e + d^2)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(c)*
log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*
sqrt(c) - 4*a*c) - 4*sqrt(c*x^4 + b*x^2 + a)*(4*c^2*d*e - (2*c^2*x^2 + b*c)
*e^2))*e^(-3)/c^2, -1/32*(16*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d*arctan(-1/2
*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 +
b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 +
a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)) - (8*c^2*d^2 - 4*b*c*d*e - (
b^2 - 4*a*c)*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*(4*c
```

```
^2*d*e - (2*c^2*x^2 + b*c)*e^2))*e^(-3)/c^2, 1/16*(4*sqrt(c*d^2 - b*d*e + a
*e^2)*c^2*d*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2 - 4*sqr
t(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e
 + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 +
(3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2)) - (8*c^2*d
^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2
+ a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - 2*sqrt(c*x^4 + b*x
^2 + a)*(4*c^2*d*e - (2*c^2*x^2 + b*c)*e^2))*e^(-3)/c^2, -1/16*(8*sqrt(-c*d
^2 + b*d*e - a*e^2)*c^2*d*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 +
b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*
x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*
b*d)*e)) + (8*c^2*d^2 - 4*b*c*d*e - (b^2 - 4*a*c)*e^2)*sqrt(-c)*arctan(1/2*
sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) +
 2*sqrt(c*x^4 + b*x^2 + a)*(4*c^2*d*e - (2*c^2*x^2 + b*c)*e^2))*e^(-3)/c^2]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

3.313 $\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$

Optimal. Leaf size=168

$$\frac{\sqrt{a+bx^2+cx^4}}{2e}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2}+\frac{\sqrt{cd^2-bde+ae^2}\,\tanh^{-1}\left(\frac{bd-2}{2\sqrt{cd^2-bde+}}\right.}{2e^2}$$

```
[Out] -1/4*(-b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^
2/c^(1/2)+1/2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^
(1/2)/(c*x^4+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/e^2+1/2*(c*x^4+b*x^2
+a)^(1/2)/e
```

Rubi [A]
time = 0.15, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1261, 748, 857, 635, 212, 738}

$$\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\,\sqrt{ae^2-bde+cd^2}}\right)}{2e^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2}+\frac{\sqrt{a+bx^2+cx^4}}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*e) - ((2*c*d - b*e)*ArcTanh[(b + 2*c*x^2)/(2*Sqr
t[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*e^2) + (Sqrt[c*d^2 - b*d*e + a*e
^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2
]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{\text{Subst}\left(\int \frac{bd-2ae+(2cd-be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{4e^2} + \frac{(cd^2-bde + \cdots}{\cdots}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2e^2} - \frac{(cd^2 - \cdots}{\cdots}\\
&= \frac{\sqrt{a+bx^2+cx^4}}{2e} - \frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,e^2} + \frac{\sqrt{cd^2-bde+\cdots}}{\cdots}
\end{aligned}
$$

Mathematica [A]
time = 0.43, size = 153, normalized size = 0.91

$$\frac{2e\sqrt{a+bx^2+cx^4}+4\sqrt{-cd^2+e(bd-ae)}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)+\frac{(2cd-be)\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{\sqrt{c}}}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[a + b*x^2 + c*x^4])/(d + e*x^2),x]
```

```
[Out] (2*e*Sqrt[a + b*x^2 + c*x^4] + 4*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqr
t[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)
]] + ((2*c*d - b*e)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]])/S
qrt[c])/(4*e^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 756 vs. $2(144) = 288$.
time = 0.16, size = 757, normalized size = 4.51 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
+1/4/e*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d
)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2/e^2*ln((1/2*(b*
e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e
^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^
2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^
2)^(1/2))/(x^2+d/e))*a+1/2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^
2+d/e))*d*b-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/
e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*c*d
^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(c*d^2-%e*b*d+%e^2*a>0)', see `assum
e?` for
```

Fricas [A]
time = 1.03, size = 1074, normalized size = 6.39

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(c*x^4 + b*x^2 + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4
- 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c
) + 2*sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (
b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*
a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)
*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*
x^2*e + d^2)))*e^(-2)/c, 1/4*((2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^4
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x
^4 + b*x^2 + a)*c*e + sqrt(c*d^2 - b*d*e + a*e^2)*c*log(-(8*c^2*d^2*x^4 + 8
*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b
*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*
a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)
/(x^4*e^2 + 2*d*x^2*e + d^2)))*e^(-2)/c, 1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2
)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)
*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x
^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)) + 4*sqrt(c*x^
4 + b*x^2 + a)*c*e - (2*c*d - b*e)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2
 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))*e^(-2)/c, 1/4*
(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*
d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 +
b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*
x^2 + a*b*d)*e)) + (2*c*d - b*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2
+ a)*c*e)*e^(-2)/c]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{a+bx^2+cx^4}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(1/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*sqrt(a + b*x**2 + c*x**4)/(d + e*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(1/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,\sqrt{c\,x^4+b\,x^2+a}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*x^2 + c*x^4)^(1/2))/(d + e*x^2), x)
```

3.314 $\int \frac{\sqrt{a+bx^2+cx^4}}{x(d+ex^2)}\,dx$

Optimal. Leaf size=186

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}-\frac{\sqrt{cd^2-bde+ae^2}\tanh^{-1}(\cdots}{\cdots}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d+1/2*a
rctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)/e-1/2*arctanh
(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)
^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/d/e
```

Rubi [A]
time = 0.17, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 909, 738, 212, 857, 635}

$$-\frac{\sqrt{ae^2-bde+cd^2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/d
 + (Sqrt[c]*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*
e) - (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)
/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*e)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x(d+ex)}\,dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{-bd+ae-cdx}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} + \frac{a\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d}$$

$$= -\frac{a\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{c\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx, x, \frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d} + \frac{\sqrt{c}\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2e} -$$

Mathematica [A]
time = 0.49, size = 174, normalized size = 0.94

$$-\frac{2\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right) - 2\sqrt{a}\,e\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right) + \sqrt{c}\,d\log\left(e\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{2de}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x*(d + e*x^2)),x]
```

```
[Out] -1/2*(2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt
[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]] - 2*Sqrt[a]*e*ArcTanh[
(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] + Sqrt[c]*d*Log[e*(b + 2*c
*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(d*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. $2(156) = 312$.
time = 0.13, size = 858, normalized size = 4.61

method	result
elliptic	$-\frac{\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{2d} - \frac{\ln\left(\frac{\frac{eb-2cd}{2e}+c\left(x^2+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}}\right.}{4d\sqrt{c}}$

default	$-\frac{e\left(\frac{\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{2e}+\frac{\ln\left(\frac{\frac{eb-2cd}{2e}+c\left(x^2+\frac{d}{e}\right)}{\sqrt{c}}+\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\right.}{e}}\right.}{4e\sqrt{c}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] -e/d*(1/2/e*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)+1/4/e*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e
-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2/e^2*ln((1
/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e
)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d
^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x^2+d/e))*d*b-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*
e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(
x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e
))*c*d^2)+1/d*(1/2*(c*x^4+b*x^2+a)^(1/2)+1/4*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*
x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^
2+a)^(1/2))/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((x^2*e + d)*x), x)
```

Fricas [A]
time = 42.79, size = 2407, normalized size = 12.94

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="fricas")

[Out] [1/4*(sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 + 8*a*
b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + sqr
t(c*d^2 - b*d*e + a*e^2)*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c
)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(
c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4
*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2
)))*e^(-1)/d, -1/4*(2*sqrt(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^
2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - sqrt(a)*e*log(-((b^2 + 4*a*c)*
x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/
x^4) - sqrt(c*d^2 - b*d*e + a*e^2)*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b
^2 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a
)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*
e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x
^2*e + d^2)))*e^(-1)/d, 1/4*(sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4
*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(a)*e*log(-((
b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt
(a) + 8*a^2)/x^4) - 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2
)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b
*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)))*e^(-1)/d, -1/4*(2*sqrt(-c)*d*arctan(1/2*
sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) -
 sqrt(a)*e*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*
(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan
(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d
^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x
^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)))*e^(-1)/d, 1/4*(2*sqrt(
-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*
b*x^2 + a^2))*e + sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) + sqrt(c*d^2 - b*d*e + a*e^2)*
log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*
x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) +
((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a
*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2)))*e^(-1)/d, -1/4*(2*sqr
t(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4
+ b*c*x^2 + a*c)) - 2*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))*e - sqrt(c*d^2 - b*d*e + a*e^2)*lo
g(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*x^
2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((
b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c
)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2)))*e^(-1)/d, 1/4*(2*sqrt(-
a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b
*x^2 + a^2))*e + sqrt(c)*d*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4
```

```
+ b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 2*sqrt(-c*d^2 + b*d*e - a*e^2
)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*s
qrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4
 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)))*e^(-1)/d, -1/2
*(sqrt(-c)*d*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2
*x^4 + b*c*x^2 + a*c)) - sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2))*e + sqrt(-c*d^2 + b*d*e - a*e^2
)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*s
qrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4
 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)))*e^(-1)/d]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x/(e*x**2+d),x)
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x*(d + e*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x/(e*x^2+d),x, algorithm="giac")
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(x*(d + e*x^2)), x)
```

3.315 $\int \frac{\sqrt{a+bx^2+cx^4}}{x^3(d+ex^2)} \, dx$

Optimal. Leaf size=361

$$-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{\sqrt{c}\,\tanh^{-1}}{}$$

```
[Out] -1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/a^(1/2)+1/2
*e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d^2-1/4*b
*e*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/c^(1/2)-1/4*(
-b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/c^(1
/2)+1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))*c^(1/2)/d+1/
2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))*(a*e^2-b*d*e+c*d^2)^(1/2)/d^2-1/2*(c*x^4+b*x^2+a)^(1/2)/d/
x^2
```

Rubi [A]
time = 0.35, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 21, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1265, 974, 746, 857, 635, 212, 738, 748}

$$\frac{\sqrt{ae^2-bde+cd^2}\,\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}-\frac{be\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,d^2}-\frac{(2cd-be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{c}\,\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(d*x^2) - (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*
Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[a]*d) + (Sqrt[a]*e*ArcTanh[(2*a + b*x^2)
/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (Sqrt[c]*ArcTanh[(b + 2*c*
x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) - (b*e*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2) - ((2*c*d - b*e)*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d^2)
+ (Sqrt[c*d^2 - b*d*e + a*e^2]*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2
*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\left(d+ex^2\right)}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2(d+ex)}\,dx,x,x^2\right)\\
&= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\sqrt{a+bx+cx^2}}{dx^2}-\frac{e\sqrt{a+bx+cx^2}}{d^2x}+\frac{e^2\sqrt{a+bx+cx^2}}{d^2(d+ex)}\right)\,dx,x,x^2\right.\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x^2}\,dx,x,x^2\right)}{2d}-\frac{e\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,dx,x,x^2\right)}{2d^2}+\frac{e^2\text{S}}{\quad}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}+\frac{\text{Subst}\left(\int \frac{b+2cx}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4d}+\frac{e\text{Subst}\left(\int \frac{-2a-}{x\sqrt{a+bx}}\right.}{4d^2}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}+\frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4d}+\frac{c\text{Subst}\left(\int \frac{1}{\sqrt{a+bx}}\right.}{2d}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx,x,\frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{c\text{Subst}\left(\int \frac{1}{4c-x^2}\right.}{\quad}\\
&= -\frac{\sqrt{a+bx^2+cx^4}}{2dx^2}-\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{a}\,e\tanh^{-1}\left(\frac{\quad}{2\sqrt{a}\sqrt{\quad}}\right.}{2d^2}
\end{aligned}$$

Mathematica [A]
time = 0.67, size = 158, normalized size = 0.44

$$\frac{-\frac{d\sqrt{a+bx^2+cx^4}}{x^2}+2\sqrt{-cd^2+bde-ae^2}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right)+\frac{(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*x^2 + c*x^4]/(x^3*(d + e*x^2)),x]
```

```
[Out] (-((d*Sqrt[a + b*x^2 + c*x^4])/x^2) + 2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcT
an[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e
- a*e^2]] + ((b*d - 2*a*e)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/
Sqrt[a]])/Sqrt[a])/(2*d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. $2(297) = 594$.
time = 0.16, size = 1004, normalized size = 2.78 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/d^2*(1/2/e*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)+1/4/e*ln((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(
b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/c^(1/2)*b-1/2/e^2*ln
((1/2*(b*e-2*c*d)/e+c*(x^2+d/e))/c^(1/2)+(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+
d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))*c^(1/2)*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b
*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+
c*d^2)/e^2)^(1/2))/(x^2+d/e))*a+1/2/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
(2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(x^2+d/e))*d*b-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b
*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(
c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+
d/e))*c*d^2)+1/d*(-1/2/a/x^2*(c*x^4+b*x^2+a)^(3/2)+1/2*b/a*(c*x^4+b*x^2+a)^
(1/2)-1/4*b/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)+1/2
*c/a*(c*x^4+b*x^2+a)^(1/2)*x^2+1/2*c^(1/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+
b*x^2+a)^(1/2)))-e/d^2*(1/2*(c*x^4+b*x^2+a)^(1/2)+1/4*b*ln((1/2*b+c*x^2)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)-1/2*a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c
*x^4+b*x^2+a)^(1/2))/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*x^4 + b*x^2 + a)/((x^2*e + d)*x^3), x)
```

Fricas [A]
time = 0.63, size = 1130, normalized size = 3.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a*x^2*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x
^2 + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^
```

```
2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 +
8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2
+ 2*d*x^2*e + d^2)) - 4*sqrt(c*x^4 + b*x^2 + a)*a*d - (b*d*x^2 - 2*a*x^2*e)
*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b
*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*d^2*x^2), 1/8*(4*sqrt(-c*d^2 + b*d*e
- a*e^2)*a*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^
2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d
^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)) -
4*sqrt(c*x^4 + b*x^2 + a)*a*d - (b*d*x^2 - 2*a*x^2*e)*sqrt(a)*log(-((b^2 +
4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) +
8*a^2)/x^4))/(a*d^2*x^2), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*x^2*log(-(8*c^
2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(
2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*
a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2
+ 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2)) - 2*sqrt(c*x^4 + b*x^2 + a)*a*d
+ (b*d*x^2 - 2*a*x^2*e)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2
+ 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*d^2*x^2), 1/4*(2*sqrt(-c*d^2
 + b*d*e - a*e^2)*a*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*
d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^
2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*
d)*e)) - 2*sqrt(c*x^4 + b*x^2 + a)*a*d + (b*d*x^2 - 2*a*x^2*e)*sqrt(-a)*arc
tan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 +
 a^2)))/(a*d^2*x^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a+bx^2+cx^4}}{x^3\,(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(1/2)/x**3/(e*x**2+d),x)
```

```
[Out] Integral(sqrt(a + b*x**2 + c*x**4)/(x**3*(d + e*x**2)), x)
```

Giac [A]
time = 4.63, size = 216, normalized size = 0.60

$$\frac{(cd^2-bde+ae^2)\arctan\left(-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}\,d^2}+\frac{(bd-2\,ae)\arctan\left(-\frac{\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,d^2}+\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)b+2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)^2-a\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(1/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] (c*d^2 - b*d*e + a*e^2)*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e
+ sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^
```

```
2) + 1/2*(b*d - 2*a*e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt
(-a))/(sqrt(-a)*d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a
*sqrt(c))/(((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{c\,x^4+b\,x^2+a}}{x^3\,\left(e\,x^2+d\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)),x)
[Out] int((a + b*x^2 + c*x^4)^(1/2)/(x^3*(d + e*x^2)), x)
```

3.316 $\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=424

$$-\frac{1}{60}x\left(13-6x^2\right)\sqrt{1+2x^2+2x^4}+\frac{109x\sqrt{1+2x^2+2x^4}}{60\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}+\frac{3}{16}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{109\left(1+\sqrt{\ldots}\right.}{\ldots}$$

[Out] 3/16*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/60*x*(-6*x^2+13)*(2*x^4+2*x^2+1)^(1/2)+109/120*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-109/120*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+15/32*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+1/120*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-70+263*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)

Rubi [A]
time = 0.37, antiderivative size = 619, normalized size of antiderivative = 1.46, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$,
Rules used = {1349, 1105, 1211, 1117, 1209, 1130, 1222, 1230, 1720}

$$\frac{3}{16}\sqrt{15}\,\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{45\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{112\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\left(1+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{139\left(1-\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{240\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{109\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{60\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{15\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{224\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{1}{30}\left(3x^2+1\right)\sqrt{2x^4+2x^2+1}\,x+\frac{109\sqrt{2x^4+2x^2+1}\,x}{60\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}-\frac{1}{4}\sqrt{2x^4+2x^2+1}\,x$$

Antiderivative was successfully verified.

[In] Int[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]

[Out] -1/4*(x*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*(1 + 3*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/30 + (109*x*Sqrt[1 + 2*x^2 + 2*x^4])/(60*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/16 - (109*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(60*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (139*(1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(240*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(4*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (45*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqr

```
t[2])/4])/(112*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (15*(3 + Sqrt[2])^2*(1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 -
 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(224*2^(1/4)*Sqrt[1
 + 2*x^2 + 2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[d*(d*x)^(m - 1)*(a + b*x^2 + c*x^4)^p*((2*b*p + c*(m + 4*p - 1)*x^
2)/(c*(m + 4*p + 1)*(m + 4*p - 1))), x] - Dist[2*p*(d^2/(c*(m + 4*p + 1)*(m
 + 4*p - 1))), Int[(d*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p - 1)*Simp[a*b*(m -
1) - (2*a*c*(m + 4*p - 1) - b^2*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a,
 b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && GtQ[m, 1] && IntegerQ[
2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = \int \left(-\frac{3}{4}\sqrt{1+2x^2+2x^4} + \frac{1}{2}x^2\sqrt{1+2x^2+2x^4} + \frac{9\sqrt{1+2x^2+2x^4}}{4\left(3+2x^2\right)}\right)dx$$

$$= \frac{1}{2}\int x^2\sqrt{1+2x^2+2x^4}\,dx - \frac{3}{4}\int \sqrt{1+2x^2+2x^4}\,dx + \frac{9}{4}\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4} + \frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4} - \frac{1}{60}\int \frac{2-4x^2}{\sqrt{1+2x^2+2}}$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4} + \frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4} - \frac{\int \frac{1-\sqrt{2}\,x^2}{\sqrt{1+2x^2+2x^4}}}{15\sqrt{2}}$$

$$= -\frac{1}{4}x\sqrt{1+2x^2+2x^4} + \frac{1}{30}x\left(1+3x^2\right)\sqrt{1+2x^2+2x^4} + \frac{109x\sqrt{1+2x^2+2x}}{60\sqrt{2}\left(1+\sqrt{2}\,x^2\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.53, size = 209, normalized size = 0.49

$$\frac{-52x-80x^3-56x^5+48x^7-218i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(199-417i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+225(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{240\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (-52*x - 80*x^3 - 56*x^5 + 48*x^7 - (218*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (199 - 41
7*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*Ar
cSinh[Sqrt[1 - I]*x], I] + 225*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
 (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(240*Sqrt
[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 528, normalized size = 1.25

method	result
risch	$\frac{x(6x^2-13)\sqrt{2x^4+2x^2+1}}{60} + \frac{\left(-\frac{109}{120}+\frac{109i}{120}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+}}$
elliptic	$\frac{x^3\sqrt{2x^4+2x^2+1}}{10} - \frac{13x\sqrt{2x^4+2x^2+1}}{60} - \frac{77\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{-1+\right.}{30\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

default	$\frac{x^3\sqrt{2x^4+2x^2+1}}{10}-\frac{13x\sqrt{2x^4+2x^2+1}}{60}-\frac{8\sqrt{1+(1-i)\,x^2}\;\sqrt{1+(1+i)\,x^2}\;\mathrm{EllipticF}\left(x\sqrt{-1+\right.}{15\sqrt{-1+i}\;\sqrt{2x^4+2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/10*x^3*(2*x^4+2*x^2+1)^(1/2)-13/60*x*(2*x^4+2*x^2+1)^(1/2)-8/15/(-1+I)^(1
/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(13/60-13/60*I)/(-1+I)^(1/2)*(1+
(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1
+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1
/2*I*2^(1/2)))-9/4/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*
x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+9/8*
I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+9/8/(-1+I)^(1/2)*(1+x
^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-9/8*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(
1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/
2)+1/2*I*2^(1/2))+15/8/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1
+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{x^4\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(x**4*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(2*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3),x)
```

```
[Out] int((x^4*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3), x)
```

3.317 $\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx$

Optimal. Leaf size=417

$$\frac{1}{6}x\sqrt{1+2x^2+2x^4}-\frac{7x\sqrt{1+2x^2+2x^4}}{6\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}-\frac{1}{8}\sqrt{15}\,\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)+\frac{7\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+}{\left(1+\sqrt{2}\right.}}}{6\,2^{3/4}}$$

```
[Out] -1/8*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/6*x*(2*x^4+2*x
^2+1)^(1/2)-7/12*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+7/12*(cos(
2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arct
an(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x
^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-5/16*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)
),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)
^(1/2)-1/12*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ell
ipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-4+17*2^(1/2))*(1+x
^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2)
)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 591, normalized size of antiderivative = 1.42, number of steps used = 13, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1349, 1105, 1211, 1117, 1209, 1222, 1230, 1720}

$$-\frac{1}{8}\sqrt{15}\,\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{15\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{56\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(1+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{3\left(1-\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}+\frac{7\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,E\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{5\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{112\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}-\frac{7\sqrt{2x^4+2x^2+1}\,x}{6\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}+\frac{1}{6}\sqrt{2x^4+2x^2+1}\,x$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/6 - (7*x*Sqrt[1 + 2*x^2 + 2*x^4])/(6*Sqrt[2]*(1
 + Sqrt[2]*x^2)) - (Sqrt[15]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/8 + (7*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + (3*(1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*
Sqrt[1 + 2*x^2 + 2*x^4]) + ((1 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(6*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (15*(3 + Sqrt[2])*(1 + Sqrt[2]*x
^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4
)*x], (2 - Sqrt[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt
```

```
[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(112*2^
(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
```

```
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx &= \int \left(\frac{1}{2}\sqrt{1+2x^2+2x^4}-\frac{3\sqrt{1+2x^2+2x^4}}{2\left(3+2x^2\right)}\right)dx\\
&= \frac{1}{2}\int \sqrt{1+2x^2+2x^4}\,dx-\frac{3}{2}\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx\\
&= \frac{1}{6}x\sqrt{1+2x^2+2x^4}+\frac{1}{6}\int \frac{2+2x^2}{\sqrt{1+2x^2+2x^4}}\,dx+\frac{3}{8}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{}\\
&= \frac{1}{6}x\sqrt{1+2x^2+2x^4}-\frac{\int \frac{1-\sqrt{2}\,x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{3\sqrt{2}}+\frac{3\int \frac{1-\sqrt{2}\,x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}}+\frac{1}{4}\Big(3\\
&= \frac{1}{6}x\sqrt{1+2x^2+2x^4}-\frac{7x\sqrt{1+2x^2+2x^4}}{6\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}-\frac{1}{8}\sqrt{15}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.92, size = 204, normalized size = 0.49

$$\frac{4x+8x^3+8x^5+14i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+(13-27i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-15(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{24\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 + 2*x^2 + 2*x^4])/(3 + 2*x^2),x]
```

```
[Out] (4*x + 8*x^3 + 8*x^5 + (14*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1
 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (13 - 27*I)*Sqrt[1 - I]
*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I
]*x], I] - 15*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Ell
ipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(24*Sqrt[1 + 2*x^2 + 2*x^4
])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 509, normalized size = 1.22

method	result
risch	$\frac{x\sqrt{2x^4+2x^2+1}}{6}+\frac{\left(\frac{7}{12}-\frac{7i}{12}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$\frac{x\sqrt{2x^4+2x^2+1}}{6}+\frac{5\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{7i\sqrt{-}}{}$
default	$\frac{x\sqrt{2x^4+2x^2+1}}{6}+\frac{\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{6}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6*x*(2*x^4+2*x^2+1)^(1/2)+1/3/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x
^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*
2^(1/2))+(-1/6+1/6*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/
(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-
EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3/2/(-1+I)^(1/2)*(1+x^
2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)
^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-3/4*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1
+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2
)+1/2*I*2^(1/2))-3/4/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(
2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+3/
4*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-5/4/(-1+I)^(1/2)*(1
```

```
+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-
1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{2x^4 + 2x^2 + 1}}{2x^2 + 3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
[Out] Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(2*x^2 + 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3),x)
[Out] int((x^2*(2*x^2 + 2*x^4 + 1)^(1/2))/(2*x^2 + 3), x)
```

3.318 $\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\, dx$

Optimal. Leaf size=381

$$\frac{x\sqrt{1+2x^2+2x^4}}{\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}+\frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,E\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)\right.}{2^{3/4}\sqrt{1+2x^2+2x^4}}$$

```
[Out] 1/12*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/2*x*(2*x^4+2*x
^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/2*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/
cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))
^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(
2*x^4+2*x^2+1)^(1/2)+5/24*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2
^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(
1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2
)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+2^(3/4)*(cos(2*arctan(2
^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)
*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2)
)^2)^(1/2)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 470, normalized size of antiderivative = 1.23, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {1222, 1211, 1117, 1209, 1230, 1720}

$$\frac{1}{4}\sqrt{\frac{5}{3}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{28\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}-\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{4\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,E\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{5\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{168\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}\,x}{\sqrt{2}\left(\sqrt{2}\,x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[5/3]*ArcTan
[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/4 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2
*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt
[2])/4])/(2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(4*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2
])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elliptic
F[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(28*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4
]) - (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqr
t[2])/4])/(168*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
```

```
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{3+2x^2}\,dx = -\left(\frac{1}{4}\int \frac{2-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx\right)+\frac{5}{2}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\int \frac{1-\sqrt{2}\,x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{\sqrt{2}} - \frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{14}\left(5\left(3+\sqrt{2}\right.\right.$$

$$= \frac{x\sqrt{1+2x^2+2x^4}}{\sqrt{2}\left(1+\sqrt{2}\,x^2\right)} + \frac{1}{4}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x}{\left(1+\right.}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.74, size = 127, normalized size = 0.33

$$-\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left((3+3i)E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(3+6i)F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+5i\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)\right)}{6\sqrt{1-i}\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(3 + 2*x^2),x]
```

```
[Out] -1/6*(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((3 + 3*I)*EllipticE[I*Ar
cSinh[Sqrt[1 - I]*x], I] - (3 + 6*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 + (5*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(Sqrt[1 - I]*
Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 341, normalized size = 0.90

method	result
default	$-\frac{\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x}}{2\sqrt{-1+}}$

elliptic	$-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+}}{2\sqrt{-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/2*I/(-1+I)^(1/2)*(
1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-
1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/2/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*
(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1
/2)+1/2*I*2^(1/2))-1/2*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)
)+5/6/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^
(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/(2*x**2+3),x)
```

```
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 + 3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 + 3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{2\,x^2+3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(2*x^2 + 3),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(2*x^2 + 3), x)
```

3.319 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx$

Optimal. Leaf size=399

$$-\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)}-\frac{1}{6}\sqrt{\frac{5}{3}}\,\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{\sqrt[4]{2}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+}{\left(1\right.}}}{}$$

```
[Out] -1/18*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*(2*x^4+2*x^
2+1)^(1/2)/x+1/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/3*(cos(2
*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arcta
n(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^
2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/42*(cos(2*arctan(2^(1/4
)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),
1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*
2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+5/504*(cos(2*arctan(2^(1/4)
*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),
1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.14, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1325, 1295, 1211, 1117, 1209, 1230, 1720}

$$-\frac{1}{6}\sqrt{\frac{5}{3}}\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{21\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\sqrt[4]{2}\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}+\frac{5\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{252\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}\,x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

```
[Out] -1/3*Sqrt[1 + 2*x^2 + 2*x^4]/x + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1
+ Sqrt[2]*x^2)) - (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])
/6 - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x
^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^
2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2
^(1/4)*x], (2 - Sqrt[2])/4])/(252*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
```

```
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1325

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[(f*x)^(m + 2)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3+2x^2)}\,dx &= \frac{1}{6}\int \frac{2+6x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx - \frac{5}{3}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx \\
&= -\frac{\sqrt{1+2x^2+2x^4}}{3x} - \frac{1}{6}\int \frac{-6-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{21}\left(5\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x^2 \cdots}} \\
&= -\frac{\sqrt{1+2x^2+2x^4}}{3x} - \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{\cdots}\right.}{\cdots} \\
&= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)} - \frac{1}{6}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}} \cdots\right.
\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.13, size = 208, normalized size = 0.52

$$\frac{-6-12x^2-12x^4-6i\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+(9-3i)\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-5(1-i)^{3/2}x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{18x\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^2*(3 + 2*x^2)),x]
```

```
[Out] (-6 - 12*x^2 - 12*x^4 - (6*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 +
(1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (9 - 3*I)*Sqrt[1 - I]
*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 -
 I]*x], I] - 5*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*
```

```
EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(18*x*Sqrt[1 + 2*x^2 +
2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 511, normalized size = 1.28

method	result
risch	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix}}{}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{2\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{2}{3}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x+2/3/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*
x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I
*2^(1/2))+(-2/3+2/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+2/3/(-1+I)^(1/2)*(1+x
^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/3*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(
1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/
2)+1/2*I*2^(1/2))-1/3/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/
(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1
/3*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-5/9/(-1+I)^(1/2)*(
1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(
-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^4 + 3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^2\cdot(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**2/(2*x**2+3),x)
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**2*(2*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^2/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^2\,\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^2*(2*x^2 + 3)),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^2*(2*x^2 + 3)), x)
```

3.320 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^4(3+2x^2)}\,dx$

Optimal. Leaf size=360

$$-\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right.\right.}{9\sqrt[4]{2}\,\sqrt{1+2x^2+2x^4}}$$

```
[Out] 1/27*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/9*(2*x^4+2*x^2
+1)^(1/2)/x^3-1/18*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*
x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2
))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+
5/126*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF
(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2)
)*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-5
/756*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi
(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/
2))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*
x^4+2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1323, 1295, 12, 1117, 1230, 1720}

$$\frac{1}{9}\sqrt{\frac{5}{3}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{5\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}-\frac{5\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{378\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

```
[Out] -1/9*Sqrt[1 + 2*x^2 + 2*x^4]/x^3 + (Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/9 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*2^(1/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (5*(3 + Sqrt[2])^2*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt
[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(378*2^(1/4)*Sqrt[1 + 2*x^2
 + 2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1323

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[(f*x)^(m + 4)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
```

```
NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^4\left(3+2x^2\right)}\,dx = \frac{1}{9}\int \frac{3+4x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{9}\int \frac{1}{\left(3+2x^2\right)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{1}{27}\int \frac{6}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{63}\left(10\left(3+\sqrt{2}\right)\right)\int \frac{1}{\sqrt{1+2x}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{5\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\right.}{}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{9}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2}{\left(1+\right.}}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.13, size = 154, normalized size = 0.43

$$-\frac{3+6x^2+6x^4+3(1-i)^{3/2}x^3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-5(1-i)^{3/2}x^3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{27x^3\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^4*(3 + 2*x^2)),x]
```

```
[Out] -1/27*(3 + 6*x^2 + 6*x^4 + 3*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1
 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 5*(1 - I)^(3/2)*x^
3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSi
nh[Sqrt[1 - I]*x], I])/(x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.21, size = 448, normalized size = 1.24

method	result
risch	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3} - \frac{2\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\mathrm{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} + \frac{10\sqrt{-}}{}$

elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{2\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{10\sqrt{-}}{}$
default	$\frac{\left(\frac{2}{9}-\frac{2i}{9}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\mathrm{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-\mathrm{EllipticE}\left(x\sqrt{-1+i}\,,\frac{\sqrt{}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] (2/9-2/9*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x
^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(
x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-4/9/(-1+I)^(1/2)*(1+x^2-I*x^2)^(
1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))+2/9*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2
)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^
(1/2))+2/9/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^
2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-2/9*I/(-1+I)
^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipt
icE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+10/27/(-1+I)^(1/2)*(1+x^2-I*x
^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/
2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-1/9*(2*x^4+2*x^2+1)^(1/2)/x^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^6 + 3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^4\cdot(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**4/(2*x**2+3),x)
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**4*(2*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^4/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^4\,\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^4*(2*x^2 + 3)),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^4*(2*x^2 + 3)), x)
```

3.321 $\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx$

Optimal. Leaf size=546

$$-\frac{\sqrt{1+2x^2+2x^4}}{15x^5}+\frac{4\sqrt{1+2x^2+2x^4}}{135x^3}-\frac{4\sqrt{1+2x^2+2x^4}}{45x}+\frac{4\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{45\left(1+\sqrt{2}\,x^2\right)}-\frac{2}{27}\sqrt{\frac{5}{3}}\tan^{-1}\left(\frac{}{\sqrt{1}}\right.$$

```
[Out] -2/81*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/15*(2*x^4+2*x
^2+1)^(1/2)/x^5+4/135*(2*x^4+2*x^2+1)^(1/2)/x^3-4/45*(2*x^4+2*x^2+1)^(1/2)/
x+4/45*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-4/45*(cos(2*arctan(2
^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)
*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2)
)^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+5/189*2^(1/4)*(cos(2*arctan(2^(1/4
)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),
1/2*(2-2^(1/2))^(1/2))*(5-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^
2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)-1/135*2^(1/4)*(cos(2*arctan(2^(1/
4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x))
,1/2*(2-2^(1/2))^(1/2))*(19-2*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+
x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)+5/1134*(cos(2*arctan(2^(1/4)*x)
)^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2
-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 546, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1323, 1295, 1211, 1117, 1209, 1343, 1728, 1722, 1720}

$$-\frac{2}{27}\sqrt{\frac{5}{3}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\sqrt[4]{2}\left(19-2\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}+\frac{5\sqrt[4]{2}\left(5-3\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{2x^4+2x^2+1}}-\frac{4\sqrt[4]{2}\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,E\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{45\sqrt{2x^4+2x^2+1}}+\frac{5\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{567\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{4\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{45\left(\sqrt{2}\,x^2+1\right)}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

```
[Out] -1/15*Sqrt[1 + 2*x^2 + 2*x^4]/x^5 + (4*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) -
 (4*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x) + (4*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])
/(45*(1 + Sqrt[2]*x^2)) - (2*Sqrt[5/3]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2
+ 2*x^4]])/27 - (4*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(45*Sqrt[1
 + 2*x^2 + 2*x^4]) + (5*2^(1/4)*(5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 +
 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sq
rt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/4)*(19 - 2*Sqrt[2])*(1 + S
```

```
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (5*(3 + Sqr
t[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(567*2
^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1323

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[(f*x)^(m + 4)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
```

```
, -2]
```

Rule 1343

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1))), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^
4]))*Simp[a*e*(m + 1) + b*d*(m + 2) + (b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e
*(m + 3)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{\sqrt{1+2x^2+2x^4}}{x^6(3+2x^2)}\,dx = \frac{1}{9}\int \frac{3+4x^2}{x^6\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{9}\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} - \frac{10\sqrt{1+2x^2+2x^4}}{27x} - \frac{1}{45}\int \frac{4+18x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx + \frac{10}{27}\int$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{10\sqrt{1+2x^2+2x^4}}{27x} + \frac{1}{135}\int \frac{-38}{x^2\sqrt{1+}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{10\sqrt{2}\,x\sqrt{1+2x^2}}{27\left(1+\sqrt{2}\right.}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{10\sqrt{2}\,x\sqrt{1+2x^2}}{27\left(1+\sqrt{2}\right.}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{15x^5} + \frac{4\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{4\sqrt{1+2x^2+2x^4}}{45x} + \frac{4\sqrt{2}\,x\sqrt{1+2x^2}}{45\left(1+\sqrt{2}\,x\right.}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 224, normalized size = 0.41

$$-\frac{27+42x^2+66x^4+48x^6+72x^8+36i\sqrt{1-i}\,x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(12+24i)\sqrt{1-i}\,x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+50(1-i)^{3/2}x^5\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{405x^5\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 2*x^2 + 2*x^4]/(x^6*(3 + 2*x^2)),x]
```

```
[Out] -1/405*(27 + 42*x^2 + 66*x^4 + 48*x^6 + 72*x^8 + (36*I)*Sqrt[1 - I]*x^5*Sqr
t[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x]
, I] - (12 + 24*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 50*(1 - I)^(3/2)*x^5*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 -
 I]*x], I])/(x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 549, normalized size = 1.01

method	result
risch	$-\frac{24x^8+16x^6+22x^4+14x^2+9}{135x^5\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{4}{45}+\frac{4i}{45}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+}}{135\sqrt{-1+i}}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}+\frac{4\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{4\sqrt{2x^4+2x^2+1}}{45x}-\frac{4\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)}}{45\sqrt{-1+i}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(2*x^4+2*x^2+1)^(1/2)/x^5+4/135*(2*x^4+2*x^2+1)^(1/2)/x^3-4/45*(2*x^4
+2*x^2+1)^(1/2)/x-4/45/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+
(-32/135+32/135*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*
x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-Ell
ipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-4/27*I/(-1+I)^(1/2)*(1+x^
2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)
^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+8/27/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+
x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)
+1/2*I*2^(1/2))+4/27*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-
4/27/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(
1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-20/81/(-1+I)^(1/2)
*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x
*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(2*x^8 + 3*x^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{2x^4+2x^2+1}}{x^6\cdot(2x^2+3)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(1/2)/x**6/(2*x**2+3),x)
[Out] Integral(sqrt(2*x**4 + 2*x**2 + 1)/(x**6*(2*x**2 + 3)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(1/2)/x^6/(2*x^2+3),x, algorithm="giac")
[Out] integrate(sqrt(2*x^4 + 2*x^2 + 1)/((2*x^2 + 3)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{2\,x^4+2\,x^2+1}}{x^6\,\left(2\,x^2+3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^6*(2*x^2 + 3)),x)
[Out] int((2*x^2 + 2*x^4 + 1)^(1/2)/(x^6*(2*x^2 + 3)), x)
```

3.322 $\int \frac{x^5\left(a+bx^2+cx^4\right)^{3/2}}{d+ex^2}\,dx$

<u>**Optimal**</u>. Leaf size=482

$$\frac{\left(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2ae)-2ce(32c^3d^3-3b^3e^3-8c\right.}{256c^3e^5}$$

```
[Out] 1/96*(16*c^2*d^2-6*b*c*d*e-3*b^2*e^2-6*c*e*(b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)
^(3/2)/c^2/e^3+1/10*(c*x^4+b*x^2+a)^(5/2)/c/e-1/512*(256*c^5*d^5+3*b^5*e^5+
6*b^3*c*e^4*(-4*a*e+b*d)-384*c^4*d^3*e*(-a*e+b*d)+96*c^3*d*e^2*(-a*e+b*d)^2
+16*b*c^2*e^3*(3*a^2*e^2-3*a*b*d*e+b^2*d^2))*arctanh(1/2*(2*c*x^2+b)/c^(1/2
)/(c*x^4+b*x^2+a)^(1/2))/c^(7/2)/e^6+1/2*d^2*(a*e^2-b*d*e+c*d^2)^(3/2)*arct
anh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2
+a)^(1/2))/e^6+1/256*(128*c^4*d^4+3*b^4*e^4-32*c^3*d^2*e*(-4*a*e+5*b*d)+8*b
*c^2*d*e^2*(-3*a*e+2*b*d)+6*b^2*c*e^3*(-2*a*e+b*d)-2*c*e*(32*c^3*d^3-3*b^3*
e^3-8*c^2*d*e*(-3*a*e+2*b*d)-6*b*c*e^2*(-2*a*e+b*d))*x^2)*(c*x^4+b*x^2+a)^(
1/2)/c^3/e^5
```

<u>**Rubi**</u> [A]
time = 0.73, antiderivative size = 482, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1265, 1667, 828, 857, 635, 212, 738}

$$-\frac{\left(16bc^2e^3(3a^2e^2-3abde+b^2d^2)+6b^3ce^4(bd-4ae)-384c^4d^3e(bd-ae)+96c^3de^2(bd-ae)^2+3b^5e^5+256c^5d^5\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{512c^{7/2}e^6}+\frac{(a+bx^2+cx^4)^{3/2}\left(-3b^2e^2-6cex^2(be+2cd)-6bcde+16c^2d^2\right)}{96c^2e^3}+\frac{\sqrt{a+bx^2+cx^4}\left(-2cex^2(-8c^2de(2bd-3ae)-6bce^2(bd-2ae)-3b^3e^3+32c^3d^3)+6b^2ce^3(bd-2ae)-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+3b^4e^4+128c^4d^4\right)}{256c^3e^5}+\frac{d^2(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^6}+\frac{(a+bx^2+cx^4)^{5/2}}{10ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((128*c^4*d^4 + 3*b^4*e^4 - 32*c^3*d^2*e*(5*b*d - 4*a*e) + 8*b*c^2*d*e^2*(2
*b*d - 3*a*e) + 6*b^2*c*e^3*(b*d - 2*a*e) - 2*c*e*(32*c^3*d^3 - 3*b^3*e^3 -
 8*c^2*d*e*(2*b*d - 3*a*e) - 6*b*c*e^2*(b*d - 2*a*e))*x^2)*Sqrt[a + b*x^2 +
 c*x^4])/(256*c^3*e^5) + ((16*c^2*d^2 - 6*b*c*d*e - 3*b^2*e^2 - 6*c*e*(2*c*
d + b*e)*x^2)*(a + b*x^2 + c*x^4)^(3/2))/(96*c^2*e^3) + (a + b*x^2 + c*x^4)
^(5/2)/(10*c*e) - ((256*c^5*d^5 + 3*b^5*e^5 + 6*b^3*c*e^4*(b*d - 4*a*e) - 3
84*c^4*d^3*e*(b*d - a*e) + 96*c^3*d*e^2*(b*d - a*e)^2 + 16*b*c^2*e^3*(b^2*d
^2 - 3*a*b*d*e + 3*a^2*e^2))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^
2 + c*x^4])])/(512*c^(7/2)*e^6) + (d^2*(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTan
h[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a +
 b*x^2 + c*x^4])])/(2*e^6)
```

<u>Rule 212</u>

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^5(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(a+bx+cx^2)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{(a+bx^2+cx^4)^{5/2}}{10ce} + \frac{\text{Subst}\left(\int \frac{\left(-\frac{5}{2}bde-\frac{5}{2}e(2cd+be)x\right)(a+bx+cx^2)^{3/2}}{d+ex}\,dx, x, x^2\right)}{10ce^2}$$

$$= \frac{(16c^2d^2-6bcde-3b^2e^2-6ce(2cd+be)x^2)(a+bx^2+cx^4)^{3/2}}{96c^2e^3} + \frac{(a+bx^2+ \cdots}{10ce \cdots}$$

$$= \frac{(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2 \cdots}{\cdots}$$

$$= \frac{(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2 \cdots}{\cdots}$$

$$= \frac{(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2 \cdots}{\cdots}$$

$$= \frac{(128c^4d^4+3b^4e^4-32c^3d^2e(5bd-4ae)+8bc^2de^2(2bd-3ae)+6b^2ce^3(bd-2 \cdots}{\cdots}$$

Mathematica [A]
time = 10.66, size = 545, normalized size = 1.13

$$\frac{1280 e^3 (a+bx^2+cx^4)^{5/2} - \cdots}{7680 e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] (1280*d^2*(a + b*x^2 + c*x^4)^(3/2) - (480*d*e*(b + 2*c*x^2)*(a + b*x^2 + c
*x^4)^(3/2))/c + (768*e^2*(a + b*x^2 + c*x^4)^(5/2))/c - (90*(b^2 - 4*a*c)*
d*e*(-2*Sqrt[c]*(b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4] + (b^2 - 4*a*c)*ArcTa
nh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]))/c^(5/2) + (15*b*e^2
*(-16*(b + 2*c*x^2)*(a + b*x^2 + c*x^4)^(3/2) + 3*(b^2 - 4*a*c)*((2*(b + 2*
c*x^2)*Sqrt[a + b*x^2 + c*x^4])/c + ((-b^2 + 4*a*c)*ArcTanh[(b + 2*c*x^2)/(
2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/c^(3/2))))/c^2 - (240*d^2*((2*c*d - b*
e)*(8*c^2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])] + 2*Sqrt[c]*(e*Sqrt[a + b*x^2 + c*x^4]*(-
(b^2*e^2) + 4*c^2*d*(-2*d + e*x^2) - 2*c*e*(-5*b*d + 4*a*e + b*e*x^2)) + 8*
c*(c*d^2 + e*(-(b*d) + a*e))^(3/2)*ArcTanh[(-(b*d) + 2*a*e - 2*c*d*x^2 + b*
e*x^2)/(2*Sqrt[c*d^2 + e*(-(b*d) + a*e)]*Sqrt[a + b*x^2 + c*x^4])])))/(c^(3
/2)*e^3))/(7680*e^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1984 vs. $2(450) = 900$.
time = 0.17, size = 1985, normalized size = 4.12

method	result	size
risch	Expression too large to display	1798
default	Expression too large to display	1985
elliptic	Expression too large to display	2068

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/160*b^2*x^4/c*(c*x^4+b*x^2+a)^(1/2)-1/128*b^3/c^2*x^2*(c*x^4+b*x^2+a
)^(1/2)-3/32*a^2*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-
5/64*a*b^2/c^2*(c*x^4+b*x^2+a)^(1/2)+3/64*a*b^3/c^(5/2)*ln((1/2*b+c*x^2)/c^
(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/10*c*x^8*(c*x^4+b*x^2+a)^(1/2)+11/80*b*x^6*(
c*x^4+b*x^2+a)^(1/2)+1/5*a*x^4*(c*x^4+b*x^2+a)^(1/2)+3/256*b^4/c^3*(c*x^4+b
*x^2+a)^(1/2)-3/512*b^5/c^(7/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1
/2))+1/10*a^2/c*(c*x^4+b*x^2+a)^(1/2)+7/160*a*b*x^2/c*(c*x^4+b*x^2+a)^(1/2)
)-d/e^2*(1/64*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)+5/32*a*b/c*(c*x^4+b*x^2+a)^(1
/2)-3/32*a*b^2/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/8*
c*x^6*(c*x^4+b*x^2+a)^(1/2)+3/16*b*x^4*(c*x^4+b*x^2+a)^(1/2)+5/16*a*x^2*(c*
x^4+b*x^2+a)^(1/2)-3/128*b^3/c^2*(c*x^4+b*x^2+a)^(1/2)+3/256*b^4/c^(5/2)*ln
((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*a^2*ln((1/2*b+c*x^2)/c^(
1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2))+1/e^2*d^2*(3/4*c^(1/2)/e^3*ln((1/2*b+c
*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b*d^2+1/6/e*c*x^4*(c*x^4+b*x^2+a)^(1/2
)+7/24/e*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/e/c*b^2*(c*x^4+b*x^2+a)^(1/2)-1/2
*c^(3/2)/e^4*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*d^3-1/4/e^2*x^
2*c*(c*x^4+b*x^2+a)^(1/2)*d-3/16/c^(1/2)/e^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
```

```
4+b*x^2+a)^(1/2))*b^2*d-5/8/e^2*b*(c*x^4+b*x^2+a)^(1/2)*d+1/2/e^3*c*(c*x^4+
b*x^2+a)^(1/2)*d^2+3/8/c^(1/2)/e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(
1/2))*a*b+2/3/e*a*(c*x^4+b*x^2+a)^(1/2)-1/32/c^(3/2)/e*ln((1/2*b+c*x^2)/c^(
1/2)+(c*x^4+b*x^2+a)^(1/2))*b^3-3/4*c^(1/2)/e^2*ln((1/2*b+c*x^2)/c^(1/2)+(c
*x^4+b*x^2+a)^(1/2))*a*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^
2+d/e))*a^2+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)
/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)
^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d*a*b
-c/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2
*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c
*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*d^2-1/2/e^3/((
a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(
x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^
2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2*d^2+c/e^4/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d^3*b-1/2*c^2/e^5/((a*e^2-b*d*e+c*
d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-
b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d^4)
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Timed out
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x^5*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

3.323 $\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2} dx$

Optimal. Leaf size=360

$$-\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b^2e^2-4ce(2bd-3ae))\,x^2)\sqrt{a}}{128c^2e^4}$$

```
[Out] -1/48*(-6*c*e*x^2-3*b*e+8*c*d)*(c*x^4+b*x^2+a)^(3/2)/c/e^2+1/256*(128*c^4*d
^4+3*b^4*e^4+8*b^2*c*e^3*(-3*a*e+b*d)-192*c^3*d^2*e*(-a*e+b*d)+48*c^2*e^2*(
-a*e+b*d)^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(5/2)
/e^5-1/2*d*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^
2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^5-1/128*(64*c^3*d^3+3
*b^3*e^3-16*c^2*d*e*(-4*a*e+5*b*d)+4*b*c*e^2*(-3*a*e+2*b*d)-2*c*e*(16*c^2*d
^2-3*b^2*e^2-4*c*e*(-3*a*e+2*b*d))*x^2)*(c*x^4+b*x^2+a)^(1/2)/c^2/e^4
```

Rubi [A]
time = 0.45, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 828, 857, 635, 212, 738}

$$\frac{(8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2+3b^4e^4+128c^4d^4)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{256c^{5/2}e^5}-\frac{\sqrt{a+bx^2+cx^4}\,(-2cex^2(-4ce(2bd-3ae)-3b^2e^2+16c^2d^2)-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)+3b^3e^3+64c^3d^3)}{128c^2e^4}-\frac{d(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^5}-\frac{(a+bx^2+cx^4)^{3/2}(-3be+8cd-6cex^2)}{48ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] -1/128*((64*c^3*d^3 + 3*b^3*e^3 - 16*c^2*d*e*(5*b*d - 4*a*e) + 4*b*c*e^2*(2
*b*d - 3*a*e) - 2*c*e*(16*c^2*d^2 - 3*b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*x^2)
*Sqrt[a + b*x^2 + c*x^4])/(c^2*e^4) - ((8*c*d - 3*b*e - 6*c*e*x^2)*(a + b*x
^2 + c*x^4)^(3/2))/(48*c*e^2) + ((128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*
d - 3*a*e) - 192*c^3*d^2*e*(b*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*ArcTanh[
(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(256*c^(5/2)*e^5) - (d*
(c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*
Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^5)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(a+bx+cx^2)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= -\frac{(8cd-3be-6cex^2)\,(a+bx^2+cx^4)^{3/2}}{48ce^2} - \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}d\left(4ace-2b\left(4cd-\frac{3be}{2}\right)\right)-\frac{1}{2}(\right.}{}\right.}{}$$

$$= -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b}{128c^2e^4}$$

$$= -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b}{128c^2e^4}$$

$$= -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b}{128c^2e^4}$$

$$= -\frac{(64c^3d^3+3b^3e^3-16c^2de(5bd-4ae)+4bce^2(2bd-3ae)-2ce(16c^2d^2-3b}{128c^2e^4}$$

Mathematica [A]
time = 1.98, size = 345, normalized size = 0.96

$$\frac{\frac{2e\sqrt{a+bx^2+cx^4}\,(-9b^3e^3+6bce^2(-4bd+10ae+bex^2)-16c^3(12d^3-6d^2ex^2+4de^2x^4-3e^3x^6)+8c^2e(ae(-32d+15ex^2)+b(30d^2-14dex^2+9e^2x^4)))}{c^2}-768d\sqrt{-cd^2+bde-ae^2}\,(cd^2+e(-bd+ae))\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)-\frac{3(128c^4d^4+3b^4e^4+8b^2ce^3(bd-3ae)-192c^3d^2e(bd-ae)+48c^2e^2(bd-ae)^2)\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)}{c^{5/2}}}{768e^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((2*e*Sqrt[a + b*x^2 + c*x^4]*(-9*b^3*e^3 + 6*b*c*e^2*(-4*b*d + 10*a*e + b*
e*x^2) - 16*c^3*(12*d^3 - 6*d^2*e*x^2 + 4*d*e^2*x^4 - 3*e^3*x^6) + 8*c^2*e*
(a*e*(-32*d + 15*e*x^2) + b*(30*d^2 - 14*d*e*x^2 + 9*e^2*x^4))))/c^2 - 768*
d*Sqrt[-(c*d^2) + b*d*e - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[c]
*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] -
 (3*(128*c^4*d^4 + 3*b^4*e^4 + 8*b^2*c*e^3*(b*d - 3*a*e) - 192*c^3*d^2*e*(b
*d - a*e) + 48*c^2*e^2*(b*d - a*e)^2)*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a +
b*x^2 + c*x^4]])/c^(5/2))/(768*e^5)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1662 vs. $2(332) = 664$.
time = 0.16, size = 1663, normalized size = 4.62

method	result	size

risch	Expression too large to display	1544
default	Expression too large to display	1663
elliptic	Expression too large to display	1696

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/64*b^2*x^2/c*(c*x^4+b*x^2+a)^(1/2)+5/32*a*b/c*(c*x^4+b*x^2+a)^(1/2)-
3/32*a*b^2/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+1/8*c*x^
6*(c*x^4+b*x^2+a)^(1/2)+3/16*b*x^4*(c*x^4+b*x^2+a)^(1/2)+5/16*a*x^2*(c*x^4+
b*x^2+a)^(1/2)-3/128*b^3/c^2*(c*x^4+b*x^2+a)^(1/2)+3/256*b^4/c^(5/2)*ln((1/
2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))+3/16*a^2*ln((1/2*b+c*x^2)/c^(1/2)
+(c*x^4+b*x^2+a)^(1/2))/c^(1/2))-d/e*(3/4*c^(1/2)/e^3*ln((1/2*b+c*x^2)/c^(1
/2)+(c*x^4+b*x^2+a)^(1/2))*b*d^2+1/6/e*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24/e*b
*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/e/c*b^2*(c*x^4+b*x^2+a)^(1/2)-1/2*c^(3/2)/e
^4*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*d^3-1/4/e^2*x^2*c*(c*x^4
+b*x^2+a)^(1/2)*d-3/16/c^(1/2)/e^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)
^(1/2))*b^2*d-5/8/e^2*b*(c*x^4+b*x^2+a)^(1/2)*d+1/2/e^3*c*(c*x^4+b*x^2+a)^(
1/2)*d^2+3/8/c^(1/2)/e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a*b+
2/3/e*a*(c*x^4+b*x^2+a)^(1/2)-1/32/c^(3/2)/e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^
4+b*x^2+a)^(1/2))*b^3-3/4*c^(1/2)/e^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2
+a)^(1/2))*a*d-1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d
^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d
/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^
2+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-
2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*
c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d*a*b-c/e^3/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a*d^2-1/2/e^3/((a*e^2-b*d*
e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2
*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*
e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*b^2*d^2+c/e^4/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d^3*b-1/2*c^2/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e
+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^
2)/e^2)^(1/2))/(x^2+d/e))*d^4)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(2*c*d-%e*b>0)', see `assume?` for m
ore det
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3(a+bx^2+cx^4)^{\frac{3}{2}}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x**3*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x^3*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

3.324 $\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx$

Optimal. Leaf size=269

$$\frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3}+\frac{(a+bx^2+cx^4)^{3/2}}{6e}-\frac{(2cd-be)\,(8c^2d^2}{}$$

```
[Out] 1/6*(c*x^4+b*x^2+a)^(3/2)/e-1/32*(-b*e+2*c*d)*(8*c^2*d^2-b^2*e^2-4*c*e*(-3*
a*e+2*b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/
e^4+1/2*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/
(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^4+1/16*(8*c^2*d^2+b^2*e^
2-2*c*e*(-4*a*e+5*b*d)-2*c*e*(-b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c/e^3
```

Rubi [A]
time = 0.29, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.259$, Rules used = {1261, 748, 828, 857, 635, 212, 738}

$$\frac{\sqrt{a+bx^2+cx^4}\,(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2)}{16ce^3}-\frac{(2cd-be)\,(-4ce(2bd-3ae)-b^2e^2+8c^2d^2)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}e^4}+\frac{(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^4}+\frac{(a+bx^2+cx^4)^{3/2}}{6e}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sq
rt[a + b*x^2 + c*x^4])/(16*c*e^3) + (a + b*x^2 + c*x^4)^(3/2)/(6*e) - ((2*c
*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d - 3*a*e))*ArcTanh[(b + 2*c*x^
2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*c^(3/2)*e^4) + ((c*d^2 - b*d*e
 + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b
*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*e^4)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(a+bx^2+cx^4)^{3/2}}{d+ex^2}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{d+ex}\,dx, x, x^2\right)$$

$$= \frac{(a+bx^2+cx^4)^{3/2}}{6e} - \frac{\text{Subst}\left(\int \frac{(bd-2ae+(2cd-be)x)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{4e}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2}{6}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2}{6}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2}{6}$$

$$= \frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce(2cd-be)x^2)\sqrt{a+bx^2+cx^4}}{16ce^3} + \frac{(a+bx^2}{6}$$

Mathematica [A]
time = 1.08, size = 260, normalized size = 0.97

$$\frac{\frac{2e\sqrt{a+bx^2+cx^4}\,(3b^2e^2+2ce(-15bd+16ae+7bex^2)+4c^2(6d^2-3dex^2+2e^2x^4))}{c} + 96\sqrt{-cd^2+bde-ae^2}\,(cd^2+e(-bd+ae))\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right) + \frac{3(2cd-be)(8c^2d^2-b^2e^2+4ce(-2bd+3ae))\log\left(c\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{c^{3/2}}}{96e^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x]
```

```
[Out] ((2*e*Sqrt[a + b*x^2 + c*x^4]*(3*b^2*e^2 + 2*c*e*(-15*b*d + 16*a*e + 7*b*e*
x^2) + 4*c^2*(6*d^2 - 3*d*e*x^2 + 2*e^2*x^4)))/c + 96*Sqrt[-(c*d^2) + b*d*e
 - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a
 + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]] + (3*(2*c*d - b*e)*(8*c^
2*d^2 - b^2*e^2 + 4*c*e*(-2*b*d + 3*a*e))*Log[c*(b + 2*c*x^2 - 2*Sqrt[c]*Sq
rt[a + b*x^2 + c*x^4])])/c^(3/2))/(96*e^4)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1410 vs. $2(241) = 482$.
time = 0.19, size = 1411, normalized size = 5.25

method	result

risch	$\frac{(8c^2e^2x^4+14bc\,e^2x^2-12c^2de\,x^2+32ac\,e^2+3e^2b^2-30bcde+24c^2d^2)\sqrt{c\,x^4+b\,x^2+a}}{48c\,e^3}+\frac{3\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right.}{8\sqrt{c}\;e}$
default	Expression too large to display
elliptic	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] 3/4*c^(1/2)/e^3*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b*d^2+1/6/e
*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24/e*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/e/c*b^
2*(c*x^4+b*x^2+a)^(1/2)-1/2*c^(3/2)/e^4*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))*d^3-1/4/e^2*x^2*c*(c*x^4+b*x^2+a)^(1/2)*d-3/16/c^(1/2)/e^2*ln(
(1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2*d-5/8/e^2*b*(c*x^4+b*x^2+a
)^(1/2)*d+1/2/e^3*c*(c*x^4+b*x^2+a)^(1/2)*d^2+3/8/c^(1/2)/e*ln((1/2*b+c*x^2
)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a*b+2/3/e*a*(c*x^4+b*x^2+a)^(1/2)-1/32/c^(
3/2)/e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^3-3/4*c^(1/2)/e^2*
ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a*d-1/2/e/((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*
e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b
*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2
)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d
^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e
^2)^(1/2))/(x^2+d/e))*d*a*b-c/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))*a*d^2-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e
+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x
^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e)
)*b^2*d^2+c/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e
^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2
+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d^3*b-1
/2*c^2/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e
-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d^4
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(c*d^2-%e*b*d+%e^2*a>0)', see `assum
e?` for
```

Fricas [A]
time = 115.14, size = 1573, normalized size = 5.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3
- 12*a*b*c)*e^3)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 +
b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c) - 48*(c^3*d^2 - b*c^2*d*e + a*c^2
*e^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^
2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)
*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e
^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^
2*e + d^2)) - 4*(24*c^3*d^2*e + (8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c + 32*a*
c^2)*e^3 - 6*(2*c^3*d*x^2 + 5*b*c^2*d)*e^2)*sqrt(c*x^4 + b*x^2 + a))*e^(-4)
/c^2, 1/96*(3*(16*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b
^3 - 12*a*b*c)*e^3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 +
b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 24*(c^3*d^2 - b*c^2*d*e + a*c^2*e^
2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 +
 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)
*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2
- 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e
 + d^2)) + 2*(24*c^3*d^2*e + (8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c + 32*a*c^2
)*e^3 - 6*(2*c^3*d*x^2 + 5*b*c^2*d)*e^2)*sqrt(c*x^4 + b*x^2 + a))*e^(-4)/c^
2, 1/192*(96*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)
*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sq
rt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4
+ a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)) - 3*(16*c^3*d^3
- 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3 - 12*a*b*c)*e^3)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(c) - 4*a*c) + 4*(24*c^3*d^2*e + (8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2*c
+ 32*a*c^2)*e^3 - 6*(2*c^3*d*x^2 + 5*b*c^2*d)*e^2)*sqrt(c*x^4 + b*x^2 + a))
*e^(-4)/c^2, 1/96*(48*(c^3*d^2 - b*c^2*d*e + a*c^2*e^2)*sqrt(-c*d^2 + b*d*e
 - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2
*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 +
(a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)) + 3*(16
*c^3*d^3 - 24*b*c^2*d^2*e + 6*(b^2*c + 4*a*c^2)*d*e^2 + (b^3 - 12*a*b*c)*e^
```

```
3)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*
x^4 + b*c*x^2 + a*c)) + 2*(24*c^3*d^2*e + (8*c^3*x^4 + 14*b*c^2*x^2 + 3*b^2
*c + 32*a*c^2)*e^3 - 6*(2*c^3*d*x^2 + 5*b*c^2*d)*e^2)*sqrt(c*x^4 + b*x^2 +
a))*e^(-4)/c^2]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(a+bx^2+cx^4)^{\frac{3}{2}}}{d+ex^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x**4+b*x**2+a)**(3/2)/(e*x**2+d),x)
```

```
[Out] Integral(x*(a + b*x**2 + c*x**4)**(3/2)/(d + e*x**2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(c*x^4+b*x^2+a)^(3/2)/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{e\,x^2+d}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2),x)
```

```
[Out] int((x*(a + b*x^2 + c*x^4)^(3/2))/(d + e*x^2), x)
```

3.325 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx$

Optimal. Leaf size=350

$$\frac{a\sqrt{a+bx^2+cx^4}}{2d}-\frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2}-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}$$

```
[Out] -1/2*a^(3/2)*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d-1/2*(
a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*
d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/e^3+1/4*a*b*arctanh(1/2*(2*c*x^2+
b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/c^(1/2)+1/16*(8*c^2*d^3+b*e^2*(-4*a*e+3
*b*d)-12*c*d*e*(-a*e+b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^
(1/2))/d/e^3/c^(1/2)+1/2*a*(c*x^4+b*x^2+a)^(1/2)/d-1/8*(4*c*d^2-e*(-4*a*e+5
*b*d)-2*c*d*e*x^2)*(c*x^4+b*x^2+a)^(1/2)/d/e^2
```

Rubi [A]
time = 0.36, antiderivative size = 350, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1265, 909, 748, 857, 635, 212, 738, 828}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d}+\frac{(-12cde(bd-ae)+be^2(3bd-4ae)+8c^2d^3)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}\,de^3}-\frac{\sqrt{a+bx^2+cx^4}\,(-e(5bd-4ae)+4cd^2-2cdex^2)}{8de^2}-\frac{(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2de^3}+\frac{a\sqrt{a+bx^2+cx^4}}{2d}+\frac{ab\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4\sqrt{c}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

```
[Out] (a*Sqrt[a + b*x^2 + c*x^4])/(2*d) - ((4*c*d^2 - e*(5*b*d - 4*a*e) - 2*c*d*e
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d*e^2) - (a^(3/2)*ArcTanh[(2*a + b*x^2)/(
2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d) + (a*b*ArcTanh[(b + 2*c*x^2)/(2*
Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*Sqrt[c]*d) + ((8*c^2*d^3 + b*e^2*(3*b
*d - 4*a*e) - 12*c*d*e*(b*d - a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a
 + b*x^2 + c*x^4])])/(16*Sqrt[c]*d*e^3) - ((c*d^2 - b*d*e + a*e^2)^(3/2)*Ar
cTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt
[a + b*x^2 + c*x^4])])/(2*d*e^3)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
```

```
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 909

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_)/(((d_.) + (e_.)*(x_))*((f_.) +
 (g_.)*(x_))), x_Symbol] :> Dist[(c*d^2 - b*d*e + a*e^2)/(e*(e*f - d*g)), I
```

```
nt[(a + b*x + c*x^2)^(p - 1)/(d + e*x), x], x] - Dist[1/(e*(e*f - d*g)), In
t[Simp[c*d*f - b*e*f + a*e*g - c*(e*f - d*g)*x, x]*((a + b*x + c*x^2)^(p -
1)/(f + g*x)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g,
0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[p]
&& GtQ[p, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x(d+ex^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x(d+ex)}\,dx, x, x^2\right)$$

$$= -\frac{\text{Subst}\left(\int \frac{(-bd+ae-cdx)\sqrt{a+bx+cx^2}}{d+ex}\,dx, x, x^2\right)}{2d} + \frac{a\text{Subst}\left(\int \frac{\sqrt{a+bx+cx^2}}{x}\,d\right.}{2d}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a\text{Subst}}{}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} + \frac{a^2\text{Subst}}{}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^2\text{Subst}}{}$$

$$= \frac{a\sqrt{a+bx^2+cx^4}}{2d} - \frac{(4cd^2-e(5bd-4ae)-2cdex^2)\sqrt{a+bx^2+cx^4}}{8de^2} - \frac{a^{3/2}\tan}{}$$

Mathematica [A]
time = 0.94, size = 258, normalized size = 0.74

$$\frac{(-4cd+5be+2cex^2)\sqrt{a+bx^2+cx^4}}{8e^2} - \frac{\sqrt{-cd^2+bde-ae^2}\,(cd^2+e(-bd+ae))\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{de^3} + \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{d} - \frac{(8c^2d^2+3b^2e^2+12ce(-bd+ae))\log\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)}{16\sqrt{c}\,e^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x]
```

[Out] ((-4*c*d + 5*b*e + 2*c*e*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*e^2) - (Sqrt[-(c*
d^2) + b*d*e - a*e^2]*(c*d^2 + e*(-(b*d) + a*e))*ArcTan[(Sqrt[c]*(d + e*x^2
) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(d*e^3) + (
a^(3/2)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]])/d - ((8*c
^2*d^2 + 3*b^2*e^2 + 12*c*e*(-(b*d) + a*e))*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqr
t[a + b*x^2 + c*x^4]])/(16*Sqrt[c]*e^3)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1612 vs. $2(302) = 604$.
time = 0.14, size = 1613, normalized size = 4.61

method	result
elliptic	$\frac{c x^2 \sqrt{c x^4+b x^2+a}}{4e}+\frac{5b\sqrt{c x^4+b x^2+a}}{8e}+\frac{3b^2 \ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{16e\sqrt{c}}+\frac{3a\sqrt{c}\,\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\right.}{}$
default	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x,method=_RETURNVERBOSE)

[Out] -e/d*(3/4*c^(1/2)/e^3*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b*d^2
+1/6/e*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24/e*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/
e/c*b^2*(c*x^4+b*x^2+a)^(1/2)-1/2*c^(3/2)/e^4*ln((1/2*b+c*x^2)/c^(1/2)+(c*x
^4+b*x^2+a)^(1/2))*d^3-1/4/e^2*x^2*c*(c*x^4+b*x^2+a)^(1/2)*d-3/16/c^(1/2)/e
^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2*d-5/8/e^2*b*(c*x^4+b
*x^2+a)^(1/2)*d+1/2/e^3*c*(c*x^4+b*x^2+a)^(1/2)*d^2+3/8/c^(1/2)/e*ln((1/2*b
+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a*b+2/3/e*a*(c*x^4+b*x^2+a)^(1/2)-1/
32/c^(3/2)/e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^3-3/4*c^(1/2
)/e^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a*d-1/2/e/((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+
2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a
*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/e^2/((a*e^2-b*d*e+c*d^2)/e^2
)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d
*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*
d^2)/e^2)^(1/2))/(x^2+d/e))*d*a*b-c/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln(
(2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e
^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(
1/2))/(x^2+d/e))*a*d^2-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2
-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
(c(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^
2+d/e))*b^2*d^2+c/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*
d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+

```
d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d
^3*b-1/2*c^2/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/
e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^
2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d^4)+1
/d*(1/6*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/c
*b^2*(c*x^4+b*x^2+a)^(1/2)-1/32/c^(3/2)*b^3*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4
+b*x^2+a)^(1/2))+3/8*a*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^
(1/2)+2/3*a*(c*x^4+b*x^2+a)^(1/2)-1/2*a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^
4+b*x^2+a)^(1/2))/x^2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="maxima")
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((x^2*e + d)*x), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a+bx^2+cx^4)^{\frac{3}{2}}}{x(d+ex^2)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x/(e*x**2+d),x)
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x*(d + e*x**2)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(c\,x^4+b\,x^2+a\right)^{3/2}}{x\,\left(e\,x^2+d\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x*(d + e*x^2)), x)
```

3.326 $\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx$

Optimal. Leaf size=562

$$\frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d}-\frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2}+\frac{(8c^2d^2+b^2e^2-2ce(5bd-4ae)-2ce\ldots}{16cd^2e\ldots}$$

```
[Out] -1/2*(c*x^4+b*x^2+a)^(3/2)/d/x^2+1/2*a^(3/2)*e*arctanh(1/2*(b*x^2+2*a)/a^(1
/2)/(c*x^4+b*x^2+a)^(1/2))/d^2+1/32*b*(-12*a*c+b^2)*e*arctanh(1/2*(2*c*x^2+
b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/d^2-1/32*(-b*e+2*c*d)*(8*c^2*d^2-
b^2*e^2-4*c*e*(-3*a*e+2*b*d))*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+
a)^(1/2))/c^(3/2)/d^2/e^2+1/2*(a*e^2-b*d*e+c*d^2)^(3/2)*arctanh(1/2*(b*d-2*
a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/
e^2-3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))*a^(1/2)/d+
3/16*(4*a*c+b^2)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/c
^(1/2)+3/8*(2*c*x^2+3*b)*(c*x^4+b*x^2+a)^(1/2)/d-1/16*e*(2*b*c*x^2+8*a*c+b^
2)*(c*x^4+b*x^2+a)^(1/2)/c/d^2+1/16*(8*c^2*d^2+b^2*e^2-2*c*e*(-4*a*e+5*b*d)
-2*c*e*(-b*e+2*c*d)*x^2)*(c*x^4+b*x^2+a)^(1/2)/c/d^2/e
```

Rubi [A]
time = 0.60, antiderivative size = 562, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1265, 974, 746, 828, 857, 635, 212, 738, 748}

$$\frac{a^{3/2}e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2d^2}+\frac{be(b^2-12ac)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2}+\frac{\sqrt{a+bx^2+cx^4}\left(-2ce(5bd-4ae)+b^2e^2-2cex^2(2cd-be)+8c^2d^2\right)}{16cd^2e}-\frac{(2cd-be)\left(-4ce(2bd-3ae)-b^2e^2+8c^2d^2\right)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{32c^{3/2}d^2e^2}-\frac{e(8ac+b^2+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2}+\frac{3(4ac+b^2)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{16\sqrt{c}\,d}+\frac{(ae^2-bde+cd^2)^{3/2}\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2e^2}-\frac{(a+bx^2+cx^4)^{3/2}}{2dx^2}+\frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d}-\frac{3\sqrt{a}\,b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] (3*(3*b + 2*c*x^2)*Sqrt[a + b*x^2 + c*x^4])/(8*d) - (e*(b^2 + 8*a*c + 2*b*c
*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^2) + ((8*c^2*d^2 + b^2*e^2 - 2*c*e*(
5*b*d - 4*a*e) - 2*c*e*(2*c*d - b*e)*x^2)*Sqrt[a + b*x^2 + c*x^4])/(16*c*d^
2*e) - (a + b*x^2 + c*x^4)^(3/2)/(2*d*x^2) - (3*Sqrt[a]*b*ArcTanh[(2*a + b*
x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*d) + (a^(3/2)*e*ArcTanh[(2*a
+ b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2) + (3*(b^2 + 4*a*c)*A
rcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(16*Sqrt[c]*d) +
 (b*(b^2 - 12*a*c)*e*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^
4])])/(32*c^(3/2)*d^2) - ((2*c*d - b*e)*(8*c^2*d^2 - b^2*e^2 - 4*c*e*(2*b*d
 - 3*a*e))*ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(32*
c^(3/2)*d^2*e^2) + ((c*d^2 - b*d*e + a*e^2)^(3/2)*ArcTanh[(b*d - 2*a*e + (2
*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/
(2*d^2*e^2)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 746

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 1))), x] - Di
st[p/(e*(m + 1)), Int[(d + e*x)^(m + 1)*(b + 2*c*x)*(a + b*x + c*x^2)^(p -
1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*
d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[p, 0] && (IntegerQ[p]
 || LtQ[m, -1]) && NeQ[m, -1] &&  !ILtQ[m + 2*p + 1, 0] && IntQuadraticQ[a,
 b, c, d, e, m, p, x]
```

Rule 748

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*((a + b*x + c*x^2)^p/(e*(m + 2*p + 1))), x
] - Dist[p/(e*(m + 2*p + 1)), Int[(d + e*x)^m*Simp[b*d - 2*a*e + (2*c*d - b
*e)*x, x]*(a + b*x + c*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e
, 0] && GtQ[p, 0] && NeQ[m + 2*p + 1, 0] && ( !RationalQ[m] || LtQ[m, 1]) &
&  !ILtQ[m + 2*p, 0] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 828

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2)
 - g*(c*d + 2*c*d*p - b*e*p) + g*c*e*(m + 2*p + 1)*x)*((a + b*x + c*x^2)^p/
(c*e^2*(m + 2*p + 1)*(m + 2*p + 2))), x] - Dist[p/(c*e^2*(m + 2*p + 1)*(m +
 2*p + 2)), Int[(d + e*x)^m*(a + b*x + c*x^2)^(p - 1)*Simp[c*e*f*(b*d - 2*a
*e)*(m + 2*p + 2) + g*(a*e*(b*e - 2*c*d*m + b*e*m) + b*d*(b*e*p - c*d - 2*c
*d*p)) + (c*e*f*(2*c*d - b*e)*(m + 2*p + 2) + g*(b^2*e^2*(p + m + 1) - 2*c^
```

```
2*d^2*(1 + 2*p) - c*e*(b*d*(m - 2*p) + 2*a*e*(m + 2*p + 1))))*x, x], x], x]
 /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] && GtQ[p, 0] && (IntegerQ[p] ||  !RationalQ[m] || (GeQ[
m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (IntegerQ[m] || IntegerQ[p]
|| IntegersQ[2*m, 2*p])
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(a+bx^2+cx^4)^{3/2}}{x^3(d+ex^2)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2(d+ex)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(a+bx+cx^2)^{3/2}}{dx^2} - \frac{e(a+bx+cx^2)^{3/2}}{d^2x} + \frac{e^2(a+bx+cx^2)^{3/2}}{d^2(d+ex)}\right) d$$

$$= \frac{\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x^2}\,dx, x, x^2\right)}{2d} - \frac{e\text{Subst}\left(\int \frac{(a+bx+cx^2)^{3/2}}{x}\,dx, x, x^2\right)}{2d^2} + \frac{e^2\text{Subs}}{}$$

$$= -\frac{(a+bx^2+cx^4)^{3/2}}{2dx^2} + \frac{3\text{Subst}\left(\int \frac{(b+2cx)\sqrt{a+bx+cx^2}}{x}\,dx, x, x^2\right)}{4d} + \frac{e\text{Subst}\left(\int\right.}{}$$

$$= \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{(8c^2d^2}{}$$

$$= \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{(8c^2d^2}{}$$

$$= \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{(8c^2d^2}{}$$

$$= \frac{3(3b+2cx^2)\sqrt{a+bx^2+cx^4}}{8d} - \frac{e(b^2+8ac+2bcx^2)\sqrt{a+bx^2+cx^4}}{16cd^2} + \frac{(8c^2d^2}{}$$

Mathematica [A]
time = 1.13, size = 230, normalized size = 0.41

$$\frac{-\frac{4(-cd^2+e(bd-ae))^{3/2}\tan^{-1}\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e^2} + 2\sqrt{a}(-3bd+2ae)\tanh^{-1}\left(\frac{-\sqrt{c}x^2+\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right) + \frac{d\left(2e(-ae+cdx^2)\sqrt{a+bx^2+cx^4}+\sqrt{c}d(2cd-3be)x^2\log\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{e^2x^2}}{4d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x]
```

```
[Out] ((-4*(-(c*d^2) + e*(b*d - a*e))^(3/2)*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[
a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/e^2 + 2*Sqrt[a]*(-3*b*
d + 2*a*e)*ArcTanh[(-(Sqrt[c]*x^2) + Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]] + (d
*(2*e*(-(a*e) + c*d*x^2)*Sqrt[a + b*x^2 + c*x^4] + Sqrt[c]*d*(2*c*d - 3*b*e
)*x^2*Log[b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4]]))/(e^2*x^2))/(4*
d^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1788 vs. 2(486) = 972.
time = 0.19, size = 1789, normalized size = 3.18 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/d^2*(3/4*c^(1/2)/e^3*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b*
d^2+1/6/e*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24/e*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/
16/e/c*b^2*(c*x^4+b*x^2+a)^(1/2)-1/2*c^(3/2)/e^4*ln((1/2*b+c*x^2)/c^(1/2)+(
c*x^4+b*x^2+a)^(1/2))*d^3-1/4/e^2*x^2*c*(c*x^4+b*x^2+a)^(1/2)*d-3/16/c^(1/2
)/e^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^2*d-5/8/e^2*b*(c*x^
4+b*x^2+a)^(1/2)*d+1/2/e^3*c*(c*x^4+b*x^2+a)^(1/2)*d^2+3/8/c^(1/2)/e*ln((1/
2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a*b+2/3/e*a*(c*x^4+b*x^2+a)^(1/2)
-1/32/c^(3/2)/e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*b^3-3/4*c^(
1/2)/e^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))*a*d-1/2/e/((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/
e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)
+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*a^2+1/e^2/((a*e^2-b*d*e+c*d^2)/
e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-
b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e
+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d*a*b-c/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*
ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2
)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2
)^(1/2))/(x^2+d/e))*a*d^2-1/2/e^3/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*
e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1
/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/
(x^2+d/e))*b^2*d^2+c/e^4/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e
+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x
^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e)
)*d^3*b-1/2*c^2/e^5/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^
2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/
e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))*d^4
)+1/d*(1/4*c*x^2*(c*x^4+b*x^2+a)^(1/2)+5/8*b*(c*x^4+b*x^2+a)^(1/2)+3/16*b^2
*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+3/4*a*c^(1/2)*ln((
1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))-1/2*a/x^2*(c*x^4+b*x^2+a)^(1/2)
-3/4*a^(1/2)*b*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2))-e/d^2*(
1/6*c*x^4*(c*x^4+b*x^2+a)^(1/2)+7/24*b*x^2*(c*x^4+b*x^2+a)^(1/2)+1/16/c*b^2
*(c*x^4+b*x^2+a)^(1/2)-1/32/c^(3/2)*b^3*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x
^2+a)^(1/2))+3/8*a*b*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2
)+2/3*a*(c*x^4+b*x^2+a)^(1/2)-1/2*a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*
x^2+a)^(1/2))/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="maxima")
```

```
[Out] integrate((c*x^4 + b*x^2 + a)^(3/2)/((x^2*e + d)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a + bx^2 + cx^4)^{\frac{3}{2}}}{x^3 (d + ex^2)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x**4+b*x**2+a)**(3/2)/x**3/(e*x**2+d),x)
```

```
[Out] Integral((a + b*x**2 + c*x**4)**(3/2)/(x**3*(d + e*x**2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*x^4+b*x^2+a)^(3/2)/x^3/(e*x^2+d),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}{x^3\,\left(e\,x^2+d\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)),x)
```

```
[Out] int((a + b*x^2 + c*x^4)^(3/2)/(x^3*(d + e*x^2)), x)
```

3.327 $\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

Optimal. Leaf size=463

$$-\frac{213}{140}x\sqrt{1+2x^2+2x^4}-\frac{27}{70}x^3\sqrt{1+2x^2+2x^4}-\frac{2211x\sqrt{1+2x^2+2x^4}}{140\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}-\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2}+\frac{17}{16}\sqrt{51}$$

```
[Out] -1/14*x*(2*x^4+2*x^2+1)^(3/2)+17/16*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^
(1/2))*51^(1/2)-213/140*x*(2*x^4+2*x^2+1)^(1/2)-27/70*x^3*(2*x^4+2*x^2+1)^(
1/2)-2211/280*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+2211/280*(cos
(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arc
tan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+
x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/32*(cos(2*arctan(2^
(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)
*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*(
(2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2^(1/2))/(2*x^4+2*x^2
+1)^(1/2)-3/280*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))
*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(514+2717*2^(1/2
))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2
^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.43, antiderivative size = 875, normalized size of antiderivative = 1.89, number of steps used = 19, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1349, 1105, 1190, 1211, 1117, 1209, 1222, 1230, 1720}

Antiderivative was successfully verified.

```
[In] Int[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-3*x*(2 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/35 - (3*x*(9 + 2*x^2)*Sqrt[1 + 2*x
^2 + 2*x^4])/20 - (309*x*Sqrt[1 + 2*x^2 + 2*x^4])/(20*Sqrt[2]*(1 + Sqrt[2]*
x^2)) - (6*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(35*(1 + Sqrt[2]*x^2)) - (x*(
1 + 2*x^2 + 2*x^4)^(3/2))/14 + (17*Sqrt[51]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/16 + (309*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
 Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(20*2^(3/
4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (6*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(35*Sqrt[1 + 2*x^2 + 2*x^4]) + (867*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
```

```
2 - Sqrt[2])/4])/(112*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (51*(5 + Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(16*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
- (3*(3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2
]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(70*2^(1/4)*Sqrt
[1 + 2*x^2 + 2*x^4]) - (3*(9 + 8*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])
/4])/(20*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt
[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*
Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(224*2^(1/4)*Sqrt[1 + 2
*x^2 + 2*x^4])
```

Rule 1105

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[x*((a + b*
x^2 + c*x^4)^p/(4*p + 1)), x] + Dist[2*(p/(4*p + 1)), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx = \int \left(-\frac{1}{2}(1+2x^2+2x^4)^{3/2} + \frac{3(1+2x^2+2x^4)^{3/2}}{2\,(3-2x^2)}\right) dx$$

$$= -\left(\frac{1}{2}\int \left(1+2x^2+2x^4\right)^{3/2}\,dx\right) + \frac{3}{2}\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$$

$$= -\frac{1}{14}x\left(1+2x^2+2x^4\right)^{3/2} - \frac{3}{14}\int \left(2+2x^2\right)\sqrt{1+2x^2+2x^4}\,dx - \frac{3}{8}\int (10+$$

$$= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4} - \frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} - \frac{1}{14}x(1+$$

$$= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4} - \frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} - \frac{1}{14}x(1+$$

$$= -\frac{3}{35}x\left(2+x^2\right)\sqrt{1+2x^2+2x^4} - \frac{3}{20}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4} - \frac{309x\sqrt{1}}{20\sqrt{2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.27, size = 214, normalized size = 0.46

$$\frac{-892x-2080x^3-2456x^5-752x^7-160x^9+4422i\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(9669-5247i)\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+10115(1-i)^{3/2}\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(-\frac{1}{3}-\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{560\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(1 + 2*x^2 + 2*x^4)^(3/2))/(3 - 2*x^2),x]
```

```
[Out] (-892*x - 2080*x^3 - 2456*x^5 - 752*x^7 - 160*x^9 + (4422*I)*Sqrt[1 - I]*Sq
rt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x
], I] - (9669 - 5247*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*
x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] + 10115*(1 - I)^(3/2)*Sqrt[1 +
(1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1
- I]*x], I])/(560*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 547, normalized size = 1.18

method	result
risch	$-\frac{x(20x^4+74x^2+223)\sqrt{2x^4+2x^2+1}}{140}+\frac{\left(\frac{2211}{280}-\frac{2211i}{280}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\text{EllipticF}\left(x\sqrt{-}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4}}$

elliptic	$-\frac{x^5\sqrt{2x^4+2x^2+1}}{7}-\frac{37x^3\sqrt{2x^4+2x^2+1}}{70}-\frac{223x\sqrt{2x^4+2x^2+1}}{140}-\frac{3729\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2}}{140\sqrt{-}}$
default	$-\frac{x^5\sqrt{2x^4+2x^2+1}}{7}-\frac{37x^3\sqrt{2x^4+2x^2+1}}{70}-\frac{223x\sqrt{2x^4+2x^2+1}}{140}-\frac{9\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(}}{35\sqrt{-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/7*x^5*(2*x^4+2*x^2+1)^(1/2)-37/70*x^3*(2*x^4+2*x^2+1)^(1/2)-223/140*x*(2
*x^4+2*x^2+1)^(1/2)-9/35/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)
)+(6/35-6/35*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-Ellipt
icE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-531/20/(-1+I)^(1/2)*(1+x^2-I
*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1
/2),1/2*2^(1/2)+1/2*I*2^(1/2))-309/40*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1
+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2
)+1/2*I*2^(1/2))-309/40/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2
)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
+309/40*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2
+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+289/8/(-1+I)^
(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipti
cPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^6 + 2*x^4 + x^2)*sqrt(2*x^4 + 2*x^2 + 1)/(2*x^2 - 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^6\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -Integral(x**2*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**6*sqrt(2*x**4 +
 2*x**2 + 1)/(2*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)*x^2/(2*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{x^2\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(x^2*(2*x^2 + 2*x^4 + 1)^(3/2))/(2*x^2 - 3),x)
```

```
[Out] -int((x^2*(2*x^2 + 2*x^4 + 1)^(3/2))/(2*x^2 - 3), x)
```

3.328 $\int \frac{(1+2x^2+2x^4)^{3/2}}{3-2x^2}\,dx$

Optimal. Leaf size=428

$$-\frac{1}{10}x(9+2x^2)\sqrt{1+2x^2+2x^4}-\frac{103x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}+\frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)+\frac{103\Big(1+$$

```
[Out] 17/24*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-1/10*x*(2*x^2+
9)*(2*x^4+2*x^2+1)^(1/2)-103/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1
/2))+103/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ell
ipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*
x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/48*
(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(
2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2+3*2^(1/
2))/(2*x^4+2*x^2+1)^(1/2)-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arc
tan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(
66+383*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2
^(1/4)/(2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.22, antiderivative size = 602, normalized size of antiderivative = 1.41, number of steps used = 12, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1222, 1190, 1211, 1117, 1209, 1230, 1720}

$$-\frac{\left(9+8\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{17\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{56\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{103\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{289\left(11-6\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{336\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{1}{10}(2x^2+9)\sqrt{2x^4+2x^2+1}\,x-\frac{103\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] -1/10*(x*(9 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]) - (103*x*Sqrt[1 + 2*x^2 + 2*x
^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)
/Sqrt[1 + 2*x^2 + 2*x^4]])/8 + (103*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x
^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(
10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*
Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x]
, (2 - Sqrt[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2]
)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF
[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
 - ((9 + 8*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[
```

```
1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x
^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[
2^(1/4)*x], (2 - Sqrt[2])/4])/(336*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1190

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(2*b*e*p + c*d*(4*p + 3) + c*e*(4*p + 1)*x^2)*((a + b*x^2 + c
*x^4)^p/(c*(4*p + 1)*(4*p + 3))), x] + Dist[2*(p/(c*(4*p + 1)*(4*p + 3))),
Int[Simp[2*a*c*d*(4*p + 3) - a*b*e + (2*a*c*e*(4*p + 1) + b*c*d*(4*p + 3) -
 b^2*e*(2*p + 1))*x^2, x]*(a + b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
GtQ[p, 0] && FractionQ[p] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{3-2x^2}\,dx &= -\left(\frac{1}{4}\int\left(10+4x^2\right)\sqrt{1+2x^2+2x^4}\,dx\right)+\frac{17}{2}\int\frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx\\ &= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{1}{120}\int\frac{192+216x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{17}{8}\int\frac{10+}{\sqrt{1+2x}}\\ &= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}+\frac{9\int\frac{1-\sqrt{2}\,x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{5\sqrt{2}}+\frac{17\int\frac{1-\sqrt{2}\,x^2}{\sqrt{1+2x^2+}}}{2\sqrt{2}}\\ &= -\frac{1}{10}x\left(9+2x^2\right)\sqrt{1+2x^2+2x^4}-\frac{103x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}+\frac{17}{8}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\right.\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.12, size = 209, normalized size = 0.49

$$\frac{-108x-240x^3-264x^5-48x^7+618i\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(1371-753i)\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+1445(1-i)^{3/2}\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(-\frac{1}{3}-\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{120\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(3 - 2*x^2),x]
```

```
[Out] (-108*x - 240*x^3 - 264*x^5 - 48*x^7 + (618*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)
*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1371
- 753*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[
I*ArcSinh[Sqrt[1 - I]*x], I] + 1445*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqr
t[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(12
0*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.17, size = 377, normalized size = 0.88

method	result
risch	$-\frac{x(2x^2+9)\sqrt{2x^4+2x^2+1}}{10}+\frac{\left(\frac{103}{20}-\frac{103i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+}}$
default	$-\frac{x^3\sqrt{2x^4+2x^2+1}}{5}-\frac{9x\sqrt{2x^4+2x^2+1}}{10}-\frac{177\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1}\right.}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{x^3\sqrt{2x^4+2x^2+1}}{5}-\frac{9x\sqrt{2x^4+2x^2+1}}{10}-\frac{177\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1}\right.}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/5*x^3*(2*x^4+2*x^2+1)^(1/2)-9/10*x*(2*x^4+2*x^2+1)^(1/2)-177/10/(-1+I)^(
1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Elliptic
F(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-103/20*I/(-1+I)^(1/2)*(1+x^2-I*
x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/
2),1/2*2^(1/2)+1/2*I*2^(1/2))-103/20/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^
2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1
/2*I*2^(1/2))+103/20*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+
289/12/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)
^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**2*sqrt
(2*x**4 + 2*x**2 + 1)/(2*x**2 - 3), x) - Integral(2*x**4*sqrt(2*x**4 + 2*x*
*2 + 1)/(2*x**2 - 3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^2 - 3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}{2\,x^2-3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(2*x^2 - 3),x)
```

```
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(2*x^2 - 3), x)
```

3.329 $\int \frac{\left(1+2x^2+2x^4\right)^{3/2}}{x^2\left(3-2x^2\right)}\,dx$

Optimal. Leaf size=722

$$-\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x}-\frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}+\frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}}{\sqrt{1+2x^2}}\right.$$

[Out] 17/36*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-1/3*(x^2+1)*(2*x^4+2*x^2+1)^(1/2)/x-5/2*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))+5/2*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/6*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/1008*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(11-6*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+289/168*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-17/24*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(5+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)

Rubi [A]
time = 0.23, antiderivative size = 722, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1325, 1285, 12, 1153, 1117, 1209, 1222, 1211, 1230, 1720}

Antiderivative was successfully verified.

[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]

[Out] -1/3*((1 + x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/x - (17*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/12 + (17*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S

```
qrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 +
 2*x^2 + 2*x^4]) + ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*2^(3/4)*Sqrt[1
 + 2*x^2 + 2*x^4]) + (289*(3 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(12*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*
Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*El
lipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(504*
2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1153

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
```

```
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1325

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[(f*x)^(m + 2)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
```

```
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^2(3-2x^2)}\,dx = -\left(\frac{1}{6}\int \frac{(-2+6x^2)\sqrt{1+2x^2+2x^4}}{x^2}\,dx\right)+\frac{17}{3}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx$$

$$= -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x}+\frac{1}{18}\int \frac{12x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{17}{12}\int \frac{10+4x^2}{\sqrt{1+2x^2+2x}}$$

$$= -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x}+\frac{2}{3}\int \frac{x^2}{\sqrt{1+2x^2+2x^4}}\,dx+\frac{17\int \frac{1-\sqrt{2}x^2}{\sqrt{1+2x^2+2x^4}}}{3\sqrt{2}}$$

$$= -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x}-\frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{17}{12}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{1+2}}\right.$$

$$= -\frac{(1+x^2)\sqrt{1+2x^2+2x^4}}{3x}-\frac{17x\sqrt{1+2x^2+2x^4}}{3\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{\sqrt{2}x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}x^2\right)}+$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 213, normalized size = 0.30

$$\frac{-12-36x^2-48x^4-24x^6+90i\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(255-165i)\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+289(1-i)^{3/2}x\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\Pi\left(-\frac{1}{3}-\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{36x\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^2*(3 - 2*x^2)),x]
```

```
[Out] (-12 - 36*x^2 - 48*x^4 - 24*x^6 + (90*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2
]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (255 - 165
*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*A
rcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(36*x
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 528, normalized size = 0.73

method	result
risch	$-\frac{2x^6+4x^4+3x^2+1}{3x\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{5}{2}-\frac{5i}{2}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\mathrm{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{x\sqrt{2x^4+2x^2+1}}{3}-\frac{35\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{-1+i}\right.}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{x\sqrt{2x^4+2x^2+1}}{3}+\frac{16\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\mathrm{EllipticF}\left(x\sqrt{-1+}\right.}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x-1/3*x*(2*x^4+2*x^2+1)^(1/2)+16/15/(-1+I)^(1/2)
*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*
(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-14/15+14/15*I)/(-1+I)^(1/2)*(1+(1
-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I
)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2
*I*2^(1/2)))-59/5/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x
^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-103/3
0*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-103/30/(-1+I)^(1/2)
*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*
(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+103/30*I/(-1+I)^(1/2)*(1+x^2-I*x^2)
^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1
/2*2^(1/2)+1/2*I*2^(1/2))+289/18/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*
x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I
)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^4 - 3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx-\int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^4-3x^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**2/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**4 - 3*x**2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^2/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}{x^2\,\left(2\,x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^2*(2*x^2 - 3)),x)
```

```
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^2*(2*x^2 - 3)), x)
```

3.330 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx$

Optimal. Leaf size=625

$$-\frac{2\sqrt{1+2x^2+2x^4}}{x}-\frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}\,x^2\right)}+\frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}\,x}{\sqrt{1+2x^2+\cdots}}\right.$$

```
[Out] 17/54*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)-2*(2*x^4+2*x^2
+1)^(1/2)/x-1/9*(-8*x^2+1)*(2*x^4+2*x^2+1)^(1/2)/x^3+1/9*x*(2*x^4+2*x^2+1)^
(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/9*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*
arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2)
)*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+
2*x^2+1)^(1/2)-289/1512*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(
1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/
2))^(1/2))*(11-6*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^
2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+289/252*(cos(2*arctan(2^(1/4)*x))^2)
^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2
^(1/2))^(1/2))*(3-2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))
^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-17/36*(cos(2*arctan(2^(1/4)*x))^2)^
(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^
(1/2))^(1/2))*(5+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^
2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)+1/9*2^(1/4)*(cos(2*arctan(2^(1/4)*x)
)^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*
(2-2^(1/2))^(1/2))*(9+5*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^
(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 625, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1323, 1285, 1295, 1211, 1117, 1209, 1222, 1230, 1720}

$$\frac{\sqrt[4]{2}\left(9+5\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{2x^4+2x^2+1}}-\frac{17\left(5+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{18\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{289\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{126\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{9\sqrt{2x^4+2x^2+1}}-\frac{289\left(11-6\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{756\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{9\left(\sqrt{2}x^2+1\right)}-\frac{2\sqrt{2x^4+2x^2+1}}{x}+\frac{17}{18}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{(1-8x^2)\sqrt{2x^4+2x^2+1}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

```
[Out] (-2*Sqrt[1 + 2*x^2 + 2*x^4])/x - ((1 - 8*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(9*x
^3) + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(9*(1 + Sqrt[2]*x^2)) + (17*Sqrt[
17/3]*ArcTanh[(Sqrt[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/18 - (2^(1/4)*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTa
n[2^(1/4)*x], (2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) + (289*(3 - Sqr
```

```
t[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) - (17*(5 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 +
Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(18*2^(1/4
)*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(1/4)*(9 + 5*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(9*Sqrt[1 + 2*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + S
qrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 +
11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(756*2^(1/4)*Sqrt[1
+ 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1222

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[-(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
```

```
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1323

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[(f*x)^(m + 4)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^4(3-2x^2)}\,dx = \frac{1}{9}\int \frac{(3+8x^2)\sqrt{1+2x^2+2x^4}}{x^4}\,dx + \frac{34}{9}\int \frac{\sqrt{1+2x^2+2x^4}}{3-2x^2}\,dx$$

$$= -\frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{1}{27}\int \frac{-54-60x^2}{x^2\sqrt{1+2x^2+2x^4}}\,dx - \frac{17}{18}\int \frac{10+4x}{\sqrt{1+2x^2}}$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{1}{27}\int \frac{60+108x^2}{\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} - \frac{17\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}\,x^2\right)} +$$

$$= -\frac{2\sqrt{1+2x^2+2x^4}}{x} - \frac{(1-8x^2)\sqrt{1+2x^2+2x^4}}{9x^3} + \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{9\left(1+\sqrt{2}\,x^2\right)} + \frac{1}{1}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.17, size = 219, normalized size = 0.35

$$\frac{-6-72x^2-132x^4-120x^6-6i\sqrt{1-i}\,x^3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(195-201i)\sqrt{1-i}\,x^3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+289(1-i)^{3/2}x^3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(-\frac{1}{3}-\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{54x^3\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^4*(3 - 2*x^2)),x]
```

```
[Out] (-6 - 72*x^2 - 132*x^4 - 120*x^6 - (6*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x
^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (195 - 2
01*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF
[I*ArcSinh[Sqrt[1 - I]*x], I] + 289*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]
*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*ArcSinh[Sqrt[1 - I]*x], I])
/(54*x^3*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 530, normalized size = 0.85

method	result

risch	$-\frac{20x^6+22x^4+12x^2+1}{9x^3\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{9}+\frac{i}{9}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}-\frac{10\sqrt{2x^4+2x^2+1}}{9x}-\frac{22\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1+\right.}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$-\frac{10\sqrt{2x^4+2x^2+1}}{9x}+\frac{44\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -10/9*(2*x^4+2*x^2+1)^(1/2)/x+44/15/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+
I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/
2*I*2^(1/2))-103/45*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/
(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1
/9*(2*x^4+2*x^2+1)^(1/2)/x^3+(-12/5+12/5*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2
)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-
118/15/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)
^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+103/45*I/(-1+I)^
(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipti
cE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-103/45/(-1+I)^(1/2)*(1+x^2-I*x
^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2
),1/2*2^(1/2)+1/2*I*2^(1/2))+289/27/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2
+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-
1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^6 - 3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx - \int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx - \int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^6-3x^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**4/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**6 - 3*x**4), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^4/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}{x^4\,\left(2\,x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^4*(2*x^2 - 3)),x)
```

```
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^4*(2*x^2 - 3)), x)
```

3.331 $\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)} \, dx$

Optimal. Leaf size=553

$$\frac{74\sqrt{1+2x^2+2x^4}}{135x^3}-\frac{262\sqrt{1+2x^2+2x^4}}{135x}-\frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5}+\frac{262\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{135\left(1+\sqrt{2}\,x^2\right)}+\frac{17}{27}\sqrt{}$$

```
[Out] 17/81*arctanh(1/3*x*51^(1/2)/(2*x^4+2*x^2+1)^(1/2))*51^(1/2)+74/135*(2*x^4+
2*x^2+1)^(1/2)/x^3-262/135*(2*x^4+2*x^2+1)^(1/2)/x-1/45*(40*x^2+3)*(2*x^4+2
*x^2+1)^(1/2)/x^5+262/135*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-2
62/135*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Elliptic
E(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2
*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-289/2268*(co
s(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*a
rctan(2^(1/4)*x)),1/2+11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(11-6*2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x
^2+1)^(1/2)+85/189*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*
x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3-2^(1/2))*(
1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x
^2+1)^(1/2)+1/135*2^(3/4)*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2
^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(37+23
*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+
2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 553, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1323, 1285, 1295, 1211, 1117, 1209, 1325, 1230, 1720}

$$\frac{2^{3/4}\left(37+23\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}+\frac{85\,2^{3/4}\left(3-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt{2x^4+2x^2+1}}-\frac{262\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{135\sqrt{2x^4+2x^2+1}}-\frac{289\left(11-6\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12+11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{1134\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{262\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{135\left(\sqrt{2}x^2+1\right)}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}+\frac{17}{27}\sqrt{\frac{17}{3}}\tanh^{-1}\left(\frac{\sqrt{\frac{17}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{(40x^2+3)\sqrt{2x^4+2x^2+1}}{45x^5}+\frac{74\sqrt{2x^4+2x^2+1}}{135x^3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

```
[Out] (74*Sqrt[1 + 2*x^2 + 2*x^4])/(135*x^3) - (262*Sqrt[1 + 2*x^2 + 2*x^4])/(135
*x) - ((3 + 40*x^2)*Sqrt[1 + 2*x^2 + 2*x^4])/(45*x^5) + (262*Sqrt[2]*x*Sqrt
[1 + 2*x^2 + 2*x^4])/(135*(1 + Sqrt[2]*x^2)) + (17*Sqrt[17/3]*ArcTanh[(Sqrt
[17/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/27 - (262*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqr
t[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (
2 - Sqrt[2])/4])/(135*Sqrt[1 + 2*x^2 + 2*x^4]) + (85*2^(3/4)*(3 - Sqrt[2])*
(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2
```

```
*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(189*Sqrt[1 + 2*x^2 + 2*x^4]) + (2^(3
/4)*(37 + 23*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[
2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(135*Sqrt[1 + 2
*x^2 + 2*x^4]) - (289*(11 - 6*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 + 11*Sqrt[2])/24, 2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(1134*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1285

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^p*((d*(m
+ 4*p + 3) + e*(m + 1)*x^2)/(f*(m + 1)*(m + 4*p + 3))), x] + Dist[2*(p/(f^2
*(m + 1)*(m + 4*p + 3))), Int[(f*x)^(m + 2)*(a + b*x^2 + c*x^4)^(p - 1)*Sim
p[2*a*e*(m + 1) - b*d*(m + 4*p + 3) + (b*e*(m + 1) - 2*c*d*(m + 4*p + 3))*x
```

```
^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && Gt
Q[p, 0] && LtQ[m, -1] && m + 4*p + 3 != 0 && IntegerQ[2*p] && (IntegerQ[p]
|| IntegerQ[m])
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1323

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/d^2, Int[(f*x)^m*(a*d + (b*d - a*e)*x^
2)*(a + b*x^2 + c*x^4)^(p - 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/(d^2*
f^4), Int[(f*x)^(m + 4)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /
; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m
, -2]
```

Rule 1325

```
Int[(((f_.)*(x_))^(m_)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[1/(d*e), Int[(f*x)^m*(a*e + c*d*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] - Dist[(c*d^2 - b*d*e + a*e^2)/(d*e*f^2),
Int[(f*x)^(m + 2)*((a + b*x^2 + c*x^4)^(p - 1)/(d + e*x^2)), x], x] /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && GtQ[p, 0] && LtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{(1+2x^2+2x^4)^{3/2}}{x^6(3-2x^2)}\,dx = \frac{1}{9}\int \frac{(3+8x^2)\sqrt{1+2x^2+2x^4}}{x^6}\,dx + \frac{34}{9}\int \frac{\sqrt{1+2x^2+2x^4}}{x^2(3-2x^2)}\,dx$$

$$= -\frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{1}{45}\int \frac{-74-68x^2}{x^4\sqrt{1+2x^2+2x^4}}\,dx - \frac{17}{27}\int \frac{-2+}{x^2\sqrt{1+2}}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{34\sqrt{1+2x^2+2x^4}}{27x} - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} - \frac{1}{135}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{17}{27}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{34}{}$$

$$= \frac{74\sqrt{1+2x^2+2x^4}}{135x^3} - \frac{262\sqrt{1+2x^2+2x^4}}{135x} - \frac{(3+40x^2)\sqrt{1+2x^2+2x^4}}{45x^5} + \frac{262}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 224, normalized size = 0.41

$$-\frac{27+192x^2+1116x^4+1848x^6+1572x^8+786i\sqrt{1-i}\,x^5\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+(543-1329i)\sqrt{1-i}\,x^5\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-1445(1-i)^{3/2}x^5\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(-\frac{1}{3}-\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{405x^5\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + 2*x^2 + 2*x^4)^(3/2)/(x^6*(3 - 2*x^2)),x]
```

```
[Out] -1/405*(27 + 192*x^2 + 1116*x^4 + 1848*x^6 + 1572*x^8 + (786*I)*Sqrt[1 - I]
*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1
 - I]*x], I] + (543 - 1329*I)*Sqrt[1 - I]*x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1
+ (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 1445*(1 - I)^(3/2)*
x^5*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[-1/3 - I/3, I*Ar
cSinh[Sqrt[1 - I]*x], I])/(x^5*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 549, normalized size = 0.99

method	result
risch	$-\frac{524x^8+616x^6+372x^4+64x^2+9}{135x^5\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{262}{135}+\frac{262i}{135}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{46\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}-\frac{208\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x}}{45\sqrt{-1+}}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{15x^5}-\frac{46\sqrt{2x^4+2x^2+1}}{135x^3}-\frac{262\sqrt{2x^4+2x^2+1}}{135x}+\frac{184\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1}}{45\sqrt{-1+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x,method=_RETURNVERBOSE)
```

```
[Out] -1/15*(2*x^4+2*x^2+1)^(1/2)/x^5-46/135*(2*x^4+2*x^2+1)^(1/2)/x^3-262/135*(2
*x^4+2*x^2+1)^(1/2)/x+184/45/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(
1/2))-206/135*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-52/15
+52/15*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2
+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*
(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+206/135*I/(-1+I)^(1/2)*(1+x^2-I*x^
2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2)
,1/2*2^(1/2)+1/2*I*2^(1/2))-236/45/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+
I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2
*I*2^(1/2))-206/135/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+578
/81/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticPi(x*(-1+I)^(1/2),-1/3-1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="maxima")
```

```
[Out] -integrate((2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="fricas")
```

```
[Out] integral(-(2*x^4 + 2*x^2 + 1)^(3/2)/(2*x^8 - 3*x^6), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx - \int \frac{2x^2\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx - \int \frac{2x^4\sqrt{2x^4+2x^2+1}}{2x^8-3x^6}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x**4+2*x**2+1)**(3/2)/x**6/(-2*x**2+3),x)
```

```
[Out] -Integral(sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**2
*sqrt(2*x**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x) - Integral(2*x**4*sqrt(2*x
**4 + 2*x**2 + 1)/(2*x**8 - 3*x**6), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((2*x^4+2*x^2+1)^(3/2)/x^6/(-2*x^2+3),x, algorithm="giac")
```

```
[Out] integrate(-(2*x^4 + 2*x^2 + 1)^(3/2)/((2*x^2 - 3)*x^6), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$-\int \frac{{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}{x^6\,\left(2\,x^2-3\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(2*x^2 + 2*x^4 + 1)^(3/2)/(x^6*(2*x^2 - 3)),x)
```

```
[Out] -int((2*x^2 + 2*x^4 + 1)^(3/2)/(x^6*(2*x^2 - 3)), x)
```

3.332 $\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=173

$$\frac{\sqrt{a+bx^2+cx^4}}{2ce}-\frac{(2cd+be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+}}\right.}{2e^2\sqrt{cd^2-bde+ae^2}}$$

[Out] -1/4*(b*e+2*c*d)*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e^2+1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e^2/(a*e^2-b*d*e+c*d^2)^(1/2)+1/2*(c*x^4+b*x^2+a)^(1/2)/c/e

Rubi [A]
time = 0.21, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 1667, 857, 635, 212, 738}

$$-\frac{(be+2cd)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2}+\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e^2\sqrt{ae^2-bde+cd^2}}+\frac{\sqrt{a+bx^2+cx^4}}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] Sqrt[a + b*x^2 + c*x^4]/(2*c*e) - ((2*c*d + b*e)*ArcTanh[(b + 2*c*x^2)/(2*S
qrt[c]*Sqrt[a + b*x^2 + c*x^4])])/(4*c^(3/2)*e^2) + (d^2*ArcTanh[(b*d - 2*a
*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x
^4])])/(2*e^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1667

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{q = Expon[Pq, x], f = Coeff[Pq, x, Expon[Pq, x]]}, S
imp[f*(d + e*x)^(m + q - 1)*((a + b*x + c*x^2)^(p + 1)/(c*e^(q - 1)*(m + q
+ 2*p + 1))), x] + Dist[1/(c*e^q*(m + q + 2*p + 1)), Int[(d + e*x)^m*(a + b
*x + c*x^2)^p*ExpandToSum[c*e^q*(m + q + 2*p + 1)*Pq - c*f*(m + q + 2*p + 1
)*(d + e*x)^q - f*(d + e*x)^(q - 2)*(b*d*e*(p + 1) + a*e^2*(m + q - 1) - c*
d^2*(m + q + 2*p + 1) - e*(2*c*d - b*e)*(m + q + p)*x), x], x], x] /; GtQ[q
, 1] && NeQ[m + q + 2*p + 1, 0]] /; FreeQ[{a, b, c, d, e, m, p}, x] && Poly
Q[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !(IGtQ
[m, 0] && RationalQ[a, b, c, d, e] && (IntegerQ[p] || ILtQ[p + 1/2, 0]))
```

Rubi steps

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{\text{Subst}\left(\int \frac{-\frac{1}{2}bde-\frac{1}{2}e(2cd+be)x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2ce^2}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} + \frac{d^2\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2e^2} - \frac{(2cd$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{d^2\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right.}{e^2}$$

$$= \frac{\sqrt{a+bx^2+cx^4}}{2ce} - \frac{(2cd+be)\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{4c^{3/2}e^2} + \frac{d^2\tan}{}$$

Mathematica [A]
time = 0.66, size = 178, normalized size = 1.03

$$\frac{\frac{2e\sqrt{a+bx^2+cx^4}}{c} + \frac{4d^2\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)} + \frac{(2cd+be)\log\left(c\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{c^{3/2}}}{4e^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ((2*e*Sqrt[a + b*x^2 + c*x^4])/c + (4*d^2*Sqrt[-(c*d^2) + b*d*e - a*e^2]*Ar
cTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b
*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e)) + ((2*c*d + b*e)*Log[c*(b + 2*c*x^2
 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])])/c^(3/2))/(4*e^2)
```

Maple [A]
time = 0.16, size = 266, normalized size = 1.54

method	result
default	$\frac{\frac{\sqrt{c\,x^4+b\,x^2+a}}{2c}-\frac{b\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{4c^{\frac{3}{2}}}}{e} - \frac{d\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e^2\sqrt{c}} - \frac{d^2\ln\left(\frac{2a\,e^2-2d}{e}\right.}{}$

risch	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ce}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)b}{4c^{\frac{3}{2}}e}-\frac{d\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e^2\sqrt{c}}-\frac{d^2\ln\left(\frac{2a\,e^2-2c}{e}\right.}{}$
elliptic	$\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ce}-\frac{\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)b}{4c^{\frac{3}{2}}e}-\frac{d\ln\left(\frac{\frac{b}{2}+c\,x^2}{\sqrt{c}}+\sqrt{c\,x^4+b\,x^2+a}\right)}{2e^2\sqrt{c}}-\frac{d^2\ln\left(\frac{2a\,e^2-2c}{e}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(1/2*(c*x^4+b*x^2+a)^(1/2)/c-1/4*b/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*
x^4+b*x^2+a)^(1/2)))-1/2*d/e^2*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/
2))/c^(1/2)-1/2/e^3*d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+
c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^
2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^5/(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 318 vs. 2(149) = 298.
time = 12.61, size = 1364, normalized size = 7.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*c^2*d^2*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2
*x^2 + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*
x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2
+ 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^
```

```
2 + 2*d*x^2*e + d^2)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*
e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(
2*c*x^2 + b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*(c^2*d^2*e - b*c*
d*e^2 + a*c*e^3))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/8*(4*sqrt(-c*d
^2 + b*d*e - a*e^2)*c^2*d^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2
+ b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^
2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 +
a*b*d)*e)) + (2*c^2*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(c
)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b
)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*(c^2*d^2*e - b*c*d*e^2 + a*c
*e^3))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*e^4), 1/4*(sqrt(c*d^2 - b*d*e + a
*e^2)*c^2*d^2*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2 + 4*s
qrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d
*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4
+ (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2)) + (2*c^2
*d^3 - b*c*d^2*e + a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(
c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sq
rt(c*x^4 + b*x^2 + a)*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3))/(c^3*d^2*e^2 - b*c
^2*d*e^3 + a*c^2*e^4), 1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c^2*d^2*arctan(-
1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2
 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2
 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)) + (2*c^2*d^3 - b*c*d^2*e
+ a*b*e^3 - (b^2 - 2*a*c)*d*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a
)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) + 2*sqrt(c*x^4 + b*x^2
+ a)*(c^2*d^2*e - b*c*d*e^2 + a*c*e^3))/(c^3*d^2*e^2 - b*c^2*d*e^3 + a*c^2*
e^4)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**5/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.333 $\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=137

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,e}-\frac{d\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2e\sqrt{cd^2-bde+ae^2}}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/e/c^(1/2)-1/2*d*
arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b
*x^2+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 857, 635, 212, 738}

$$\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,e}-\frac{d\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2e\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*Sqrt[c]*e) -
(d*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]
*Sqrt[a + b*x^2 + c*x^4])])/(2*e*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)\\
&= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2e} - \frac{d\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,d\right.}{2e}\\
&= \frac{\text{Subst}\left(\int \frac{1}{4c-x^2}\,dx,x,\frac{b+2cx^2}{\sqrt{a+bx^2+cx^4}}\right)}{e} + \frac{d\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx\right.}{e}\\
&= \frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{c}\,e} - \frac{d\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+}}\right.}{2e\sqrt{cd^2-bde+ae^2}}
\end{aligned}$$

Mathematica [A]
time = 0.44, size = 146, normalized size = 1.07

$$-\frac{\frac{2d\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{\log\left(e\left(b+2cx^2-2\sqrt{c}\,\sqrt{a+bx^2+cx^4}\right)\right)}{\sqrt{c}}}{2e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*((2*d*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*S
qrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d)
 + a*e)) + Log[e*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/Sqrt[c]
)/e
```

Maple [A]
time = 0.13, size = 204, normalized size = 1.49

method	result
default	$\frac{\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{2e\sqrt{c}}+\frac{d\ln\left(\frac{\frac{2a e^2-2deb+2c d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\cdots}}{x^2+\frac{d}{e}}\right.}{2e^2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}}$
elliptic	$\frac{\ln\left(\frac{\frac{b}{2}+c x^2}{\sqrt{c}}+\sqrt{c x^4+b x^2+a}\right)}{2e\sqrt{c}}+\frac{d\ln\left(\frac{\frac{2a e^2-2deb+2c d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\cdots}}{x^2+\frac{d}{e}}\right.}{2e^2\sqrt{\frac{a e^2-deb+c d^2}{e^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/e*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2))/c^(1/2)+1/2*d/e^2/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(117) = 234$.
time = 1.03, size = 1100, normalized size = 8.03

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*c*d*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 +
 (b^2 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 +
2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^
2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*
d*x^2*e + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^
2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c) - 4*a*c))/(c^2*d^
2*e - b*c*d*e^2 + a*c*e^3), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan
(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d
^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x
^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)) - (c*d^2 - b*d*e + a*e^
2)*sqrt(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(c) - 4*a*c))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3), 1/4*(sqrt(c
*d^2 - b*d*e + a*e^2)*c*d*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*
c)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt
(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(
4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^
2)) - 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)
*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)))/(c^2*d^2*e - b*c*d*e^2
+ a*c*e^3), -1/2*(sqrt(-c*d^2 + b*d*e - a*e^2)*c*d*arctan(-1/2*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2
)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b
*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)) + (c*d^2 - b*d*e + a*e^2)*sqrt(-c)*arctan
(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*
c)))/(c^2*d^2*e - b*c*d*e^2 + a*c*e^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(x**3/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(e\,x^2+d)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.334 $\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=86

$$\frac{\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^2-bde+ae^2}}$$

```
[Out] 1/2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x
^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1261, 738, 212}

$$\frac{\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2\sqrt{ae^2-bde+cd^2}}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sq
rt[a + b*x^2 + c*x^4])]/(2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{x}{(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)\\ &= -\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-x^2}\,dx, x, \frac{-bd+2ae-(2cd-be)x^2}{\sqrt{a+bx^2+cx^4}}\right)\\ &= -\frac{\tanh^{-1}\left(\frac{-bd+2ae-(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{cd^2-bde+ae^2}}\end{aligned}$$

Mathematica [A]
time = 0.30, size = 96, normalized size = 1.12

$$\frac{\sqrt{-cd^2+e(bd-ae)}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
[Out] (Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*
x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(76) = 152.
time = 0.12, size = 165, normalized size = 1.92

method	result
default	$-\frac{\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2e\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}$
elliptic	$-\frac{\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2e\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/e/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2
*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c
*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x/(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(78) = 156$.
time = 0.41, size = 367, normalized size = 4.27

$$\left[\frac{\log\left(-\frac{8\,c^2d^2x^4+8\,bcd^2x^2+(b^2+4\,ac)d^2+4\,\sqrt{cx^4+bx^2+a}\,(2\,cdx^2+bd-(bx^2+2\,a)e)\sqrt{cd^2-bde+ae^2}+((b^2+4\,ac)x^4+8\,abx^2+8\,a^2)e^2-2\,(4\,bcdx^4+(3\,b^2+4\,ac)dx^2+4\,abd)e}{x^4e^2+2\,dx^2e+d^2}\right)}{4\,\sqrt{cd^2-bde+ae^2}}, \frac{\sqrt{-cd^2+bde-ae^2}\,\arctan\left(-\frac{\sqrt{cx^4+bx^2+a}\,(2\,cdx^2+bd-(bx^2+2\,a)e)\sqrt{-cd^2+bde-ae^2}}{2\,(c^2d^2x^4+bcd^2x^2+acd^2+(acx^4+abx^2+a^2)e^2-(bcdx^4+b^2dx^2+abd)e)}\right)}{2\,(cd^2-bde+ae^2)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4
 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^
2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2
+ 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2))/sqrt(c*d^2 - b*d*
e + a*e^2), 1/2*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2
 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2
*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x
^4 + b^2*d*x^2 + a*b*d)*e))/(c*d^2 - b*d*e + a*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(d+ex^2)\,\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

[Out] Integral(x/((d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)

Giac [A]
time = 4.19, size = 75, normalized size = 0.87

$$\frac{\arctan\left(-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 + b*d*e - a*e^2))/sqrt(-c*d^2 + b*d*e - a*e^2)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.335 $\int \frac{1}{x(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=138

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d}-\frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2d\sqrt{cd^2-bde+ae^2}}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d/a^(1/2)-1/2*e
*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+
b*x^2+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1265, 974, 738, 212}

$$-\frac{e\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\sqrt{ae^2-bde+cd^2}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])]/(Sqrt[a]*d)
 - (e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e
^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
```

```
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx\sqrt{a+bx+cx^2}} - \frac{e}{d(d+ex)\sqrt{a+bx+cx^2}}\right)\,dx, x, \cdots$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx, x, x^2\right)}{2d} - \frac{e\text{Subst}\left(\int \frac{1}{(d+ex)\sqrt{a+bx+c\cdots}}\cdots\right.}{2d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{4a-x^2}\,dx, x, \frac{2a+bx^2}{\sqrt{a+bx^2+cx^4}}\right)}{d} + \frac{e\text{Subst}\left(\int \frac{1}{4cd^2-4bde+4ae^2-\cdots}\cdots\right.}{\cdots}$$

$$= -\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d} - \frac{e\tanh^{-1}\left(\frac{bd-2ae+(2cd\cdots}{2\sqrt{cd^2-bde+ae^2}\cdots}\right.}{2d\sqrt{cd^2-bde+\cdots}}$$

Mathematica [A]
time = 0.48, size = 144, normalized size = 1.04

$$\frac{-\frac{e\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\left(d+ex^2\right)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{cd^2+e(-bd+ae)}+\frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{\sqrt{a}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] (-((e*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a
 + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*
e))) + ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/Sqrt[a])/d
```

Maple [A]
time = 0.13, size = 207, normalized size = 1.50

method	result
default	$\frac{\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2d\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}} - \ln$
elliptic	$\frac{\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2d\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}} - \ln$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*
d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-1/2/d/a^(1/2)*ln(
(2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(121) = 242$.
time = 0.53, size = 1121, normalized size = 8.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 +
 (b^2 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 +
```

```
2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^
2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*
d*x^2*e + d^2)) + (c*d^2 - b*d*e + a*e^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 +
 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))
/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2), -1/4*(2*sqrt(-c*d^2 + b*d*e - a*e^2)*a*
arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqr
t(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 +
 a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e))*e - (c*d^2 - b*d*
e + a*e^2)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x
^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4))/(a*c*d^3 - a*b*d^2*e + a^2*d*e
^2), 1/4*(sqrt(c*d^2 - b*d*e + a*e^2)*a*e*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x
^2 + (b^2 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^
2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 +
8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2
+ 2*d*x^2*e + d^2)) + 2*(c*d^2 - b*d*e + a*e^2)*sqrt(-a)*arctan(1/2*sqrt(c*
x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)))/(a*c*d^
3 - a*b*d^2*e + a^2*d*e^2), -1/2*(sqrt(-c*d^2 + b*d*e - a*e^2)*a*arctan(-1/
2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 +
 b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 +
 a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e))*e - (c*d^2 - b*d*e + a*e^2)
*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^
4 + a*b*x^2 + a^2)))/(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{x\,\left(e\,x^2+d\right)\,\sqrt{c\,x^4+b\,x^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.336 $\int \frac{1}{x^3(d+ex^2)\sqrt{a+bx^2+cx^4}}\,dx$

Optimal. Leaf size=218

$$-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}+\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}+\frac{e^2\tanh^{-1}\left(\cdots\right.}{\cdots}$$

```
[Out] 1/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d+1/2*
e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/a^(1/2)+1/2*e^
2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(1/2)-1/2*(c*x^4+b*x^2+a)^(1/2)/a/
d/x^2
```

Rubi [A]
time = 0.18, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 974, 744, 738, 212}

$$\frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d}+\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2\sqrt{ae^2-bde+cd^2}}+\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2}-\frac{\sqrt{a+bx^2+cx^4}}{2adx^2}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*Sqrt[a + b*x^2 + c*x^4]/(a*d*x^2) + (b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a
]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(3/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*Sq
rt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*Sqrt[a]*d^2) + (e^2*ArcTanh[(b*d - 2*a*
e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^
4])])/(2*d^2*Sqrt[c*d^2 - b*d*e + a*e^2])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 744

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[e*(d + e*x)^(m + 1)*((a + b*x + c*x^2)^(p + 1)/((m + 1)*(c*d
^2 - b*d*e + a*e^2))), x] + Dist[(2*c*d - b*e)/(2*(c*d^2 - b*d*e + a*e^2)),
 Int[(d + e*x)^(m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e,
 m, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2
*c*d - b*e, 0] && EqQ[m + 2*p + 3, 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\sqrt{a+bx^2+cx^4}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx^2\sqrt{a+bx+cx^2}} - \frac{e}{d^2x\sqrt{a+bx+cx^2}} + \frac{}{d^2(d+ex)}\right.\right.$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{2d} - \frac{e\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,d\right.}{2d^2}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2} - \frac{b\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx+cx^2}}\,dx,x,x^2\right)}{4ad} + \frac{e\text{Subst}}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2} + \frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2\sqrt{a}\,d^2} + \frac{e^2\tanh^{-1}}{}$$

$$= -\frac{\sqrt{a+bx^2+cx^4}}{2adx^2} + \frac{b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{3/2}d} + \frac{e\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.88, size = 163, normalized size = 0.75

$$-\frac{\frac{d\sqrt{a+bx^2+cx^4}}{ax^2}+\frac{2e^2\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+bde-ae^2}}\right)}{\sqrt{-cd^2+bde-ae^2}}+\frac{(bd+2ae)\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{3/2}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]),x]
```

```
[Out] -1/2*((d*Sqrt[a + b*x^2 + c*x^4])/(a*x^2) + (2*e^2*ArcTan[(Sqrt[c]*(d + e*x
^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + b*d*e - a*e^2]])/Sqrt[-(c*
d^2) + b*d*e - a*e^2] + ((b*d + 2*a*e)*ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^
2 + c*x^4])/Sqrt[a]])/a^(3/2))/d^2
```

Maple [A]
time = 0.16, size = 275, normalized size = 1.26

method	result
default	$-\frac{e\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{2d^2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}$
risch	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ad\,x^2}+\frac{e\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2d^2\sqrt{a}}+\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)b}{4d\,a^{\frac{3}{2}}}$
elliptic	$-\frac{\sqrt{c\,x^4+b\,x^2+a}}{2ad\,x^2}+\frac{e\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)}{2d^2\sqrt{a}}+\frac{\ln\left(\frac{2a+b\,x^2+2\sqrt{a}\,\sqrt{c\,x^4+b\,x^2+a}}{x^2}\right)b}{4d\,a^{\frac{3}{2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e/d^2/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b
*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e
-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))+1/d*(-1/2*(c
```

```
*x^4+b*x^2+a)^(1/2)/a/x^2+1/4*b/a^(3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^
2+a)^(1/2))/x^2))+1/2*e/d^2/a^(1/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)
^(1/2))/x^2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(c*x^4 + b*x^2 + a)*(x^2*e + d)*x^3), x)
```

Fricas [A]
time = 0.79, size = 1486, normalized size = 6.82

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*sqrt(c*d^2 - b*d*e + a*e^2)*a^2*x^2*e^2*log(-(8*c^2*d^2*x^4 + 8*b*c
*d^2*x^2 + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d -
 (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*
x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^
4*e^2 + 2*d*x^2*e + d^2)) + (b*c*d^3*x^2 - a*b*d*x^2*e^2 - (b^2 - 2*a*c)*d^
2*x^2*e + 2*a^2*x^2*e^3)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 4*sq
rt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*c*d^3 - a*
b*d^2*e + a^2*d*e^2)*sqrt(c*x^4 + b*x^2 + a))/(a^2*c*d^4*x^2 - a^2*b*d^3*x^
2*e + a^3*d^2*x^2*e^2), 1/8*(4*sqrt(-c*d^2 + b*d*e - a*e^2)*a^2*x^2*arctan(
-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^
2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^
2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e))*e^2 + (b*c*d^3*x^2 - a*b
*d*x^2*e^2 - (b^2 - 2*a*c)*d^2*x^2*e + 2*a^2*x^2*e^3)*sqrt(a)*log(-((b^2 +
4*a*c)*x^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) +
8*a^2)/x^4) - 4*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^4 + b*x^2 + a))/
(a^2*c*d^4*x^2 - a^2*b*d^3*x^2*e + a^3*d^2*x^2*e^2), 1/4*(sqrt(c*d^2 - b*d*
e + a*e^2)*a^2*x^2*e^2*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*
d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*
d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b
*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2))
 - (b*c*d^3*x^2 - a*b*d*x^2*e^2 - (b^2 - 2*a*c)*d^2*x^2*e + 2*a^2*x^2*e^3)*
sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4
 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqrt(c*x^4 + b*x^2
 + a))/(a^2*c*d^4*x^2 - a^2*b*d^3*x^2*e + a^3*d^2*x^2*e^2), 1/4*(2*sqrt(-c*
```

```
d^2 + b*d*e - a*e^2)*a^2*x^2*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2
 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d
^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 +
 a*b*d)*e))*e^2 - (b*c*d^3*x^2 - a*b*d*x^2*e^2 - (b^2 - 2*a*c)*d^2*x^2*e +
2*a^2*x^2*e^3)*sqrt(-a)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sq
rt(-a)/(a*c*x^4 + a*b*x^2 + a^2)) - 2*(a*c*d^3 - a*b*d^2*e + a^2*d*e^2)*sqr
t(c*x^4 + b*x^2 + a))/(a^2*c*d^4*x^2 - a^2*b*d^3*x^2*e + a^3*d^2*x^2*e^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (d + ex^2) \sqrt{a + bx^2 + cx^4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(1/2),x)
```

```
[Out] Integral(1/(x**3*(d + e*x**2)*sqrt(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 3.46, size = 208, normalized size = 0.95

$$\frac{\arctan\left(-\frac{\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)e + \sqrt{c}\,d}{\sqrt{-cd^2 + bde - ae^2}}\right)e^2}{\sqrt{-cd^2 + bde - ae^2}\,d^2} - \frac{(bd + 2\,ae)\arctan\left(-\frac{\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{2\sqrt{-a}\,ad^2} + \frac{\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)b + 2\,a\sqrt{c}}{2\left(\left(\sqrt{c}\,x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2
 + b*d*e - a*e^2))*e^2/(sqrt(-c*d^2 + b*d*e - a*e^2)*d^2) - 1/2*(b*d + 2*a*
e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a*d^
2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqrt(
c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a*d)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(e\,x^2 + d\right)\,\sqrt{c\,x^4 + b\,x^2 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)),x)
```

```
[Out] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(1/2)), x)
```

3.337 $\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=418

$$\frac{x\sqrt{1+2x^2+2x^4}}{2\sqrt{2}\left(1+\sqrt{2}x^2\right)}-\frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{4\left(2-3\sqrt{2}\right)}-\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}E\left(2\text{t}\right.}{2\,2^{3/4}\sqrt{1+2x^2}}$$

```
[Out] -3/40*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*30^(1/2)*(3-2^(1/2))/(2-
3*2^(1/2))+1/4*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/4*(cos(2*a
rctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(
2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*
2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/4*(cos(2*arctan(2^(1/4)*x
))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2
*(2-2^(1/2))^(1/2))*(1-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2
^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+3/16*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {1339, 1117, 1209, 1720}

$$-\frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{4\left(2-3\sqrt{2}\right)}+\frac{\left(1-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{2\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{3\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{8\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}\,x}{2\sqrt{2}\left(\sqrt{2}x^2+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (x*Sqrt[1 + 2*x^2 + 2*x^4])/(2*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (3*Sqrt[3/10]*(
3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(4*(2 - 3*Sqrt[
2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ell
ipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*Sqrt[1 + 2*x^2 + 2
*x^4]) + ((1 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + S
qrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(2*2^(3/4)*
(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) + (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2
])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(8*2^(3/4)*(2 - 3*Sqrt[2])*Sq
rt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1339

```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[-(2*c*d - a*e*q)/(c*e*(e - d*q))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Dist[1/(e*q), Int[(1 - q*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] + Dist[d^2/(e*(e - d*q)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\int \frac{1-\sqrt{2}\,x^2}{\sqrt{1+2x^2+2x^4}}\,dx}{2\sqrt{2}} + \frac{9\int \frac{1+\sqrt{2}\,x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx}{2\left(2-3\sqrt{2}\right)} - \frac{\left(12-2\sqrt{2}\right.}{}$$

$$= \frac{x\sqrt{1+2x^2+2x^4}}{2\sqrt{2}\left(1+\sqrt{2}\,x^2\right)} - \frac{3\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)}{4\left(2-3\sqrt{2}\right)} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 127, normalized size = 0.30

$$-\frac{\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left((1+i)E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(1+4i)F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+3i\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)\right)}{4\sqrt{1-i}\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -1/4*(Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*((1 + I)*EllipticE[I*ArcS
inh[Sqrt[1 - I]*x], I] - (1 + 4*I)*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] +
 (3*I)*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(Sqrt[1 - I]*Sq
rt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 222, normalized size = 0.53

method	result
default	$\frac{\left(-\frac{1}{4}+\frac{i}{4}\right)\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-\mathrm{EllipticE}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2+x^2+1}\sqrt{ix^2+x}}{4\sqrt{-1+}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] (-1/4+1/4*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*
x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-3/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^
```

```
(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/
2*2^(1/2)+1/2*I*2^(1/2))+3/4/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)
^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/
2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
[Out] Integral(x**4/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^4/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.338 $\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=247

$$-\frac{1}{4}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2\right.\right.}{14\,2^{3/4}\sqrt{1+2x^2+2x^4}}$$

```
[Out] -1/20*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/28*(cos(2*arc
tan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^
(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+
1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/112*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^
(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/
2)
```

Rubi [A]
time = 0.08, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1333, 1117, 1720}

$$-\frac{1}{4}\sqrt{\frac{3}{5}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{56\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -1/4*(Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]) - ((3 + Sqrt
[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellipt
icF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x
^4]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqr
t[2])/4])/(56*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1333

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(-a)*((e + d*q)/(c*d^2 - a*e^2))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[a*d*((e + d*q)/(c*d^2 - a*e^
2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\left(\frac{1}{14}\left(2+3\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx\right) + \frac{1}{14}\left(3\left(2+3\sqrt{2}\right)\right)\int \frac{}{(3}$$

$$= -\frac{1}{4}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+}{(1+}}}{14\ 2^{3/}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.11, size = 99, normalized size = 0.40

$$\frac{(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\left(F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)\right)}{4\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*(EllipticF[I*Arc
Sinh[Sqrt[1 - I]*x], I] - EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I
]))/(4*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 134, normalized size = 0.54

method	result
default	$\frac{\sqrt{1+(1-i)\,x^{2}}\,\sqrt{1+(1+i)\,x^{2}}\,\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^{4}+2x^{2}+1}}-\frac{\sqrt{-i x^{2}+x^{2}+1}\,\sqrt{i x^{2}+x}}{2\sqrt{-1}}$
elliptic	$\frac{\sqrt{-i x^{2}+x^{2}+1}\,\sqrt{i x^{2}+x^{2}+1}\,\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^{4}+2x^{2}+1}}-\frac{\sqrt{-i x^{2}+x^{2}+1}\,\sqrt{i x^{2}+x^{2}}}{2\sqrt{-1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/2/(-1+I)^(1/2)*(1
+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-
1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^{2}}{(2x^{2}+3)\sqrt{2x^{4}+2x^{2}+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
[Out] Integral(x**2/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(x^2/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.339 $\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=245

$$\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)}{2\sqrt{15}}+\frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}\,x^2\right)^2}}\,F\left(2\tan^{-1}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right.}{14\sqrt[4]{2}\,\sqrt{1+2x^2+2x^4}}$$

```
[Out] 1/30*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/28*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(
1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1
)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)-1/168*(cos(2*arcta
n(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(
1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1
/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2
)
```

Rubi [A]
time = 0.07, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1230, 1117, 1720}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)}{2\sqrt{15}}+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{14\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}-\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{84\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(2*Sqrt[15]) + ((3 + Sqrt[2])
*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[
2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(14*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
 - ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]
*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2]
)/4])/(84*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = \frac{1}{7}\left(3+\sqrt{2}\right)\int \frac{1}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{7}\left(2+3\sqrt{2}\right)\int \frac{1+\sqrt{2}}{(3+2x^2)\sqrt{1+}}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)}{2\sqrt{15}} + \frac{\left(3+\sqrt{2}\right)\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+2x}{\left(1+\sqrt{2}\,x^2\right)}}}{14\sqrt[4]{2}\,\sqrt{1+2}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.04, size = 80, normalized size = 0.33

$$-\frac{i\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{3\sqrt{1-i}\,\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((-1/3*I)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3,
 I*ArcSinh[Sqrt[1 - I]*x], I])/(Sqrt[1 - I]*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.12, size = 70, normalized size = 0.29

method	result	size
default	$\frac{\sqrt{-i x^{2}+x^{2}+1}\, \sqrt{i x^{2}+x^{2}+1}\, \mathrm{EllipticPi}\left(x \sqrt{-1+i}, \frac{1}{3}+\frac{i}{3}, \frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{3 \sqrt{-1+i}\, \sqrt{2 x^{4}+2 x^{2}+1}}$	70
elliptic	$\frac{\sqrt{-i x^{2}+x^{2}+1}\, \sqrt{i x^{2}+x^{2}+1}\, \mathrm{EllipticPi}\left(x \sqrt{-1+i}, \frac{1}{3}+\frac{i}{3}, \frac{\sqrt{-1-i}}{\sqrt{-1+i}}\right)}{3 \sqrt{-1+i}\, \sqrt{2 x^{4}+2 x^{2}+1}}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
[Out] 1/3/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1
/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^6 + 10*x^4 + 8*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(2 x^{2}+3\right) \sqrt{2 x^{4}+2 x^{2}+1}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/((2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(2\,x^2+3)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.340 $\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=399

$$-\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)}{3\sqrt{15}}-\frac{\sqrt[4]{2}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1+2x^2+}{\left(1+\sqrt{2}\,x\right.}}}{3\sqrt{1}}$$

[Out] -1/45*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*(2*x^4+2*x^2+1)^(1/2)/x+1/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/3*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/42*2^(1/4)*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(5-3*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)+1/252*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/4)/(2*x^4+2*x^2+1)^(1/2)

Rubi [A]

time = 0.22, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1343, 1728, 1209, 1722, 1117, 1720}

$$-\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)}{3\sqrt{15}}+\frac{\left(5-3\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{21\;2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\sqrt[4]{2}\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,E\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}+\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{126\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2}\,\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}\,x^2+1\right)}-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

Antiderivative was successfully verified.

[In] Int[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]

[Out] -1/3*Sqrt[1 + 2*x^2 + 2*x^4]/x + (Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2)) - ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]/(3*Sqrt[15]) - (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) + ((5 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(21*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(126*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1343

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1))), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^
4]))*Simp[a*e*(m + 1) + b*d*(m + 2) + (b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e
*(m + 3)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{1}{3}\int \frac{-2+6x^2+4x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{1}{12}\int \frac{-8+12\sqrt{2}+\left(24-4\left(6-2\sqrt{2}\right)\right)x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)} - \frac{\sqrt[4]{2}\left(1+\sqrt{2}\,x^2\right)\sqrt{}}{}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)} - \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+}}\right.}{3\sqrt{15}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 147, normalized size = 0.37

$$-\frac{i\left(-3i(1+2x^2+2x^4)+\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(3E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-3F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(1+i)\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)\right)\right)}{9x\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] ((-1/9*I)*((-3*I)*(1 + 2*x^2 + 2*x^4) + Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]
*Sqrt[1 + (1 + I)*x^2]*(3*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - 3*Ellipt
icF[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + I)*EllipticPi[1/3 + I/3, I*ArcSinh[
Sqrt[1 - I]*x], I])))/(x*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 178, normalized size = 0.45

method	result
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
risch	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(-\frac{1}{3}+\frac{i}{3}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix^2}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x+(-1/3+1/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*
(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(
1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))-2/
9/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2
)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^8 + 10*x^6 + 8*x^4 + 3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{x^2\cdot(2x^2+3)\,\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
```

```
[Out] Integral(1/(x**2*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
```

```
[Out] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.341 $\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$

Optimal. Leaf size=422

$$-\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)}+\frac{2\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)}{9\sqrt{15}}+\frac{2\sqrt[4]{2}\,\left(1+\right.}{}$$

```
[Out] 2/135*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/9*(2*x^4+2*x^
2+1)^(1/2)/x^3+2/3*(2*x^4+2*x^2+1)^(1/2)/x-2/3*x*(2*x^4+2*x^2+1)^(1/2)*2^(1
/2)/(1+x^2*2^(1/2))+2/3*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(
1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2
^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(
1/2)-1/378*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Elli
pticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3
+2^(1/2))^2*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(3/
4)/(2*x^4+2*x^2+1)^(1/2)-1/126*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arc
tan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(
1+19*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(
3/4)/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1343, 1697, 1728, 1209, 1722, 1117, 1720}

$$\frac{2\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)}{9\sqrt{15}}-\frac{\left(1+19\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{63\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}+\frac{2\sqrt[4]{2}\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,E\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{3\sqrt{2x^4+2x^2+1}}-\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{189\sqrt[4]{2}\,\sqrt{2x^4+2x^2+1}}-\frac{2\sqrt{2}\,\sqrt{2x^4+2x^2+1}\,x}{3\left(\sqrt{2}\,x^2+1\right)}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] -1/9*Sqrt[1 + 2*x^2 + 2*x^4]/x^3 + (2*Sqrt[1 + 2*x^2 + 2*x^4])/(3*x) - (2*S
qrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4])/(3*(1 + Sqrt[2]*x^2)) + (2*ArcTan[(Sqrt[5
/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(9*Sqrt[15]) + (2*2^(1/4)*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(3*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 19*Sqrt[2])*(1 +
Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcT
an[2^(1/4)*x], (2 - Sqrt[2])/4])/(63*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3
 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^
2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/
(189*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1343

```
Int[(x_)^(m_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4]), x_Symbol] :> Simp[x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1))), x
] - Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^
4]))*Simp[a*e*(m + 1) + b*d*(m + 2) + (b*e*(m + 2) + c*d*(m + 3))*x^2 + c*e
*(m + 3)*x^4, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && ILtQ[m/2, 0]
```

Rule 1697

```
Int[((Px_)*(x_)^(m_))/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_
.)*(x_)^4]), x_Symbol] :> With[{A = Coeff[Px, x, 0], B = Coeff[Px, x, 2], C
 = Coeff[Px, x, 4]}, Simp[A*x^(m + 1)*(Sqrt[a + b*x^2 + c*x^4]/(a*d*(m + 1)
)), x] + Dist[1/(a*d*(m + 1)), Int[(x^(m + 2)/((d + e*x^2)*Sqrt[a + b*x^2 +
 c*x^4]))*Simp[a*B*d*(m + 1) - A*(a*e*(m + 1) + b*d*(m + 2)) + (a*C*d*(m +
1) - A*(b*e*(m + 2) + c*d*(m + 3)))*x^2 - A*c*e*(m + 3)*x^4, x], x], x]] /;
 FreeQ[{a, b, c, d, e}, x] && PolyQ[Px, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && I
LtQ[m/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
```

```
[c*A^2 - a*B^2, 0]
```

Rule 1722

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[a*(B*d - A*e)*((e + d*q)/(c*d^2 - a*e^2)), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1728

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
, 2], C = Coeff[P4x, x, 4]}, Dist[-C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rubi steps

$$\int \frac{1}{x^4(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx = -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{1}{9}\int \frac{-18-14x^2-4x^4}{x^2(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{1}{27}\int \frac{6+120x^2+72}{(3+2x^2)\sqrt{1+2x^2}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{1}{108}\int \frac{24+216\sqrt{2}+(4}{(3+2x^2)\sqrt{}}$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)}+$$

$$= -\frac{\sqrt{1+2x^2+2x^4}}{9x^3}+\frac{2\sqrt{1+2x^2+2x^4}}{3x}-\frac{2\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{3\left(1+\sqrt{2}\,x^2\right)}+$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.13, size = 219, normalized size = 0.52

$$\frac{-3+12x^2+30x^4+36x^6+18i\sqrt{1-i}\,x^3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(3+15i)\sqrt{1-i}\,x^3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+2(1-i)^{3/2}x^3\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{27x^3\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*(3 + 2*x^2)*Sqrt[1 + 2*x^2 + 2*x^4]),x]
```

```
[Out] (-3 + 12*x^2 + 30*x^4 + 36*x^6 + (18*I)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^
2]*Sqrt[1 + (1 + I)*x^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (3 + 15*I
)*Sqrt[1 - I]*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*A
rcSinh[Sqrt[1 - I]*x], I] + 2*(1 - I)^(3/2)*x^3*Sqrt[1 + (1 - I)*x^2]*Sqrt[
1 + (1 + I)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(27*x^
3*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.13, size = 260, normalized size = 0.62

method	result
risch	$\frac{12x^6+10x^4+4x^2-1}{9x^3\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\mathrm{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$\frac{2\sqrt{2x^4+2x^2+1}}{3x}+\frac{\left(\frac{2}{3}-\frac{2i}{3}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\mathrm{EllipticF}\left(x\sqrt{-1+i}\,,\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{\sqrt{2x^4+2x^2+1}}{9x^3}+\frac{2\sqrt{2x^4+2x^2+1}}{3x}+\frac{4\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{-1+i}\right.}{9\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3*(2*x^4+2*x^2+1)^(1/2)/x+(2/3-2/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1
+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/
2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+4/27
/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)
*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))-1/9*(2*x^4+
2*x^2+1)^(1/2)/x^3-2/9/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)
/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="maxima")
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(4*x^10 + 10*x^8 + 8*x^6 + 3*x^4), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 \cdot (2x^2+3)\sqrt{2x^4+2x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(1/2),x)
[Out] Integral(1/(x**4*(2*x**2 + 3)*sqrt(2*x**4 + 2*x**2 + 1)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(1/2),x, algorithm="giac")
[Out] integrate(1/(sqrt(2*x^4 + 2*x^2 + 1)*(2*x^2 + 3)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\left(2\,x^2+3\right)\,\sqrt{2\,x^4+2\,x^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)),x)
[Out] int(1/(x^4*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(1/2)), x)
```

3.342 $\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{a(b^2d-2acd-abe)+(b^3d-3abcd-ab^2e+2a^2ce)\,x^2}{c\,(b^2-4ac)\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{\ }{2\sqrt{\ }}\right.}{2}$$

```
[Out] 1/2*arctanh(1/2*(2*c*x^2+b)/c^(1/2)/(c*x^4+b*x^2+a)^(1/2))/c^(3/2)/e-1/2*d^
3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4
+b*x^2+a)^(1/2))/e/(a*e^2-b*d*e+c*d^2)^(3/2)+(a*(-a*b*e-2*a*c*d+b^2*d)+(2*a
^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)*x^2)/c/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*
x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 1660, 857, 635, 212, 738}

$$\frac{x^2(2a^2ce-ab^2e-3abcd+b^3d)+a(-abe-2acd+b^2d)}{c\,(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}+\frac{\tanh^{-1}\left(\frac{b+2cx^2}{2\sqrt{c}\,\sqrt{a+bx^2+cx^4}}\right)}{2c^{3/2}e}-\frac{d^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\,\sqrt{ae^2-bde+cd^2}}\right)}{2e\,(ae^2-bde+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(b^2*d - 2*a*c*d - a*b*e) + (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)*x^
2)/(c*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) + ArcT
anh[(b + 2*c*x^2)/(2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(2*c^(3/2)*e) - (d^3
*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*S
qrt[a + b*x^2 + c*x^4])])/(2*e*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 635

```
Int[1/Sqrt[(a_) + (b_.)*(x_) + (c_.)*(x_)^2], x_Symbol] :> Dist[2, Subst[In
t[1/(4*c - x^2), x], x, (b + 2*c*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a,
 b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 857

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + b*x +
c*x^2)^p, x], x] + Dist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + b*x + c*x^2)^p,
 x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x] && NeQ[b^2 - 4*a*c, 0] &&
NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^7}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a(b^2d-2acd-abe)+(b^3d-3abcd-ab^2e+2a^2ce)\,x^2}{c\,(b^2-4ac)\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{a(b^2d-2acd-abe)+(b^3d-3abcd-ab^2e+2a^2ce)\,x^2}{c\,(b^2-4ac)\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{a(b^2d-2acd-abe)+(b^3d-3abcd-ab^2e+2a^2ce)\,x^2}{c\,(b^2-4ac)\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{\text{Subst}\left(\int \frac{\cdots}{4c\cdots}\right.}{\cdots}$$

$$= \frac{a(b^2d-2acd-abe)+(b^3d-3abcd-ab^2e+2a^2ce)\,x^2}{c\,(b^2-4ac)\,(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{\tanh^{-1}\left(\frac{\cdots}{2\sqrt{\cdots}}\right.}{\cdots}$$

Mathematica [A]
time = 0.97, size = 251, normalized size = 1.06

$$\frac{-b^3dx^2+ab(-bd+3cdx^2+bex^2)+a^2(be+2c(d-ex^2))}{c\,(-b^2+4ac)\,(cd^2+e(-bd+ae))\sqrt{a+bx^2+cx^4}} - \frac{d^3\sqrt{-cd^2+e(bd-ae)}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{e\,(cd^2+e(-bd+ae))^2} - \frac{\log\left(ce\left(b+2cx^2-2\sqrt{c}\sqrt{a+bx^2+cx^4}\right)\right)}{2c^{3/2}e}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b^3*d*x^2) + a*b*(-(b*d) + 3*c*d*x^2 + b*e*x^2) + a^2*(b*e + 2*c*(d - e*
x^2)))/(c*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4]
) - (d^3*Sqrt[-(c*d^2) + e*(b*d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqr
t[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(e*(c*d^2 + e*(-(b*d
) + a*e))^2) - Log[c*e*(b + 2*c*x^2 - 2*Sqrt[c]*Sqrt[a + b*x^2 + c*x^4])]/(
2*c^(3/2)*e)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(214) = 428$.
time = 0.14, size = 696, normalized size = 2.95

method	result

elliptic	$\frac{\ln\left(\frac{\frac{b}{2}+c\,x^{2}}{\sqrt{c}}+\sqrt{c\,x^{4}+b\,x^{2}+a}\right)}{2c^{\frac{3}{2}}e}-\frac{2c\,d^{3}\ln\left(\frac{\frac{2a\,e^{2}-2deb+2c\,d^{2}}{e^{2}}+\frac{(eb-2cd)\left(x^{2}+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^{2}-deb+c\,d^{2}}{e^{2}}}\sqrt{c\left(x^{2}+\frac{d}{e}\right)^{2}+}}{x^{2}+\frac{d}{e}}\right)}{e^{2}\left(e\sqrt{-4ac+b^{2}}-eb+2cd\right)\left(e\sqrt{-4ac+b^{2}}+eb-2cd\right)}$
default	$\frac{-\frac{x^{2}}{2c\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{b}{4c^{2}\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{b^{2}x^{2}}{2c\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}+\frac{b^{3}}{4c^{2}\left(4ac-b^{2}\right)\sqrt{c\,x^{4}+b\,x^{2}+a}}}{e}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e*(-1/2*x^2/c/(c*x^4+b*x^2+a)^(1/2)+1/4*b/c^2/(c*x^4+b*x^2+a)^(1/2)+1/2*b
^2/c/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)*x^2+1/4*b^3/c^2/(4*a*c-b^2)/(c*x^4+b
*x^2+a)^(1/2)+1/2/c^(3/2)*ln((1/2*b+c*x^2)/c^(1/2)+(c*x^4+b*x^2+a)^(1/2)))+
d/e^2/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4*a*c-b^2)+d^2/e^3/(c*x^4+b*x^2+a)
^(1/2)*(2*c*x^2+b)/(4*a*c-b^2)-d^3/e^3*(2*c*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c
*d)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*
(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)
^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2
))/(x^2+d/e))-2*c/(-4*a*c+b^2)/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/(x^2-1/2/c*
(-b+(-4*a*c+b^2)^(1/2)))*((x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*c+(-4*a*c+b
^2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)+2*c/(-4*a*c+b^2)/(e*(-
4*a*c+b^2)^(1/2)+e*b-2*c*d)/(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x^2+1/2*(b
+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/
2))/c))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^7/((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(217) = 434$.
time = 38.18, size = 4905, normalized size = 20.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c^3 - 4*a*c^4)*d^4*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^4*x^2 + (a*b^2
*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 +
(a^2*b^3 - 4*a^3*b*c)*x^2)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^4 + (a*b^4
- 4*a^2*b^2*c)*d*x^2 + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2
 - 8*a^2*c^3)*d^2*x^4 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x^2 + (a*b^4 -
2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^4 + (b^4
*c - 4*a*b^2*c^2)*d^3*x^2 + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e)*sqrt(c)*log(-8*
c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2 + b)*sqrt(c)
 - 4*a*c) + ((b^2*c^3 - 4*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 +
(a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-(8*c^2*d^2*x^
4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^
2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4
 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*
d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2)) - 4*sqrt(c*x^4 + b*x^2 + a)*((a^3*b*c +
(a^2*b^2*c - 2*a^3*c^2)*x^2)*e^4 - ((2*a*b^3*c - 5*a^2*b*c^2)*d*x^2 + 2*(a^
2*b^2*c - a^3*c^2)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*x^2 + (a
*b^3*c - a^2*b*c^2)*d^2)*e^2 - ((b^3*c^2 - 3*a*b*c^3)*d^3*x^2 + (a*b^2*c^2
- 2*a^2*c^3)*d^3)*e))/((a^3*b^2*c^2 - 4*a^4*c^3 + (a^2*b^2*c^3 - 4*a^3*c^4)
*x^4 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*x^2)*e^5 - 2*((a*b^3*c^3 - 4*a^2*b*c^4)*
d*x^4 + (a*b^4*c^2 - 4*a^2*b^2*c^3)*d*x^2 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*d)*
e^4 + ((b^4*c^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*x^4 + (b^5*c^2 - 2*a*b^3*c^3
 - 8*a^2*b*c^4)*d^2*x^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2)*e^3
- 2*((b^3*c^4 - 4*a*b*c^5)*d^3*x^4 + (b^4*c^3 - 4*a*b^2*c^4)*d^3*x^2 + (a*b
^3*c^3 - 4*a^2*b*c^4)*d^3)*e^2 + ((b^2*c^5 - 4*a*c^6)*d^4*x^4 + (b^3*c^4 -
4*a*b*c^5)*d^4*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*d^4)*e), -1/4*(2*((b^2*c^3 - 4
*a*c^4)*d^3*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d
^3)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d
*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b
*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x
^2 + a*b*d)*e)) - ((b^2*c^3 - 4*a*c^4)*d^4*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^4*
x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3
*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x
^4 + (a*b^4 - 4*a^2*b^2*c)*d*x^2 + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c -
 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^4 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x^2
 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^
3*x^4 + (b^4*c - 4*a*b^2*c^2)*d^3*x^2 + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e)*sqr
```

```
t(c)*log(-8*c^2*x^4 - 8*b*c*x^2 - b^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*x^2
+ b)*sqrt(c) - 4*a*c) + 4*sqrt(c*x^4 + b*x^2 + a)*((a^3*b*c + (a^2*b^2*c -
2*a^3*c^2)*x^2)*e^4 - ((2*a*b^3*c - 5*a^2*b*c^2)*d*x^2 + 2*(a^2*b^2*c - a^3
*c^2)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 2*a^2*c^3)*d^2*x^2 + (a*b^3*c - a^2*
b*c^2)*d^2)*e^2 - ((b^3*c^2 - 3*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 2*a^2*c^3)*
d^3)*e))/((a^3*b^2*c^2 - 4*a^4*c^3 + (a^2*b^2*c^3 - 4*a^3*c^4)*x^4 + (a^2*b
^3*c^2 - 4*a^3*b*c^3)*x^2)*e^5 - 2*((a*b^3*c^3 - 4*a^2*b*c^4)*d*x^4 + (a*b^
4*c^2 - 4*a^2*b^2*c^3)*d*x^2 + (a^2*b^3*c^2 - 4*a^3*b*c^3)*d)*e^4 + ((b^4*c
^3 - 2*a*b^2*c^4 - 8*a^2*c^5)*d^2*x^4 + (b^5*c^2 - 2*a*b^3*c^3 - 8*a^2*b*c^
4)*d^2*x^2 + (a*b^4*c^2 - 2*a^2*b^2*c^3 - 8*a^3*c^4)*d^2)*e^3 - 2*((b^3*c^4
 - 4*a*b*c^5)*d^3*x^4 + (b^4*c^3 - 4*a*b^2*c^4)*d^3*x^2 + (a*b^3*c^3 - 4*a^
2*b*c^4)*d^3)*e^2 + ((b^2*c^5 - 4*a*c^6)*d^4*x^4 + (b^3*c^4 - 4*a*b*c^5)*d^
4*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*d^4)*e), -1/4*(2*((b^2*c^3 - 4*a*c^4)*d^4*x
^4 + (b^3*c^2 - 4*a*b*c^3)*d^4*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2
 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*e^4 -
 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^4 + (a*b^4 - 4*a^2*b^2*c)*d*x^2 + (a^2*b^3
- 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^4 + (b^5 - 2
*a*b^3*c - 8*a^2*b*c^2)*d^2*x^2 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^
2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^4 + (b^4*c - 4*a*b^2*c^2)*d^3*x^2 + (a*b
^3*c - 4*a^2*b*c^2)*d^3)*e)*sqrt(-c)*arctan(1/2*sqrt(c*x^4 + b*x^2 + a)*(2*
c*x^2 + b)*sqrt(-c)/(c^2*x^4 + b*c*x^2 + a*c)) - ((b^2*c^3 - 4*a*c^4)*d^3*x
^4 + (b^3*c^2 - 4*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^3)*sqrt(c*d^
2 - b*d*e + a*e^2)*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d^2
- 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d^2
- b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d
*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2)) + 4
*sqrt(c*x^4 + b*x^2 + a)*((a^3*b*c + (a^2*b^2*c - 2*a^3*c^2)*x^2)*e^4 - ((2
*a*b^3*c - 5*a^2*b*c^2)*d*x^2 + 2*(a^2*b^2*c - a^3*c^2)*d)*e^3 + ((b^4*c -
2*a*b^2*c^2 - 2*a^2*c^3)*d^2*x^2 + (a*b^3*c - a^2*b*c^2)*d^2)*e^2 - ((b^3*c
^2 - 3*a*b*c^3)*d^3*x^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^3)*e))/((a^3*b^2*c^2 -
4*a^4*c^3 + (a^2*b^2*c^3 - 4*a^3*c^4)*x^4 + (a^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**7/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x^7/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.343 $\int \frac{x^5}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=167

$$-\frac{a(bd-2ae)+(b^2d-2acd-abe)\,x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}}+\frac{d^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\,\sqrt{a+bx^2+cx^4}}\right)}{2\,(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] 1/2*d^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(-a*(-2*a*e+b*d)-(-a*b*e-2
*a*c*d+b^2*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 1660, 12, 738, 212}

$$\frac{d^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\,\sqrt{ae^2-bde+cd^2}}\right)}{2\,(ae^2-bde+cd^2)^{3/2}}-\frac{x^2(-abe-2acd+b^2d)+a(bd-2ae)}{(b^2-4ac)\,\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((a*(b*d - 2*a*e) + (b^2*d - 2*a*c*d - a*b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 -
 b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (d^2*ArcTanh[(b*d - 2*a*e + (2*
c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(
2*(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1660

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> With[{Q = PolynomialQuotient[(d + e*x)^m*Pq, a + b*x + c*x
^2, x], f = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x],
x, 0], g = Coeff[PolynomialRemainder[(d + e*x)^m*Pq, a + b*x + c*x^2, x], x
, 1]}, Simp[(b*f - 2*a*g + (2*c*f - b*g)*x)*((a + b*x + c*x^2)^(p + 1)/((p
+ 1)*(b^2 - 4*a*c))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*ExpandToSum[((p + 1)*(b^2 - 4*a*c)*Q)/(d + e*x)^m
 - ((2*p + 3)*(2*c*f - b*g))/(d + e*x)^m, x], x], x]] /; FreeQ[{a, b, c, d,
 e}, x] && PolyQ[Pq, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2
, 0] && LtQ[p, -1] && ILtQ[m, 0]
```

Rubi steps

$$\int \frac{x^5}{(d+ex^2)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2}{(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int -\frac{\cdots}{2(cd^2-bde\cdots}\right.\cdots}{\cdots}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{d^2\text{Subst}\left(\int \frac{\cdots}{(d+ex)\sqrt{\cdots}}\right.\cdots}{2\left(cd^2-\cdots\right.}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} - \frac{d^2\text{Subst}\left(\int \frac{\cdots}{4cd^2-4bde\cdots}\right.\cdots}{c\cdots}$$

$$= -\frac{a(bd-2ae)+\left(b^2d-2acd-abe\right)x^2}{\left(b^2-4ac\right)\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}} + \frac{d^2\tanh^{-1}\left(\frac{\cdots}{2\sqrt{cd^2-\cdots}}\right.\cdots}{2\left(cd^2\cdots\right.}$$

Mathematica [A]

time = 0.65, size = 180, normalized size = 1.08

$$\frac{-2a^2e+b^2dx^2-2acdx^2+ab(d-ex^2)}{(b^2-4ac)\left(-cd^2+e(bd-ae)\right)\sqrt{a+bx^2+cx^4}}+\frac{d^2\sqrt{-cd^2+e(bd-ae)}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\left(cd^2+e(-bd+ae)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-2*a^2*e + b^2*d*x^2 - 2*a*c*d*x^2 + a*b*(d - e*x^2))/((b^2 - 4*a*c)*(-(c*
d^2) + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (d^2*Sqrt[-(c*d^2) + e*(b*
d - a*e)]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c
*d^2) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(156) = 312$.
time = 0.14, size = 542, normalized size = 3.25

method	result
elliptic	$\frac{2c\,d^2\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-eb+2cd\right)\left(e\sqrt{-4ac+b^2}+eb-2cd\right)e\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}$
default	$-\frac{b\,x^2+2a}{e\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}-\frac{d(2c\,x^2+b)}{e^2\sqrt{c\,x^4+b\,x^2+a}\,(4ac-b^2)}+\frac{d^2\left(\frac{2ce\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{}}{}\right)}{\left(e\sqrt{-4ac+b^2}-\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/e/(c*x^4+b*x^2+a)^(1/2)*(b*x^2+2*a)/(4*a*c-b^2)-d/e^2/(c*x^4+b*x^2+a)^(1
/2)*(2*c*x^2+b)/(4*a*c-b^2)+1/e^2*d^2*(2*c*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*
d)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(
a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^
(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2)
)/(x^2+d/e))-2*c/(-4*a*c+b^2)/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/(x^2-1/2/c*(
-b+(-4*a*c+b^2)^(1/2)))*((x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*c+(-4*a*c+b^
```

```
2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)+2*c/(-4*a*c+b^2)/(e*(-4
*a*c+b^2)^(1/2)+e*b-2*c*d)/(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x^2+1/2*(b+
(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2
))/c))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
[Out] integrate(x^5/((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(159) = 318$.
time = 0.60, size = 1399, normalized size = 8.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
[Out] [1/4*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3 - 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2
*c)*d^2)*sqrt(c*d^2 - b*d*e + a*e^2)*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 +
(b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2
*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2
)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d
*x^2*e + d^2)) - 4*(a*b*c*d^3 + (b^2*c - 2*a*c^2)*d^3*x^2 - (a^2*b*x^2 + 2*
a^3)*e^3 + (3*a^2*b*d + 2*(a*b^2 - a^2*c)*d*x^2)*e^2 - ((b^3 - a*b*c)*d^2*x
^2 + (a*b^2 + 2*a^2*c)*d^2)*e)*sqrt(c*x^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4
)*d^4*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^4*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (
a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2
)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^4 + (a*b^4 - 4*a^2*b^2*c)*d*x^2 + (a
^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^4 + (
b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x^2 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*
d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^4 + (b^4*c - 4*a*b^2*c^2)*d^3*x^2
 + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e), 1/2*(((b^2*c - 4*a*c^2)*d^2*x^4 + (b^3
- 4*a*b*c)*d^2*x^2 + (a*b^2 - 4*a^2*c)*d^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*ar
ctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(
-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a
*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a*b*d)*e)) - 2*(a*b*c*d^3 + (b
^2*c - 2*a*c^2)*d^3*x^2 - (a^2*b*x^2 + 2*a^3)*e^3 + (3*a^2*b*d + 2*(a*b^2 -
 a^2*c)*d*x^2)*e^2 - ((b^3 - a*b*c)*d^2*x^2 + (a*b^2 + 2*a^2*c)*d^2)*e)*sqr
t(c*x^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4)*d^4*x^4 + (b^3*c^2 - 4*a*b*c^3)*
```

```
d^4*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4
*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)
*d*x^4 + (a*b^4 - 4*a^2*b^2*c)*d*x^2 + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4
*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^4 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2
*x^2 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3
)*d^3*x^4 + (b^4*c - 4*a*b^2*c^2)*d^3*x^2 + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x**5/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 458 vs. 2(159) = 318.
time = 4.11, size = 458, normalized size = 2.74

$$\frac{d^2\arctan\left(-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}}-\frac{\frac{(b^2cd^3-2\,ac^2d^3-b^3d^2e+abcd^2e+2\,ab^2de^2-2\,a^2cde^2-a^2be^3)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{abcd^3-ab^2d^2e-2\,a^2cd^2e+3\,a^2bde^2-2\,a^3e^3}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] d^2*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c
*d^2 + b*d*e - a*e^2))/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2
)) - ((b^2*c*d^3 - 2*a*c^2*d^3 - b^3*d^2*e + a*b*c*d^2*e + 2*a*b^2*d*e^2 -
2*a^2*c*d*e^2 - a^2*b*e^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e +
 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*
a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (a*b*c*d^3 - a
*b^2*d^2*e - 2*a^2*c*d^2*e + 3*a^2*b*d*e^2 - 2*a^3*e^3)/(b^2*c^2*d^4 - 4*a*
c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2
 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^
3*c*e^4))/sqrt(c*x^4 + b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^5}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x^5/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.344 $\int \frac{x^3}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=159

$$\frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}-\frac{de\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] -1/2*d*e*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)
/(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(a*(-b*e+2*c*d)+c*(-2*a*e
+b*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.12, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 836, 12, 738, 212}

$$\frac{cx^2(bd-2ae)+a(2cd-be)}{(b^2-4ac)\sqrt{a+bx^2+cx^4}\,(ae^2-bde+cd^2)}-\frac{de\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2(ae^2-bde+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (a*(2*c*d - b*e) + c*(b*d - 2*a*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e
^2)*Sqrt[a + b*x^2 + c*x^4]) - (d*e*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^
2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*(c*d^2 - b*
d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 836

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(f*(b*c*d - b^2*e + 2
*a*c*e) - a*g*(2*c*d - b*e) + c*(f*(2*c*d - b*e) - g*(b*d - 2*a*e))*x)*((a
+ b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2))), x]
 + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^m*
(a + b*x + c*x^2)^(p + 1)*Simp[f*(b*c*d*e*(2*p - m + 2) + b^2*e^2*(p + m +
2) - 2*c^2*d^2*(2*p + 3) - 2*a*c*e^2*(m + 2*p + 3)) - g*(a*e*(b*e - 2*c*d*m
 + b*e*m) - b*d*(3*c*d - b*e + 2*c*d*p - b*e*p)) + c*e*(g*(b*d - 2*a*e) - f
*(2*c*d - b*e))*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && LtQ[p, -1
] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3}{(d+ex^2)\,(a+bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x}{(d+ex)\,(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int \frac{(b^2-4ac}{2(d+ex)\sqrt{a+b}}\right.}{(b^2-4ac)\,(cd^2}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}} - \frac{(de)\text{Subst}\left(\int \frac{}{(d+ex)\sqrt{a}}\right.}{2\,(cd^2-b}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}} + \frac{(de)\text{Subst}\left(\int \frac{1}{4cd^2-4bde+}\right.}{cd^2}$$

$$= \frac{a(2cd-be)+c(bd-2ae)x^2}{(b^2-4ac)\,(cd^2-bde+ae^2)\,\sqrt{a+bx^2+cx^4}} - \frac{de\tanh^{-1}\left(\frac{}{2\sqrt{cd^2-b}}\right.}{2\,(cd^2-}$$

Mathematica [A]
time = 0.71, size = 172, normalized size = 1.08

$$\frac{-bcdx^2+a(-2cd+be+2cex^2)}{(b^2-4ac)\,(-cd^2+e(bd-ae))\sqrt{a+bx^2+cx^4}}-\frac{de\sqrt{-cd^2+e(bd-ae)}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{(cd^2+e(-bd+ae))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b*c*d*x^2) + a*(-2*c*d + b*e + 2*c*e*x^2))/((b^2 - 4*a*c)*(-(c*d^2) + e*
(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) - (d*e*Sqrt[-(c*d^2) + e*(b*d - a*e)]
*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e
*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(147) = 294$.
time = 0.14, size = 499, normalized size = 3.14

method	result
elliptic	$-\frac{2cd\ln\left(\frac{\frac{2ae^2-2deb+2cd^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{ae^2-deb+cd^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-eb+2cd\right)\left(e\sqrt{-4ac+b^2}+eb-2cd\right)\sqrt{\frac{ae^2-deb+cd^2}{e^2}}}$
default	$\frac{2cx^2+b}{e\sqrt{cx^4+bx^2+a}\,(4ac-b^2)}-\frac{d\left(\frac{2ce\ln\left(\frac{\frac{2ae^2-2deb+2cd^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{ae^2-deb+cd^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-eb+2cd\right)\left(e\sqrt{-4ac+b^2}+eb-2cd\right)\sqrt{}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/e/(c*x^4+b*x^2+a)^(1/2)*(2*c*x^2+b)/(4*a*c-b^2)-d/e*(2*c*e/(e*(-4*a*c+b^2
)^(1/2)-e*b+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/((a*e^2-b*d*e+c*d^2)/e^
2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x^2+d/e)+2*((a*e^2-b*
d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2+d/e)+(a*e^2-b*d*e+c
*d^2)/e^2)^(1/2))/(x^2+d/e))-2*c/(-4*a*c+b^2)/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c
*d)/(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*((x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2))
```

```
)^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2))))^(1/2)+2*c/(-4
*a*c+b^2)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/
c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)^(1/2)*(x^2+1/2*(b+(
-4*a*c+b^2)^(1/2))/c))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^3/((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 661 vs. $2(152) = 304$.
time = 0.65, size = 1367, normalized size = 8.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*d*x^4 + (b^3 - 4*a*b*c)*d*x^2 + (a*b^2 - 4*a^2*c)*
d)*sqrt(c*d^2 - b*d*e + a*e^2)*e*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2
 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*
e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^
2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2
*e + d^2)) + 4*(b*c^2*d^3*x^2 + 2*a*c^2*d^3 - (2*a^2*c*x^2 + a^2*b)*e^3 + (
3*a*b*c*d*x^2 + (a*b^2 + 2*a^2*c)*d)*e^2 - (3*a*b*c*d^2 + (b^2*c + 2*a*c^2)
*d^2*x^2)*e)*sqrt(c*x^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4)*d^4*x^4 + (b^3*c
^2 - 4*a*b*c^3)*d^4*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c
+ (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*e^4 - 2*((a*b^3*
c - 4*a^2*b*c^2)*d*x^4 + (a*b^4 - 4*a^2*b^2*c)*d*x^2 + (a^2*b^3 - 4*a^3*b*c
)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^4 + (b^5 - 2*a*b^3*c -
8*a^2*b*c^2)*d^2*x^2 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3
*c^2 - 4*a*b*c^3)*d^3*x^4 + (b^4*c - 4*a*b^2*c^2)*d^3*x^2 + (a*b^3*c - 4*a^
2*b*c^2)*d^3)*e), -1/2*(((b^2*c - 4*a*c^2)*d*x^4 + (b^3 - 4*a*b*c)*d*x^2 +
(a*b^2 - 4*a^2*c)*d)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 +
b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)
/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*
c*d*x^4 + b^2*d*x^2 + a*b*d)*e))*e - 2*(b*c^2*d^3*x^2 + 2*a*c^2*d^3 - (2*a^
2*c*x^2 + a^2*b)*e^3 + (3*a*b*c*d*x^2 + (a*b^2 + 2*a^2*c)*d)*e^2 - (3*a*b*c
*d^2 + (b^2*c + 2*a*c^2)*d^2*x^2)*e)*sqrt(c*x^4 + b*x^2 + a))/((b^2*c^3 - 4
*a*c^4)*d^4*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^4*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d
```

```
^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*
c)*x^2)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^4 + (a*b^4 - 4*a^2*b^2*c)*d*x^
2 + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x
^4 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x^2 + (a*b^4 - 2*a^2*b^2*c - 8*a^3
*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^4 + (b^4*c - 4*a*b^2*c^2)*d
^3*x^2 + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
[Out] Integral(x**3/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 441 vs. 2(152) = 304.
time = 2.94, size = 441, normalized size = 2.77

$$-\frac{d\arctan\left(-\frac{\left(\sqrt{c}\,x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}\,d}{\sqrt{-cd^2+bde-ae^2}}\right)e}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}}+\frac{\frac{(bc^2d^3-b^2cd^2e-2\,ac^2d^2e+3\,abcde^2-2\,a^2ce^3)x^2}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}+\frac{2\,ac^2d^3-3\,abcd^2e+ab^2de^2+2\,a^2cde^2-a^2be^3}{b^2c^2d^4-4\,ac^3d^4-2\,b^3cd^3e+8\,abc^2d^3e+b^4d^2e^2-2\,ab^2cd^2e^2-8\,a^2c^2d^2e^2-2\,ab^3de^3+8\,a^2bcde^3+a^2b^2e^4-4\,a^3ce^4}}{\sqrt{cx^4+bx^2+a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
[Out] -d*arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*
d^2 + b*d*e - a*e^2))*e/((c*d^2 - b*d*e + a*e^2)*sqrt(-c*d^2 + b*d*e - a*e^
2)) + ((b*c^2*d^3 - b^2*c*d^2*e - 2*a*c^2*d^2*e + 3*a*b*c*d*e^2 - 2*a^2*c*e
^3)*x^2/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*
d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c
*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4) + (2*a*c^2*d^3 - 3*a*b*c*d^2*e + a*b^2*
d*e^2 + 2*a^2*c*d*e^2 - a^2*b*e^3)/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3
*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2
- 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*e^4))/sqrt(c*x^4
+ b*x^2 + a)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^3}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int(x^3/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.345 $\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=166

$$-\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}}+\frac{e^2\tanh^{-1}\left(\frac{bd-2ae+(2cd-be)x^2}{2\sqrt{cd^2-bde+ae^2}\sqrt{a+bx^2+cx^4}}\right)}{2(cd^2-bde+ae^2)^{3/2}}$$

```
[Out] 1/2*e^2*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/
(c*x^4+b*x^2+a)^(1/2))/(a*e^2-b*d*e+c*d^2)^(3/2)+(-b*c*d+b^2*e-2*a*c*e-c*(-
b*e+2*c*d)*x^2)/(-4*a*c+b^2)/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1261, 754, 12, 738, 212}

$$\frac{e^2\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2(ae^2-bde+cd^2)^{3/2}}-\frac{2ace+b^2(-e)+cx^2(2cd-be)+bcd}{(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -((b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2)/((b^2 - 4*a*c)*(c*d^2 - b
*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4])) + (e^2*ArcTanh[(b*d - 2*a*e + (2*c*
d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
```

```
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x}{(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{\text{Subst}\left(\int -\frac{(\ldots}{2(d+ex)\sqrt{\ldots}}\right.}{(b^2-4ac)(\ldots}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{e^2\text{Subst}\left(\int \frac{\ldots}{(d+ex)\sqrt{\ldots}}\right.}{2(cd^2-\ldots}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} - \frac{e^2\text{Subst}\left(\int \frac{\ldots}{4cd^2-4bde\ldots}\right.}{c\ldots}$$

$$= -\frac{bcd-b^2e+2ace+c(2cd-be)x^2}{(b^2-4ac)(cd^2-bde+ae^2)\sqrt{a+bx^2+cx^4}} + \frac{e^2\tanh^{-1}\left(\frac{\ldots}{2\sqrt{cd^2-\ldots}}\right.}{2(cd^2\ldots}$$

Mathematica [A]

time = 0.62, size = 177, normalized size = 1.07

$$\frac{-b^2e+2c(ae+cdx^2)+bc(d-ex^2)}{(b^2-4ac)\left(-cd^2+e(bd-ae)\right)\sqrt{a+bx^2+cx^4}}+\frac{e^2\sqrt{-cd^2+e(bd-ae)}\,\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\left(cd^2+e(-bd+ae)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (-(b^2*e) + 2*c*(a*e + c*d*x^2) + b*c*(d - e*x^2))/((b^2 - 4*a*c)*(-(c*d^2)
 + e*(b*d - a*e))*Sqrt[a + b*x^2 + c*x^4]) + (e^2*Sqrt[-(c*d^2) + e*(b*d -
a*e)]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2
) + e*(b*d - a*e)]])/(c*d^2 + e*(-(b*d) + a*e))^2
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(154) = 308$.
time = 0.13, size = 454, normalized size = 2.73

method	result
default	$\frac{2ce\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-eb+2cd\right)\left(e\sqrt{-4ac+b^2}+eb-2cd\right)\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}-$
elliptic	$\frac{2ce\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-eb+2cd\right)\left(e\sqrt{-4ac+b^2}+eb-2cd\right)\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2*c*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/((a
*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d)/e*(x
^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/e*(x^2
+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-2*c/(-4*a*c+b^2)/(e*(-4*a*
c+b^2)^(1/2)-e*b+2*c*d)/(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*((x^2-1/2/c*(-b
+(-4*a*c+b^2)^(1/2)))^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1
/2))))^(1/2)+2*c/(-4*a*c+b^2)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(x^2+1/2*(b+
(-4*a*c+b^2)^(1/2))/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*c+b^2)
^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x/((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 666 vs. 2(158) = 316.
time = 0.63, size = 1377, normalized size = 8.30

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^3 - 4*a*b*c)*x^2)*sqrt(
c*d^2 - b*d*e + a*e^2)*e^2*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a
*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqr
t(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*
(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d
^2)) - 4*(2*c^3*d^3*x^2 + b*c^2*d^3 - (a*b*c*x^2 + a*b^2 - 2*a^2*c)*e^3 + (
(b^2*c + 2*a*c^2)*d*x^2 + (b^3 - a*b*c)*d)*e^2 - (3*b*c^2*d^2*x^2 + 2*(b^2*
c - a*c^2)*d^2)*e)*sqrt(c*x^4 + b*x^2 + a))/((b^2*c^3 - 4*a*c^4)*d^4*x^4 +
(b^3*c^2 - 4*a*b*c^3)*d^4*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*
a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*e^4 - 2*((
a*b^3*c - 4*a^2*b*c^2)*d*x^4 + (a*b^4 - 4*a^2*b^2*c)*d*x^2 + (a^2*b^3 - 4*a
^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^4 + (b^5 - 2*a*b^
3*c - 8*a^2*b*c^2)*d^2*x^2 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2
*((b^3*c^2 - 4*a*b*c^3)*d^3*x^4 + (b^4*c - 4*a*b^2*c^2)*d^3*x^2 + (a*b^3*c
- 4*a^2*b*c^2)*d^3)*e), 1/2*(((b^2*c - 4*a*c^2)*x^4 + a*b^2 - 4*a^2*c + (b^
3 - 4*a*b*c)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x
^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c
^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d
*x^4 + b^2*d*x^2 + a*b*d)*e))*e^2 - 2*(2*c^3*d^3*x^2 + b*c^2*d^3 - (a*b*c*x
^2 + a*b^2 - 2*a^2*c)*e^3 + ((b^2*c + 2*a*c^2)*d*x^2 + (b^3 - a*b*c)*d)*e^2
 - (3*b*c^2*d^2*x^2 + 2*(b^2*c - a*c^2)*d^2)*e)*sqrt(c*x^4 + b*x^2 + a))/((
b^2*c^3 - 4*a*c^4)*d^4*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^4*x^2 + (a*b^2*c^2 - 4
*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3
 - 4*a^3*b*c)*x^2)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^4 + (a*b^4 - 4*a^2*
b^2*c)*d*x^2 + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 - 8*a^2
*c^3)*d^2*x^4 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x^2 + (a*b^4 - 2*a^2*b^
```

```
2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^4 + (b^4*c - 4*a
*b^2*c^2)*d^3*x^2 + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x}{(d+ex^2)\,(a+bx^2+cx^4)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(x/((d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 454 vs. 2(158) = 316.
time = 3.08, size = 454, normalized size = 2.73

$$-\frac{\frac{(2c^3d^3-3bc^2d^2e+b^2cde^2+2ac^2de^2-abce^3)x^2}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}+\frac{bc^2d^3-2b^2cd^2e+2ac^2d^2e+b^3de^2-abcde^2-ab^2e^3+2a^2ce^3}{b^2c^2d^4-4ac^3d^4-2b^3cd^3e+8abc^2d^3e+b^4d^2e^2-2ab^2cd^2e^2-8a^2c^2d^2e^2-2ab^3de^3+8a^2bcde^3+a^2b^2e^4-4a^3ce^4}}{\sqrt{cx^4+bx^2+a}}+\frac{\arctan\left(-\frac{\left(\sqrt{c}x^2-\sqrt{cx^4+bx^2+a}\right)e+\sqrt{c}d}{\sqrt{-cd^2+bde-ae^2}}\right)e^2}{(cd^2-bde+ae^2)\sqrt{-cd^2+bde-ae^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((2*c^3*d^3 - 3*b*c^2*d^2*e + b^2*c*d*e^2 + 2*a*c^2*d*e^2 - a*b*c*e^3)*x^2
/(b^2*c^2*d^4 - 4*a*c^3*d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2
 - 2*a*b^2*c*d^2*e^2 - 8*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3
+ a^2*b^2*e^4 - 4*a^3*c*e^4) + (b*c^2*d^3 - 2*b^2*c*d^2*e + 2*a*c^2*d^2*e +
 b^3*d*e^2 - a*b*c*d*e^2 - a*b^2*e^3 + 2*a^2*c*e^3)/(b^2*c^2*d^4 - 4*a*c^3*
d^4 - 2*b^3*c*d^3*e + 8*a*b*c^2*d^3*e + b^4*d^2*e^2 - 2*a*b^2*c*d^2*e^2 - 8
*a^2*c^2*d^2*e^2 - 2*a*b^3*d*e^3 + 8*a^2*b*c*d*e^3 + a^2*b^2*e^4 - 4*a^3*c*
e^4))/sqrt(c*x^4 + b*x^2 + a) + arctan(-((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2
+ a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^2/((c*d^2 - b*d*e + a*
e^2)*sqrt(-c*d^2 + b*d*e - a*e^2))
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x}{(e\,x^2+d)\,\left(c\,x^4+b\,x^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(x/((d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.346 $\int \frac{1}{x(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=266

$$\frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}}+\frac{e(bcd-b^2e+2ace+c(2cd-be)x^2)}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a+bx^2+cx^4}}-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+}}\right.}{2a^{3/2}d}$$

```
[Out] -1/2*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d-1/2*e
^3*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^
4+b*x^2+a)^(1/2))/d/(a*e^2-b*d*e+c*d^2)^(3/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c
+b^2)/d/(c*x^4+b*x^2+a)^(1/2)+e*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^2)/(-
4*a*c+b^2)/d/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)
```

Rubi [A]

time = 0.24, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 974, 754, 12, 738, 212}

$$-\frac{\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d}+\frac{e\left(2ace+b^2(-e)+cx^2(2cd-be)+bcd\right)}{d\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}\left(ae^2-bde+cd^2\right)}+\frac{-2ac+b^2+bcx^2}{ad\left(b^2-4ac\right)\sqrt{a+bx^2+cx^4}}-\frac{e^3\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d\left(ae^2-bde+cd^2\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*Sqrt[a + b*x^2 + c*x^4]) + (e*(b
*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d*(c*d^2 - b*
d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sq
rt[a + b*x^2 + c*x^4])]/(2*a^(3/2)*d) - (e^3*ArcTanh[(b*d - 2*a*e + (2*c*d
- b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d*
(c*d^2 - b*d*e + a*e^2)^(3/2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
```

```
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx\left(a+bx+cx^2\right)^{3/2}} - \frac{e}{d(d+ex)\left(a+bx+cx^2\right)^{3/2}}\right) \ldots$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)}{2d} - \frac{e\text{Subst}\left(\int \frac{1}{(d+ex)(a+bx+cx^2)^{3/2}}\,dx, x \ldots\right.}{2d}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd-b^2e+2ace+c(2cd- \ldots}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a\ldots}}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd-b^2e+2ace+c(2cd- \ldots}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a\ldots}}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd-b^2e+2ace+c(2cd- \ldots}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a\ldots}}$$

$$= \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)d\sqrt{a+bx^2+cx^4}} + \frac{e(bcd-b^2e+2ace+c(2cd- \ldots}{\left(b^2-4ac\right)d\left(cd^2-bde+ae^2\right)\sqrt{a\ldots}}$$

Mathematica [**A**]
time = 1.12, size = 247, normalized size = 0.93

$$\frac{b^3e-bc(3ae+cdx^2)+2ac^2(d-ex^2)+b^2c(-d+ex^2)}{a\left(-b^2+4ac\right)\left(cd^2+e(-bd+ae)\right)\sqrt{a+bx^2+cx^4}} - \frac{e^3\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\,(d+ex^2)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{d\left(cd^2+e(-bd+ae)\right)^2} + \frac{\tanh^{-1}\left(\frac{\sqrt{c}\,x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] (b^3*e - b*c*(3*a*e + c*d*x^2) + 2*a*c^2*(d - e*x^2) + b^2*c*(-d + e*x^2))/
(a*(-b^2 + 4*a*c)*(c*d^2 + e*(-(b*d) + a*e))*Sqrt[a + b*x^2 + c*x^4]) - (e^
3*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[c]*(d + e*x^2) - e*Sqrt[a + b
*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]])/(d*(c*d^2 + e*(-(b*d) + a*e
))^2) + ArcTanh[(Sqrt[c]*x^2 - Sqrt[a + b*x^2 + c*x^4])/Sqrt[a]]/(a^(3/2)*d
)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 562 vs. $2(242) = 484$.
time = 0.34, size = 563, normalized size = 2.12

method	result
default	$-\frac{e\left(\frac{2ce\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-eb+2cd\right)\left(e\sqrt{-4ac+b^2}+eb-2cd\right)\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}\right.}{}$
elliptic	$-\frac{2c\,e^2\ln\left(\frac{\frac{2a\,e^2-2deb+2c\,d^2}{e^2}+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+2\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}\sqrt{c\left(x^2+\frac{d}{e}\right)^2+\frac{(eb-2cd)\left(x^2+\frac{d}{e}\right)}{e}+\frac{a\,e^2-deb+c\,d^2}{e^2}}}{x^2+\frac{d}{e}}\right)}{\left(e\sqrt{-4ac+b^2}-eb+2cd\right)\left(e\sqrt{-4ac+b^2}+eb-2cd\right)d\sqrt{\frac{a\,e^2-deb+c\,d^2}{e^2}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -e/d*(2*c*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*
d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*c*d
)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*d)/
e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-2*c/(-4*a*c+b^2)/(e*
(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*((x^2-1/2
/c*(-b+(-4*a*c+b^2)^(1/2)))^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*c+b
^2)^(1/2))))^(1/2)+2*c/(-4*a*c+b^2)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(x^2+1
/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4*a*
c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2))+1/d*(1/2/a/(c*x^4+b
*x^2+a)^(1/2)-1/2*b/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2/a^(
3/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1203 vs. $2(247) = 494$.

time = 2.48, size = 4917, normalized size = 18.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b
*c)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*e^3*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^
2 + (b^2 + 4*a*c)*d^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2
 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8
*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 +
 2*d*x^2*e + d^2)) + ((b^2*c^3 - 4*a*c^4)*d^4*x^4 + (b^3*c^2 - 4*a*b*c^3)*d
^4*x^2 + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*
a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*
d*x^4 + (a*b^4 - 4*a^2*b^2*c)*d*x^2 + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*
c - 2*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^4 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*
x^2 + (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)
*d^3*x^4 + (b^4*c - 4*a*b^2*c^2)*d^3*x^2 + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e)*
sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*
x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) + 4*(a*b*c^3*d^4*x^2 + (a*b^2*c^2 - 2*a^2*
c^3)*d^4 - ((a^2*b^2*c - 2*a^3*c^2)*d*x^2 + (a^2*b^3 - 3*a^3*b*c)*d)*e^3 +
((a*b^3*c - a^2*b*c^2)*d^2*x^2 + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2)*e^2
 - (2*(a*b^2*c^2 - a^2*c^3)*d^3*x^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d^3)*e)*sqr
t(c*x^4 + b*x^2 + a))/((a^2*b^2*c^3 - 4*a^3*c^4)*d^5*x^4 + (a^2*b^3*c^2 - 4
*a^3*b*c^3)*d^5*x^2 + (a^3*b^2*c^2 - 4*a^4*c^3)*d^5 + ((a^4*b^2*c - 4*a^5*c
^2)*d*x^4 + (a^4*b^3 - 4*a^5*b*c)*d*x^2 + (a^5*b^2 - 4*a^6*c)*d)*e^4 - 2*((
a^3*b^3*c - 4*a^4*b*c^2)*d^2*x^4 + (a^3*b^4 - 4*a^4*b^2*c)*d^2*x^2 + (a^4*b
^3 - 4*a^5*b*c)*d^2)*e^3 + ((a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*x^4
 + (a^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*x^2 + (a^3*b^4 - 2*a^4*b^2*c -
 8*a^5*c^2)*d^3)*e^2 - 2*((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*x^4 + (a^2*b^4*c
 - 4*a^3*b^2*c^2)*d^4*x^2 + (a^3*b^3*c - 4*a^4*b*c^2)*d^4)*e), -1/4*(2*(a^3*
b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*sq
rt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 +
 b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2)/(c^2*d^2*x^4 + b*c*d^2
*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b*c*d*x^4 + b^2*d*x^2 + a
*b*d)*e))*e^3 - ((b^2*c^3 - 4*a*c^4)*d^4*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^4*x^
2 + (a*b^2*c^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c
^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^4
 + (a*b^4 - 4*a^2*b^2*c)*d*x^2 + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2
*a*b^2*c^2 - 8*a^2*c^3)*d^2*x^4 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x^2 +
 (a*b^4 - 2*a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*
x^4 + (b^4*c - 4*a*b^2*c^2)*d^3*x^2 + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e)*sqrt(
a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 +
 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*(a*b*c^3*d^4*x^2 + (a*b^2*c^2 - 2*a^2*c^3)*
```

```
d^4 - ((a^2*b^2*c - 2*a^3*c^2)*d*x^2 + (a^2*b^3 - 3*a^3*b*c)*d)*e^3 + ((a*b
^3*c - a^2*b*c^2)*d^2*x^2 + (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2)*e^2 - (2
*(a*b^2*c^2 - a^2*c^3)*d^3*x^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d^3)*e)*sqrt(c*x
^4 + b*x^2 + a))/((a^2*b^2*c^3 - 4*a^3*c^4)*d^5*x^4 + (a^2*b^3*c^2 - 4*a^3*
b*c^3)*d^5*x^2 + (a^3*b^2*c^2 - 4*a^4*c^3)*d^5 + ((a^4*b^2*c - 4*a^5*c^2)*d
*x^4 + (a^4*b^3 - 4*a^5*b*c)*d*x^2 + (a^5*b^2 - 4*a^6*c)*d)*e^4 - 2*((a^3*b
^3*c - 4*a^4*b*c^2)*d^2*x^4 + (a^3*b^4 - 4*a^4*b^2*c)*d^2*x^2 + (a^4*b^3 -
4*a^5*b*c)*d^2)*e^3 + ((a^2*b^4*c - 2*a^3*b^2*c^2 - 8*a^4*c^3)*d^3*x^4 + (a
^2*b^5 - 2*a^3*b^3*c - 8*a^4*b*c^2)*d^3*x^2 + (a^3*b^4 - 2*a^4*b^2*c - 8*a^
5*c^2)*d^3)*e^2 - 2*((a^2*b^3*c^2 - 4*a^3*b*c^3)*d^4*x^4 + (a^2*b^4*c - 4*a
^3*b^2*c^2)*d^4*x^2 + (a^3*b^3*c - 4*a^4*b*c^2)*d^4)*e), 1/4*((a^3*b^2 - 4*
a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a^2*b^3 - 4*a^3*b*c)*x^2)*sqrt(c*d^2
 - b*d*e + a*e^2)*e^3*log(-(8*c^2*d^2*x^4 + 8*b*c*d^2*x^2 + (b^2 + 4*a*c)*d
^2 - 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(c*d
^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b*x^2 + 8*a^2)*e^2 - 2*(4*b*
c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x^4*e^2 + 2*d*x^2*e + d^2))
+ 2*((b^2*c^3 - 4*a*c^4)*d^4*x^4 + (b^3*c^2 - 4*a*b*c^3)*d^4*x^2 + (a*b^2*c
^2 - 4*a^2*c^3)*d^4 + (a^3*b^2 - 4*a^4*c + (a^2*b^2*c - 4*a^3*c^2)*x^4 + (a
^2*b^3 - 4*a^3*b*c)*x^2)*e^4 - 2*((a*b^3*c - 4*a^2*b*c^2)*d*x^4 + (a*b^4 -
4*a^2*b^2*c)*d*x^2 + (a^2*b^3 - 4*a^3*b*c)*d)*e^3 + ((b^4*c - 2*a*b^2*c^2 -
 8*a^2*c^3)*d^2*x^4 + (b^5 - 2*a*b^3*c - 8*a^2*b*c^2)*d^2*x^2 + (a*b^4 - 2*
a^2*b^2*c - 8*a^3*c^2)*d^2)*e^2 - 2*((b^3*c^2 - 4*a*b*c^3)*d^3*x^4 + (b^4*c
 - 4*a*b^2*c^2)*d^3*x^2 + (a*b^3*c - 4*a^2*b*c^2)*d^3)*e)*sqrt(-a)*arctan(1
/2*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(-a)/(a*c*x^4 + a*b*x^2 + a^2)
) + 4*(a*b*c^3*d^4*x^2 + (a*b^2*c^2 - 2*a^2*c^3)*d^4 - ((a^2*b^2*c - 2*a^3*
c^2)*d*x^2 + (a^2*b^3 - 3*a^3*b*c)*d)*e^3 + ((a*b^3*c - a^2*b*c^2)*d^2*x^2
+ (a*b^4 - 2*a^2*b^2*c - 2*a^3*c^2)*d^2)*e^2 - (2*(a*b^2*c^2 - a^2*c^3)*d^3
*x^2 + (2*a*b^3*c - 5*a^2*b*c^2)*d^3)*e)*sqrt(c*x^4 + b*x^2 + a))/((a^2*b^2
*c^3 - 4*a^3*c^4)*d^5*x^4 + (a^2*b^3*c^2 - 4*a^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Error: Bad Argument Type
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x\,\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
```

```
[Out] int(1/(x*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.347 $\int \frac{1}{x^3(d+ex^2)(a+bx^2+cx^4)^{3/2}}\,dx$

Optimal. Leaf size=419

$$-\frac{e(b^2-2ac+bcx^2)}{a(b^2-4ac)d^2\sqrt{a+bx^2+cx^4}}+\frac{b^2-2ac+bcx^2}{a(b^2-4ac)dx^2\sqrt{a+bx^2+cx^4}}-\frac{e^2(bcd-b^2e+2ace+c(2cd-be)}{(b^2-4ac)d^2(cd^2-bde+ae^2)\sqrt{a+b}}$$

[Out] 3/4*b*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(5/2)/d+1/2*e*arctanh(1/2*(b*x^2+2*a)/a^(1/2)/(c*x^4+b*x^2+a)^(1/2))/a^(3/2)/d^2+1/2*e^4*arctanh(1/2*(b*d-2*a*e+(-b*e+2*c*d)*x^2)/(a*e^2-b*d*e+c*d^2)^(1/2)/(c*x^4+b*x^2+a)^(1/2))/d^2/(a*e^2-b*d*e+c*d^2)^(3/2)-e*(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d^2/(c*x^4+b*x^2+a)^(1/2)+(b*c*x^2-2*a*c+b^2)/a/(-4*a*c+b^2)/d/x^2/(c*x^4+b*x^2+a)^(1/2)-e^2*(b*c*d-b^2*e+2*a*c*e+c*(-b*e+2*c*d)*x^2)/(-4*a*c+b^2)/d^2/(a*e^2-b*d*e+c*d^2)/(c*x^4+b*x^2+a)^(1/2)-1/2*(-8*a*c+3*b^2)*(c*x^4+b*x^2+a)^(1/2)/a^2/(-4*a*c+b^2)/d/x^2

Rubi [A]
time = 0.35, antiderivative size = 419, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1265, 974, 754, 820, 738, 212, 12}

$$\frac{e\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{2a^{3/2}d^2}+\frac{3b\tanh^{-1}\left(\frac{2a+bx^2}{2\sqrt{a}\sqrt{a+bx^2+cx^4}}\right)}{4a^{5/2}d}-\frac{(3b^2-8ac)\sqrt{a+bx^2+cx^4}}{2a^2dx^2(b^2-4ac)}-\frac{e^2(2ace+b^2(-e)+cx^2(2cd-be)+bcd)}{d^2(b^2-4ac)\sqrt{a+bx^2+cx^4}(ae^2-bde+cd^2)}-\frac{e(-2ac+b^2+bcx^2)}{ad^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{-2ac+b^2+bcx^2}{adx^2(b^2-4ac)\sqrt{a+bx^2+cx^4}}+\frac{e^4\tanh^{-1}\left(\frac{-2ae+x^2(2cd-be)+bd}{2\sqrt{a+bx^2+cx^4}\sqrt{ae^2-bde+cd^2}}\right)}{2d^2(ae^2-bde+cd^2)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]

[Out] -((e*(b^2 - 2*a*c + b*c*x^2))/(a*(b^2 - 4*a*c)*d^2*Sqrt[a + b*x^2 + c*x^4])) + (b^2 - 2*a*c + b*c*x^2)/(a*(b^2 - 4*a*c)*d*x^2*Sqrt[a + b*x^2 + c*x^4]) - (e^2*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)*x^2))/((b^2 - 4*a*c)*d^2*(c*d^2 - b*d*e + a*e^2)*Sqrt[a + b*x^2 + c*x^4]) - ((3*b^2 - 8*a*c)*Sqrt[a + b*x^2 + c*x^4])/(2*a^2*(b^2 - 4*a*c)*d*x^2) + (3*b*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(4*a^(5/2)*d) + (e*ArcTanh[(2*a + b*x^2)/(2*Sqrt[a]*Sqrt[a + b*x^2 + c*x^4])])/(2*a^(3/2)*d^2) + (e^4*ArcTanh[(b*d - 2*a*e + (2*c*d - b*e)*x^2)/(2*Sqrt[c*d^2 - b*d*e + a*e^2]*Sqrt[a + b*x^2 + c*x^4])])/(2*d^2*(c*d^2 - b*d*e + a*e^2)^(3/2))

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 738

```
Int[1/(((d_.) + (e_.)*(x_))*Sqrt[(a_.) + (b_.)*(x_) + (c_.)*(x_)^2]), x_Sym
bol] :> Dist[-2, Subst[Int[1/(4*c*d^2 - 4*b*d*e + 4*a*e^2 - x^2), x], x, (2
*a*e - b*d - (2*c*d - b*e)*x)/Sqrt[a + b*x + c*x^2]], x] /; FreeQ[{a, b, c,
 d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 754

```
Int[((d_.) + (e_.)*(x_))^(m_)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_), x_S
ymbol] :> Simp[(d + e*x)^(m + 1)*(b*c*d - b^2*e + 2*a*c*e + c*(2*c*d - b*e)
*x)*((a + b*x + c*x^2)^(p + 1)/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^
2))), x] + Dist[1/((p + 1)*(b^2 - 4*a*c)*(c*d^2 - b*d*e + a*e^2)), Int[(d +
 e*x)^m*Simp[b*c*d*e*(2*p - m + 2) + b^2*e^2*(m + p + 2) - 2*c^2*d^2*(2*p +
 3) - 2*a*c*e^2*(m + 2*p + 3) - c*e*(2*c*d - b*e)*(m + 2*p + 4)*x, x]*(a +
b*x + c*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && LtQ[p,
-1] && IntQuadraticQ[a, b, c, d, e, m, p, x]
```

Rule 820

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)^(p_.), x_Symbol] :> Simp[(-(e*f - d*g))*(d + e*x)^(m + 1)*((a +
 b*x + c*x^2)^(p + 1)/(2*(p + 1)*(c*d^2 - b*d*e + a*e^2))), x] - Dist[(b*(e
*f + d*g) - 2*(c*d*f + a*e*g))/(2*(c*d^2 - b*d*e + a*e^2)), Int[(d + e*x)^(
m + 1)*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, p}, x]
 && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && EqQ[Simplify[m
+ 2*p + 3], 0]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{1}{x^3\left(d+ex^2\right)\left(a+bx^2+cx^4\right)^{3/2}}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{1}{x^2(d+ex)\left(a+bx+cx^2\right)^{3/2}}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{1}{dx^2\left(a+bx+cx^2\right)^{3/2}} - \frac{e}{d^2x\left(a+bx+cx^2\right)^{3/2}} + \frac{}{d^2(d\ \cdots}\right.\right.$$

$$= \frac{\text{Subst}\left(\int \frac{1}{x^2(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right)}{2d} - \frac{e\text{Subst}\left(\int \frac{1}{x(a+bx+cx^2)^{3/2}}\,dx, x, x^2\right.}{2d^2}$$

$$= -\frac{e(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{e(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{e(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}}$$

$$= -\frac{e(b^2-2ac+bcx^2)}{a\left(b^2-4ac\right)d^2\sqrt{a+bx^2+cx^4}} + \frac{b^2-2ac+bcx^2}{a\left(b^2-4ac\right)dx^2\sqrt{a+bx^2+cx^4}}$$

Mathematica [A]
time = 1.71, size = 363, normalized size = 0.87

$$-\frac{\frac{d\left(4a^3ce^2+3b^2d(-cd+be)x^2\left(b+cx^2\right)+a^2\left(-b^2e^2+4bce\left(-d+ex^2\right)+4c^2\left(d^2+dex^2+e^2x^4\right)\right)+a\left(8c^3d^2x^4+b^3e\left(d-ex^2\right)+10bc^2dx^2\left(d-ex^2\right)-b^2c\left(d^2+12dex^2+e^2x^4\right)\right)\right)}{a^2(b^2-4ac)(-cd^2+e(bd-ae))x^2\sqrt{a+bx^2+cx^4}} - \frac{2e^4\sqrt{-cd^2+bde-ae^2}\tan^{-1}\left(\frac{\sqrt{c}\left(d+ex^2\right)-e\sqrt{a+bx^2+cx^4}}{\sqrt{-cd^2+e(bd-ae)}}\right)}{\left(cd^2+e(-bd+ae)\right)^2} + \frac{(3bd+2ae)\tanh^{-1}\left(\frac{\sqrt{c}x^2-\sqrt{a+bx^2+cx^4}}{\sqrt{a}}\right)}{a^{5/2}}}{2d^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x]
```

```
[Out] -1/2*((d*(4*a^3*c*e^2 + 3*b^2*d*(-(c*d) + b*e)*x^2*(b + c*x^2) + a^2*(-(b^2
*e^2) + 4*b*c*e*(-d + e*x^2) + 4*c^2*(d^2 + d*e*x^2 + e^2*x^4)) + a*(8*c^3*
d^2*x^4 + b^3*e*(d - e*x^2) + 10*b*c^2*d*x^2*(d - e*x^2) - b^2*c*(d^2 + 12*
d*e*x^2 + e^2*x^4))))/(a^2*(b^2 - 4*a*c)*(-(c*d^2) + e*(b*d - a*e))*x^2*Sqr
t[a + b*x^2 + c*x^4]) - (2*e^4*Sqrt[-(c*d^2) + b*d*e - a*e^2]*ArcTan[(Sqrt[
c]*(d + e*x^2) - e*Sqrt[a + b*x^2 + c*x^4])/Sqrt[-(c*d^2) + e*(b*d - a*e)]]
)/(c*d^2 + e*(-(b*d) + a*e))^2 + ((3*b*d + 2*a*e)*ArcTanh[(Sqrt[c]*x^2 - Sq
rt[a + b*x^2 + c*x^4])/Sqrt[a]])/a^(5/2))/d^2
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 763 vs. $2(379) = 758$.
time = 0.41, size = 764, normalized size = 1.82 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] e^2/d^2*(2*c*e/(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/(e*(-4*a*c+b^2)^(1/2)+e*b-2
*c*d)/((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*ln((2*(a*e^2-b*d*e+c*d^2)/e^2+(b*e-2*
c*d)/e*(x^2+d/e)+2*((a*e^2-b*d*e+c*d^2)/e^2)^(1/2)*(c*(x^2+d/e)^2+(b*e-2*c*
d)/e*(x^2+d/e)+(a*e^2-b*d*e+c*d^2)/e^2)^(1/2))/(x^2+d/e))-2*c/(-4*a*c+b^2)/
(e*(-4*a*c+b^2)^(1/2)-e*b+2*c*d)/(x^2-1/2/c*(-b+(-4*a*c+b^2)^(1/2)))*((x^2-
1/2/c*(-b+(-4*a*c+b^2)^(1/2)))^2*c+(-4*a*c+b^2)^(1/2)*(x^2-1/2/c*(-b+(-4*a*
c+b^2)^(1/2))))^(1/2)+2*c/(-4*a*c+b^2)/(e*(-4*a*c+b^2)^(1/2)+e*b-2*c*d)/(x^
2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)*((x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c)^2*c-(-4
*a*c+b^2)^(1/2)*(x^2+1/2*(b+(-4*a*c+b^2)^(1/2))/c))^(1/2))+1/d*(-1/2/a/x^2/
(c*x^4+b*x^2+a)^(1/2)-3/4*b/a^2/(c*x^4+b*x^2+a)^(1/2)+3/2*b^2/a^2/(4*a*c-b^
2)/(c*x^4+b*x^2+a)^(1/2)*x^2*c+3/4*b^3/a^2/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2
)+3/4*b/a^(5/2)*ln((2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2)-2*c/a*(
2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2))-e/d^2*(1/2/a/(c*x^4+b*x^2+a)^
(1/2)-1/2*b/a*(2*c*x^2+b)/(4*a*c-b^2)/(c*x^4+b*x^2+a)^(1/2)-1/2/a^(3/2)*ln(
(2*a+b*x^2+2*a^(1/2)*(c*x^4+b*x^2+a)^(1/2))/x^2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)^(3/2)*(x^2*e + d)*x^3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(384) = 768$.
time = 5.42, size = 6590, normalized size = 15.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2
 - 4*a^5*c)*x^2)*sqrt(c*d^2 - b*d*e + a*e^2)*e^4*log(-(8*c^2*d^2*x^4 + 8*b*
c*d^2*x^2 + (b^2 + 4*a*c)*d^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(2*c*d*x^2 + b*d
- (b*x^2 + 2*a)*e)*sqrt(c*d^2 - b*d*e + a*e^2) + ((b^2 + 4*a*c)*x^4 + 8*a*b
```

```
*x^2 + 8*a^2)*e^2 - 2*(4*b*c*d*x^4 + (3*b^2 + 4*a*c)*d*x^2 + 4*a*b*d)*e)/(x
^4*e^2 + 2*d*x^2*e + d^2)) + (3*(b^3*c^3 - 4*a*b*c^4)*d^5*x^6 + 3*(b^4*c^2
 - 4*a*b^2*c^3)*d^5*x^4 + 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5*x^2 + 2*((a^3*b^2*
c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^5*c)*x^2)*e
^5 - ((a^2*b^3*c - 4*a^3*b*c^2)*d*x^6 + (a^2*b^4 - 4*a^3*b^2*c)*d*x^4 + (a^
3*b^3 - 4*a^4*b*c)*d*x^2)*e^4 - 4*((a*b^4*c - 5*a^2*b^2*c^2 + 4*a^3*c^3)*d^
2*x^6 + (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*x^4 + (a^2*b^4 - 5*a^3*b^2*
c + 4*a^4*c^2)*d^2*x^2)*e^3 + ((3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b*c^3)*d^3*x
^6 + (3*b^6 - 10*a*b^4*c - 8*a^2*b^2*c^2)*d^3*x^4 + (3*a*b^5 - 10*a^2*b^3*c
 - 8*a^3*b*c^2)*d^3*x^2)*e^2 - 2*((3*b^4*c^2 - 13*a*b^2*c^3 + 4*a^2*c^4)*d^
4*x^6 + (3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*x^4 + (3*a*b^4*c - 13*a^
2*b^2*c^2 + 4*a^3*c^3)*d^4*x^2)*e)*sqrt(a)*log(-((b^2 + 4*a*c)*x^4 + 8*a*b*
x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x^4) - 4*((3
*a*b^2*c^3 - 8*a^2*c^4)*d^5*x^4 + (3*a*b^3*c^2 - 10*a^2*b*c^3)*d^5*x^2 + (a
^2*b^2*c^2 - 4*a^3*c^3)*d^5 + ((a^3*b^2*c - 4*a^4*c^2)*d*x^4 + (a^3*b^3 - 4
*a^4*b*c)*d*x^2 + (a^4*b^2 - 4*a^5*c)*d)*e^4 - 2*((2*a^2*b^3*c - 7*a^3*b*c^
2)*d^2*x^4 + 2*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^2)*d^2*x^2 + (a^3*b^3 - 4*a^4
*b*c)*d^2)*e^3 + (3*(a*b^4*c - 2*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*x^4 + (3*a*b^
5 - 8*a^2*b^3*c - 10*a^3*b*c^2)*d^3*x^2 + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^
2)*d^3)*e^2 - 2*(3*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*x^4 + (3*a*b^4*c - 11*a^2*
b^2*c^2 + 2*a^3*c^3)*d^4*x^2 + (a^2*b^3*c - 4*a^3*b*c^2)*d^4)*e)*sqrt(c*x^4
 + b*x^2 + a))/((a^3*b^2*c^3 - 4*a^4*c^4)*d^6*x^6 + (a^3*b^3*c^2 - 4*a^4*b*
c^3)*d^6*x^4 + (a^4*b^2*c^2 - 4*a^5*c^3)*d^6*x^2 + ((a^5*b^2*c - 4*a^6*c^2)
*d^2*x^6 + (a^5*b^3 - 4*a^6*b*c)*d^2*x^4 + (a^6*b^2 - 4*a^7*c)*d^2*x^2)*e^4
 - 2*((a^4*b^3*c - 4*a^5*b*c^2)*d^3*x^6 + (a^4*b^4 - 4*a^5*b^2*c)*d^3*x^4 +
 (a^5*b^3 - 4*a^6*b*c)*d^3*x^2)*e^3 + ((a^3*b^4*c - 2*a^4*b^2*c^2 - 8*a^5*c
^3)*d^4*x^6 + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*b*c^2)*d^4*x^4 + (a^4*b^4 - 2*
a^5*b^2*c - 8*a^6*c^2)*d^4*x^2)*e^2 - 2*((a^3*b^3*c^2 - 4*a^4*b*c^3)*d^5*x^
6 + (a^3*b^4*c - 4*a^4*b^2*c^2)*d^5*x^4 + (a^4*b^3*c - 4*a^5*b*c^2)*d^5*x^2
)*e), 1/8*(4*((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^
4*b^2 - 4*a^5*c)*x^2)*sqrt(-c*d^2 + b*d*e - a*e^2)*arctan(-1/2*sqrt(c*x^4 +
 b*x^2 + a)*(2*c*d*x^2 + b*d - (b*x^2 + 2*a)*e)*sqrt(-c*d^2 + b*d*e - a*e^2
)/(c^2*d^2*x^4 + b*c*d^2*x^2 + a*c*d^2 + (a*c*x^4 + a*b*x^2 + a^2)*e^2 - (b
*c*d*x^4 + b^2*d*x^2 + a*b*d)*e))*e^4 + (3*(b^3*c^3 - 4*a*b*c^4)*d^5*x^6 +
3*(b^4*c^2 - 4*a*b^2*c^3)*d^5*x^4 + 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5*x^2 + 2
*((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*x^4 + (a^4*b^2 - 4*a^
5*c)*x^2)*e^5 - ((a^2*b^3*c - 4*a^3*b*c^2)*d*x^6 + (a^2*b^4 - 4*a^3*b^2*c)*
d*x^4 + (a^3*b^3 - 4*a^4*b*c)*d*x^2)*e^4 - 4*((a*b^4*c - 5*a^2*b^2*c^2 + 4*
a^3*c^3)*d^2*x^6 + (a*b^5 - 5*a^2*b^3*c + 4*a^3*b*c^2)*d^2*x^4 + (a^2*b^4 -
 5*a^3*b^2*c + 4*a^4*c^2)*d^2*x^2)*e^3 + ((3*b^5*c - 10*a*b^3*c^2 - 8*a^2*b
*c^3)*d^3*x^6 + (3*b^6 - 10*a*b^4*c - 8*a^2*b^2*c^2)*d^3*x^4 + (3*a*b^5 - 1
0*a^2*b^3*c - 8*a^3*b*c^2)*d^3*x^2)*e^2 - 2*((3*b^4*c^2 - 13*a*b^2*c^3 + 4*
a^2*c^4)*d^4*x^6 + (3*b^5*c - 13*a*b^3*c^2 + 4*a^2*b*c^3)*d^4*x^4 + (3*a*b^
4*c - 13*a^2*b^2*c^2 + 4*a^3*c^3)*d^4*x^2)*e)*sqrt(a)*log(-((b^2 + 4*a*c)*x
^4 + 8*a*b*x^2 + 4*sqrt(c*x^4 + b*x^2 + a)*(b*x^2 + 2*a)*sqrt(a) + 8*a^2)/x
```

```
^4) - 4*((3*a*b^2*c^3 - 8*a^2*c^4)*d^5*x^4 + (3*a*b^3*c^2 - 10*a^2*b*c^3)*d
^5*x^2 + (a^2*b^2*c^2 - 4*a^3*c^3)*d^5 + ((a^3*b^2*c - 4*a^4*c^2)*d*x^4 + (
a^3*b^3 - 4*a^4*b*c)*d*x^2 + (a^4*b^2 - 4*a^5*c)*d)*e^4 - 2*((2*a^2*b^3*c -
 7*a^3*b*c^2)*d^2*x^4 + 2*(a^2*b^4 - 4*a^3*b^2*c + a^4*c^2)*d^2*x^2 + (a^3*
b^3 - 4*a^4*b*c)*d^2)*e^3 + (3*(a*b^4*c - 2*a^2*b^2*c^2 - 4*a^3*c^3)*d^3*x^
4 + (3*a*b^5 - 8*a^2*b^3*c - 10*a^3*b*c^2)*d^3*x^2 + (a^2*b^4 - 2*a^3*b^2*c
 - 8*a^4*c^2)*d^3)*e^2 - 2*(3*(a*b^3*c^2 - 3*a^2*b*c^3)*d^4*x^4 + (3*a*b^4*
c - 11*a^2*b^2*c^2 + 2*a^3*c^3)*d^4*x^2 + (a^2*b^3*c - 4*a^3*b*c^2)*d^4)*e)
*sqrt(c*x^4 + b*x^2 + a))/((a^3*b^2*c^3 - 4*a^4*c^4)*d^6*x^6 + (a^3*b^3*c^2
 - 4*a^4*b*c^3)*d^6*x^4 + (a^4*b^2*c^2 - 4*a^5*c^3)*d^6*x^2 + ((a^5*b^2*c -
 4*a^6*c^2)*d^2*x^6 + (a^5*b^3 - 4*a^6*b*c)*d^2*x^4 + (a^6*b^2 - 4*a^7*c)*d
^2*x^2)*e^4 - 2*((a^4*b^3*c - 4*a^5*b*c^2)*d^3*x^6 + (a^4*b^4 - 4*a^5*b^2*c
)*d^3*x^4 + (a^5*b^3 - 4*a^6*b*c)*d^3*x^2)*e^3 + ((a^3*b^4*c - 2*a^4*b^2*c^
2 - 8*a^5*c^3)*d^4*x^6 + (a^3*b^5 - 2*a^4*b^3*c - 8*a^5*b*c^2)*d^4*x^4 + (a
^4*b^4 - 2*a^5*b^2*c - 8*a^6*c^2)*d^4*x^2)*e^2 - 2*((a^3*b^3*c^2 - 4*a^4*b*
c^3)*d^5*x^6 + (a^3*b^4*c - 4*a^4*b^2*c^2)*d^5*x^4 + (a^4*b^3*c - 4*a^5*b*c
^2)*d^5*x^2)*e), 1/4*((((a^3*b^2*c - 4*a^4*c^2)*x^6 + (a^3*b^3 - 4*a^4*b*c)*
x^4 + (a^4*b^2 - 4*a^5*c)*x^2)*sqrt(c*d^2 - b*d...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^3 (d + ex^2) (a + bx^2 + cx^4)^{\frac{3}{2}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**3/(e*x**2+d)/(c*x**4+b*x**2+a)**(3/2),x)
```

```
[Out] Integral(1/(x**3*(d + e*x**2)*(a + b*x**2 + c*x**4)**(3/2)), x)
```

Giac [**A**]
time = 5.00, size = 762, normalized size = 1.82

$$-\frac{\frac{(a^2b^2c^3d^3 - 2a^3c^4d^3 - 2a^2b^3c^2d^2e + 5a^3bc^3d^2e + a^2b^4cde^2 - 2a^3b^2c^2de^2 - 2a^4c^3de^2 - a^3b^3ce^3 + 3a^4bc^2e^3)x^2}{a^4b^2c^2d^4 - 4a^5c^3d^4 - 2a^4b^3cd^3e + 8a^5bc^2d^3e + a^4b^4d^2e^2 - 2a^5b^2cd^2e^2 - 8a^6c^2d^2e^2 - 2a^5b^3de^3 + 8a^6bcde^3 + a^6b^2e^4 - 4a^7ce^4} + \frac{a^2b^3c^2d^3 - 3a^3bc^3d^3 - 2a^2b^4cd^2e + 7a^3b^2c^2d^2e - 2a^4c^3d^2e + a^2b^5de^2 - 3a^3b^3cde^2 - a^4bc^2de^2 - a^3b^4e^3 + 4a^4b^2ce^3 - 2a^5c^2e^3}{a^4b^2c^2d^4 - 4a^5c^3d^4 - 2a^4b^3cd^3e + 8a^5bc^2d^3e + a^4b^4d^2e^2 - 2a^5b^2cd^2e^2 - 8a^6c^2d^2e^2 - 2a^5b^3de^3 + 8a^6bcde^3 + a^6b^2e^4 - 4a^7ce^4}}{\sqrt{cx^4 + bx^2 + a}} + \frac{\arctan\left(-\frac{\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)e + \sqrt{c}d}{\sqrt{-cd^2 + bde - ae^2}}\right)e^4}{(cd^4 - bd^3e + ad^2e^2)\sqrt{-cd^2 + bde - ae^2}} - \frac{(3bd + 2ae)\arctan\left(-\frac{\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}}{\sqrt{-a}}\right)}{2\sqrt{-a}a^2d^2} + \frac{\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)b + 2a\sqrt{c}}{2\left(\left(\sqrt{c}x^2 - \sqrt{cx^4 + bx^2 + a}\right)^2 - a\right)a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^3/(e*x^2+d)/(c*x^4+b*x^2+a)^(3/2),x, algorithm="giac")
```

```
[Out] -((a^2*b^2*c^3*d^3 - 2*a^3*c^4*d^3 - 2*a^2*b^3*c^2*d^2*e + 5*a^3*b*c^3*d^2*
e + a^2*b^4*c*d*e^2 - 2*a^3*b^2*c^2*d*e^2 - 2*a^4*c^3*d*e^2 - a^3*b^3*c*e^3
 + 3*a^4*b*c^2*e^3)*x^2/(a^4*b^2*c^2*d^4 - 4*a^5*c^3*d^4 - 2*a^4*b^3*c*d^3*
e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 - 2*a^5*b^2*c*d^2*e^2 - 8*a^6*c^2*d
^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3 + a^6*b^2*e^4 - 4*a^7*c*e^4) + (
a^2*b^3*c^2*d^3 - 3*a^3*b*c^3*d^3 - 2*a^2*b^4*c*d^2*e + 7*a^3*b^2*c^2*d^2*e
 - 2*a^4*c^3*d^2*e + a^2*b^5*d*e^2 - 3*a^3*b^3*c*d*e^2 - a^4*b*c^2*d*e^2 -
```

```
a^3*b^4*e^3 + 4*a^4*b^2*c*e^3 - 2*a^5*c^2*e^3)/(a^4*b^2*c^2*d^4 - 4*a^5*c^3
*d^4 - 2*a^4*b^3*c*d^3*e + 8*a^5*b*c^2*d^3*e + a^4*b^4*d^2*e^2 - 2*a^5*b^2*
c*d^2*e^2 - 8*a^6*c^2*d^2*e^2 - 2*a^5*b^3*d*e^3 + 8*a^6*b*c*d*e^3 + a^6*b^2
*e^4 - 4*a^7*c*e^4))/sqrt(c*x^4 + b*x^2 + a) + arctan(-((sqrt(c)*x^2 - sqrt
(c*x^4 + b*x^2 + a))*e + sqrt(c)*d)/sqrt(-c*d^2 + b*d*e - a*e^2))*e^4/((c*d
^4 - b*d^3*e + a*d^2*e^2)*sqrt(-c*d^2 + b*d*e - a*e^2)) - 1/2*(3*b*d + 2*a*
e)*arctan(-(sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))/sqrt(-a))/(sqrt(-a)*a^2*
d^2) + 1/2*((sqrt(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))*b + 2*a*sqrt(c))/(((sqr
t(c)*x^2 - sqrt(c*x^4 + b*x^2 + a))^2 - a)*a^2*d)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^3\,\left(e\,x^2+d\right)\,{\left(c\,x^4+b\,x^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)),x)
[Out] int(1/(x^3*(d + e*x^2)*(a + b*x^2 + c*x^4)^(3/2)), x)
```

3.348 $\int \frac{x^8}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx$

Optimal. Leaf size=449

$$\frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{20}x\sqrt{1+2x^2+2x^4}+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}+\frac{27}{80}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)-$$

```
[Out] 27/400*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/20*x^3*(-2*x
^2+1)/(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^
2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/
cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))
^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(
2*x^4+2*x^2+1)^(1/2)+27/160*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan
(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2
^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))
^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)+1/16*(cos(2*arctan(2^
(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*
x)),1/2*(2-2^(1/2))^(1/2))*(-2+7*2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/
(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 566, normalized size of antiderivative = 1.26, number of steps used = 10, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1327, 1289, 1293, 1211, 1117, 1209, 1339, 1720}

$$-\frac{27\sqrt{\frac{3}{10}}\left(3-\sqrt{2}\right)\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{40\left(2-3\sqrt{2}\right)}-\frac{\left(7+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{40\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{9\left(1-3\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{27\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{80\,2^{3/4}\left(2-3\sqrt{2}\right)\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{1}{20}\sqrt{2x^4+2x^2+1}\,x+\frac{\left(1-2x^2\right)x^3}{20\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x^3*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4]
)/20 + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (27*Sqr
t[3/10]*(3 - Sqrt[2])*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]])/(40*(2
 - 3*Sqrt[2])) - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1
+ 2*x^2 + 2*x^4]) + (9*(1 - 3*Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(20*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4]) - ((7 + Sqrt[2])*(1
+ Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*Ar
cTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(40*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (
27*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^
```

```
2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(80*2^(3/4)*(2 - 3*Sqrt[2])*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1293

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[e*f*(f*x)^(m - 1)*((a + b*x^2 + c*x^4)^(p +
1)/(c*(m + 4*p + 3))), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1327

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[-f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2)), Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1)
/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1339

```
Int[(x_)^4/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[-(2*c*d - a*e*q)/(c*e*(e - d*q))
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + (-Dist[1/(e*q), Int[(1 - q*x^2)/S
qrt[a + b*x^2 + c*x^4], x], x] + Dist[d^2/(e*(e - d*q)), Int[(1 + q*x^2)/((
d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x])] /; FreeQ[{a, b, c, d, e}, x]
&& NeQ[b^2 - 4*a*c, 0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^8}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\left(\frac{1}{10}\int \frac{x^4(3+4x^2)}{(1+2x^2+2x^4)^{3/2}}\,dx\right) + \frac{9}{10}\int \frac{x^4}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,d$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{x^2(-6+12x^2)}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{9\int \frac{1-\sqrt{2}\,x^2}{\sqrt{1+2x^2+2x}}}{20\sqrt{2}}$$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4} + \frac{9x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}\left(1+\sqrt{2}\,x^2\right)} - $$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4} + \frac{9x\sqrt{1+2x^2+2x^4}}{20\sqrt{2}\left(1+\sqrt{2}\,x^2\right)} - $$

$$= \frac{x^3(1-2x^2)}{20\sqrt{1+2x^2+2x^4}} + \frac{1}{20}x\sqrt{1+2x^2+2x^4} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}\,x^2\right)} - $$

Mathematica [C] Result contains complex when optimal does not.
time = 10.20, size = 199, normalized size = 0.44

$$\frac{4x+12x^3-4i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(29-33i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+27(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{80\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (4*x + 12*x^3 - (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^
2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (29 - 33*I)*Sqrt[1 - I]*Sqrt[1
+ (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
 + 27*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[
1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(80*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 603, normalized size = 1.34

method	result
risch	$\frac{x(3x^2+1)}{20\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{20}+\frac{i}{20}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\left(\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{4\left(-\frac{3}{80}x^3-\frac{1}{80}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{31\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{40\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-}}{}$
default	$\frac{x}{8\sqrt{2x^4+2x^2+1}}-\frac{11\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{243i}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/8*x/(2*x^4+2*x^2+1)^(1/2)-11/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*
x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I
*2^(1/2))+243/160*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+3*(
1/8*x^3+1/8*x)/(2*x^4+2*x^2+1)^(1/2)-243/160*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(
1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2
*2^(1/2)+1/2*I*2^(1/2))-9/2*(-1/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/2)+(47/32-4
7/32*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1
)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-
1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+27/16*x^3/(2*x^4+2*x^2+1)^(1/2)-81/4
*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+81/160/(-1+I)^(1/2)*(1+x^2-I*x^2)^
(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/
2*2^(1/2)+1/2*I*2^(1/2))+243/160/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*
x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I
*2^(1/2))+27/40/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4
+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/
2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^8/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{(2x^2+3)(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**8/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^8/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

```
[Out] int(x^8/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.349 $\int \frac{x^6}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=423

$$\frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)}-\frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2}{\left(1+\sqrt{2}\right.}}}{10\ 2}$$

```
[Out] -9/200*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/20*x*(-2*x^2
+1)/(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/
2))-1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*Ellipt
icE(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4
+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-9/80*(cos(
2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arc
tan(2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2
*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(
2*x^4+2*x^2+1)^(1/2)-1/8*(2^(1/4)+2^(3/4))*(cos(2*arctan(2^(1/4)*x))^2)^(1/
2)/cos(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/
2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(-2+3*
2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 503, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1327, 1289, 1211, 1117, 1209, 1333, 1720}

$$-\frac{9}{40}\sqrt{\frac{3}{5}}\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{9\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\ 2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{40\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\ 2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{9\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{560\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}+\frac{(1-2x^2)x}{20\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(1 - 2*x^2))/(20*Sqrt[1 + 2*x^2 + 2*x^4]) + (x*Sqrt[1 + 2*x^2 + 2*x^4])/
(10*Sqrt[2]*(1 + Sqrt[2]*x^2)) - (9*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
 2*x^2 + 2*x^4]])/40 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqr
t[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*S
qrt[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2
+ 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(40*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (9*(3 + Sqrt[2])*(1 + Sqrt[2]*x^
2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)
*x], (2 - Sqrt[2])/4])/(140*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(3 + Sqrt
[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Elli
pticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(560*2^
(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1289

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1)
*((b*d - 2*a*e - (b*e - 2*c*d)*x^2)/(2*(p + 1)*(b^2 - 4*a*c))), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1327

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[-f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2)), Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1)
/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1333

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(-a)*((e + d*q)/(c*d^2 - a*e^2))
```

```
, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[a*d*((e + d*q)/(c*d^2 - a*e^
2)), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; Free
Q[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
0] && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{x^6}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx &= -\left(\frac{1}{10}\int \frac{x^2(3+4x^2)}{\left(1+2x^2+2x^4\right)^{3/2}}\,dx\right)+\frac{9}{10}\int \frac{x^2}{(3+2x^2)\sqrt{1+2x^2+2x^4}} \\ &= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{1}{40}\int \frac{-2+4x^2}{\sqrt{1+2x^2+2x^4}}\,dx-\frac{1}{140}\Big(9\Big(2+3\sqrt{2} \\ &= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}-\frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{9\Big(3+}{} \\ &= \frac{x(1-2x^2)}{20\sqrt{1+2x^2+2x^4}}+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}\,x^2\right)}-\frac{9}{40}\sqrt{\frac{3}{5}}\tan^{-1}\Bigg(\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.65, size = 199, normalized size = 0.47

$$\frac{2x-4x^3-2i\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+(8-6i)\sqrt{1-i}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-9(1-i)^{3/2}\sqrt{1+(1-i)x^2}\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{40\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (2*x - 4*x^3 - (2*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (8 - 6*I)*Sqrt[1 - I]*Sqrt[1 + (
1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] -
9*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3
+ I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(40*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.14, size = 586, normalized size = 1.39

method	result
risch	$-\frac{x(2x^2-1)}{20\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{20}+\frac{i}{20}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{4\left(\frac{1}{40}x^3-\frac{1}{80}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{7\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-ix}}{}$
default	$-\frac{2\left(\frac{1}{8}x^3+\frac{1}{8}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{7\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{4\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{17}{16}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(1/8*x^3+1/8*x)/(2*x^4+2*x^2+1)^(1/2)+7/4/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/
2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2
^(1/2)+1/2*I*2^(1/2))+(-17/16+17/16*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+
(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2
)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3*(-1
/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/2)-9/8*x^3/(2*x^4+2*x^2+1)^(1/2)+27/2*(3/2
0*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)-27/80/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*
(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1
/2)+1/2*I*2^(1/2))-81/80*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(
1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/
2))-81/80/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2
+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+81/80*I/(-1+I
)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellip
ticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-9/20/(-1+I)^(1/2)*(1+x^2-I*x
^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/
2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^6/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(2x^2+3)\,(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**6/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^6/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

```
[Out] int(x^6/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.350 $\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=422

$$-\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}}+\frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)-\frac{\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2}{\left(1+\sqrt{\right.}}}}{10}$$

```
[Out] 3/100*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/10*x*(x^2+2)/
(2*x^4+2*x^2+1)^(1/2)+1/20*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-
1/20*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+3/40*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)+1/8*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4
)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(2+2^(1/2))
*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^
(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 501, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1327, 1192, 1211, 1117, 1209, 1230, 1720}

$$\frac{3}{20}\sqrt{\frac{3}{5}}\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)+\frac{9\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{3\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{280\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{2x^4+2x^2+1}\,x}{10\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{(x^2+2)x}{10\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -1/10*(x*(2 + x^2))/Sqrt[1 + 2*x^2 + 2*x^4] + (x*Sqrt[1 + 2*x^2 + 2*x^4])/(
10*Sqrt[2]*(1 + Sqrt[2]*x^2)) + (3*Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 +
2*x^2 + 2*x^4]])/20 - ((1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sq
rt[1 + 2*x^2 + 2*x^4]) + ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4
])/(20*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + (9*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2
)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*
x], (2 - Sqrt[2])/4])/(140*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (3*(3 + Sqrt[
2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*Ellip
ticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(280*2^(
1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1327

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[-f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
```

```
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[d^2*(f
^4/(c*d^2 - b*d*e + a*e^2)), Int[(f*x)^(m - 4)*((a + b*x^2 + c*x^4)^(p + 1)
/(d + e*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^4}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = -\left(\frac{1}{10}\int \frac{3+4x^2}{(1+2x^2+2x^4)^{3/2}}\,dx\right) + \frac{9}{10}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,d$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{1}{40}\int \frac{4-4x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{70}\left(9\left(3+\sqrt{2}\right)\right.$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) + \frac{9\left(3+\right.}{}$$

$$= -\frac{x(2+x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{x\sqrt{1+2x^2+2x^4}}{10\sqrt{2}\left(1+\sqrt{2}\,x^2\right)} + \frac{3}{20}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{}{\sqrt{1+}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.13, size = 199, normalized size = 0.47

$$-\frac{4x+2x^3+i\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+(1-2i)\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-3(1-i)^{3/2}\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{20\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -1/20*(4*x + 2*x^3 + I*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (1 - 2*I)*Sqrt[1 - I]*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
- 3*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/
3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/Sqrt[1 + 2*x^2 + 2*x^4]
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.14, size = 561, normalized size = 1.33

method	result
risch	$-\frac{x(x^2+2)}{10\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{20}+\frac{i}{20}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\operatorname{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)\ldots\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$ …
elliptic	$-\frac{4\left(\frac{1}{40}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{3\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\operatorname{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{20\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{i\sqrt{-\ldots}}{\ldots}$ …
default	$-\frac{2\left(-\frac{1}{4}x^3-\frac{1}{8}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\operatorname{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{5}{8}-\frac{5}{8}\ldots\right)}{\ldots}$ …

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*(-1/4*x^3-1/8*x)/(2*x^4+2*x^2+1)^(1/2)-1/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2
)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^
(1/2)+1/2*I*2^(1/2))+(5/8-5/8*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*
x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+3/4*x^3/(2*
x^4+2*x^2+1)^(1/2)-9*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+9/40/(-1+I)^(1
/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF
(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+27/40*I/(-1+I)^(1/2)*(1+x^2-I*x^
2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2)
,1/2*2^(1/2)+1/2*I*2^(1/2))+27/40/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I
*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*
I*2^(1/2))-27/40*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*
x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+3/10
/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)
*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^4/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(2x^2+3)\,(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**4/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^4/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

```
[Out] int(x^4/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.351 $\int \frac{x^2}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=423

$$\frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{5\left(1+\sqrt{2}\,x^2\right)}-\frac{1}{10}\sqrt{\frac{3}{5}}\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)+\frac{\sqrt[4]{2}\left(1+\sqrt{2}\,x^2\right)\sqrt{\frac{1}{(}}}{}$$

```
[Out] -1/50*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)+1/10*x*(4*x^2+3
)/(2*x^4+2*x^2+1)^(1/2)-1/5*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))
+1/5*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-1/20*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)-1/4*(2^(1/4)+2^(3/4))*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/c
os(2*arctan(2^(1/4)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^
(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)/(-2+3*2^(1
/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 503, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1329, 1192, 1211, 1117, 1209, 1230, 1720}

$$-\frac{1}{10}\sqrt{\frac{3}{5}}\,\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{2x^4+2x^2+1}}\right)-\frac{\left(1+2\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{20\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{3\left(3+\sqrt{2}\right)\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,F\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{70\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{\sqrt[4]{2}\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,E\left(2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\sqrt{2x^4+2x^2+1}}+\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}\,x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}\,x^2+1\right)^2}}\,\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}\,x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{140\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{5\left(\sqrt{2}\,x^2+1\right)}+\frac{(4x^2+3)\,x}{10\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (x*(3 + 4*x^2))/(10*Sqrt[1 + 2*x^2 + 2*x^4]) - (Sqrt[2]*x*Sqrt[1 + 2*x^2 +
2*x^4])/(5*(1 + Sqrt[2]*x^2)) - (Sqrt[3/5]*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*
x^2 + 2*x^4]])/10 + (2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*Sqrt[
1 + 2*x^2 + 2*x^4]) - (3*(3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(70*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((1 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x
], (2 - Sqrt[2])/4])/(20*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])^
2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticP
i[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(140*2^(1/4)
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1329

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> Dist[f^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^(
```

```
m - 2)*(a*e + c*d*x^2)*(a + b*x^2 + c*x^4)^p, x], x] - Dist[d*e*(f^2/(c*d^2
 - b*d*e + a*e^2)), Int[(f*x)^(m - 2)*((a + b*x^2 + c*x^4)^(p + 1)/(d + e*x
^2)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[
p, -1] && GtQ[m, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{x^2}{(3+2x^2)\left(1+2x^2+2x^4\right)^{3/2}}\,dx = \frac{1}{10}\int \frac{2+6x^2}{\left(1+2x^2+2x^4\right)^{3/2}}\,dx - \frac{3}{5}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{-4-16x^2}{\sqrt{1+2x^2+2x^4}}\,dx - \frac{1}{35}\Bigl(3\bigl(3+\sqrt{2}\,\bigr)\Bigr)$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{1}{10}\sqrt{\frac{3}{5}}\,\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right) - \frac{3\bigl(3+\sqrt{\ }}{}$$

$$= \frac{x(3+4x^2)}{10\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{5\left(1+\sqrt{2}\,x^2\right)} - \frac{1}{10}\sqrt{\frac{3}{5}}\,\tan^{-1}\left(\frac{}{\sqrt{1-}}\right.$$

Mathematica [C] Result contains complex when optimal does not.
time = 7.03, size = 199, normalized size = 0.47

$$\frac{6x+8x^3+4i\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(1+3i)\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-2(1-i)^{3/2}\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{20\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

[Out] (6*x + 8*x^3 + (4*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2
]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] - (1 + 3*I)*Sqrt[1 - I]*Sqrt[1 + (
1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] -
2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/3
+ I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(20*Sqrt[1 + 2*x^2 + 2*x^4])

Maple [C] Result contains complex when optimal does not.
time = 0.16, size = 536, normalized size = 1.27

method	result
risch	$\frac{x(4x^2+3)}{10\sqrt{2x^4+2x^2+1}}+\frac{\left(\frac{1}{5}-\frac{i}{5}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)-\mathrm{El}\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{4\left(-\frac{1}{10}x^3-\frac{3}{40}x\right)}{\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}-\frac{i\sqrt{-ix^2}}{}$
default	$-\frac{x^3}{2\sqrt{2x^4+2x^2+1}}+\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\mathrm{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{2\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{1}{4}+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)

[Out] -1/2*x^3/(2*x^4+2*x^2+1)^(1/2)+1/2/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I
)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2
*I*2^(1/2))+(-1/4+1/4*I)/(-1+I)^(1/2)*(1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/
2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2
))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2)))+6*(3/20*x^3+1/20*x)
/(2*x^4+2*x^2+1)^(1/2)-3/20/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^
(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1
/2))-9/20*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x
^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-9/20/(-1+I)
^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*Ellipt
icE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+9/20*I/(-1+I)^(1/2)*(1+x^2-I*
x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/
2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/5/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I
*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^(1/2),1/3+1/3*I,(-1-I
)^(1/2)/(-1+I)^(1/2))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)*x^2/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 1
4*x^2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(2x^2+3)(2x^4+2x^2+1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(x**2/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(x^2/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

```
[Out] int(x^2/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.352 $\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=422

$$-\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}}+\frac{3x\sqrt{1+2x^2+2x^4}}{5\sqrt{2}\left(1+\sqrt{2}x^2\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}}-\frac{3\left(1+\sqrt{2}x^2\right)\sqrt{\frac{1+2x^2+2x^4}{\left(1+\sqrt{2}x^2\right)^2}}\,E}{5\,2^{3/4}\sqrt{1+}}$$

```
[Out] 1/75*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/5*x*(3*x^2+1)/
(2*x^4+2*x^2+1)^(1/2)+3/10*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-
3/10*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x
^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)+1/30*(cos(2*ar
ctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(
2^(1/4)*x)),1/2-11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(
1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^
4+2*x^2+1)^(1/2)+1/4*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4
)*x))*EllipticF(sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(2+2^(1/2))
*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^
(1/2))/(2*x^4+2*x^2+1)^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 501, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1235, 1192, 1211, 1117, 1209, 1230, 1720}

$$\frac{\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{5\sqrt{15}}+\frac{\left(3+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{10\,2^{3/4}\sqrt{2x^4+2x^2+1}}+\frac{\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{35\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{3\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{5\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{210\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{3\sqrt{2x^4+2x^2+1}\,x}{5\sqrt{2}\left(\sqrt{2}x^2+1\right)}-\frac{(3x^2+1)x}{5\sqrt{2x^4+2x^2+1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -1/5*(x*(1 + 3*x^2))/Sqrt[1 + 2*x^2 + 2*x^4] + (3*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(5*Sqrt[2]*(1 + Sqrt[2]*x^2)) + ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x
^4]]/(5*Sqrt[15]) - (3*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt
[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(5*2^(3/4)*Sqr
t[1 + 2*x^2 + 2*x^4]) + ((3 + Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 +
2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4]
)/(35*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) + ((3 + 2*Sqrt[2])*(1 + Sqrt[2]*x^2)
*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x
], (2 - Sqrt[2])/4])/(10*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + Sqrt[2])^
2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticP
```

```
i[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(210*2^(1/4)
*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx = \frac{1}{10}\int \frac{2-4x^2}{(1+2x^2+2x^4)^{3/2}}\,dx + \frac{2}{5}\int \frac{1}{(3+2x^2)\sqrt{1+2x^2+2x^4}}\,dx$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{1}{40}\int \frac{16+24x^2}{\sqrt{1+2x^2+2x^4}}\,dx + \frac{1}{35}\left(2\left(3+\sqrt{2}\right)\right)$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right)}{5\sqrt{15}} + \frac{\left(3+\sqrt{2}\right)\left(1+\right.}{}$$

$$= -\frac{x(1+3x^2)}{5\sqrt{1+2x^2+2x^4}} + \frac{3x\sqrt{1+2x^2+2x^4}}{5\sqrt{2}\left(1+\sqrt{2}\,x^2\right)} + \frac{\tan^{-1}\left(\frac{\sqrt{\frac{5}{3}}\,x}{\sqrt{1+2x^2+2x^4}}\right.}{5\sqrt{15}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.11, size = 199, normalized size = 0.47

$$\frac{-6x-18x^3-9i\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+(6+3i)\sqrt{1-i}\,\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)+2(1-i)^{3/2}\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)}{30\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/((3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] (-6*x - 18*x^3 - (9*I)*Sqrt[1 - I]*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x
^2]*EllipticE[I*ArcSinh[Sqrt[1 - I]*x], I] + (6 + 3*I)*Sqrt[1 - I]*Sqrt[1 +
 (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I]
+ 2*(1 - I)^(3/2)*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticPi[1/
3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I])/(30*Sqrt[1 + 2*x^2 + 2*x^4])
```

Maple [**C**] Result contains complex when optimal does not.
time = 0.14, size = 366, normalized size = 0.87

method	result
risch	$-\frac{x(3x^2+1)}{5\sqrt{2x^4+2x^2+1}} + \frac{\left(-\frac{3}{10}+\frac{3i}{10}\right)\sqrt{1+(1-i)\,x^2}\,\sqrt{1+(1+i)\,x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i}\,, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$-\frac{4\left(\frac{3}{20}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}} + \frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1+i}\,, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} + \frac{3i\sqrt{-}}{}$
elliptic	$-\frac{4\left(\frac{3}{20}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}} + \frac{\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1+i}\,, \frac{\sqrt{2}}{2}+\frac{i\sqrt{2}}{2}\right)}{10\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}} + \frac{3i\sqrt{-}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -4*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)+1/10/(-1+I)^(1/2)*(1+x^2-I*x^2)^
(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/
2*2^(1/2)+1/2*I*2^(1/2))+3/10*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x
^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*
2^(1/2))+3/10/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2
*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-3/10*I/(-
1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*El
lipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+2/15/(-1+I)^(1/2)*(1+x^2-
I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-1+I)^
(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^10 + 28*x^8 + 40*x^6 + 32*x^4 + 14*x^
2 + 3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(2x^2 + 3)(2x^4 + 2x^2 + 1)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
[Out] Integral(1/((2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
[Out] int(1/((2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.353 $\int \frac{1}{x^2(3+2x^2)(1+2x^2+2x^4)^{3/2}}\,dx$

Optimal. Leaf size=468

$$-\frac{x}{3\sqrt{1+2x^2+2x^4}}+\frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}}-\frac{\sqrt{1+2x^2+2x^4}}{3x}+\frac{2\sqrt{2}\,x\sqrt{1+2x^2+2x^4}}{15\left(1+\sqrt{2}\,x^2\right)}-\frac{2\tan^{-1}\left(\frac{\sqrt{\ }}{\sqrt{1+2x}}\right.}{15\sqrt{15}}$$

```
[Out] -2/225*arctan(1/3*x*15^(1/2)/(2*x^4+2*x^2+1)^(1/2))*15^(1/2)-1/3*x/(2*x^4+2
*x^2+1)^(1/2)+2/15*x*(3*x^2+1)/(2*x^4+2*x^2+1)^(1/2)-1/3*(2*x^4+2*x^2+1)^(1
/2)/x+2/15*x*(2*x^4+2*x^2+1)^(1/2)*2^(1/2)/(1+x^2*2^(1/2))-2/15*(cos(2*arct
an(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticE(sin(2*arctan(2^(
1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*x^2+1)/(1+x^2*2^(
1/2))^2)^(1/2)*2^(1/4)/(2*x^4+2*x^2+1)^(1/2)-1/45*(cos(2*arctan(2^(1/4)*x))
^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticPi(sin(2*arctan(2^(1/4)*x)),1/2-
11/24*2^(1/2),1/2*(2-2^(1/2))^(1/2))*(3+2^(1/2))*(1+x^2*2^(1/2))*((2*x^4+2*
x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(2-3*2^(1/2))/(2*x^4+2*x^2+1)^(1/2)
+1/6*(cos(2*arctan(2^(1/4)*x))^2)^(1/2)/cos(2*arctan(2^(1/4)*x))*EllipticF(
sin(2*arctan(2^(1/4)*x)),1/2*(2-2^(1/2))^(1/2))*(-7+3*2^(1/2))*(1+x^2*2^(1/
2))*((2*x^4+2*x^2+1)/(1+x^2*2^(1/2))^2)^(1/2)*2^(1/4)/(-2+3*2^(1/2))/(2*x^4
+2*x^2+1)^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 644, normalized size of antiderivative = 1.38, number of steps used = 15, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1349, 1135, 1295, 1211, 1117, 1209, 1235, 1192, 1230, 1720}

$$-\frac{2\text{ArcTan}\left(\frac{\sqrt{\frac{5}{3}}x}{\sqrt{2x^4+2x^2+1}}\right)}{15\sqrt{15}}-\frac{\left(3+2\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{15\,2^{3/4}\sqrt{2x^4+2x^2+1}}-\frac{2^{3/4}\left(3+\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{105\sqrt{2x^4+2x^2+1}}-\frac{\left(1-\sqrt{2}\right)\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}F\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{6\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}-\frac{2\sqrt[4]{2}\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}E\left(2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{15\sqrt{2x^4+2x^2+1}}+\frac{\left(3+\sqrt{2}\right)^2\left(\sqrt{2}x^2+1\right)\sqrt{\frac{2x^4+2x^2+1}{\left(\sqrt{2}x^2+1\right)^2}}\Pi\left(\frac{1}{24}\left(12-11\sqrt{2}\right);2\text{ArcTan}\left(\sqrt[4]{2}x\right)|\frac{1}{4}\left(2-\sqrt{2}\right)\right)}{315\sqrt[4]{2}\sqrt{2x^4+2x^2+1}}+\frac{2\sqrt{2}\sqrt{2x^4+2x^2+1}\,x}{15\left(\sqrt{2}x^2+1\right)}+\frac{2(3x^2+1)x}{15\sqrt{2x^4+2x^2+1}}-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] -1/3*x/Sqrt[1 + 2*x^2 + 2*x^4] + (2*x*(1 + 3*x^2))/(15*Sqrt[1 + 2*x^2 + 2*x
^4]) - Sqrt[1 + 2*x^2 + 2*x^4]/(3*x) + (2*Sqrt[2]*x*Sqrt[1 + 2*x^2 + 2*x^4]
)/(15*(1 + Sqrt[2]*x^2)) - (2*ArcTan[(Sqrt[5/3]*x)/Sqrt[1 + 2*x^2 + 2*x^4]]
)/(15*Sqrt[15]) - (2*2^(1/4)*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1
+ Sqrt[2]*x^2)^2]*EllipticE[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*Sqrt
[1 + 2*x^2 + 2*x^4]) - ((1 - Sqrt[2])*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2
*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])
/(6*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4]) - (2^(3/4)*(3 + Sqrt[2])*(1 + Sqrt[2]*
x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*ArcTan[2^(1/
4)*x], (2 - Sqrt[2])/4])/(105*Sqrt[1 + 2*x^2 + 2*x^4]) - ((3 + 2*Sqrt[2])*(
```

```
1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x^2)^2]*EllipticF[2*
ArcTan[2^(1/4)*x], (2 - Sqrt[2])/4])/(15*2^(3/4)*Sqrt[1 + 2*x^2 + 2*x^4]) +
 ((3 + Sqrt[2])^2*(1 + Sqrt[2]*x^2)*Sqrt[(1 + 2*x^2 + 2*x^4)/(1 + Sqrt[2]*x
^2)^2]*EllipticPi[(12 - 11*Sqrt[2])/24, 2*ArcTan[2^(1/4)*x], (2 - Sqrt[2])/
4])/(315*2^(1/4)*Sqrt[1 + 2*x^2 + 2*x^4])
```

Rule 1117

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]/
(2*q*Sqrt[a + b*x^2 + c*x^4]))*EllipticF[2*ArcTan[q*x], 1/2 - b*(q^2/(4*c))
], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1135

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(-(d*x)^(m + 1))*(b^2 - 2*a*c + b*c*x^2)*((a + b*x^2 + c*x^4)^(p +
 1)/(2*a*d*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c))
, Int[(d*x)^m*(a + b*x^2 + c*x^4)^(p + 1)*Simp[b^2*(m + 2*p + 3) - 2*a*c*(m
 + 4*p + 5) + b*c*(m + 4*p + 7)*x^2, x], x], x] /; FreeQ[{a, b, c, d, m}, x
] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p] && (IntegerQ[p] ||
IntegerQ[m])
```

Rule 1192

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*((a + b*x^2 +
 c*x^4)^(p + 1)/(2*a*(p + 1)*(b^2 - 4*a*c))), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1209

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, Simp[(-d)*x*(Sqrt[a + b*x^2 + c*x^4]/(a*(1 + q
^2*x^2))), x] + Simp[d*(1 + q^2*x^2)*(Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*
x^2)^2)]/(q*Sqrt[a + b*x^2 + c*x^4]))*EllipticE[2*ArcTan[q*x], 1/2 - b*(q^2
/(4*c))], x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[c/a]
```

Rule 1211

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
```

```
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1230

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1235

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2
 + c*x^4)^p, x], x] + Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(a + b*x^2 + c*
x^4)^(p + 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && ILtQ[p + 1/2, 0]
```

Rule 1295

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[d*(f*x)^(m + 1)*((a + b*x^2 + c*x^4)^(p + 1)
/(a*f*(m + 1))), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1349

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, p, q}, x] && N
eQ[b^2 - 4*a*c, 0] && (IGtQ[p, 0] || IGtQ[q, 0] || IntegersQ[m, q])
```

Rule 1720

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, Simp[(-(B*d - A*e))*(A
rcTan[Rt[-b + c*(d/e) + a*(e/d), 2]*(x/Sqrt[a + b*x^2 + c*x^4])]/(2*d*e*Rt[
-b + c*(d/e) + a*(e/d), 2])), x] + Simp[(B*d + A*e)*(A + B*x^2)*(Sqrt[A^2*(
(a + b*x^2 + c*x^4)/(a*(A + B*x^2)^2))]/(4*d*e*A*q*Sqrt[a + b*x^2 + c*x^4])
)*EllipticPi[Cancel[-(B*d - A*e)^2/(4*d*e*A*B)], 2*ArcTan[q*x], 1/2 - b*(A/
(4*a*B))], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
```

```
[c*A^2 - a*B^2, 0]
```

Rubi steps

$$\int \frac{1}{x^2 (3+2x^2) (1+2x^2+2x^4)^{3/2}} \, dx = \int \left(\frac{1}{3x^2 (1+2x^2+2x^4)^{3/2}} - \frac{2}{3 (3+2x^2) (1+2x^2+2x^4)^{3/2}}\right) dx$$

$$= \frac{1}{3} \int \frac{1}{x^2 (1+2x^2+2x^4)^{3/2}} \, dx - \frac{2}{3} \int \frac{1}{(3+2x^2) (1+2x^2+2x^4)^{3/2}} \, dx$$

$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}} - \frac{1}{15} \int \frac{2-4x^2}{(1+2x^2+2x^4)^{3/2}} \, dx + \frac{1}{12} \int \frac{4 \cdots}{x^2\sqrt{1+ \cdots}}$$

$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}} + \frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{1+2x^2+2x^4}}{3x} - \frac{1}{60 \cdots}$$

$$2 \cdots$$

$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}} + \frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{1+2x^2+2x^4}}{3x} - \cdots$$

$$= -\frac{x}{3\sqrt{1+2x^2+2x^4}} + \frac{2x(1+3x^2)}{15\sqrt{1+2x^2+2x^4}} - \frac{\sqrt{1+2x^2+2x^4}}{3x} + \frac{2 \cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 10.16, size = 211, normalized size = 0.45

$$\frac{-12i\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,E\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-(27-39i)\sqrt{1-i}\,x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,F\left(i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)-2\left(15+39x^2+12x^4+2(1-i)^{3/2}x\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\Pi\left(\frac{1}{3}+\frac{i}{3};i\sinh^{-1}\left(\sqrt{1-i}\,x\right)\middle|i\right)\right)}{90x\sqrt{1+2x^2+2x^4}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*(3 + 2*x^2)*(1 + 2*x^2 + 2*x^4)^(3/2)),x]
```

```
[Out] ((-12*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I)*x^2]*Elliptic
E[I*ArcSinh[Sqrt[1 - I]*x], I] - (27 - 39*I)*Sqrt[1 - I]*x*Sqrt[1 + (1 - I)
*x^2]*Sqrt[1 + (1 + I)*x^2]*EllipticF[I*ArcSinh[Sqrt[1 - I]*x], I] - 2*(15
+ 39*x^2 + 12*x^4 + 2*(1 - I)^(3/2)*x*Sqrt[1 + (1 - I)*x^2]*Sqrt[1 + (1 + I
)*x^2]*EllipticPi[1/3 + I/3, I*ArcSinh[Sqrt[1 - I]*x], I]))/(90*x*Sqrt[1 +
2*x^2 + 2*x^4])
```

Maple [C] Result contains complex when optimal does not.
time = 0.15, size = 553, normalized size = 1.18

method	result
risch	$-\frac{4x^4+13x^2+5}{15x\sqrt{2x^4+2x^2+1}}+\frac{\left(-\frac{2}{15}+\frac{2i}{15}\right)\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\left(\text{EllipticF}\left(x\sqrt{-1+i},\frac{\sqrt{2}}{2}+\frac{i\sqrt{}}{}\right.\right.}{\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
elliptic	$-\frac{4\left(-\frac{1}{10}x^3+\frac{1}{20}x\right)}{\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{11\sqrt{-ix^2+x^2+1}\,\sqrt{ix^2+x^2+1}\,\text{EllipticF}\left(x\sqrt{-1+i}\right.}{15\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$
default	$-\frac{\sqrt{2x^4+2x^2+1}}{3x}-\frac{x}{3\sqrt{2x^4+2x^2+1}}-\frac{\sqrt{1+(1-i)x^2}\,\sqrt{1+(1+i)x^2}\,\text{EllipticF}\left(x\sqrt{-1+i}\right.}{3\sqrt{-1+i}\,\sqrt{2x^4+2x^2+1}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(2*x^4+2*x^2+1)^(1/2)/x-1/3*x/(2*x^4+2*x^2+1)^(1/2)-1/3/(-1+I)^(1/2)*(
1+(1-I)*x^2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-
1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+(-1/3+1/3*I)/(-1+I)^(1/2)*(1+(1-I)*x^
2)^(1/2)*(1+(1+I)*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*(EllipticF(x*(-1+I)^(1/2
),1/2*2^(1/2)+1/2*I*2^(1/2))-EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(
1/2)))+8/3*(3/20*x^3+1/20*x)/(2*x^4+2*x^2+1)^(1/2)-1/15/(-1+I)^(1/2)*(1+x^2
-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^
(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-1/5*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+
x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticF(x*(-1+I)^(1/2),1/2*2^(1/2)
+1/2*I*2^(1/2))-1/5/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2
*x^4+2*x^2+1)^(1/2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))+1/5
*I/(-1+I)^(1/2)*(1+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/
2)*EllipticE(x*(-1+I)^(1/2),1/2*2^(1/2)+1/2*I*2^(1/2))-4/45/(-1+I)^(1/2)*(1
+x^2-I*x^2)^(1/2)*(1+x^2+I*x^2)^(1/2)/(2*x^4+2*x^2+1)^(1/2)*EllipticPi(x*(-
1+I)^(1/2),1/3+1/3*I,(-1-I)^(1/2)/(-1+I)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*x^4 + 2*x^2 + 1)/(8*x^12 + 28*x^10 + 40*x^8 + 32*x^6 + 14*x
^4 + 3*x^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \cdot (2x^2+3) (2x^4+2x^2+1)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(2*x**2+3)/(2*x**4+2*x**2+1)**(3/2),x)
```

```
[Out] Integral(1/(x**2*(2*x**2 + 3)*(2*x**4 + 2*x**2 + 1)**(3/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(2*x^2+3)/(2*x^4+2*x^2+1)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(1/((2*x^4 + 2*x^2 + 1)^(3/2)*(2*x^2 + 3)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\left(2\,x^2+3\right)\,{\left(2\,x^4+2\,x^2+1\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)),x)
```

```
[Out] int(1/(x^2*(2*x^2 + 3)*(2*x^2 + 2*x^4 + 1)^(3/2)), x)
```

3.354 $\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=406

$$\frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)\left(d+ex^2\right)^{3/2}}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}-\frac{\left(b^2cd-ac^2d-b^3e+2abce-\frac{b^3cd-3abc^2d-b^4e+4c}{\sqrt{b^2-4c}}\right.}{\sqrt{2}\,c^{7/2}\sqrt{2cd-}}$$

```
[Out] -1/3*(b*e+c*d)*(e*x^2+d)^(3/2)/c^2/e^2+1/5*(e*x^2+d)^(5/2)/c/e^2+(-a*c+b^2)
*(e*x^2+d)^(1/2)/c^3-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(
b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b^2*c*d-a*c^2*d-b^3*e+2*a*b*c*e+(2*a^2*c^2*e
-4*a*b^2*c*e+3*a*b*c^2*d+b^4*e-b^3*c*d)/(-4*a*c+b^2)^(1/2))/c^(7/2)*2^(1/2)
/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+
d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b^2*c*d-a*c^2*d-b^3*e+2*a
*b*c*e+(-2*a^2*c^2*e+4*a*b^2*c*e-3*a*b*c^2*d-b^4*e+b^3*c*d)/(-4*a*c+b^2)^(1
/2))/c^(7/2)*2^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 5.44, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 911, 1301, 1180, 214}

$$-\frac{\left(-\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{-2a^2c^2e+4ab^2ce-3abc^2d+b^4(-e)+b^3cd}{\sqrt{b^2-4ac}}+2abce-ac^2d+b^3(-e)+b^2cd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{7/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{\left(d+ex^2\right)^{3/2}(be+cd)}{3c^2e^2}+\frac{\left(d+ex^2\right)^{5/2}}{5ce^2}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c)*Sqrt[d + e*x^2])/c^3 - ((c*d + b*e)*(d + e*x^2)^(3/2))/(3*c^2*
e^2) + (d + e*x^2)^(5/2)/(5*c*e^2) - ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*
e - (b^3*c*d - 3*a*b*c^2*d - b^4*e + 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 -
4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^
2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])
- ((b^2*c*d - a*c^2*d - b^3*e + 2*a*b*c*e + (b^3*c*d - 3*a*b*c^2*d - b^4*e
+ 4*a*b^2*c*e - 2*a^2*c^2*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(7/2)*S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^7\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^3\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{(b^2-ac)e}{c^3}-\frac{(cd+be)x^2}{c^2e}+\frac{x^4}{ce}-\frac{(b^2-ac)(cd^2-bde+ae^2)-(b^2cd-ac^2d-b^3e+2abce)x^2}{c^3e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)\,dx, \ldots}{e}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2}+\frac{(d+ex^2)^{5/2}}{5ce^2}-\frac{\text{Subst}\left(\int \frac{(b^2-ac)(cd^2-\ldots}{cd\ldots}\ldots\right.}{\ldots}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2}+\frac{(d+ex^2)^{5/2}}{5ce^2}+\frac{\left(b^2cd-ac^2d-b^3e\ldots\right.}{\ldots}$$

$$= \frac{(b^2-ac)\sqrt{d+ex^2}}{c^3}-\frac{(cd+be)(d+ex^2)^{3/2}}{3c^2e^2}+\frac{(d+ex^2)^{5/2}}{5ce^2}-\frac{\left(b^2cd-ac^2d-b^3e\ldots\right.}{\ldots}$$

Mathematica [A]
time = 1.44, size = 475, normalized size = 1.17

$$\frac{\frac{2\sqrt{c}\sqrt{d+ex^2}\left(15b^2e^2+c^2(-2d^2+dex^2+3e^2x^4)-5ce(3ae+b(d+ex^2))\right)}{e^2}-\frac{15\sqrt{2}\left(-b^4e+ac^2\left(\sqrt{b^2-4ac}\,d-2ae\right)+b^2c\left(-\sqrt{b^2-4ac}\,d+4ae\right)+b^3\left(cd+\sqrt{b^2-4ac}\,e\right)-abc\left(3cd+2\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{15\sqrt{2}\left(b^4e+ac^2\left(\sqrt{b^2-4ac}\,d+2ae\right)-b^2c\left(\sqrt{b^2-4ac}\,d+4ae\right)+abc\left(3cd-2\sqrt{b^2-4ac}\,e\right)+b^3\left(-cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}}{30c^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x^2]*(15*b^2*e^2 + c^2*(-2*d^2 + d*e*x^2 + 3*e^2*x^4
) - 5*c*e*(3*a*e + b*(d + e*x^2))))/e^2 - (15*Sqrt[2]*(-(b^4*e) + a*c^2*(Sq
rt[b^2 - 4*a*c]*d - 2*a*e) + b^2*c*(-(Sqrt[b^2 - 4*a*c]*d) + 4*a*e) + b^3*(
c*d + Sqrt[b^2 - 4*a*c]*e) - a*b*c*(3*c*d + 2*Sqrt[b^2 - 4*a*c]*e))*ArcTan[
(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) - (15*Sqrt[2
]*(b^4*e + a*c^2*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) - b^2*c*(Sqrt[b^2 - 4*a*c]*d
 + 4*a*e) + a*b*c*(3*c*d - 2*Sqrt[b^2 - 4*a*c]*e) + b^3*(-(c*d) + Sqrt[b^2
- 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + (b + Sq
```

```
rt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*
c])*e]))/(30*c^(7/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 457, normalized size = 1.13

method	result
risch	$-\frac{\left(-3c^2e^2x^4+5bc\,e^2x^2-c^2de\,x^2+15ac\,e^2-15e^2b^2+5bcde+2c^2d^2\right)\sqrt{e\,x^2+d}}{15e^2c^3}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a\,e^2-8deb\right.\right.}}{}$
default	$\frac{\frac{x^2\left(e\,x^2+d\right)^{\frac{3}{2}}}{5e}-\frac{2d\left(e\,x^2+d\right)^{\frac{3}{2}}}{15e^2}}{c}-\frac{b\left(e\,x^2+d\right)^{\frac{3}{2}}}{3c^2e}+\frac{-\frac{\left(ac-b^2\right)\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)}{2c}-\frac{d\left(ac-b^2\right)}{2c\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)}-\frac{\_R=\mathrm{RootOf}(c}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/5*x^2*(e*x^2+d)^(3/2)/e-2/15*d/e^2*(e*x^2+d)^(3/2))-1/3*b/c^2*(e*x^2
+d)^(3/2)/e+1/c^2*(-1/2*(a*c-b^2)/c*((e*x^2+d)^(1/2)-e^(1/2)*x)-1/2/c*d*(a*
c-b^2)/((e*x^2+d)^(1/2)-e^(1/2)*x)-1/4/c*sum(((-2*a*b*c*e+a*c^2*d+b^3*e-b^2
*c*d)*_R^6+(-4*a^2*c*e^2+4*a*b^2*e^2+2*a*b*c*d*e-3*a*c^2*d^2-3*b^3*d*e+3*b^
2*c*d^2)*_R^4+d*(4*a^2*c*e^2-4*a*b^2*e^2-2*a*b*c*d*e+3*a*c^2*d^2+3*b^3*d*e-
3*b^2*c*d^2)*_R^2+2*a*b*c*d^3*e-a*c^2*d^4-b^3*d^3*e+b^2*c*d^4)/(_R^7*c+3*_R
^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^
3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16
*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+d^4*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)*x^7/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^7 \sqrt{d + ex^2}}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**7*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(374) = 748.
time = 4.07, size = 928, normalized size = 2.29

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -(((b^4*c - 5*a*b^2*c^2 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*
e^2)*c^2 + 2*(b^3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*
c^5)*d*e - 2*((b^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)
*sqrt(b^2 - 4*a*c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c
) + (b^5*c^2 - 4*a*b^3*c^3 + 2*a^2*b*c^4)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*
e + d)/sqrt(-(2*c^6*d*e^12 - b*c^5*e^13 + sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e
^13 + a*c^5*e^14)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))*e^(-12)/c^6))/
((2*sqrt(b^2 - 4*a*c)*c^4*d + (b^2*c^3 - 4*a*c^4 - sqrt(b^2 - 4*a*c)*b*c^3)
*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2) + (((b^4*c - 5*a*
b^2*c^2 + 4*a^2*c^3)*d*e - (b^5 - 6*a*b^3*c + 8*a^2*b*c^2)*e^2)*c^2 + 2*(b^
3*c^4 - 3*a*b*c^5)*d^2 - (3*b^4*c^3 - 11*a*b^2*c^4 + 4*a^2*c^5)*d*e + 2*((b
^2*c^3 - a*c^4)*sqrt(b^2 - 4*a*c)*d^2 - (b^3*c^2 - a*b*c^3)*sqrt(b^2 - 4*a*
c)*d*e + (a*b^2*c^2 - a^2*c^3)*sqrt(b^2 - 4*a*c)*e^2)*abs(c) + (b^5*c^2 - 4
*a*b^3*c^3 + 2*a^2*b*c^4)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*
c^6*d*e^12 - b*c^5*e^13 - sqrt(-4*(c^6*d^2*e^12 - b*c^5*d*e^13 + a*c^5*e^14
)*c^6*e^12 + (2*c^6*d*e^12 - b*c^5*e^13)^2))*e^(-12)/c^6))/((2*sqrt(b^2 - 4
*a*c)*c^4*d - (b^2*c^3 - 4*a*c^4 + sqrt(b^2 - 4*a*c)*b*c^3)*e)*sqrt(-4*c^2*
d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2) + 1/15*(3*(x^2*e + d)^(5/2)*c^4*e
^8 - 5*(x^2*e + d)^(3/2)*c^4*d*e^8 - 5*(x^2*e + d)^(3/2)*b*c^3*e^9 + 15*sqr
t(x^2*e + d)*b^2*c^2*e^10 - 15*sqrt(x^2*e + d)*a*c^3*e^10)*e^(-10)/c^5
```

Mupad [B]
time = 2.48, size = 2500, normalized size = 6.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(1/2)*((3*d^2)/(c*e^2) - (a*e^4 + c*d^2*e^2 - b*d*e^3)/(c^2*e^4
) + (((3*d)/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(c^2*e^4))*(b*e^3 - 2*c*d*e^2))/(
c*e^2)) - (d + e*x^2)^(3/2)*(d/(c*e^2) + (b*e^3 - 2*c*d*e^2)/(3*c^2*e^4)) +
 atan(((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c
^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a
^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1/2)*(-(b^9*e
- 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d
 + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*
a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*
c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b
^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2)
- 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*
b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c
^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b
^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^
3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*
d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*
c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*
(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16
*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^8*e^4 +
 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^
2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2
 + 14*a*b^5*c^2*d*e^3 + 14*a^3*b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b
^3*c^3*d*e^3))/c^5)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2)
 - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^
3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*
c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(
4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^
3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8
*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*1i - (((16*a^3*c^6*e^4 + 4*a*
b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 - 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4
*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d
^2*e^2)/c^5 + (2*(d + e*x^2)^(1/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c
- b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^
5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*
c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) +
5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/
2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3
 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^
4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d + 38*
```

```
a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*c -
b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*d*(-
(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3*c^2
*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a^
2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^2*c^
8)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^8*e^4 + 2*a^4*c^4*e^4 + 20*a^2*b^4*c^2
*e^4 - 16*a^3*b^2*c^3*e^4 - 2*a^3*c^5*d^2*e^2 + b^6*c^2*d^2*e^2 - 8*a*b^6*c
*e^4 - 2*b^7*c*d*e^3 + 9*a^2*b^2*c^4*d^2*e^2 + 14*a*b^5*c^2*d*e^3 + 14*a^3*
b*c^4*d*e^3 - 6*a*b^4*c^3*d^2*e^2 - 28*a^2*b^3*c^3*d*e^3))/c^5)*(-(b^9*e -
8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*a^2*b^4*c^3*d +
 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^3*c^3*e*(-(4*a*
c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b*c^4*e + b^5*c*
d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^(1/2) - 4*a*b^3
*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b^2)^3)^(1/2) -
6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b^4*c^7 - 8*a*b^
2*c^8)))^(1/2)*1i)/((((16*a^3*c^6*e^4 + 4*a*b^4*c^4*e^4 - 4*b^5*c^4*d*e^3 -
 20*a^2*b^2*c^5*e^4 + 16*a^2*c^7*d^2*e^2 + 4*b^4*c^5*d^2*e^2 + 20*a*b^3*c^5
*d*e^3 - 16*a^2*b*c^6*d*e^3 - 20*a*b^2*c^6*d^2*e^2)/c^5 - (2*(d + e*x^2)^(1
/2)*(-(b^9*e - 8*a^4*c^5*d - b^6*e*(-(4*a*c - b^2)^3)^(1/2) - b^8*c*d - 33*
a^2*b^4*c^3*d + 38*a^3*b^2*c^4*d + 42*a^2*b^5*c^2*e - 63*a^3*b^3*c^3*e + a^
3*c^3*e*(-(4*a*c - b^2)^3)^(1/2) - 11*a*b^7*c*e + 10*a*b^6*c^2*d + 28*a^4*b
*c^4*e + b^5*c*d*(-(4*a*c - b^2)^3)^(1/2) + 5*a*b^4*c*e*(-(4*a*c - b^2)^3)^
(1/2) - 4*a*b^3*c^2*d*(-(4*a*c - b^2)^3)^(1/2) + 3*a^2*b*c^3*d*(-(4*a*c - b
^2)^3)^(1/2) - 6*a^2*b^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^9 + b
^4*c^7 - 8*a*b^2*c^8)))^(1/2)*(4*b^3*c^7*e^3 - 8*b^2*c^8*d*e^2 - 16*a*b*c^8
*e^3 + 32*a*c^9*d*e^2))/c^5)*(-(b^9*e - 8*a^4*c...
```

3.355 $\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=324

$$-\frac{b\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3ce}+\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)/c/e-b*(e*x^2+d)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^(1/2)*(
e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+a*c*e+(
-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(2*
c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(
1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-
2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(2*c*d-e*(b+(-
4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 2.49, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 911, 1301, 1180, 214}

$$\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3ce}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[d + e*x^2])/c^2) + (d + e*x^2)^(3/2)/(3*c*e) + ((b*c*d - b^2*e +
a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]
])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((b*c*d - b^
2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^2\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right) \\
&= \frac{\text{Subst}\left(\int \frac{x^2\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex^2}\right)}{e} \\
&= \frac{\text{Subst}\left(\int \left(-\frac{be}{c^2}+\frac{x^2}{c}+\frac{b(cd^2-bde+ae^2)-(bcd-b^2e+ace)x^2}{c^2e\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\right)\,dx, x, \sqrt{d+ex^2}\right)}{e} \\
&= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3ce}+\frac{\text{Subst}\left(\int \frac{b(cd^2-bde+ae^2)+(-bcd+b^2e-ace)x^2}{\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}}\,dx, x, \sqrt{d+ex\ldots}\right.}{c^2e^2} \\
&= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3ce}-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \ldots\right.}{2c^2e^2} \\
&= -\frac{b\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3ce}+\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\ldots\right.}{\sqrt{2}\,c^{5/2}\sqrt{2cd-\left(b-\sqrt{b^2-4a\ldots}\right.}}
\end{aligned}
$$

<u>Mathematica</u> [A]
time = 1.06, size = 383, normalized size = 1.18

$$
\frac{\frac{2\sqrt{c}\sqrt{d+ex^2}(-3be+c(d+ex^2))}{e}+\frac{3\sqrt{2}\left(-b^3e+bc\left(-\sqrt{b^2-4ac}\,d+3ae\right)+b^2\left(cd+\sqrt{b^2-4ac}\,e\right)-ac\left(2cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{3\sqrt{2}\left(b^3e-bc\left(\sqrt{b^2-4ac}\,d+3ae\right)+ac\left(2cd-\sqrt{b^2-4ac}\,e\right)+b^2\left(-cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}}{6c^{5/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*Sqrt[c]*Sqrt[d + e*x^2]*(-3*b*e + c*(d + e*x^2)))/e + (3*Sqrt[2]*(-(b^3
*e) + b*c*(-(Sqrt[b^2 - 4*a*c]*d) + 3*a*e) + b^2*(c*d + Sqrt[b^2 - 4*a*c]*e
) - a*c*(2*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x
^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*
c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (3*Sqrt[2]*(b^3*e - b*c*(Sqrt[b^2 - 4*a
*c]*d + 3*a*e) + a*c*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b^2*(-(c*d) + Sqrt[b^2
 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + (b + S
qrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a
*c])*e]))/(6*c^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 335, normalized size = 1.03

method	result
risch	$-\frac{\left(-c e\,x^{2}+3 e b-c d\right) \sqrt{e\,x^{2}+d}}{3 e\,c^{2}}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\left(4 e b-4 c d\right)\_Z^{6}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{4}+\left(4 d^{2} e b-4 c\,d^{3}\right)\_Z^{2}+d^{4} c\right)} \frac{\left(\left(a c e-\right.\right.}{}}{}$
default	$\frac{\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{3 c e}-\frac{\frac{b\left(\sqrt{e\,x^{2}+d}-\sqrt{e}\,x\right)}{2 c}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+\left(4 e b-4 c d\right)\_Z^{6}+\left(16 a\,e^{2}-8 d e b+6 c\,d^{2}\right)\_Z^{4}+\left(4 d^{2} e b-4 c\,d^{3}\right)\_Z^{2}+d^{4} c\right)}}{}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/3*(e*x^2+d)^(3/2)/c/e-1/c*(1/2*b/c*((e*x^2+d)^(1/2)-e^(1/2)*x)-1/4/c*sum(
((-a*c*e+b^2*e-b*c*d)*_R^6+(4*a*b*e^2-a*c*d*e-3*b^2*d*e+3*b*c*d^2)*_R^4+d*(
-4*a*b*e^2+a*c*d*e+3*b^2*d*e-3*b*c*d^2)*_R^2+a*c*d^3*e-b^2*d^3*e+c*d^4*b)/(
_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^
2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*
d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+d^4*c))+1/
2/c*b*d/((e*x^2+d)^(1/2)-e^(1/2)*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)*x^5/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4208 vs. 2(295) = 590.
time = 103.30, size = 4208, normalized size = 12.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*c^2*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5
*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^
```

3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c
^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)
/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*e*log(-(2*(a^2*b^4*c - 2*a^3*
b^2*c^2)*d^2 + 2*sqrt(1/2)*sqrt(x^2*e + d)*((b^7*c - 7*a*b^5*c^2 + 14*a^2*b
^3*c^3 - 8*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^
3 + 4*a^4*c^4)*e - (b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*sqrt(((b^6*c^2 -
4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 -
 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4
*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*
d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2
 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3
 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a
^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) + (2*a^3*b^4 - 6*
a^4*b^2*c + 2*a^5*c^2 - (a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*x^2)*e^2 + ((a^
2*b^4*c - 2*a^3*b^2*c^2)*d*x^2 - 2*(a^2*b^5 - 2*a^3*b^3*c - a^4*b*c^2)*d)*e
 - ((a^2*b^2*c^5 - 4*a^3*c^6)*x^2*e + 2*(a^2*b^2*c^5 - 4*a^3*c^6)*d)*sqrt((
(b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^
2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^
2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/x^2) - 3*sqrt(1/2)*c^2*sqrt((
(b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e + (
b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b
^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c +
11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2
*c^5 - 4*a*c^6))*e*log(-(2*(a^2*b^4*c - 2*a^3*b^2*c^2)*d^2 - 2*sqrt(1/2)*sq
rt(x^2*e + d)*((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 8*a^3*b*c^4)*d - (b^
8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e - (b^5*c^5 -
 7*a*b^3*c^6 + 12*a^2*b*c^7)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*
d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*
a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^
11)))*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*
b*c^2)*e + (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4
)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 -
6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*
c^11)))/(b^2*c^5 - 4*a*c^6)) + (2*a^3*b^4 - 6*a^4*b^2*c + 2*a^5*c^2 - (a^2*
b^5 - 3*a^3*b^3*c + a^4*b*c^2)*x^2)*e^2 + ((a^2*b^4*c - 2*a^3*b^2*c^2)*d*x^
2 - 2*(a^2*b^5 - 2*a^3*b^3*c - a^4*b*c^2)*d)*e - ((a^2*b^2*c^5 - 4*a^3*c^6)
*x^2*e + 2*(a^2*b^2*c^5 - 4*a^3*c^6)*d)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^
2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^
10 - 4*a*c^11)))/x^2) + 3*sqrt(1/2)*c^2*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*
c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6
*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^
3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^
3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*e*log(-(2*(a
^2*b^4*c - 2*a^3*b^2*c^2)*d^2 + 2*sqrt(1/2)*sqrt(x^2*e + d)*((b^7*c - 7*a*b

```
^5*c^2 + 14*a^2*b^3*c^3 - 8*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^
2 - 17*a^3*b^2*c^3 + 4*a^4*c^4)*e + (b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*
sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2
+ 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))*sqrt(((b^4*c - 4*a*b^2*
c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6
)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^
2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 -
6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))
+ (2*a^3*b^4 - 6*a^4*b^2*c + 2*a^5*c^2 - (a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2
)*x^2)*e^2 + ((a^2*b^4*c - 2*a^3*b^2*c^2)*d*x^2 - 2*(a^2*b^5 - 2*a^3*b^3*c
- a^4*b*c^2)*d)*e + ((a^2*b^2*c^5 - 4*a^3*c^6)*x^2*e + 2*(a^2*b^2*c^5 - 4*a
^3*c^6)*d)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^2*c^4)*d^2 - 2*(b^7*c - 5
*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b^8 - 6*a*b^6*c + 11*a^2*b
^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 - 4*a*c^11)))/x^2) - 3*sqr
t(1/2)*c^2*sqrt(((b^4*c - 4*a*b^2*c^2 + 2*a^2*c^3)*d - (b^5 - 5*a*b^3*c + 5
*a^2*b*c^2)*e - (b^2*c^5 - 4*a*c^6)*sqrt(((b^6*c^2 - 4*a*b^4*c^3 + 4*a^2*b^
2*c^4)*d^2 - 2*(b^7*c - 5*a*b^5*c^2 + 7*a^2*b^3*c^3 - 2*a^3*b*c^4)*d*e + (b
^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*e^2)/(b^2*c^10 -
 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*e*log(-(2*(a^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**5*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 745 vs. 2(295) = 590.
time = 3.63, size = 745, normalized size = 2.30

$$\frac{\left((x^2e+d)^{\frac{3}{2}}c^2e^2-3\sqrt{x^2e+d}\,bce^3\right)e^{(-3)}}{3c^3}+\frac{\left(((b^3c-4abc^2)de-(b^4-5ab^2c+4a^2c^2)e^2)c^2+2(b^2c^4-2ac^5)d^2-(3b^3c^3-8abc^4)de-2\left(\sqrt{b^2-4ac}\,bc^3d^2-\sqrt{b^2-4ac}\,b^2c^2de+\sqrt{b^2-4ac}\,abc^2e^2\right)|c|+(b^4c^2-3ab^2c^3)e^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{\left(2c^4de^4-bc^3e^5+\sqrt{-4(c^4d^2e^4-bc^3de^5+ac^3e^6)c^4e^4+(2c^4de^4-bc^3e^5)^2}\right)e^{(-4)}}{c^4}}}\right)}{\left(2\sqrt{b^2-4ac}\,c^3d+\left(b^2c^2-4ac^3-\sqrt{b^2-4ac}\,bc^2\right)e\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,c^2}-\frac{\left(((b^3c-4abc^2)de-(b^4-5ab^2c+4a^2c^2)e^2)c^2+2(b^2c^4-2ac^5)d^2-(3b^3c^3-8abc^4)de+2\left(\sqrt{b^2-4ac}\,bc^3d^2-\sqrt{b^2-4ac}\,b^2c^2de+\sqrt{b^2-4ac}\,abc^2e^2\right)|c|+(b^4c^2-3ab^2c^3)e^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{\left(2c^4de^4-bc^3e^5-\sqrt{-4(c^4d^2e^4-bc^3de^5+ac^3e^6)c^4e^4+(2c^4de^4-bc^3e^5)^2}\right)e^{(-4)}}{c^4}}}\right)}{\left(2\sqrt{b^2-4ac}\,c^3d-\left(b^2c^2-4ac^3+\sqrt{b^2-4ac}\,bc^2\right)e\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/3*((x^2*e + d)^(3/2)*c^2*e^2 - 3*sqrt(x^2*e + d)*b*c*e^3)*e^(-3)/c^3 + ((
(b^3*c - 4*a*b*c^2)*d*e - (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(b^2*c
^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*a*b*c^4)*d*e - 2*(sqrt(b^2 - 4*a*c)*b*c^
3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*abs(
```

```
c) + (b^4*c^2 - 3*a*b^2*c^3)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-
(2*c^4*d*e^4 - b*c^3*e^5 + sqrt(-4*(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*
c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)^2))*e^(-4)/c^4))/((2*sqrt(b^2 - 4*a*c)*
c^3*d + (b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*
(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2) - (((b^3*c - 4*a*b*c^2)*d*e - (b^4 - 5*
a*b^2*c + 4*a^2*c^2)*e^2)*c^2 + 2*(b^2*c^4 - 2*a*c^5)*d^2 - (3*b^3*c^3 - 8*
a*b*c^4)*d*e + 2*(sqrt(b^2 - 4*a*c)*b*c^3*d^2 - sqrt(b^2 - 4*a*c)*b^2*c^2*d
*e + sqrt(b^2 - 4*a*c)*a*b*c^2*e^2)*abs(c) + (b^4*c^2 - 3*a*b^2*c^3)*e^2)*a
rctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d*e^4 - b*c^3*e^5 - sqrt(-4*
(c^4*d^2*e^4 - b*c^3*d*e^5 + a*c^3*e^6)*c^4*e^4 + (2*c^4*d*e^4 - b*c^3*e^5)
^2))*e^(-4)/c^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d - (b^2*c^2 - 4*a*c^3 + sqrt(b
^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2)
```

Mupad [B]
time = 1.99, size = 2500, normalized size = 7.72

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(3/2)/(3*c*e) - atan(((((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4
*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^
3)/c^3 - (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2
)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4
*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c
*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^
2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(
1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c
^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*
a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a
*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2
) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(
1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2
)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^
2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c
^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a
*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^
2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e
 - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2)
- 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*
c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 +
 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 + (2*(d +
 e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6
*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(
```

1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i)/((2*(a^4*c*e^5 - a^3*b^2*e^5 + a^2*b^3*d*e^4 + a^3*c^2*d^2*e^3 + a^2*b*c^2*d^3*e^2 - 2*a^2*b^2*c*d^2*e^3))/c^3 + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 - (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^6*e^4 - 2*a^3*c^3*e^4 + 9*a^2*b^2*c^2*e^4 + 2*a^2*c^4*d^2*e^2 + b^4*c^2*d^2*e^2 - 6*a*b^4*c*e^4 - 2*b^5*c*d*e^3 + 10*a*b^3*c^2*d*e^3 - 10*a^2*b*c^3*d*e^3 - 4*a*b^2*c^3*d^2*e^2))/c^3)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) + (((4*a*b^3*c^3*e^4 - 16*a^2*b*c^4*e^4 - 4*b^4*c^3*d*e^3 + 4*b^3*c^4*d^2*e^2 - 16*a*b*c^5*d^2*e^2 + 16*a*b^2*c^4*d*e^3)/c^3 + (2*(d + e*x^2)^(1/2)*(-(b^7*e + 8*a^3*c^4*d + b^4*e*(-(4*a*c - b^2)^3)^(1/2) - b^6*c*d - 18*a^2*b^2*c^3*d + 25*a^2*b^3*c^2*e + a^2*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 9*a*b^5*c*e + 8*a*b^4*c^2*d - 20*a^3*b*c^3*e - b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) + 2*a*b*c^2*d*(-(4*a*c - b^2)^3)^...

3.356 $\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=292

$$\frac{\sqrt{d+ex^2}}{c}+\frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\left(bcd-b\right.}{}$$

```
[Out] (e*x^2+d)^(1/2)/c+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(
-4*a*c+b^2)^(1/2)))^(1/2))*(b*c*d-b^2*e+2*a*c*e-(-b*e+c*d)*(-4*a*c+b^2)^(1/
2))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/
2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/
2)))^(1/2))*(b*c*d-b^2*e+2*a*c*e+(-b*e+c*d)*(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(
1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 2.43, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 838, 840, 1180, 214}

$$\frac{\left(-\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\sqrt{b^2-4ac}\,(cd-be)+2ace+b^2(-e)+bcd\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[d + e*x^2]/c + ((b*c*d - b^2*e + 2*a*c*e - Sqrt[b^2 - 4*a*c]*(c*d - b*
e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - ((b*c*d - b^2*e + 2*a*c*e + Sqrt[b^2 - 4*a*c]*(c*d - b*e))*
ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-ae+(cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx, x, x^2\right)}{2c}$$

$$= \frac{\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-ae^2-d(cd-be)+(cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}$$

$$= \frac{\sqrt{d+ex^2}}{c} - \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-}\right.}{2c\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{d+ex^2}}{c} + \frac{\left(bcd-b^2e+2ace-\sqrt{b^2-4ac}\,(cd-be)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+}}{\sqrt{2cd-\left(b-\sqrt{b^2}\right.}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.94, size = 350, normalized size = 1.20

$$\frac{2\sqrt{c}\sqrt{d+ex^2} - \frac{\left(-ibcd-c\sqrt{-b^2+4ac}\,d+ib^2e-2iace+b\sqrt{-b^2+4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} - \frac{\left(ibcd-c\sqrt{-b^2+4ac}\,d-ib^2e+2iace+b\sqrt{-b^2+4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x^2] - (((-I)*b*c*d - c*Sqrt[-b^2 + 4*a*c]*d + I*b^2*
e - (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d +
e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*
c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) - ((I*b*c*d - c*Sqrt[-b^2 +
 4*a*c]*d - I*b^2*e + (2*I)*a*c*e + b*Sqrt[-b^2 + 4*a*c]*e)*ArcTan[(Sqrt[2]
*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sq
rt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]))/(2*c^(3/
2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 273, normalized size = 0.93

method	result
risch	$\frac{\sqrt{e\,x^2+d}}{c} - \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^4+\left(4d^2eb-4c\,d^3\right)\_Z^2+d^4c\right)} \frac{\left((eb-cd)\_R^6+\left(4a\,e^2-3deb\right.\right.}{\_R^7c+3\_}}{4c}$
default	$\frac{\sqrt{e\,x^2+d}-\sqrt{e}\,x}{2c} - \frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^4+\left(4d^2eb-4c\,d^3\right)\_Z^2+d^4c\right)} \frac{\left((eb-cd)\_R^6+\left(4\right.\right.}{-}}{4c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/c*((e*x^2+d)^(1/2)-e^(1/2)*x)-1/4/c*sum(((b*e-c*d)*_R^6+(4*a*e^2-3*b*d*
e+3*c*d^2)*_R^4+d*(-4*a*e^2+3*b*d*e-3*c*d^2)*_R^2-d^3*e*b+d^4*c)/(_R^7*c+3*
_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*
d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(
16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+d^4*c))+1/2/c*d/((e
*x^2+d)^(1/2)-e^(1/2)*x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2452 vs. $2(260) = 520$.
time = 29.46, size = 2452, normalized size = 8.40

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 -
 4*a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c +
a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2
 + 2*sqrt(1/2)*sqrt(x^2*e + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c
+ 4*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 -
```

```
2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c
^7)))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*s
qrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^
2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + (2*a^2*b^2 - 2*a^3*c - (a*b
^3 - a^2*b*c)*x^2)*e^2 + (a*b^2*c*d*x^2 - 2*a*b^3*d)*e - ((a*b^2*c^3 - 4*a^
2*c^4)*x^2*e + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c -
a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2)
- sqrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*
a*c^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2
*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 -
2*sqrt(1/2)*sqrt(x^2*e + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c + 4
*a^2*b*c^2)*e - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 - 2*(
b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)
))*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e + (b^2*c^3 - 4*a*c^4)*sqrt
((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/
(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + (2*a^2*b^2 - 2*a^3*c - (a*b^3
- a^2*b*c)*x^2)*e^2 + (a*b^2*c*d*x^2 - 2*a*b^3*d)*e - ((a*b^2*c^3 - 4*a^2*c
^4)*x^2*e + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b
*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + s
qrt(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c
^4)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^
2)*e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 + 2*s
qrt(1/2)*sqrt(x^2*e + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c + 4*a^
2*b*c^2)*e + (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 - 2*(b^3
*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*
sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b
^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^
2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + (2*a^2*b^2 - 2*a^3*c - (a*b^3 - a
^2*b*c)*x^2)*e^2 + (a*b^2*c*d*x^2 - 2*a*b^3*d)*e + ((a*b^2*c^3 - 4*a^2*c^4)
*x^2*e + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^
2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt
(1/2)*c*sqrt(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)
*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*
e^2)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a*b^2*c*d^2 - 2*sqrt
(1/2)*sqrt(x^2*e + d)*((b^4*c - 4*a*b^2*c^2)*d - (b^5 - 5*a*b^3*c + 4*a^2*b
*c^2)*e + (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*sqrt((b^2*c^2*d^2 - 2*(b^3*c
- a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))*sqr
t(((b^2*c - 2*a*c^2)*d - (b^3 - 3*a*b*c)*e - (b^2*c^3 - 4*a*c^4)*sqrt((b^2*
c^2*d^2 - 2*(b^3*c - a*b*c^2)*d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c
^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) + (2*a^2*b^2 - 2*a^3*c - (a*b^3 - a^2*
b*c)*x^2)*e^2 + (a*b^2*c*d*x^2 - 2*a*b^3*d)*e + ((a*b^2*c^3 - 4*a^2*c^4)*x^
2*e + 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*sqrt((b^2*c^2*d^2 - 2*(b^3*c - a*b*c^2)*
d*e + (b^4 - 2*a*b^2*c + a^2*c^2)*e^2)/(b^2*c^6 - 4*a*c^7)))/x^2) + 4*sqrt(
x^2*e + d))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 619 vs. 2(260) = 520.
time = 4.24, size = 619, normalized size = 2.12

$$\frac{\sqrt{x^2e+d}}{c}-\frac{\left(2\,bc^4d^2+((b^2c-4\,ac^2)de-(b^3-4\,abc)e^2)c^2-(3\,b^2c^3-4\,ac^4)de-2\left(\sqrt{b^2-4\,ac}\,c^3d^2-\sqrt{b^2-4\,ac}\,bc^2de+\sqrt{b^2-4\,ac}\,ac^2e^2\right)|c|+(b^3c^2-2\,abc^3)e^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,c^2d-bce+\sqrt{-4\,(c^2d^2-bcde+ace^2)c^2+(2\,c^2d-bce)^2}}{c^2}}}\right)}{\left(2\sqrt{b^2-4\,ac}\,c^2d-\left(b^2c-4\,ac^2+\sqrt{b^2-4\,ac}\,bc\right)e\right)\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\,c^2}+\frac{\left(2\,bc^4d^2+((b^2c-4\,ac^2)de-(b^3-4\,abc)e^2)c^2-(3\,b^2c^3-4\,ac^4)de+2\left(\sqrt{b^2-4\,ac}\,c^3d^2-\sqrt{b^2-4\,ac}\,bc^2de+\sqrt{b^2-4\,ac}\,ac^2e^2\right)|c|+(b^3c^2-2\,abc^3)e^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,c^2d-bce-\sqrt{-4\,(c^2d^2-bcde+ace^2)c^2+(2\,c^2d-bce)^2}}{c^2}}}\right)}{\left(2\sqrt{b^2-4\,ac}\,c^2d+\left(b^2c-4\,ac^2-\sqrt{b^2-4\,ac}\,bc\right)e\right)\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(x^2*e + d)/c - (2*b*c^4*d^2 + ((b^2*c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)
*e^2)*c^2 - (3*b^2*c^3 - 4*a*c^4)*d*e - 2*(sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt
(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*c)*a*c^2*e^2)*abs(c) + (b^3*c^2 -
2*a*b*c^3)*e^2)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^2*d - b*c*e +
 sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2))/((
2*sqrt(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + sqrt(b^2 - 4*a*c)*b*c)*e)*sq
rt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2) + (2*b*c^4*d^2 + ((b^2*
c - 4*a*c^2)*d*e - (b^3 - 4*a*b*c)*e^2)*c^2 - (3*b^2*c^3 - 4*a*c^4)*d*e + 2
*(sqrt(b^2 - 4*a*c)*c^3*d^2 - sqrt(b^2 - 4*a*c)*b*c^2*d*e + sqrt(b^2 - 4*a*
c)*a*c^2*e^2)*abs(c) + (b^3*c^2 - 2*a*b*c^3)*e^2)*arctan(2*sqrt(1/2)*sqrt(x
^2*e + d)/sqrt(-(2*c^2*d - b*c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^
2 + (2*c^2*d - b*c*e)^2))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d + (b^2*c - 4*a*
c^2 - sqrt(b^2 - 4*a*c)*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*
c)*e)*c^2)
```

Mupad [B]
time = 2.34, size = 2500, normalized size = 8.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(1/2)/c - atan(((((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*
d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d
```

```
 + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^
4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2
)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a
*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*
c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4
*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*
b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*
d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3
))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*
a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2)
 - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4))
)^(1/2)*1i - (((16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3
*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)
*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3
*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*
a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/
2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*
((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*
c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a
*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2
) + (2*(d + e*x^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c
^2*d^2*e^2 - 4*a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c
^3*d - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c
*e*(-(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*
d - 12*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/(((
(16*a^2*c^3*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*
b^2*c^3*d^2*e^2 - 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^(1/2)*((8*a^2*c^3*d
 - b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-
(4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 1
2*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e
^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((8*a^2*c^3*d -
 b^5*e - b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(
4*a*c - b^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12
*a^2*b*c^2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x
^2)^(1/2)*(b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*
a*b^2*c*e^4 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e -
b^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b
^2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^
2*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(a*c^2*d^3*e^2 -
a^2*b*e^5 + a*b^2*d*e^4 + a^2*c*d*e^4 - 2*a*b*c*d^2*e^3))/c + (((16*a^2*c^3
*e^4 - 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2
*e^2 - 16*a*b*c^3*d*e^3)/c + (2*(d + e*x^2)^(1/2)*((8*a^2*c^3*d - b^5*e - b
^2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^
2)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2
```

```
*e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*
c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*((8*a^2*c^3*d - b^5*e - b^
2*e*(-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2
)^3)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*
e)/(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(
b^4*e^4 + 2*a^2*c^2*e^4 - 2*a*c^3*d^2*e^2 + b^2*c^2*d^2*e^2 - 4*a*b^2*c*e^4
 - 2*b^3*c*d*e^3 + 6*a*b*c^2*d*e^3))/c)*((8*a^2*c^3*d - b^5*e - b^2*e*(-(4*
a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^(1/2
) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8*(16
*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)))*((8*a^2*c^3*d - b^5*e - b^2*e*(
-(4*a*c - b^2)^3)^(1/2) + b^4*c*d + 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3)^
(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c^2*d - 12*a^2*b*c^2*e)/(8
*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*2i - atan(((((16*a^2*c^3*e^4
- 4*a*b^2*c^2*e^4 + 16*a*c^4*d^2*e^2 + 4*b^3*c^2*d*e^3 - 4*b^2*c^3*d^2*e^2
- 16*a*b*c^3*d*e^3)/c - (2*(d + e*x^2)^(1/2)*(-(b^5*e - 8*a^2*c^3*d - b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c*e + a*c*e*(-(4*a*c - b^2)^3
)^(1/2) + b*c*d*(-(4*a*c - b^2)^3)^(1/2) + 6*a*b^2*c^2*d + 12*a^2*b*c^2*e)/
(8*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*
d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(b^5*e - 8*a^2*c^3*d - b^2*e
*(-(4*a*c - b^2)^3)^(1/2) - b^4*c*d - 7*a*b^3*c...
```

3.357 $\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\tan}{\sqrt{2}\sqrt{c}}$$

```
[Out] -1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^
2)^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2)))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)*2^(1/2)/c^(1/2)/(-
4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1261, 713, 1144, 214}

$$\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d
+ e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*Sqrt[c]*Sqrt[b
^2 - 4*a*c])) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*ArcTanh[(Sqrt[2]*S
qrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*
Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 713

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= e\text{Subst}\left(\int \frac{x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)$$
$$= -\left(\frac{1}{2}\left(-e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{-\frac{1}{2}\sqrt{b^2-4ac}\,e+\frac{1}{2}(-2cd+be)+cx^2}\,dx, x, \sqrt{\right.\right.$$
$$= -\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{2cd-}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.57, size = 256, normalized size = 1.27

$$\frac{\frac{\left(-2icd+\left(ib+\sqrt{-b^2+4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\left(2icd+\left(-ib+\sqrt{-b^2+4ac}\right)e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{\sqrt{2}\sqrt{c}\sqrt{-b^2+4ac}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((((-2*I)*c*d + (I*b + Sqrt[-b^2 + 4*a*c])*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/Sqrt[-2*c*d + (b
```

```
- I*Sqrt[-b^2 + 4*a*c])*e] + (((2*I)*c*d + ((-I)*b + Sqrt[-b^2 + 4*a*c])*e)
*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 +
 4*a*c]*e]])/Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e])/(Sqrt[2]*Sqrt[c]*
Sqrt[-b^2 + 4*a*c])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 177, normalized size = 0.88

method	result
default	$-\frac{e\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{8}+(4eb-4cd)\_Z^{6}+\left(16a\,e^{2}-8deb+6c\,d^{2}\right)\_Z^{4}+\left(4d^{2}eb-4c\,d^{3}\right)\_Z^{2}+d^{4}c\right)}\frac{\left(-\_R^{6}-\_R^{4}d+\_R^{2}d^{2}+d^{3}\right)\ln}{\_R^{7}c+3\_R^{5}be-3\_R^{5}cd+8\_R^{3}a\,e^{2}-4}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*e*sum((-_R^6-_R^4*d+_R^2*d^2+d^3)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3
*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^
(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*
_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+d^4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(169) = 338$.
time = 6.19, size = 1037, normalized size = 5.13

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c
^3))/(b^2*c - 4*a*c^2))*log((2*b*d*e + 2*sqrt(1/2)*sqrt(x^2*e + d)*((b^2 -
4*a*c)*e + (b^3*c - 4*a*b*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e
 + (b^2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) + (b*x^2
 - 2*a)*e^2 + ((b^2*c - 4*a*c^2)*x^2*e + 2*(b^2*c - 4*a*c^2)*d)*e/sqrt(b^2*
```

```
c^2 - 4*a*c^3))/x^2) + 1/4*sqrt(1/2)*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*
e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((2*b*d*e - 2*sqrt(1/2)*sq
rt(x^2*e + d)*((b^2 - 4*a*c)*e + (b^3*c - 4*a*b*c^2)*e/sqrt(b^2*c^2 - 4*a*c
^3))*sqrt((2*c*d - b*e + (b^2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*
c - 4*a*c^2)) + (b*x^2 - 2*a)*e^2 + ((b^2*c - 4*a*c^2)*x^2*e + 2*(b^2*c - 4
*a*c^2)*d)*e/sqrt(b^2*c^2 - 4*a*c^3))/x^2) - 1/4*sqrt(1/2)*sqrt((2*c*d - b*
e - (b^2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((2*
b*d*e + 2*sqrt(1/2)*sqrt(x^2*e + d)*((b^2 - 4*a*c)*e - (b^3*c - 4*a*b*c^2)*
e/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*e/sqrt(b^2
*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) + (b*x^2 - 2*a)*e^2 - ((b^2*c - 4*a*c^2
)*x^2*e + 2*(b^2*c - 4*a*c^2)*d)*e/sqrt(b^2*c^2 - 4*a*c^3))/x^2) + 1/4*sqrt
(1/2)*sqrt((2*c*d - b*e - (b^2*c - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2
*c - 4*a*c^2))*log((2*b*d*e - 2*sqrt(1/2)*sqrt(x^2*e + d)*((b^2 - 4*a*c)*e
- (b^3*c - 4*a*b*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((2*c*d - b*e - (b^2*c
 - 4*a*c^2)*e/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) + (b*x^2 - 2*a)*e
^2 - ((b^2*c - 4*a*c^2)*x^2*e + 2*(b^2*c - 4*a*c^2)*d)*e/sqrt(b^2*c^2 - 4*a
*c^3))/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 3.96, size = 228, normalized size = 1.13

$$-\frac{\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,\sqrt{x^2e+d}}{\sqrt{-\frac{2\,cd-be+\sqrt{(2\,cd-be)^2-4\,(cd^2-bde+ae^2)c}}{c}}}\right)}{2\sqrt{b^2-4\,ac}\,|c|}+\frac{\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\,\arctan\left(\frac{2\sqrt{\frac{1}{2}}\,\sqrt{x^2e+d}}{\sqrt{-\frac{2\,cd-be-\sqrt{(2\,cd-be)^2-4\,(cd^2-bde+ae^2)c}}{c}}}\right)}{2\sqrt{b^2-4\,ac}\,|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sq
rt(x^2*e + d)/sqrt(-(2*c*d - b*e + sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e
+ a*e^2)*c))/c))/(sqrt(b^2 - 4*a*c)*abs(c)) + 1/2*sqrt(-4*c^2*d + 2*(b*c +
sqrt(b^2 - 4*a*c)*c)*e)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c*d - b
*e - sqrt((2*c*d - b*e)^2 - 4*(c*d^2 - b*d*e + a*e^2)*c))/c))/(sqrt(b^2 - 4
*a*c)*abs(c))
```

Mupad [B]
time = 1.72, size = 717, normalized size = 3.55

$$-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{e\,x^2+d}\,\left(-2\,b^2\,c\,e^4+4\,b\,c^2\,d\,e^3-4\,c^3\,d^2\,e^2+4\,a\,c^2\,e^4\right)+\frac{\sqrt{e\,x^2+d}\,\left(8\,b^3\,c^2\,e^3-16\,b^2\,c^3\,d\,e^2-32\,a\,b\,c^3\,e^3+64\,a\,c^4\,d\,e^2\right)\,\left(b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{-\frac{b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{2\,c^2\,d^2\,e^3-2\,b\,c\,d\,e^4+2\,a\,c\,e^5}\right)\,\sqrt{-\frac{b^3\,e+e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}+8\,a\,c^2\,d-2\,b^2\,c\,d-4\,a\,b\,c\,e}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}-2\,\mathrm{atanh}\left(\frac{2\left(\sqrt{e\,x^2+d}\,\left(-2\,b^2\,c\,e^4+4\,b\,c^2\,d\,e^3-4\,c^3\,d^2\,e^2+4\,a\,c^2\,e^4\right)-\frac{\sqrt{e\,x^2+d}\,\left(8\,b^3\,c^2\,e^3-16\,b^2\,c^3\,d\,e^2-32\,a\,b\,c^3\,e^3+64\,a\,c^4\,d\,e^2\right)\,\left(e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e\right)}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}\right)\,\sqrt{\frac{e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}}{2\,c^2\,d^2\,e^3-2\,b\,c\,d\,e^4+2\,a\,c\,e^5}\right)\,\sqrt{\frac{e\,\sqrt{-{\left(4\,a\,c-b^2\right)}^3}-b^3\,e-8\,a\,c^2\,d+2\,b^2\,c\,d+4\,a\,b\,c\,e}{8\,\left(16\,a^2\,c^3-8\,a\,b^2\,c^2+b^4\,c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] - 2*atanh((2*((d + e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2
+ 4*b*c^2*d*e^3) + ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 3
2*a*b*c^3*e^3 + 64*a*c^4*d*e^2)*(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c
^2*d - 2*b^2*c*d - 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*(-(b
^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d - 4*a*b*c*e)/(8*(
b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b
*c*d*e^4))*(-(b^3*e + e*(-(4*a*c - b^2)^3)^(1/2) + 8*a*c^2*d - 2*b^2*c*d -
4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2) - 2*atanh((2*((d +
 e*x^2)^(1/2)*(4*a*c^2*e^4 - 2*b^2*c*e^4 - 4*c^3*d^2*e^2 + 4*b*c^2*d*e^3) -
 ((d + e*x^2)^(1/2)*(8*b^3*c^2*e^3 - 16*b^2*c^3*d*e^2 - 32*a*b*c^3*e^3 + 64
*a*c^4*d*e^2)*(e*(-(4*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d +
 4*a*b*c*e))/(8*(b^4*c + 16*a^2*c^3 - 8*a*b^2*c^2)))*((e*(-(4*a*c - b^2)^3)
^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c + 16*a^2*c^3
- 8*a*b^2*c^2)))^(1/2))/(2*c^2*d^2*e^3 + 2*a*c*e^5 - 2*b*c*d*e^4))*((e*(-(4
*a*c - b^2)^3)^(1/2) - b^3*e - 8*a*c^2*d + 2*b^2*c*d + 4*a*b*c*e)/(8*(b^4*c
 + 16*a^2*c^3 - 8*a*b^2*c^2)))^(1/2)
```

3.358 $\int \frac{\sqrt{d+ex^2}}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=281

$$-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}+\frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-$$

```
[Out] -arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/a+1/2*arctanh(2^(1/2)*c^(1/2)*(e*
x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b*d-2*a*e+d*(
-4*a*c+b^2)^(1/2))/a*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1
/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))^(1/2))*c^(1/2)*(b*d-2*a*e-d*(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(-4
*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.83, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 911, 1301, 212, 1180, 214}

$$\frac{\sqrt{c}\left(d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-d\sqrt{b^2-4ac}-2ae+bd\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) + (Sqrt[c]*(b*d + Sqrt[b^2
- 4*a*c]*d - 2*a*e)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b -
 Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c]*(b*d - Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcT
anh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{de}{a(d-x^2)}+\frac{e\left(cd^2-bde+ae^2-cdx^2\right)}{a\left(cd^2-bde+ae^2-(2cd-be)x^2+cx^4\right)}\right)\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{cd^2-bde+ae^2-cdx^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{c\right.}}{a}$$

$$= -\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\left(c\left(bd-\sqrt{b^2-4ac}\,d-2ae\right)\right)\text{Subst}\left(\int \frac{}{\frac{1}{2}\sqrt{b^2-}}\right.}{2a\sqrt{b^2-4a}}$$

$$= -\frac{\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{2cd-}}\right.}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2}\right.}}$$

Mathematica [A]
time = 0.67, size = 274, normalized size = 0.98

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-bd+\sqrt{b^2-4ac}\,d+2ae\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+2\sqrt{d}\,\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((Sqrt[2]*Sqrt[c]*(b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*ArcTan[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b
^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-
(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2
])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*
d + (b + Sqrt[b^2 - 4*a*c])*e]) + 2*Sqrt[d]*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]
])/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 299, normalized size = 1.06

method	result
default	$-\frac{\frac{\sqrt{e\,x^2+d}}{2}-\frac{\sqrt{e}\,x}{2}+\frac{d}{2\sqrt{e\,x^2+d}-2\sqrt{e}\,x}-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^4+\left(4d^2eb-4c\,d^3\right)\ldots\right.}\right.}{}}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(1/2*(e*x^2+d)^(1/2)-1/2*e^(1/2)*x+1/2*d/((e*x^2+d)^(1/2)-e^(1/2)*x)-1
/4*sum((-c*d*_R^6+(4*a*e^2-4*b*d*e+3*c*d^2)*_R^4+d*(-4*a*e^2+4*b*d*e-3*c*d^
2)*_R^2+d^4*c)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R
^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*
_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)
*_Z^2+d^4*c))+1/a*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/
2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1578 vs. $2(241) = 482$.
time = 49.98, size = 3169, normalized size = 11.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqr
t((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)
)*log(-(2*b^2*d^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(x^2*e + d)*sqrt((b
^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*
a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2
- 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a*b*x^2 - 2*a^2)*e^2 + (b^2*d*x^2 - 4*
a*b*d)*e - ((a^2*b^2 - 4*a^3*c)*x^2*e + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*
d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(
```

```
a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a
^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*sqrt
(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(x^2*e + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e
^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*
c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*
a^3*c)) - (a*b*x^2 - 2*a^2)*e^2 + (b^2*d*x^2 - 4*a*b*d)*e - ((a^2*b^2 - 4*a
^3*c)*x^2*e + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)
/(a^4*b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d -
(a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*
sqrt(x^2*e + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*s
qrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d
*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a*b*x^2 - 2*a^2
)*e^2 + (b^2*d*x^2 - 4*a*b*d)*e + ((a^2*b^2 - 4*a^3*c)*x^2*e + 2*(a^2*b^2 -
 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2
) + sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*l
og(-(2*b^2*d^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(x^2*e + d)*sqrt((b^2*
d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c
)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4
*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a*b*x^2 - 2*a^2)*e^2 + (b^2*d*x^2 - 4*a*b
*d)*e + ((a^2*b^2 - 4*a^3*c)*x^2*e + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2
 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - 2*sqrt(d)*log(-(x^2*e
- 2*sqrt(x^2*e + d)*sqrt(d) + 2*d)/x^2))/a, -1/4*(sqrt(1/2)*a*sqrt(-(a*b*e
- (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2
)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 + 4*sqrt(1/2)*
(a^3*b^2 - 4*a^4*c)*sqrt(x^2*e + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a
^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqr
t((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)
) - (a*b*x^2 - 2*a^2)*e^2 + (b^2*d*x^2 - 4*a*b*d)*e - ((a^2*b^2 - 4*a^3*c)*
x^2*e + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*
b^2 - 4*a^5*c)))/x^2) - sqrt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d + (a^2*b
^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a
^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(x
^2*e + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(
a*b*e - (b^2 - 2*a*c)*d + (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a
^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a*b*x^2 - 2*a^2)*e^2
+ (b^2*d*x^2 - 4*a*b*d)*e - ((a^2*b^2 - 4*a^3*c)*x^2*e + 2*(a^2*b^2 - 4*a^3
*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) - sq
rt(1/2)*a*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^
2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2
*b^2*d^2 + 4*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*sqrt(x^2*e + d)*sqrt((b^2*d^2 -
2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d -
(a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c)) - (a*b*x^2 - 2*a^2)*e^2 + (b^2*d*x^2 - 4*a*b*d)*e
```

```
+ ((a^2*b^2 - 4*a^3*c)*x^2*e + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a
*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/x^2) + sqrt(1/2)*a*sqrt(-(a*b*e - (
b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(
a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*b^2*d^2 - 4*sqrt(1/2)*(a^
3*b^2 - 4*a^4*c)*sqrt(x^2*e + d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*
b^2 - 4*a^5*c))*sqrt(-(a*b*e - (b^2 - 2*a*c)*d - (a^2*b^2 - 4*a^3*c)*sqrt((
b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) -
 (a*b*x^2 - 2*a^2)*e^2 + (b^2*d*x^2 - 4*a*b*d)*e + ((a^2*b^2 - 4*a^3*c)*x^2
*e + 2*(a^2*b^2 - 4*a^3*c)*d)*sqrt((b^2*d^2 - 2*a*b*d*e + a^2*e^2)/(a^4*b^2
 - 4*a^5*c)))/x^2) - 4*sqrt(-d)*arctan(sqrt(-d)/sqrt(x^2*e + d)))/a]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x*(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 717 vs. 2(241) = 482.
time = 3.95, size = 717, normalized size = 2.55

$$\frac{d\arctan\left(\frac{\sqrt{x^2e+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}} - \frac{\left(\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,(b^2-4ac)a^2de - 2\left(\sqrt{b^2-4ac}\,acd^2-\sqrt{b^2-4ac}\,abde+\sqrt{b^2-4ac}\,a^2e^2\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,|a| - \left(2a^2bcd^2+2a^3be^2-(a^2b^2+4a^3c)de\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2acd-abe+\sqrt{-4(acd^2-abde+a^2e^2)ac+(2acd-abe)^2}}{ac}}}\right)}{8\left(\sqrt{b^2-4ac}\,a^2cd^2-\sqrt{b^2-4ac}\,a^2bde+\sqrt{b^2-4ac}\,a^3e^2\right)|a||c|} + \frac{\left(\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,(b^2-4ac)a^2de + 2\left(\sqrt{b^2-4ac}\,acd^2-\sqrt{b^2-4ac}\,abde+\sqrt{b^2-4ac}\,a^2e^2\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,|a| - \left(2a^2bcd^2+2a^3be^2-(a^2b^2+4a^3c)de\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2acd-abe-\sqrt{-4(acd^2-abde+a^2e^2)ac+(2acd-abe)^2}}{ac}}}\right)}{8\left(\sqrt{b^2-4ac}\,a^2cd^2-\sqrt{b^2-4ac}\,a^2bde+\sqrt{b^2-4ac}\,a^3e^2\right)|a||c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(sqrt(-4*c^2*d + 2*(b
*c - sqrt(b^2 - 4*a*c)*c)*e)*(b^2 - 4*a*c)*a^2*d*e - 2*(sqrt(b^2 - 4*a*c)*a
*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2
*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2
 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c
*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 -
4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)
*abs(a)*abs(c)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*(b^
2 - 4*a*c)*a^2*d*e + 2*(sqrt(b^2 - 4*a*c)*a*c*d^2 - sqrt(b^2 - 4*a*c)*a*b*d
*e + sqrt(b^2 - 4*a*c)*a^2*e^2)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*
c)*e)*abs(a) - (2*a^2*b*c*d^2 + 2*a^3*b*e^2 - (a^2*b^2 + 4*a^3*c)*d*e)*sqrt
(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x^2*e
 + d)/sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c +
```

```
(2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c*d^2 - sqrt(b^2 - 4*
a*c)*a^2*b*d*e + sqrt(b^2 - 4*a*c)*a^3*e^2)*abs(a)*abs(c))
```

Mupad [B]
time = 6.87, size = 2500, normalized size = 8.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9
+ 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) + ((b^4*d + 8*a^2*c^2*d - a*b^3*e
+ a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d
 + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((((b^4*d +
 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)
^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2
*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a
*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c
*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*
a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*
c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^
2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) - 192*a^4*c^4*d*e^10 - 192*a^3*c^5*d^3*e^8
 + 48*a^2*b^2*c^4*d^3*e^8 - 48*a^2*b^3*c^3*d^2*e^9 + 192*a^3*b*c^4*d^2*e^9
+ 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^
4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 -
16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*
c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a
^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^
(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c))
)^(1/2) + 12*a*c^5*d^4*e^8 + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^
4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^
11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c
 - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e
)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(
2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a
*b*c^3*d*e^11) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(
1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^
4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*
c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/
2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(
1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^
2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*
(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4
*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2
```

*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2) + 12*a^2*c^4*d^2*e^10 - 4*b^2*c^4*d^4*e^8 + 4*b^4*c^2*d^2*e^10 + 8*a*b*c^4*d^3*e^9 - 4*a*b^3*c^2*d*e^11 + 20*a^2*b*c^3*d*e^11 - 24*a*b^2*c^3*d^2*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*1i)/(((d + e*x^2)^(1/2)*(2*a^2*c^3*e^12 + 6*c^5*d^4*e^8 - 8*b*c^4*d^3*e^9 + 4*b^2*c^3*d^2*e^10 - 4*a*b*c^3*d*e^11) - ((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(12*a*c^5*d^4*e^8 - (((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*((d + e*x^2)^(1/2)*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*c^3*d*e^9) + 192*a^4*c^4*d*e^10 + 192*a^3*c^5*d^3*e^8 - 48*a^2*b^2*c^4*d^3*e^8 + 48*a^2*b^3*c^3*d^2*e^9 - 192*a^3*b*c^4*d^2*e^9 - 48*a^3*b^2*c^3*d*e^10) - (d + e*x^2)^(1/2)*(32*a^3*b*c^3*e^11 + 48*a^3*c^4*d*e^10 - 8*a^2*b^3*c^2*e^11 + 144*a^2*c^5*d^3*e^8 + 16*b^4*c^3*d^3*e^8 - 16*b^5*c^2*d^2*e^9 + 16*a*b^4*c^2*d*e^10 - 96*a*b^2*c^4*d^3*e^8 + 96*a*b^3*c^3*d^2*e^9 - 144*a^2*b*c^4*d^2*e^9 - 72*a^2*b^2*c^3*d*e^10))*((b^4*d + 8*a^2*c^2*d - a*b^3*e + a*e*(-(4*a*c - b^2)^3)^(1/2) - b*d*(-(4*a*c - b^2)^3)^(1/2) - 6*a*b^2*c*d + 4*a^2*b*c*e)/(8*(a^2*b^4 + 16*a^4*c^2 - 8*a^3*b^2*c)))^(1/...

3.359 $\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=382

$$-\frac{\sqrt{d+ex^2}}{2ax^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}}+\frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{c}\left(b^2d-2acd-abe+\sqrt{b^2-\cdots}\right.}{\sqrt{2}\,a^2\sqrt{b^2-\cdots}}$$

```
[Out] 1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a/d^(1/2)+(-a*e+b*d)*arctanh((e*x^2+
d)^(1/2)/d^(1/2))/a^2/d^(1/2)-1/2*(e*x^2+d)^(1/2)/a/x^2-1/2*arctanh(2^(1/2)
*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b
^2*d-2*a*c*d-a*b*e+(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(-4*a*c+b^2)^
(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e
*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^2*d-2*a*c*
d-a*b*e-(-a*e+b*d)*(-4*a*c+b^2)^(1/2))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*
d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 2.76, antiderivative size = 370, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1265, 911, 1301, 205, 212, 1180, 214}

$$-\frac{\sqrt{c}\left(\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}\,(bd-ae)-abe-2acd+b^2d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}}-\frac{\sqrt{d+ex^2}}{2ax^2}+\frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*Sqrt[d + e*x^2]/(a*x^2) + (e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a*Sq
rt[d]) + ((b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^2*Sqrt[d]) - (Sq
rt[c]*(b^2*d - 2*a*c*d - a*b*e + Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sq
rt[c]*(b^2*d - 2*a*c*d - a*b*e - Sqrt[b^2 - 4*a*c]*(b*d - a*e))*ArcTanh[(Sq
rt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(S
qrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
```

```
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{de^2}{a(d-x^2)^2}-\frac{e(-bd+ae)}{a^2(d-x^2)}+\frac{e(-b(cd^2-bde+ae^2)+c(bd-ae)x^2)}{a^2(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{-b(cd^2-bde+ae^2)+c(bd-ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^2} + \frac{(de)\text{Subst}\left(\int \frac{1}{(d-x^2)^2}\,dx\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} + \frac{e\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d+e\right.}{2a}$$

$$= -\frac{\sqrt{d+ex^2}}{2ax^2} + \frac{e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a\sqrt{d}} + \frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2\sqrt{d}} - \cdots$$

Mathematica [A]
time = 1.19, size = 348, normalized size = 0.91

$$\frac{-\frac{a\sqrt{d+ex^2}}{x^2}+\frac{\sqrt{2}\sqrt{c}\left(b^2d-2acd+b\sqrt{b^2-4ac}\,d-abe-a\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(-b^2d+2acd+b\sqrt{b^2-4ac}\,d+abe-a\sqrt{b^2-4ac}\,e\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+\frac{(2bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{\sqrt{d}}}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(b^2*d - 2*a*c*d + b*Sqrt[b^
2 - 4*a*c]*d - a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[
d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*S
qrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(-(b^2*d) + 2*a
*c*d + b*Sqrt[b^2 - 4*a*c]*d + a*b*e - a*Sqrt[b^2 - 4*a*c]*e)*ArcTan[(Sqrt[
2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqr
t[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((2*b*d - a*e)*A
rcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/Sqrt[d])/(2*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 404, normalized size = 1.06

method	result
risch	$-\frac{\sqrt{e\,x^2+d}}{2a\,x^2}-\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)e}{2a\sqrt{d}}+\frac{\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^2+d}}{x}\right)b}{a^2}-\frac{\_R=\text{RootOf}\left(c\_Z^8+(4eb-4cd)\right.}{}$ $-\frac{b\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)}{2}-\frac{\left(\sum_{\_R=\text{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^4+\left(4d^2eb-4c\,d^3\right)\_Z^2+d^4c\right)}\frac{\left(c(-ae+bd)\_\right.}{}\right.}{}$
default	$-\frac{}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(-1/2*b*((e*x^2+d)^(1/2)-e^(1/2)*x)-1/4*sum((c*(-a*e+b*d)*_R^6+(-4*a
*b*e^2-a*c*d*e+4*b^2*d*e-3*b*c*d^2)*_R^4+d*(4*a*b*e^2+a*c*d*e-4*b^2*d*e+3*b
*c*d^2)*_R^2+a*c*d^3*e-c*d^4*b)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-
4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x
-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4
*b*d^2*e-4*c*d^3)*_Z^2+d^4*c))-1/2*b*d/((e*x^2+d)^(1/2)-e^(1/2)*x))+1/a*(-1
/2/d/x^2*(e*x^2+d)^(3/2)+1/2*e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)
*(e*x^2+d)^(1/2))/x)))-b/a^2*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*
x^2+d)^(1/2))/x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x^3(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x**3*(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] sage0*x
```

Mupad [B]
time = 5.46, size = 2500, normalized size = 6.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)
[Out] (atan((((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e
^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*
a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) - (
((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*
e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9
 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^
10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a
^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b
*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a*e - 2*b*d)*(((d + e*x^2)^(
1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^
5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*
c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b
^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*
b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) -
 ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^
11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9
```

+ 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2))*(a*e - 2*b*d))/(4*a^2*d^(1/2)))*1i)/(4*a^2*d^(1/2)) + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) + (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 + ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) + ((a*e - 2*b*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 + ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6*d^(1/2))))/(4*a^2*d^(1/2))))/(4*a^2*d^(1/2))*(a*e - 2*b*d))/(4*a^2*d^(1/2)))*1i)/(4*a^2*d^(1/2))))/(((a^3*c^5*e^13)/2 + a*c^7*d^4*e^9 - 2*b*c^7*d^5*e^8 + (3*a^2*c^6*d^2*e^11)/2 + 2*b^2*c^6*d^4*e^9 - 4*a*b*c^6*d^3*e^10 - (3*a^2*b*c^5*d*e^12)/2 + a*b^2*c^5*d^2*e^11)/a^4 - ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(6*a^4*c^5*e^12 + 4*a^2*c^7*d^4*e^8 + 6*a^3*c^6*d^2*e^10 + 4*b^4*c^5*d^4*e^8 + 21*a^2*b^2*c^5*d^2*e^10 - 18*a^3*b*c^5*d*e^11 - 8*a*b^2*c^6*d^4*e^8 - 12*a*b^3*c^5*d^3*e^9))/(2*a^4) - (((16*a^5*b*c^4*e^12 + 20*a^5*c^5*d*e^11 + a^3*b^5*c^2*e^12 - 8*a^4*b^3*c^3*e^12 + 20*a^4*c^6*d^3*e^9 + 40*a^2*b^3*c^5*d^4*e^8 - 20*a^2*b^4*c^4*d^3*e^9 - 27*a^2*b^5*c^3*d^2*e^10 - 20*a^3*b^2*c^5*d^3*e^9 + 84*a^3*b^3*c^4*d^2*e^10 - 8*a*b^5*c^4*d^4*e^8 + 6*a*b^6*c^3*d^3*e^9 + 2*a*b^7*c^2*d^2*e^10 - 3*a^2*b^6*c^2*d*e^11 - 32*a^3*b*c^6*d^4*e^8 + 28*a^3*b^4*c^3*d*e^11 - 36*a^4*b*c^5*d^2*e^10 - 68*a^4*b^2*c^4*d*e^11)/a^4 - ((a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(240*a^6*b*c^4*e^11 + 64*a^6*c^5*d*e^10 + 20*a^4*b^5*c^2*e^11 - 140*a^5*b^3*c^3*e^11 + 160*a^5*c^6*d^3*e^8 - 32*a^2*b^6*c^3*d^3*e^8 + 32*a^2*b^7*c^2*d^2*e^9 + 224*a^3*b^4*c^4*d^3*e^8 - 208*a^3*b^5*c^3*d^2*e^9 - 432*a^4*b^2*c^5*d^3*e^8 + 272*a^4*b^3*c^4*d^2*e^9 - 48*a^3*b^6*c^2*d*e^10 + 348*a^4*b^4*c^3*d*e^10 + 224*a^5*b*c^5*d^2*e^9 - 648*a^5*b^2*c^4*d*e^10))/(2*a^4) - ((a*e - 2*b

```
*d)*((128*a^8*c^4*e^11 + 8*a^6*b^4*c^2*e^11 - 64*a^7*b^2*c^3*e^11 + 128*a^7
*c^5*d^2*e^9 + 32*a^5*b^3*c^4*d^3*e^8 - 24*a^5*b^4*c^3*d^2*e^9 + 64*a^6*b^2
*c^4*d^2*e^9 - 256*a^7*b*c^4*d*e^10 - 8*a^5*b^5*c^2*d*e^10 - 128*a^6*b*c^5*
d^3*e^8 + 96*a^6*b^3*c^3*d*e^10)/a^4 - ((d + e*x^2)^(1/2)*(a*e - 2*b*d)*(10
24*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5
*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^...
```

3.360 $\int \frac{\sqrt{d+ex^2}}{x^5(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=552

$$-\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right.}{2a^2d^{3/2}}$$

```
[Out] -3/8*e^2*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a/d^(3/2)-1/2*e*(-a*e+b*d)*arctan
h((e*x^2+d)^(1/2)/d^(1/2))/a^2/d^(3/2)-(-a*b*e-a*c*d+b^2*d)*arctanh((e*x^2+
d)^(1/2)/d^(1/2))/a^3/d^(1/2)-1/4*(e*x^2+d)^(1/2)/a/x^4+3/8*e*(e*x^2+d)^(1/
2)/a/d/x^2+1/2*(-a*e+b*d)*(e*x^2+d)^(1/2)/a^2/d/x^2+1/2*arctanh(2^(1/2)*c^(
1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^3*d
-a*c*(-2*a*e+d*(-4*a*c+b^2)^(1/2))+b^2*(-a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c
*d+e*(-4*a*c+b^2)^(1/2)))/a^3*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2)*(b^3*d-b^2*(a*e+d*(-4*a*c+b^2)^(1/2))
+a*c*(2*a*e+d*(-4*a*c+b^2)^(1/2))-a*b*(3*c*d-e*(-4*a*c+b^2)^(1/2)))/a^3*2^(
1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 2.81, antiderivative size = 552, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1265, 911, 1301, 205, 212, 1180, 214}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)(-abe-acd+b^2d)}{a^3\sqrt{d}}+\frac{\sqrt{c}\left(b^2\left(d\sqrt{b^2-4ac}-ae\right)-ab\left(e\sqrt{b^2-4ac}+3cd\right)-ac\left(d\sqrt{b^2-4ac}-2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\sqrt{c}\left(-b^2\left(d\sqrt{b^2-4ac}+ae\right)-ab\left(3cd-e\sqrt{b^2-4ac}\right)+ac\left(d\sqrt{b^2-4ac}+2ae\right)+b^3d\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}a^3\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2d^{3/2}}+\frac{\sqrt{d+ex^2}(bd-ae)}{2a^2dx^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}+\frac{3e\sqrt{d+ex^2}}{8adx^2}-\frac{\sqrt{d+ex^2}}{4ax^4}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*Sqrt[d + e*x^2]/(a*x^4) + (3*e*Sqrt[d + e*x^2])/(8*a*d*x^2) + ((b*d -
a*e)*Sqrt[d + e*x^2])/(2*a^2*d*x^2) - (3*e^2*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d
]])/(8*a*d^(3/2)) - (e*(b*d - a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2
*d^(3/2)) - ((b^2*d - a*c*d - a*b*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(a^3
*Sqrt[d]) + (Sqrt[c]*(b^3*d - a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(Sqrt
[b^2 - 4*a*c]*d - a*e) - a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*a^3*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (Sqrt[c
]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a
*e) - a*b*(3*c*d - Sqrt[b^2 - 4*a*c]*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d +
```

```
e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^3*Sqrt[b^2 - 4
*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 + 
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^5\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{d+ex}}{x^3\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^2}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^3\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{de^3}{a(d-x^2)^3}+\frac{e^2(-bd+ae)}{a^2(d-x^2)^2}+\frac{e(-b^2d+acd+abe)}{a^3(d-x^2)}+\frac{e\left((b^2-ac)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd\right.\right.}{a^3\left(cd^2-bde+ae^2-(2cd-be)x^2+c\right.}\right.\right.}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-ac)\left(cd^2-bde+ae^2\right)-c\left(b^2d-acd-abe\right)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^3} - \frac{(de^2)\,\text{Subst}\left(\int \frac{}{(a}\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{(b^2d-acd-abe)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^3\sqrt{d}} -$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{e(bd-ae)\tanh^{-1}\left(\frac{\sqrt{d+}}{\sqrt{}}\right.}{2a^2d^{3/2}}$$

$$= -\frac{\sqrt{d+ex^2}}{4ax^4}+\frac{3e\sqrt{d+ex^2}}{8adx^2}+\frac{(bd-ae)\sqrt{d+ex^2}}{2a^2dx^2}-\frac{3e^2\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{8ad^{3/2}}$$

Mathematica [A]
time = 1.85, size = 445, normalized size = 0.81

$$\frac{\frac{a\sqrt{d+ex^2}\left(4bdx^2-a\left(2d+ex^2\right)\right)}{dx^4}+\frac{4\sqrt{2}\sqrt{c}\left(-b^3d+ac\left(\sqrt{b^2-4ac}\,d-2ae\right)+b^2\left(-\sqrt{b^2-4ac}\,d+ae\right)+ab\left(3cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-\sqrt{b^2-4ac}\,e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{4\sqrt{2}\sqrt{c}\left(b^3d-b^2\left(\sqrt{b^2-4ac}\,d+ae\right)+ac\left(\sqrt{b^2-4ac}\,d+2ae\right)+ab\left(-3cd+\sqrt{b^2-4ac}\,e\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}\right)}{\sqrt{b^2-4ac}\sqrt{-2cd+\left(b+\sqrt{b^2-4ac}\right)e}}+\frac{\left(-8b^2d^2+4abde+a\left(8cd^2+ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{d^{3/2}}}{8a^3}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[d + e*x^2]/(x^5*(a + b*x^2 + c*x^4)),x]

[Out] ((a*Sqrt[d + e*x^2]*(4*b*d*x^2 - a*(2*d + e*x^2)))/(d*x^4) + (4*Sqrt[2]*Sqrt[c]*(-(b^3*d) + a*c*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) + b^2*(-(Sqrt[b^2 - 4*a*c]*d) + a*e) + a*b*(3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - Sqrt[b^2 - 4*a*c]*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b - Sqrt[b^2 - 4*a*c])*e]) + (4*Sqrt[2]*Sqrt[c]*(b^3*d - b^2*(Sqrt[b^2 - 4*a*c]*d + a*e) + a*c*(Sqrt[b^2 - 4*a*c]*d + 2*a*e) + a*b*(-3*c*d + Sqrt[b^2 - 4*a*c]*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e]) + ((-8*b^2*d^2 + 4*a*b*d*e + a*(8*c*d^2 + a*e^2))*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/d^(3/2))/(8*a^3)

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 597, normalized size = 1.08

method	result
risch	$-\frac{\sqrt{e x^2+d}\,\left(a e x^2-4 b d x^2+2 a d\right)}{8 d a^2 x^4}+\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e x^2+d}}{x}\right) e^2}{8 d^{\frac{3}{2}} a}+\frac{\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e x^2+d}}{x}\right) b e}{2\sqrt{d}\, a^2}+\frac{\sqrt{d}\,\ln\left(\ldots\right.}{\ldots}$
default	$\frac{\frac{ac\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)}{2}-\frac{b^2\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)}{2}-\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a e^2-8deb+6c d^2\right)\_Z^4+\left(4d^2eb-\ldots\right.\right.}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/a^3*(1/2*a*c*((e*x^2+d)^(1/2)-e^(1/2)*x)-1/2*b^2*((e*x^2+d)^(1/2)-e^(1/2)*x)-1/4*sum((c*(-a*b*e-a*c*d+b^2*d)*_R^6+(4*a^2*c*e^2-4*a*b^2*e^2-5*a*b*c*d*e+3*a*c^2*d^2+4*b^3*d*e-3*b^2*c*d^2)*_R^4+d*(-4*a^2*c*e^2+4*a*b^2*e^2+5*a*b*c*d*e-3*a*c^2*d^2-4*b^3*d*e+3*b^2*c*d^2)*_R^2+a*b*c*d^3*e+a*c^2*d^4-b^2*c*d^4)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+d^4*c))+1/2*d*(a*c-b^2)/((e*x^2+d)^(1/2)-e^(1/2)*x))+1/a*(-1/4/d/x^4*(e*x^2+d)^(3/2)-1/4*e/d*(-1/2/d/x^2*(e*x^2+d)^(3/2)+1/2*e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))))-b/a^2*(-1/2/d/x^2*(e*x^2+d)^(3/2)+1/2*e/d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)))+(-a*c+b^2)/a^3*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)/((c*x^4 + b*x^2 + a)*x^5), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x^5 \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**5/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**5*(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1055 vs. 2(490) = 980.
time = 5.38, size = 1055, normalized size = 1.91

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^5/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c +
4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*b*c)*e^2)*a^2 - 2*((a*b^2*c - a^2*c^2)*sqrt
(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^
3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e
)*abs(a) - sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c -
 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^
```

```
4*b*c)*e^2))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e
+ sqrt(-4*(a^3*c*d^2 - a^3*b*d*e + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2
))/(a^3*c)))/((sqrt(b^2 - 4*a*c)*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e +
sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a)*abs(c)) + 1/8*(sqrt(-4*c^2*d + 2*(b*c + s
qrt(b^2 - 4*a*c)*c)*e)*((b^4 - 5*a*b^2*c + 4*a^2*c^2)*d*e - (a*b^3 - 4*a^2*
b*c)*e^2)*a^2 + 2*((a*b^2*c - a^2*c^2)*sqrt(b^2 - 4*a*c)*d^2 - (a*b^3 - a^2
*b*c)*sqrt(b^2 - 4*a*c)*d*e + (a^2*b^2 - a^3*c)*sqrt(b^2 - 4*a*c)*e^2)*sqrt
(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - sqrt(-4*c^2*d + 2*(b*
c + sqrt(b^2 - 4*a*c)*c)*e)*(2*(a^2*b^3*c - 3*a^3*b*c^2)*d^2 - (a^2*b^4 - a
^3*b^2*c - 4*a^4*c^2)*d*e + (a^3*b^3 - 2*a^4*b*c)*e^2))*arctan(2*sqrt(1/2)*
sqrt(x^2*e + d)/sqrt(-(2*a^3*c*d - a^3*b*e - sqrt(-4*(a^3*c*d^2 - a^3*b*d*e
 + a^4*e^2)*a^3*c + (2*a^3*c*d - a^3*b*e)^2))/(a^3*c)))/((sqrt(b^2 - 4*a*c)
*a^4*c*d^2 - sqrt(b^2 - 4*a*c)*a^4*b*d*e + sqrt(b^2 - 4*a*c)*a^5*e^2)*abs(a
)*abs(c)) + 1/8*(8*b^2*d^2 - 8*a*c*d^2 - 4*a*b*d*e - a^2*e^2)*arctan(sqrt(x
^2*e + d)/sqrt(-d))/(a^3*sqrt(-d)*d) + 1/8*(4*(x^2*e + d)^(3/2)*b*d*e - 4*s
qrt(x^2*e + d)*b*d^2*e - (x^2*e + d)^(3/2)*a*e^2 - sqrt(x^2*e + d)*a*d*e^2)
*e^(-2)/(a^2*d*x^4)
```

Mupad [B]
time = 7.30, size = 2500, normalized size = 4.53

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^5*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan(((((((2048*a^12*c^4*d*e^12 + 12288*a^10*c^6*d^5*e^8 + 14336*a^11*c^5*d
^3*e^10 + 2048*a^8*b^4*c^4*d^5*e^8 - 1536*a^8*b^5*c^3*d^4*e^9 - 512*a^8*b^6
*c^2*d^3*e^10 - 11264*a^9*b^2*c^5*d^5*e^8 + 7168*a^9*b^3*c^4*d^4*e^9 + 6272
*a^9*b^4*c^3*d^3*e^10 + 384*a^9*b^5*c^2*d^2*e^11 - 20480*a^10*b^2*c^4*d^3*e
^10 - 3072*a^10*b^3*c^3*d^2*e^11 - 4096*a^10*b*c^5*d^4*e^9 + 128*a^10*b^4*c
^2*d*e^12 + 6144*a^11*b*c^4*d^2*e^11 - 1024*a^11*b^2*c^3*d*e^12)/(64*a^8*d^
2) - ((d + e*x^2)^(1/2)*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1
/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^
3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3
)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(
1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^
2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*(24576*a^12*c^
5*d^4*e^8 + 16384*a^13*c^4*d^2*e^10 + 2048*a^10*b^4*c^3*d^4*e^8 - 2048*a^10
*b^5*c^2*d^3*e^9 - 14336*a^11*b^2*c^4*d^4*e^8 + 15360*a^11*b^3*c^3*d^3*e^9
+ 1024*a^11*b^4*c^2*d^2*e^10 - 8192*a^12*b^2*c^3*d^2*e^10 - 28672*a^12*b*c^
4*d^3*e^9))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^
(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e +
a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)
^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)
```

^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + ((d + e*x^2)^(1/2)*(32*a^10*c^5*d*e^12 - 48*a^10*b*c^4*e^13 - 4*a^8*b^5*c^2*e^13 + 28*a^9*b^3*c^3*e^13 + 4608*a^8*c^7*d^5*e^8 + 2048*a^9*c^6*d^3*e^10 + 512*a^4*b^8*c^3*d^5*e^8 - 512*a^4*b^9*c^2*d^4*e^9 - 4608*a^5*b^6*c^4*d^5*e^8 + 4352*a^5*b^7*c^3*d^4*e^9 + 768*a^5*b^8*c^2*d^3*e^10 + 14080*a^6*b^4*c^5*d^5*e^8 - 11264*a^6*b^5*c^4*d^4*e^9 - 6912*a^6*b^6*c^3*d^3*e^10 - 256*a^6*b^7*c^2*d^2*e^11 - 16384*a^7*b^2*c^6*d^5*e^8 + 7168*a^7*b^3*c^5*d^4*e^9 + 19776*a^7*b^4*c^4*d^3*e^10 + 2272*a^7*b^5*c^3*d^2*e^11 - 18048*a^8*b^2*c^5*d^3*e^10 - 6144*a^8*b^3*c^4*d^2*e^11 - 32*a^7*b^6*c^2*d*e^12 + 3584*a^8*b*c^6*d^4*e^9 + 228*a^8*b^4*c^3*d*e^12 + 4608*a^9*b*c^5*d^2*e^11 - 408*a^9*b^2*c^4*d*e^12))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) + (16*a^9*c^5*e^14 - 4*a^6*b^6*c^2*e^14 + 28*a^7*b^4*c^3*e^14 - 52*a^8*b^2*c^4*e^14 - 768*a^6*c^8*d^6*e^8 - 768*a^7*c^7*d^4*e^10 + 16*a^8*c^6*d^2*e^12 - 512*a^2*b^8*c^4*d^6*e^8 + 384*a^2*b^9*c^3*d^5*e^9 + 128*a^2*b^10*c^2*d^4*e^10 + 3840*a^3*b^6*c^5*d^6*e^8 - 2048*a^3*b^7*c^4*d^5*e^9 - 2208*a^3*b^8*c^3*d^4*e^10 - 224*a^3*b^9*c^2*d^3*e^11 - 8704*a^4*b^4*c^6*d^6*e^8 + 896*a^4*b^5*c^5*d^5*e^9 + 10752*a^4*b^6*c^4*d^4*e^10 + 2688*a^4*b^7*c^3*d^3*e^11 + 96*a^4*b^8*c^2*d^2*e^12 + 6400*a^5*b^2*c^7*d^6*e^8 + 5632*a^5*b^3*c^6*d^5*e^9 - 18144*a^5*b^4*c^5*d^4*e^10 - 10464*a^5*b^5*c^4*d^3*e^11 - 836*a^5*b^6*c^3*d^2*e^12 + 9344*a^6*b^2*c^6*d^4*e^10 + 14592*a^6*b^3*c^5*d^3*e^11 + 2236*a^6*b^4*c^4*d^2*e^12 - 1716*a^7*b^2*c^5*d^2*e^12 - 528*a^8*b*c^5*d*e^13 + 4*a^5*b^7*c^2*d*e^13 - 4352*a^6*b*c^7*d^5*e^9 - 92*a^6*b^5*c^3*d*e^13 - 5632*a^7*b*c^6*d^3*e^11 + 436*a^7*b^3*c^4*d*e^13)/(64*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e + 20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4 + 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2) - ((d + e*x^2)^(1/2)*(a^6*b^2*c^5*e^14 - 2*a^7*c^6*e^14 + 192*a^4*c^9*d^6*e^8 + 32*a^5*c^8*d^4*e^10 + 34*a^6*c^7*d^2*e^12 + 64*b^8*c^5*d^6*e^8 + 704*a^2*b^4*c^7*d^6*e^8 + 960*a^2*b^5*c^6*d^5*e^9 + 192*a^2*b^6*c^5*d^4*e^10 - 512*a^3*b^2*c^8*d^6*e^8 - 1280*a^3*b^3*c^7*d^5*e^9 - 752*a^3*b^4*c^6*d^4*e^10 - 56*a^3*b^5*c^5*d^3*e^11 + 704*a^4*b^2*c^7*d^4*e^10 + 128*a^4*b^3*c^6*d^3*e^11 - 15*a^4*b^4*c^5*d^2*e^12 + 60*a^5*b^2*c^6*d^2*e^12 - 10*a^6*b*c^6*d*e^13 - 384*a*b^6*c^6*d^6*e^8 - 192*a*b^7*c^5*d^5*e^9 + 384*a^4*b*c^8*d^5*e^9 - 144*a^5*b*c^7*d^3*e^11 + 6*a^5*b^3*c^5*d*e^13))/(32*a^8*d^2))*((b^8*d + 8*a^4*c^4*d - b^5*d*(-(4*a*c - b^2)^3)^(1/2) - a*b^7*e + 33*a^2*b^4*c^2*d - 38*a^3*b^2*c^3*d - 25*a^3*b^3*c^2*e + a^3*c^2*e*(-(4*a*c - b^2)^3)^(1/2) - 10*a*b^6*c*d + a*b^4*e*(-(4*a*c - b^2)^3)^(1/2) + 9*a^2*b^5*c*e +

```
20*a^4*b*c^3*e + 4*a*b^3*c*d*(-(4*a*c - b^2)^3)^(1/2) - 3*a^2*b*c^2*d*(-(4
*a*c - b^2)^3)^(1/2) - 3*a^2*b^2*c*e*(-(4*a*c - b^2)^3)^(1/2))/(8*(a^6*b^4
+ 16*a^8*c^2 - 8*a^7*b^2*c)))^(1/2)*1i - (((((2...
```

3.361 $\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=390

$$\frac{x\sqrt{d+ex^2}}{2c}-\frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\left(bcd-b^2e\right.}{}$$

```
[Out] 1/2*(-2*b*e+c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c^2/e^(1/2)+1/2*x*(e*x^
2+d)^(1/2)/c-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+b^
3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*
c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))
^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 2.18, antiderivative size = 390, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1305, 396, 223, 212, 1706, 385, 211}

$$-\frac{\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}}+\frac{x\sqrt{d+ex^2}}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c) - ((b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d
- b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b
 - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b*c*d -
b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c]
)*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4
*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b
+ Sqrt[b^2 - 4*a*c])*e]) + ((c*d - 2*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(2*c^2*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1305

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[(f*x)^(m - 4)*(d +
e*x^2)^(q - 1)*(Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x]/(a + b
*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{\int \frac{cd-be+cex^2}{\sqrt{d+ex^2}}\,dx}{c^2} - \frac{\int \frac{a(cd-be)+(bcd-b^2e+ace)x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{bcd-b^2e+ace-\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} \cdots\right.}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c^2} - \frac{\left(bcd-b^2e+ace-\frac{b^2c\cdots}{}\right.}{\cdots}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} + \frac{(cd-2be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2\sqrt{e}} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+\cdots}{\sqrt{b^2-4ac}}\right.}{\cdots}$$

$$= \frac{x\sqrt{d+ex^2}}{2c} - \frac{\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-\cdots}\right.}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{\cdots}}\right.}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 10915 vs. $2(390) = 780$.
time = 16.33, size = 10915, normalized size = 27.99

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 294, normalized size = 0.75

method	result
risch	$\frac{x\sqrt{e\,x^2+d}}{2c} - \frac{\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)\sqrt{e}\,b}{c^2} + \frac{\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)d}{2c\sqrt{e}} + \frac{\sqrt{e}\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4+(4eb-4cd)\cdots\right.}\right.}{\cdots}$

default	$\frac{\frac{x\sqrt{e\,x^2+d}}{2}+\frac{d\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{2\sqrt{e}}}{c}+\frac{\sqrt{e}\left(\frac{b\ln\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)}{c}-\frac{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2)))+1/c
*e^(1/2)*(b/c*ln((e*x^2+d)^(1/2)-e^(1/2)*x)-1/2/c*sum(((-a*c*e+b^2*e-b*c*d)
*_R^2+2*(2*a*b*e^2-a*c*d*e-b^2*d*e+b*c*d^2)*_R-a*d^2*e*c+b^2*d^2*e-b*c*d^3)
/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d
^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^
3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3310 vs. $2(353) = 706$.
time = 15.60, size = 3310, normalized size = 8.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*c^2*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*
c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 -
2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^
2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*e*log(-((a*b^3*c - a^2*b
*c^2)*d^2*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3
+ a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4
*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + 4*(a^2*b^3 - 2*a^3*b*c)*x^2
*e^2 - 2*(a^2*b^2*c - a^3*c^2)*d^2 + 2*sqrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*
a^2*b*c^3)*d*x - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*x*e + (b^4*c^4 - 6*a*b^2
```

```
*c^5 + 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
- 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(
b^2*c^8 - 4*a*c^9)))*sqrt(x^2*e + d)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 -
4*a*b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3
 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^
4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - ((a*
b^4 + 2*a^2*b^2*c - 4*a^3*c^2)*d*x^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d)*e)/x^2) -
 sqrt(1/2)*c^2*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
*e - (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b
^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e
^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*e*log(-((a*b^3*c - a^2*b*c^2
)*d^2*x^2 + (a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^
2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c +
 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + 4*(a^2*b^3 - 2*a^3*b*c)*x^2*e^2
 - 2*(a^2*b^2*c - a^3*c^2)*d^2 - 2*sqrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a^2*
b*c^3)*d*x - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*x*e + (b^4*c^4 - 6*a*b^2*c^5
 + 8*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*
a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*
c^8 - 4*a*c^9)))*sqrt(x^2*e + d)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*
b^2*c + 2*a^2*c^2)*e - (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a
^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c
+ 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - ((a*b^4
+ 2*a^2*b^2*c - 4*a^3*c^2)*d*x^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d)*e)/x^2) + sqr
t(1/2)*c^2*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e +
 (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
 - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/
(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*e*log(-((a*b^3*c - a^2*b*c^2)*d^
2*x^2 - (a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^
4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a
^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + 4*(a^2*b^3 - 2*a^3*b*c)*x^2*e^2 - 2
*(a^2*b^2*c - a^3*c^2)*d^2 + 2*sqrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^
3)*d*x - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*x*e - (b^4*c^4 - 6*a*b^2*c^5 + 8
*a^2*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^
3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8
- 4*a*c^9)))*sqrt(x^2*e + d)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*
c + 2*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c
^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*
a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - ((a*b^4 + 2*
a^2*b^2*c - 4*a^3*c^2)*d*x^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d)*e)/x^2) - sqrt(1/
2)*c^2*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + (b^
2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3
*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2
*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*e*log(-((a*b^3*c - a^2*b*c^2)*d^2*x^
2 - (a*b^2*c^4 - 4*a^2*c^5)*d*x^2*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d
^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b
```

```
^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)) + 4*(a^2*b^3 - 2*a^3*b*c)*x^2*e^2 - 2*(a^
2*b^2*c - a^3*c^2)*d^2 - 2*sqrt(1/2)*((b^5*c - 5*a*b^3*c^2 + 4*a^2*b*c^3)*d
*x - (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*x*e - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2
*c^6)*x*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^
2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(b^2*c^8 - 4*
a*c^9)))*sqrt(x^2*e + d)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c +
2*a^2*c^2)*e + (b^2*c^4 - 4*a*c^5)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*
d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*
b^2*c^2)*e^2)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - ((a*b^4 + 2*a^2*
b^2*c - 4*a^3*c^2)*d*x^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d)*e)/x^2) + 2*sqrt(x^2*
e + d)*c*x*e - (c*d - 2*b*e)*e^(1/2)*log(-2*x^2*e + 2*sqrt(x^2*e + d)*x*e^(
1/2) - d))*e^(-1)/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 5.44, size = 53, normalized size = 0.14

$$-\frac{(cd-2\,be)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}+\frac{\sqrt{x^2e+d}\,x}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(c*d - 2*b*e)*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c^2 + 1/2*
sqrt(x^2*e + d)*x/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^4*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

3.362 $\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=324

$$\frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2c\cdots}}{\sqrt{b\cdots}}\right.}{c\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b\cdots\right.}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/c+arctan(x*(2*c*d-e*(b-(-4*a*c+b
^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-b*e+(-
2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))
^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(c*d-b*e+(2*a*c*e-b^
2*e+b*c*d)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(
-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 1.02, antiderivative size = 324, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1307, 223, 212, 1706, 385, 211}

$$\frac{\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c
*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*
x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]) + ((c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt
[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 -
4*a*c])*e]) + (Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/c
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{ae-(cd-be)x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{c} + \frac{e\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c}$$

$$= -\frac{\int \left(\frac{-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)dx}{c} + \frac{e\text{Sub}\cdots}{\cdots}$$

$$= \frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+e\cdots}}}{c}$$

$$= \frac{\sqrt{e}\,\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{b+\sqrt{b^2-4ac}-\left(-2cd\cdots\right.}\right.}{c}$$

$$= \frac{\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\left(cd-be+\frac{bcd\cdots}{\sqrt{b\cdots}}\right.}{c\sqrt{b+\cdots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4331 vs. $2(324) = 648$.
time = 16.03, size = 4331, normalized size = 13.37

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[d + e*x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*
e + 2*a*c*e + b*Sqrt[b^2 - 4*a*c]*e)*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)
/c]*Log[-(Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]/Sqrt[2]) + x]) + Sqrt[-((b + Sqr
t[b^2 - 4*a*c])/c)]*(b*c*d - c*Sqrt[b^2 - 4*a*c]*d - b^2*e + 2*a*c*e + b*Sq
rt[b^2 - 4*a*c]*e)*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[Sqrt[(-b +
 Sqrt[b^2 - 4*a*c])/c]/Sqrt[2] + x] + b*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*
d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 -
4*a*c])/c)]/Sqrt[2]) + x] + c*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c
])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[
b^2 - 4*a*c])/c)]/Sqrt[2]) + x] - b^2*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sq
rt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*
```

c])/c)]/Sqrt[2]) + x] + 2*a*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2]) + x] - b*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[-(Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2]) + x] - b*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] - c*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] + b^2*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] - 2*a*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] + b*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]/Sqrt[2] + x] + 2*c*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*Sqrt[e]*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[e*x + Sqrt[e]*Sqrt[d + e*x^2]] - b*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] + c*Sqrt[b^2 - 4*a*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] + b^2*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - 2*a*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - b*Sqrt[b^2 - 4*a*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d - Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] + b*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - c*Sqrt[b^2 - 4*a*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*d*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] - b^2*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] + 2*a*c*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*e)/c]*Sqrt[d + e*x^2]] + b*Sqrt[b^2 - 4*a*c]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*Sqrt[2*d - ((b + Sqrt[b^2 - 4*a*c])*e)/c]*Log[2*d + Sqrt[2]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*x + Sqrt[(4*c*d - 2*b*e + 2*Sqrt[b^2 - 4*a*c]*

```
e)/c]*Sqrt[d + e*x^2]] + b*c*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d
 - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4
*a*c])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2
]] + c*Sqrt[b^2 - 4*a*c]*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*d*Sqrt[(2*c*d - b
*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c
])/c)]*e*x + Sqrt[4*d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] -
 b^2*Sqrt[(-b + Sqrt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*
c]*e)/c]*Log[2*d - Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x + Sqrt[4*
d - (2*(b + Sqrt[b^2 - 4*a*c])*e)/c]*Sqrt[d + e*x^2]] + 2*a*c*Sqrt[(-b + Sq
rt[b^2 - 4*a*c])/c]*e*Sqrt[(2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e)/c]*Log[2*d -
 Sqrt[2]*Sqrt[-((b + Sqrt[b^2 - 4*a*c])/c)]*e*x...
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.13, size = 222, normalized size = 0.69

method	result
default	$-\sqrt{e}\left(-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left((eb-cd)\_R^2+2\left(2a\,e^2-deb+c\,d^2\right)\_\right.}{c\_R^3+3\_R^2be-3\_R^2}}{2c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -e^(1/2)*(-1/2/c*sum(((b*e-c*d)*_R^2+2*(2*a*e^2-b*d*e+c*d^2)*_R+d^2*e*b-c*d
^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-
c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*
_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))+1/c*ln(
(e*x^2+d)^(1/2)-e^(1/2)*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1663 vs. 2(289) = 578.
time = 2.66, size = 1663, normalized size = 5.13

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt
((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))
*log((b*c*d^2*x^2 + 4*a*b*x^2*e^2 + (b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2
 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 + 2*sqrt(1/2)*((b^
2*c - 4*a*c^2)*d*x - (b^3 - 4*a*b*c)*x*e + (b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^
2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(x^2*e + d)*sqrt(-(b
*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^
2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) - ((b^2 + 4*a*c)*d*x^2 -
2*a*b*d)*e)/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 -
4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^
2 - 4*a*c^3))*log((b*c*d^2*x^2 + 4*a*b*x^2*e^2 + (b^2*c^2 - 4*a*c^3)*d*x^2*
sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*a*c*d^2 - 2*s
qrt(1/2)*((b^2*c - 4*a*c^2)*d*x - (b^3 - 4*a*b*c)*x*e + (b^3*c^2 - 4*a*b*c^
3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(x^2*e
+ d)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2
*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) - ((b^2 + 4*
a*c)*d*x^2 - 2*a*b*d)*e)/x^2) + sqrt(1/2)*c*sqrt(-(b*c*d - (b^2 - 2*a*c)*e
+ (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c
^5)))/(b^2*c^2 - 4*a*c^3))*log((b*c*d^2*x^2 + 4*a*b*x^2*e^2 - (b^2*c^2 - 4*
a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)) - 2*
a*c*d^2 + 2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d*x - (b^3 - 4*a*b*c)*x*e - (b^3*c
^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5))
)*sqrt(x^2*e + d)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt
((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))
 - ((b^2 + 4*a*c)*d*x^2 - 2*a*b*d)*e)/x^2) - sqrt(1/2)*c*sqrt(-(b*c*d - (b^
2 - 2*a*c)*e + (b^2*c^2 - 4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^
2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log((b*c*d^2*x^2 + 4*a*b*x^2*e^2 -
(b^2*c^2 - 4*a*c^3)*d*x^2*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4
*a*c^5)) - 2*a*c*d^2 - 2*sqrt(1/2)*((b^2*c - 4*a*c^2)*d*x - (b^3 - 4*a*b*c)
*x*e - (b^3*c^2 - 4*a*b*c^3)*x*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^
4 - 4*a*c^5)))*sqrt(x^2*e + d)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + (b^2*c^2 -
4*a*c^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^
2 - 4*a*c^3)) - ((b^2 + 4*a*c)*d*x^2 - 2*a*b*d)*e)/x^2) - 2*e^(1/2)*log(-2*
x^2*e - 2*sqrt(x^2*e + d)*x*e^(1/2) - d))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [**A**]
time = 3.85, size = 27, normalized size = 0.08

$$-\frac{e^{\frac{1}{2}}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*e^(1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^2*(d + e*x^2)^(1/2))/(a + b*x^2 + c*x^4), x)
```

3.363 $\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=240

$$\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,\tan^{-}}{\sqrt{b^2-4ac}\,\sqrt{}}$$

```
[Out] arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(-4*a*c+b^2)^(1/2
)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1
/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^
(1/2)))^(1/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$,
Rules used = {1188, 399, 223, 212, 385, 211}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2
- 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} + \frac{\left(-2cd+\left(b+\right.\right.}{}$$

$$= \frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx,\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{2cd-\left(\right.}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 8.66, size = 250, normalized size = 1.04

$$\frac{1}{2}e^{3/2}\text{RootSum}\left[cd^4-4cd^3\#1^2+4bd^2e\#1^2+6cd^2\#1^4-8bde\#1^4+16ae^2\#1^4-4cd\#1^6+4be\#1^6+c\#1^8\&,\frac{d^2\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}-\#1\right)+2d\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}-\#1\right)\#1^2+\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}-\#1\right)\#1^4}{cd^3-bd^2e-3cd^2\#1^2+4bde\#1^2-8ae^2\#1^2+3cd\#1^4-3be\#1^4-c\#1^6}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b
*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (d^2*Log[-
(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] + 2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2
] - #1]*#1^2 + Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^
2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4
- c*#1^6) & ])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 161, normalized size = 0.67

method	result
default	$-\frac{e^{\frac{3}{2}}\left(\sum_{\_R=\text{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(\_R^2+2\_Rd+d^2\right)\ln\left(\left(\sqrt{e\,x^2}\right.\right.}{c\_R^3+3\_R^2be-3\_R^2cd+8\_Ra\,e^2-}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^(3/2)*sum((_R^2+2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2
-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),
_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^
2*e-4*c*d^3)*_Z+d^4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1007 vs. 2(206) = 412.
time = 0.87, size = 1007, normalized size = 4.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*
a^3*c)))/(a*b^2 - 4*a^2*c))*log(-(b*d^2*x^2 - 4*a*d*x^2*e + (a*b^2 - 4*a^2*
c)*d*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)*sq
rt(x^2*e + d)*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^2 -
 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2)/x^2)
 - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 -
 4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-(b*d^2*x^2 - 4*a*d*x^2*e + (a*b^2 - 4*a
^2*c)*d*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*a^3*c)
*sqrt(x^2*e + d)*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e + (a*b^
2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2)/x
^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^
2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-(b*d^2*x^2 - 4*a*d*x^2*e - (a*b^2 -
4*a^2*c)*d*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x^2 + 4*sqrt(1/2)*(a^2*b^2 - 4*a^3
*c)*sqrt(x^2*e + d)*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e - (a
*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*d^2
)/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2
```

```
*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c))*log(-(b*d^2*x^2 - 4*a*d*x^2*e - (a*b^2
 - 4*a^2*c)*d*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x^2 - 4*sqrt(1/2)*(a^2*b^2 - 4*
a^3*c)*sqrt(x^2*e + d)*sqrt(d^2/(a^2*b^2 - 4*a^3*c))*x*sqrt(-(b*d - 2*a*e -
 (a*b^2 - 4*a^2*c)*sqrt(d^2/(a^2*b^2 - 4*a^3*c)))/(a*b^2 - 4*a^2*c)) - 2*a*
d^2)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(a + b*x**2 + c*x**4), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{c\,x^4+b\,x^2+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4),x)
[Out] int((d + e*x^2)^(1/2)/(a + b*x^2 + c*x^4), x)
```

3.364 $\int \frac{\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=291

$$-\frac{\sqrt{d+ex^2}}{ax}-\frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{a\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2}}$$

```
[Out] -(e*x^2+d)^(1/2)/a/x-c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x
^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(d+(-2*a*e+b*d)/(-4*a*c+b^2)^(1/2
))/a/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*
arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2))*(d+(2*a*e-b*d)/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/
2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.49, antiderivative size = 291, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1309, 270, 1706, 385, 211}

$$-\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (c*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[
(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
- 4*a*c])*e]) - (c*(d - (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^
2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{bd-ae+cdx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{a} + \frac{d\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax} - \frac{\int \left(\frac{cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+\right.}}\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{a} - \cdots$$

$$= -\frac{\sqrt{d+ex^2}}{ax} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-2cd+\left(b+\sqrt{b^2-\right.\right.}}\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{ax} - \frac{c\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \cdots$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4644 vs. $2(291) = 582$.
time = 16.25, size = 4644, normalized size = 15.96

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*x)) - (-1/2*(b*d*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d -
Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt
[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*
c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2
- 4*a*c]/c]/Sqrt[2])) + (Sqrt[b^2 - 4*a*c]*d*(Log[Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log
[2*d - Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c)
 - Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2
```

```
 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqr
t[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) +
Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (a*e*(Log[Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/
c]/Sqrt[2] + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d -
Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt
[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*
c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
])*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + Sqrt[-(b/c) + Sqrt[b^2
- 4*a*c]/c]/Sqrt[2])) + (b*d*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2]) + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2
]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b^2 -
 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*
e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) - Sqrt
[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqr
t[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2])) - (Sqrt[b^2 - 4*a*c]*d*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c
]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d +
Sqrt[2]*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt
[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*
c]/c)*e)/2]))/(2*Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c)
- Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]
)*(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*
a*c]/c]/Sqrt[2])) - (a*e*(Log[-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
 + x]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sq
rt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a
*c]/c)*e)/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) - Sqrt[b^2 - 4*a*c]/c)*e)/2
]))/(Sqrt[2]*c*Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]*(Sqrt[-(b/c) - Sqrt[b^2 -
 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/
c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt
[2])) - (b*d*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[d +
((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c) + Sqr
t[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sq
rt[d + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sqrt[2]*
c*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/
Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[
b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) - (S
qrt[b^2 - 4*a*c]*d*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqr
t[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c)
 + Sqrt[b^2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)
/2]*Sqrt[d + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(2*Sq
rt[2]*c*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*
c]/c]/Sqrt[2]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) -
 Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])
```

```
) + (a*e*(Log[Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2] + x]/Sqrt[d + ((-(
b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d - Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^
2 - 4*a*c]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d
 + e*x^2]]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]))/(Sqrt[2]*c*Sqrt
[-(b/c) + Sqrt[b^2 - 4*a*c]/c]*(-(Sqrt[-(b/c) - Sqrt[b^2 - 4*a*c]/c]/Sqrt[2
]) - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])*(Sqrt[-(b/c) - Sqrt[b^2 -
4*a*c]/c]/Sqrt[2] - Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2])) + (b*d*(Lo
g[-(Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c]/c]/Sqrt[2]) + x]/Sqrt[d + ((-(b/c) + Sq
rt[b^2 - 4*a*c]/c)*e)/2] - Log[2*d + Sqrt[2]*Sqrt[-(b/c) + Sqrt[b^2 - 4*a*c
]/c]*e*x + 2*Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2]*Sqrt[d + e*x^2]
]/Sqrt[d + ((-(b/c) + Sqrt[b^2 - 4*a*c]/c)*e)/2...
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 270, normalized size = 0.93

method	result
risch	$-\frac{\sqrt{e\,x^2+d}}{ax}+\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(\_R^2cd+2\left(-2a\,e^2 \cdots\right.\right.}{c\_R^3+3\_R \cdots}\right.}{2a}$
default	$\frac{\sqrt{e}\left(\ln\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(-\_R^2 \cdots\right.}{c\_ \cdots}}{2}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*e^(1/2)*(ln((e*x^2+d)^(1/2)-e^(1/2)*x)-1/2*sum((-_R^2*c*d+2*(2*a*e^2-2*
b*d*e+c*d^2)*_R-c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+
3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*
_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)
*_Z+d^4*c)))+1/a*(-1/d/x*(e*x^2+d)^(3/2)+2*e/d*(1/2*x*(e*x^2+d)^(1/2)+1/2*d
/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2435 vs. $2(258) = 516$.
time = 1.64, size = 2435, normalized size = 8.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^
2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3
 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((4*a^2*b*c*
x^2*e^2 + (b^3*c - a*b*c^2)*d^2*x^2 + (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a
^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^
6*b^2 - 4*a^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 + 2*sqrt(1/2)*((a*b^4 - 5*a^2
*b^2*c + 4*a^3*c^2)*d*x - (a^2*b^3 - 4*a^3*b*c)*x*e - (a^4*b^3 - 4*a^5*b*c)
*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/(a^6*b^2 - 4*a^7*c)))*sqrt(x^2*e + d)*sqrt(-((b^3 - 3*a*b*c)*d - (a*
b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c
 + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 -
 4*a^4*c)) + (2*a^2*b*c*d - (5*a*b^2*c - 4*a^2*c^2)*d*x^2)*e)/x^2) - sqrt(1
/2)*a*x*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + (a^3*b^2 - 4*a^4*c
)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)
*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((4*a^2*b*c*x^2*e^2 + (
b^3*c - a*b*c^2)*d^2*x^2 + (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2
+ (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a
^7*c)) - 2*(a*b^2*c - a^2*c^2)*d^2 - 2*sqrt(1/2)*((a*b^4 - 5*a^2*b^2*c + 4*
a^3*c^2)*d*x - (a^2*b^3 - 4*a^3*b*c)*x*e - (a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^
2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6
*b^2 - 4*a^7*c)))*sqrt(x^2*e + d)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2
*c)*e + (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)
*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))
+ (2*a^2*b*c*d - (5*a*b^2*c - 4*a^2*c^2)*d*x^2)*e)/x^2) + sqrt(1/2)*a*x*sqr
t(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2
*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*
b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((4*a^2*b*c*x^2*e^2 + (b^3*c - a*b
*c^2)*d^2*x^2 - (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*
a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*
(a*b^2*c - a^2*c^2)*d^2 + 2*sqrt(1/2)*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d*
x - (a^2*b^3 - 4*a^3*b*c)*x*e + (a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 +
 (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^
7*c)))*sqrt(x^2*e + d)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^
3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a
```

```
*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) + (2*a^2*b*
c*d - (5*a*b^2*c - 4*a^2*c^2)*d*x^2)*e)/x^2) - sqrt(1/2)*a*x*sqrt(-((b^3 -
3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a^4*c)*sqrt((a^2*b^2*e^2 +
(b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7
*c)))/(a^3*b^2 - 4*a^4*c))*log((4*a^2*b*c*x^2*e^2 + (b^3*c - a*b*c^2)*d^2*x
^2 - (a^3*b^2*c - 4*a^4*c^2)*d*x^2*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a
^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)) - 2*(a*b^2*c -
a^2*c^2)*d^2 - 2*sqrt(1/2)*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d*x - (a^2*b^
3 - 4*a^3*b*c)*x*e + (a^4*b^3 - 4*a^5*b*c)*x*sqrt((a^2*b^2*e^2 + (b^4 - 2*a
*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))*sqrt
(x^2*e + d)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - (a^3*b^2 - 4*a
^4*c)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*
b*c)*d*e)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) + (2*a^2*b*c*d - (5*a*
b^2*c - 4*a^2*c^2)*d*x^2)*e)/x^2) - 4*sqrt(x^2*e + d))/(a*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d+ex^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

3.365 $\int \frac{\sqrt{d+ex^2}}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=373

$$-\frac{\sqrt{d+ex^2}}{3ax^3}+\frac{2e\sqrt{d+ex^2}}{3adx}+\frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx}+\frac{c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right.}}{\sqrt{b-\sqrt{b^2-4ac}}}\right.}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right.}}$$

```
[Out] -1/3*(e*x^2+d)^(1/2)/a/x^3+2/3*e*(e*x^2+d)^(1/2)/a/d/x+(-a*e+b*d)*(e*x^2+d)
^(1/2)/a^2/d/x+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c
+b^2)^(1/2))/a^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/
2))^(1/2)+c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)
/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d)/(-4*a*c+b^2)^
(1/2))/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1
/2)
```

Rubi [A]

time = 1.71, antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1309, 277, 270, 6860, 1706, 385, 211}

$$\frac{c\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}(bd-ae)}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3adx}-\frac{\sqrt{d+ex^2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*Sqrt[d + e*x^2]/(a*x^3) + (2*e*Sqrt[d + e*x^2])/(3*a*d*x) + ((b*d - a*
e)*Sqrt[d + e*x^2])/(a^2*d*x) + (c*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/S
qrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (c*(b*d - a*e - (b^2*d - 2*a*c*d
 - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2
 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^4\left(a+bx^2+cx^4\right)}dx = -\frac{\int \frac{bd-ae+cdx^2}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}dx}{a} + \frac{d\int \frac{1}{x^4\sqrt{d+ex^2}}dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} - \frac{\int \left(\frac{bd-ae}{ax^2\sqrt{d+ex^2}} + \frac{-b^2d+acd+abe-c(bd-ae)x^2}{a\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right)dx}{a} - \frac{(2e)\int \frac{1}{x^2\sqrt{d-}}}{3a}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} - \frac{\int \frac{-b^2d+acd+abe-c(bd-ae)x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}dx}{a^2} - \frac{(bd-ae)\int \frac{}{x^2\sqrt{d}}}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} - \frac{\int \left(\frac{-c(bd-ae)-\frac{c\left(b^2d-2\right.}{\sqrt{b^2-}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{\left(c\left(bd-ae-\frac{b^2d-2acd-}{\sqrt{b^2-4a}}\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{\left(c\left(bd-ae-\frac{b^2d-2acd-}{\sqrt{b^2-4a}}\right.\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{3ax^3} + \frac{2e\sqrt{d+ex^2}}{3adx} + \frac{(bd-ae)\sqrt{d+ex^2}}{a^2dx} + \frac{c\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right.}{a^2\sqrt{b-\sqrt{b^2-}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7777 vs. $2(373) = 746$.
time = 16.31, size = 7777, normalized size = 20.85

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 318, normalized size = 0.85

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(ae\,x^2-3bd\,x^2+ad)}{3a^2x^3d}+\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\right.}{}$
default	$\frac{\sqrt{e}\left(-b\ln\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(c(ae\right.}{}\right.}{}\right.}{a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a^2*e^(1/2)*(-b*ln((e*x^2+d)^(1/2)-e^(1/2)*x)+1/2*sum((c*(a*e-b*d)*_R^2+2
*(2*a*b*e^2+a*c*d*e-2*b^2*d*e+b*c*d^2)*_R+a*d^2*e*c-b*c*d^3)/(_R^3*c+3*_R^2
*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+
d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b
*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c)))-b/a^2*(-1/d/x*(e*x^2+d)^
(3/2)+2*e/d*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/
2))))-1/3/a/d/x^3*(e*x^2+d)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)/((c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4132 vs. $2(338) = 676$.
time = 6.47, size = 4132, normalized size = 11.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*
b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6
```

*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(-((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2*x^2 + (a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*x^2*e^2 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 + 2*sqrt(1/2)*((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d*x - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*x*e - (a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))*sqrt(x^2*e + d)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)) - ((5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*x^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d)*e)/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(-((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2*x^2 + (a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 4*(a^2*b^3*c^2 - 2*a^3*b*c^3)*x^2*e^2 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^2 - 2*sqrt(1/2)*((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d*x - (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*x*e - (a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))*sqrt(x^2*e + d)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)) - ((5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*x^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d)*e)/x^2) + 3*sqrt(1/2)*a^2*d*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log(-((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2*x^2 - (a^5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^

```
4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a
^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)) + 4*(a^2*
b^3*c^2 - 2*a^3*b*c^3)*x^2*e^2 - 2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^
2 + 2*sqrt(1/2)*((a*b^7 - 7*a^2*b^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d*x -
 (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*x*e + (a^6*b^4 - 6*a^7*b^2*c + 8*a
^8*c^2)*x*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4
)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^
6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))*sqrt(x^2*e +
d)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*
c^2)*e - (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^
3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b
*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11
*c)))/(a^5*b^2 - 4*a^6*c)) - ((5*a*b^4*c^2 - 14*a^2*b^2*c^3 + 4*a^3*c^4)*d*
x^2 - 2*(a^2*b^3*c^2 - 2*a^3*b*c^3)*d)*e)/x^2) - 3*sqrt(1/2)*a^2*d*x^3*sqrt
(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e
- (a^5*b^2 - 4*a^6*c)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c
^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d
*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(a^10*b^2 - 4*a^11*c)))/(
a^5*b^2 - 4*a^6*c))*log(-((b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^2*x^2 - (a^
5*b^2*c^2 - 4*a^6*c^3)*d*x^2*sqrt(((b^8 - 6*a*b...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x^4 \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(d + e*x**2)/(x**4*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(1/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

3.366 $\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=512

$$-\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}+\frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}-\frac{2e(bd-ae)\sqrt{d+ex^2}}{3a^2d^2x}-\frac{(b^2d-acd-ab}{a^3dx}$$

```
[Out] -1/5*(e*x^2+d)^(1/2)/a/x^5+4/15*e*(e*x^2+d)^(1/2)/a/d/x^3+1/3*(-a*e+b*d)*(e
*x^2+d)^(1/2)/a^2/d/x^3-8/15*e^2*(e*x^2+d)^(1/2)/a/d^2/x-2/3*e*(-a*e+b*d)*(
e*x^2+d)^(1/2)/a^2/d^2/x-(-a*b*e-a*c*d+b^2*d)*(e*x^2+d)^(1/2)/a^3/d/x-c*arc
tan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2
)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(2*a^2*c*e-a*b^2*e-3*a*b*c*d+b^3*d)/(-4*
a*c+b^2)^(1/2))/a^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1
/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2*d-a*c*d-a*b*e+(-2*a^2*c*e+a*b^2*e+3*
a*b*c*d-b^3*d)/(-4*a*c+b^2)^(1/2))/a^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-
e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 3.50, antiderivative size = 512, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1309, 277, 270, 6860, 1706, 385, 211}

$$-\frac{\sqrt{d+ex^2}\left(-abe-acd+b^2d\right)}{a^3dx}-\frac{2e\sqrt{d+ex^2}(bd-ae)}{3a^2d^2x}+\frac{\sqrt{d+ex^2}(bd-ae)}{3a^2dx^3}-\frac{c\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{8e^2\sqrt{d+ex^2}}{15ad^2x}-\frac{\sqrt{d+ex^2}}{5ax^5}+\frac{4e\sqrt{d+ex^2}}{15adx^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*Sqrt[d + e*x^2]/(a*x^5) + (4*e*Sqrt[d + e*x^2])/(15*a*d*x^3) + ((b*d -
 a*e)*Sqrt[d + e*x^2])/(3*a^2*d*x^3) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^2*x)
 - (2*e*(b*d - a*e)*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - ((b^2*d - a*c*d - a*b*
e)*Sqrt[d + e*x^2])/(a^3*d*x) - (c*(b^2*d - a*c*d - a*b*e + (b^3*d - 3*a*b*
c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*
Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (c*(
b^2*d - a*c*d - a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2
- 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c
*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{d+ex^2}}{x^6\left(a+bx^2+cx^4\right)}\,dx = -\frac{\int \frac{bd-ae+cdx^2}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{a} + \frac{d\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} - \frac{\int \left(\frac{bd-ae}{ax^4\sqrt{d+ex^2}} + \frac{-b^2d+acd+abe}{a^2x^2\sqrt{d+ex^2}} + \frac{b^3d-2abcd-ab^2e+a^2ce+c(b^2d-acd-}{a^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} - \frac{\int \frac{b^3d-2abcd-ab^2e+a^2ce+c(b^2d-acd-abe)x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{a^3} + \frac{(8e^2)\int}{}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{(b^2d-a}{}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-}{3}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-}{3}$$

$$= -\frac{\sqrt{d+ex^2}}{5ax^5} + \frac{4e\sqrt{d+ex^2}}{15adx^3} + \frac{(bd-ae)\sqrt{d+ex^2}}{3a^2dx^3} - \frac{8e^2\sqrt{d+ex^2}}{15ad^2x} - \frac{2e(bd-}{3}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 10933 vs. $2(512) = 1024$.
time = 16.41, size = 10933, normalized size = 21.35

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[d + e*x^2]/(x^6*(a + b*x^2 + c*x^4)),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 417, normalized size = 0.81

method	result
risch	$-\frac{\sqrt{e\,x^{2}+d}\,\left(-2a^{2}e^{2}x^{4}-5abde\,x^{4}-15ac\,d^{2}x^{4}+15b^{2}d^{2}x^{4}+a^{2}de\,x^{2}-5ab\,d^{2}x^{2}+3a^{2}d^{2}\right)}{15d^{2}a^{3}x^{5}}-\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(4eb-4cd)\ldots\right.}\right.}{\ldots}$
default	$-\frac{\sqrt{e}\left((ac-b^{2})\ln\left(\sqrt{e\,x^{2}+d}-\sqrt{e}\,x\right)-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(4eb-4cd)\_Z^{3}+\left(16a\,e^{2}-8deb+6c\,d^{2}\right)\_Z^{2}+\left(4d^{2}eb-4c\,d^{3}\right)\_Z+\ldots\right.}\right.}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^3*e^(1/2)*((a*c-b^2)*ln((e*x^2+d)^(1/2)-e^(1/2)*x)-1/2*sum((c*(-a*b*e-
a*c*d+b^2*d)*_R^2+2*(2*a^2*c*e^2-2*a*b^2*e^2-3*a*b*c*d*e+a*c^2*d^2+2*b^3*d*
e-b^2*c*d^2)*_R-a*b*c*d^2*e-a*c^2*d^3+b^2*c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*
c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(
1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2
)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))+1/a*(-1/5/d/x^5*(e*x^2+d)^(3/2)+2/15
*e/d^2/x^3*(e*x^2+d)^(3/2))+(-a*c+b^2)/a^3*(-1/d/x*(e*x^2+d)^(3/2)+2*e/d*(1
/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))))+1/3*b/a^
2/d/x^3*(e*x^2+d)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(x^2*e + d)/((c*x^4 + b*x^2 + a)*x^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5811 vs. 2(472) = 944.
time = 15.80, size = 5811, normalized size = 11.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/60*(15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7
*a^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*
b^2 - 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3
+ 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c
+ 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^
2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2
)/(a^14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(((b^7*c^3 - 5*a*b^5*c^4
+ 6*a^2*b^3*c^5 - a^3*b*c^6)*d^2*x^2 + (a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt
(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 -
12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 -
40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*
c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^1
5*c)) + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*x^2*e^2 - 2*(a*b^6*c^
3 - 5*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 + 2*sqrt(1/2)*((a*b^10 - 1
0*a^2*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a^6*c^5)
*d*x - (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*
c^4)*x*e - (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b^1
0*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a
^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 2
2*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2
 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))*sqrt(x^2*e
+ d)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6
*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12
 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*
b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*
b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*
a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/
(a^7*b^2 - 4*a^8*c)) - ((5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*
a^4*c^6)*d*x^2 - 2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d)*e)/x^2) -
 15*sqrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*
b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 -
 4*a^8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*
a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*
a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^1
0 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^
14*b^2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(((b^7*c^3 - 5*a*b^5*c^4 + 6*a
^2*b^3*c^5 - a^3*b*c^6)*d^2*x^2 + (a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^
12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^
5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^
4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 2
2*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c))
 + 4*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*x^2*e^2 - 2*(a*b^6*c^3 - 5
*a^2*b^4*c^4 + 6*a^3*b^2*c^5 - a^4*c^6)*d^2 - 2*sqrt(1/2)*((a*b^10 - 10*a^2
*b^8*c + 35*a^3*b^6*c^2 - 51*a^4*b^4*c^3 + 29*a^5*b^2*c^4 - 4*a^6*c^5)*d*x
- (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*
```

```
x*e - (a^8*b^5 - 7*a^9*b^3*c + 12*a^10*b*c^2)*x*sqrt(((b^12 - 10*a*b^10*c +
 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^
6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5
*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24
*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))*sqrt(x^2*e + d)*
sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 - 6*a^2*
b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + (a^7*b^2 - 4*a^8*c)*sqrt(((b^12 - 10
*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c
^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c
^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b
^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^2 - 4*a^15*c)))/(a^7*
b^2 - 4*a^8*c)) - ((5*a*b^6*c^3 - 24*a^2*b^4*c^4 + 27*a^3*b^2*c^5 - 4*a^4*c
^6)*d*x^2 - 2*(a^2*b^5*c^3 - 4*a^3*b^3*c^4 + 3*a^4*b*c^5)*d)*e)/x^2) + 15*s
qrt(1/2)*a^3*d^2*x^5*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3
)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e - (a^7*b^2 - 4*a^
8*c)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b
^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b
^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8
*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/(a^14*b^
2 - 4*a^15*c)))/(a^7*b^2 - 4*a^8*c))*log(((b^7*c^3 - 5*a*b^5*c^4 + 6*a^2*b^
3*c^5 - a^3*b*c^6)*d^2*x^2 - (a^7*b^2*c^3 - 4*a^8*c^4)*d*x^2*sqrt(((b^12 -
10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 +...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{d + ex^2}}{x^6 \left(a + bx^2 + cx^4\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(1/2)/x**6/(c*x**4+b*x**2+a),x)
[Out] Integral(sqrt(d + e*x**2)/(x**6*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(1/2)/x^6/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{e\,x^2+d}}{x^6\,\left(c\,x^4+b\,x^2+a\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(1/2)/(x^6*(a + b*x^2 + c*x^4)), x)
```

3.367 $\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=460

$$\frac{(cd-be)\sqrt{d+ex^2}}{c^2}+\frac{(d+ex^2)^{3/2}}{3c}+\frac{\left(b^3e^2-b^2e\left(2cd+\sqrt{b^2-4ac}\,e\right)+c\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\right.\right.\right.}{\sqrt{2}\,c^{5/2}\sqrt{b^2-}}$$

```
[Out] 1/3*(e*x^2+d)^(3/2)/c+(-b*e+c*d)*(e*x^2+d)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^
(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(b^3*e^2-b^2*
e*(2*c*d+e*(-4*a*c+b^2)^(1/2))+c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-
4*a*c+b^2)^(1/2)))+b*c*(c*d^2+e*(-3*a*e+2*d*(-4*a*c+b^2)^(1/2))))/c^(5/2)*2
^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctan
h(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(
b^3*e^2-b^2*e*(2*c*d-e*(-4*a*c+b^2)^(1/2))-c*(a*e^2*(-4*a*c+b^2)^(1/2)-c*d*
(4*a*e+d*(-4*a*c+b^2)^(1/2)))+b*c*(c*d^2-e*(3*a*e+2*d*(-4*a*c+b^2)^(1/2))))
/c^(5/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 3.58, antiderivative size = 460, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 838, 840, 1180, 214}

$$\frac{\left(bc\left(e\left(2d\sqrt{b^2-4ac}-3ae\right)+cd^2\right)+c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}-4ae\right)\right)-b^2e\left(e\sqrt{b^2-4ac}+2cd\right)+b^3e^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(bc\left(cd^2-e\left(2d\sqrt{b^2-4ac}+3ae\right)\right)-c\left(ae^2\sqrt{b^2-4ac}-cd\left(d\sqrt{b^2-4ac}+4ae\right)\right)-b^2e\left(2cd-e\sqrt{b^2-4ac}\right)+b^3e^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d+ex^2}(cd-be)}{c^2}+\frac{(d+ex^2)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((c*d - b*e)*Sqrt[d + e*x^2])/c^2 + (d + e*x^2)^(3/2)/(3*c) + ((b^3*e^2 - b
^2*e*(2*c*d + Sqrt[b^2 - 4*a*c]*e) + c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt
[b^2 - 4*a*c]*d - 4*a*e)) + b*c*(c*d^2 + e*(2*Sqrt[b^2 - 4*a*c]*d - 3*a*e))
)*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 -
4*a*c])*e]) - ((b^3*e^2 - b^2*e*(2*c*d - Sqrt[b^2 - 4*a*c]*e) + b*c*(c*d^2
- e*(2*Sqrt[b^2 - 4*a*c]*d + 3*a*e)) - c*(a*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sq
rt[b^2 - 4*a*c]*d + 4*a*e)))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(5/2)*Sqrt[b^2 - 4*a*c]*Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex)^{3/2}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{(d+ex^2)^{3/2}}{3c} + \frac{\text{Subst}\left(\int \frac{\sqrt{d+ex}\,(-ae+(cd-be)x)}{a+bx+cx^2}\,dx, x, x^2\right)}{2c}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3c} + \frac{\text{Subst}\Big(\int \frac{-ae(2cd-be)+(c^2d^2+b^2e^2-ce(2bd+ae))x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx,}{2c^2}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3c} + \frac{\text{Subst}\Big(\int \frac{-ae^2(2cd-be)-d(c^2d^2+b^2e^2-ce(2bd+ae))+(c^2}{cd^2-bde+ae^2+(-2cd+be)x^2}}{c^2}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3c} - \frac{\Big(b^3e^2 - b^2e\big(2cd+\sqrt{b^2-4ac}\,e\big) + c\Big(a\sqrt{b^2-}}{}$$

$$= \frac{(cd-be)\sqrt{d+ex^2}}{c^2} + \frac{(d+ex^2)^{3/2}}{3c} + \frac{\Big(b^3e^2 - b^2e\big(2cd+\sqrt{b^2-4ac}\,e\big) + c\Big(a\sqrt{b^2-}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.60, size = 501, normalized size = 1.09

$$\frac{2\sqrt{c}\sqrt{d+ex^2}\,(4cd-3be+cex^2) + \frac{3\left(ib^3e^2+b^2e\left(-2icd+\sqrt{-b^2+4ac}\,e\right)+ibc\left(cd^2+e\left(2i\sqrt{-b^2+4ac}\,d-3ae\right)\right)+c\left(-a\sqrt{-b^2+4ac}\,e^2+cd\left(\sqrt{-b^2+4ac}\,d+4iae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}} + \frac{3\left(-ib^3e^2+b^2e\left(2icd+\sqrt{-b^2+4ac}\,e\right)+bc\left(-icd^2+e\left(-2\sqrt{-b^2+4ac}\,d+3iae\right)\right)+c\left(-a\sqrt{-b^2+4ac}\,e^2+cd\left(\sqrt{-b^2+4ac}\,d-4iae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[d + e*x^2]*(4*c*d - 3*b*e + c*e*x^2) + (3*(I*b^3*e^2 + b^2*
e*((-2*I)*c*d + Sqrt[-b^2 + 4*a*c]*e) + I*b*c*(c*d^2 + e*((2*I)*Sqrt[-b^2 +
 4*a*c]*d - 3*a*e)) + c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a
*c]*d + (4*I)*a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d +
 b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b -
 I*Sqrt[-b^2 + 4*a*c])*e]) + (3*((-I)*b^3*e^2 + b^2*e*((2*I)*c*d + Sqrt[-b^
2 + 4*a*c]*e) + b*c*((-I)*c*d^2 + e*(-2*Sqrt[-b^2 + 4*a*c]*d + (3*I)*a*e))
+ c*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e)))
*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 +
 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c
])*e]))/(6*c^(5/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 444, normalized size = 0.97

method	result
risch	$-\frac{(-c e x^2+3eb-4cd)\sqrt{e x^2+d}}{3c^2}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a e^2-8deb+6c d^2\right)\_Z^4+\left(4d^2eb-4c d^3\right)\_Z^2+d^4c\right)}\frac{\left(\left(a c e^2\right.\right.}{}}{}$
default	$-\frac{-\frac{\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)^3 c}{3}+4eb\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)-5cd\left(\sqrt{e x^2+d}-\sqrt{e}\,x\right)}{8c^2}-\frac{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/8/c^2*(-1/3*((e*x^2+d)^(1/2)-e^(1/2)*x)^3*c+4*e*b*((e*x^2+d)^(1/2)-e^(1/
2)*x)-5*c*d*((e*x^2+d)^(1/2)-e^(1/2)*x))-1/4/c^2*sum(((a*c*e^2-b^2*e^2+2*b*
c*d*e-c^2*d^2)*_R^6+(-4*a*b*e^3+5*a*c*d*e^2+3*b^2*d*e^2-6*b*c*d^2*e+3*c^2*d
^3)*_R^4+d*(4*a*b*e^3-5*a*c*d*e^2-3*b^2*d*e^2+6*b*c*d^2*e-3*c^2*d^3)*_R^2-a
*c*d^3*e^2+b^2*d^3*e^2-2*b*c*e*d^4+c^2*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8
*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/
2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*
d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+d^4*c))+1/24/c*d^3/((e*x^2+d)^(1/2)-e^(1
/2)*x)^3-1/8/c^2*d*(4*b*e-5*c*d)/((e*x^2+d)^(1/2)-e^(1/2)*x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 857 vs. 2(414) = 828.
time = 4.96, size = 857, normalized size = 1.86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -(2*b*c^5*d^3 - (5*b^2*c^4 - 8*a*c^5)*d^2*e + ((b^2*c^2 - 4*a*c^3)*d^2*e -
2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^2*c + 4*a^2*c^2)*e^3)*c^2 + 2*(2
*b^3*c^3 - 5*a*b*c^4)*d*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4
*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(
b^2 - 4*a*c)*d*e^2)*abs(c) - (b^4*c^2 - 3*a*b^2*c^3)*e^3)*arctan(2*sqrt(1/2
)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d - b*c^3*e + sqrt(-4*(c^4*d^2 - b*c^3*d*e +
 a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d
 - (b^2*c^2 - 4*a*c^3 + sqrt(b^2 - 4*a*c)*b*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c
- sqrt(b^2 - 4*a*c)*c)*e)*c^2) + (2*b*c^5*d^3 - (5*b^2*c^4 - 8*a*c^5)*d^2*e
 + ((b^2*c^2 - 4*a*c^3)*d^2*e - 2*(b^3*c - 4*a*b*c^2)*d*e^2 + (b^4 - 5*a*b^
2*c + 4*a^2*c^2)*e^3)*c^2 + 2*(2*b^3*c^3 - 5*a*b*c^4)*d*e^2 + 2*(sqrt(b^2 -
 4*a*c)*c^4*d^3 - 2*sqrt(b^2 - 4*a*c)*b*c^3*d^2*e - sqrt(b^2 - 4*a*c)*a*b*c
^2*e^3 + (b^2*c^2 + a*c^3)*sqrt(b^2 - 4*a*c)*d*e^2)*abs(c) - (b^4*c^2 - 3*a
*b^2*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^4*d - b*c^3*e
- sqrt(-4*(c^4*d^2 - b*c^3*d*e + a*c^3*e^2)*c^4 + (2*c^4*d - b*c^3*e)^2))/c
^4))/((2*sqrt(b^2 - 4*a*c)*c^3*d + (b^2*c^2 - 4*a*c^3 - sqrt(b^2 - 4*a*c)*b
*c^2)*e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2) + 1/3*((x^2*
e + d)^(3/2)*c^2 + 3*sqrt(x^2*e + d)*c^2*d - 3*sqrt(x^2*e + d)*b*c*e)/c^3
```

Mupad [**B**]
time = 3.41, size = 2500, normalized size = 5.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (d + e*x^2)^(3/2)/(3*c) - atan(((((4*a*b^3*c^3*e^5 - 16*a^2*b*c^4*e^5 + 16*
a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c^5*d^3*e^2 + 8*
b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c^3 - (2*(d + e*
x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d
^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c
^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^
2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 +
 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2
*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^
2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3
 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 4
8*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6
*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e
 - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*
(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7*d*e^2))/c^3)*(
-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*
b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*
a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 14
4*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5
- 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3
 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*
e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^
5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*
e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2
 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b
^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2) - (2*(d + e*
x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*e^6 - 2*a*c^5*d^4*e^2 + 9*a^2*b^2*c^2*e^6 +
 12*a^2*c^4*d^2*e^4 + b^2*c^4*d^4*e^2 - 4*b^3*c^3*d^3*e^3 + 6*b^4*c^2*d^2*e
^4 - 6*a*b^4*c*e^6 - 4*b^5*c*d*e^5 + 12*a*b*c^4*d^3*e^3 + 20*a*b^3*c^2*d*e^
5 - 20*a^2*b*c^3*d*e^5 - 24*a*b^2*c^3*d^2*e^4))/c^3)*(-(((4*b^7*e^3 - 32*a^
2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^
4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*
c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 2
16*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5
*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4
+ 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3
 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3
 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e
 + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e
 + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16
*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/2)*1i - (((4*a*b^3*c^3*e^5 - 16*a^2*
b*c^4*e^5 + 16*a*c^6*d^3*e^2 + 16*a^2*c^5*d*e^4 - 4*b^4*c^3*d*e^4 - 4*b^2*c
^5*d^3*e^2 + 8*b^3*c^4*d^2*e^3 - 32*a*b*c^5*d^2*e^3 + 12*a*b^2*c^4*d*e^4)/c
^3 + (2*(d + e*x^2)^(1/2)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 +
 24*a*b^2*c^4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e
```

```
+ 100*a^2*b^3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*
e + 96*a*b^4*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 -
 (256*a^2*c^7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*
c*d^2*e^4 - a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4
*b*d*e^5 - 3*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1
/2) + 2*b^7*e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^
3*b*c^3*e^3 + 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*
a*b^5*c*e^3 - 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*
a^2*b*c^4*d^2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^
2*c^6)))^(1/2)*(4*b^3*c^5*e^3 - 8*b^2*c^6*d*e^2 - 16*a*b*c^6*e^3 + 32*a*c^7
*d*e^2))/c^3)*(-(((4*b^7*e^3 - 32*a^2*c^5*d^3 - 4*b^4*c^3*d^3 + 24*a*b^2*c^
4*d^3 - 80*a^3*b*c^3*e^3 + 96*a^3*c^4*d*e^2 + 12*b^5*c^2*d^2*e + 100*a^2*b^
3*c^2*e^3 - 36*a*b^5*c*e^3 - 12*b^6*c*d*e^2 - 84*a*b^3*c^3*d^2*e + 96*a*b^4
*c^2*d*e^2 + 144*a^2*b*c^4*d^2*e - 216*a^2*b^2*c^3*d*e^2)^2/4 - (256*a^2*c^
7 + 16*b^4*c^5 - 128*a*b^2*c^6)*(a^5*e^6 + a^2*c^3*d^6 + 3*a^4*c*d^2*e^4 -
a^2*b^3*d^3*e^3 + 3*a^3*b^2*d^2*e^4 + 3*a^3*c^2*d^4*e^2 - 3*a^4*b*d*e^5 - 3
*a^2*b*c^2*d^5*e - 6*a^3*b*c*d^3*e^3 + 3*a^2*b^2*c*d^4*e^2))^(1/2) + 2*b^7*
e^3 - 16*a^2*c^5*d^3 - 2*b^4*c^3*d^3 + 12*a*b^2*c^4*d^3 - 40*a^3*b*c^3*e^3
+ 48*a^3*c^4*d*e^2 + 6*b^5*c^2*d^2*e + 50*a^2*b^3*c^2*e^3 - 18*a*b^5*c*e^3
- 6*b^6*c*d*e^2 - 42*a*b^3*c^3*d^2*e + 48*a*b^4*c^2*d*e^2 + 72*a^2*b*c^4*d^
2*e - 108*a^2*b^2*c^3*d*e^2)/(16*(16*a^2*c^7 + b^4*c^5 - 8*a*b^2*c^6)))^(1/
2) + (2*(d + e*x^2)^(1/2)*(b^6*e^6 - 2*a^3*c^3*...
```

3.368 $\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=327

$$\frac{e\sqrt{d+ex^2}}{c}-\frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+}}{\sqrt{2cd-\left(b-\sqrt{b^2-}\right.}}\right.}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

```
[Out] e*(e*x^2+d)^(1/2)/c-1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b
-(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*
(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e
*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1
/2))-2*c*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c^(3/2)*2^(1/2)/(-4*a*c+b^2)^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.96, antiderivative size = 327, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1261, 717, 840, 1180, 214}

$$-\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{e\sqrt{d+ex^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*Sqrt[d + e*x^2])/c - ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e
*(b*d - Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2
])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*
a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + ((2*c^2*d^2 + b*(b + Sqrt[b
^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTanh[(Sqrt[2
]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[
2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 717

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Simp[e*((d + e*x)^(m - 1)/(c*(m - 1))), x] + Dist[1/c, Int[(d + e*x)^(
m - 2)*(Simp[c*d^2 - a*e^2 + e*(2*c*d - b*e)*x, x]/(a + b*x + c*x^2)), x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && NeQ[2*c*d - b*e, 0] && GtQ[m, 1]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{cd^2-ae^2+e(2cd-be)x}{\sqrt{d+ex}\,(a+bx+cx^2)}\,dx, x, x^2\right)}{2c}$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\text{Subst}\left(\int \frac{-de(2cd-be)+e(cd^2-ae^2)+e(2cd-be)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{c}$$

$$= \frac{e\sqrt{d+ex^2}}{c} + \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\text{Subst}}{2c\sqrt{b^2-4ac}}$$

$$= \frac{e\sqrt{d+ex^2}}{c} - \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tanh^{}}{\sqrt{2}\,c^{3/2}\sqrt{b^2-4ac}\sqrt{2cd-\left(b-\sqrt{b^2-4\right.}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.14, size = 373, normalized size = 1.14

$$\frac{2\sqrt{c}\,e\sqrt{d+ex^2}+\frac{\left(-2ic^2d^2-b\left(ib+\sqrt{-b^2+4ac}\right)e^2+2ce\left(ibd+\sqrt{-b^2+4ac}\,d+iae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\left(2ic^2d^2-b\left(-ib+\sqrt{-b^2+4ac}\right)e^2+2ce\left(-ibd+\sqrt{-b^2+4ac}\,d-iae\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-\frac{b^2}{2}+2ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}}{2c^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*e*Sqrt[d + e*x^2] + (((-2*I)*c^2*d^2 - b*(I*b + Sqrt[-b^2 + 4*a*
c])*e^2 + 2*c*e*(I*b*d + Sqrt[-b^2 + 4*a*c]*d + I*a*e))*ArcTan[(Sqrt[2]*Sqr
t[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-
1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (((2*I)*c^2
*d^2 - b*((-I)*b + Sqrt[-b^2 + 4*a*c])*e^2 + 2*c*e*((-I)*b*d + Sqrt[-b^2 +
4*a*c]*d - I*a*e))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b
*e + I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-1/2*b^2 + 2*a*c]*Sqrt[-2*c*d + (b + I
*Sqrt[-b^2 + 4*a*c])*e]))/(2*c^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 277, normalized size = 0.85

method	result

risch	$\frac{e\sqrt{e\,x^2+d}}{c}-\frac{e\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^4+\left(4d^2eb-4c\,d^3\right)\_Z^2+d^4c\right)}\frac{\left((eb-2cd)\_R^6+\left(4a\,e^2 \ldots\right.\right.}{\_R^7 \ldots}\right.}{4c}$
default	$-\frac{e\left(-\frac{\sqrt{e\,x^2+d}-\sqrt{e}\,x}{c}+\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^4+\left(4d^2eb-4c\,d^3\right)\_Z^2+d^4c\right)}\frac{\left((eb-2cd)\_R \ldots\right.}{\ldots}\right.}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e*(-1/c*((e*x^2+d)^(1/2)-e^(1/2)*x)+1/2/c*sum(((b*e-2*c*d)*_R^6+(4*a*e
^2-3*b*d*e+2*c*d^2)*_R^4+d*(-4*a*e^2+3*b*d*e-2*c*d^2)*_R^2-d^3*e*b+2*d^4*c)
/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*
d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*
c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+d^4*c))-
1/c*d/((e*x^2+d)^(1/2)-e^(1/2)*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4378 vs. 2(288) = 576.
time = 68.19, size = 4378, normalized size = 13.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e
^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c
^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 +
(b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))
*log(-(6*b*c^3*d^5*e + 2*sqrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^
```

2 - 4*a*b*c^3)*d^2*e^2 + (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^4 + ((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt(x^2*e + d)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - (2*a^2*b^2 - 2*a^3*c - (a*b^3 - a^2*b*c)*x^2)*e^6 - ((b^4 + 2*a*b^2*c)*d*x^2 - 2*(2*a*b^3 + a^2*b*c)*d)*e^5 + 2*((2*b^3*c + a*b*c^2)*d^2*x^2 - (b^4 + 6*a*b^2*c + 2*a^2*c^2)*d^2)*e^4 - 2*(3*b^2*c^2*d^3*x^2 - 4*(b^3*c + 2*a*b*c^2)*d^3)*e^3 + 3*(b*c^3*d^4*x^2 - 2*(2*b^2*c^2 + a*c^3)*d^4)*e^2 + (2*(b^2*c^4 - 4*a*c^5)*d^3 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*e^3 - ((b^3*c^3 - 4*a*b*c^4)*d*x^2 - 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*e^2 + ((b^2*c^4 - 4*a*c^5)*d^2*x^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d^2)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/x^2) - sqrt(1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(6*b*c^3*d^5*e - 2*sqrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2 + (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^4 + ((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt(x^2*e + d)*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 + (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - (2*a^2*b^2 - 2*a^3*c - (a*b^3 - a^2*b*c)*x^2)*e^6 - ((b^4 + 2*a*b^2*c)*d*x^2 - 2*(2*a*b^3 + a^2*b*c)*d)*e^5 + 2*((2*b^3*c + a*b*c^2)*d^2*x^2 - (b^4 + 6*a*b^2*c + 2*a^2*c^2)*d^2)*e^4 - 2*(3*b^2*c^2*d^3*x^2 - 4*(b^3*c + 2*a*b*c^2)*d^3)*e^3 + 3*(b*c^3*d^4*x^2 - 2*(2*b^2*c^2 + a*c^3)*d^4)*e^2 + (2*(b^2*c^4 - 4*a*c^5)*d^3 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*e^3 - ((b^3*c^3 - 4*a*b*c^4)*d*x^2 - 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*e^2 + ((b^2*c^4 - 4*a*c^5)*d^2*x^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d^2)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/x^2) + sqrt(1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log(-(6*b*c^3*d^5*e + 2*sqrt(1/2)*(3*(b^2*c^3 - 4*a*c^4)*d^3*e - 6*(b^3*c^2 - 4*a*b*c^3)*d^2*e^2 + (4*b^4*c - 17*a*b^2*c^2 + 4*a^2*c^3)*d*e^3 - (b^

```
5 - 5*a*b^3*c + 4*a^2*b*c^2)*e^4 - ((b^3*c^4 - 4*a*b*c^5)*d - (b^4*c^3 - 6*
a*b^2*c^4 + 8*a^2*c^5)*e)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2
*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^
2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7)))*sqrt(x^2*e + d)*sqrt((2*c^3*d^3 - 3*b*c^2
*d^2*e + 3*(b^2*c - 2*a*c^2)*d*e^2 - (b^3 - 3*a*b*c)*e^3 - (b^2*c^3 - 4*a*c
^4)*sqrt((9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^
4 - 6*(b^3*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - (2*a^2*b^2 - 2*a^3*c - (a*b^3 - a^2*b*c)
*x^2)*e^6 - ((b^4 + 2*a*b^2*c)*d*x^2 - 2*(2*a*b^3 + a^2*b*c)*d)*e^5 + 2*((2
*b^3*c + a*b*c^2)*d^2*x^2 - (b^4 + 6*a*b^2*c + 2*a^2*c^2)*d^2)*e^4 - 2*(3*b
^2*c^2*d^3*x^2 - 4*(b^3*c + 2*a*b*c^2)*d^3)*e^3 + 3*(b*c^3*d^4*x^2 - 2*(2*b
^2*c^2 + a*c^3)*d^4)*e^2 - (2*(b^2*c^4 - 4*a*c^5)*d^3 + (a*b^2*c^3 - 4*a^2*
c^4)*x^2*e^3 - ((b^3*c^3 - 4*a*b*c^4)*d*x^2 - 2*(a*b^2*c^3 - 4*a^2*c^4)*d)*
e^2 + ((b^2*c^4 - 4*a*c^5)*d^2*x^2 - 2*(b^3*c^3 - 4*a*b*c^4)*d^2)*e)*sqrt((
9*c^4*d^4*e^2 - 18*b*c^3*d^3*e^3 + 3*(5*b^2*c^2 - 2*a*c^3)*d^2*e^4 - 6*(b^3
*c - a*b*c^2)*d*e^5 + (b^4 - 2*a*b^2*c + a^2*c^2)*e^6)/(b^2*c^6 - 4*a*c^7))
)/x^2) - sqrt(1/2)*c*sqrt((2*c^3*d^3 - 3*b*c^2*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. 2(288) = 576.
time = 7.19, size = 649, normalized size = 1.98

$$\frac{\sqrt{x^2e+d}\,e}{c}+\frac{\left(4c^5d^3-6bc^4d^2e-(2(b^2c-4ac^2)de^2-(b^3-4abc)e^3)c^2+4(b^2c^3-ac^4)de^2+2\left(\sqrt{b^2-4ac}\,c^3d^2e-\sqrt{b^2-4ac}\,bc^2de^2+\sqrt{b^2-4ac}\,ac^2e^3\right)|c|-(b^3c^2-2abc^3)e^3\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2c^2d-bce+\sqrt{-4(c^2d^2-bcde+ace^2)c^2+(2c^2d-bce)^2}}{c^2}}}\right)}{\left(2\sqrt{b^2-4ac}\,c^2d-\left(b^2c-4ac^2+\sqrt{b^2-4ac}\,bc\right)e\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,c^2}-\frac{\left(4c^5d^3-6bc^4d^2e-(2(b^2c-4ac^2)de^2-(b^3-4abc)e^3)c^2+4(b^2c^3-ac^4)de^2-2\left(\sqrt{b^2-4ac}\,c^3d^2e-\sqrt{b^2-4ac}\,bc^2de^2+\sqrt{b^2-4ac}\,ac^2e^3\right)|c|-(b^3c^2-2abc^3)e^3\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2c^2d-bce-\sqrt{-4(c^2d^2-bcde+ace^2)c^2+(2c^2d-bce)^2}}{c^2}}}\right)}{\left(2\sqrt{b^2-4ac}\,c^2d+\left(b^2c-4ac^2-\sqrt{b^2-4ac}\,bc\right)e\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(x^2*e + d)*e/c + (4*c^5*d^3 - 6*b*c^4*d^2*e - (2*(b^2*c - 4*a*c^2)*d*e
^2 - (b^3 - 4*a*b*c)*e^3)*c^2 + 4*(b^2*c^3 - a*c^4)*d*e^2 + 2*(sqrt(b^2 - 4
*a*c)*c^3*d^2*e - sqrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e
^3)*abs(c) - (b^3*c^2 - 2*a*b*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/
sqrt(-(2*c^2*d - b*c*e + sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2
*d - b*c*e)^2))/c^2))/((2*sqrt(b^2 - 4*a*c)*c^2*d - (b^2*c - 4*a*c^2 + sqrt
(b^2 - 4*a*c)*b*c)*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*c^2)
```

```
 - (4*c^5*d^3 - 6*b*c^4*d^2*e - (2*(b^2*c - 4*a*c^2)*d*e^2 - (b^3 - 4*a*b*c
)*e^3)*c^2 + 4*(b^2*c^3 - a*c^4)*d*e^2 - 2*(sqrt(b^2 - 4*a*c)*c^3*d^2*e - s
qrt(b^2 - 4*a*c)*b*c^2*d*e^2 + sqrt(b^2 - 4*a*c)*a*c^2*e^3)*abs(c) - (b^3*c
^2 - 2*a*b*c^3)*e^3)*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*c^2*d - b*
c*e - sqrt(-4*(c^2*d^2 - b*c*d*e + a*c*e^2)*c^2 + (2*c^2*d - b*c*e)^2))/c^2
))/((2*sqrt(b^2 - 4*a*c)*c^2*d + (b^2*c - 4*a*c^2 - sqrt(b^2 - 4*a*c)*b*c)*
e)*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*c^2)
```

Mupad [B]
time = 4.13, size = 2500, normalized size = 7.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (e*(d + e*x^2)^(1/2))/c - atan(((((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*
c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c - (
2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*
b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c
*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4
*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 +
3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c
^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d
^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3
 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 +
 b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c
^4*e^3 + 32*a*c^5*d*e^2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3
+ 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 -
 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5
 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d
^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^
5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*
b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*
b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*
a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2) - (2*(d + e*x^2)^(1/2)*(b^4*e^6 +
2*a^2*c^2*e^6 + 2*c^4*d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*
c^2*d^2*e^4 - 4*a*b^2*c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*
b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^
2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e +
 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*
e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d
^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3)
)^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*
a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c
```

^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(
1/2)*1i - (((16*a^2*c^3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^
2*d*e^4 - 4*b^2*c^3*d^2*e^3 - 16*a*b*c^3*d*e^4)/c + (2*(d + e*x^2)^(1/2)*(-
(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3
*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^
2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*
(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^
2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3
*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3
- 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*
a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4
)))^(1/2)*(4*b^3*c^3*e^3 - 8*b^2*c^4*d*e^2 - 16*a*b*c^4*e^3 + 32*a*c^5*d*e^
2))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 9
6*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a
*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*
b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 + 16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b
*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*
e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*
a*b^2*c^4)))^(1/2) + (2*(d + e*x^2)^(1/2)*(b^4*e^6 + 2*a^2*c^2*e^6 + 2*c^4*
d^4*e^2 - 12*a*c^3*d^2*e^4 - 4*b*c^3*d^3*e^3 + 6*b^2*c^2*d^2*e^4 - 4*a*b^2*
c*e^6 - 4*b^3*c*d*e^5 + 12*a*b*c^2*d*e^5))/c)*(-(((4*b^5*e^3 + 32*a*c^4*d^3
- 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^2*e -
28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e^2)^2
/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b^3*d^
3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e
^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) + 2*b^5*e^3 +
16*a*c^4*d^3 - 4*b^2*c^3*d^3 + 24*a^2*b*c^2*e^3 - 48*a^2*c^3*d*e^2 + 6*b^3*
c^2*d^2*e - 14*a*b^3*c*e^3 - 6*b^4*c*d*e^2 - 24*a*b*c^3*d^2*e + 36*a*b^2*c^
2*d*e^2)/(16*(16*a^2*c^5 + b^4*c^3 - 8*a*b^2*c^4)))^(1/2)*1i)/((((16*a^2*c^
3*e^5 - 4*a*b^2*c^2*e^5 + 16*a*c^4*d^2*e^3 + 4*b^3*c^2*d*e^4 - 4*b^2*c^3*d^
2*e^3 - 16*a*b*c^3*d*e^4)/c - (2*(d + e*x^2)^(1/2)*(-(((4*b^5*e^3 + 32*a*c^
4*d^3 - 8*b^2*c^3*d^3 + 48*a^2*b*c^2*e^3 - 96*a^2*c^3*d*e^2 + 12*b^3*c^2*d^
2*e - 28*a*b^3*c*e^3 - 12*b^4*c*d*e^2 - 48*a*b*c^3*d^2*e + 72*a*b^2*c^2*d*e
^2)^2/4 - (256*a^2*c^5 + 16*b^4*c^3 - 128*a*b^2*c^4)*(a^3*e^6 + c^3*d^6 - b
^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*
d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b...

3.369 $\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=346

$$-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}-\frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b\left(cd^2+ae^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\ldots}{\sqrt{2cd-\ldots}}\right.}{\sqrt{2}\,a\sqrt{c}\,\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

```
[Out] -d^(3/2)*arctanh((e*x^2+d)^(1/2)/d^(1/2))/a-1/2*arctanh(2^(1/2)*c^(1/2)*(e*
x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*(-b*(a*e^2+c*d^2)+a*e^
2*(-4*a*c+b^2)^(1/2)-c*d*(-4*a*e+d*(-4*a*c+b^2)^(1/2)))/a*2^(1/2)/c^(1/2)/(
-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)-1/2*arctanh(2^(1/2
)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2))*(b*(a*e^2
+c*d^2)+a*e^2*(-4*a*c+b^2)^(1/2)-c*d*(4*a*e+d*(-4*a*c+b^2)^(1/2)))/a*2^(1/2
)/c^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 1.10, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 911, 1301, 212, 1180, 214}

$$-\frac{\left(-cd\left(d\sqrt{b^2-4ac}-4ae\right)+ae^2\sqrt{b^2-4ac}-b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a\sqrt{c}\,\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-cd\left(d\sqrt{b^2-4ac}+4ae\right)+ae^2\sqrt{b^2-4ac}+b\left(ae^2+cd^2\right)\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a\sqrt{c}\,\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a) - ((a*Sqrt[b^2 - 4*a*c]*e^2
 - c*d*(Sqrt[b^2 - 4*a*c]*d - 4*a*e) - b*(c*d^2 + a*e^2))*ArcTanh[(Sqrt[2]*
Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]
*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((a
*Sqrt[b^2 - 4*a*c]*e^2 - c*d*(Sqrt[b^2 - 4*a*c]*d + 4*a*e) + b*(c*d^2 + a*e
^2))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e]])/(Sqrt[2]*a*Sqrt[c]*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}}{x(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{d^2e}{a(d-x^2)}+\frac{e(d(cd^2-bde+ae^2)-(cd^2-ae^2)x^2)}{a(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{d(cd^2-bde+ae^2)+(-cd^2+ae^2)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a} - \frac{d^2\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, x, \sqrt{d}\right.}{a}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} + \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b(cd\right.}{}$$

$$= -\frac{d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a} - \frac{\left(a\sqrt{b^2-4ac}\,e^2-cd\left(\sqrt{b^2-4ac}\,d-4ae\right)-b(cd\right.}{\sqrt{2}\,a\sqrt{c}\,\sqrt{b^2-4ac}\,\sqrt{2c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.27, size = 379, normalized size = 1.10

$$-\frac{\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}\,e^2+cd\left(\sqrt{-b^2+4ac}\,d+4iae\right)-ib(cd^2+ae^2)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\sqrt{2}\left(-a\sqrt{-b^2+4ac}\,e^2+cd\left(\sqrt{-b^2+4ac}\,d-4iae\right)+ib(cd^2+ae^2)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{c}\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}+2d^{3/2}\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d + (
4*I)*a*e) - I*b*(c*d^2 + a*e^2))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/S
qrt[-2*c*d + b*e - I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sq
rt[-2*c*d + (b - I*Sqrt[-b^2 + 4*a*c])*e]) + (Sqrt[2]*(-(a*Sqrt[-b^2 + 4*a*
c]*e^2) + c*d*(Sqrt[-b^2 + 4*a*c]*d - (4*I)*a*e) + I*b*(c*d^2 + a*e^2))*Arc
Tan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a
*c]*e]])/(Sqrt[c]*Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c
])*e]) + 2*d^(3/2)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.13, size = 381, normalized size = 1.10

method	result
default	$-\frac{\frac{\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)^3}{24}+\frac{5d\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)}{8}-\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^8+(4eb-4cd)\_Z^6+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^4+\left(4d^2e\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(1/24*((e*x^2+d)^(1/2)-e^(1/2)*x)^3+5/8*d*((e*x^2+d)^(1/2)-e^(1/2)*x)-
1/4*sum(((a*e^2-c*d^2)*_R^6+d*(5*a*e^2-4*b*d*e+3*c*d^2)*_R^4+d^2*(-5*a*e^2+
4*b*d*e-3*c*d^2)*_R^2-a*d^3*e^2+c*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3
*a*e^2-4*_R^3*b*d*e+3*_R^3*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^
(1/2)*x-_R),_R=RootOf(c*_Z^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*
_Z^4+(4*b*d^2*e-4*c*d^3)*_Z^2+d^4*c))+1/24*d^3/((e*x^2+d)^(1/2)-e^(1/2)*x)^
3+5/8*d^2/((e*x^2+d)^(1/2)-e^(1/2)*x))+1/a*(1/3*(e*x^2+d)^(3/2)+d*((e*x^2+d
)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{\frac{3}{2}}}{x\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(x*(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 827 vs. 2(298) = 596.
time = 7.13, size = 827, normalized size = 2.39

$$\frac{d^2\arctan\left(\frac{\sqrt{x^2e+d}}{\sqrt{-d}}\right)}{a\sqrt{-d}} - \frac{\left(\left((b^2c-4ac^2)d^2e-(ab^2-4a^2c)e^3\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,a^2-2\left(\sqrt{b^2-4ac}\,ac^2d^3-\sqrt{b^2-4ac}\,abcd^2e+\sqrt{b^2-4ac}\,a^2cde^2\right)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\,|a|-(2a^2bc^2d^3+6a^3bcde^2-a^3b^2e^3-(a^2b^2c+8a^3c^2)d^2e)\sqrt{-4c^2d+2\left(bc-\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2acd-abe+\sqrt{-4(acd^2-abde+a^2e^2)ac+(2acd-abe)^2}}{ac}}}\right)}{8\left(\sqrt{b^2-4ac}\,a^2c^2d^2-\sqrt{b^2-4ac}\,a^2bcde+\sqrt{b^2-4ac}\,a^3ce^2\right)|a||c|} + \frac{\left(\left((b^2c-4ac^2)d^2e-(ab^2-4a^2c)e^3\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,a^2+2\left(\sqrt{b^2-4ac}\,ac^2d^3-\sqrt{b^2-4ac}\,abcd^2e+\sqrt{b^2-4ac}\,a^2cde^2\right)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\,|a|-(2a^2bc^2d^3+6a^3bcde^2-a^3b^2e^3-(a^2b^2c+8a^3c^2)d^2e)\sqrt{-4c^2d+2\left(bc+\sqrt{b^2-4ac}\,c\right)e}\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2acd-abe-\sqrt{-4(acd^2-abde+a^2e^2)ac+(2acd-abe)^2}}{ac}}}\right)}{8\left(\sqrt{b^2-4ac}\,a^2c^2d^2-\sqrt{b^2-4ac}\,a^2bcde+\sqrt{b^2-4ac}\,a^3ce^2\right)|a||c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] d^2*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a*sqrt(-d)) - 1/8*(((b^2*c - 4*a*c^2)
*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*
c)*e)*a^2 - 2*(sqrt(b^2 - 4*a*c)*a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e
+ sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)
*c)*e)*abs(a) - (2*a^2*b*c^2*d^3 + 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2
*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e))*ar
ctan(2*sqrt(1/2)*sqrt(x^2*e + d)/sqrt(-(2*a*c*d - a*b*e + sqrt(-4*(a*c*d^2
- a*b*d*e + a^2*e^2)*a*c + (2*a*c*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c
)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2
)*abs(a)*abs(c)) + 1/8*(((b^2*c - 4*a*c^2)*d^2*e - (a*b^2 - 4*a^2*c)*e^3)*s
qrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*a^2 + 2*(sqrt(b^2 - 4*a*c)*
a*c^2*d^3 - sqrt(b^2 - 4*a*c)*a*b*c*d^2*e + sqrt(b^2 - 4*a*c)*a^2*c*d*e^2)*
sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*abs(a) - (2*a^2*b*c^2*d^3
+ 6*a^3*b*c*d*e^2 - a^3*b^2*e^3 - (a^2*b^2*c + 8*a^3*c^2)*d^2*e)*sqrt(-4*c^
2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e))*arctan(2*sqrt(1/2)*sqrt(x^2*e + d)/
sqrt(-(2*a*c*d - a*b*e - sqrt(-4*(a*c*d^2 - a*b*d*e + a^2*e^2)*a*c + (2*a*c
*d - a*b*e)^2))/(a*c)))/((sqrt(b^2 - 4*a*c)*a^2*c^2*d^2 - sqrt(b^2 - 4*a*c)
*a^2*b*c*d*e + sqrt(b^2 - 4*a*c)*a^3*c*e^2)*abs(a)*abs(c))
```

Mupad [B]
time = 7.67, size = 2500, normalized size = 7.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] atan((((d + e*x^2)^(1/2)*(2*a^4*c*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10
- 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2
*e^14 + 24*b^2*c^3*d^6*e^10 - 16*b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*
b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2
```

*c*d^2*e^14) + (-((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2
*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*
c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3
*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*
e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6
*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*
a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2
*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*
c^2)))^(1/2)*(((-((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2
*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*
c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3
*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*
e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6
*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*
a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2
*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*
c^2)))^(1/2)*((d + e*x^2)^(1/2)*(-((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c
^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*
d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^
2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e
^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 -
3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*
a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3
*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b
^4*c - 8*a^3*b^2*c^2)))^(1/2)*(512*a^5*c^4*e^10 + 32*a^3*b^4*c^2*e^10 - 256
*a^4*b^2*c^3*e^10 + 768*a^4*c^5*d^2*e^8 + 64*a^2*b^4*c^3*d^2*e^8 - 448*a^3*
b^2*c^4*d^2*e^8 - 896*a^4*b*c^4*d*e^9 - 64*a^2*b^5*c^2*d*e^9 + 480*a^3*b^3*
c^3*d*e^9) - 192*a^3*c^5*d^4*e^8 - 192*a^4*c^4*d^2*e^10 + 48*a^2*b^2*c^4*d^
4*e^8 - 64*a^2*b^3*c^3*d^3*e^9 + 16*a^2*b^4*c^2*d^2*e^10 - 16*a^3*b^2*c^3*d
^2*e^10 + 64*a^4*b*c^3*d*e^11 + 256*a^3*b*c^4*d^3*e^9 - 16*a^3*b^3*c^2*d*e^
11) + (d + e*x^2)^(1/2)*(8*a^3*b^3*c*e^13 - 32*a^4*b*c^2*e^13 + 176*a^4*c^3
*d*e^12 - 144*a^2*c^5*d^5*e^8 + 224*a^3*c^4*d^3*e^10 - 16*b^4*c^3*d^5*e^8 +
16*b^5*c^2*d^4*e^9 + 48*a^2*b^2*c^3*d^3*e^10 + 112*a^2*b^3*c^2*d^2*e^11 -
16*a^2*b^4*c*d*e^12 + 96*a*b^2*c^4*d^5*e^8 - 80*a*b^3*c^3*d^4*e^9 - 32*a*b^
4*c^2*d^3*e^10 + 96*a^2*b*c^4*d^4*e^9 - 416*a^3*b*c^3*d^2*e^11 + 16*a^3*b^2
*c^2*d*e^12))*(-((((4*b^4*c*d^3 - 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*
c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c
^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*
b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 + 3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e
^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*
a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*b^3*e^3 - 16*a^2*c^3*d^3 + 12*a
*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*e^3 + 6*a*b^3*c*d^2*e - 24*a^2*
b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^4*c^3 + a^2*b^4*c - 8*a^3*b^2*c
^2)))^(1/2) + 12*a*c^5*d^7*e^8 + 4*a^4*c^2*d*e^14 - 84*a^2*c^4*d^5*e^10 - 9
2*a^3*c^3*d^3*e^12 - 4*b^2*c^4*d^7*e^8 - 4*b^3*c^3*d^6*e^9 + 8*b^4*c^2*d^5*

```
e^10 - 12*a^2*b^2*c^2*d^3*e^12 + 32*a*b*c^4*d^6*e^9 - 4*a^3*b^2*c*d*e^14 -
36*a*b^2*c^3*d^5*e^10 - 20*a*b^3*c^2*d^4*e^11 + 160*a^2*b*c^3*d^4*e^11 + 4*
a^2*b^3*c*d^2*e^13 + 16*a^3*b*c^2*d^2*e^13))*(-(((4*b^4*c*d^3 - 4*a^2*b^3*e
^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*a^3*b*c*e^3
- 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2/4 - (256*a^
4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*c^2)*(a^3*e^6 + c^3*d^6 - b^3*d^3*e^3 +
3*a*b^2*d^2*e^4 + 3*a*c^2*d^4*e^2 + 3*a^2*c*d^2*e^4 + 3*b^2*c*d^4*e^2 - 3*a
^2*b*d*e^5 - 3*b*c^2*d^5*e - 6*a*b*c*d^3*e^3))^(1/2) - 2*b^4*c*d^3 + 2*a^2*
b^3*e^3 - 16*a^2*c^3*d^3 + 12*a*b^2*c^2*d^3 + 48*a^3*c^2*d*e^2 - 8*a^3*b*c*
e^3 + 6*a*b^3*c*d^2*e - 24*a^2*b*c^2*d^2*e - 12*a^2*b^2*c*d*e^2)/(16*(16*a^
4*c^3 + a^2*b^4*c - 8*a^3*b^2*c^2)))^(1/2)*1i + ((d + e*x^2)^(1/2)*(2*a^4*c
*e^16 + 6*c^5*d^8*e^8 - 16*a*c^4*d^6*e^10 - 16*b*c^4*d^7*e^9 + 4*b^4*c*d^4*
e^12 + 16*a^2*c^3*d^4*e^12 + 8*a^3*c^2*d^2*e^14 + 24*b^2*c^3*d^6*e^10 - 16*
b^3*c^2*d^5*e^11 - 8*a^3*b*c*d*e^15 - 8*a*b^3*c*d^3*e^13 + 16*a*b^2*c^2*d^4
*e^12 - 24*a^2*b*c^2*d^3*e^13 + 12*a^2*b^2*c*d^2*e^14) + (-(((4*b^4*c*d^3 -
 4*a^2*b^3*e^3 + 32*a^2*c^3*d^3 - 24*a*b^2*c^2*d^3 - 96*a^3*c^2*d*e^2 + 16*
a^3*b*c*e^3 - 12*a*b^3*c*d^2*e + 48*a^2*b*c^2*d^2*e + 24*a^2*b^2*c*d*e^2)^2
/4 - (256*a^4*c^3 + 16*a^2*b^4*c - 128*a^3*b^2*...
```

3.370 $\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=417

$$-\frac{d\sqrt{d+ex^2}}{2ax^2}+\frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}+\frac{\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}-\frac{\sqrt{c}\left(b^2d^2+bd\left(\sqrt{b^2-\right.\right.}{}$$

```
[Out] 1/2*e*arctanh((e*x^2+d)^(1/2)/d^(1/2))*d^(1/2)/a+(-2*a*e+b*d)*arctanh((e*x^
2+d)^(1/2)/d^(1/2))*d^(1/2)/a^2-1/2*d*(e*x^2+d)^(1/2)/a/x^2-1/2*arctanh(2^(
1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2))*c^(1/2
)*(b^2*d^2+b*d*(-2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d^2+e*(-a*e+d*(-4*a*c+b
^2)^(1/2))))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(e*x^2+d)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2
)^(1/2)))^(1/2))*c^(1/2)*(b^2*d^2-b*d*(2*a*e+d*(-4*a*c+b^2)^(1/2))-2*a*(c*d
^2-e*(a*e+d*(-4*a*c+b^2)^(1/2)))))/a^2*2^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 2.27, antiderivative size = 416, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1265, 911, 1301, 205, 212, 1180, 214}

$$-\frac{\sqrt{c}\left(bd\left(d\sqrt{b^2-4ac}-2ae\right)-2ae\left(d\sqrt{b^2-4ac}-ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\sqrt{c}\left(-bd\left(d\sqrt{b^2-4ac}+2ae\right)+2ae\left(d\sqrt{b^2-4ac}+ae\right)-2acd^2+b^2d^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}\right)}{\sqrt{2}\,a^2\sqrt{b^2-4ac}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2}-\frac{d\sqrt{d+ex^2}}{2ax^2}+\frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(d*Sqrt[d + e*x^2])/(a*x^2) + (Sqrt[d]*e*ArcTanh[Sqrt[d + e*x^2]/Sqrt[
d]])/(2*a) + (Sqrt[d]*(b*d - 2*a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/a^2 -
 (Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + b*d*(Sqrt[b^2 - 4*a*c]*d - 2*a*e) - 2*a*e*
(Sqrt[b^2 - 4*a*c]*d - a*e))*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]])/(Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2
*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[c]*(b^2*d^2 - 2*a*c*d^2 + 2*a*e*
(Sqrt[b^2 - 4*a*c]*d + a*e) - b*d*(Sqrt[b^2 - 4*a*c]*d + 2*a*e))*ArcTanh[(S
qrt[2]*Sqrt[c]*Sqrt[d + e*x^2]])/Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]])/(
Sqrt[2]*a^2*Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^n)^(p +
 1)/(a*n*(p + 1))), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n
```

```
)^(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integ
erQ[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denom
inator[p + 1/n] < Denominator[p])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}}{x^3(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^{3/2}}{x^2(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-\frac{d}{e}+\frac{x^2}{e}\right)^2\left(\frac{cd^2-bde+ae^2}{e^2}-\frac{(2cd-be)x^2}{e^2}+\frac{cx^4}{e^2}\right)}\,dx, x, \sqrt{d+ex^2}\right)}{e}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{d^2e^2}{a(d-x^2)^2}-\frac{de(-bd+2ae)}{a^2(d-x^2)}+\frac{e(-(bd-ae)(cd^2-bde+ae^2)+cd(bd-2ae)x^2)}{a^2(cd^2-bde+ae^2-(2cd-be)x^2+cx^4)}\right)\,dx, x, \sqrt{d\right.}}{e}$$

$$= \frac{\text{Subst}\left(\int \frac{-(bd-ae)(cd^2-bde+ae^2)+cd(bd-2ae)x^2}{cd^2-bde+ae^2+(-2cd+be)x^2+cx^4}\,dx, x, \sqrt{d+ex^2}\right)}{a^2} + \frac{(d^2e)\,\text{Subst}\left(\int \frac{}{(d-}\right.}{}$$

$$= -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{a^2} + \frac{(de)\text{Subst}\left(\int \frac{1}{d-x^2}\,dx, \right.}{2a}$$

$$= -\frac{d\sqrt{d+ex^2}}{2ax^2} + \frac{\sqrt{d}\,e\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a} + \frac{\sqrt{d}\,(bd-2ae)\tanh^{-1}\left(\frac{\sqrt{d+e}}{\sqrt{d}}\right.}{a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.55, size = 427, normalized size = 1.02

$$\frac{-\frac{ad\sqrt{d+ex^2}}{x^2}+\frac{\sqrt{2}\sqrt{c}\left(-ib^2d^2+bd\left(\sqrt{-b^2+4ac}\,d+2iae\right)-2ia\left(-cd^2+e\left(-i\sqrt{-b^2+4ac}\,d+ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be-i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b-i\sqrt{-b^2+4ac}\right)e}}+\frac{\sqrt{2}\sqrt{c}\left(ib^2d^2+bd\left(\sqrt{-b^2+4ac}\,d-2iae\right)+2ia\left(-cd^2+e\left(i\sqrt{-b^2+4ac}\,d+ae\right)\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{d+ex^2}}{\sqrt{-2cd+be+i\sqrt{-b^2+4ac}\,e}}\right)}{\sqrt{-b^2+4ac}\sqrt{-2cd+\left(b+i\sqrt{-b^2+4ac}\right)e}}+\sqrt{d}\,(2bd-3ae)\tanh^{-1}\left(\frac{\sqrt{d+ex^2}}{\sqrt{d}}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*d*Sqrt[d + e*x^2])/x^2) + (Sqrt[2]*Sqrt[c]*((-I)*b^2*d^2 + b*d*(Sqrt[
-b^2 + 4*a*c]*d + (2*I)*a*e) - (2*I)*a*(-(c*d^2) + e*((-I)*Sqrt[-b^2 + 4*a*
c]*d + a*e)))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e -
I*Sqrt[-b^2 + 4*a*c]*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b - I*Sqrt[-b^
2 + 4*a*c])*e]) + (Sqrt[2]*Sqrt[c]*(I*b^2*d^2 + b*d*(Sqrt[-b^2 + 4*a*c]*d -
 (2*I)*a*e) + (2*I)*a*(-(c*d^2) + e*(I*Sqrt[-b^2 + 4*a*c]*d + a*e)))*ArcTan
[(Sqrt[2]*Sqrt[c]*Sqrt[d + e*x^2])/Sqrt[-2*c*d + b*e + I*Sqrt[-b^2 + 4*a*c]
*e]])/(Sqrt[-b^2 + 4*a*c]*Sqrt[-2*c*d + (b + I*Sqrt[-b^2 + 4*a*c])*e]) + Sq
rt[d]*(2*b*d - 3*a*e)*ArcTanh[Sqrt[d + e*x^2]/Sqrt[d]])/(2*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 535, normalized size = 1.28

method	result
risch	$-\frac{d\sqrt{e\,x^{2}+d}}{2a\,x^{2}}-\frac{3\sqrt{d}\,\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)e}{2a}+\frac{d^{\frac{3}{2}}\ln\left(\frac{2d+2\sqrt{d}\,\sqrt{e\,x^{2}+d}}{x}\right)b}{a^{2}}-\frac{\_R=\mathrm{RootOf}\left(c\_Z^{8}+(4eb\right.}{}$
default	$-\frac{-\frac{b\left(\sqrt{e\,x^{2}+d}-\sqrt{e}\,x\right)^{3}}{24}+\frac{ae\left(\sqrt{e\,x^{2}+d}-\sqrt{e}\,x\right)}{2}-\frac{5bd\left(\sqrt{e\,x^{2}+d}-\sqrt{e}\,x\right)}{8}+\frac{d(4ae-5bd)}{8\sqrt{e\,x^{2}+d}-8\sqrt{e}\,x}-\frac{}{24\left(\sqrt{e\right.}}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(-1/24*b*((e*x^2+d)^(1/2)-e^(1/2)*x)^3+1/2*a*e*((e*x^2+d)^(1/2)-e^(1
/2)*x)-5/8*b*d*((e*x^2+d)^(1/2)-e^(1/2)*x)+1/8*d*(4*a*e-5*b*d)/((e*x^2+d)^(
1/2)-e^(1/2)*x)-1/24*b*d^3/((e*x^2+d)^(1/2)-e^(1/2)*x)^3-1/4*sum((c*d*(-2*a
*e+b*d)*_R^6+(4*a^2*e^3-8*a*b*d*e^2+2*a*c*d^2*e+4*b^2*d^2*e-3*b*c*d^3)*_R^4
+d*(-4*a^2*e^3+8*a*b*d*e^2-2*a*c*d^2*e-4*b^2*d^2*e+3*b*c*d^3)*_R^2+2*a*c*d^
4*e-b*c*d^5)/(_R^7*c+3*_R^5*b*e-3*_R^5*c*d+8*_R^3*a*e^2-4*_R^3*b*d*e+3*_R^3
*c*d^2+_R*b*d^2*e-_R*c*d^3)*ln((e*x^2+d)^(1/2)-e^(1/2)*x-_R),_R=RootOf(c*_Z
^8+(4*b*e-4*c*d)*_Z^6+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^4+(4*b*d^2*e-4*c*d^3)*_
Z^2+d^4*c)))+1/a*(-1/2/d/x^2*(e*x^2+d)^(5/2)+3/2*e/d*(1/3*(e*x^2+d)^(3/2)+d
*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(1/2))/x))))-b/a^2*(1
/3*(e*x^2+d)^(3/2)+d*((e*x^2+d)^(1/2)-d^(1/2)*ln((2*d+2*d^(1/2)*(e*x^2+d)^(
1/2))/x)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{\frac{3}{2}}}{x^3(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(x**3*(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 5.54, size = 433, normalized size = 1.04

$$-\frac{(2\,bd^2-3\,ade)\arctan\left(\frac{\sqrt{x^2e+d}}{\sqrt{-d}}\right)}{2\,a^2\sqrt{-d}}-\frac{\sqrt{-4\,c^2d+2\left(bc-\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^2-2\,ac+\sqrt{b^2-4\,ac}\,b\right)d-\left(ab+\sqrt{b^2-4\,ac}\,a\right)e\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,a^2cd-a^2be+\sqrt{-4\,(a^2cd^2-a^2bde+a^3e^2)a^2c+(2\,a^2cd-a^2be)^2}}{a^2c}}}\right)}{4\sqrt{b^2-4\,ac}\,a^2|c|}+\frac{\sqrt{-4\,c^2d+2\left(bc+\sqrt{b^2-4\,ac}\,c\right)e}\left(\left(b^2-2\,ac-\sqrt{b^2-4\,ac}\,b\right)d-\left(ab-\sqrt{b^2-4\,ac}\,a\right)e\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{x^2e+d}}{\sqrt{-\frac{2\,a^2cd-a^2be-\sqrt{-4\,(a^2cd^2-a^2bde+a^3e^2)a^2c+(2\,a^2cd-a^2be)^2}}{a^2c}}}\right)}{4\sqrt{b^2-4\,ac}\,a^2|c|}-\frac{\sqrt{x^2e+d}\,d}{2\,ax^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*d^2 - 3*a*d*e)*arctan(sqrt(x^2*e + d)/sqrt(-d))/(a^2*sqrt(-d)) -
1/4*sqrt(-4*c^2*d + 2*(b*c - sqrt(b^2 - 4*a*c)*c)*e)*((b^2 - 2*a*c + sqrt(b
^2 - 4*a*c)*b)*d - (a*b + sqrt(b^2 - 4*a*c)*a)*e)*arctan(2*sqrt(1/2)*sqrt(x
^2*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e + sqrt(-4*(a^2*c*d^2 - a^2*b*d*e + a^3
*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sqrt(b^2 - 4*a*c)*a^2*ab
s(c)) + 1/4*sqrt(-4*c^2*d + 2*(b*c + sqrt(b^2 - 4*a*c)*c)*e)*((b^2 - 2*a*c
- sqrt(b^2 - 4*a*c)*b)*d - (a*b - sqrt(b^2 - 4*a*c)*a)*e)*arctan(2*sqrt(1/2
)*sqrt(x^2*e + d)/sqrt(-(2*a^2*c*d - a^2*b*e - sqrt(-4*(a^2*c*d^2 - a^2*b*d
*e + a^3*e^2)*a^2*c + (2*a^2*c*d - a^2*b*e)^2))/(a^2*c)))/(sqrt(b^2 - 4*a*c
)*a^2*abs(c)) - 1/2*sqrt(x^2*e + d)*d/(a*x^2)
```

Mupad [B]
time = 6.10, size = 2500, normalized size = 6.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (d^(1/2)*atan(((d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16
 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^
4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*
d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^
```

3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) - (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3*d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4) + (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4 - (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2))*(3*a*e - 2*b*d))/(4*a^2))*1i)/(4*a^2) + (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4*d^5*e^11 + 8*a^2*b^4*c^3*d^4*e^12 + 88*a^3*b^2*c^4*d^4*e^12 - 28*a^3*b^3*c^3*d^3*e^13 + 33*a^4*b^2*c^3*d^2*e^14 - 16*a^5*b*c^3*d*e^15 - 8*a*b^2*c^6*d^8*e^8 - 28*a*b^3*c^5*d^7*e^9 + 8*a^2*b*c^6*d^7*e^9 - 228*a^3*b*c^5*d^5*e^11 - 60*a^4*b*c^4*d^3*e^13))/(2*a^4) + (d^(1/2)*((56*a^4*c^6*d^6*e^9 - 44*a^5*c^5*d^4*e^11 - 100*a^6*c^4*d^2*e^13 + 40*a^2*b^3*c^5*d^7*e^8 - 39*a^2*b^5*c^3*d^5*e^10 - 11*a^2*b^6*c^2*d^4*e^11 - 108*a^3*b^2*c^5*d^6*e^9 + 96*a^3*b^3*c^4*d^5*e^10 + 111*a^3*b^4*c^3*d^4*e^11 + 22*a^3*b^5*c^2*d^3*e^12 - 237*a^4*b^2*c^4*d^4*e^11 - 161*a^4*b^3*c^3*d^3*e^12 - 19*a^4*b^4*c^2*d^2*e^13 + 111*a^5*b^2*c^3*d^2*e^13 - 28*a^6*b*c^3*d*e^14 - 8*a*b^5*c^4*d^7*e^8 + 6*a*b^6*c^3*d^6*e^9 + 2*a*b^7*c^2*d^5*e^10 - 32*a^3*b*c^6*d^7*e^8 + 92*a^4*b*c^5*d^5*e^10 + 252*a^5*b*c^4*d^3*e^12 + 6*a^5*b^3*c^2*d*e^14)/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(64*a^7*b*c^3*e^13 + 352*a^7*c^4*d*e^12 - 16*a^6*b^3*c^2*e^13 - 160*a^5*c^6*d^5*e^8 + 736*a^6*c^5*d^3*e^10 + 32*a^2*b^6*c^3*d^5*e^8 - 32*a^2*b^7*c^2*d^4*e^9 - 224*a^3*b^4*c^4*d^5*e^8 + 144*a^3*b^5*c^3*d^4*e^9 + 112*a^3*b^6*c^2*d^3*e^10 + 432*a^4*b^2*c^5*d^5*e^8 + 144*a^4*b^3*c^4*d^4*e^9 - 716*a^4*b^4*c^3*d^3*e^10 - 132*a^4*b^5*c^2*d^2*e^11 + 936*a^5*b^2*c^4*d^3*e^10 + 860*a^5*b^3*c^3 *d^2*e^11 - 896*a^5*b*c^5*d^4*e^9 + 64*a^5*b^4*c^2*d*e^12 - 1392*a^6*b*c^4*d^2*e^11 - 336*a^6*b^2*c^3*d*e^12))/(2*a^4) - (d^(1/2)*((320*a^8*c^4*d*e^11 + 320*a^7*c^5*d^3*e^9 + 32*a^5*b^3*c^4*d^4*e^8 - 24*a^5*b^4*c^3*d^3*e^9 - 8*a^5*b^5*c^2*d^2*e^10 + 16*a^6*b^2*c^4*d^3*e^9 + 144*a^6*b^3*c^3*d^2*e^10 - 128*a^6*b*c^5*d^4*e^8 + 8*a^6*b^4*c^2*d*e^11 - 448*a^7*b*c^4*d^2*e^10 - 112*a^7*b^2*c^3*d*e^11)/a^4 + (d^(1/2)*(d + e*x^2)^(1/2)*(3*a*e - 2*b*d)*(1024*a^9*c^4*e^10 + 64*a^7*b^4*c^2*e^10 - 512*a^8*b^2*c^3*e^10 + 1536*a^8*c^5*d^2*e^8 + 128*a^6*b^4*c^3*d^2*e^8 - 896*a^7*b^2*c^4*d^2*e^8 - 1792*a^8*b*c^4*d*e^9 - 128*a^6*b^5*c^2*d*e^9 + 960*a^7*b^3*c^3*d*e^9))/(8*a^6))*(3*a*e - 2*b*d))/(4*a^2)))/(4*a^2))*(3*a*e - 2*b*d))/(4*a^2))*1i)/(4*a^2))/((3*a*c^7*d^9*e^9 + 3*a^5*c^3*d*e^17 - 2*b*c^7*d^10*e^8 + 3*a^2*c^6*d^7*e^11 + 3*a^4*c^4*d^3*e^15 + 4*b^2*c^6*d^9*e^9 - 2*b^3*c^5*d^8*e^10 + 2*a^2*b^2*c^4*d^5*e^13 - (11*a^2*b^3*c^3*d^4*e^14)/2 + 11*a^3*b^2*c^3*d^3*e^15 - 8*a*b*c^6*d^8*e^10 + 4*a*b^2*c^5*d^7*e^11 + a*b^4*c^3*d^5*e^13 - (3*a^2*b*c^5*d^6*e^12)/2 - 5*a^3*b*c^4*d^4*e^14 - (19*a^4*b*c^3*d^2*e^16)/2)/a^4 - (d^(1/2)*(3*a*e - 2*b*d)*(((d + e*x^2)^(1/2)*(4*a^6*c^3*e^16 + 4*a^2*c^7*d^8*e^8 - 2*a^3*c^6*d^6*e^10 + 132*a^4*c^5*d^4*e^12 - 2*a^5*c^4*d^2*e^14 + 4*b^4*c^5*d^8*e^8 + 129*a^2*b^2*c^5*d^6*e^10 - 32*a^2*b^3*c^4...

3.371 $\int \frac{x^4 (d+ex^2)^{3/2}}{a+bx^2+cx^4} \, dx$

Optimal. Leaf size=595

$$\frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2}+\frac{x(d+ex^2)^{3/2}}{4c}-\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bcd-b^2e+ace-\frac{b^2cd-2ac^2d-b^3e+3abce}{\sqrt{b^2-4ac}}\right.}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/4*x*(e*x^2+d)^(3/2)/c+1/8*d*(-4*b*e+3*c*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1
/2))/c^2/e^(1/2)-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*c*d-b^2*e+a*c*e+
(-3*a*b*c*e+2*a*c^2*d+b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/c^3-1/2*ar
ctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*c*d-b^2*e+a*c*e+(3*a*b*c*e-2*a*c^2*d-b^
3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/c^3+1/8*(-4*b*e+3*c*d)*x*(e*x^2+d)
^(1/2)/c^2-1/2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(-3*a*b*c*e+2*a*c^2*d+
b^3*e-b^2*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/c
^3/(b-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)
))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*c*d-b^2*e+a*c*e+(
3*a*b*c*e-2*a*c^2*d-b^3*e+b^2*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+
b^2)^(1/2)))^(1/2)/c^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 2.38, antiderivative size = 595, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 9, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.310$, Rules used = {1305, 396, 201, 223, 212, 1706, 399, 385, 211}

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^3\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)}{2c^3}+\frac{x\sqrt{d+ex^2}(3cd-4be)}{8c^2}+\frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}}+\frac{x(d+ex^2)^{3/2}}{4c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((3*c*d - 4*b*e)*x*Sqrt[d + e*x^2])/(8*c^2) + (x*(d + e*x^2)^(3/2))/(4*c) -
 (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d
 - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(2*c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b
 + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*c^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]
) + (d*(3*c*d - 4*b*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c^2*Sqrt[e]
) - (Sqrt[e]*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
```

```
c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3) - (Sq
rt[e]*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*c*e)/Sq
rt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^3)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1305

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[(f*x)^(m - 4)*(d +
e*x^2)^(q - 1)*(Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x]/(a + b
*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{\int \sqrt{d+ex^2}\,(cd-be+cex^2)\,dx}{c^2} - \frac{\int \frac{\sqrt{d+ex^2}\,(a(cd-be)+(bcd-b^2e+ace)x^2)}{a+bx^2+cx^4}\,dx}{c^2}$$

$$= \frac{x(d+ex^2)^{3/2}}{4c} - \frac{\int \left(\frac{\left(bcd-b^2e+ace+\frac{-b^2cd+2ac^2d+b^3e-3abce}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(bcd-b^2e+ace-\frac{-b^2cd+2\cdots}{\sqrt{b\cdots}}\right.}{b+\sqrt{b^2\cdots}} \right.}{c^2}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x(d+ex^2)^{3/2}}{4c} + \frac{(d(3cd-4be))\int \frac{1}{\sqrt{d+ex^2}}\,dx}{8c^2} - \frac{\left(bcd-\cdots\right.}{\cdots}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x(d+ex^2)^{3/2}}{4c} + \frac{(d(3cd-4be))\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+\cdots}}\right.}{8c^2}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x(d+ex^2)^{3/2}}{4c} + \frac{d(3cd-4be)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8c^2\sqrt{e}} - \frac{\left(e\cdots\right.}{\cdots}$$

$$= \frac{(3cd-4be)x\sqrt{d+ex^2}}{8c^2} + \frac{x(d+ex^2)^{3/2}}{4c} - \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\left(bcd-b^2e\cdots\right.}{\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 18689 vs. $2(595) = 1190$.
time = 16.35, size = 18689, normalized size = 31.41

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 459, normalized size = 0.77

method	result
risch	$-\frac{x\left(-2ce\,x^{2}+4eb-5cd\right)\sqrt{e\,x^{2}+d}}{8c^{2}}-\frac{\ln\left(\sqrt{e}\,x+\sqrt{e\,x^{2}+d}\right)e^{\frac{3}{2}}a}{c^{2}}+\frac{\ln\left(\sqrt{e}\,x+\sqrt{e\,x^{2}+d}\right)e^{\frac{3}{2}}b^{2}}{c^{3}}-\frac{3\ln\left(\sqrt{e}\right.}{} \cdots$
default	$\frac{\frac{x\left(e\,x^{2}+d\right)^{\frac{3}{2}}}{4}+\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2}+\frac{d\ln\left(\sqrt{e}\,x+\sqrt{e\,x^{2}+d}\right)}{2\sqrt{e}}\right)}{4}}{c}+\frac{\sqrt{e}\left(\frac{b\left(\sqrt{e\,x^{2}+d}-\sqrt{e}\,x\right)^{2}}{2c}+\frac{\left(4ace-4b^{2}e+6bcd\right)\ln}{}\cdots\right.}{} \cdots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/4*x*(e*x^2+d)^(3/2)+3/4*d*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(e^
(1/2)*x+(e*x^2+d)^(1/2))))+1/4/c*e^(1/2)*(1/2*b*((e*x^2+d)^(1/2)-e^(1/2)*x)
^2/c+1/c^2*(4*a*c*e-4*b^2*e+6*b*c*d)*ln((e*x^2+d)^(1/2)-e^(1/2)*x)-1/2/c*b*
d^2/((e*x^2+d)^(1/2)-e^(1/2)*x)^2-2/c^2*sum(((2*a*b*c*e^2-2*a*c^2*d*e-b^3*e
^2+2*b^2*c*d*e-b*c^2*d^2)*_R^2+2*(2*a^2*c*e^3-2*a*b^2*e^3+2*a*b*c*d*e^2+b^3
*d*e^2-2*b^2*c*d^2*e+b*c^2*d^3)*_R+2*a*b*c*d^2*e^2-2*a*c^2*d^3*e-b^3*d^2*e^
2+2*b^2*c*d^3*e-b*c^2*d^4)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*
d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootO
f(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*
d^3)*_Z+d^4*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 6.19, size = 104, normalized size = 0.17

$$\frac{1}{8}\sqrt{x^2e+d}\left(\frac{2\,x^2e}{c}+\frac{(5\,c^5de^2-4\,bc^4e^3)e^{(-2)}}{c^6}\right)x-\frac{(3\,c^2d^2-12\,bcde+8\,b^2e^2-8\,ace^2)e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^2*e + d)*(2*x^2*e/c + (5*c^5*d*e^2 - 4*b*c^4*e^3)*e^(-2)/c^6)*x
- 1/16*(3*c^2*d^2 - 12*b*c*d*e + 8*b^2*e^2 - 8*a*c*e^2)*e^(-1/2)*log((x*e^(
1/2) - sqrt(x^2*e + d))^2)/c^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(e\,x^2+d\right)}^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^4*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

3.372 $\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=491

$$\frac{ex\sqrt{d+ex^2}}{2c}+\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right.}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] 1/2*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^(1/2)/c+1/2*arctanh(x*e^(1/2)/(e
*x^2+d)^(1/2))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/
c^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(
-4*a*c+b^2)^(1/2))*e^(1/2)/c^2+1/2*e*x*(e*x^2+d)^(1/2)/c+1/2*arctan(x*(2*c*
d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1
/2))*(c*d-b*e+(-2*a*c*e+b^2*e-b*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/c^2/(b-(-4*a*c+b^2)^(1/2))^(1/2)+1/2*arctan(x*(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
)*(c*d-b*e+(2*a*c*e-b^2*e+b*c*d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^
2)^(1/2)))^(1/2)/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.20, antiderivative size = 491, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1307, 201, 223, 212, 1706, 399, 385, 211}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2c^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)}{2c^2}+\frac{ex\sqrt{d+ex^2}}{2c}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
```

```
[Out] (e*x*Sqrt[d + e*x^2])/(2*c) + (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d
 - b*e - (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d -
(b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])
])/(2*c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])])/(2*c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqr
t[e]*x)/Sqrt[d + e*x^2]])/(2*c) + (Sqrt[e]*(c*d - b*e - (b*c*d - b^2*e + 2*
a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2) + (
Sqrt[e]*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(
Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c^2)
```

Rule 201

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^p/(n*p
+ 1)), x] + Dist[a*n*(p/(n*p + 1)), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{\sqrt{d+ex^2}\,(ae-(cd-be)x^2)}{a+bx^2+cx^4}\,dx}{c} + \frac{e\int \sqrt{d+ex^2}\,dx}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} - \frac{\int \left(\frac{\left(-cd+be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(-cd+be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\sqrt{d+}}{b+\sqrt{b^2-4ac}+2cx^2}\right.}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{(de)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(cd-be+\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)}{c}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(e\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{\sqrt{d}}}{2c^2}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c} + \frac{\left(e\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}}{2c^2}$$

$$= \frac{ex\sqrt{d+ex^2}}{2c} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(cd-be-\frac{bcd-b^2e+2ace}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{}}\right.}{2c^2\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 14032 vs. $2(491) = 982$.
time = 16.20, size = 14032, normalized size = 28.58

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x]
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 347, normalized size = 0.71

method	result
risch	$\frac{ex\sqrt{e\,x^2+d}}{2c}-\frac{e^{\frac{3}{2}}\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)b}{c^2}+\frac{3\sqrt{e}\,\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)d}{2c}+\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4e\right.}\right.}{}$
default	$-\frac{\sqrt{e}\left(\frac{\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)^2}{2c}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(\left(-ac\right.\right.}{}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*e^(1/2)*(1/2*((e*x^2+d)^(1/2)-e^(1/2)*x)^2/c+2/c^2*sum(((-a*c*e^2+b^2*
e^2-2*b*c*d*e+c^2*d^2)*_R^2+2*(2*a*b*e^3-3*a*c*d*e^2-b^2*d*e^2+2*b*c*d^2*e-
c^2*d^3)*_R-a*c*d^2*e^2+b^2*d^2*e^2-2*b*c*d^3*e+c^2*d^4)/(_R^3*c+3*_R^2*b*e
-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(
1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e
+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))-1/2/c*d^2/((e*x^2+d)^(1/2)-e^
(1/2)*x)^2+1/c^2*(-4*b*e+6*c*d)*ln((e*x^2+d)^(1/2)-e^(1/2)*x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5968 vs. $2(432) = 864$.
time = 52.90, size = 5968, normalized size = 12.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c^2*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3
*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + (b^2*c^4 - 4*a*
c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(1
0*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*
d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c +
4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(b*c^4*
d^6*x^2 - 2*a*c^4*d^6 + 4*(a^2*b^3 - 2*a^3*b*c)*x^2*e^6 + 2*sqrt(1/2)*((b^2
*c^4 - 4*a*c^5)*d^4*x - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*x*e + 3*(2*b^4*c^2 - 9*
a*b^2*c^3 + 4*a^2*c^4)*d^2*x*e^2 - (4*b^5*c - 21*a*b^3*c^2 + 20*a^2*b*c^3)*
d*x*e^3 + (b^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*x*e^4 - ((b^3*c^5 - 4*a*b*c^6)*
d*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*e)*sqrt((c^6*d^6 - 6*b*c^5*d^5*
e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 +
 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2
+ 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*
a*c^9)))*sqrt(x^2*e + d)*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3
*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + (b^2*c^4 -
 4*a*c^5)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 -
 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*
c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4
*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - ((5*a
*b^4 + 2*a^2*b^2*c - 12*a^3*c^2)*d*x^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d)*e^5 + (
(b^5 + 17*a*b^3*c - 11*a^2*b*c^2)*d^2*x^2 - 2*(a*b^4 + a^2*b^2*c - 3*a^3*c^
2)*d^2)*e^4 - 2*((2*b^4*c + 11*a*b^2*c^2 - 4*a^2*c^3)*d^3*x^2 - 2*(2*a*b^3*
c - a^2*b*c^2)*d^3)*e^3 + 2*((3*b^3*c^2 + 7*a*b*c^3)*d^4*x^2 - 2*(3*a*b^2*c
^2 - a^2*c^3)*d^4)*e^2 + 4*(2*a*b*c^3*d^5 - (b^2*c^3 + a*c^4)*d^5*x^2)*e -
((b^2*c^5 - 4*a*c^6)*d^3*x^2 - (b^3*c^4 - 4*a*b*c^5)*d^2*x^2*e + (a*b^2*c^4
 - 4*a^2*c^5)*d*x^2*e^2)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a
*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b
^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 +
 (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/x^2) - sqrt(1
/2)*c^2*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c
^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + (b^2*c^4 - 4*a*c^5)*sqrt((c
^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 -
11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*
(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^
2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(b*c^4*d^6*x^2 - 2*
a*c^4*d^6 + 4*(a^2*b^3 - 2*a^3*b*c)*x^2*e^6 - 2*sqrt(1/2)*((b^2*c^4 - 4*a*c
^5)*d^4*x - 4*(b^3*c^3 - 4*a*b*c^4)*d^3*x*e + 3*(2*b^4*c^2 - 9*a*b^2*c^3 +
4*a^2*c^4)*d^2*x*e^2 - (4*b^5*c - 21*a*b^3*c^2 + 20*a^2*b*c^3)*d*x*e^3 + (b
^6 - 6*a*b^4*c + 8*a^2*b^2*c^2)*x*e^4 - ((b^3*c^5 - 4*a*b*c^6)*d*x - (b^4*c
^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x*e)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2
*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^
2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^
3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))*sqr
t(x^2*e + d)*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*
```

```
a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + (b^2*c^4 - 4*a*c^5)*sq
rt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c
^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4
 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b
^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - ((5*a*b^4 + 2*a^2
*b^2*c - 12*a^3*c^2)*d*x^2 - 2*(a^2*b^3 - 2*a^3*b*c)*d)*e^5 + ((b^5 + 17*a*
b^3*c - 11*a^2*b*c^2)*d^2*x^2 - 2*(a*b^4 + a^2*b^2*c - 3*a^3*c^2)*d^2)*e^4
- 2*((2*b^4*c + 11*a*b^2*c^2 - 4*a^2*c^3)*d^3*x^2 - 2*(2*a*b^3*c - a^2*b*c^
2)*d^3)*e^3 + 2*((3*b^3*c^2 + 7*a*b*c^3)*d^4*x^2 - 2*(3*a*b^2*c^2 - a^2*c^3
)*d^4)*e^2 + 4*(2*a*b*c^3*d^5 - (b^2*c^3 + a*c^4)*d^5*x^2)*e - ((b^2*c^5 -
4*a*c^6)*d^3*x^2 - (b^3*c^4 - 4*a*b*c^5)*d^2*x^2*e + (a*b^2*c^4 - 4*a^2*c^5
)*d*x^2*e^2)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^
2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a
^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*
b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2*c^8 - 4*a*c^9)))/x^2) + sqrt(1/2)*c^2*sqrt
(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 -
(b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 - (b^2*c^4 - 4*a*c^5)*sqrt((c^6*d^6 - 6*b
*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*
d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a
*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/(b^2
*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(b*c^4*d^6*x^2 - 2*a*c^4*d^6 +
4*(a^2*b^3 - 2*a^3*b*c)*x^2*e^6 + 2*sqrt(1/2)*(...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*(d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 4.50, size = 56, normalized size = 0.11

$$\frac{\sqrt{x^2e+d}\,xe}{2\,c} - \frac{\left(3\,cde^{\frac{1}{2}} - 2\,be^{\frac{3}{2}}\right)\log\left(\left(xe^{\frac{1}{2}} - \sqrt{x^2e+d}\right)^2\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2*e + d)*x*e/c - 1/4*(3*c*d*e^(1/2) - 2*b*e^(3/2))*log((x*e^(1/2
) - sqrt(x^2*e + d))^2)/c^2
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2\,{\left(e\,x^2+d\right)}^{3/2}}{c\,x^4+b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4),x)
[Out] int((x^2*(d + e*x^2)^(3/2))/(a + b*x^2 + c*x^4), x)
```

3.373 $\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=487

$$\frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-$$

```
[Out] 1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(3*c*d-e*(b-(-4*a*c+b^2)^(1/2)))*e^(
1/2)/c/(-4*a*c+b^2)^(1/2)-1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(3*c*d-e*(
b+(-4*a*c+b^2)^(1/2)))*e^(1/2)/c/(-4*a*c+b^2)^(1/2)+arctan(x*(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(2*c
^2*d^2+b*e^2*(b-(-4*a*c+b^2)^(1/2))-2*c*e*(b*d+a*e-d*(-4*a*c+b^2)^(1/2)))/c
/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^
(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c^2*d^2+b*e^2*(b+(-4*a*c+b^2)^(1/2))-2*c
*e*(b*d+a*e+d*(-4*a*c+b^2)^(1/2)))/c/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/
2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 1.07, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {1188, 427, 537, 223, 212, 385, 211}

$$\frac{\left(-2ce\left(-d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(b-\sqrt{b^2-4ac}\right)+2c^2d^2\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(-2ce\left(d\sqrt{b^2-4ac}+ae+bd\right)+be^2\left(\sqrt{b^2-4ac}+b\right)+2c^2d^2\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}{2c\sqrt{b^2-4ac}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\left(3cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}{2c\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((2*c^2*d^2 + b*(b - Sqrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d - Sqrt[b^2 - 4*a*c
]*d + a*e))*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sq
rt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((2*c^2*d^2 + b*(b + S
qrt[b^2 - 4*a*c])*e^2 - 2*c*e*(b*d + Sqrt[b^2 - 4*a*c]*d + a*e))*ArcTan[(Sq
rt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[
d + e*x^2])]])/(c*Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e]) + (Sqrt[e]*(3*c*d - (b - Sqrt[b^2 - 4*a*c])*e)
*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*c*Sqrt[b^2 - 4*a*c]) - (Sqrt[e]*(
3*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*Sqrt[b^2 - 4*a*c])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 427

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[d*x*(a + b*x^n)^(p + 1)*((c + d*x^n)^(q - 1)/(b*(n*(p + q) + 1))),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 537

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{(d+ex^2)^{3/2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^2)^{3/2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\int \frac{d\left(4cd-\left(b-\sqrt{b^2-4ac}\right)e\right)+2e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{2\sqrt{b^2-4ac}} - \frac{\int \frac{d\left(4cd-\left(b+\sqrt{b^2-4ac}\right)e \cdots\right.}{\left(b+\sqrt{b^2-4a\cdots}\right.}}{2\sqrt{\cdots}}$$

$$= \frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\int \frac{1}{\sqrt{d+ex^2}}\,dx}{2c\sqrt{b^2-4ac}} - \frac{\left(e\left(3cd-\left(b+\sqrt{b^2-4ac}\right)e\right)\right)\cdots}{2c\sqrt{b^2-4ac}}$$

$$= \frac{\left(e\left(3cd-\left(b-\sqrt{b^2-4ac}\right)e\right)\right)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{2c\sqrt{b^2-4ac}} - \frac{\left(e\left(3cd-\left(b+\cdots\right.\right.\right.}{\cdots}$$

$$= \frac{\left(2c^2d^2+b\left(b-\sqrt{b^2-4ac}\right)e^2-2ce\left(bd-\sqrt{b^2-4ac}\,d+ae\right)\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b\cdots\right.}}{\sqrt{b-\sqrt{b^2\cdots}}}\right.}{c\sqrt{b^2-4ac}\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 9290 vs. $2(487) = 974$.
time = 16.13, size = 9290, normalized size = 19.08

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.13, size = 215, normalized size = 0.44

method	result

default	$-e^{\frac{3}{2}}\left(\frac{\ln\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)}{c}-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left((eb-\right.}{}}{2c}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -e^(3/2)*(1/c*ln((e*x^2+d)^(1/2)-e^(1/2)*x)-1/2/c*sum(((b*e-2*c*d)*_R^2+2*e
*(2*a*e-b*d)*_R+d^2*e*b-2*c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4
*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R
=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*
e-4*c*d^3)*_Z+d^4*c)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3855 vs. $2(423) = 846$.
time = 15.37, size = 3855, normalized size = 7.92

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2
 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 +
 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^
2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log(-(b*c^3*d^6*x^2 - 2*a
*c^3*d^6 - 4*a^3*b*x^2*e^6 + 2*sqrt(1/2)*((a*b^2*c^2 - 4*a^2*c^3)*d^3*x*e -
 3*(a^2*b^2*c - 4*a^3*c^2)*d*x*e^3 + (a^2*b^3 - 4*a^3*b*c)*x*e^4 - (2*(a^2*
b^2*c^3 - 4*a^3*c^4)*d*x - (a^2*b^3*c^2 - 4*a^3*b*c^3)*x*e)*sqrt((c^4*d^6 -
 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5
+ a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))*sqrt(x^2*e + d)*sqrt(-(b*c^2*d^3
```

```
 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a
^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2
*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^
2 - 4*a^2*c^3)) - (2*a^3*b*d - (5*a^2*b^2 + 12*a^3*c)*d*x^2)*e^5 - ((a*b^3
+ 19*a^2*b*c)*d^2*x^2 - 2*(a^2*b^2 + 3*a^3*c)*d^2)*e^4 - 4*(2*a^2*b*c*d^3 -
 (a*b^2*c + 2*a^2*c^2)*d^3*x^2)*e^3 + 2*(a*b*c^2*d^4*x^2 + 2*a^2*c^2*d^4)*e
^2 + (2*a*b*c^2*d^5 - (b^2*c^2 + 4*a*c^3)*d^5*x^2)*e + ((a*b^2*c^3 - 4*a^2*
c^4)*d^3*x^2 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*x^2*e + (a^2*b^2*c^2 - 4*a^3*c
^3)*d*x^2*e^2)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*
c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/x^
2) - sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2
- 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 +
2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2
*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log(-(b*c^3*d^6*x^2 - 2*a*
c^3*d^6 - 4*a^3*b*x^2*e^6 - 2*sqrt(1/2)*((a*b^2*c^2 - 4*a^2*c^3)*d^3*x*e -
3*(a^2*b^2*c - 4*a^3*c^2)*d*x*e^3 + (a^2*b^3 - 4*a^3*b*c)*x*e^4 - (2*(a^2*b
^2*c^3 - 4*a^3*c^4)*d*x - (a^2*b^3*c^2 - 4*a^3*b*c^3)*x*e)*sqrt((c^4*d^6 -
6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 +
 a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))*sqrt(x^2*e + d)*sqrt(-(b*c^2*d^3
- 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + (a*b^2*c^2 - 4*a^
2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*
e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2
 - 4*a^2*c^3)) - (2*a^3*b*d - (5*a^2*b^2 + 12*a^3*c)*d*x^2)*e^5 - ((a*b^3 +
 19*a^2*b*c)*d^2*x^2 - 2*(a^2*b^2 + 3*a^3*c)*d^2)*e^4 - 4*(2*a^2*b*c*d^3 -
(a*b^2*c + 2*a^2*c^2)*d^3*x^2)*e^3 + 2*(a*b*c^2*d^4*x^2 + 2*a^2*c^2*d^4)*e^
2 + (2*a*b*c^2*d^5 - (b^2*c^2 + 4*a*c^3)*d^5*x^2)*e + ((a*b^2*c^3 - 4*a^2*c
^4)*d^3*x^2 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*x^2*e + (a^2*b^2*c^2 - 4*a^3*c^
3)*d*x^2*e^2)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c
^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/x^2
) + sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 -
 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2
*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*
b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log(-(b*c^3*d^6*x^2 - 2*a*c
^3*d^6 - 4*a^3*b*x^2*e^6 + 2*sqrt(1/2)*((a*b^2*c^2 - 4*a^2*c^3)*d^3*x*e - 3
*(a^2*b^2*c - 4*a^3*c^2)*d*x*e^3 + (a^2*b^3 - 4*a^3*b*c)*x*e^4 + (2*(a^2*b^
2*c^3 - 4*a^3*c^4)*d*x - (a^2*b^3*c^2 - 4*a^3*b*c^3)*x*e)*sqrt((c^4*d^6 - 6
*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 +
a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))*sqrt(x^2*e + d)*sqrt(-(b*c^2*d^3 -
 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2
*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e
^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2
- 4*a^2*c^3)) - (2*a^3*b*d - (5*a^2*b^2 + 12*a^3*c)*d*x^2)*e^5 - ((a*b^3 +
19*a^2*b*c)*d^2*x^2 - 2*(a^2*b^2 + 3*a^3*c)*d^2)*e^4 - 4*(2*a^2*b*c*d^3 - (
a*b^2*c + 2*a^2*c^2)*d^3*x^2)*e^3 + 2*(a*b*c^2*d^4*x^2 + 2*a^2*c^2*d^4)*e^2
 + (2*a*b*c^2*d^5 - (b^2*c^2 + 4*a*c^3)*d^5*x^2)*e - ((a*b^2*c^3 - 4*a^2*c^
```

```
4)*d^3*x^2 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^2*x^2*e + (a^2*b^2*c^2 - 4*a^3*c^3
)*d*x^2*e^2)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^
2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))/x^2)
 - sqrt(1/2)*c*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 -
2*a^2*c)*e^3 - (a*b^2*c^2 - 4*a^2*c^3)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*
a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/(a^2*b
^2*c^4 - 4*a^3*c^5)))/(a*b^2*c^2 - 4*a^2*c^3))*log(-(b*c^3*d^6*x^2 - 2*a*c^
3*d^6 - 4*a^3*b*x^2*e^6 - 2*sqrt(1/2)*((a*b^2*c^2 - 4*a^2*c^3)*d^3*x*e - 3*
(a^2*b^2*c - 4*a^3*c^2)*d*x*e^3 + (a^2*b^3 - 4*a^3*b*c)*x*e^4 + (2*(a^2*b^2
*c^3 - 4*a^3*c^4)*d*x - (a^2*b^3*c^2 - 4*a^3*b*c^3)*x*e)*sqrt((c^4*d^6 - 6*
a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a
^2*b^2*e^6)/(a^2*b^2*c^4 - 4*a^3*c^5)))*sqrt(x^2*e + d)*sqrt(-(b*c^2*d^3 -
6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{\frac{3}{2}}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral((d + e*x**2)**(3/2)/(a + b*x**2 + c*x**4), x)
```

Giac [A]
time = 7.77, size = 27, normalized size = 0.06

$$-\frac{e^{\frac{3}{2}}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] -1/2*e^(3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4),x)
[Out] int((d + e*x^2)^(3/2)/(a + b*x^2 + c*x^4), x)
```

3.374 $\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=260

$$-\frac{d\sqrt{d+ex^2}}{ax}-\frac{\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)^{3/2}}+\frac{\left(2cd-\left(b+\sqrt{b^2\right.\right.}{}$$

```
[Out] -arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^
(1/2))^(3/2)/(-4*a*c+b^2)^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)
^(1/2)))^(3/2)/(-4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(3/2)-d*(e*x^2+d)^
(1/2)/a/x
```

Rubi [A]

time = 0.58, antiderivative size = 432, normalized size of antiderivative = 1.66, number of steps used = 16, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.276, Rules used = {1309, 283, 223, 212, 1706, 399, 385, 211}

$$-\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{2a\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{2a\sqrt{\sqrt{b^2-4ac}+b}}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)}{2a}-\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)}{2a}-\frac{d\sqrt{d+ex^2}}{ax}+\frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d*Sqrt[d + e*x^2])/(a*x)) - (Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(d
+ (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b - Sq
rt[b^2 - 4*a*c]]) - (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(d - (b*d - 2*
a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a*Sqrt[b + Sqrt[b^2 - 4*
a*c]]) + (d*Sqrt[e]*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/a - (Sqrt[e]*(d -
 (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*
a) - (Sqrt[e]*(d + (b*d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]])/(2*a)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}}{x^2(a+bx^2+cx^4)}\,dx = -\frac{\int \frac{(bd-ae+cdx^2)\sqrt{d+ex^2}}{a+bx^2+cx^4}\,dx}{a} + \frac{d\int \frac{\sqrt{d+ex^2}}{x^2}\,dx}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} - \frac{\int \left(\frac{\left(cd+\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(cd-\frac{c(bd-2ae)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b+\sqrt{b^2-4ac}+2cx^2}\right.}{a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} + \frac{(de)\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right)}{a} - \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right.}{}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a} - \frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{d+}}}{2a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} + \frac{d\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a} - \frac{\left(e\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int\right.}{2a}$$

$$= -\frac{d\sqrt{d+ex^2}}{ax} - \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(d+\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd}}{\sqrt{b-}}\right.}{2a\sqrt{b-\sqrt{b^2-4ac}}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 7789 vs. $2(260) = 520$.
time = 16.26, size = 7789, normalized size = 29.96

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

Maple [**C**] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 351, normalized size = 1.35

method	result

risch	$-\frac{d\sqrt{e\,x^2+d}}{ax}-\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(\left(a\,e^2-c\,d^2\right)\_R \cdots\right.}{c\_ \cdots}\right.}{2a}$
default	$\frac{\sqrt{e}\left(\frac{\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)^2}{2}-2\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(\left(a\,e^2 \cdots\right.\right.}{\cdots}\right.\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a*e^(1/2)*(1/2*((e*x^2+d)^(1/2)-e^(1/2)*x)^2-2*sum(((a*e^2-c*d^2)*_R^2+
2*d*(3*a*e^2-2*b*d*e+c*d^2)*_R+d^2*e^2*a-d^4*c)/(_R^3*c+3*_R^2*b*e-3*_R^2*c
*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1
/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)
*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))-1/2*d^2/((e*x^2+d)^(1/2)-e^(1/2)*x)^2+
6*d*ln((e*x^2+d)^(1/2)-e^(1/2)*x))+1/a*(-1/d/x*(e*x^2+d)^(5/2)+4*e/d*(1/4*x
*(e*x^2+d)^(3/2)+3/4*d*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(e^(1/2)*x+(e
*x^2+d)^(1/2)))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4096 vs. 2(225) = 450.
time = 8.47, size = 4096, normalized size = 15.75

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 + (b^3 - 3*a*b*c)*d^3 - 2*a^3*e^3 -
 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 -
```

```
(b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e
 - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*
c))*log(-((b^3*c - a*b*c^2)*d^6*x^2 - 12*a^4*d*x^2*e^5 - 2*(a*b^2*c - a^2*c
^2)*d^6 + 2*sqrt(1/2)*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d^4*x - 3*(a^2*b^3
 - 4*a^3*b*c)*d^3*x*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*x*e^2 - ((a^4*b^3 - 4*a^5
*b*c)*d*x - 2*(a^5*b^2 - 4*a^6*c)*x*e)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a
*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^
2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(x^2*e + d)*sqrt(-(3*a^
2*b*d*e^2 + (b^3 - 3*a*b*c)*d^3 - 2*a^3*e^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + (
a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^
6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4
*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) + 3*(9*a^3*b*d^2*x^2 - 2*a
^4*d^2)*e^4 + 2*(6*a^3*b*d^3 - (11*a^2*b^2 + 4*a^3*c)*d^3*x^2)*e^3 + 2*((4*
a*b^3 + 5*a^2*b*c)*d^4*x^2 - 2*(2*a^2*b^2 + a^3*c)*d^4)*e^2 - ((b^4 + 6*a*b
^2*c - 4*a^2*c^2)*d^5*x^2 - 2*(a*b^3 + 2*a^2*b*c)*d^5)*e + ((a^3*b^2*c - 4*
a^4*c^2)*d^3*x^2 - (a^3*b^3 - 4*a^4*b*c)*d^2*x^2*e + (a^4*b^2 - 4*a^5*c)*d*
x^2*e^2)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*
d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6
*b^2 - 4*a^7*c)))/x^2) - sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d*e^2 + (b^3 - 3*a*b*
c)*d^3 - 2*a^3*e^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-
(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*
b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)
))/(a^3*b^2 - 4*a^4*c))*log(-((b^3*c - a*b*c^2)*d^6*x^2 - 12*a^4*d*x^2*e^5
 - 2*(a*b^2*c - a^2*c^2)*d^6 - 2*sqrt(1/2)*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2
)*d^4*x - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*x*e + 3*(a^3*b^2 - 4*a^4*c)*d^2*x*e^2
 - ((a^4*b^3 - 4*a^5*b*c)*d*x - 2*(a^5*b^2 - 4*a^6*c)*x*e)*sqrt(-(18*a^3*b*
d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*
b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))*sqrt(x^
2*e + d)*sqrt(-(3*a^2*b*d*e^2 + (b^3 - 3*a*b*c)*d^3 - 2*a^3*e^3 - 3*(a*b^2
 - 2*a^2*c)*d^2*e + (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a
*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^
2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c)) + 3*(9
*a^3*b*d^2*x^2 - 2*a^4*d^2)*e^4 + 2*(6*a^3*b*d^3 - (11*a^2*b^2 + 4*a^3*c)*d
^3*x^2)*e^3 + 2*((4*a*b^3 + 5*a^2*b*c)*d^4*x^2 - 2*(2*a^2*b^2 + a^3*c)*d^4)
*e^2 - ((b^4 + 6*a*b^2*c - 4*a^2*c^2)*d^5*x^2 - 2*(a*b^3 + 2*a^2*b*c)*d^5)*
e + ((a^3*b^2*c - 4*a^4*c^2)*d^3*x^2 - (a^3*b^3 - 4*a^4*b*c)*d^2*x^2*e + (a
^4*b^2 - 4*a^5*c)*d*x^2*e^2)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*c + a
^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*
a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/x^2) + sqrt(1/2)*a*x*sqrt(-(3*a^2*b*d
*e^2 + (b^3 - 3*a*b*c)*d^3 - 2*a^3*e^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - (a^3*b
^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9
*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)
/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-((b^3*c - a*b*c^2)*d^6*x^2
 - 12*a^4*d*x^2*e^5 - 2*(a*b^2*c - a^2*c^2)*d^6 + 2*sqrt(1/2)*((a*b^4 - 5*a
^2*b^2*c + 4*a^3*c^2)*d^4*x - 3*(a^2*b^3 - 4*a^3*b*c)*d^3*x*e + 3*(a^3*b^2
```

```
- 4*a^4*c)*d^2*x*e^2 + ((a^4*b^3 - 4*a^5*b*c)*d*x - 2*(a^5*b^2 - 4*a^6*c)*x
*e)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e
^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2
- 4*a^7*c)))*sqrt(x^2*e + d)*sqrt(-(3*a^2*b*d*e^2 + (b^3 - 3*a*b*c)*d^3 - 2
*a^3*e^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*
d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*
b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^
2 - 4*a^4*c)) + 3*(9*a^3*b*d^2*x^2 - 2*a^4*d^2)*e^4 + 2*(6*a^3*b*d^3 - (11*
a^2*b^2 + 4*a^3*c)*d^3*x^2)*e^3 + 2*((4*a*b^3 + 5*a^2*b*c)*d^4*x^2 - 2*(2*a
^2*b^2 + a^3*c)*d^4)*e^2 - ((b^4 + 6*a*b^2*c - 4*a^2*c^2)*d^5*x^2 - 2*(a*b^
3 + 2*a^2*b*c)*d^5)*e - ((a^3*b^2*c - 4*a^4*c^2)*d^3*x^2 - (a^3*b^3 - 4*a^4
*b*c)*d^2*x^2*e + (a^4*b^2 - 4*a^5*c)*d*x^2*e^2)*sqrt(-(18*a^3*b*d^3*e^3 -
(b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e
 - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/x^2) - sqrt(1/2)*
a*x*sqrt(-(3*a^2*b*d*e^2 + (b^3 - 3*a*b*c)*d^3 - 2*a^3*e^3 - 3*(a*b^2 - 2*a
^2*c)*d^2*e - (a^3*b^2 - 4*a^4*c)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*
c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2
 - 2*a^3*c)*d^4*e^2)/(a^6*b^2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log(-((b^3*
c - a*b*c^2)*d^6*x^2 - 12*a^4*d*x^2*e^5 - 2*(a*b^2*c - a^2*c^2)*d^6 - 2*sqr
t(1/2)*((a*b^4 - 5*a^2*b^2*c + 4*a^3*c^2)*d^4*x...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^{\frac{3}{2}}}{x^2\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**(3/2)/(x**2*(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^(3/2)/(x^2*(a + b*x^2 + c*x^4)), x)
```

3.375 $\int \frac{\left(d+ex^2\right)^{3/2}}{x^4\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=523

$$\frac{(bd-ae)\sqrt{d+ex^2}}{a^2x}-\frac{\left(d+ex^2\right)^{3/2}}{3ax^3}+\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\left(bd-ae+\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd}}{\sqrt{b-}}\right.}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}$$

```
[Out] -1/3*(e*x^2+d)^(3/2)/a/x^3-(-a*e+b*d)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*e^
(1/2)/a^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^
2*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/a^2+1/2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))
*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))*e^(1/2)/a^2+(-a*e+b*d)
*(e*x^2+d)^(1/2)/a^2/x+1/2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/
(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(-a*b*e-2*a*c*d+b^2*
d)/(-4*a*c+b^2)^(1/2))*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(b-(-4*a*
c+b^2)^(1/2))^(1/2)+1/2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*
x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+2*a*c*d-b^2*d)/(
-4*a*c+b^2)^(1/2))*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/a^2/(b+(-4*a*c+b^
2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.80, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 19, number of rules used = 10, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.345$, Rules used = {1309, 270, 6860, 283, 223, 212, 1706, 399, 385, 211}

$$\frac{\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{b-\sqrt{b^2-4ac}}}+\frac{\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{2a^2\sqrt{\sqrt{b^2-4ac}+b}}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}+\frac{\sqrt{e}\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)}{2a^2}+\frac{\sqrt{d+ex^2}(bd-ae)}{a^2x}-\frac{\sqrt{e}(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}x}{\sqrt{d+ex^2}}\right)}{a^2}-\frac{\left(d+ex^2\right)^{3/2}}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((b*d - a*e)*Sqrt[d + e*x^2])/(a^2*x) - (d + e*x^2)^(3/2)/(3*a*x^3) + (Sqrt
[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b - Sqrt[b^2 - 4*a*c
]]) + (Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(b*d - a*e - (b^2*d - 2*a*c*
d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(2*a^2*Sqrt[b + Sqrt[
b^2 - 4*a*c]]) - (Sqrt[e]*(b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])
/a^2 + (Sqrt[e]*(b*d - a*e - (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*A
rcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2*a^2) + (Sqrt[e]*(b*d - a*e + (b^2*d
```

```
 - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]
)/(2*a^2)
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 283

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + 1))), x] - Dist[b*n*(p/(c^n*(m + 1))), In
t[(c*x)^(m + n)*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && IGtQ[
n, 0] && GtQ[p, 0] && LtQ[m, -1] &&  !ILtQ[(m + n*p + n + 1)/n, 0] && IntBi
nomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{(d+ex^2)^{3/2}}{x^4(a+bx^2+cx^4)}\,dx = -\frac{\int \frac{(bd-ae+cdx^2)\sqrt{d+ex^2}}{x^2(a+bx^2+cx^4)}\,dx}{a} + \frac{d\int \frac{\sqrt{d+ex^2}}{x^4}\,dx}{a}$$

$$= -\frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\int \left(\frac{(bd-ae)\sqrt{d+ex^2}}{ax^2} + \frac{\sqrt{d+ex^2}\,(-b^2d+acd+abe-c(bd-ae)x^2)}{a(a+bx^2+cx^4)}\right)dx}{a}$$

$$= -\frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\int \frac{\sqrt{d+ex^2}\,(-b^2d+acd+abe-c(bd-ae)x^2)}{a+bx^2+cx^4}\,dx}{a^2} - \frac{(bd-ae)\int \frac{\sqrt{d+ex^2}}{x^2}}{a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\int \left(\frac{\left(-c(bd-ae)-\frac{c(b^2d-2acd-abe)}{\sqrt{b^2-4ac}}\right)\sqrt{d+ex^2}}{b-\sqrt{b^2-4ac}+2cx^2} + \right.}{a}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{(e(bd-ae))\text{Subst}\left(\int \frac{1}{1-ex^2}\,dx, x, \frac{x}{\sqrt{d+ex}}\right.}{a^2}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\sqrt{e}\,(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{(e}{}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} - \frac{\sqrt{e}\,(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{a^2} + \frac{(e}{}$$

$$= \frac{(bd-ae)\sqrt{d+ex^2}}{a^2x} - \frac{(d+ex^2)^{3/2}}{3ax^3} + \frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,\left(bd-ae+\right.}{2a^2\sqrt{}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 9321 vs. $2(523) = 1046$.

time = 16.31, size = 9321, normalized size = 17.82

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x]
[Out] Result too large to show
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 478, normalized size = 0.91

method	result
risch	$-\frac{\sqrt{e\,x^{2}+d}\,\left(4ae\,x^{2}-3bd\,x^{2}+ad\right)}{3a^{2}x^{3}}+\frac{\sqrt{e}\,\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(4eb-4cd)\_Z^{3}+\left(16a\,e^{2}-8deb+6c\,d^{2}\right)\_Z^{2}+\left(4d^{2}eb-4c\,d^{3}\right)\_Z+d^{4}\right.}\right.}{}$
default	$\frac{\sqrt{e}\,\left(-\frac{b\left(\sqrt{e\,x^{2}+d}-\sqrt{e}\,x\right)^{2}}{2}-2\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(4eb-4cd)\_Z^{3}+\left(16a\,e^{2}-8deb+6c\,d^{2}\right)\_Z^{2}+\left(4d^{2}eb-4c\,d^{3}\right)\_Z+d^{4}c\right)}\frac{\left(cd\right.}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/a^2*e^(1/2)*(-1/2*b*((e*x^2+d)^(1/2)-e^(1/2)*x)^2-2*sum((c*d*(-2*a*e+b*
d)*_R^2+2*(2*a^2*e^3-4*a*b*d*e^2+2*b^2*d^2*e-b*c*d^3)*_R-2*a*c*d^3*e+c*d^4*
b)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c
*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_
Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))+(4*a*e-6
*b*d)*ln((e*x^2+d)^(1/2)-e^(1/2)*x)+1/2*b*d^2/((e*x^2+d)^(1/2)-e^(1/2)*x)^2
)-b/a^2*(-1/d/x*(e*x^2+d)^(5/2)+4*e/d*(1/4*x*(e*x^2+d)^(3/2)+3/4*d*(1/2*x*(
e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2)))))+1/a*(-1/3/d/x
^3*(e*x^2+d)^(5/2)+2/3*e/d*(-1/d/x*(e*x^2+d)^(5/2)+4*e/d*(1/4*x*(e*x^2+d)^(
3/2)+3/4*d*(1/2*x*(e*x^2+d)^(1/2)+1/2*d/e^(1/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2
))))))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^(3/2)/((c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7833 vs. $2(462) = 924$.
time = 32.03, size = 7833, normalized size = 14.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(
a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a
^3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^
5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20
*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b
*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b
^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c))*log((4*a^
5*b*c*x^2*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6*x^2 - 2*(a*b^4*c^2
- 3*a^2*b^2*c^3 + a^3*c^4)*d^6 + 2*sqrt(1/2)*((a*b^7 - 7*a^2*b^5*c + 13*a^3
*b^3*c^2 - 4*a^4*b*c^3)*d^4*x - (4*a^2*b^6 - 25*a^3*b^4*c + 37*a^4*b^2*c^2
- 4*a^5*c^3)*d^3*x*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*c^2)*d^2*x*e^
2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*x*e^3 + (a^5*b^3 - 4*a^6*b*c)
*x*e^4 - ((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d*x - (a^7*b^3 - 4*a^8*b*c)*x
*e)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 +
 a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e
 + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(1
0*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^
2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*
c)))*sqrt(x^2*e + d)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^3 - 3*(a*b^4
- 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e^2 - (a^3*b^2
 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c
+ 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c +
7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b
^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*
d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a
^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)) + (2*a^5*b*c*d
- (17*a^4*b^2*c - 12*a^5*c^2)*d*x^2)*e^5 + (7*(4*a^3*b^3*c - 5*a^4*b*c^2)*d
^2*x^2 - 2*(4*a^4*b^2*c - 3*a^5*c^2)*d^2)*e^4 - 2*((11*a^2*b^4*c - 16*a^3*b
^2*c^2 - 4*a^4*c^3)*d^3*x^2 - 2*(3*a^3*b^3*c - 4*a^4*b*c^2)*d^3)*e^3 + 2*((
4*a*b^5*c - 3*a^2*b^3*c^2 - 11*a^3*b*c^3)*d^4*x^2 - 2*(2*a^2*b^4*c - 3*a^3*
b^2*c^2 - a^4*c^3)*d^4)*e^2 - ((b^6*c + 4*a*b^4*c^2 - 17*a^2*b^2*c^3 + 4*a^
3*c^4)*d^5*x^2 - 2*(a*b^5*c - 5*a^3*b*c^3)*d^5)*e + ((a^5*b^2*c^2 - 4*a^6*c
^3)*d^3*x^2 - (a^5*b^3*c - 4*a^6*b*c^2)*d^2*x^2*e + (a^6*b^2*c - 4*a^7*c^2)
*d*x^2*e^2)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b
^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^
```

```
3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^
2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 1
0*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 -
 4*a^11*c)))/x^2) - 3*sqrt(1/2)*a^2*x^3*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c
^2)*d^3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*
c)*d*e^2 - (a^3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6
+ (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b
^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a
^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3
*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e
^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*
c))*log((4*a^5*b*c*x^2*e^6 + (b^5*c^2 - 3*a*b^3*c^3 + a^2*b*c^4)*d^6*x^2 -
2*(a*b^4*c^2 - 3*a^2*b^2*c^3 + a^3*c^4)*d^6 - 2*sqrt(1/2)*((a*b^7 - 7*a^2*b
^5*c + 13*a^3*b^3*c^2 - 4*a^4*b*c^3)*d^4*x - (4*a^2*b^6 - 25*a^3*b^4*c + 37
*a^4*b^2*c^2 - 4*a^5*c^3)*d^3*x*e + 3*(2*a^3*b^5 - 11*a^4*b^3*c + 12*a^5*b*
c^2)*d^2*x*e^2 - (4*a^4*b^4 - 19*a^5*b^2*c + 12*a^6*c^2)*d*x*e^3 + (a^5*b^3
 - 4*a^6*b*c)*x*e^4 - ((a^6*b^4 - 6*a^7*b^2*c + 8*a^8*c^2)*d*x - (a^7*b^3 -
 4*a^8*b*c)*x*e)*sqrt((a^6*b^2*e^6 + (b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*
a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4
*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d
^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 19*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^
4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*
b^2 - 4*a^11*c)))*sqrt(x^2*e + d)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d^
3 - 3*(a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*d^2*e + 3*(a^2*b^3 - 3*a^3*b*c)*d*e
^2 - (a^3*b^2 - 2*a^4*c)*e^3 + (a^5*b^2 - 4*a^6*c)*sqrt((a^6*b^2*e^6 + (b^8
 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^6 - 6*(a*b^7 - 5
*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d^5*e + 3*(5*a^2*b^6 - 20*a^3*b^4
*c + 20*a^4*b^2*c^2 - 2*a^5*c^3)*d^4*e^2 - 2*(10*a^3*b^5 - 30*a^4*b^3*c + 1
9*a^5*b*c^2)*d^3*e^3 + 3*(5*a^4*b^4 - 10*a^5*b^2*c + 3*a^6*c^2)*d^2*e^4 - 6
*(a^5*b^3 - a^6*b*c)*d*e^5)/(a^10*b^2 - 4*a^11*c)))/(a^5*b^2 - 4*a^6*c)) +
(2*a^5*b*c*d - (17*a^4*b^2*c - 12*a^5*c^2)*d*x^...
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**(3/2)/x**4/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^(3/2)/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\left(e\,x^2+d\right)}^{3/2}}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int((d + e*x^2)^(3/2)/(x^4*(a + b*x^2 + c*x^4)), x)
```

3.376 $\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=281

$$-\frac{b\sqrt{1-x^2}}{c^2}-\frac{(1-x^2)^{3/2}}{3c}+\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{\left(b^2-c\right.}{}$$

```
[Out] -1/3*(-x^2+1)^(3/2)/c-b*(-x^2+1)^(1/2)/c^2+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^
2+1)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(3*a*b*c+2*a*c^2-
b^3-b^2*c)/(-4*a*c+b^2)^(1/2))/c^(5/2)*2^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(
1/2)+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^
(1/2))*(b^2-a*c+b*c+(-3*a*b*c-2*a*c^2+b^3+b^2*c)/(-4*a*c+b^2)^(1/2))/c^(5/2
)*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 4.88, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 911, 1301, 1180, 214}

$$\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,c^{5/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{b\sqrt{1-x^2}}{c^2}-\frac{(1-x^2)^{3/2}}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((b*Sqrt[1 - x^2])/c^2) - (1 - x^2)^(3/2)/(3*c) + ((b^2 - a*c + b*c - (b^3
 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*S
qrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b +
 2*c - Sqrt[b^2 - 4*a*c]]) + ((b^2 - a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2
*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b +
 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(5/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*
c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
```

```
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^5\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}\,x^2}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)\\
&= -\text{Subst}\left(\int \left(\frac{b}{c^2}+\frac{x^2}{c}-\frac{b(a+b+c)-\left(b^2-ac+bc\right)x^2}{c^2\left(a+b+c+(-b-2c)x^2+cx^4\right)}\right)dx, x, \sqrt{1-x^2}\right)\\
&= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}+\frac{\text{Subst}\left(\int \frac{b(a+b+c)+\left(-b^2+ac-bc\right)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{c^2}\\
&= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}-\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{\frac{1}{2}(-b-2c)+\frac{1}{2}}\right.}{2c^2}\\
&= -\frac{b\sqrt{1-x^2}}{c^2}-\frac{\left(1-x^2\right)^{3/2}}{3c}+\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}}{\sqrt{b+2c-}}\right.}{\sqrt{2}\,c^{5/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}
\end{aligned}$$

Mathematica [A]
time = 0.85, size = 346, normalized size = 1.23

$$\frac{2\sqrt{c}\sqrt{1-x^2}\left(-3b+c\left(-1+x^2\right)\right)-\frac{3\sqrt{2}\left(b^3+bc\left(-3a+\sqrt{b^2-4ac}\right)+b^2\left(c+\sqrt{b^2-4ac}\right)-ac\left(2c+\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}}-\frac{3\sqrt{2}\left(-b^3+ac\left(2c-\sqrt{b^2-4ac}\right)+bc\left(3a+\sqrt{b^2-4ac}\right)+b^2\left(-c+\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c+\sqrt{b^2-4ac}}}}{6c^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*Sqrt[c]*Sqrt[1 - x^2]*(-3*b + c*(-1 + x^2)) - (3*Sqrt[2]*(b^3 + b*c*(-3*
a + Sqrt[b^2 - 4*a*c]) + b^2*(c + Sqrt[b^2 - 4*a*c]) - a*c*(2*c + Sqrt[b^2
- 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2
- 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) - (3*Sqr
t[2]*(-b^3 + a*c*(2*c - Sqrt[b^2 - 4*a*c]) + b*c*(3*a + Sqrt[b^2 - 4*a*c])
+ b^2*(-c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt
[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^
2 - 4*a*c]]))/(6*c^(5/2))
```

Maple [A]
time = 0.16, size = 463, normalized size = 1.65

method	result

	$2a\left(\frac{\left(-2ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+\sqrt{-4ac+b^2}\,bc+4abc+4c^2a-b^3-b^2c\right)\arctan\left(\frac{2}{2\sqrt{4ac-2}}\right.}{\left(8ac-2b^2\right)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-}}}\right.$
default	$-\frac{\left(-x^2+1\right)^{\frac{3}{2}}}{3c}-\frac{}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/3*(-x^2+1)^(3/2)/c-1/c*(2/c*a*((-2*a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*a*c+b^
2)^(1/2)+(-4*a*c+b^2)^(1/2)*b*c+4*a*b*c+4*c^2*a-b^3-b^2*c)/(8*a*c-2*b^2)/(4
*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arcta
n(1/2*(2*a*((-x^2+1)^(1/2)-1)^2/x^2+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*
b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))-(2*a*c*(-4*
a*c+b^2)^(1/2)-b^2*(-4*a*c+b^2)^(1/2)-(-4*a*c+b^2)^(1/2)*b*c+4*a*b*c+4*c^2*
a-b^3-b^2*c)/(8*a*c-2*b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+
b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*a*((-x^2+1)^(1/2)-1)^2/x^2+2*(-4*a*c
+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(
1/2)-2*a*b)^(1/2)))+2*b/c/(((-x^2+1)^(1/2)-1)^2/x^2+1))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^5/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3615 vs. $2(237) = 474$.
time = 3.73, size = 3615, normalized size = 12.86

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] -1/6*(3*sqrt(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*
a*b^3 - b^4)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2
)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 +
b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)
)*log(-(2*a^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*
b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4
- 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^
5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^
4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 +
 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b
^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*
b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a
^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*
x^2)*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c -
(b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*
b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a
*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 +
(a^5 - 2*a^4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqr
t(1/2)*c^2*sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4
)*c - (b^2*c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(
3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 -
2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a
^3*b^4 + (a^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b
^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5
+ b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b
)*c^2 + (a^2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 -
 2*(3*a^4*b^2 - a^3*b^3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c
^7)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*
b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*
c)/(b^2*c^10 - 4*a*c^11)) + (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14
*a^2*b^3)*c^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((
b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c - (b^2*c^5 -
4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2
*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)
*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^
4*b)*c^2 - (3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) - 3*sqrt(1/2)*c^2*
sqrt((b^5 + 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*
c^5 - 4*a*c^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 -
 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6
- b^7)*c)/(b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a
^2*b^2*c^5 - 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2
*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2
- 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^
```

```
2*b^5 + (a^4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b
^2 - a^3*b^3)*c + sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqr
t((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b
^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^1
0 - 4*a*c^11)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c
^3 + (20*a^2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*sqrt((b^5 + 2*a^2
*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c^6)*sq
rt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*
b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^
10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6)) - 2*(a^3*b^4 + (a^5 - 2*a^4*b)*c^2 -
(3*a^4*b^2 - a^3*b^3)*c)*sqrt(-x^2 + 1))/x^2) + 3*sqrt(1/2)*c^2*sqrt((b^5 +
 2*a^2*c^3 + (5*a^2*b - 4*a*b^2)*c^2 - (5*a*b^3 - b^4)*c + (b^2*c^5 - 4*a*c
^6)*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3
+ 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(
b^2*c^10 - 4*a*c^11)))/(b^2*c^5 - 4*a*c^6))*log(-(2*a^3*b^4 - (a^2*b^2*c^5
- 4*a^3*c^6)*x^2*sqrt((b^8 + (a^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2
 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^
6 - b^7)*c)/(b^2*c^10 - 4*a*c^11)) + 2*(a^5 - 2*a^4*b)*c^2 + (a^2*b^5 + (a^
4*b - 2*a^3*b^2)*c^2 - (3*a^3*b^3 - a^2*b^4)*c)*x^2 - 2*(3*a^4*b^2 - a^3*b^
3)*c - sqrt(1/2)*((b^5*c^5 - 7*a*b^3*c^6 + 12*a^2*b*c^7)*x^2*sqrt((b^8 + (a
^4 - 4*a^3*b + 4*a^2*b^2)*c^4 - 2*(3*a^3*b^2 - 7*a^2*b^3 + 2*a*b^4)*c^3 + (
11*a^2*b^4 - 10*a*b^5 + b^6)*c^2 - 2*(3*a*b^6 - b^7)*c)/(b^2*c^10 - 4*a*c^1
1)) - (b^8 + 4*(a^4 - 2*a^3*b)*c^4 - (17*a^3*b^2 - 14*a^2*b^3)*c^3 + (20*a^
2*b^4 - 7*a*b^5)*c^2 - (8*a*b^6 - b^7)*c)*x^2)*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^5\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**5*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4637 vs. $2(237) = 474$.
time = 7.63, size = 4637, normalized size = 16.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

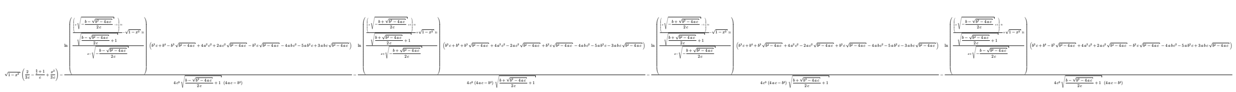
```
[In] integrate(x^5*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^5*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6
+ 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2*c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2 -
```

4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 43*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^6 + 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^6 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^7 - 2*(b^2 - 4*a*c)*b^4*c^4 + 6*(b^2 - 4*a*c)*a*b^2*c^5 - 6*(b^2 - 4*a*c)*b^3*c^5 + 16*(b^2 - 4*a*c)*a*b*c^6 - 4*(b^2 - 4*a*c)*b^2*c^6 + 8*(b^2 - 4*a*c)*a*c^7 - (2*b^6*c^2 - 18*a*b^4*c^3 + 2*b^5*c^3 + 48*a^2*b^2*c^4 - 16*a*b^3*c^4 - 32*a^3*c^5 + 32*a^2*b*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^6 + 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 18*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 24*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 + 33*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^3 - 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^4 - 2*(b^2 - 4*a*c)*b^4*c^2 + 10*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(b^2 - 4*a*c)*b^3*c^3 - 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4)*c^2 - 2*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^5*c^2 + sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^6*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c^3 - 6*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^3 + 2*a*b^5*c^3 + 2*b^6*c^3 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 16*a^2*b^3*c^4 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 16*a*b^4*c^4 + 2*b^5*c^4 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 32*a^3*b*c^5 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 + 32*a^2*b^2*c^5 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*a*b^3*c^5 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^6 + 32*a^2*b*c^6 - 2*(b^2 - 4*a*c)*a*b^3*c^3 - 2*(b^2 - 4*a*c)*b^4*

```
c^3 + 8*(b^2 - 4*a*c)*a^2*b*c^4 + 8*(b^2 - 4*a*c)*a*b^2*c^4 - 2*(b^2 - 4*a*
c)*b^3*c^4 + 8*(b^2 - 4*a*c)*a*b*c^5)*abs(c))*arctan(2*sqrt(1/2)*sqrt(-x^2
+ 1)/sqrt(-(b*c^3 + 2*c^4 + sqrt(-4*(a*c^3 + b*c^3 + c^4)*c^4 + (b*c^3 + 2*
c^4)^2))/c^4))/((a*b^4*c^4 + b^5*c^4 - 8*a^2*b^2*c^5 - 6*a*b^3*c^5 + 3*b^4*
c^5 + 16*a^3*c^6 + 8*a^2*b*c^6 - 11*a*b^2*c^6 + 7*b^3*c^6 - 4*a^2*c^7 - 28*
a*b*c^7 + 5*b^2*c^7 - 20*a*c^8)*c^2) + 1/8*(2*b^6*c^4 - 14*a*b^4*c^5 + 6*b^
5*c^5 + 24*a^2*b^2*c^6 - 40*a*b^3*c^6 + 4*b^4*c^6 + 64*a^2*b*c^7 - 24*a*b^2
*c^7 + 32*a^2*c^8 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2
- 4*a*c)*c)*b^6*c^2 + 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(
b^2 - 4*a*c)*c)*a*b^4*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 +
 sqrt(b^2 - 4*a*c)*c)*b^5*c^3 - 12*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*
c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^4 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(
-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^4 - 32*sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^5 + 43*sqrt(2)*sqrt
(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^5 - 19*sqrt(
2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^5 - 16*
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^6
+ 48*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b
*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -...
```

Mupad [B]
time = 1.45, size = 917, normalized size = 3.26



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (1 - x^2)^(1/2)*(2/(3*c) - (b/c + 1)/c + x^2/(3*c)) - (log((((x*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1
)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
))*(b^3*c + b^4 - b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 + 2*a*c^2*(b^2 - 4*a*
c)^(1/2) - b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c + 3*a*b*c*(b^2
 - 4*a*c)^(1/2)))/(4*c^3*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c
 - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b
+ (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 + b^3*(b^2 - 4*a*c)^(1/2) +
 4*a^2*c^2 - 2*a*c^2*(b^2 - 4*a*c)^(1/2) + b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*
b*c^2 - 5*a*b^2*c - 3*a*b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^3*(4*a*c - b^2)*((b
+ (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (
1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*c +
 b^4 + b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2 - 2*a*c^2*(b^2 - 4*a*c)^(1/2) +
b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a*b^2*c - 3*a*b*c*(b^2 - 4*a*c)^(
```

```
1/2)))/(4*c^3*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) -
(log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^3*c + b^4 - b^3*(b^2 - 4*a*c)^(1/2) + 4*a^2*c^2
+ 2*a*c^2*(b^2 - 4*a*c)^(1/2) - b^2*c*(b^2 - 4*a*c)^(1/2) - 4*a*b*c^2 - 5*a
*b^2*c + 3*a*b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^3*((b - (b^2 - 4*a*c)^(1/2))/(2
*c) + 1)^(1/2)*(4*a*c - b^2))
```

3.377 $\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=229

$$\frac{\sqrt{1-x^2}}{c}-\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\right.}{\sqrt{2}\,c^{3/2}\sqrt{b+2c+\sqrt{}}}$$

```
[Out] (-x^2+1)^(1/2)/c-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+
b^2)^(1/2))^(1/2))*(b+c+(2*a*c-b^2-b*c)/(-4*a*c+b^2)^(1/2))/c^(3/2)*2^(1/2)
/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2
)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b+c+(-2*a*c+b^2+b*c)/(-4*a*c+b^2)^(1/2
))/c^(3/2)*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 1.20, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1265, 838, 840, 1180, 214}

$$-\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\sqrt{1-x^2}}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[1 - x^2]/c - ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTanh[
(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2
]*c^(3/2)*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b + c + (b^2 - 2*a*c + b*c
)/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 838

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Simp[g*((d + e*x)^m/(c*m)), x] + Dist[1/c, Int[
(d + e*x)^(m - 1)*(Simp[c*d*f - a*e*g + (g*c*d - b*e*g + c*e*f)*x, x]/(a +
b*x + c*x^2)), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && FractionQ[m] && GtQ[m, 0]
```

Rule 840

```
Int[((f_.) + (g_.)*(x_))/(Sqrt[(d_.) + (e_.)*(x_)]*((a_.) + (b_.)*(x_) + (c
_.)*(x_)^2)), x_Symbol] :> Dist[2, Subst[Int[(e*f - d*g + g*x^2)/(c*d^2 - b
*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*x^4), x], x, Sqrt[d + e*x]], x] /; Fre
eQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x^3\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\mathrm{Subst}\left(\int \frac{\sqrt{1-x}\,x}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= \frac{\sqrt{1-x^2}}{c} + \frac{\mathrm{Subst}\left(\int \frac{a+(b+c)x}{\sqrt{1-x}\,(a+bx+cx^2)}\,dx, x, x^2\right)}{2c}\\
&= \frac{\sqrt{1-x^2}}{c} + \frac{\mathrm{Subst}\left(\int \frac{-a-b-c+(b+c)x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{c}\\
&= \frac{\sqrt{1-x^2}}{c} + \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)+\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-\right.}{2c}\\
&= \frac{\sqrt{1-x^2}}{c} - \frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tanh^{-1}\left(\frac{\sqrt{2}\,\sqrt{c}\,\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,c^{3/2}\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{\left(b+c+\right.}{}
\end{aligned}
$$

Mathematica [A]
time = 0.58, size = 285, normalized size = 1.24

$$\frac{\sqrt{1-x^2}}{c}+\frac{\left(b^2-2ac+bc+b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}}+\frac{\left(-b^2+2ac-bc+b\sqrt{b^2-4ac}+c\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}c^{3/2}\sqrt{b^2-4ac}\sqrt{-b-2c+\sqrt{b^2-4ac}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] Sqrt[1 - x^2]/c + ((b^2 - 2*a*c + b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 - 
4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4
*a*c]]])/(Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*
c]]) + ((-b^2 + 2*a*c - b*c + b*Sqrt[b^2 - 4*a*c] + c*Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*c^(3/2)*Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(189) = 378$.
time = 0.15, size = 383, normalized size = 1.67

method	result
default	$\frac{2a\left(-\frac{\left(-b\sqrt{-4ac+b^2}-2\sqrt{-4ac+b^2}\,c+4ac-b^2\right)\arctan\left(\frac{-\frac{2a\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\sqrt{-4ac+b^2}-2a-2\cdots}{2\sqrt{4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2b\sqrt{-4ac+\cdots}}}\right)}{\left(8ac-2b^2\right)\sqrt{4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2b\sqrt{-4ac+b^2}-2ab}}\cdots\right.}{\cdots}$
risch	$-\frac{x^2-1}{c\sqrt{-x^2+1}}+\frac{2a\arctan\left(\frac{\frac{2a\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\sqrt{-4ac+b^2}+2a+2b}{2\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}\right)\sqrt{-4ac+b^2}\cdots}{c\left(8ac-2b^2\right)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c*a*(-(-b*(-4*a*c+b^2)^(1/2)-2*(-4*a*c+b^2)^(1/2)*c+4*a*c-b^2)/(8*a*c-2*b
^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)
*arctan(1/2*(-2*a*((-x^2+1)^(1/2)-1)^2/x^2+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4
*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))+(b*(
```

```
-4*a*c+b^2)^(1/2)+2*(-4*a*c+b^2)^(1/2)*c+4*a*c-b^2)/(8*a*c-2*b^2)/(4*a*c-2*
b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(
2*a*((-x^2+1)^(1/2)-1)^2/x^2+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(
-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)))+2/c/(((-x^2+1)^(1
/2)-1)^2/x^2+1)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2053 vs. $2(187) = 374$.
time = 1.49, size = 2053, normalized size = 8.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4
)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*sqr
t((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7))
+ (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4*c^3
 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*
b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*b^
3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c^4
)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c
^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1))/
x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 + (a*b^2*c^3 - 4*a^2*c^4)*x^2*s
qrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)
) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b^4*c
^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(
a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) + (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*a*
b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c - (b^2*c^3 - 4*a*c
^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a
*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 + 1)
)/x^2) - sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
```

```
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x^2
*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^
7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c + sqrt(1/2)*((b^4
*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2
*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (5*
a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4*a
*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4
*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2 +
1))/x^2) + sqrt(1/2)*c*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4))*log((2*a^2*b^2 - (a*b^2*c^3 - 4*a^2*c^4)*x
^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*
c^7)) + (a*b^3 - (a^2*b - a*b^2)*c)*x^2 - 2*(a^3 - a^2*b)*c - sqrt(1/2)*((b
^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*x^2*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 -
 2*(a*b^2 - b^3)*c)/(b^2*c^6 - 4*a*c^7)) - (b^5 + 4*(a^2*b - a*b^2)*c^2 - (
5*a*b^3 - b^4)*c)*x^2)*sqrt((b^3 - 2*a*c^2 - (3*a*b - b^2)*c + (b^2*c^3 - 4
*a*c^4)*sqrt((b^4 + (a^2 - 2*a*b + b^2)*c^2 - 2*(a*b^2 - b^3)*c)/(b^2*c^6 -
 4*a*c^7)))/(b^2*c^3 - 4*a*c^4)) - 2*(a^2*b^2 - (a^3 - a^2*b)*c)*sqrt(-x^2
+ 1))/x^2) + 2*sqrt(-x^2 + 1))/c
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^3\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**3*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 4060 vs. $2(187) = 374$.
time = 6.05, size = 4060, normalized size = 17.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] sqrt(-x^2 + 1)/c + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6*b^4*c^5 + 16*a^2*b*c^6
 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^7 - sqrt(2)*sqrt(b^2 -
4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 + 6*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 - 5*sqrt(2)*s
qrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(
```

2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 2
0*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*
c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)
*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a^2*c^5 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
4*a*c)*c)*a*b*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(
b^2 - 4*a*c)*c)*b^2*c^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*a*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a
*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6 - 4*(b^2 - 4*a*c)*
b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a^2*b*c^4 - 16*a*b^2*c^4
+ 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*a*b^3*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2
- 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b
^2 - 4*a*c)*c)*a^2*b*c^2 + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 - 7*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2
*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^2 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*
c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^3 + 28*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)
*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^3 + 20*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c^4 - 2*(b^2 - 4*a*c)*b^3*
c^2 + 8*(b^2 - 4*a*c)*a*b*c^3 - 2*(b^2 - 4*a*c)*b^2*c^3 + 8*(b^2 - 4*a*c)*a
*c^4)*c^2 - 2*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^4*c^2 +
sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5*c^2 - 8*sqrt(2)*sqrt(
-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^3 - 6*sqrt(2)*sqrt(-b*c - 2*c
^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c^3 + 3*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^
2 - 4*a*c)*c)*b^4*c^3 + 2*a*b^4*c^3 + 2*b^5*c^3 + 16*sqrt(2)*sqrt(-b*c - 2*
c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^4 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2
- 4*a*c)*c)*a^2*b*c^4 - 11*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c
)*a*b^2*c^4 - 16*a^2*b^2*c^4 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a
*c)*c)*b^3*c^4 - 16*a*b^3*c^4 + 2*b^4*c^4 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*a^2*c^5 + 32*a^3*c^5 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*a*b*c^5 + 32*a^2*b*c^5 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 -
sqrt(b^2 - 4*a*c)*c)*b^2*c^5 - 16*a*b^2*c^5 - 20*sqrt(2)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*a*c^6 + 32*a^2*c^6 - 2*(b^2 - 4*a*c)*a*b^2*c^3 - 2*(
b^2 - 4*a*c)*b^3*c^3 + 8*(b^2 - 4*a*c)*a^2*c^4 + 8*(b^2 - 4*a*c)*a*b*c^4 -
2*(b^2 - 4*a*c)*b^2*c^4 + 8*(b^2 - 4*a*c)*a*c^5)*abs(c))*arctan(2*sqrt(1/2)
*sqrt(-x^2 + 1)/sqrt(-(b*c + 2*c^2 + sqrt(-4*(a*c + b*c + c^2)*c^2 + (b*c +
2*c^2)^2))/c^2))/((a*b^4*c^3 + b^5*c^3 - 8*a^2*b^2*c^4 - 6*a*b^3*c^4 + 3*b
^4*c^4 + 16*a^3*c^5 + 8*a^2*b*c^5 - 11*a*b^2*c^5 + 7*b^3*c^5 - 4*a^2*c^6 -
28*a*b*c^6 + 5*b^2*c^6 - 20*a*c^7)*c^2) + 1/8*(2*b^5*c^4 - 12*a*b^3*c^5 + 6
*b^4*c^5 + 16*a^2*b*c^6 - 32*a*b^2*c^6 + 4*b^3*c^6 + 32*a^2*c^7 - 16*a*b*c^
7 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5*
c^2 + 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*

a*b^3*c^3 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c^3 - 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^4 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^4 - 13*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^4 - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^5 + 26*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^5 - 19*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^5 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^6 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b*c^6 - 2*(b^2 - 4*a*c)*b^3*c^4 + 4*(b^2 - 4*a*c)*a*b*c^5 - 6*(b^2 - 4*a*c)*b^2*c^5 + 8*(b^2 - 4*a*c)*a*c^6 - 4*(b^2 - 4*a*c)*b*c^6 - (2*b^5*c^2 - 16*a*b^3*c^3 + 2*b^4*c^3 + 32*a^2*b*c^4 - 16*a*b^2*c^4 + 32*a^2*c^5 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^5 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c - 3*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c - 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2...

Mupad [B]
time = 1.31, size = 776, normalized size = 3.39

$$\frac{\sqrt{1-x^2}}{c}-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1{}\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(4\,a\,c^2-b^2\,c-b^3+b^2\,\sqrt{b^2-4\,a\,c}+4\,a\,b\,c-2\,a\,c\,\sqrt{b^2-4\,a\,c}+b\,c\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1{}\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(b^2\,c-4\,a\,c^2+b^3+b^2\,\sqrt{b^2-4\,a\,c}-4\,a\,b\,c-2\,a\,c\,\sqrt{b^2-4\,a\,c}+b\,c\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1{}\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(4\,a\,c^2-b^2\,c-b^3+b^2\,\sqrt{b^2-4\,a\,c}+4\,a\,b\,c-2\,a\,c\,\sqrt{b^2-4\,a\,c}+b\,c\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1{}\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(b^2\,c-4\,a\,c^2+b^3+b^2\,\sqrt{b^2-4\,a\,c}-4\,a\,b\,c-2\,a\,c\,\sqrt{b^2-4\,a\,c}+b\,c\,\sqrt{b^2-4\,a\,c}\right)}{4\,c^2\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (1 - x^2)^(1/2)/c - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)
*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
 (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2*(b
^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)
^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
+ (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 -
4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^2*c - 4*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2)
- 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4
*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b -
 (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c
) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2)))*(4*a*c^2 - b^2*c - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c - 2*a*c*
(b^2 - 4*a*c)^(1/2) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*((b - (b^2 - 4*a*c)^
(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/
2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1
- x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*c - 4
*a*c^2 + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c - 2*a*c*(b^2 - 4*a*c)^(1/2
```

```
) + b*c*(b^2 - 4*a*c)^(1/2)))/(4*c^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2
))/(2*c) + 1)^(1/2))
```

3.378 $\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=182

$$-\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

```
[Out] -1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2
))*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)+1/2*
arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*(b
+2*c+(-4*a*c+b^2)^(1/2))^(1/2)*2^(1/2)/c^(1/2)/(-4*a*c+b^2)^(1/2)
```

Rubi [A]
time = 0.18, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {1261, 713, 1144, 214}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}-\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c]))
+ (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2]
)/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 713

```
Int[Sqrt[(d_.) + (e_.)*(x_)]/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol]
 :> Dist[2*e, Subst[Int[x^2/(c*d^2 - b*d*e + a*e^2 - (2*c*d - b*e)*x^2 + c*
x^4), x], x, Sqrt[d + e*x]], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4
*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[2*c*d - b*e, 0]
```

Rule 1144

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2/2)*(b/q + 1), Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2/2)*(b/q - 1), Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{x\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{a+bx+cx^2}\,dx, x, x^2\right)\\
&= -\text{Subst}\left(\int \frac{x^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)\\
&= \frac{1}{2}\left(-1-\frac{b+2c}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4ac}+cx^2}\,dx, x, \sqrt{1-x^2}\right.\\
&= -\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}+\frac{\sqrt{b+2c+\sqrt{b^2-}}}{}
\end{aligned}
$$

Mathematica [A]
time = 0.24, size = 169, normalized size = 0.93

$$
\frac{\sqrt{-b-2c-\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)-\sqrt{-b-2c+\sqrt{b^2-4ac}}\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\sqrt{c}\sqrt{b^2-4ac}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/
Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]] - Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]*Ar
cTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]])/(
Sqrt[2]*Sqrt[c]*Sqrt[b^2 - 4*a*c])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 359 vs. $2(143) = 286$.
time = 0.13, size = 360, normalized size = 1.98

method	result
default	$2a\left(\frac{\left(-2\sqrt{-4ac+b^2}\,a-b\sqrt{-4ac+b^2}+4ac-b^2\right)\arctan\left(\frac{\frac{2a\left(\sqrt{-x^2+1}-1\right)^2}{x^2}+2\sqrt{-4ac+b^2}+2a}{2\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac}}}\right.}{2a(4ac-b^2)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{-4ac+b^2}-2ab}}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2*a*(1/2*(-2*(-4*a*c+b^2)^(1/2)*a-b*(-4*a*c+b^2)^(1/2)+4*a*c-b^2)/a/(4*a*c-
b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2
)*arctan(1/2*(2*a*((-x^2+1)^(1/2)-1)^2/x^2+2*(-4*a*c+b^2)^(1/2)+2*a+2*b)/(4
*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))-1/2*
(2*(-4*a*c+b^2)^(1/2)*a+b*(-4*a*c+b^2)^(1/2)+4*a*c-b^2)/a/(4*a*c-b^2)/(4*a*
c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1
/2*(-2*a*((-x^2+1)^(1/2)-1)^2/x^2+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^
2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 871 vs. $2(143) = 286$.
time = 0.81, size = 871, normalized size = 4.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(1/2)*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(
b^2*c - 4*a*c^2))*log((b*x^2 + (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3
) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4
*a*c^3))*sqrt((b + 2*c - (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c
- 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/2)*sqrt((b + 2*c
- (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2
+ (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2)*((b^2 - 4*a*c)*
x^2 + (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c - (b^2
*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1
)*a + 2*a)/x^2) - 1/2*sqrt(1/2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*
c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(
b^2*c^2 - 4*a*c^3) + sqrt(1/2)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2
/sqrt(b^2*c^2 - 4*a*c^3))*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 -
4*a*c^3))/(b^2*c - 4*a*c^2)) - 2*sqrt(-x^2 + 1)*a + 2*a)/x^2) + 1/2*sqrt(1/
2)*sqrt((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*
c^2))*log((b*x^2 - (b^2*c - 4*a*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3) - sqrt(1/2
)*((b^2 - 4*a*c)*x^2 - (b^3*c - 4*a*b*c^2)*x^2/sqrt(b^2*c^2 - 4*a*c^3))*sqr
t((b + 2*c + (b^2*c - 4*a*c^2)/sqrt(b^2*c^2 - 4*a*c^3))/(b^2*c - 4*a*c^2))
- 2*sqrt(-x^2 + 1)*a + 2*a)/x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 591 vs. 2(143) = 286.
time = 5.10, size = 591, normalized size = 3.25

$$-\frac{\left(2b^2c^2-8ac^3-\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2-\sqrt{b^2-4ac}\,c}\,b^2+4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2-\sqrt{b^2-4ac}\,c}\,ac-2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2-\sqrt{b^2-4ac}\,c}\,bc-5\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2-\sqrt{b^2-4ac}\,c}\,c^2-2(b^2-4ac)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{-x^2+1}}{\sqrt{-\frac{b+2c+\sqrt{(b+2c)^2-4(a+b+c)c}}{c}}}\right)}{2(b^4-8ab^2c+2b^3c+16a^2c^2-8abc^2+5b^2c^2-20ac^3)|c|}+\frac{\left(2b^2c^2-8ac^3-\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2+\sqrt{b^2-4ac}\,c}\,b^2+4\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2+\sqrt{b^2-4ac}\,c}\,ac-2\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2+\sqrt{b^2-4ac}\,c}\,bc-5\sqrt{2}\sqrt{b^2-4ac}\sqrt{-bc-2c^2+\sqrt{b^2-4ac}\,c}\,c^2-2(b^2-4ac)c^2\right)\arctan\left(\frac{2\sqrt{\frac{1}{2}}\sqrt{-x^2+1}}{\sqrt{-\frac{b+2c-\sqrt{(b+2c)^2-4(a+b+c)c}}{c}}}\right)}{2(b^4-8ab^2c+2b^3c+16a^2c^2-8abc^2+5b^2c^2-20ac^3)|c|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*sqrt(-x^2
```

```
 + 1)/sqrt(-(b + 2*c + sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))/((b^4 - 8*a
*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*abs(c)) +
 1/2*(2*b^2*c^2 - 8*a*c^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*b^2 + 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*a*c - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*b*c - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + s
qrt(b^2 - 4*a*c)*c)*c^2 - 2*(b^2 - 4*a*c)*c^2)*arctan(2*sqrt(1/2)*sqrt(-x^2
 + 1)/sqrt(-(b + 2*c - sqrt((b + 2*c)^2 - 4*(a + b + c)*c))/c))/((b^4 - 8*a
*b^2*c + 2*b^3*c + 16*a^2*c^2 - 8*a*b*c^2 + 5*b^2*c^2 - 20*a*c^3)*abs(c))
```

Mupad [B]

time = 1.29, size = 649, normalized size = 3.57

$$\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}-1\right)\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac+b\sqrt{b^2-4ac}+2c\sqrt{b^2-4ac}-b^2\right)}{4c\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4ac-b^2\right)}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}-1\right)\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}-\sqrt{1-x^2}\,\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b\sqrt{b^2-4ac}-4ac+2c\sqrt{b^2-4ac}+b^2\right)}{4c\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}+1\right)\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4ac}}{2c}}}\right)\left(4ac+b\sqrt{b^2-4ac}+2c\sqrt{b^2-4ac}-b^2\right)}{4c\sqrt{\frac{b-\sqrt{b^2-4ac}}{2c}+1}\,\left(4ac-b^2\right)}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}+1\right)\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}+\sqrt{1-x^2}\,\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4ac}}{2c}}}\right)\left(b\sqrt{b^2-4ac}-4ac+2c\sqrt{b^2-4ac}+b^2\right)}{4c\left(4ac-b^2\right)\sqrt{\frac{b+\sqrt{b^2-4ac}}{2c}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) + 2*c*(b^2 - 4*a*c)^
(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2
)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2
 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + 2*c*(b^2 - 4*
a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)
^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b -
 (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b
^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c + b*(b^2 - 4*a*c)^(1/2) + 2*c*(b^2
 - 4*a*c)^(1/2) - b^2))/(4*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4
*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b*(b^2 - 4*a*c)^(1/2) - 4*a*c + 2*c
*(b^2 - 4*a*c)^(1/2) + b^2))/(4*c*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/
(2*c) + 1)^(1/2))
```

3.379 $\int \frac{\sqrt{1-x^2}}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=241

$$-\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a}+\frac{\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{b+2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{\sqrt{c}\left(2a+b-\sqrt{\right.}}{\sqrt{2}\,a}$$

```
[Out] -arctanh((-x^2+1)^(1/2))/a+1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*
c-(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*a+b+(-4*a*c+b^2)^(1/2))/a*2^(1/2)/(
-4*a*c+b^2)^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1
/2)*(-x^2+1)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(2*a+b-(-4*a*c
+b^2)^(1/2))/a*2^(1/2)/(-4*a*c+b^2)^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.03, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 911, 1301, 213, 1180, 214}

$$\frac{\sqrt{c}\left(\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\sqrt{b^2-4ac}+2a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(ArcTanh[Sqrt[1 - x^2]]/a) + (Sqrt[c]*(2*a + b + Sqrt[b^2 - 4*a*c])*ArcTan
h[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt
[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*(2*a
+ b - Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2
*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a*Sqrt[b^2 - 4*a*c]*Sqrt[b + 2*c + Sqrt[
b^2 - 4*a*c]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{x\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= -\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx, x, \sqrt{1-x^2}\right)$$

$$= -\text{Subst}\left(\int \left(-\frac{1}{a\left(-1+x^2\right)}+\frac{-a-b-c+cx^2}{a\left(a+b+c-(b+2c)x^2+cx^4\right)}\right)dx, x, \sqrt{1-x^2}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sqrt{1-x^2}\right)}{a} - \frac{\text{Subst}\left(\int \frac{-a-b-c+cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx, x, \sqrt{1-x^2}\right)}{a}$$

$$= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a} + \frac{\left(c\left(2a+b-\sqrt{b^2-4ac}\right)\right)\text{Subst}\left(\int \frac{1}{\frac{1}{2}(-b-2c)-\frac{1}{2}\sqrt{b^2-4}}\right.}{2a\sqrt{b^2-4ac}}$$

$$= -\frac{\tanh^{-1}\left(\sqrt{1-x^2}\right)}{a} + \frac{\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-}}{\sqrt{b+2c-\sqrt{b^2-}}}\right.}{\sqrt{2}\,a\sqrt{b^2-4ac}\sqrt{b+2c-\sqrt{b^2-4ac}}}$$

Mathematica [A]
time = 0.57, size = 264, normalized size = 1.10

$$-\frac{\frac{\sqrt{2}\sqrt{c}\left(-2a-b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(2a+b+\sqrt{b^2-4ac}\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c+\sqrt{b^2-4ac}}}-\log\left(-1+\sqrt{1-x^2}\right)+\log\left(a\left(1+\sqrt{1-x^2}\right)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*((Sqrt[2]*Sqrt[c]*(-2*a - b + Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[
c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*S
qrt[-b - 2*c - Sqrt[b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(2*a + b + Sqrt[b^2 -
 4*a*c])*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c + Sqrt[b^2 -
4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]) - Log[-1 +
 Sqrt[1 - x^2]] + Log[a*(1 + Sqrt[1 - x^2])])/a
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(196) = 392$.
time = 0.14, size = 475, normalized size = 1.97

method	result

default	$-\frac{2a\left(-\frac{\left(\sqrt{-4ac+b^2}\,ab-2ac\sqrt{-4ac+b^2}+b^2\sqrt{-4ac+b^2}+4a^2c-a\,b^2+4abc-b^3\right)\arctan\left(\frac{-\frac{2a\left(\sqrt{-x^2+\cdots}\right.}{x^2}\cdots}{2\sqrt{4ac-2b^2+2\sqrt{-\cdots}}}\right.}{2a\left(4ac-b^2\right)\sqrt{4ac-2b^2+2\sqrt{-4ac+b^2}\,a+2b\sqrt{-4ac+b^2}\,-\cdots}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a*(2*a*(-1/2*((-4*a*c+b^2)^(1/2)*a*b-2*a*c*(-4*a*c+b^2)^(1/2)+b^2*(-4*a*
c+b^2)^(1/2)+4*a^2*c-a*b^2+4*a*b*c-b^3)/a/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*
c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*a*((-x^2+
1)^(1/2)-1)^2/x^2+2*(-4*a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)
^(1/2)*a+2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1/2))+1/2*(-(-4*a*c+b^2)^(1/2)*a*b+
2*a*c*(-4*a*c+b^2)^(1/2)-b^2*(-4*a*c+b^2)^(1/2)+4*a^2*c-a*b^2+4*a*b*c-b^3)/
a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*
a*b)^(1/2)*arctan(1/2*(2*a*((-x^2+1)^(1/2)-1)^2/x^2+2*(-4*a*c+b^2)^(1/2)+2*
a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/2)-2*a*b)^(1
/2)))+2/(((-x^2+1)^(1/2)-1)^2/x^2+1))+1/a*((-x^2+1)^(1/2)-arctanh(1/(-x^2+1
)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1232 vs. $2(196) = 392$.
time = 2.65, size = 1232, normalized size = 5.11

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 +
2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a
```

```
^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a
*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 -
4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b
 + b^2)/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a
*b)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*
b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c + (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2
*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c
)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)) - (a*b + b^2)*x^2 - 2*a
^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a*sqrt((a*b + b
^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*
c)))/(a^2*b^2 - 4*a^3*c))*log(-(2*sqrt(1/2)*(a^3*b^2 - 4*a^4*c)*x^2*sqrt((a
^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 -
 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c
)) + (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))
- (a*b + b^2)*x^2 - 2*a^2 - 2*a*b + 2*(a^2 + a*b)*sqrt(-x^2 + 1))/x^2) - sq
rt(1/2)*a*sqrt((a*b + b^2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b +
 b^2)/(a^4*b^2 - 4*a^5*c)))/(a^2*b^2 - 4*a^3*c))*log((2*sqrt(1/2)*(a^3*b^2
- 4*a^4*c)*x^2*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c))*sqrt((a*b + b^
2 - 2*a*c - (a^2*b^2 - 4*a^3*c)*sqrt((a^2 + 2*a*b + b^2)/(a^4*b^2 - 4*a^5*c
)))/(a^2*b^2 - 4*a^3*c)) - (a^2*b^2 - 4*a^3*c)*x^2*sqrt((a^2 + 2*a*b + b^2)
/(a^4*b^2 - 4*a^5*c)) + (a*b + b^2)*x^2 + 2*a^2 + 2*a*b - 2*(a^2 + a*b)*sqr
t(-x^2 + 1))/x^2) + 2*log((sqrt(-x^2 + 1) - 1)/x))/a
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x*(a + b*x**2 + c*x**4)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3639 vs. $2(196) = 392$.
time = 8.10, size = 3639, normalized size = 15.10

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*log(sqrt(-x^2 + 1) + 1)/a + 1/2*log(-sqrt(-x^2 + 1) + 1)/a + 1/8*(4*a^
3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*
a^3*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*
c)*c)*a^3*b^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*
a*c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2
- 4*a*c)*c)*a^4*b*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(
b^2 - 4*a*c)*c)*a^3*b^2*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 +
 sqrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*s
qrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 -
 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 10*sqrt(2)*sqrt(
b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 4*(b^2 -
4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 -
4*(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2
)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sq
rt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c - 16
*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*c^2
 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b
*c^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)
*b^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*
c)*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 + 2*(sqr
t(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + sqrt(2)*sqrt(-b*c -
 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c - 6*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c
)*a^2*b^3*c + 3*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^4*c -
2*a^2*b^4*c - 2*a*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*
c)*a^4*c^2 + 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 -
 11*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 + 16*a^3*b
^2*c^2 + 7*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^3*c^2 + 16*
a^2*b^3*c^2 - 2*a*b^4*c^2 - 4*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)
*c)*a^3*c^3 - 32*a^4*c^3 - 28*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)
*c)*a^2*b*c^3 - 32*a^3*b*c^3 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a
*c)*c)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2
 - 4*a*c)*c)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2*(b^2 - 4*
a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^2 + 2*(b^2
 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a))*arctan(2*sqrt(1/2)*s
qrt(-x^2 + 1)/sqrt(-(a*b + 2*a*c + sqrt(-4*(a^2 + a*b + a*c)*a*c + (a*b + 2
*a*c)^2))/(a*c)))/((a^3*b^4 + a^2*b^5 - 8*a^4*b^2*c - 6*a^3*b^3*c + 3*a^2*b
^4*c + 16*a^5*c^2 + 8*a^4*b*c^2 - 11*a^3*b^2*c^2 + 7*a^2*b^3*c^2 - 4*a^4*c^
3 - 28*a^3*b*c^3 + 5*a^2*b^2*c^3 - 20*a^3*c^4)*abs(a)*abs(c)) - 1/8*(4*a^3*
b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^
3*b*c^4 - 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)
*c)*a^3*b^3 - sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
```

```
c)*c)*a^2*b^4 + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 -
4*a*c)*c)*a^4*b*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^
2 - 4*a*c)*c)*a^3*b^2*c - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - s
qrt(b^2 - 4*a*c)*c)*a^2*b^3*c + 16*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*
c^2 - sqrt(b^2 - 4*a*c)*c)*a^4*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c
 - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*b*c^2 - 9*sqrt(2)*sqrt(b^2 - 4*a*c)*sqr
t(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^2*c^2 - 20*sqrt(2)*sqrt(b^2 - 4
*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^3*c^3 - 10*sqrt(2)*sqrt(b^
2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b*c^3 - 4*(b^2 - 4*
a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*
(b^2 - 4*a*c)*a^2*b*c^3 + (2*b^4*c^2 - 16*a*b^2*c^3 + 32*a^2*c^4 - sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c - 16*s
qrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 +
 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c
^2 - 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b
^2*c^2 + 20*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)
*c)*a*c^3 - 2*(b^2 - 4*a*c)*b^2*c^2 + 8*(b^2 - 4*a*c)*a*c^3)*a^2 - 2*(sqrt(
2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*b^4 + sqrt(2)*sqrt(-b*c - 2
*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2
- 4*a*c)*c)*a^3*b^2*c - 6*sqrt(2)*sqrt(-b*c - 2...
```

Mupad [B]
time = 1.30, size = 669, normalized size = 2.78

$$\frac{\ln\left(\sqrt{\frac{1}{x^2}-1}-\sqrt{\frac{1}{x^2}}\right)}{a}+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1{}\mathrm{i}}{x+\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(4\,a\,c+2\,a\,\sqrt{b^2-4\,a\,c}+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{4\,a\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}+1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}+\sqrt{1-x^2}\,1{}\mathrm{i}}{x+\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(2\,a\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+b^2\right)}{4\,a\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}+\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1{}\mathrm{i}}{x-\sqrt{-\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(4\,a\,c+2\,a\,\sqrt{b^2-4\,a\,c}+b\,\sqrt{b^2-4\,a\,c}-b^2\right)}{4\,a\,\left(4\,a\,c-b^2\right)\,\sqrt{\frac{b+\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\frac{\ln\left(\frac{\frac{\left(x\,\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}-1\right)\,1{}\mathrm{i}}{\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}}-\sqrt{1-x^2}\,1{}\mathrm{i}}{x-\sqrt{-\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}}}\right)\,\left(2\,a\,\sqrt{b^2-4\,a\,c}-4\,a\,c+b\,\sqrt{b^2-4\,a\,c}+b^2\right)}{4\,a\,\sqrt{\frac{b-\sqrt{b^2-4\,a\,c}}{2\,c}+1}\,\left(4\,a\,c-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/a + (log((((x*(-(b + (b^2 - 4*a*c)^(
1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (
1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a*c +
 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^2))/(4*a*(4*a*c - b^2)
*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*
a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(1/2) + b^2))/(4*a*((b - (b
^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b + (b^2
- 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)
^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2))
)*(4*a*c + 2*a*(b^2 - 4*a*c)^(1/2) + b*(b^2 - 4*a*c)^(1/2) - b^2))/(4*a*(4*
```

```
a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b -
(b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c)
 + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(
1/2)))*(2*a*(b^2 - 4*a*c)^(1/2) - 4*a*c + b*(b^2 - 4*a*c)^(1/2) + b^2))/(4*
a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

3.380 $\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=290

$$-\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{\sqrt{c}\left(a+b+\frac{b^2+a(b-2c)}{\sqrt{b^2-4ac}}\right)\tan}{\sqrt{2}\,a^2\sqrt{b+2c}}$$

```
[Out] 1/2*(a+2*b)*arctanh((-x^2+1)^(1/2))/a^2-1/4/a/(1-(-x^2+1)^(1/2))+1/4/a/(1+(
-x^2+1)^(1/2))-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1/2)/(b+2*c-(-4*a*c+b^
2)^(1/2))^(1/2))*c^(1/2)*(a+b+(b^2+a*(b-2*c))/(-4*a*c+b^2)^(1/2))/a^2*2^(1/
2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-1/2*arctanh(2^(1/2)*c^(1/2)*(-x^2+1)^(1
/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2))*c^(1/2)*(a+b+(-b^2-a*(b-2*c))/(-4*a*c
+b^2)^(1/2))/a^2*2^(1/2)/(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.52, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1265, 911, 1301, 213, 1180, 214}

$$-\frac{\sqrt{c}\left(\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{c}\left(-\frac{a(b-2c)+b^2}{\sqrt{b^2-4ac}}+a+b\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{\sqrt{b^2-4ac}+b+2c}}\right)}{\sqrt{2}\,a^2\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(\sqrt{1-x^2}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/4*1/(a*(1 - Sqrt[1 - x^2])) + 1/(4*a*(1 + Sqrt[1 - x^2])) + ((a + 2*b)*A
rcTanh[Sqrt[1 - x^2]])/(2*a^2) - (Sqrt[c]*(a + b + (b^2 + a*(b - 2*c))/Sqrt
[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[b + 2*c - Sqrt[
b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (Sqrt[c]*
(a + b - (b^2 + a*(b - 2*c))/Sqrt[b^2 - 4*a*c])*ArcTanh[(Sqrt[2]*Sqrt[c]*Sq
rt[1 - x^2])/Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]])/(Sqrt[2]*a^2*Sqrt[b + 2*c
+ Sqrt[b^2 - 4*a*c]])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 911

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> With[{q = Denominator[m]}, Dist[q/e, S
ubst[Int[x^(q*(m + 1) - 1)*((e*f - d*g)/e + g*(x^q/e))^n*((c*d^2 - b*d*e +
a*e^2)/e^2 - (2*c*d - b*e)*(x^q/e^2) + c*(x^(2*q)/e^2))^p, x], x, (d + e*x)
^(1/q)], x]] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[e*f - d*g, 0] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegersQ[n, p] && Fra
ctionQ[m]
```

Rule 1180

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1301

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*((d + e*x^2)^q/(a
+ b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{x^3\left(a+bx^2+cx^4\right)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{\sqrt{1-x}}{x^2\left(a+bx+cx^2\right)}\,dx, x, x^2\right)$$

$$= -\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^2\left(a+b+c+(-b-2c)x^2+cx^4\right)}\,dx, x, \sqrt{1-x^2}\right)$$

$$= -\text{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}+\frac{a+2b}{2a^2\left(-1+x^2\right)}+\frac{b(a+b+c)-}{a^2\left(a+b+c-(b\right.}\right.\right.$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}-\frac{\text{Subst}\left(\int \frac{b(a+b+c)-(a+b)cx^2}{a+b+c+(-b-2c)x^2+cx^4}\,dx,\right.}{a^2}$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}+$$

$$= -\frac{1}{4a\left(1-\sqrt{1-x^2}\right)}+\frac{1}{4a\left(1+\sqrt{1-x^2}\right)}+\frac{(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}-$$

Mathematica [**A**]
time = 1.54, size = 307, normalized size = 1.06

$$\frac{-\frac{a\sqrt{1-x^2}}{x^2}+\frac{\sqrt{2}\sqrt{c}\left(b\left(-b+\sqrt{b^2-4ac}\right)+a\left(-b+2c+\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c-\sqrt{b^2-4ac}}}+\frac{\sqrt{2}\sqrt{c}\left(b\left(b+\sqrt{b^2-4ac}\right)+a\left(b-2c+\sqrt{b^2-4ac}\right)\right)\tan^{-1}\left(\frac{\sqrt{2}\sqrt{c}\sqrt{1-x^2}}{\sqrt{-b-2c+\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\sqrt{-b-2c+\sqrt{b^2-4ac}}}+(a+2b)\tanh^{-1}\left(\sqrt{1-x^2}\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-((a*Sqrt[1 - x^2])/x^2) + (Sqrt[2]*Sqrt[c]*(b*(-b + Sqrt[b^2 - 4*a*c]) +
a*(-b + 2*c + Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sq
rt[-b - 2*c - Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c - Sqrt[
b^2 - 4*a*c]]) + (Sqrt[2]*Sqrt[c]*(b*(b + Sqrt[b^2 - 4*a*c]) + a*(b - 2*c +
 Sqrt[b^2 - 4*a*c]))*ArcTan[(Sqrt[2]*Sqrt[c]*Sqrt[1 - x^2])/Sqrt[-b - 2*c +
 Sqrt[b^2 - 4*a*c]]])/(Sqrt[b^2 - 4*a*c]*Sqrt[-b - 2*c + Sqrt[b^2 - 4*a*c]]
) + (a + 2*b)*ArcTanh[Sqrt[1 - x^2]])/(2*a^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(240) = 480$.
time = 0.17, size = 580, normalized size = 2.00

method	result
default	$-\frac{-2a\left(\frac{\left(2\sqrt{-4ac+b^2}\,a^2c-\sqrt{-4ac+b^2}\,a\,b^2+3\sqrt{-4ac+b^2}\,abc-\sqrt{-4ac+b^2}\,b^3+4a^2bc-4a^2c^2-a\,b^3+5a\,b^2c-b^4\right)}{2a\left(4ac-b^2\right)\sqrt{4ac-2b^2-2\sqrt{-4ac+b^2}\,a-2b\sqrt{\ldots}}}\right.}{\ldots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/a^2*(-2*a*(1/2*(2*(-4*a*c+b^2)^(1/2)*a^2*c-(-4*a*c+b^2)^(1/2)*a*b^2+3*(-
4*a*c+b^2)^(1/2)*a*b*c-(-4*a*c+b^2)^(1/2)*b^3+4*a^2*b*c-4*a^2*c^2-a*b^3+5*a
*b^2*c-b^4)/a/(4*a*c-b^2)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b
^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(2*a*((-x^2+1)^(1/2)-1)^2/x^2+2*(-4*a*c+b
^2)^(1/2)+2*a+2*b)/(4*a*c-2*b^2-2*(-4*a*c+b^2)^(1/2)*a-2*b*(-4*a*c+b^2)^(1/
2)-2*a*b)^(1/2))-1/2*(-2*(-4*a*c+b^2)^(1/2)*a^2*c+(-4*a*c+b^2)^(1/2)*a*b^2-
3*(-4*a*c+b^2)^(1/2)*a*b*c+(-4*a*c+b^2)^(1/2)*b^3+4*a^2*b*c-4*a^2*c^2-a*b^3
+5*a*b^2*c-b^4)/a/(4*a*c-b^2)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a
*c+b^2)^(1/2)-2*a*b)^(1/2)*arctan(1/2*(-2*a*((-x^2+1)^(1/2)-1)^2/x^2+2*(-4*
a*c+b^2)^(1/2)-2*a-2*b)/(4*a*c-2*b^2+2*(-4*a*c+b^2)^(1/2)*a+2*b*(-4*a*c+b^2
)^(1/2)-2*a*b)^(1/2)))-2*b/(((-x^2+1)^(1/2)-1)^2/x^2+1))+1/a*(-1/2/x^2*(-x^
2+1)^(3/2)-1/2*(-x^2+1)^(1/2)+1/2*arctanh(1/(-x^2+1)^(1/2)))-b/a^2*((-x^2+1
)^(1/2)-arctanh(1/(-x^2+1)^(1/2)))
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2799 vs. $2(236) = 472$.
time = 7.20, size = 2799, normalized size = 9.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)
*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4
*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/
(a^4*b^2 - 4*a^5*c))*log(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b
^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b
^4)*c)/(a^8*b^2 - 4*a^9*c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^
2 - (a*b^3 + b^4)*c)*x^2 - 2*(a^2*b^2 + a*b^3)*c + sqrt(1/2)*((a^5*b^3 - 4*
a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^
2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 +
a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3
 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*
b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*
b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*
a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*a^2*x^2*
sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)
*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b
^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log
(((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*
b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*
c)) + 2*(a^3 + 2*a^2*b)*c^2 + ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2
 - 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b
^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b
^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) + (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b
)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*
a^2*b + 4*a*b^2)*c - (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) - 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 +
a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^
2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5
 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4
)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^
2)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*
(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)
*c^2 - ((a^2*b + 2*a*b^2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*
c + sqrt(1/2)*((a^5*b^3 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a
^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b
^2 - 4*a^9*c)) - (a^2*b^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*
a^2*b^3)*c)*x^2)*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a
^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^
2 - 4*a^5*c)) + 2*((a^3 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1
))/x^2) - sqrt(1/2)*a^2*x^2*sqrt((a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*
b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b
```

```
 + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c
)))/(a^4*b^2 - 4*a^5*c))*log(-((a^4*b^2*c - 4*a^5*c^2)*x^2*sqrt((a^2*b^4 +
2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 +
2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - 2*(a^3 + 2*a^2*b)*c^2 - ((a^2*b + 2*a*b^
2)*c^2 - (a*b^3 + b^4)*c)*x^2 + 2*(a^2*b^2 + a*b^3)*c - sqrt(1/2)*((a^5*b^3
 - 4*a^6*b*c)*x^2*sqrt((a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^
2)*c^2 - 2*(a^3*b^2 + 3*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)) - (a^2*b
^4 + a*b^5 + 4*(a^4 + 2*a^3*b)*c^2 - (5*a^3*b^2 + 6*a^2*b^3)*c)*x^2)*sqrt((
a*b^3 + b^4 + 2*a^2*c^2 - (3*a^2*b + 4*a*b^2)*c + (a^4*b^2 - 4*a^5*c)*sqrt(
(a^2*b^4 + 2*a*b^5 + b^6 + (a^4 + 4*a^3*b + 4*a^2*b^2)*c^2 - 2*(a^3*b^2 + 3
*a^2*b^3 + 2*a*b^4)*c)/(a^8*b^2 - 4*a^9*c)))/(a^4*b^2 - 4*a^5*c)) + 2*((a^3
 + 2*a^2*b)*c^2 - (a^2*b^2 + a*b^3)*c)*sqrt(-x^2 + 1))/x^2) + (a + 2*b)*x^2
*log((sqrt(-x^2 + 1) - 1)/x) + sqrt(-x^2 + 1)*a)/(a^2*x^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{x^3(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**3*(a + b*x**2 + c*x**4)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1675 vs. 2(236) = 472.
time = 6.68, size = 1675, normalized size = 5.78

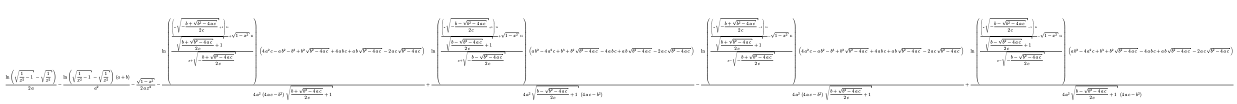
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt
(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2
- sqrt(b^2 - 4*a*c)*c)*b^4*c + 2*b^5*c + 16*sqrt(2)*sqrt(-b*c - 2*c^2 - sqr
t(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*
c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c^
2 - 16*a*b^3*c^2 + 2*b^4*c^2 - 20*sqrt(2)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*
a*c)*c)*a*b*c^3 + 32*a^2*b*c^3 - 12*a*b^2*c^3 + 16*a^2*c^4 - sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^4 + 6*sqrt(2)*sqrt(b
^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b^2*c - 2*sqrt(2)*sq
rt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^3*c - 8*sqrt(2)*
sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a^2*c^2 + 4*sqrt
(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*b*c^2 - 5*
```

```
sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*b^2*c^2
+ 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 - sqrt(b^2 - 4*a*c)*c)*a*c
^3 - 2*(b^2 - 4*a*c)*b^3*c + 8*(b^2 - 4*a*c)*a*b*c^2 - 2*(b^2 - 4*a*c)*b^2*
c^2 + 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b
 + 2*a^2*c + sqrt(-4*(a^3 + a^2*b + a^2*c)*a^2*c + (a^2*b + 2*a^2*c)^2))/(a
^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 +
5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) - 1/4*(sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt
(b^2 - 4*a*c)*c)*b^5 - 8*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a
*b^3*c + 2*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^4*c - 2*b^5*c
 + 16*sqrt(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a^2*b*c^2 - 8*sqrt(2
)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b^2*c^2 + 5*sqrt(2)*sqrt(-b*c
- 2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^3*c^2 + 16*a*b^3*c^2 - 2*b^4*c^2 - 20*sqrt
(2)*sqrt(-b*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^3 - 32*a^2*b*c^3 + 12*a*
b^2*c^3 - 16*a^2*c^4 + sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b
^2 - 4*a*c)*c)*b^4 - 6*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sqrt(b
^2 - 4*a*c)*c)*a*b^2*c + 2*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 + sq
rt(b^2 - 4*a*c)*c)*b^3*c + 8*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^2 +
sqrt(b^2 - 4*a*c)*c)*a^2*c^2 - 4*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c - 2*c^
2 + sqrt(b^2 - 4*a*c)*c)*a*b*c^2 + 5*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b*c -
2*c^2 + sqrt(b^2 - 4*a*c)*c)*b^2*c^2 - 10*sqrt(2)*sqrt(b^2 - 4*a*c)*sqrt(-b
*c - 2*c^2 + sqrt(b^2 - 4*a*c)*c)*a*c^3 + 2*(b^2 - 4*a*c)*b^3*c - 8*(b^2 -
4*a*c)*a*b*c^2 + 2*(b^2 - 4*a*c)*b^2*c^2 - 4*(b^2 - 4*a*c)*a*c^3)*arctan(2*
sqrt(1/2)*sqrt(-x^2 + 1)/sqrt(-(a^2*b + 2*a^2*c - sqrt(-4*(a^3 + a^2*b + a^
2*c)*a^2*c + (a^2*b + 2*a^2*c)^2))/(a^2*c)))/((a^2*b^4 - 8*a^3*b^2*c + 2*a^
2*b^3*c + 16*a^4*c^2 - 8*a^3*b*c^2 + 5*a^2*b^2*c^2 - 20*a^3*c^3)*abs(c)) +
1/4*(a + 2*b)*log(sqrt(-x^2 + 1) + 1)/a^2 - 1/4*(a + 2*b)*log(-sqrt(-x^2 +
1) + 1)/a^2 - 1/2*sqrt(-x^2 + 1)/(a*x^2)
```

Mupad [B]
time = 1.41, size = 825, normalized size = 2.84



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x^3*(a + b*x^2 + c*x^4)),x)
```

```
[Out] log((1/x^2 - 1)^(1/2) - (1/x^2)^(1/2))/(2*a) - (log((1/x^2 - 1)^(1/2) - (1/
x^2)^(1/2))*(a + b))/a^2 - (1 - x^2)^(1/2)/(2*a*x^2) - (log((((x*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) +
1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2
)))*(4*a^2*c - a*b^2 - b^3 + b^2*(b^2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(b^2 -
 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*(4*a*c - b^2)*((b + (b^2
 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^
```

```
2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(a*b^2 - 4*a^2
*c + b^3 + b^2*(b^2 - 4*a*c)^(1/2) - 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*
a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
)*(4*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*
1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x -
(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(4*a^2*c - a*b^2 - b^3 + b^2*(b^
2 - 4*a*c)^(1/2) + 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^
(1/2)))/(4*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) +
 (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4
*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*
c)^(1/2))/(2*c))^(1/2)))*(a*b^2 - 4*a^2*c + b^3 + b^2*(b^2 - 4*a*c)^(1/2) -
 4*a*b*c + a*b*(b^2 - 4*a*c)^(1/2) - 2*a*c*(b^2 - 4*a*c)^(1/2)))/(4*a^2*((b
 - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

3.381 $\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=325

$$\frac{x\sqrt{1-x^2}}{2c}+\frac{(2b+c)\sin^{-1}(x)}{2c^2}-\frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}}-$$

```
[Out] 1/2*(2*b+c)*arcsin(x)/c^2+1/2*x*(-x^2+1)^(1/2)/c-arctan(x*(b+2*c-(-4*a*c+b^
2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(
3*a*b*c+2*a*c^2-b^3-b^2*c)/(-4*a*c+b^2)^(1/2))/c^2/(b-(-4*a*c+b^2)^(1/2))^(
1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(b+2*c+(-4*a*c+b^2)^(1/2))^(
1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*c+(-3*a*b*c-2*
a*c^2+b^3+b^2*c)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*
c+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 3.68, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1305, 396, 222, 1706, 385, 211}

$$-\frac{\left(-\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\text{ArcTan}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{\left(\frac{-3abc-2ac^2+b^3+b^2c}{\sqrt{b^2-4ac}}-ac+b^2+bc\right)\text{ArcTan}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}+\frac{\text{ArcSin}(x)(2b+c)}{2c^2}+\frac{\sqrt{1-x^2}\,x}{2c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (x*Sqrt[1 - x^2])/(2*c) + ((2*b + c)*ArcSin[x])/(2*c^2) - ((b^2 - a*c + b*c
 - (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*
c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c^
2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - ((b^2 -
a*c + b*c + (b^3 - 3*a*b*c + b^2*c - 2*a*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sq
rt[b + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^
2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 396

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[d*x*((a + b*x^n)^(p + 1)/(b*(n*(p + 1) + 1))), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 1305

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[f^4/c^2, Int[(f*x)^(m - 4)*(c*d - b*e + c
*e*x^2)*(d + e*x^2)^(q - 1), x], x] - Dist[f^4/c^2, Int[(f*x)^(m - 4)*(d +
e*x^2)^(q - 1)*(Simp[a*(c*d - b*e) + (b*c*d - b^2*e + a*c*e)*x^2, x]/(a + b
*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c,
0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{\int \frac{b+c-cx^2}{\sqrt{1-x^2}}\,dx}{c^2} - \frac{\int \frac{a(b+c)+(b^2-ac+bc)x^2}{\sqrt{1-x^2}\,(a+bx^2+cx^4)}\,dx}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} - \frac{\int \left(\frac{b^2-ac+bc+\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{b^2-ac+bc-\frac{-b^3+3abc-b^2c+2ac^2}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right)}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\int \frac{}{\sqrt{1-x^2}\,(b-\cdots}}{c^2}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{b-\sqrt{b^2\cdots}}\right.}{c^2\cdots}$$

$$= \frac{x\sqrt{1-x^2}}{2c} + \frac{(2b+c)\sin^{-1}(x)}{2c^2} - \frac{\left(b^2-ac+bc-\frac{b^3-3abc+b^2c-2ac^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+\cdots}}{\sqrt{b-\cdots}}\right.}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2\cdots}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.64, size = 588, normalized size = 1.81

$$2cx\sqrt{1-x^2}+4(2b+c)\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)+\text{RootSum}\left[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4+4a\#1^6+4b\#1^6+a\#1^8\&,\ \frac{-ab\log(x)-ac\log(x)+ab\log\left(-1+\sqrt{1-x^2}-x\#1\right)+ac\log\left(-1+\sqrt{1-x^2}-x\#1\right)-3ab\log(x)\#1^2-4b^2\log(x)\#1^2+ac\log(x)\#1^2-4bc\log(x)\#1^2\cdots}{\cdots}\right.$$

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(x^4*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*x*Sqrt[1 - x^2] + 4*(2*b + c)*ArcTan[x/(-1 + Sqrt[1 - x^2])] + RootSum
[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b
*#1^6 + a*#1^8 & , (-(a*b*Log[x]) - a*c*Log[x] + a*b*Log[-1 + Sqrt[1 - x^2]
 - x*#1] + a*c*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a*b*Log[x]*#1^2 - 4*b^2*L
og[x]*#1^2 + a*c*Log[x]*#1^2 - 4*b*c*Log[x]*#1^2 + 3*a*b*Log[-1 + Sqrt[1 -
x^2] - x*#1]*#1^2 + 4*b^2*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 - a*c*Log[-1
+ Sqrt[1 - x^2] - x*#1]*#1^2 + 4*b*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 -
3*a*b*Log[x]*#1^4 - 4*b^2*Log[x]*#1^4 + a*c*Log[x]*#1^4 - 4*b*c*Log[x]*#1^4
 + 3*a*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*b^2*Log[-1 + Sqrt[1 - x^2]
 - x*#1]*#1^4 - a*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*b*c*Log[-1 + Sq
rt[1 - x^2] - x*#1]*#1^4 - a*b*Log[x]*#1^6 - a*c*Log[x]*#1^6 + a*b*Log[-1 +
 Sqrt[1 - x^2] - x*#1]*#1^6 + a*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^6)/(a*#
```

```
1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 + 3*b*#1^5 + a*#1^7) &
 ])/(4*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 226, normalized size = 0.70

method	result
risch	$-\frac{x(x^2-1)}{2c\sqrt{-x^2+1}}+\frac{\arcsin(x)b}{c^2}+\frac{\arcsin(x)}{2c}+\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)} \frac{\left(a(b+c)\_R^6 \right.}{}}{}$
default	$\frac{\frac{x\sqrt{-x^2+1}}{2}+\frac{\arcsin(x)}{2}}{c}+\frac{-\frac{2b\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{c}+\sum_{\_R=\text{RootOf}\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*x*(-x^2+1)^(1/2)+1/2*arcsin(x))+1/c*(-2*b/c*arctan(((-x^2+1)^(1/2)
-1)/x)+1/4/c*sum((a*(b+c)*_R^6+(3*a*b-a*c+4*b^2+4*b*c)*_R^4+(3*a*b-a*c+4*b^
2+4*b*c)*_R^2+a*b+a*c)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c
+_R*a+_R*b)*ln(((-x^2+1)^(1/2)-1)/x-_R),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*
a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 2860 vs. 2(279) = 558.
time = 1.34, size = 2860, normalized size = 8.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c +
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
```

```
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + sqrt(1/2)*((b^6 + 4*a^2*b*
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*
c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4
+ (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c^2*sqrt(
-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt(
(b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2
- 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a
^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*
(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^
3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*
b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x - ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2
*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (
4*a*b^2 - b^3)*c + (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a
*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8
 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*
b^2)*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c
^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2
*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b
^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a
^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c + s
qrt(1/2)*((b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*
c)*sqrt(-x^2 + 1)*x - (b^6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a
*b^4 - b^5)*c)*x + ((b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x -
(b^4*c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a
*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8
 - 4*a*c^9)))*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c
^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 -
4*a*c^5)) - 2*(a^2*b^3 - a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x
^2) - sqrt(1/2)*c^2*sqrt(-(b^4 + (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c -
(b^2*c^4 - 4*a*c^5)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*
b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*
c^4 - 4*a*c^5))*log(-(2*a^2*b^3 - 2*a^3*c^2 - 2*(a^2*b^3 - a^3*c^2 - (2*a^3
*b - a^2*b^2)*c)*x^2 - 2*(2*a^3*b - a^2*b^2)*c - sqrt(1/2)*((b^6 + 4*a^2*b*
c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*sqrt(-x^2 + 1)*x - (b^
```

```
6 + 4*a^2*b*c^3 + (8*a^2*b^2 - 5*a*b^3)*c^2 - (6*a*b^4 - b^5)*c)*x + ((b^4*
c^4 - 6*a*b^2*c^5 + 8*a^2*c^6)*sqrt(-x^2 + 1)*x - (b^4*c^4 - 6*a*b^2*c^5 +
8*a^2*c^6)*x)*sqrt((b^6 + a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 -
6*a*b^3 + b^4)*c^2 - 2*(2*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))*sqrt(-(b^4
+ (2*a^2 - 3*a*b)*c^2 - (4*a*b^2 - b^3)*c - (b^2*c^4 - 4*a*c^5)*sqrt((b^6 +
 a^2*c^4 + 2*(2*a^2*b - a*b^2)*c^3 + (4*a^2*b^2 - 6*a*b^3 + b^4)*c^2 - 2*(2
*a*b^4 - b^5)*c)/(b^2*c^8 - 4*a*c^9)))/(b^2*c^4 - 4*a*c^5)) - 2*(a^2*b^3 -
a^3*c^2 - (2*a^3*b - a^2*b^2)*c)*sqrt(-x^2 + 1))/x^2) - sqrt(-x^2 + 1)*c*x
+ 2*(2*b + c)*arctan((sqrt(-x^2 + 1) - 1)/x))/c^2
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4 \sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**4*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1709 vs. 2(279) = 558.
time = 7.54, size = 1709, normalized size = 5.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/4*(3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3 + 2*sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^4 - 2*a^2*b^4 - sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*b^5 + 2*a*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*
b + sqrt(b^2 - 4*a*c)*a)*a^3*b*c - 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 -
4*a*c)*a)*a^2*b^2*c + 12*a^3*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*a*b^3*c - 16*a^2*b^3*c - 16*a^4*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a
*b + sqrt(b^2 - 4*a*c)*a)*a^2*b*c^2 + 32*a^3*b*c^2 - 3*sqrt(2)*sqrt(2*a^2 +
 a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 - 2*sqrt(2)*sqrt(2*a^
2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 + sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 + 6*sqrt(2)*sqrt(2*a^2
+ a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c + 4*sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c - 6*sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c + 8*sqrt(2)*sqrt
(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 + 2*(b^2 - 4*
a*c)*a^2*b^2 - 2*(b^2 - 4*a*c)*a*b^3 - 4*(b^2 - 4*a*c)*a^3*c + 8*(b^2 - 4*a
*c)*a^2*b*c)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^
```

```
2 + 1) - 1)/x)/sqrt((2*a*c^2 + b*c^2 + sqrt(-4*(a*c^2 + b*c^2 + c^3)*a*c^2
+ (2*a*c^2 + b*c^2)^2))/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*
c^2 - 12*a^5*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/4*(3*sqrt(
2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^3 + 2*sqrt(2)*sqrt(2*a^2 +
 a*b - sqrt(b^2 - 4*a*c)*a)*a*b^4 - 2*a^2*b^4 - sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*b^5 - 2*a*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
 - 4*a*c)*a)*a^3*b*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^
2*b^2*c + 12*a^3*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*
a*b^3*c + 16*a^2*b^3*c - 16*a^4*c^2 - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^
2 - 4*a*c)*a)*a^2*b*c^2 - 32*a^3*b*c^2 - 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(
b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 - 2*sqrt(2)*sqrt(2*a^2 + a*b - sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 + sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c + 4*sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c - 6*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c + 8*sqrt(2)*sqrt(2*a^2 + a*b
 - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2 + 2*(b^2 - 4*a*c)*a^2*b^2
 + 2*(b^2 - 4*a*c)*a*b^3 - 4*(b^2 - 4*a*c)*a^3*c - 8*(b^2 - 4*a*c)*a^2*b*c)
*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/
x)/sqrt((2*a*c^2 + b*c^2 - sqrt(-4*(a*c^2 + b*c^2 + c^3)*a*c^2 + (2*a*c^2 +
 b*c^2)^2))/(a*c^2)))/(3*a^4*b^2*c^2 + 2*a^3*b^3*c^2 - a^2*b^4*c^2 - 12*a^5
*c^3 - 8*a^4*b*c^3 + 8*a^3*b^2*c^3 - 16*a^4*c^4) + 1/2*sqrt(-x^2 + 1)*x/c +
 1/4*(pi*sgn(x) + 2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x
^2 + 1) - 1)))*(2*b + c)/c^2
```

Mupad [B]
time = 1.30, size = 1024, normalized size = 3.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] asin(x)*((b/c + 1)/c - 1/(2*c)) + (x*(1 - x^2)^(1/2))/(2*c) - (log((((x*(-(
b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(
2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2
- 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/
2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*
(4*a*c - b^2)) + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i
)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-
(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(
2*c))^(3/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (
b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))
```

```
^(3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) + (log((((x*(-(b - (b^2 - 4*a*
c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)
 + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2
*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))
/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b
 - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(3/2)))/(2*c*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
- (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 -
4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + a
*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1
/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-
(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*c*(4*a*c - b^2)*((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2))
```

3.382 $\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=263

$$-\frac{\sin^{-1}(x)}{c}+\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{\left(b+c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}}{c\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{}}$$

```
[Out] -arcsin(x)/c+arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-
4*a*c+b^2)^(1/2))^(1/2))*(b+c+(2*a*c-b^2-b*c)/(-4*a*c+b^2)^(1/2))/c/(b-(-4*
a*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(b+2*c+(-4*
a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+c+(-2
*a*c+b^2+b*c)/(-4*a*c+b^2)^(1/2))/c/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4
*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 1.30, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1307, 222, 1706, 385, 211}

$$\frac{\left(-\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\text{ArcTan}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\,\sqrt{b-\sqrt{b^2-4ac}}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{-\sqrt{b^2-4ac}+b+2c}}+\frac{\left(\frac{-2ac+b^2+bc}{\sqrt{b^2-4ac}}+b+c\right)\text{ArcTan}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\,\sqrt{\sqrt{b^2-4ac}+b}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\text{ArcSin}(x)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(ArcSin[x]/c) + ((b + c - (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(S
qrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x
^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) +
((b + c + (b^2 - 2*a*c + b*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqr
t[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(c*Sqrt[b
+ Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = -\frac{\int \frac{1}{\sqrt{1-x^2}}\,dx}{c}-\frac{\int \frac{-a-(b+c)x^2}{\sqrt{1-x^2}\,(a+bx^2+cx^4)}\,dx}{c}$$

$$= -\frac{\sin^{-1}(x)}{c}-\frac{\int \left(\frac{-b-c+\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}+\frac{-b-c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right) d}{c}$$

$$= -\frac{\sin^{-1}(x)}{c}+\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{c}+\frac{\Big(b+c+}{}$$

$$= -\frac{\sin^{-1}(x)}{c}+\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\Big(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,d}{c}$$

$$= -\frac{\sin^{-1}(x)}{c}+\frac{\left(b+c-\frac{b^2-2ac+bc}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}}+\frac{\Big(b+c}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.43, size = 412, normalized size = 1.57

$$-\frac{8\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)+\text{RootSum}\left[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4+4a\#1^6+4b\#1^6+a\#1^8\&,\frac{-a\log(x)+a\log\left(-1+\sqrt{1-x^2}-x\#1\right)-3a\log(x)\#1^2-4b\log(x)\#1^2-4c\log(x)\#1^2+3a\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^2+4b\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^2+4c\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^2-3a\log(x)\#1^4-4b\log(x)\#1^4-4c\log(x)\#1^4+3a\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^4+4b\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^4+4c\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^4-a\log(x)\#1^6+a\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^6}{a\#1+b\#1+3a\#1^3+4b\#1^3+8c\#1^3+3a\#1^5+3b\#1^5+a\#1^7}\&\right]}{4c}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/4*(8*ArcTan[x/(-1 + Sqrt[1 - x^2])] + RootSum[a + 4*a*#1^2 + 4*b*#1^2 +
6*a*#1^4 + 8*b*#1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-(a*Log
[x]) + a*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a*Log[x]*#1^2 - 4*b*Log[x]*#1^2
 - 4*c*Log[x]*#1^2 + 3*a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*b*Log[-1 +
 Sqrt[1 - x^2] - x*#1]*#1^2 + 4*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 - 3*a
*Log[x]*#1^4 - 4*b*Log[x]*#1^4 - 4*c*Log[x]*#1^4 + 3*a*Log[-1 + Sqrt[1 - x^
2] - x*#1]*#1^4 + 4*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 4*c*Log[-1 + Sq
rt[1 - x^2] - x*#1]*#1^4 - a*Log[x]*#1^6 + a*Log[-1 + Sqrt[1 - x^2] - x*#1]
*#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5 + 3*b*#1^5
+ a*#1^7) & ])/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.13, size = 175, normalized size = 0.67

method	result
default	$\frac{2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)}{c}-\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)}\frac{\left(a\_R^6+(4c+3a+4b)\_R^4+(\dots\right.}{\_R^7a+3\_R^5a+3\_R\dots}}{4c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 2/c*arctan(((-x^2+1)^(1/2)-1)/x)-1/4/c*sum((a*_R^6+(4*c+3*a+4*b)*_R^4+(4*c+
3*a+4*b)*_R^2+a)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+
_R*b)*ln(((-x^2+1)^(1/2)-1)/x-_R),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+
16*c)*_Z^4+(4*a+4*b)*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

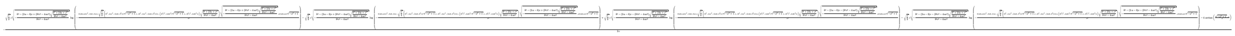
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1430 vs. $2(223) = 446$.
time = 0.73, size = 1430, normalized size = 5.44



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/2*(sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2
+ 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*
c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(
-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2 - 4*a*b*c^3)*
sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c
^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3)*sqrt((b^2 +
2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqr
t(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c
^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3))*log
(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2)*((b^3 - 4*a*c^2 - (4*a*b -
 b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*x - ((b^3*c^2
 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqrt((b^2 + 2*b*c
 + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c + (b^2*c^2 - 4*a*c^3
)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4*a*c^3)) + 2*(
a*b + a*c)*sqrt(-x^2 + 1))/x^2) + sqrt(1/2)*c*sqrt(-(b^2 - (2*a - b)*c - (b
^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 -
 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c + sqrt(1/2)*((b^3 - 4*a*
c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4*a*b - b^2)*c)
*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a*b*c^3)*x)*sqr
t((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a - b)*c - (b^2
*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))/(b^2*c^2 - 4
*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - sqrt(1/2)*c*sqrt(-(b^2 - (2
*a - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5
)))/(b^2*c^2 - 4*a*c^3))*log(-(2*(a*b + a*c)*x^2 - 2*a*b - 2*a*c - sqrt(1/2
)*((b^3 - 4*a*c^2 - (4*a*b - b^2)*c)*sqrt(-x^2 + 1)*x - (b^3 - 4*a*c^2 - (4
*a*b - b^2)*c)*x + ((b^3*c^2 - 4*a*b*c^3)*sqrt(-x^2 + 1)*x - (b^3*c^2 - 4*a
*b*c^3)*x)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5)))*sqrt(-(b^2 - (2*a
 - b)*c - (b^2*c^2 - 4*a*c^3)*sqrt((b^2 + 2*b*c + c^2)/(b^2*c^4 - 4*a*c^5))
)/(b^2*c^2 - 4*a*c^3)) + 2*(a*b + a*c)*sqrt(-x^2 + 1))/x^2) - 4*arctan((sqr
t(-x^2 + 1) - 1)/x))/c
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2 \sqrt{-(x-1)(x+1)}}{a + bx^2 + cx^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 3580 vs. $2(223) = 446$.
time = 5.38, size = 3580, normalized size = 13.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

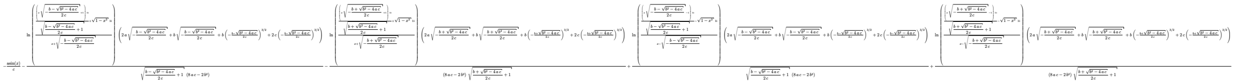
```
[In] integrate(x^2*(-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(pi*sgn(x) + 2*arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)^2/x^2 - 1)/(sqrt(-x
^2 + 1) - 1)))/c - 1/8*((2*a^2*b^4 - 16*a^3*b^2*c + 32*a^4*c^2 + 3*sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2 + 2*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3 - sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4 - 12*sqrt(2
)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c - 8*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c + 8*sq
rt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^2*c - 1
6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*c^2
 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3*c)*c^2*abs(a) + 2*(3*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c + 5*sqrt(2)*sqrt(2*a^2
 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^3*c + sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b
^2 - 4*a*c)*a)*a*b^4*c + 2*a^2*b^4*c - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2
- 4*a*c)*a)*b^5*c + 2*a*b^5*c - 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*
a*c)*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*b*
c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b^2*c^2 - 16*a^
3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^3*c^2 -
16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*b^4*c^2 +
2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^3*c^3 +
32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*b*c^3 +
 32*a^3*b*c^3 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a*b^2*c^3
 - 16*a^2*b^2*c^3 - 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*a^2*
c^4 + 32*a^3*c^4 - 2*(b^2 - 4*a*c)*a^2*b^2*c - 2*(b^2 - 4*a*c)*a*b^3*c + 8*
(b^2 - 4*a*c)*a^3*c^2 + 8*(b^2 - 4*a*c)*a^2*b*c^2 - 2*(b^2 - 4*a*c)*a*b^2*c
^2 + 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4*a^3*b^3*c^2 + 2*a^2*b^4*c^
2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 - 16*a^3*b*c^4 + 6*sqrt(2)*sq
rt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c^2 + 7*sqrt(
2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^2*c^2 -
sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^4*c^2 +
 12*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c
^3 + 22*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a
```

```
^2*b*c^3 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a
*c)*a*b^2*c^3 - 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2
- 4*a*c)*b^3*c^3 + 16*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(
b^2 - 4*a*c)*a^2*c^4 + 8*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sq
rt(b^2 - 4*a*c)*a*b*c^4 - 4*(b^2 - 4*a*c)*a^3*b*c^2 - 2*(b^2 - 4*a*c)*a^2*b
^2*c^2 - 8*(b^2 - 4*a*c)*a^3*c^3 - 4*(b^2 - 4*a*c)*a^2*b*c^3)*abs(a))*arcta
n(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a*
c + b*c + sqrt(-4*(a*c + b*c + c^2)*a*c + (2*a*c + b*c)^2))/(a*c)))/((3*a^5
*b^2*c^2 + 5*a^4*b^3*c^2 + a^3*b^4*c^2 - a^2*b^5*c^2 - 12*a^6*c^3 - 20*a^5*
b*c^3 + 3*a^4*b^2*c^3 + 10*a^3*b^3*c^3 - a^2*b^4*c^3 - 28*a^5*c^4 - 24*a^4*
b*c^4 + 8*a^3*b^2*c^4 - 16*a^4*c^5)*abs(c)) - 1/8*((2*a^2*b^4 - 16*a^3*b^2*
c + 32*a^4*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2
 - 4*a*c)*a^2*b^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(
b^2 - 4*a*c)*a*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b
^2 - 4*a*c)*b^4 - 12*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b
^2 - 4*a*c)*a^3*c - 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(
b^2 - 4*a*c)*a^2*b*c + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sq
rt(b^2 - 4*a*c)*a*b^2*c - 16*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*a^2*c^2 - 2*(b^2 - 4*a*c)*a^2*b^2 + 8*(b^2 - 4*a*c)*a^3
*c)*c^2*abs(a) + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b
^2*c + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^3*c + sqrt(2
)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a*b^4*c - 2*a^2*b^4*c - sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*b^5*c - 2*a*b^5*c - 12*sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*c^2 - 20*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*a^3*b*c^2 + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*
a*c)*a)*a^2*b^2*c^2 + 16*a^3*b^2*c^2 + 10*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b
^2 - 4*a*c)*a)*a*b^3*c^2 + 16*a^2*b^3*c^2 - sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*b^4*c^2 - 2*a*b^4*c^2 - 28*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*a^3*c^3 - 32*a^4*c^3 - 24*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(
b^2 - 4*a*c)*a)*a^2*b*c^3 - 32*a^3*b*c^3 + 8*sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*a*b^2*c^3 + 16*a^2*b^2*c^3 - 16*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c)*a)*a^2*c^4 - 32*a^3*c^4 + 2*(b^2 - 4*a*c)*a^2*b^2*c + 2
*(b^2 - 4*a*c)*a*b^3*c - 8*(b^2 - 4*a*c)*a^3*c^2 - 8*(b^2 - 4*a*c)*a^2*b*c^
2 + 2*(b^2 - 4*a*c)*a*b^2*c^2 - 8*(b^2 - 4*a*c)*a^2*c^3)*abs(a)*abs(c) + (4
*a^3*b^3*c^2 + 2*a^2*b^4*c^2 - 16*a^4*b*c^3 + 4*a^2*b^3*c^3 - 32*a^4*c^4 -
16*a^3*b*c^4 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqr...
```

Mupad [B]

time = 1.27, size = 870, normalized size = 3.31



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - x^2)^(1/2))/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(
-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(3/2) + 2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2)*(8*a*c - 2*b^2)) - (log((((x*(-(b - (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*
a*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/
(2*c))^(1/2) + b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b - (b^2
 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2
)*(8*a*c - 2*b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1
)*1i)/((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x
+ (-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/
2)))/((8*a*c - 2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - asin(x
)/c + (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^
2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 -
 4*a*c)^(1/2))/(2*c))^(1/2)))*(2*a*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + b*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(3/2) + 2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/((8*a*c -
2*b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

3.383 $\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=220

$$\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{b+2c+\sqrt{b^2-4ac}}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}}$$

```
[Out] arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b-(-4*a*c+b^2)^(1
/2))^(1/2))*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)^(1/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2)-arctan(x*(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/
(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+2*c+(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)
^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1188, 399, 222, 385, 211}

$$\frac{\sqrt{-\sqrt{b^2-4ac}+b+2c}\,\text{ArcTan}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\,\sqrt{b-\sqrt{b^2-4ac}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}}-\frac{\sqrt{\sqrt{b^2-4ac}+b+2c}\,\text{ArcTan}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\,\sqrt{\sqrt{b^2-4ac}+b}}\right)}{\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]
]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[
b - Sqrt[b^2 - 4*a*c]]) - (Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[b
 + 2*c + Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])]
)/(Sqrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 399

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Di
st[b/d, Int[(a + b*x^n)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^
n)^(p - 1)/(c + d*x^n), x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*
d, 0] && EqQ[n*(p - 1) + 1, 0] && IntegerQ[n]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{\sqrt{1-x^2}}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{\sqrt{1-x^2}}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\int \frac{1}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(b+2c+\sqrt{b^2-4ac}\right.}{}$$

$$= \frac{\left(b+2c-\sqrt{b^2-4ac}\right)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-b-2c+\sqrt{b^2-4ac}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x}}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}} - \frac{\sqrt{b+2c+\sqrt{b^2-}}}{\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.12, size = 155, normalized size = 0.70

$$\frac{1}{2}i\text{RootSum}\left[16a+16b+16c-32a\#1-32b\#1-32c\#1+16a\#1^2+20b\#1^2+24c\#1^2-4b\#1^3-8c\#1^3+c\#1^4\&, \frac{\log\left(2-2x^2-2ix\sqrt{1-x^2}-\#1\right)\#1^2}{-8a-8b-8c+8a\#1+10b\#1+12c\#1-3b\#1^2-6c\#1^2+c\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(a + b*x^2 + c*x^4),x]
```

```
[Out] (I/2)*RootSum[16*a + 16*b + 16*c - 32*a*#1 - 32*b*#1 - 32*c*#1 + 16*a*#1^2
+ 20*b*#1^2 + 24*c*#1^2 - 4*b*#1^3 - 8*c*#1^3 + c*#1^4 & , (Log[2 - 2*x^2 -
 (2*I)*x*Sqrt[1 - x^2] - #1]*#1^2)/(-8*a - 8*b - 8*c + 8*a*#1 + 10*b*#1 + 1
2*c*#1 - 3*b*#1^2 - 6*c*#1^2 + c*#1^3) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 130, normalized size = 0.59

method	result
default	$-\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)}\frac{\left(\_R^6-\_R^4-\_R^2+1\right)\ln\left(\frac{\sqrt{-x^2+1}-1}{x}-\_R\right)}{\_R^7a+3\_R^5a+3\_R^5b+3\_R^3a+4\_R^3b+8c\_R^3+\_Ra}\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4*sum((_R^6-_R^4-_R^2+1)/(_R^7*a+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_
R^3*c+_R*a+_R*b)*ln(((-x^2+1)^(1/2)-1)/x-_R),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^
6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^2+a))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/(c*x^4 + b*x^2 + a), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 759 vs. 2(180) = 360.
time = 0.45, size = 759, normalized size = 3.45

$$\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2a+b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(-\frac{x^2+\frac{\sqrt{\frac{1}{2}}\left((ab^2-4a^2c)\sqrt{-x^2+1}\,x-(ab^2-4a^2c)x\right)\sqrt{-\frac{2a+b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}}{\sqrt{a^2b^2-4a^3c}}+\sqrt{-x^2+1}-1}{x^2}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2a+b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(-\frac{x^2-\frac{\sqrt{\frac{1}{2}}\left((ab^2-4a^2c)\sqrt{-x^2+1}\,x-(ab^2-4a^2c)x\right)\sqrt{-\frac{2a+b+\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}}{\sqrt{a^2b^2-4a^3c}}+\sqrt{-x^2+1}-1}{x^2}\right)-\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2a+b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(-\frac{x^2+\frac{\sqrt{\frac{1}{2}}\left((ab^2-4a^2c)\sqrt{-x^2+1}\,x-(ab^2-4a^2c)x\right)\sqrt{-\frac{2a+b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}}{\sqrt{a^2b^2-4a^3c}}+\sqrt{-x^2+1}-1}{x^2}\right)+\frac{1}{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{2a+b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}\log\left(-\frac{x^2-\frac{\sqrt{\frac{1}{2}}\left((ab^2-4a^2c)\sqrt{-x^2+1}\,x-(ab^2-4a^2c)x\right)\sqrt{-\frac{2a+b-\frac{ab^2-4a^2c}{\sqrt{a^2b^2-4a^3c}}}{ab^2-4a^2c}}}{\sqrt{a^2b^2-4a^3c}}+\sqrt{-x^2+1}-1}{x^2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3*c))/(
a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2 + 1)*x
```

```
 - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4
*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1) - 1)/x
^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 - 4*a^3
*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt(-x^2
+ 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b + (a*b^2 - 4*a^2*c)/sqrt(a^2*b
^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2 + 1)
- 1)/x^2) - 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2*b^2 -
 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 + sqrt(1/2)*((a*b^2 - 4*a^2*c)*sqrt
(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt
(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqrt(-x^2
 + 1) - 1)/x^2) + 1/2*sqrt(1/2)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c)/sqrt(a^2
*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))*log(-(x^2 - sqrt(1/2)*((a*b^2 - 4*a^2*c
)*sqrt(-x^2 + 1)*x - (a*b^2 - 4*a^2*c)*x)*sqrt(-(2*a + b - (a*b^2 - 4*a^2*c
)/sqrt(a^2*b^2 - 4*a^3*c))/(a*b^2 - 4*a^2*c))/sqrt(a^2*b^2 - 4*a^3*c) + sqr
t(-x^2 + 1) - 1)/x^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-(x-1)(x+1)}}{a+bx^2+cx^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(a + b*x**2 + c*x**4), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(180) = 360.
time = 5.62, size = 641, normalized size = 2.91

$$-\frac{\left(2a^2b^2-8a^3c+3\sqrt{2}\sqrt{2a^2+ab+\sqrt{b^2-4ac}\,a}\sqrt{b^2-4ac}\,a^2+2\sqrt{2}\sqrt{2a^2+ab+\sqrt{b^2-4ac}\,a}\sqrt{b^2-4ac}\,ab-\sqrt{2}\sqrt{2a^2+ab+\sqrt{b^2-4ac}\,a}\sqrt{b^2-4ac}\,b^2+4\sqrt{2}\sqrt{2a^2+ab+\sqrt{b^2-4ac}\,a}\sqrt{b^2-4ac}\,ac-2(b^2-4ac)a^2\right)|a|\arctan\left(-\frac{\sqrt{2}\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)}{2\sqrt{\frac{2a+b+\sqrt{(2a+b)^2-4(a+b+c)a}}{a}}}\right)}{2(3a^4b^2+2a^3b^3-a^2b^4-12a^5c-8a^4bc+8a^3b^2c-16a^4c^2)}-\frac{\left(2a^2b^2-8a^3c+3\sqrt{2}\sqrt{2a^2+ab-\sqrt{b^2-4ac}\,a}\sqrt{b^2-4ac}\,a^2+2\sqrt{2}\sqrt{2a^2+ab-\sqrt{b^2-4ac}\,a}\sqrt{b^2-4ac}\,ab-\sqrt{2}\sqrt{2a^2+ab-\sqrt{b^2-4ac}\,a}\sqrt{b^2-4ac}\,b^2+4\sqrt{2}\sqrt{2a^2+ab-\sqrt{b^2-4ac}\,a}\sqrt{b^2-4ac}\,ac-2(b^2-4ac)a^2\right)|a|\arctan\left(-\frac{\sqrt{2}\left(\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}\right)}{2\sqrt{\frac{2a+b-\sqrt{(2a+b)^2-4(a+b+c)a}}{a}}}\right)}{2(3a^4b^2+2a^3b^3-a^2b^4-12a^5c-8a^4bc+8a^3b^2c-16a^4c^2)}$$

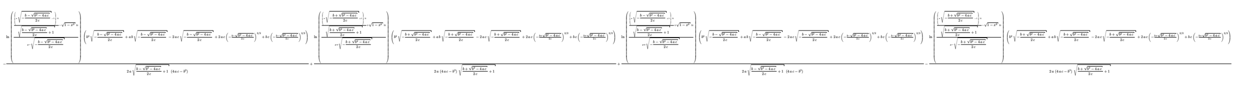
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2*b^2 - 8*a^3*c + 3*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*
a)*sqrt(b^2 - 4*a*c)*a^2 + 2*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a
)*sqrt(b^2 - 4*a*c)*a*b - sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*s
qrt(b^2 - 4*a*c)*b^2 + 4*sqrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sq
rt(b^2 - 4*a*c)*a*c - 2*(b^2 - 4*a*c)*a^2)*abs(a)*arctan(-1/2*sqrt(2)*(x/(s
qrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a + b + sqrt((2*a + b)
^2 - 4*(a + b + c)*a))/a))/(3*a^4*b^2 + 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*
a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2) - 1/2*(2*a^2*b^2 - 8*a^3*c + 3*sqrt(2)*
sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2 + 2*sqrt(2)*s
```

```
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b - sqrt(2)*sqrt
(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^2 + 4*sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*c - 2*(b^2 - 4*a*c)*
a^2)*abs(a)*arctan(-1/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) -
 1)/x)/sqrt((2*a + b - sqrt((2*a + b)^2 - 4*(a + b + c)*a))/a))/(3*a^4*b^2
+ 2*a^3*b^3 - a^2*b^4 - 12*a^5*c - 8*a^4*b*c + 8*a^3*b^2*c - 16*a^4*c^2)
```

Mupad [B]
time = 1.27, size = 989, normalized size = 4.50



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(a + b*x^2 + c*x^4),x)
```

```
[Out] (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b
*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(4*a*c - b^2)*((b + (b^2 - 4*a*
c)^(1/2))/(2*c) + 1)^(1/2)) - (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^
(1/2) - 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2
)*1i)/(x - (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b - (b^2 - 4*
a*c)^(1/2))/(2*c))^(1/2) + a*b*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2
*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b - (b^2 - 4*a*c)^
(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(
(b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)) + (log((((x*(-(b
- (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(1/2))/(2*
c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2))/(2*c))
^(1/2)))*(b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*(-(b - (b^2 -
4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)
 + 2*a*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b*c*(-(b - (b^2 - 4*a*c
)^(1/2))/(2*c))^(3/2)))/(2*a*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4
*a*c - b^2)) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/
((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b
 + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*
c))^(1/2) + a*b*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c*(-(b + (b^
2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 2*a*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(
3/2) + b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a*(4*a*c - b^2)*((
b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2))
```

3.384 $\int \frac{\sqrt{1-x^2}}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=265

$$-\frac{\sqrt{1-x^2}}{ax}-\frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{\cdots}}{\sqrt{\cdots}}\right)}{a\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{b\cdots}}$$

```
[Out] -(-x^2+1)^(1/2)/a/x-c*arctan(x*(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+(2*a+b)/(-4*a*c+b^2)^(1/2))/a/(b-(-4*a
*c+b^2)^(1/2))^(1/2)/(b+2*c-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(b+2*c+(-4
*a*c+b^2)^(1/2))^(1/2)/(-x^2+1)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(1+(-2*
a-b)/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(b+2*c+(-4*a*c+b^2)
^(1/2))^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 265, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1309, 270, 1706, 385, 211}

$$-\frac{c\left(\frac{2a+b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{x\sqrt{-\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{b-\sqrt{b^2-4ac}}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{-\sqrt{b^2-4ac}+b+2c}}-\frac{c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{x\sqrt{\sqrt{b^2-4ac}+b+2c}}{\sqrt{1-x^2}\sqrt{\sqrt{b^2-4ac}+b}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{\sqrt{b^2-4ac}+b+2c}}-\frac{\sqrt{1-x^2}}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[1 - x^2]/(a*x)) - (c*(1 + (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[
b + 2*c - Sqrt[b^2 - 4*a*c]]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])
])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[b + 2*c - Sqrt[b^2 - 4*a*c]]) - (c*(
1 - (2*a + b)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]]*
x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[1 - x^2])])/(a*Sqrt[b + Sqrt[b^2 - 4*a
*c]]*Sqrt[b + 2*c + Sqrt[b^2 - 4*a*c]])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
```

```
p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1309

```
Int[(((f_.)*(x_))^(m_)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[d/a, Int[(f*x)^m*(d + e*x^2)^(q - 1), x],
x] - Dist[1/(a*f^2), Int[(f*x)^(m + 2)*(d + e*x^2)^(q - 1)*(Simp[b*d - a*e
+ c*d*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && LtQ[m, 0]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2+cx^4\right)}\,dx = \frac{\int \frac{1}{x^2\sqrt{1-x^2}}\,dx}{a} - \frac{\int \frac{a+b+cx^2}{\sqrt{1-x^2}\,(a+bx^2+cx^4)}\,dx}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\int \left(\frac{c+\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b-\sqrt{b^2-4ac}+2cx^2\right)} + \frac{c-\frac{(2a+b)c}{\sqrt{b^2-4ac}}}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right.}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\,dx}{a} - \frac{\left(c\right.}{}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{\left(c\left(1-\frac{2a+b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{1}{b+\sqrt{b^2-4ac}-\left(-b-2c-\sqrt{b^2-4ac}\right.}\right.}{a}$$

$$= -\frac{\sqrt{1-x^2}}{ax} - \frac{c\left(1+\frac{2a+b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{b+2c-\sqrt{b^2-4ac}}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{1-x^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{b+2c-\sqrt{b^2-4ac}}} - \frac{c\left(1\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.46, size = 471, normalized size = 1.78

$$-\frac{\sqrt{1-x^2}}{ax}+\frac{\text{RootSum}\left[a+4a\#1^2+4b\#1^2+6a\#1^4+8b\#1^4+16c\#1^4+4a\#1^6+4b\#1^6+a\#1^8\&,\frac{-a\log(x)-b\log(x)+a\log\left(-1+\sqrt{1-x^2}-x\#1\right)+b\log\left(-1+\sqrt{1-x^2}-x\#1\right)-3a\log(x)\#1^2-3b\log(x)\#1^2-4c\log(x)\#1^2+3a\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^2+3b\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^2+4c\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^2-3a\log(x)\#1^4-3b\log(x)\#1^4-4c\log(x)\#1^4+3a\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^4+3b\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^4+4c\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^4-a\log(x)\#1^6-b\log(x)\#1^6+a\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^6+b\log\left(-1+\sqrt{1-x^2}-x\#1\right)\#1^6}{a\#1+b\#1+3a\#1^3+4b\#1^3+8c\#1^3+3a\#1^5+3b\#1^5+a\#1^7}\&\right]}{4a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - x^2]/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[1 - x^2]/(a*x)) + RootSum[a + 4*a*#1^2 + 4*b*#1^2 + 6*a*#1^4 + 8*b*#
1^4 + 16*c*#1^4 + 4*a*#1^6 + 4*b*#1^6 + a*#1^8 & , (-(a*Log[x]) - b*Log[x]
+ a*Log[-1 + Sqrt[1 - x^2] - x*#1] + b*Log[-1 + Sqrt[1 - x^2] - x*#1] - 3*a
*Log[x]*#1^2 - 3*b*Log[x]*#1^2 - 4*c*Log[x]*#1^2 + 3*a*Log[-1 + Sqrt[1 - x^
2] - x*#1]*#1^2 + 3*b*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^2 + 4*c*Log[-1 + Sq
rt[1 - x^2] - x*#1]*#1^2 - 3*a*Log[x]*#1^4 - 3*b*Log[x]*#1^4 - 4*c*Log[x]*#
1^4 + 3*a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 + 3*b*Log[-1 + Sqrt[1 - x^2]
- x*#1]*#1^4 + 4*c*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^4 - a*Log[x]*#1^6 - b*
Log[x]*#1^6 + a*Log[-1 + Sqrt[1 - x^2] - x*#1]*#1^6 + b*Log[-1 + Sqrt[1 - x
^2] - x*#1]*#1^6)/(a*#1 + b*#1 + 3*a*#1^3 + 4*b*#1^3 + 8*c*#1^3 + 3*a*#1^5
+ 3*b*#1^5 + a*#1^7) & ]/(4*a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 212, normalized size = 0.80

method	result
risch	$\frac{x^2-1}{ax\sqrt{-x^2+1}}+\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)}\frac{\left((a+b)\_R^6+(3a+3b+4c)\_R^4+(3a+3b+4c\ldots\right.}{\_R^7a+3\_R^5a+3\_R^5b+3\_R\ldots}}{4a}$
default	$\frac{\left(\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^8+(4a+4b)\_Z^6+(6a+8b+16c)\_Z^4+(4a+4b)\_Z^2+a\right)}\frac{\left((a+b)\_R^6+(3a+3b+4c)\_R^4+(3a+3b+4c)\_R^2+a+b\right)\ln\left(\sqrt{\ldots}\right.}{\_R^7a+3\_R^5a+3\_R^5b+3\_R^3a+4\_R^3b+8c\_R\ldots}}{4}\right.}{a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/a*(1/4*sum(((a+b)*_R^6+(3*a+3*b+4*c)*_R^4+(3*a+3*b+4*c)*_R^2+a+b)/(_R^7*a
+3*_R^5*a+3*_R^5*b+3*_R^3*a+4*_R^3*b+8*_R^3*c+_R*a+_R*b)*ln(((-x^2+1)^(1/2)
-1)/x-_R),_R=RootOf(a*_Z^8+(4*a+4*b)*_Z^6+(6*a+8*b+16*c)*_Z^4+(4*a+4*b)*_Z^
2+a))-2*arctan(((-x^2+1)^(1/2)-1)/x))+1/a*(-1/x*(-x^2+1)^(3/2)-x*(-x^2+1)^(
1/2)-arcsin(x))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1998 vs. $2(221) = 442$.
time = 0.56, size = 1998, normalized size = 7.54

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/2*(sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x - ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c + (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) + sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c + sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
```

```
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - sqrt(1/2)*a*x*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^
4*c)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^
2 - 4*a^7*c)))/(a^3*b^2 - 4*a^4*c))*log((2*a*c^2 - 2*(a*c^2 - (a*b + b^2)*c
)*x^2 - 2*(a*b + b^2)*c - sqrt(1/2)*((a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b +
5*a*b^2)*c)*sqrt(-x^2 + 1)*x - (a*b^3 + b^4 + 4*a^2*c^2 - (4*a^2*b + 5*a*b^
2)*c)*x + ((a^3*b^3 - 4*a^4*b*c)*sqrt(-x^2 + 1)*x - (a^3*b^3 - 4*a^4*b*c)*x
)*sqrt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 -
 4*a^7*c)))*sqrt(-(a*b^2 + b^3 - (2*a^2 + 3*a*b)*c - (a^3*b^2 - 4*a^4*c)*sq
rt((a^2*b^2 + 2*a*b^3 + b^4 + a^2*c^2 - 2*(a^2*b + a*b^2)*c)/(a^6*b^2 - 4*a
^7*c)))/(a^3*b^2 - 4*a^4*c)) - 2*(a*c^2 - (a*b + b^2)*c)*sqrt(-x^2 + 1))/x^
2) - 2*sqrt(-x^2 + 1))/(a*x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{-\left(x-1\right)\left(x+1\right)}}{x^{2}\left(a+bx^{2}+cx^{4}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-x**2+1)**(1/2)/x**2/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(sqrt(-(x - 1)*(x + 1))/(x**2*(a + b*x**2 + c*x**4)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3965 vs. 2(221) = 442.
time = 9.25, size = 3965, normalized size = 14.96

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

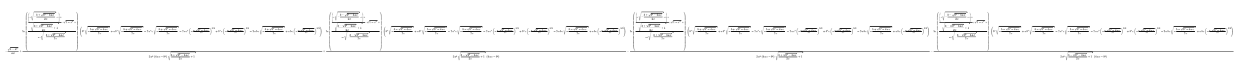
```
[In] integrate((-x^2+1)^(1/2)/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] 1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 12*a^5
*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
- 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^
2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b - sqr
t(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*b - sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a^2 + a*b
 - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqrt(2*a^2 +
 a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(2)*sqrt(2*
```

a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 16*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*c^2 - 8*sqrt(2
)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b*c^2 - 4*(
b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c)*a^4*b^3 + 8*
(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 + 2*a^2*b^5 - 16
*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*sqrt(2)*sqrt(2*a^
2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 + 5*sqrt(2)*sqrt(2
*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^3 + sqrt(2)*sqrt(
2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^4 - sqrt(2)*sqrt(2
*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^5 - 12*sqrt(2)*sqrt(2
*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*c - 20*sqrt(2)*sqrt
(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b*c + 8*sqrt(2)*s
qrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^3*c - 16*sqrt(
2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*c^2 - 16*s
qrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b*c^2
- 2*(b^2 - 4*a*c)*a^3*b^2 - 2*(b^2 - 4*a*c)*a^2*b^3 + 8*(b^2 - 4*a*c)*a^4*c
 + 8*(b^2 - 4*a*c)*a^3*b*c)*a^2 + 2*(3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2
- 4*a*c)*a)*a^5*b^2 + 5*sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4
*b^3 + sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^3*b^4 - 2*a^4*b^4
- sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^2*b^5 - 2*a^3*b^5 - 12*
sqrt(2)*sqrt(2*a^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^6*c - 20*sqrt(2)*sqrt(2*a
^2 + a*b - sqrt(b^2 - 4*a*c)*a)*a^5*b*c + 3*sqrt(2)*sqrt(2*a^2 + a*b - sqrt
(b^2 - 4*a*c)*a)*a^4*b^2*c + 16*a^5*b^2*c + 10*sqrt(2)*sqrt(2*a^2 + a*b - s
qrt(b^2 - 4*a*c)*a)*a^3*b^3*c + 16*a^4*b^3*c - sqrt(2)*sqrt(2*a^2 + a*b - s
qrt(b^2 - 4*a*c)*a)*a^2*b^4*c - 2*a^3*b^4*c - 28*sqrt(2)*sqrt(2*a^2 + a*b -
 sqrt(b^2 - 4*a*c)*a)*a^5*c^2 - 32*a^6*c^2 - 24*sqrt(2)*sqrt(2*a^2 + a*b -
sqrt(b^2 - 4*a*c)*a)*a^4*b*c^2 - 32*a^5*b*c^2 + 8*sqrt(2)*sqrt(2*a^2 + a*b
- sqrt(b^2 - 4*a*c)*a)*a^3*b^2*c^2 + 16*a^4*b^2*c^2 - 16*sqrt(2)*sqrt(2*a^2
 + a*b - sqrt(b^2 - 4*a*c)*a)*a^4*c^3 - 32*a^5*c^3 + 2*(b^2 - 4*a*c)*a^4*b^
2 + 2*(b^2 - 4*a*c)*a^3*b^3 - 8*(b^2 - 4*a*c)*a^5*c - 8*(b^2 - 4*a*c)*a^4*b
c + 2(b^2 - 4*a*c)*a^3*b^2*c - 8*(b^2 - 4*a*c)*a^4*c^2)*abs(a))*arctan(-1
/2*sqrt(2)*(x/(sqrt(-x^2 + 1) - 1) - (sqrt(-x^2 + 1) - 1)/x)/sqrt((2*a^2 +
a*b + sqrt(-4*(a^2 + a*b + a*c)*a^2 + (2*a^2 + a*b)^2))/a^2))/(3*a^8*b^2 +
5*a^7*b^3 + a^6*b^4 - a^5*b^5 - 12*a^9*c - 20*a^8*b*c + 3*a^7*b^2*c + 10*a^
6*b^3*c - a^5*b^4*c - 28*a^8*c^2 - 24*a^7*b*c^2 + 8*a^6*b^2*c^2 - 16*a^7*c^
3) + 1/8*(4*a^6*b^3 + 6*a^5*b^4 + 2*a^4*b^5 - 16*a^7*b*c - 32*a^6*b^2*c - 1
2*a^5*b^3*c + 32*a^7*c^2 + 16*a^6*b*c^2 + 6*sqrt(2)*sqrt(2*a^2 + a*b + sqrt
(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*b + 13*sqrt(2)*sqrt(2*a^2 + a*b + sq
rt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b^2 + 7*sqrt(2)*sqrt(2*a^2 + a*b +
 sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^3 - sqrt(2)*sqrt(2*a^2 + a*b
+ sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^4 - sqrt(2)*sqrt(2*a^2 + a*b
 + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^5 - 12*sqrt(2)*sqrt(2*a^2 +
 a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^6*c - 6*sqrt(2)*sqrt(2*a^2
+ a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*b*c + 12*sqrt(2)*sqrt(2*
a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b^2*c + 6*sqrt(2)*sq

```
rt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^3*c - 16*sqrt
(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^5*c^2 - 8*s
qrt(2)*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*b*c^2
- 4*(b^2 - 4*a*c)*a^6*b - 6*(b^2 - 4*a*c)*a^5*b^2 - 2*(b^2 - 4*a*c)*a^4*b^3
 + 8*(b^2 - 4*a*c)*a^6*c + 4*(b^2 - 4*a*c)*a^5*b*c - (2*a^3*b^4 + 2*a^2*b^5
 - 16*a^4*b^2*c - 16*a^3*b^3*c + 32*a^5*c^2 + 32*a^4*b*c^2 + 3*sqrt(2)*sqrt
(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^3*b^2 + 5*sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^2*b^3 + sqrt(2)*
sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a*b^4 - sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*b^5 - 12*sqrt(2)*s
qrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b^2 - 4*a*c)*a^4*c - 20*sqrt(2)
*sqrt(2*a^2 + a*b + sqrt(b^2 - 4*a*c)*a)*sqrt(b...
```

Mupad [B]
time = 1.21, size = 1234, normalized size = 4.66



Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1 - x^2)^(1/2)/(x^2*(a + b*x^2 + c*x^4)),x)
```

```
[Out] (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/((b + (b^2 - 4*
a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b + (b^2 - 4*a*c
)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b
^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b + (b^2 - 4*a*c)^
(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b^
2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b + (b^2 - 4*a*c)
^(1/2))/(2*c))^(1/2) + a*b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*
a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (1 - x^2)^
(1/2)/(a*x) + (log((((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 1)*1i)/(
(b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2) - (1 - x^2)^(1/2)*1i)/(x - (-(b
- (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(1/2) + a*b^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b -
(b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*
c))^(3/2) + b^2*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b -
 (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c
))^(3/2)))/(2*a^2*((b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2)
) - (log((((x*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b + (b^2
- 4*a*c)^(1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b + (b^2 - 4
*a*c)^(1/2))/(2*c))^(1/2)))*(b^3*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) +
 a*b^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b + (b^2 - 4*a
*c)^(1/2))/(2*c))^(1/2) - 2*a*c^2*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)
+ b^2*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b + (b^2 - 4*
a*c)^(1/2))/(2*c))^(1/2) + a*b*c*(-(b + (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))
/(2*a^2*(4*a*c - b^2)*((b + (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)) - (log((
```

```
((x*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + 1)*1i)/((b - (b^2 - 4*a*c)^(
1/2))/(2*c) + 1)^(1/2) + (1 - x^2)^(1/2)*1i)/(x + (-(b - (b^2 - 4*a*c)^(1/2
))/(2*c))^(1/2)))*(b^3*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) + a*b^2*(-(
b - (b^2 - 4*a*c)^(1/2))/(2*c))^(1/2) - 2*a^2*c*(-(b - (b^2 - 4*a*c)^(1/2))
/(2*c))^(1/2) - 2*a*c^2*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) + b^2*c*(-
(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2) - 3*a*b*c*(-(b - (b^2 - 4*a*c)^(1/2)
)/(2*c))^(1/2) + a*b*c*(-(b - (b^2 - 4*a*c)^(1/2))/(2*c))^(3/2)))/(2*a^2*((
b - (b^2 - 4*a*c)^(1/2))/(2*c) + 1)^(1/2)*(4*a*c - b^2))
```

3.385 $\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx$

Optimal. Leaf size=96

$$-\sin^{-1}(x)+\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(-1+\sqrt{5}\right)}}{\sqrt{1-x^2}}\right.$$

```
[Out] -arcsin(x)-1/5*arctanh(1/2*x*(-2+2*5^(1/2))^(1/2)/(-x^2+1)^(1/2))*(-10+5*5^
(1/2))^(1/2)+1/5*arctan(1/2*x*(2+2*5^(1/2))^(1/2)/(-x^2+1)^(1/2))*(10+5*5^(
1/2))^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {1307, 222, 1706, 385, 213, 209}

$$-\text{ArcSin}(x)+\sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\,\text{ArcTan}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right)-\sqrt{\frac{1}{5}\left(\sqrt{5}-2\right)}\tanh^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(\sqrt{5}-1\right)}\,x}{\sqrt{1-x^2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] -ArcSin[x] + Sqrt[(2 + Sqrt[5])/5]*ArcTan[(Sqrt[(1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]] - Sqrt[(-2 + Sqrt[5])/5]*ArcTanh[(Sqrt[(-1 + Sqrt[5])/2]*x)/Sqrt[1
- x^2]]
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 222

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1307

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[e*(f^2/c), Int[(f*x)^(m - 2)*(d + e*x^2)^
(q - 1), x], x] - Dist[f^2/c, Int[(f*x)^(m - 2)*(d + e*x^2)^(q - 1)*(Simp[a
*e - (c*d - b*e)*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d,
 e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && GtQ[q, 0] && GtQ[m, 1
] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{x^2\sqrt{1-x^2}}{-1+x^2+x^4}\,dx &= -\int \frac{1}{\sqrt{1-x^2}}\,dx - \int \frac{1-2x^2}{\sqrt{1-x^2}\,(-1+x^2+x^4)}\,dx \\
&= -\sin^{-1}(x) - \int \left(\frac{-2+\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\left(1-\sqrt{5}+2x^2\right)} + \frac{-2-\frac{4}{\sqrt{5}}}{\sqrt{1-x^2}\left(1+\sqrt{5}+2x^2\right)}\right) dx \\
&= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\int \frac{1}{\sqrt{1-x^2}\left(1-\sqrt{5}+2x^2\right)}\,dx + \frac{1}{5}\left(2\left(5+2\sqrt{5}\right)\right. \\
&= -\sin^{-1}(x) + \frac{1}{5}\left(2\left(5-2\sqrt{5}\right)\right)\text{Subst}\left(\int \frac{1}{1-\sqrt{5}-\left(-3+\sqrt{5}\right)x^2}\,dx, x, \frac{x}{\sqrt{1-x^2}}\right. \\
&= -\sin^{-1}(x) + \sqrt{\frac{1}{5}\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{\frac{1}{2}\left(1+\sqrt{5}\right)}\,x}{\sqrt{1-x^2}}\right) - \sqrt{\frac{1}{5}\left(-2+\sqrt{5}\right)}\tanh
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 199 vs. 2(96) = 192.

time = 0.42, size = 199, normalized size = 2.07

$$\frac{1}{5}\left(-10\tan^{-1}\left(\frac{x}{-1+\sqrt{1-x^2}}\right)+\sqrt{5\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{-2+\sqrt{5}}\,x}{-1+\sqrt{1-x^2}}\right)+\sqrt{5\left(2+\sqrt{5}\right)}\tan^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\,x}{-1+\sqrt{1-x^2}}\right)+\sqrt{5\left(-2+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{2+\sqrt{5}}\,x}{1-\sqrt{1-x^2}}\right)+\sqrt{5\left(-2+\sqrt{5}\right)}\tanh^{-1}\left(\frac{\sqrt{-2+\sqrt{5}}\,x}{-1+\sqrt{1-x^2}}\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^2*Sqrt[1 - x^2])/(-1 + x^2 + x^4),x]
```

```
[Out] (-10*ArcTan[x/(-1 + Sqrt[1 - x^2])] + Sqrt[5*(2 + Sqrt[5])]*ArcTan[(Sqrt[-2
 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[5*(2 + Sqrt[5])]*ArcTan[(Sqrt[2
 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])] + Sqrt[5*(-2 + Sqrt[5])]*ArcTanh[(Sqrt
[2 + Sqrt[5]]*x)/(1 - Sqrt[1 - x^2])] + Sqrt[5*(-2 + Sqrt[5])]*ArcTanh[(Sqr
t[-2 + Sqrt[5]]*x)/(-1 + Sqrt[1 - x^2])])/5
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(71) = 142$.
time = 0.57, size = 160, normalized size = 1.67

method	result
default	$2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)-\frac{\sqrt{5}\,\sqrt{2+\sqrt{5}}\,\arctan\left(\frac{\sqrt{-x^2+1}-1}{x\sqrt{2+\sqrt{5}}}\right)}{5}+\frac{\sqrt{-2+\sqrt{5}}\,\sqrt{5}\,\operatorname{arctanh}\left(\right.}{5}$
trager	$\operatorname{RootOf}\left(\_Z^2+1\right)\ln\left(-\operatorname{RootOf}\left(\_Z^2+1\right)\sqrt{-x^2+1}+x\right)-\frac{\operatorname{RootOf}\left(\_Z^2+25\operatorname{RootOf}\left(400\_Z^4+80\_\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x,method=_RETURNVERBOSE)
```

```
[Out] 2*arctan(((-x^2+1)^(1/2)-1)/x)-1/5*5^(1/2)*(2+5^(1/2))^(1/2)*arctan(((-x^2+
1)^(1/2)-1)/x/(2+5^(1/2))^(1/2))+1/5*(-2+5^(1/2))^(1/2)*5^(1/2)*arctanh(((-
x^2+1)^(1/2)-1)/x/(-2+5^(1/2))^(1/2))-1/5*5^(1/2)/(-2+5^(1/2))^(1/2)*arctan
(((-x^2+1)^(1/2)-1)/x/(-2+5^(1/2))^(1/2))-1/5*5^(1/2)/(2+5^(1/2))^(1/2)*arc
tanh(((-x^2+1)^(1/2)-1)/x/(2+5^(1/2))^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-x^2 + 1)*x^2/(x^4 + x^2 - 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 290 vs. $2(71) = 142$.
time = 0.38, size = 290, normalized size = 3.02

$$\frac{2}{5}\sqrt{5}\sqrt{\sqrt{5}+2}\arctan\left(\frac{\sqrt{2}\left(\sqrt{-x^2+1}\left(\sqrt{5}-3\right)+\sqrt{5}-3\right)\sqrt{\sqrt{5}+2}\sqrt{\frac{x^4-4x^2-\sqrt{5}\left(x^4-2x^2\right)-2\left(\sqrt{5}x^2-x^2+2\right)\sqrt{-x^2+1}+4}{x^4}}+2\sqrt{-x^2+1}\sqrt{\sqrt{5}+2}\left(\sqrt{5}-3\right)}{4x}\right)+\frac{1}{10}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\frac{2x^2+\left(\sqrt{-x^2+1}\left(\sqrt{5}x+x\right)-\sqrt{5}x-x\right)\sqrt{\sqrt{5}-2}+2\sqrt{-x^2+1}-2}{x^2}\right)-\frac{1}{10}\sqrt{5}\sqrt{\sqrt{5}-2}\log\left(-\frac{2x^2-\left(\sqrt{-x^2+1}\left(\sqrt{5}x+x\right)-\sqrt{5}x-x\right)\sqrt{\sqrt{5}-2}+2\sqrt{-x^2+1}-2}{x^2}\right)+2\arctan\left(\frac{\sqrt{-x^2+1}-1}{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="fricas")
```

```
[Out] 2/5*sqrt(5)*sqrt(sqrt(5) + 2)*arctan(1/4*(sqrt(2)*(sqrt(-x^2 + 1)*(sqrt(5)
- 3) + sqrt(5) - 3)*sqrt(sqrt(5) + 2)*sqrt((x^4 - 4*x^2 - sqrt(5)*(x^4 - 2*
x^2) - 2*(sqrt(5)*x^2 - x^2 + 2)*sqrt(-x^2 + 1) + 4)/x^4) + 2*sqrt(-x^2 + 1
)*sqrt(sqrt(5) + 2)*(sqrt(5) - 3))/x) + 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(
-(2*x^2 + (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2
) + 2*sqrt(-x^2 + 1) - 2)/x^2) - 1/10*sqrt(5)*sqrt(sqrt(5) - 2)*log(-(2*x^2
 - (sqrt(-x^2 + 1)*(sqrt(5)*x + x) - sqrt(5)*x - x)*sqrt(sqrt(5) - 2) + 2*s
qrt(-x^2 + 1) - 2)/x^2) + 2*arctan((sqrt(-x^2 + 1) - 1)/x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2\sqrt{-(x-1)(x+1)}}{x^4+x^2-1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(-x**2+1)**(1/2)/(x**4+x**2-1),x)
```

```
[Out] Integral(x**2*sqrt(-(x - 1)*(x + 1))/(x**4 + x**2 - 1), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(71) = 142$.
time = 4.80, size = 209, normalized size = 2.18

$$-\frac{1}{2}\pi\mathrm{sgn}(x)-\frac{1}{5}\sqrt{5\sqrt{5}+10}\arctan\left(-\frac{\frac{x}{\sqrt{-x^2+1}-1}-\frac{\sqrt{-x^2+1}-1}{x}}{\sqrt{2\sqrt{5}+2}}\right)-\frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}\right|\right)+\frac{1}{10}\sqrt{5\sqrt{5}-10}\log\left(\left|-\sqrt{2\sqrt{5}-2}-\frac{x}{\sqrt{-x^2+1}-1}+\frac{\sqrt{-x^2+1}-1}{x}\right|\right)-\arctan\left(-\frac{x\left(\frac{\left(\sqrt{-x^2+1}-1\right)^2}{x^2}-1\right)}{2\left(\sqrt{-x^2+1}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(-x^2+1)^(1/2)/(x^4+x^2-1),x, algorithm="giac")
```

```
[Out] -1/2*pi*sgn(x) - 1/5*sqrt(5*sqrt(5) + 10)*arctan(-(x/(sqrt(-x^2 + 1) - 1) -
 (sqrt(-x^2 + 1) - 1)/x)/sqrt(2*sqrt(5) + 2)) - 1/10*sqrt(5*sqrt(5) - 10)*l
og(abs(sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/
x)) + 1/10*sqrt(5*sqrt(5) - 10)*log(abs(-sqrt(2*sqrt(5) - 2) - x/(sqrt(-x^2
 + 1) - 1) + (sqrt(-x^2 + 1) - 1)/x)) - arctan(-1/2*x*((sqrt(-x^2 + 1) - 1)
^2/x^2 - 1)/(sqrt(-x^2 + 1) - 1))
```

Mupad [B]
time = 1.50, size = 383, normalized size = 3.99

$$-\operatorname{asin}(x)-\frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}-2\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}-1\right)1\mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}-\sqrt{1-x^2}\,1\mathrm{i}}{x-\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}+2\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}+\frac{\ln\left(\frac{\frac{\left(x\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+1\right)1\mathrm{i}}{\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}-2\right)}{\left(2\sqrt{\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{3}{2}-\frac{\sqrt{5}}{2}}}-\frac{\ln\left(\frac{\frac{\left(x\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+1\right)1\mathrm{i}}{\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}+\sqrt{1-x^2}\,1\mathrm{i}}{x+\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}}\right)\left(\sqrt{5}+2\right)}{\left(2\sqrt{-\frac{\sqrt{5}}{2}-\frac{1}{2}}+4\left(-\frac{\sqrt{5}}{2}-\frac{1}{2}\right)^{3/2}\right)\sqrt{\frac{\sqrt{5}}{2}+\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(1 - x^2)^(1/2))/(x^2 + x^4 - 1),x)
```

```
[Out] (log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(5^(1/2)/2 + 3/2)^(1/2) - (1 -
 x^2)^(1/2)*1i)/(x - (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/((2*(- 5^(1
/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 3/2)^(1/2))
- (log((((x*(5^(1/2)/2 - 1/2)^(1/2) - 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2) - (1 -
 x^2)^(1/2)*1i)/(x - (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/((2*(5^(1/2)/
2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^(1/2)) - asin
(x) + (log((((x*(5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(3/2 - 5^(1/2)/2)^(1/2) +
(1 - x^2)^(1/2)*1i)/(x + (5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) - 2))/((2*(5^(1
/2)/2 - 1/2)^(1/2) + 4*(5^(1/2)/2 - 1/2)^(3/2))*(3/2 - 5^(1/2)/2)^(1/2)) -
(log((((x*(- 5^(1/2)/2 - 1/2)^(1/2) + 1)*1i)/(5^(1/2)/2 + 3/2)^(1/2) + (1 -
 x^2)^(1/2)*1i)/(x + (- 5^(1/2)/2 - 1/2)^(1/2)))*(5^(1/2) + 2))/((2*(- 5^(1
/2)/2 - 1/2)^(1/2) + 4*(- 5^(1/2)/2 - 1/2)^(3/2))*(5^(1/2)/2 + 3/2)^(1/2))
```

3.386 $\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=479

$$-\frac{3dx\sqrt{d+ex^2}}{8ce^2}-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{x^3\sqrt{d+ex^2}}{4ce}-\frac{\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4}\right.}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d}}\right.}{c^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 3/8*d^2*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(5/2)+1/2*b*d*arctanh(x*e^(1
/2)/(e*x^2+d)^(1/2))/c^2/e^(3/2)+(-a*c+b^2)*arctanh(x*e^(1/2)/(e*x^2+d)^(1/
2))/c^3/e^(1/2)-3/8*d*x*(e*x^2+d)^(1/2)/c/e^2-1/2*b*x*(e*x^2+d)^(1/2)/c^2/e
+1/4*x^3*(e*x^2+d)^(1/2)/c/e-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2
)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-2*a*b*c+(-2*a^2*c^2+4*
a*b^2*c-b^4)/(-4*a*c+b^2)^(1/2))/c^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/
2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^3-2*a*b*c+(2*a^2*c^2-4*
a*b^2*c+b^4)/(-4*a*c+b^2)^(1/2))/c^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 1.25, antiderivative size = 479, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.241, Rules used = {1317, 223, 212, 327, 1706, 385, 211}

$$-\frac{\left(-\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{2a^2c^2-4ab^2c+b^4}{\sqrt{b^2-4ac}}-2abc+b^3\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c^3\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{(b^2-ac)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c^3\sqrt{e}}+\frac{bd\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2c^2e^{3/2}}-\frac{bx\sqrt{d+ex^2}}{2c^2e}+\frac{3d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{8ce^{5/2}}-\frac{3dx\sqrt{d+ex^2}}{8ce^2}+\frac{x^3\sqrt{d+ex^2}}{4ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-3*d*x*Sqrt[d + e*x^2])/(8*c*e^2) - (b*x*Sqrt[d + e*x^2])/(2*c^2*e) + (x^3
*Sqrt[d + e*x^2])/(4*c*e) - ((b^3 - 2*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b - Sqrt[b^2 - 4*a*c]
]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - ((b^3 - 2*a*b*c + (b^4 - 4*a*b
^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^3*Sqrt[b +
Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + (3*d^2*ArcTan
h[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(8*c*e^(5/2)) + (b*d*ArcTanh[(Sqrt[e]*x)/Sq
rt[d + e*x^2]])/(2*c^2*e^(3/2)) + ((b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d +
 e*x^2]])/(c^3*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \left(\frac{b^2-ac}{c^3\sqrt{d+ex^2}} - \frac{bx^2}{c^2\sqrt{d+ex^2}} + \frac{x^4}{c\sqrt{d+ex^2}} - \frac{a(b^2-ac)+b(b^2-2\ldots}{c^3\sqrt{d+ex^2}\,(a+bx^2\ldots}\right.$$

$$= -\frac{\int \frac{a(b^2-ac)+b(b^2-2ac)x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{c^3} - \frac{b\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{c^2} + \frac{\int \frac{x^4}{\sqrt{d+ex^2}}\,dx}{c} + \ldots$$

$$= -\frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\int \left(\frac{b(b^2-2ac)+\frac{-b^4+4ab^2c-2a^2c^2}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \ldots\right.}{c^3 \ldots}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{(b^2-ac)\tanh^{-1}\left(\frac{\ldots}{\sqrt{\ldots}}\right.}{c^3\sqrt{e}}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} + \frac{bd\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex\ldots}}\right.}{2c^2e^{3/2}}$$

$$= -\frac{3dx\sqrt{d+ex^2}}{8ce^2} - \frac{bx\sqrt{d+ex^2}}{2c^2e} + \frac{x^3\sqrt{d+ex^2}}{4ce} - \frac{\left(b^3-2abc-\frac{b^4-4ab^2\ldots}{\sqrt{b^2-\ldots}}\right.}{c^3\sqrt{b-\sqrt{\ldots}}}$$

Mathematica [A]
time = 11.29, size = 461, normalized size = 0.96

$$\frac{-\frac{4bcx\sqrt{d+ex^2}}{e} + \frac{2c^2x^3\sqrt{d+ex^2}}{e} - \frac{8\left(b^3-2abc-\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}\,e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}} - \frac{8\left(b^3-2abc+\frac{b^4-4ab^2c+2a^2c^2}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{4bcd\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}} + \frac{8(b^2-ac)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}} + \frac{3c^2d\left(-\sqrt{e}\,x\sqrt{d+ex^2}+d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)\right)}{e^{5/2}}}{8c^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^8/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((-4*b*c*x*Sqrt[d + e*x^2])/e + (2*c^2*x^3*Sqrt[d + e*x^2])/e - (8*(b^3 - 2
*a*b*c - (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
]) - (8*(b^3 - 2*a*b*c + (b^4 - 4*a*b^2*c + 2*a^2*c^2)/Sqrt[b^2 - 4*a*c])*A
rcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*
c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[
```

```
b^2 - 4*a*c])*e]) + (4*b*c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2)
+ (8*(b^2 - a*c)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e] + (3*c^2*d*(
-(Sqrt[e]*x*Sqrt[d + e*x^2]) + d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]]))/e^(
5/2))/(8*c^3)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 375, normalized size = 0.78

method	result
risch	$-\frac{x\left(-2ce\,x^{2}+4eb+3cd\right)\sqrt{e\,x^{2}+d}}{8e^{2}c^{2}}-\frac{\ln\left(\sqrt{e}\,x+\sqrt{e\,x^{2}+d}\right)a}{c^{2}\sqrt{e}}+\frac{\ln\left(\sqrt{e}\,x+\sqrt{e\,x^{2}+d}\right)b^{2}}{c^{3}\sqrt{e}}+\frac{\ln\left(\sqrt{e}\,x+\sqrt{e\ldots}\right.}{2c^{2}\ldots}$
default	$-\frac{-c^{2}\left(\frac{x^{3}\sqrt{e\,x^{2}+d}}{4e}-\frac{3d\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(\sqrt{e}\,x+\sqrt{e\,x^{2}+d}\right)}{2e^{\frac{3}{2}}}\right)}{4e}\right)+bc\left(\frac{x\sqrt{e\,x^{2}+d}}{2e}-\frac{d\ln\left(\sqrt{e}\,x+\sqrt{e\,x^{2}\ldots}\right.}{2e^{\frac{3}{2}}}\ldots\right.}{c^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/c^3*(-c^2*(1/4*x^3/e*(e*x^2+d)^(1/2)-3/4*d/e*(1/2*x/e*(e*x^2+d)^(1/2)-1/
2*d/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2))))+b*c*(1/2*x/e*(e*x^2+d)^(1/2)-1/
2*d/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2)))+a*c*ln(e^(1/2)*x+(e*x^2+d)^(1/2)
)/e^(1/2)-b^2*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2))-1/2/c^3*e^(1/2)*sum((b
*(2*a*c-b^2)*_R^2+2*(2*a^2*c*e-2*a*b^2*e-2*a*b*c*d+b^3*d)*_R+2*a*b*c*d^2-b^
3*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2
*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*
d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^8/((c*x^4 + b*x^2 + a)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 9162 vs. 2(408) = 816.
time = 135.04, size = 9162, normalized size = 19.13

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/16*(4*sqrt(1/2)*c^3*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^
3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + ((b^2*c^7 - 4*
a*c^8)*d^2 - (b^3*c^6 - 4*a*b*c^7)*d*e + (a*b^2*c^6 - 4*a^2*c^7)*e^2)*sqrt(
((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 1
2*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 4
0*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c
 + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/((b^2*c^14 - 4*a*c
^15)*d^4 - 2*(b^3*c^13 - 4*a*b*c^14)*d^3*e + (b^4*c^12 - 2*a*b^2*c^13 - 8*a
^2*c^14)*d^2*e^2 - 2*(a*b^3*c^12 - 4*a^2*b*c^13)*d*e^3 + (a^2*b^2*c^12 - 4*
a^3*c^13)*e^4)))/((b^2*c^7 - 4*a*c^8)*d^2 - (b^3*c^6 - 4*a*b*c^7)*d*e + (a*
b^2*c^6 - 4*a^2*c^7)*e^2))*e^3*log(((a^3*b^7 - 5*a^4*b^5*c + 6*a^5*b^3*c^2
- a^6*b*c^3)*d^2*x^2 + 4*(a^5*b^5 - 4*a^6*b^3*c + 3*a^7*b*c^2)*x^2*e^2 - 2*
(a^4*b^6 - 5*a^5*b^4*c + 6*a^6*b^2*c^2 - a^7*c^3)*d^2 + 2*sqrt(1/2)*((b^11
- 11*a*b^9*c + 44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*c^4 - 8*a^5*b*c
^5)*d^2*x - (2*a*b^10 - 20*a^2*b^8*c + 70*a^3*b^6*c^2 - 101*a^4*b^4*c^3 + 5
3*a^5*b^2*c^4 - 4*a^6*c^5)*d*x*e + (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2
- 31*a^5*b^3*c^3 + 12*a^6*b*c^4)*x*e^2 - ((b^6*c^7 - 8*a*b^4*c^8 + 18*a^2*b
^2*c^9 - 8*a^3*c^10)*d^3*x - (b^7*c^6 - 7*a*b^5*c^7 + 11*a^2*b^3*c^8 + 4*a^
3*b*c^9)*d^2*x*e + (2*a*b^6*c^6 - 15*a^2*b^4*c^7 + 30*a^3*b^2*c^8 - 8*a^4*c
^9)*d*x*e^2 - (a^2*b^5*c^6 - 7*a^3*b^3*c^7 + 12*a^4*b*c^8)*x*e^3)*sqrt(((b^
12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^
5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^
4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 2
2*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/((b^2*c^14 - 4*a*c^15)
*d^4 - 2*(b^3*c^13 - 4*a*b*c^14)*d^3*e + (b^4*c^12 - 2*a*b^2*c^13 - 8*a^2*c
^14)*d^2*e^2 - 2*(a*b^3*c^12 - 4*a^2*b*c^13)*d*e^3 + (a^2*b^2*c^12 - 4*a^3*
c^13)*e^4)))*sqrt(x^2*e + d)*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a
^3*b*c^3)*d - (a*b^6 - 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + ((b^2*c
^7 - 4*a*c^8)*d^2 - (b^3*c^6 - 4*a*b*c^7)*d*e + (a*b^2*c^6 - 4*a^2*c^7)*e^2
)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*
c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*
c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^
3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/((b^2*c^14
- 4*a*c^15)*d^4 - 2*(b^3*c^13 - 4*a*b*c^14)*d^3*e + (b^4*c^12 - 2*a*b^2*c^1
3 - 8*a^2*c^14)*d^2*e^2 - 2*(a*b^3*c^12 - 4*a^2*b*c^13)*d*e^3 + (a^2*b^2*c^
12 - 4*a^3*c^13)*e^4)))/((b^2*c^7 - 4*a*c^8)*d^2 - (b^3*c^6 - 4*a*b*c^7)*d*
e + (a*b^2*c^6 - 4*a^2*c^7)*e^2)) - ((5*a^4*b^6 - 24*a^5*b^4*c + 27*a^6*b^2
*c^2 - 4*a^7*c^3)*d*x^2 - 2*(a^5*b^5 - 4*a^6*b^3*c + 3*a^7*b*c^2)*d)*e - ((
a^3*b^2*c^7 - 4*a^4*c^8)*d^3*x^2 - (a^3*b^3*c^6 - 4*a^4*b*c^7)*d^2*x^2*e +
(a^4*b^2*c^6 - 4*a^5*c^7)*d*x^2*e^2)*sqrt(((b^12 - 10*a*b^10*c + 37*a^2*b^8
*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 + a^6*c^6)*d^2 - 2*
```

```
(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 + 22*a^5*b^3*c^4 -
3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c^2 - 24*a^5*b^4*c^
3 + 9*a^6*b^2*c^4)*e^2)/((b^2*c^14 - 4*a*c^15)*d^4 - 2*(b^3*c^13 - 4*a*b*c^
14)*d^3*e + (b^4*c^12 - 2*a*b^2*c^13 - 8*a^2*c^14)*d^2*e^2 - 2*(a*b^3*c^12
- 4*a^2*b*c^13)*d*e^3 + (a^2*b^2*c^12 - 4*a^3*c^13)*e^4)))/x^2) - 4*sqrt(1/
2)*c^3*sqrt(-((b^7 - 7*a*b^5*c + 14*a^2*b^3*c^2 - 7*a^3*b*c^3)*d - (a*b^6 -
 6*a^2*b^4*c + 9*a^3*b^2*c^2 - 2*a^4*c^3)*e + ((b^2*c^7 - 4*a*c^8)*d^2 - (b
^3*c^6 - 4*a*b*c^7)*d*e + (a*b^2*c^6 - 4*a^2*c^7)*e^2)*sqrt(((b^12 - 10*a*b
^10*c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^4*c^4 - 12*a^5*b^2*c^5 +
 a^6*c^6)*d^2 - 2*(a*b^11 - 9*a^2*b^9*c + 29*a^3*b^7*c^2 - 40*a^4*b^5*c^3 +
 22*a^5*b^3*c^4 - 3*a^6*b*c^5)*d*e + (a^2*b^10 - 8*a^3*b^8*c + 22*a^4*b^6*c
^2 - 24*a^5*b^4*c^3 + 9*a^6*b^2*c^4)*e^2)/((b^2*c^14 - 4*a*c^15)*d^4 - 2*(b
^3*c^13 - 4*a*b*c^14)*d^3*e + (b^4*c^12 - 2*a*b^2*c^13 - 8*a^2*c^14)*d^2*e^
2 - 2*(a*b^3*c^12 - 4*a^2*b*c^13)*d*e^3 + (a^2*b^2*c^12 - 4*a^3*c^13)*e^4))
)/((b^2*c^7 - 4*a*c^8)*d^2 - (b^3*c^6 - 4*a*b*c^7)*d*e + (a*b^2*c^6 - 4*a^2
*c^7)*e^2))*e^3*log(((a^3*b^7 - 5*a^4*b^5*c + 6*a^5*b^3*c^2 - a^6*b*c^3)*d^
2*x^2 + 4*(a^5*b^5 - 4*a^6*b^3*c + 3*a^7*b*c^2)*x^2*e^2 - 2*(a^4*b^6 - 5*a^
5*b^4*c + 6*a^6*b^2*c^2 - a^7*c^3)*d^2 - 2*sqrt(1/2)*((b^11 - 11*a*b^9*c +
44*a^2*b^7*c^2 - 77*a^3*b^5*c^3 + 54*a^4*b^3*c^4 - 8*a^5*b*c^5)*d^2*x - (2*
a*b^10 - 20*a^2*b^8*c + 70*a^3*b^6*c^2 - 101*a^4*b^4*c^3 + 53*a^5*b^2*c^4 -
 4*a^6*c^5)*d*x*e + (a^2*b^9 - 9*a^3*b^7*c + 27*a^4*b^5*c^2 - 31*a^5*b^3*c^
3 + 12*a^6*b*c^4)*x*e^2 - ((b^6*c^7 - 8*a*b^4*c^8 + 18*a^2*b^2*c^9 - 8*a^3*
c^10)*d^3*x - (b^7*c^6 - 7*a*b^5*c^7 + 11*a^2*b^3*c^8 + 4*a^3*b*c^9)*d^2*x*
e + (2*a*b^6*c^6 - 15*a^2*b^4*c^7 + 30*a^3*b^2*c^8 - 8*a^4*c^9)*d*x*e^2 - (
a^2*b^5*c^6 - 7*a^3*b^3*c^7 + 12*a^4*b*c^8)*x*e^3)*sqrt(((b^12 - 10*a*b^10*
c + 37*a^2*b^8*c^2 - 62*a^3*b^6*c^3 + 46*a^4*b^...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^8}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**8/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**8/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 6.42, size = 105, normalized size = 0.22

$$\frac{1}{8}\sqrt{x^2e+d}\left(\frac{2\,x^2e^{(-1)}}{c}-\frac{(3\,c^5de+4\,bc^4e^2)e^{(-3)}}{c^6}\right)x-\frac{(3\,c^2d^2+4\,bcde+8\,b^2e^2-8\,ace^2)e^{\left(-\frac{5}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{16\,c^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^8/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(x^2*e + d)*(2*x^2*e^(-1)/c - (3*c^5*d*e + 4*b*c^4*e^2)*e^(-3)/c^6)
*x - 1/16*(3*c^2*d^2 + 4*b*c*d*e + 8*b^2*e^2 - 8*a*c*e^2)*e^(-5/2)*log((x*e
^(1/2) - sqrt(x^2*e + d))^2)/c^3
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^8}{\sqrt{e\,x^2+d}\,\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^8/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.387 $\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=366

$$\frac{x\sqrt{d+ex^2}}{2ce}+\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right.}{c^2\sqrt{b+\sqrt{b^2-}}}$$

```
[Out] -1/2*d*arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-b*arctanh(x*e^(1/2)/(e*
x^2+d)^(1/2))/c^2/e^(1/2)+1/2*x*(e*x^2+d)^(1/2)/c/e+arctan(x*(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2
-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))
)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c+b*(-3*a*c+
b^2)/(-4*a*c+b^2)^(1/2))/c^2/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a
*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.82, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {1317, 223, 212, 327, 1706, 385, 211}

$$\frac{\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}}-\frac{d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}}+\frac{x\sqrt{d+ex^2}}{2ce}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (x*Sqrt[d + e*x^2])/(2*c*e) + ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*
a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d -
 (b - Sqrt[b^2 - 4*a*c])*e]) + ((b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4
*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^
2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e]) - (d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(2
*c*e^(3/2)) - (b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(c^2*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 327

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(n
 - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*(m + n*p + 1))), x] - Dist[
a*c^n*((m - n + 1)/(b*(m + n*p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \left(-\frac{b}{c^2\sqrt{d+ex^2}} + \frac{x^2}{c\sqrt{d+ex^2}} + \frac{ab+(b^2-ac)\,x^2}{c^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{ab+(b^2-ac)x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{c^2} - \frac{b\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c^2} + \frac{\int \frac{x^2}{\sqrt{d+ex^2}}\,dx}{c}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\int \left(\frac{b^2-ac+\frac{b(-b^2+3ac)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b^2-ac-\frac{b(-b}{\sqrt{b^2}}}{\left(b+\sqrt{b^2-4ac}+2c\right.}\right.}{c^2}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\int}{}$$

$$= \frac{x\sqrt{d+ex^2}}{2ce} - \frac{d\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{2ce^{3/2}} - \frac{b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c^2\sqrt{e}} + $$

$$= \frac{x\sqrt{d+ex^2}}{2ce} + \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4\right.}}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d}}\right.}{c^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)}}$$

Mathematica [A]
time = 10.73, size = 355, normalized size = 0.97

$$\frac{\frac{cx\sqrt{d+ex^2}}{e} + \frac{2\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}\,e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}} + \frac{2\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} - \frac{cd\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}} - \frac{2b\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^6/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((c*x*Sqrt[d + e*x^2])/e + (2*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e]) + (2*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a
*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b +
```

```
Sqrt[b^2 - 4*a*c])*e]) - (c*d*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/e^(3/2
) - (2*b*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/Sqrt[e])/(2*c^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.16, size = 267, normalized size = 0.73

method	result
default	$\frac{\frac{x\sqrt{e x^2+d}}{2e}-\frac{d\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{2e^{\frac{3}{2}}}}{c}-\frac{b\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{c^2\sqrt{e}}-\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+(1\right.}\right.}{}$
risch	$\frac{x\sqrt{e x^2+d}}{2ce}-\frac{b\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)}{c^2\sqrt{e}}-\frac{\ln\left(\sqrt{e}\,x+\sqrt{e x^2+d}\right)d}{2ce^{\frac{3}{2}}}+\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(1/2*x/e*(e*x^2+d)^(1/2)-1/2*d/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2)))-b
/c^2*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)-1/2/c^2*e^(1/2)*sum(((-a*c+b^2)*
_R^2+2*(2*a*b*e+a*c*d-b^2*d)*_R-a*c*d^2+b^2*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*
c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(
1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2
)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 7362 vs. $2(312) = 624$.
time = 55.00, size = 7362, normalized size = 20.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*c^2*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*a
^2*b^2*c + 2*a^3*c^2)*e + ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*
d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
- 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2
*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 -
 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 -
8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*
a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b
^2*c^4 - 4*a^2*c^5)*e^2))*e^2*log(-((a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*d^2
*x^2 + 4*(a^4*b^3 - 2*a^5*b*c)*x^2*e^2 - 2*(a^3*b^4 - 3*a^4*b^2*c + a^5*c^2
)*d^2 + 2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 + 4
*a^4*c^4)*d^2*x - (2*a*b^7 - 14*a^2*b^5*c + 27*a^3*b^3*c^2 - 12*a^4*b*c^3)*
d*x*e + (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*x*e^2 - ((b^5*c^5 - 7*a*b^3
*c^6 + 12*a^2*b*c^7)*d^3*x - (b^6*c^4 - 6*a*b^4*c^5 + 6*a^2*b^2*c^6 + 8*a^3
*c^7)*d^2*x*e + (2*a*b^5*c^4 - 13*a^2*b^3*c^5 + 20*a^3*b*c^6)*d*x*e^2 - (a^
2*b^4*c^4 - 6*a^3*b^2*c^5 + 8*a^4*c^6)*x*e^3)*sqrt(((b^8 - 6*a*b^6*c + 11*a
^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*
b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/(
(b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a
*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b
^2*c^8 - 4*a^3*c^9)*e^4)))*sqrt(x^2*e + d)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*
b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + ((b^2*c^5 - 4*a*c^6)*d^2 -
 (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*
b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^
5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2
*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b
^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*
e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4
- 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)) - ((5*a^3*b^4 - 14*a^4*b^2
*c + 4*a^5*c^2)*d*x^2 - 2*(a^4*b^3 - 2*a^5*b*c)*d)*e - ((a^2*b^2*c^5 - 4*a^
3*c^6)*d^3*x^2 - (a^2*b^3*c^4 - 4*a^3*b*c^5)*d^2*x^2*e + (a^3*b^2*c^4 - 4*a
^4*c^5)*d*x^2*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3
+ a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e
+ (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 -
 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*
e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/
x^2) - sqrt(1/2)*c^2*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2*b*c^2)*d - (a*b^4 - 4*
a^2*b^2*c + 2*a^3*c^2)*e + ((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)
*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a*b^6*c + 11*a^2*b^4*c^2
 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3*b^3*c^2 -
2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/((b^2*c^10
- 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*a*b^2*c^9 -
 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*b^2*c^8 - 4
*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4 - 4*a*b*c^5)*d*e + (a*
b^2*c^4 - 4*a^2*c^5)*e^2))*e^2*log(-((a^2*b^5 - 3*a^3*b^3*c + a^4*b*c^2)*d^
```

```
2*x^2 + 4*(a^4*b^3 - 2*a^5*b*c)*x^2*e^2 - 2*(a^3*b^4 - 3*a^4*b^2*c + a^5*c^
2)*d^2 - 2*sqrt(1/2)*((b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 17*a^3*b^2*c^3 +
4*a^4*c^4)*d^2*x - (2*a*b^7 - 14*a^2*b^5*c + 27*a^3*b^3*c^2 - 12*a^4*b*c^3)
*d*x*e + (a^2*b^6 - 6*a^3*b^4*c + 8*a^4*b^2*c^2)*x*e^2 - ((b^5*c^5 - 7*a*b^
3*c^6 + 12*a^2*b*c^7)*d^3*x - (b^6*c^4 - 6*a*b^4*c^5 + 6*a^2*b^2*c^6 + 8*a^
3*c^7)*d^2*x*e + (2*a*b^5*c^4 - 13*a^2*b^3*c^5 + 20*a^3*b*c^6)*d*x*e^2 - (a
^2*b^4*c^4 - 6*a^3*b^2*c^5 + 8*a^4*c^6)*x*e^3)*sqrt(((b^8 - 6*a*b^6*c + 11*
a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b^5*c + 7*a^3
*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^2*c^2)*e^2)/
((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (b^4*c^8 - 2*
a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d*e^3 + (a^2*
b^2*c^8 - 4*a^3*c^9)*e^4)))*sqrt(x^2*e + d)*sqrt(-((b^5 - 5*a*b^3*c + 5*a^2
*b*c^2)*d - (a*b^4 - 4*a^2*b^2*c + 2*a^3*c^2)*e + ((b^2*c^5 - 4*a*c^6)*d^2
- (b^3*c^4 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)*sqrt(((b^8 - 6*a
*b^6*c + 11*a^2*b^4*c^2 - 6*a^3*b^2*c^3 + a^4*c^4)*d^2 - 2*(a*b^7 - 5*a^2*b
^5*c + 7*a^3*b^3*c^2 - 2*a^4*b*c^3)*d*e + (a^2*b^6 - 4*a^3*b^4*c + 4*a^4*b^
2*c^2)*e^2)/((b^2*c^10 - 4*a*c^11)*d^4 - 2*(b^3*c^9 - 4*a*b*c^10)*d^3*e + (
b^4*c^8 - 2*a*b^2*c^9 - 8*a^2*c^10)*d^2*e^2 - 2*(a*b^3*c^8 - 4*a^2*b*c^9)*d
*e^3 + (a^2*b^2*c^8 - 4*a^3*c^9)*e^4)))/((b^2*c^5 - 4*a*c^6)*d^2 - (b^3*c^4
 - 4*a*b*c^5)*d*e + (a*b^2*c^4 - 4*a^2*c^5)*e^2)) - ((5*a^3*b^4 - 14*a^4*b^
2*c + 4*a^5*c^2)*d*x^2 - 2*(a^4*b^3 - 2*a^5*b*c)*d)*e - ((a^2*b^2*c^5 - 4*a
^3*c^6)*d^3*x^2 - (a^2*b^3*c^4 - 4*a^3*b*c^5)*d...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
```

```
[Out] Integral(x**6/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 6.63, size = 55, normalized size = 0.15

$$\frac{\sqrt{x^2e+d}\,xe^{(-1)}}{2\,c}+\frac{(cd+2\,be)e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{4\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(x^2*e + d)*x*e^(-1)/c + 1/4*(c*d + 2*b*e)*e^(-3/2)*log((x*e^(1/2)
- sqrt(x^2*e + d))^2)/c^2
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{\sqrt{e\,x^2+d}\,\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^6/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.388 $\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=298

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{\quad}\right.}}{\sqrt{b+\sqrt{b^2-4a}}}\right.}{c\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b+\sqrt{b^2}\right.}}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(1/2)-arctan(x*(2*c*d-e*(b-(-4*a*c+b
^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b
^2)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d
)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c
/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.51, antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1317, 223, 212, 1706, 385, 211}

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^2
 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(c*Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqr
t[d + e*x^2]]/(c*Sqrt[e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \left(\frac{1}{c\sqrt{d+ex^2}} - \frac{a+bx^2}{c\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right)dx$$

$$= \frac{\int \frac{1}{\sqrt{d+ex^2}}\,dx}{c} - \frac{\int \frac{a+bx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{c}$$

$$= -\frac{\int \left(\frac{b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\right)}{c}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+}}}{c}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{}{b-\sqrt{b^2-4ac}-\left(-2\right.}\right.}{c}$$

$$= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{\left(b+\right.}{c}$$

Mathematica [A]
time = 10.45, size = 292, normalized size = 0.98

$$\frac{-\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}\,e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}} - \frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}} + \frac{\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{\sqrt{e}}}{c}$$

Antiderivative was successfully verified.

```
[In] Integrate[x^4/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (-(((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqrt[
b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b
- Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])) - ((b + (b^
2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
```

```
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]) + ArcTanh[(Sqrt[e]*x)/Sqrt
[d + e*x^2]]/Sqrt[e])/c
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 200, normalized size = 0.67

method	result
default	$\frac{\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{c\sqrt{e}}+\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(b\_\right.}{c\_R}\right.}{2c}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*ln(e^(1/2)*x+(e*x^2+d)^(1/2))/e^(1/2)+1/2/c*e^(1/2)*sum((b*_R^2+2*(2*a*
e-b*d)*_R+d^2*b)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c
*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(
4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^
4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 5585 vs. $2(257) = 514$.
time = 9.72, size = 5585, normalized size = 18.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^3
 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)*
sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d
*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2
*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*
```

b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3
)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*e*log((4*a^3*b*x^2*e^2 + (a*b^3 - a^2
*b*c)*d^2*x^2 - 2*(a^2*b^2 - a^3*c)*d^2 + 2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4
*a^2*b*c^2)*d^2*x - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*x*e + (a^2*b^3 -
4*a^3*b*c)*x*e^2 - ((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3*x - (b^5*c^2 -
5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*x*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c
^4)*d*x*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*x*e^3)*sqrt((a^2*b^2*e^2 + (b^4 -
 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d
^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^
2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4))
)*sqrt(x^2*e + d)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c^
3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)
*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*
d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 -
2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2
*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^
3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)) + (2*a^3*b*d - (5*a^2*b^2 - 4*a^3*c)
*d*x^2)*e - ((a*b^2*c^3 - 4*a^2*c^4)*d^3*x^2 - (a*b^3*c^2 - 4*a^2*b*c^3)*d^
2*x^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*x^2*e^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2
*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4
 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*
e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/
x^2) - sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*c
^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2
)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)
*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 -
 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^
2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c
^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*e*log((4*a^3*b*x^2*e^2 + (a*b^3 - a
^2*b*c)*d^2*x^2 - 2*(a^2*b^2 - a^3*c)*d^2 - 2*sqrt(1/2)*((b^5 - 5*a*b^3*c +
 4*a^2*b*c^2)*d^2*x - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*x*e + (a^2*b^3
- 4*a^3*b*c)*x*e^2 - ((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3*x - (b^5*c^2
- 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*x*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3
*c^4)*d*x*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*x*e^3)*sqrt((a^2*b^2*e^2 + (b^4
 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)
d^4 - 2(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*
d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4
)))*sqrt(x^2*e + d)*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e + ((b^2*
c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^
2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c
)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4
- 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a
^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*
c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2)) + (2*a^3*b*d - (5*a^2*b^2 - 4*a^3*
c)*d*x^2)*e - ((a*b^2*c^3 - 4*a^2*c^4)*d^3*x^2 - (a*b^3*c^2 - 4*a^2*b*c^3)*

```
d^2*x^2*e + (a^2*b^2*c^2 - 4*a^3*c^3)*d*x^2*e^2)*sqrt((a^2*b^2*e^2 + (b^4 -
 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*c)*d*e)/((b^2*c^6 - 4*a*c^7)*d
^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^
2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (a^2*b^2*c^4 - 4*a^3*c^5)*e^4))
)/x^2) + sqrt(1/2)*c*sqrt(-((b^3 - 3*a*b*c)*d - (a*b^2 - 2*a^2*c)*e - ((b^2
*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e
^2)*sqrt((a^2*b^2*e^2 + (b^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*b*
c)*d*e)/((b^2*c^6 - 4*a*c^7)*d^4 - 2*(b^3*c^5 - 4*a*b*c^6)*d^3*e + (b^4*c^4
 - 2*a*b^2*c^5 - 8*a^2*c^6)*d^2*e^2 - 2*(a*b^3*c^4 - 4*a^2*b*c^5)*d*e^3 + (
a^2*b^2*c^4 - 4*a^3*c^5)*e^4)))/((b^2*c^3 - 4*a*c^4)*d^2 - (b^3*c^2 - 4*a*b
*c^3)*d*e + (a*b^2*c^2 - 4*a^2*c^3)*e^2))*e*log((4*a^3*b*x^2*e^2 + (a*b^3 -
 a^2*b*c)*d^2*x^2 - 2*(a^2*b^2 - a^3*c)*d^2 + 2*sqrt(1/2)*((b^5 - 5*a*b^3*c
 + 4*a^2*b*c^2)*d^2*x - (2*a*b^4 - 9*a^2*b^2*c + 4*a^3*c^2)*d*x*e + (a^2*b^
3 - 4*a^3*b*c)*x*e^2 + ((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^3*x - (b^5*c^
2 - 5*a*b^3*c^3 + 4*a^2*b*c^4)*d^2*x*e + 2*(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a
^3*c^4)*d*x*e^2 - (a^2*b^3*c^2 - 4*a^3*b*c^3)*x*e^3)*sqrt((a^2*b^2*e^2 + (b
^4 - 2*a*b^2*c + a^2*c^2)*d^2 - 2*(a*b^3 - a^2*...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(x**4/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 7.41, size = 27, normalized size = 0.09

$$-\frac{e^{\left(-\frac{1}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] -1/2*e^(-1/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^4/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.389 $\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=240

$$-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+\frac{\sqrt{b+\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\cdots\right.}}{\sqrt{b+\sqrt{b^2-4\cdots}}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b+\sqrt{b^2-\cdots}\right.}}$$

```
[Out] -arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c
+b^2)^(1/2))^(1/2))*(b-(-4*a*c+b^2)^(1/2))^(1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-
e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)+arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(
1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-4*a*c+b^2)^(1/2))^(
1/2)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1317, 385, 211}

$$\frac{\sqrt{\sqrt{b^2-4ac}+b}\,\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[b - Sqrt[b^2 - 4*a*c]]*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*S
qrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e])) + (Sqrt[b + Sqrt[b^2 - 4*a*c]]*Arc
Tan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]
]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \left(\frac{1-\frac{b}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{1+\frac{b}{\sqrt{b^2-4ac}}}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{}}\right.$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx + \left(1+\frac{}{\sqrt{}}\right.$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\mathrm{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4a\right.\right.}}\right.$$

$$= -\frac{\sqrt{b-\sqrt{b^2-4ac}}\,\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} + \frac{\sqrt{b+}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 5.21, size = 250, normalized size = 1.04

$$\frac{1}{2}\sqrt{e}\,\mathrm{RootSum}\left[cd^4-4cd^3\#1^2+4bd^2e\#1^2+6cd^2\#1^4-8bde\#1^4+16ae^2\#1^4-4cd\#1^6+4be\#1^6+c\#1^8\&,\frac{d^2\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}-\#1\right)-2d\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}-\#1\right)\#1^2+\log\left(-\sqrt{e}\,x+\sqrt{d+ex^2}-\#1\right)\#1^4}{cd^3-bd^2e-3cd^2\#1^2+4bde\#1^2-8ae^2\#1^2+3cd\#1^4-3be\#1^4-c\#1^6}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[x^2/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (Sqrt[e]*RootSum[c*d^4 - 4*c*d^3*#1^2 + 4*b*d^2*e*#1^2 + 6*c*d^2*#1^4 - 8*b
*d*e*#1^4 + 16*a*e^2*#1^4 - 4*c*d*#1^6 + 4*b*e*#1^6 + c*#1^8 & , (d^2*Log[-
(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1] - 2*d*Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2
```

```
] - #1]*#1^2 + Log[-(Sqrt[e]*x) + Sqrt[d + e*x^2] - #1]*#1^4)/(c*d^3 - b*d^
2*e - 3*c*d^2*#1^2 + 4*b*d*e*#1^2 - 8*a*e^2*#1^2 + 3*c*d*#1^4 - 3*b*e*#1^4
- c*#1^6) & ])/2
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 161, normalized size = 0.67

method	result
default	$-\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(\_R^2-2\_Rd+d^2\right)\ln\left(\left(\sqrt{e}\right.\right.}{c\_R^3+3\_R^2be-3\_R^2cd+8\_Ra}\right.}{2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2*e^(1/2)*sum((_R^2-2*_R*d+d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2
-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),
_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^
2*e-4*c*d^3)*_Z+d^4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 3417 vs. $2(206) = 412$.
time = 2.78, size = 3417, normalized size = 14.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c
)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c
 - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*
a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 -
 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(-(b*d^2*x^2 - 4*a*d*x^2*e - 2*a
*d^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^3*c - 4*a*b*c^2)*d^3*x - (b^4
```

- 2*a*b^2*c - 8*a^2*c^2)*d^2*x*e + 3*(a*b^3 - 4*a^2*b*c)*d*x*e^2 - 2*(a^2*b^2 - 4*a^3*c)*x*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))*sqrt(x^2*e + d)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)) - ((b^2*c - 4*a*c^2)*d^3*x^2 - (b^3 - 4*a*b*c)*d^2*x^2*e + (a*b^2 - 4*a^2*c)*d*x^2*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(-(b*d^2*x^2 - 4*a*d*x^2*e - 2*a*d^2 - 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x - ((b^3*c - 4*a*b*c^2)*d^3*x - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*x*e + 3*(a*b^3 - 4*a^2*b*c)*d*x*e^2 - 2*(a^2*b^2 - 4*a^3*c)*x*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))*sqrt(x^2*e + d)*sqrt(-(b*d - 2*a*e + ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)) - ((b^2*c - 4*a*c^2)*d^3*x^2 - (b^3 - 4*a*b*c)*d^2*x^2*e + (a*b^2 - 4*a^2*c)*d*x^2*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(-(b*d^2*x^2 - 4*a*d*x^2*e - 2*a*d^2 + 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x + ((b^3*c - 4*a*b*c^2)*d^3*x - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*x*e + 3*(a*b^3 - 4*a^2*b*c)*d*x*e^2 - 2*(a^2*b^2 - 4*a^3*c)*x*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))*sqrt(x^2*e + d)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)) + ((b^2*c - 4*a*c^2)*d^3*x^2 - (b^3 - 4*a*b*c)*d^2*x^2*e + (a*b^2 - 4*a^2*c)*d*x^2*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4

```
 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2
- 4*a^3*c)*e^4)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c
^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 -
 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)
*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c
 - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2 - 4*a^2*c)*e^2))*log(-(b*d^2
*x^2 - 4*a*d*x^2*e - 2*a*d^2 - 2*sqrt(1/2)*((b^2 - 4*a*c)*d^2*x + ((b^3*c -
 4*a*b*c^2)*d^3*x - (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*x*e + 3*(a*b^3 - 4*a^
2*b*c)*d*x*e^2 - 2*(a^2*b^2 - 4*a^3*c)*x*e^3)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)
*d^4 - 2*(b^3*c - 4*a*b*c^2)*d^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2
- 2*(a*b^3 - 4*a^2*b*c)*d*e^3 + (a^2*b^2 - 4*a^3*c)*e^4)))*sqrt(x^2*e + d)*
sqrt(-(b*d - 2*a*e - ((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e + (a*b^2
- 4*a^2*c)*e^2)*sqrt(d^2/((b^2*c^2 - 4*a*c^3)*d^4 - 2*(b^3*c - 4*a*b*c^2)*d
^3*e + (b^4 - 2*a*b^2*c - 8*a^2*c^2)*d^2*e^2 - 2*(a*b^3 - 4*a^2*b*c)*d*e^3
+ (a^2*b^2 - 4*a^3*c)*e^4)))/((b^2*c - 4*a*c^2)*d^2 - (b^3 - 4*a*b*c)*d*e +
 (a*b^2 - 4*a^2*c)*e^2)) + ((b^2*c - 4*a*c^2)*d...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(x**2/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^2/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.390 $\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=243

$$\frac{2c\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{2c\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}$$

```
[Out] 2*c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*
a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1
/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))/(-4*a*c+b^2)^(1/2
)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {1188, 385, 211}

$$\frac{2c\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{2c\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2
- 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b^2 - 4*a*c]*Sqrt[b - Sqrt[b^2 - 4*a*c]]
*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]) - (2*c*ArcTan[(Sqrt[2*c*d - (b +
Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(S
qrt[b^2 - 4*a*c]*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4
*a*c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \frac{(2c)\int \frac{1}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)}}{\sqrt{b^2-4ac}}$$

$$= \frac{(2c)\text{Subst}\left(\int \frac{1}{b-\sqrt{b^2-4ac}-\left(-2cd+\left(b-\sqrt{b^2-4ac}\right)e\right)x^2}\,dx, x, \frac{x}{\sqrt{d+ex^2}}\right.}{\sqrt{b^2-4ac}}$$

$$= \frac{2c\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b^2-4ac}\,\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}} - \frac{2}{\sqrt{b^2-4ac}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.
time = 0.13, size = 179, normalized size = 0.74

$$-2e^{3/2}\text{RootSum}\left[cd^4-4cd^3\#1+4bd^2e\#1+6cd^2\#1^2-8bde\#1^2+16ae^2\#1^2-4cd\#1^3+4be\#1^3+c\#1^4\&, \frac{\log\left(d+2ex^2-2\sqrt{e}\,x\sqrt{d+ex^2}-\#1\right)\#1}{-cd^3+bd^2e+3cd^2\#1-4bde\#1+8ae^2\#1-3cd\#1^2+3be\#1^2+c\#1^3}\&\right]$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -2*e^(3/2)*RootSum[c*d^4 - 4*c*d^3*#1 + 4*b*d^2*e*#1 + 6*c*d^2*#1^2 - 8*b*d
*e*#1^2 + 16*a*e^2*#1^2 - 4*c*d*#1^3 + 4*b*e*#1^3 + c*#1^4 & , (Log[d + 2*e
*x^2 - 2*Sqrt[e]*x*Sqrt[d + e*x^2] - #1]*#1)/(-(c*d^3) + b*d^2*e + 3*c*d^2*
#1 - 4*b*d*e*#1 + 8*a*e^2*#1 - 3*c*d*#1^2 + 3*b*e*#1^2 + c*#1^3) & ]
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 151, normalized size = 0.62

method	result
default	$-2e^{\frac{3}{2}}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+(16a\,e^2-8deb+6c\,d^2)\_Z^2+(4d^2eb-4c\,d^3)\_Z+d^4c\right)}\frac{\_R\ln\left(\left(\sqrt{e\,x^2}\right.\right.}{c\_R^3+3\_R^2be-3\_R^2cd+8}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2*e^(3/2)*sum(_R/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*
c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+
(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d
^4*c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x^2*e + d)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 4577 vs. 2(209) = 418.
time = 3.69, size = 4577, normalized size = 18.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 -
 (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d
*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*
d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*
c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 -
 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log((b*c^2*d^2*x^2 + 4*a*b*c*x^
2*e^2 - 2*a*c^2*d^2 + 2*sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*d*x*e - (a*b^3 - 4
*a^2*b*c)*x*e^2 + (2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3*x - 3*(a^2*b^3*c - 4*a^3
*b*c^2)*d^2*x*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*x*e^2 - (a^3*b^3 -
```

4*a^4*b*c)*x*e^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))*sqrt(x^2*e + d)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)) + (2*a*b*c*d - (b^2*c + 4*a*c^2)*d*x^2)*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*x^2 - (a*b^3*c - 4*a^2*b*c^2)*d^2*x^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*x^2*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/x^2) - 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log((b*c^2*d^2*x^2 + 4*a*b*c*x^2*e^2 - 2*a*c^2*d^2 - 2*sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*d*x*e - (a*b^3 - 4*a^2*b*c)*x*e^2 + (2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3*x - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*x*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*x*e^2 - (a^3*b^3 - 4*a^4*b*c)*x*e^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))*sqrt(x^2*e + d)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e + ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)) + (2*a*b*c*d - (b^2*c + 4*a*c^2)*d*x^2)*e + ((a*b^2*c^2 - 4*a^2*c^3)*d^3*x^2 - (a*b^3*c - 4*a^2*b*c^2)*d^2*x^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*x^2*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/x^2) + 1/4*sqrt(1/2)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2))*log((b*c^2*d^2*x^2 + 4*a*b*c*x^2*e^2 - 2*a*c^2*d^2 + 2*sqrt(1/2)*((a*b^2*c - 4*a^2*c^2)*d*x*e - (a*b^3 - 4*a^2*b*c)*x*e^2 - (2*(a^2*b^2*c^2 - 4*a^3*c^3)*d^3*x - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*x*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d*x*e^2 -

```
(a^3*b^3 - 4*a^4*b*c)*x*e^3)*sqrt((c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2
*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^
3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4
*a^5*c)*e^4)))*sqrt(x^2*e + d)*sqrt(-(b*c*d - (b^2 - 2*a*c)*e - ((a*b^2*c -
 4*a^2*c^2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)*sqrt((
c^2*d^2 - 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*
c - 4*a^3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4*c^2)*d^2*e^2 - 2*(a
^3*b^3 - 4*a^4*b*c)*d*e^3 + (a^4*b^2 - 4*a^5*c)*e^4)))/((a*b^2*c - 4*a^2*c^
2)*d^2 - (a*b^3 - 4*a^2*b*c)*d*e + (a^2*b^2 - 4*a^3*c)*e^2)) + (2*a*b*c*d -
 (b^2*c + 4*a*c^2)*d*x^2)*e - ((a*b^2*c^2 - 4*a^2*c^3)*d^3*x^2 - (a*b^3*c -
 4*a^2*b*c^2)*d^2*x^2*e + (a^2*b^2*c - 4*a^3*c^2)*d*x^2*e^2)*sqrt((c^2*d^2
- 2*b*c*d*e + b^2*e^2)/((a^2*b^2*c^2 - 4*a^3*c^3)*d^4 - 2*(a^2*b^3*c - 4*a^
3*b*c^2)*d^3*e + (a^2*b^4 - 2*a^3*b^2*c - 8*a^4...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(1/(sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/((d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.391 $\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=280

$$-\frac{\sqrt{d+ex^2}}{adx}-\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\right.}{a\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{}}$$

```
[Out] -(e*x^2+d)^(1/2)/a/d/x-c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e
*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/a/(2*c
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/a/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {1317, 270, 1706, 385, 211}

$$-\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{\sqrt{d+ex^2}}{adx}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(Sqrt[d + e*x^2]/(a*d*x)) - (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*
e]) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b + Sqrt
[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \left(\frac{1}{ax^2\sqrt{d+ex^2}} + \frac{-b-cx^2}{a\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right) dx$$

$$= \frac{\int \frac{1}{x^2\sqrt{d+ex^2}}\,dx}{a} + \frac{\int \frac{-b-cx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} + \frac{\int \left(\frac{-c-\frac{bc}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \frac{-c+\cdots}{\left(b+\sqrt{b^2-4ac}\cdots\right.}\cdots\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{1}{\left(b+\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+\cdots}}\cdots}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{\cdots}{b+\sqrt{b^2-4ac}-\left(-2c\cdots\right.}\cdots\right.}{a}$$

$$= -\frac{\sqrt{d+ex^2}}{adx} - \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4a\cdots}\right.}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d-\cdots}}\cdots\right.}{a\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)\cdots}}$$

Mathematica [A]
time = 10.82, size = 271, normalized size = 0.97

$$-\frac{\frac{\sqrt{d+ex^2}}{dx} + \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}\,e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^2*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((Sqrt[d + e*x^2]/(d*x) + (c*(1 + b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
- b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2
])]])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e])
 + (c*(1 - b/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]])/(Sqrt[b + Sqrt[b^2
- 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/a)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 197, normalized size = 0.70

method	result
default	$\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(4eb-4cd)\_Z^{3}+\left(16a\,e^{2}-8deb+6c\,d^{2}\right)\_Z^{2}+\left(4d^{2}eb-4c\,d^{3}\right)\_Z+d^{4}c\right)}\frac{\left(c\_R^{2}+2(2eb-cd)\_R+c\,d^{2}\right)\ln\left(\left(\sqrt{e}\right.\right.}{c\_R^{3}+3\_R^{2}be-3\_R^{2}cd+8\_Ra\,e^{2}-4}\right.}{2a}$
risch	$\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(4eb-4cd)\_Z^{3}+\left(16a\,e^{2}-8deb+6c\,d^{2}\right)\_Z^{2}+\left(4d^{2}eb-4c\,d^{3}\right)\_Z+d^{4}c\right)}\frac{\left(c\_R^{2}+2(2eb-cd)\_R+c\,d^{2}\right)\ln\left(\left(\sqrt{e}\right.\right.}{c\_R^{3}+3\_R^{2}be-3\_R^{2}cd+8\_Ra\,e^{2}-4}\right.}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a*e^(1/2)*sum((c*_R^2+2*(2*b*e-c*d)*_R+c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2
*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^
(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^
2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))-(e*x^2+d)^(1/2)/a/d/x
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x^2*e + d)*x^2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 6466 vs. $2(243) = 486$.
time = 13.25, size = 6466, normalized size = 23.09

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^
2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*
b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c
- 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/(
(a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^
```

```
4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8
*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)
*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log(-((b^3*c^3 - a*b*c^4)*d^2*x^2 + 4*(a*b
^3*c^2 - 2*a^2*b*c^3)*x^2*e^2 - 2*(a*b^2*c^3 - a^2*c^4)*d^2 + 2*sqrt(1/2)*(
(a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^2*x - (2*a*b^5*c - 11*a^2*b^3*c^2
 + 12*a^3*b*c^3)*d*x*e + (a*b^6 - 6*a^2*b^4*c + 8*a^3*b^2*c^2)*x*e^2 - ((a^
4*b^3*c^2 - 4*a^5*b*c^3)*d^3*x - 2*(a^4*b^4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*
d^2*x*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b*c^2)*d*x*e^2 - (a^5*b^4 - 6*a^6*
b^2*c + 8*a^7*c^2)*x*e^3)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(
b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*
e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (
a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3
+ (a^8*b^2 - 4*a^9*c)*e^4)))*sqrt(x^2*e + d)*sqrt(-((b^3*c - 3*a*b*c^2)*d -
 (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3
- 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 +
a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c
 + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^
7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 -
 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2
 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)) - ((b^4*c^2 + 2*a*
b^2*c^3 - 4*a^2*c^4)*d*x^2 - 2*(a*b^3*c^2 - 2*a^2*b*c^3)*d)*e + ((a^3*b^2*c
^3 - 4*a^4*c^4)*d^3*x^2 - (a^3*b^3*c^2 - 4*a^4*b*c^3)*d^2*x^2*e + (a^4*b^2*
c^2 - 4*a^5*c^3)*d*x^2*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2
*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2
)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e +
 (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^
3 + (a^8*b^2 - 4*a^9*c)*e^4)))/x^2) - sqrt(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b
*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b^2*c - 4*a^4*c^2)*d^2 -
(a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b
^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (b^6 -
4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a^6*b^3
*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2 - 2*(
a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/((a^3*b^2*c - 4*a^4
*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2))*log(-((b^
3*c^3 - a*b*c^4)*d^2*x^2 + 4*(a*b^3*c^2 - 2*a^2*b*c^3)*x^2*e^2 - 2*(a*b^2*c
^3 - a^2*c^4)*d^2 - 2*sqrt(1/2)*((a*b^4*c^2 - 5*a^2*b^2*c^3 + 4*a^3*c^4)*d^
2*x - (2*a*b^5*c - 11*a^2*b^3*c^2 + 12*a^3*b*c^3)*d*x*e + (a*b^6 - 6*a^2*b^
4*c + 8*a^3*b^2*c^2)*x*e^2 - ((a^4*b^3*c^2 - 4*a^5*b*c^3)*d^3*x - 2*(a^4*b^
4*c - 5*a^5*b^2*c^2 + 4*a^6*c^3)*d^2*x*e + (a^4*b^5 - 5*a^5*b^3*c + 4*a^6*b
*c^2)*d*x*e^2 - (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2)*x*e^3)*sqrt(((b^4*c^2 -
 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e + (
b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*(a
^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e^2
 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))*sqrt(x^2*e +
d)*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e + ((a^3*b
```

```
^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a^5*c)*e^2
)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*
a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a
^7*c^3)*d^4 - 2*(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c -
8*a^8*c^2)*d^2*e^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^
4)))/((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 -
4*a^5*c)*e^2)) - ((b^4*c^2 + 2*a*b^2*c^3 - 4*a^2*c^4)*d*x^2 - 2*(a*b^3*c^2
- 2*a^2*b*c^3)*d)*e + ((a^3*b^2*c^3 - 4*a^4*c^4)*d^3*x^2 - (a^3*b^3*c^2 - 4
*a^4*b*c^3)*d^2*x^2*e + (a^4*b^2*c^2 - 4*a^5*c^3)*d*x^2*e^2)*sqrt(((b^4*c^2
 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e +
 (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^2)/((a^6*b^2*c^2 - 4*a^7*c^3)*d^4 - 2*
(a^6*b^3*c - 4*a^7*b*c^2)*d^3*e + (a^6*b^4 - 2*a^7*b^2*c - 8*a^8*c^2)*d^2*e
^2 - 2*(a^7*b^3 - 4*a^8*b*c)*d*e^3 + (a^8*b^2 - 4*a^9*c)*e^4)))/x^2) + sqrt
(1/2)*a*d*x*sqrt(-((b^3*c - 3*a*b*c^2)*d - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e
- ((a^3*b^2*c - 4*a^4*c^2)*d^2 - (a^3*b^3 - 4*a^4*b*c)*d*e + (a^4*b^2 - 4*a
^5*c)*e^2)*sqrt(((b^4*c^2 - 2*a*b^2*c^3 + a^2*c^4)*d^2 - 2*(b^5*c - 3*a*b^3
*c^2 + 2*a^2*b*c^3)*d*e + (b^6 - 4*a*b^4*c + 4*...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(1/(x**2*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^2*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.392 $\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=341

$$-\frac{\sqrt{d+ex^2}}{3adx^3}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x}+\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}}+$$

```
[Out] -1/3*(e*x^2+d)^(1/2)/a/d/x^3+b*(e*x^2+d)^(1/2)/a^2/d/x+2/3*e*(e*x^2+d)^(1/2
)/a/d^2/x+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)
/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/(2*c
*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+c*arctan(x*
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2
))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(b+(-4*a*c+b^2)^(1/2))^(1/
2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1317, 277, 270, 1706, 385, 211}

$$\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}+\frac{b\sqrt{d+ex^2}}{a^2dx}+\frac{2e\sqrt{d+ex^2}}{3ad^2x}-\frac{\sqrt{d+ex^2}}{3adx^3}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*Sqrt[d + e*x^2]/(a*d*x^3) + (b*Sqrt[d + e*x^2])/(a^2*d*x) + (2*e*Sqrt[
d + e*x^2])/(3*a*d^2*x) + (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(
Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqr
t[d + e*x^2])])/(a^2*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x
^2])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c]
)*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \left(\frac{1}{ax^4\sqrt{d+ex^2}} - \frac{b}{a^2x^2\sqrt{d+ex^2}} + \frac{b^2-ac+bcx^2}{a^2\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\right.$$

$$= \frac{\int \frac{b^2-ac+bcx^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}dx}{a^2} + \frac{\int \frac{1}{x^4\sqrt{d+ex^2}}dx}{a} - \frac{b\int \frac{1}{x^2\sqrt{d+ex^2}}dx}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{\int \left(\frac{bc+\frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \right.}{a^2}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{\left(c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\right.}{}$$

$$= -\frac{\sqrt{d+ex^2}}{3adx^3} + \frac{b\sqrt{d+ex^2}}{a^2dx} + \frac{2e\sqrt{d+ex^2}}{3ad^2x} + \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{ta}}{a^2\sqrt{b-\sqrt{b^2-4}}}$$

<u>Mathematica</u> [A]
time = 10.53, size = 320, normalized size = 0.94

$$\frac{\frac{3b\sqrt{d+ex^2}}{dx} - \frac{a(d-2ex^2)\sqrt{d+ex^2}}{d^2x^3} + \frac{3c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}\,e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}} + \frac{3c\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^4*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((3*b*Sqrt[d + e*x^2])/(d*x) - (a*(d - 2*e*x^2)*Sqrt[d + e*x^2])/(d^2*x^3)
+ (3*c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - b*e + Sqr
t[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[
b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e]) + (3*c*(b
+ (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*
```

```
a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[
b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]))/(3*a^2)
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 248, normalized size = 0.73

method	result
risch	$-\frac{\sqrt{e\,x^2+d}\,(-2ae\,x^2-3bd\,x^2+ad)}{3d^2a^2x^3}-\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4\right.}\right.}{2a}$
default	$\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(-bc\_R^2+2\left(2ace-2b^2e+bcd\right)\_R-bc\,d^2\right)}{c\_R^3+3\_R^2be-3\_R^2cd+8\_R a}\right.}{2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/a^2*e^(1/2)*sum((-b*c*_R^2+2*(2*a*c*e-2*b^2*e+b*c*d)*_R-b*c*d^2)/(_R^3*
c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(
((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a
*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))+b*(e*x^2+d)^(1/2)
/a^2/d/x+1/a*(-1/3/d/x^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x^2*e + d)*x^4), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 8220 vs. $2(301) = 602$.
time = 62.84, size = 8220, normalized size = 24.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/12*(3*sqrt(1/2)*a^2*d^2*x^3*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d
 - (b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e + ((a^5*b^2*c - 4*a^6*c^
```

2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)*sqrt(((b^8*c^
2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c
- 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10
- 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*
b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4
 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (
a^12*b^2 - 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6
*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2))*log(((b^5*c^4 - 3*a*b^3*c^5 + a^2*b*c
^6)*d^2*x^2 + 4*(a*b^5*c^3 - 4*a^2*b^3*c^4 + 3*a^3*b*c^5)*x^2*e^2 - 2*(a*b^
4*c^4 - 3*a^2*b^2*c^5 + a^3*c^6)*d^2 + 2*sqrt(1/2)*((a*b^7*c^2 - 7*a^2*b^5*
c^3 + 13*a^3*b^3*c^4 - 4*a^4*b*c^5)*d^2*x - (2*a*b^8*c - 16*a^2*b^6*c^2 + 3
9*a^3*b^4*c^3 - 29*a^4*b^2*c^4 + 4*a^5*c^5)*d*x*e + (a*b^9 - 9*a^2*b^7*c +
27*a^3*b^5*c^2 - 31*a^4*b^3*c^3 + 12*a^5*b*c^4)*x*e^2 - ((a^6*b^4*c^2 - 6*a
^7*b^2*c^3 + 8*a^8*c^4)*d^3*x - (2*a^6*b^5*c - 13*a^7*b^3*c^2 + 20*a^8*b*c^
3)*d^2*x*e + (a^6*b^6 - 6*a^7*b^4*c + 6*a^8*b^2*c^2 + 8*a^9*c^3)*d*x*e^2 -
(a^7*b^5 - 7*a^8*b^3*c + 12*a^9*b*c^2)*x*e^3)*sqrt(((b^8*c^2 - 6*a*b^6*c^3
+ 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 +
16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22
*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11
*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c
- 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^1
3*c)*e^4)))*sqrt(x^2*e + d)*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d -
(b^6 - 6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e + ((a^5*b^2*c - 4*a^6*c^2)*
d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)*sqrt(((b^8*c^2 -
 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7
*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8
*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2
*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 -
2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^1
2*b^2 - 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*
c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)) - ((b^6*c^3 - 9*a^2*b^2*c^5 + 4*a^3*c^6)
*d*x^2 - 2*(a*b^5*c^3 - 4*a^2*b^3*c^4 + 3*a^3*b*c^5)*d)*e + ((a^5*b^2*c^4 -
 4*a^6*c^5)*d^3*x^2 - (a^5*b^3*c^3 - 4*a^6*b*c^4)*d^2*x^2*e + (a^6*b^2*c^3
- 4*a^7*c^4)*d*x^2*e^2)*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b^4*c^4 - 6*a
^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5*c^3 - 13*a^
3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c^2 - 24*a^3*
b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4 - 2*(a^10*b^
3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c^2)*d^2*e^2
- 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4)))/x^2) - 3*s
qrt(1/2)*a^2*d^2*x^3*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d - (b^6 -
6*a*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e + ((a^5*b^2*c - 4*a^6*c^2)*d^2 - (
a^5*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2)*sqrt(((b^8*c^2 - 6*a*b^
6*c^3 + 11*a^2*b^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*
c^2 + 16*a^2*b^5*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*
c + 22*a^2*b^6*c^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 -

```
4*a^11*c^3)*d^4 - 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*
b^2*c - 8*a^12*c^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 -
 4*a^13*c)*e^4)))/((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5*b^3 - 4*a^6*b*c)*d*e
+ (a^6*b^2 - 4*a^7*c)*e^2))*log(((b^5*c^4 - 3*a*b^3*c^5 + a^2*b*c^6)*d^2*x^
2 + 4*(a*b^5*c^3 - 4*a^2*b^3*c^4 + 3*a^3*b*c^5)*x^2*e^2 - 2*(a*b^4*c^4 - 3*
a^2*b^2*c^5 + a^3*c^6)*d^2 - 2*sqrt(1/2)*((a*b^7*c^2 - 7*a^2*b^5*c^3 + 13*a
^3*b^3*c^4 - 4*a^4*b*c^5)*d^2*x - (2*a*b^8*c - 16*a^2*b^6*c^2 + 39*a^3*b^4*
c^3 - 29*a^4*b^2*c^4 + 4*a^5*c^5)*d*x*e + (a*b^9 - 9*a^2*b^7*c + 27*a^3*b^5
*c^2 - 31*a^4*b^3*c^3 + 12*a^5*b*c^4)*x*e^2 - ((a^6*b^4*c^2 - 6*a^7*b^2*c^3
 + 8*a^8*c^4)*d^3*x - (2*a^6*b^5*c - 13*a^7*b^3*c^2 + 20*a^8*b*c^3)*d^2*x*e
 + (a^6*b^6 - 6*a^7*b^4*c + 6*a^8*b^2*c^2 + 8*a^9*c^3)*d*x*e^2 - (a^7*b^5 -
 7*a^8*b^3*c + 12*a^9*b*c^2)*x*e^3)*sqrt(((b^8*c^2 - 6*a*b^6*c^3 + 11*a^2*b
^4*c^4 - 6*a^3*b^2*c^5 + a^4*c^6)*d^2 - 2*(b^9*c - 7*a*b^7*c^2 + 16*a^2*b^5
*c^3 - 13*a^3*b^3*c^4 + 3*a^4*b*c^5)*d*e + (b^10 - 8*a*b^8*c + 22*a^2*b^6*c
^2 - 24*a^3*b^4*c^3 + 9*a^4*b^2*c^4)*e^2)/((a^10*b^2*c^2 - 4*a^11*c^3)*d^4
- 2*(a^10*b^3*c - 4*a^11*b*c^2)*d^3*e + (a^10*b^4 - 2*a^11*b^2*c - 8*a^12*c
^2)*d^2*e^2 - 2*(a^11*b^3 - 4*a^12*b*c)*d*e^3 + (a^12*b^2 - 4*a^13*c)*e^4))
)*sqrt(x^2*e + d)*sqrt(-((b^5*c - 5*a*b^3*c^2 + 5*a^2*b*c^3)*d - (b^6 - 6*a
*b^4*c + 9*a^2*b^2*c^2 - 2*a^3*c^3)*e + ((a^5*b^2*c - 4*a^6*c^2)*d^2 - (a^5
*b^3 - 4*a^6*b*c)*d*e + (a^6*b^2 - 4*a^7*c)*e^2...
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(1/(x**4*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^4*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.393 $\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=443

$$-\frac{\sqrt{d+ex^2}}{5adx^5}+\frac{b\sqrt{d+ex^2}}{3a^2dx^3}+\frac{4e\sqrt{d+ex^2}}{15ad^2x^3}-\frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}-\frac{2be\sqrt{d+ex^2}}{3a^2d^2x}-\frac{8e^2\sqrt{d+ex^2}}{15ad^3x}-\frac{c\left(b^2-ac\right.}{a^3}$$

```
[Out] -1/5*(e*x^2+d)^(1/2)/a/d/x^5+1/3*b*(e*x^2+d)^(1/2)/a^2/d/x^3+4/15*e*(e*x^2+
d)^(1/2)/a/d^2/x^3-(-a*c+b^2)*(e*x^2+d)^(1/2)/a^3/d/x-2/3*b*e*(e*x^2+d)^(1/
2)/a^2/d^2/x-8/15*e^2*(e*x^2+d)^(1/2)/a/d^3/x-c*arctan(x*(2*c*d-e*(b-(-4*a*
c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c
+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^3/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1
/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))
)^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b*(-3*a*c+b^
2)/(-4*a*c+b^2)^(1/2))/a^3/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 1.04, antiderivative size = 443, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {1317, 277, 270, 1706, 385, 211}

$$-\frac{c\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^3\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}-\frac{c\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^3\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}-\frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx}-\frac{2be\sqrt{d+ex^2}}{3a^2d^2x}+\frac{b\sqrt{d+ex^2}}{3a^2dx^3}-\frac{8e^2\sqrt{d+ex^2}}{15ad^3x}+\frac{4e\sqrt{d+ex^2}}{15ad^2x^3}-\frac{\sqrt{d+ex^2}}{5adx^5}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/5*Sqrt[d + e*x^2]/(a*d*x^5) + (b*Sqrt[d + e*x^2])/(3*a^2*d*x^3) + (4*e*S
qrt[d + e*x^2])/(15*a*d^2*x^3) - ((b^2 - a*c)*Sqrt[d + e*x^2])/(a^3*d*x) -
(2*b*e*Sqrt[d + e*x^2])/(3*a^2*d^2*x) - (8*e^2*Sqrt[d + e*x^2])/(15*a*d^3*x
) - (c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^
2])])/(a^3*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e]) - (c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e
*x^2])])/(a^3*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*
c])*e])
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx = \int \left(\frac{1}{ax^6\sqrt{d+ex^2}} - \frac{b}{a^2x^4\sqrt{d+ex^2}} + \frac{b^2-ac}{a^3x^2\sqrt{d+ex^2}} + \frac{-b(b^2-2a\cdots}{a^3\sqrt{d+ex^2}\cdots}\right.$$

$$= \frac{\int \frac{-b(b^2-2ac)-c(b^2-ac)x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{a^3} + \frac{\int \frac{1}{x^6\sqrt{d+ex^2}}\,dx}{a} - \frac{b\int \frac{1}{x^4\sqrt{d+ex^2}}\,dx}{a^2} \cdots$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx} + \frac{\int \left(\frac{-\frac{bc(b^2\cdots}{\sqrt{b^2-\cdots}}}{(b-\sqrt{b^2-4a\cdots}}\right.}{\cdots}$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx} - \cdots$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx} - \cdots$$

$$= -\frac{\sqrt{d+ex^2}}{5adx^5} + \frac{b\sqrt{d+ex^2}}{3a^2dx^3} + \frac{4e\sqrt{d+ex^2}}{15ad^2x^3} - \frac{(b^2-ac)\sqrt{d+ex^2}}{a^3dx} - \cdots$$

Mathematica [A]
time = 11.25, size = 383, normalized size = 0.86

$$-\frac{\frac{15(b^2-ac)\sqrt{d+ex^2}}{dx} - \frac{5ab(d-2ex^2)\sqrt{d+ex^2}}{d^2x^3} + \frac{a^2\sqrt{d+ex^2}\,(3d^2-4dex^2+8e^2x^4)}{d^3x^5} + \frac{15c\left(b^2-ac+\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-be+\sqrt{b^2-4ac}\,e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}} + \frac{15c\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b+\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b+\sqrt{b^2-4ac}\right)e}}}{15a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/(x^6*Sqrt[d + e*x^2]*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/15*((15*(b^2 - a*c)*Sqrt[d + e*x^2])/(d*x) - (5*a*b*(d - 2*e*x^2)*Sqrt[d
 + e*x^2])/(d^2*x^3) + (a^2*Sqrt[d + e*x^2]*(3*d^2 - 4*d*e*x^2 + 8*e^2*x^4)
)/(d^3*x^5) + (15*c*(b^2 - a*c + (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*ArcTa
n[(Sqrt[2*c*d - b*e + Sqrt[b^2 - 4*a*c]*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*
Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d + (-b + Sqrt[b^2
 - 4*a*c])*e]) + (15*c*(b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*Ar
cTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c
```

```
]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e]))/a^3
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.15, size = 349, normalized size = 0.79

method	result
risch	$-\frac{\sqrt{e\,x^{2}+d}\,\left(8a^{2}e^{2}x^{4}+10abde\,x^{4}-15ac\,d^{2}x^{4}+15b^{2}d^{2}x^{4}-4a^{2}de\,x^{2}-5ab\,d^{2}x^{2}+3a^{2}d^{2}\right)}{15d^{3}a^{3}x^{5}}-\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(4eb-4c\right.}\right.}{}$
default	$-\frac{\sqrt{e}\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^{4}+(4eb-4cd)\_Z^{3}+\left(16a\,e^{2}-8deb+6c\,d^{2}\right)\_Z^{2}+\left(4d^{2}eb-4c\,d^{3}\right)\_Z+d^{4}c\right)}\frac{\left(c\left(ac-b^{2}\right)\_R^{2}+2\left(4abce-a\,c^{2}d-2b^{3}e\right.\right.}{c\_R^{3}+3\_R^{2}be-3\_}\right.}{2a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/2/a^3*e^(1/2)*sum((c*(a*c-b^2)*_R^2+2*(4*a*b*c*e-a*c^2*d-2*b^3*e+b^2*c*d
)*_R+a*c^2*d^2-b^2*c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d
*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf
(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d
^3)*_Z+d^4*c))+1/a*(-1/5/d/x^5*(e*x^2+d)^(1/2)-4/5*e/d*(-1/3/d/x^3*(e*x^2+d
)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2)))-(-a*c+b^2)*(e*x^2+d)^(1/2)/a^3/d/x-b/
a^2*(-1/3/d/x^3*(e*x^2+d)^(1/2)+2/3*e/d^2/x*(e*x^2+d)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*sqrt(x^2*e + d)*x^6), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 10039 vs. 2(394) = 788.
time = 157.67, size = 10039, normalized size = 22.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/60*(15*sqrt(1/2)*a^3*d^3*x^5*sqrt(-((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3
 - 7*a^3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*
a^4*c^4)*e + ((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^
8*b^2 - 4*a^9*c)*e^2)*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62
*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c -
 11*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*
b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^
3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*
b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4
 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (
a^16*b^2 - 4*a^17*c)*e^4)))/((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8
*b*c)*d*e + (a^8*b^2 - 4*a^9*c)*e^2))*log(-((b^7*c^5 - 5*a*b^5*c^6 + 6*a^2*
b^3*c^7 - a^3*b*c^8)*d^2*x^2 + 4*(a*b^7*c^4 - 6*a^2*b^5*c^5 + 10*a^3*b^3*c^
6 - 4*a^4*b*c^7)*x^2*e^2 - 2*(a*b^6*c^5 - 5*a^2*b^4*c^6 + 6*a^3*b^2*c^7 - a
^4*c^8)*d^2 + 2*sqrt(1/2)*((a*b^10*c^2 - 10*a^2*b^8*c^3 + 35*a^3*b^6*c^4 -
51*a^4*b^4*c^5 + 29*a^5*b^2*c^6 - 4*a^6*c^7)*d^2*x - (2*a*b^11*c - 22*a^2*b
^9*c^2 + 88*a^3*b^7*c^3 - 155*a^4*b^5*c^4 + 114*a^5*b^3*c^5 - 24*a^6*b*c^6)
*d*x*e + (a*b^12 - 12*a^2*b^10*c + 54*a^3*b^8*c^2 - 112*a^4*b^6*c^3 + 104*a
^5*b^4*c^4 - 32*a^6*b^2*c^5)*x*e^2 - ((a^8*b^5*c^2 - 7*a^9*b^3*c^3 + 12*a^1
0*b*c^4)*d^3*x - (2*a^8*b^6*c - 15*a^9*b^4*c^2 + 30*a^10*b^2*c^3 - 8*a^11*c
^4)*d^2*x*e + (a^8*b^7 - 7*a^9*b^5*c + 11*a^10*b^3*c^2 + 4*a^11*b*c^3)*d*x*
e^2 - (a^9*b^6 - 8*a^10*b^4*c + 18*a^11*b^2*c^2 - 8*a^12*c^3)*x*e^3)*sqrt((
(b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*b^4*c^
6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2 + 46*a^2*b^9*
c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e +
 (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4
- 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 -
2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2
)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4)))*
sqrt(x^2*e + d)*sqrt(-((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^3*b*c^4)
*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4)*e + ((
a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 - 4*a^9*c
)*e^2)*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 +
46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2
+ 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^6
*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148
*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^1
5*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c
 - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^
17*c)*e^4)))/((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^
8*b^2 - 4*a^9*c)*e^2)) - ((b^8*c^4 - 2*a*b^6*c^5 - 10*a^2*b^4*c^6 + 20*a^3*
b^2*c^7 - 4*a^4*c^8)*d*x^2 - 2*(a*b^7*c^4 - 6*a^2*b^5*c^5 + 10*a^3*b^3*c^6
- 4*a^4*b*c^7)*d)*e + ((a^7*b^2*c^5 - 4*a^8*c^6)*d^3*x^2 - (a^7*b^3*c^4 - 4
*a^8*b*c^5)*d^2*x^2*e + (a^8*b^2*c^4 - 4*a^9*c^5)*d*x^2*e^2)*sqrt(((b^12*c^
2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^6*c^5 + 46*a^4*b^4*c^6 - 12*a
```

```
^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b^11*c^2 + 46*a^2*b^9*c^3 - 91
*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6 + 4*a^6*b*c^7)*d*e + (b^14 -
 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c^3 + 148*a^4*b^6*c^4 - 80*a^5
*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2 - 4*a^15*c^3)*d^4 - 2*(a^14*
b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^15*b^2*c - 8*a^16*c^2)*d^2*e^
2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^2 - 4*a^17*c)*e^4)))/x^2) - 1
5*sqrt(1/2)*a^3*d^3*x^5*sqrt(-((b^7*c - 7*a*b^5*c^2 + 14*a^2*b^3*c^3 - 7*a^
3*b*c^4)*d - (b^8 - 8*a*b^6*c + 20*a^2*b^4*c^2 - 16*a^3*b^2*c^3 + 2*a^4*c^4
)*e + ((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d*e + (a^8*b^2 -
 4*a^9*c)*e^2)*sqrt(((b^12*c^2 - 10*a*b^10*c^3 + 37*a^2*b^8*c^4 - 62*a^3*b^
6*c^5 + 46*a^4*b^4*c^6 - 12*a^5*b^2*c^7 + a^6*c^8)*d^2 - 2*(b^13*c - 11*a*b
^11*c^2 + 46*a^2*b^9*c^3 - 91*a^3*b^7*c^4 + 86*a^4*b^5*c^5 - 34*a^5*b^3*c^6
 + 4*a^6*b*c^7)*d*e + (b^14 - 12*a*b^12*c + 56*a^2*b^10*c^2 - 128*a^3*b^8*c
^3 + 148*a^4*b^6*c^4 - 80*a^5*b^4*c^5 + 16*a^6*b^2*c^6)*e^2)/((a^14*b^2*c^2
 - 4*a^15*c^3)*d^4 - 2*(a^14*b^3*c - 4*a^15*b*c^2)*d^3*e + (a^14*b^4 - 2*a^
15*b^2*c - 8*a^16*c^2)*d^2*e^2 - 2*(a^15*b^3 - 4*a^16*b*c)*d*e^3 + (a^16*b^
2 - 4*a^17*c)*e^4)))/((a^7*b^2*c - 4*a^8*c^2)*d^2 - (a^7*b^3 - 4*a^8*b*c)*d
*e + (a^8*b^2 - 4*a^9*c)*e^2))*log(-((b^7*c^5 - 5*a*b^5*c^6 + 6*a^2*b^3*c^7
 - a^3*b*c^8)*d^2*x^2 + 4*(a*b^7*c^4 - 6*a^2*b^5*c^5 + 10*a^3*b^3*c^6 - 4*a
^4*b*c^7)*x^2*e^2 - 2*(a*b^6*c^5 - 5*a^2*b^4*c^6 + 6*a^3*b^2*c^7 - a^4*c^8)
*d^2 - 2*sqrt(1/2)*((a*b^10*c^2 - 10*a^2*b^8*c^3 + 35*a^3*b^6*c^4 - 51*a^4*
b^4*c^5 + 29*a^5*b^2*c^6 - 4*a^6*c^7)*d^2*x - (...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^6\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**6/(c*x**4+b*x**2+a)/(e*x**2+d)**(1/2),x)
[Out] Integral(1/(x**6*sqrt(d + e*x**2)*(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^6/(c*x^4+b*x^2+a)/(e*x^2+d)^(1/2),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^6\,\sqrt{e\,x^2+d}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^6*(d + e*x^2)^(1/2)*(a + b*x^2 + c*x^4)), x)
```

3.394 $\int \frac{x^6}{\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=350

$$-\frac{d^2x}{e\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}+\frac{2\left(b^2-ac-\frac{b\left(b^2-3ac\right)}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)^{3/2}}+\frac{2\left(\right.}{}$$

```
[Out] arctanh(x*e^(1/2)/(e*x^2+d)^(1/2))/c/e^(3/2)-d^2*x/e/(a*e^2-b*d*e+c*d^2)/(e
*x^2+d)^(1/2)+2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(
1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/
2))/c/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+2
*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c
+b^2)^(1/2))^(1/2))*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]
time = 2.86, antiderivative size = 507, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$,
Rules used = {1311, 294, 223, 212, 1706, 385, 211}

$$\frac{\left(-\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,\left(ae^2-bde+cd^2\right)}+\frac{\left(\frac{2a^2ce-ab^2e-3abcd+b^3d}{\sqrt{b^2-4ac}}-abe-acd+b^2d\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{c\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,\left(ae^2-bde+cd^2\right)}-\frac{d^2x}{e\sqrt{d+ex^2}\,\left(ae^2-bde+cd^2\right)}-\frac{(bd-ae)\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{c\sqrt{e}\,\left(ae^2-bde+cd^2\right)}+\frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d^2*x)/(e*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + ((b^2*d - a*c*d -
a*b*e - (b^3*d - 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan
[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*S
qrt[d + e*x^2])])/(c*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2
 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + ((b^2*d - a*c*d - a*b*e + (b^3*d -
 3*a*b*c*d - a*b^2*e + 2*a^2*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (
b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])]
)/(c*Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c
*d^2 - b*d*e + a*e^2)) + (d^2*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^2]])/(e^(3/2
)*(c*d^2 - b*d*e + a*e^2)) - ((b*d - a*e)*ArcTanh[(Sqrt[e]*x)/Sqrt[d + e*x^
2]])/(c*Sqrt[e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 223

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 294

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Dist[c^n
*((m - n + 1)/(b*n*(p + 1))), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[d^2*(f^4/(c*d^2 - b*d*e + a*e^2)), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
f*x)^(m - 4)*(d + e*x^2)^(q + 1)*(Simp[a*d + (b*d - a*e)*x^2, x]/(a + b*x^2
 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^6}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = -\frac{\int \frac{x^2(ad+(bd-ae)x^2)}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{x^2}{(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{d^2x}{e(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\int \left(\frac{bd-ae}{c\sqrt{d+ex^2}} - \frac{a(bd-ae)+(b^2d-acd-}{c\sqrt{d+ex^2}\,(a+bx}\right.}{cd^2-bde+ae^2}$$

$$= -\frac{d^2x}{e(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{\int \frac{a(bd-ae)+(b^2d-acd-abe)x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{c(cd^2-bde+ae^2)} + \frac{d^2\mathrm{Su}}{}$$

$$= -\frac{d^2x}{e(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}(cd^2-bde+ae^2)} + \frac{\int \left(\frac{b^2}{(b-}\right.}{}$$

$$= -\frac{d^2x}{e(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}(cd^2-bde+ae^2)} - \frac{(bd-}{c\sqrt{}}$$

$$= -\frac{d^2x}{e(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{d^2\tanh^{-1}\left(\frac{\sqrt{e}\,x}{\sqrt{d+ex^2}}\right)}{e^{3/2}(cd^2-bde+ae^2)} - \frac{(bd-}{c\sqrt{}}$$

$$= -\frac{d^2x}{e(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{\left(b^2d-acd-abe-\frac{b^3d-3abcd-ab^2e+}{\sqrt{b^2-4ac}}\right.}{c\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 20.34, size = 2281, normalized size = 6.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((b*x)/(c^2*d*Sqrt[d + e*x^2])) - (Sqrt[d]*Sqrt[1 + (e*x^2)/d]*(Sqrt[d]*Sq
rt[e]*x*Sqrt[(d + e*x^2)/d] - d*ArcSinh[(Sqrt[e]*x)/Sqrt[d]] - e*x^2*ArcSin
h[(Sqrt[e]*x)/Sqrt[d]]))/(c*e^(3/2)*(d + e*x^2)^(3/2)) + ((b^2 - a*c + (b*(
```

-b^2 + 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*c^2*(b - Sqrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]) + ((b^2 - a*c - (b*(-b^2 + 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*c^2*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +

```
2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sq
rt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^
2))])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 369, normalized size = 1.05

method	result
default	$\frac{-\frac{x}{e\sqrt{e\,x^2+d}}+\frac{\ln\left(\sqrt{e}\,x+\sqrt{e\,x^2+d}\right)}{e^{\frac{3}{2}}}}{c}-\frac{bx}{c^2 d\sqrt{e\,x^2+d}}-\frac{16\sqrt{e}\left(-\frac{c\left(\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\right.\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 1/c*(-x/e/(e*x^2+d)^(1/2)+1/e^(3/2)*ln(e^(1/2)*x+(e*x^2+d)^(1/2)))-b/c^2*x/
d/(e*x^2+d)^(1/2)-16/c^2*e^(1/2)*(-1/8*c/(4*a*e^2-4*b*d*e+4*c*d^2)*sum(((a*
b*e+a*c*d-b^2*d)*_R^2+2*(2*a^2*e^2-3*a*b*d*e-a*c*d^2+b^2*d^2)*_R+d^2*e*a*b+
a*c*d^3-b^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c
*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(
4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^
4*c))-1/2*(a*b*e+a*c*d-b^2*d)/(4*a*e^2-4*b*d*e+4*c*d^2)/(((e*x^2+d)^(1/2)-e
^(1/2)*x)^2+d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^6/((c*x^4 + b*x^2 + a)*(x^2*e + d)^(3/2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^6}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**6/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [A]
time = 5.52, size = 75, normalized size = 0.21

$$-\frac{c^2d^2x}{(c^3d^2e-bc^2de^2+ac^2e^3)\sqrt{x^2e+d}}-\frac{e^{\left(-\frac{3}{2}\right)}\log\left(\left(xe^{\frac{1}{2}}-\sqrt{x^2e+d}\right)^2\right)}{2\,c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] -c^2*d^2*x/((c^3*d^2*e - b*c^2*d*e^2 + a*c^2*e^3)*sqrt(x^2*e + d)) - 1/2*e^
(-3/2)*log((x*e^(1/2) - sqrt(x^2*e + d))^2)/c
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6}{{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^6/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.395 $\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=360

$$\frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}}-\frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}-\frac{\left(bd-\cdots\right.}{\sqrt{b+\cdots}}$$

```
[Out] d*x/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)-arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(
1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e+(a*b*e+
2*a*c*d-b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-arctan(x*(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b*d-a*e
+(-a*b*e-2*a*c*d+b^2*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+
b^2)^(1/2))^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 360, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1311, 197, 1706, 385, 211}

$$-\frac{\left(-\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}-\frac{\left(\frac{-abe-2acd+b^2d}{\sqrt{b^2-4ac}}-ae+bd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}+\frac{dx}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (d*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((b*d - a*e - (b^2*d - 2*
a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*
c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^
2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)
) - ((b*d - a*e + (b^2*d - 2*a*c*d - a*b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt
[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d
+ e*x^2])])/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1311

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[d^2*(f^4/(c*d^2 - b*d*e + a*e^2)), Int[(f
*x)^(m - 4)*(d + e*x^2)^q, x], x] - Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(
f*x)^(m - 4)*(d + e*x^2)^(q + 1)*(Simp[a*d + (b*d - a*e)*x^2, x]/(a + b*x^2
 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &
&  !IntegerQ[q] && LtQ[q, -1] && GtQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = -\frac{\int \frac{ad+(bd-ae)x^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2} + \frac{d^2\int \frac{1}{(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\int \left(\frac{bd-ae+\frac{-b^2d+2acd+abe}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \right.}{cd^2-bd}$$

$$= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\int \frac{}{\left(b-\sqrt{b^2-\right.}}}{cd^2-bde+ae}$$

$$= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{b-}\right.}{}$$

$$= \frac{dx}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} - \frac{\left(bd-ae-\frac{b^2d-2acd-abe}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2}}{\sqrt{}}\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2\right.}}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 17.76, size = 2162, normalized size = 6.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] x/(c*d*Sqrt[d + e*x^2]) - ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqr
t[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d +
 e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((-
b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2)
)/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[-(((2*c*d
 + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))
]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(
-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2
```

```
)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[-(((2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]]
)/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b + Sqrt[b^2
- 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b
+ Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*H
ypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((
-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))
]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*c*(b - Sqrt[b^2 - 4*a*c]
)*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x
^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]) - ((b - (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((b +
Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d
^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2
 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*Ar
cSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*
c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sq
rt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + Sqr
t[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2
)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*
c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4
*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b +
 Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b +
 Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x
^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2
 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2
 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*c*(b + S
qrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt
[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))*Sq
rt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2))])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 284, normalized size = 0.79

method	result

default	$\frac{x}{cd\sqrt{e\,x^2+d}}+\frac{16\sqrt{e}\left(-\frac{ae-bd}{2(4a\,e^2-4deb+4c\,d^2)\left(\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)^2+d\right)}-\frac{c\left(\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\right.\right.}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)

[Out] 1/c*x/d/(e*x^2+d)^(1/2)+16/c*e^(1/2)*(-1/2*(a*e-b*d)/(4*a*e^2-4*b*d*e+4*c*d^2)/(((e*x^2+d)^(1/2)-e^(1/2)*x)^2+d)-1/8*c/(4*a*e^2-4*b*d*e+4*c*d^2)*sum(((a*e-b*d)*_R^2+2*d*(-3*a*e+b*d)*_R+a*d^2*e-d^3*b)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c)))

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")

[Out] integrate(x^4/((c*x^4 + b*x^2 + a)*(x^2*e + d)^(3/2)), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 14319 vs. 2(329) = 658.
time = 165.06, size = 14319, normalized size = 39.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")

[Out] -1/4*(sqrt(1/2)*(c*d^3 + a*x^2*e^3 - (b*d*x^2 - a*d)*e^2 + (c*d^2*x^2 - b*d^2)*e)*sqrt(-(3*a^2*b*d*e^2 + (b^3 - 3*a*b*c)*d^3 - 2*a^3*e^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(

5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6))*log((12*a^4*d*x^2*e^3 - (a*b^3 - a^2*b*c)*d^4*x^2 + 2*(a^2*b^2 - a^3*c)*d^4 + 2*sqrt(1/2)*((b^5 - 5*a*b^3*c + 4*a^2*b*c^2)*d^5*x - (5*a*b^4 - 22*a^2*b^2*c + 8*a^3*c^2)*d^4*x*e + 9*(a^2*b^3 - 4*a^3*b*c)*d^3*x*e^2 - 6*(a^3*b^2 - 4*a^4*c)*d^2*x*e^3 - ((b^4*c^3 - 6*a*b^2*c^4 + 8*a^2*c^5)*d^8*x - (3*b^5*c^2 - 16*a*b^3*c^3 + 16*a^2*b*c^4)*d^7*x*e + (3*b^6*c - 9*a*b^4*c^2 - 16*a^2*b^2*c^3 + 16*a^3*c^4)*d^6*x*e^2 - (b^7 + 6*a*b^5*c - 40*a^2*b^3*c^2)*d^5*x*e^3 + 5*(a*b^6 - a^2*b^4*c - 12*a^3*b^2*c^2)*d^4*x*e^4 - (11*a^2*b^5 - 32*a^3*b^3*c - 48*a^4*b*c^2)*d^3*x*e^5 + (13*a^3*b^4 - 48*a^4*b^2*c - 16*a^5*c^2)*d^2*x*e^6 - 8*(a^4*b^3 - 4*a^5*b*c)*d*x*e^7 + 2*(a^5*b^2 - 4*a^6*c)*x*e^8)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))*sqrt(x^2*e + d)*sqrt(-(3*a^2*b*d*e^2 + (b^3 - 3*a*b*c)*d^3 - 2*a^3*e^3 - 3*(a*b^2 - 2*a^2*c)*d^2*e + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt(-(18*a^3*b*d^3*e^3 - (b^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e - 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5

```
 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^
6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12
)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3
*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3
+ 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e
^5 + (a^3*b^2 - 4*a^4*c)*e^6)) - 3*(5*a^3*b*d^2*x^2 - 2*a^4*d^2)*e^2 - (6*a
^3*b*d^3 - (7*a^2*b^2 - 4*a^3*c)*d^3*x^2)*e + ((a*b^2*c^3 - 4*a^2*c^4)*d^7*
x^2 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^6*x^2*e + 3*(a*b^4*c - 3*a^2*b^2*c^2 -
4*a^3*c^3)*d^5*x^2*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)*d^4*x^2*e^3 +
 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*d^3*x^2*e^4 - 3*(a^3*b^3 - 4*a^4*b*c
)*d^2*x^2*e^5 + (a^4*b^2 - 4*a^5*c)*d*x^2*e^6)*sqrt(-(18*a^3*b*d^3*e^3 - (b
^4 - 2*a*b^2*c + a^2*c^2)*d^6 - 9*a^4*d^2*e^4 + 6*(a*b^3 - a^2*b*c)*d^5*e -
 3*(5*a^2*b^2 - 2*a^3*c)*d^4*e^2)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 -
4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*
(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^4}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(x**4/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4}{{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(x^4/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.396 $\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=333

$$-\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}}+\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,x}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\,(cd^2-bde+ae^2)}+\frac{c\left(d+\right.}{\sqrt{b+\right.}}$$

```
[Out] -e*x/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)+c*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2
)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(d+(2*a*e-b*d
)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^
(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)+c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2
)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(d+(-2*a*e+b*d)/(-4
*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d-e*
(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]
time = 0.48, antiderivative size = 333, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {1313, 197, 1706, 385, 211}

$$\frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\,\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}+\frac{c\left(\frac{bd-2ae}{\sqrt{b^2-4ac}}+d\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\,\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}-\frac{ex}{\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((e*x)/((c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2])) + (c*(d - (b*d - 2*a*e)/
Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt
[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqr
t[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) + (c*(d + (b*
d - 2*a*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*
e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((d_.) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Dist[(-d)*e*(f^2/(c*d^2 - b*d*e + a*e^2)), Int
[(f*x)^(m - 2)*(d + e*x^2)^q, x], x] + Dist[f^2/(c*d^2 - b*d*e + a*e^2), In
t[(f*x)^(m - 2)*(d + e*x^2)^(q + 1)*(Simp[a*e + c*d*x^2, x]/(a + b*x^2 + c*
x^4)), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] &&  !I
ntegerQ[q] && LtQ[q, -1] && GtQ[m, 1] && LeQ[m, 3]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^2}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \frac{\int \frac{ae+cdx^2}{\sqrt{d+ex^2}(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2} - \frac{(de)\int \frac{1}{(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= -\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd+\frac{c(-bd+2ae)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} + \cdots\right.}{cd^2-bd\cdots}$$

$$= -\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\cdots}{\left(b-\sqrt{b^2-\cdots}\right.}}{cd^2-bde+ae\cdots}$$

$$= -\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{\left(c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \frac{\cdots}{b-\sqrt{\cdots}}\right.}{\cdots}$$

$$= -\frac{ex}{(cd^2-bde+ae^2)\sqrt{d+ex^2}} + \frac{c\left(d-\frac{bd-2ae}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd\cdots}}{\sqrt{b-\cdots}}\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2\cdots}\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 16.50, size = 2119, normalized size = 6.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((1 - b/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] +
2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + S
qrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2
)^2))])/d - 45*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*
(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d +
(-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])
/d - (45*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/
(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*
```

```
a*c])*e)*x^4*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^
2)) + 4*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4
*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-
b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d +
 (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]
+ (4*e*x^2*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2
- 4*a*c] - 2*c*x^2))))^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d
*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*
d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))
)])/d))/(15*(b - Sqrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b
+ Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e
*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))]) + ((1 + b/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d
 + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30
*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])
*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2
, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
))])/d))/(15*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e
)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqr
t[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.14, size = 246, normalized size = 0.74

method	result

default	$-16\sqrt{e}\left(\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(\_R^2cd+2\left(2a\,e^2-c\,d^2\right)\_R+c\,d^3\right)}{c\_R^3+3\_R^2be-3\_R^2cd+8\_}}{32a\,e^2-32deb+32c\,d^2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -16*e^(1/2)*(1/8/(4*a*e^2-4*b*d*e+4*c*d^2)*sum((_R^2*c*d+2*(2*a*e^2-c*d^2)*
_R+c*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*
d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4
*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))+1
/2*d/(4*a*e^2-4*b*d*e+4*c*d^2)/(((e*x^2+d)^(1/2)-e^(1/2)*x)^2+d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(x^2/((c*x^4 + b*x^2 + a)*(x^2*e + d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 13948 vs. $2(301) = 602$.
time = 116.39, size = 13948, normalized size = 41.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] -1/4*(sqrt(1/2)*(c*d^3 + a*x^2*e^3 - (b*d*x^2 - a*d)*e^2 + (c*d^2*x^2 - b*d
^2)*e)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b*c*d*e^2 - (a*b^2 - 2*a^2*c)
*e^3 + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c
- 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e
^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*
d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^
2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 -
4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^
5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e
```

^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5
*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*
b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c -
30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a
^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(
5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*
e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 -
4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a
*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^
4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6))*log(-(b*c^3*d
^4*x^2 - 4*a*c^3*d^3*x^2*e - 2*a*c^3*d^4 - 4*a^2*b*c*x^2*e^4 + 2*sqrt(1/2)*
((b^2*c^3 - 4*a*c^4)*d^5*x - 4*(a*b^2*c^2 - 4*a^2*c^3)*d^3*x*e^2 + (a*b^3*c
 - 4*a^2*b*c^2)*d^2*x*e^3 + 3*(a^2*b^2*c - 4*a^3*c^2)*d*x*e^4 - (a^2*b^3 -
4*a^3*b*c)*x*e^5 - ((b^3*c^4 - 4*a*b*c^5)*d^8*x - (3*b^4*c^3 - 8*a*b^2*c^4
- 16*a^2*c^5)*d^7*x*e + (3*b^5*c^2 + 4*a*b^3*c^3 - 64*a^2*b*c^4)*d^6*x*e^2
- (b^6*c + 17*a*b^4*c^2 - 72*a^2*b^2*c^3 - 48*a^3*c^4)*d^5*x*e^3 + 10*(a*b^
5*c - a^2*b^3*c^2 - 12*a^3*b*c^3)*d^4*x*e^4 - (a*b^6 + 17*a^2*b^4*c - 72*a^
3*b^2*c^2 - 48*a^4*c^3)*d^3*x*e^5 + (3*a^2*b^5 + 4*a^3*b^3*c - 64*a^4*b*c^2
)*d^2*x*e^6 - (3*a^3*b^4 - 8*a^4*b^2*c - 16*a^5*c^2)*d*x*e^7 + (a^4*b^3 - 4
*a^5*b*c)*x*e^8)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^
2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 -
6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d
^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2
 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^
3*c^3 - 40*a^3*b*c^4)*d^7*e^5 + (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^
3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 -
 40*a^4*b*c^3)*d^5*e^7 + 15*(a^2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8
- 10*(2*a^3*b^5 - 5*a^4*b^3*c - 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a
^5*b^2*c - 8*a^6*c^2)*d^2*e^10 - 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2
- 4*a^7*c)*e^12)))*sqrt(x^2*e + d)*sqrt(-(b*c^2*d^3 - 6*a*c^2*d^2*e + 3*a*b
*c*d*e^2 - (a*b^2 - 2*a^2*c)*e^3 + ((b^2*c^3 - 4*a*c^4)*d^6 - 3*(b^3*c^2 -
4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a^2*c^3)*d^4*e^2 - (b^5 + 2*a
*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a^2*b^2*c - 4*a^3*c^2)*d^2*e^
4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 - 4*a^4*c)*e^6)*sqrt((c^4*d^6
- 6*a*c^3*d^4*e^2 + 2*a*b*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5
 + a^2*b^2*e^6)/((b^2*c^6 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e
+ 3*(5*b^4*c^4 - 18*a*b^2*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b
^3*c^4 - 12*a^2*b*c^5)*d^9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*
d^8*e^4 - 6*(b^7*c + 6*a*b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^5 +
 (b^8 + 26*a*b^6*c - 30*a^2*b^4*c^2 - 340*a^3*b^2*c^3 - 80*a^4*c^4)*d^6*e^6
 - 6*(a*b^7 + 6*a^2*b^5*c - 30*a^3*b^3*c^2 - 40*a^4*b*c^3)*d^5*e^7 + 15*(a^
2*b^6 - 15*a^4*b^2*c^2 - 4*a^5*c^3)*d^4*e^8 - 10*(2*a^3*b^5 - 5*a^4*b^3*c -
 12*a^5*b*c^2)*d^3*e^9 + 3*(5*a^4*b^4 - 18*a^5*b^2*c - 8*a^6*c^2)*d^2*e^10
- 6*(a^5*b^3 - 4*a^6*b*c)*d*e^11 + (a^6*b^2 - 4*a^7*c)*e^12)))/((b^2*c^3 -
4*a*c^4)*d^6 - 3*(b^3*c^2 - 4*a*b*c^3)*d^5*e + 3*(b^4*c - 3*a*b^2*c^2 - 4*a

```
^2*c^3)*d^4*e^2 - (b^5 + 2*a*b^3*c - 24*a^2*b*c^2)*d^3*e^3 + 3*(a*b^4 - 3*a
^2*b^2*c - 4*a^3*c^2)*d^2*e^4 - 3*(a^2*b^3 - 4*a^3*b*c)*d*e^5 + (a^3*b^2 -
4*a^4*c)*e^6)) - (2*a^2*b*c*d - (a*b^2*c + 12*a^2*c^2)*d*x^2)*e^3 - 3*(a*b*
c^2*d^2*x^2 - 2*a^2*c^2*d^2)*e^2 - ((b^2*c^4 - 4*a*c^5)*d^7*x^2 - 3*(b^3*c^
3 - 4*a*b*c^4)*d^6*x^2*e + 3*(b^4*c^2 - 3*a*b^2*c^3 - 4*a^2*c^4)*d^5*x^2*e^
2 - (b^5*c + 2*a*b^3*c^2 - 24*a^2*b*c^3)*d^4*x^2*e^3 + 3*(a*b^4*c - 3*a^2*b
^2*c^2 - 4*a^3*c^3)*d^3*x^2*e^4 - 3*(a^2*b^3*c - 4*a^3*b*c^2)*d^2*x^2*e^5 +
 (a^3*b^2*c - 4*a^4*c^2)*d*x^2*e^6)*sqrt((c^4*d^6 - 6*a*c^3*d^4*e^2 + 2*a*b
*c^2*d^3*e^3 + 9*a^2*c^2*d^2*e^4 - 6*a^2*b*c*d*e^5 + a^2*b^2*e^6)/((b^2*c^6
 - 4*a*c^7)*d^12 - 6*(b^3*c^5 - 4*a*b*c^6)*d^11*e + 3*(5*b^4*c^4 - 18*a*b^2
*c^5 - 8*a^2*c^6)*d^10*e^2 - 10*(2*b^5*c^3 - 5*a*b^3*c^4 - 12*a^2*b*c^5)*d^
9*e^3 + 15*(b^6*c^2 - 15*a^2*b^2*c^4 - 4*a^3*c^5)*d^8*e^4 - 6*(b^7*c + 6*a*
b^5*c^2 - 30*a^2*b^3*c^3 - 40*a^3*b*c^4)*d^7*e^...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{x^2}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(x**2/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^2}{{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(x^2/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.397 $\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=341

$$\frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}}-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\;x}{\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\;\left(cd^2-bde+ae^2\right)}-\frac{c\left(e\right.}{\sqrt{b+}}$$

```
[Out] e^2*x/d/(a*e^2-b*d*e+c*d^2)/(e*x^2+d)^(1/2)-c*arctan(x*(2*c*d-e*(b-(-4*a*c+
b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(e+(b*e-2*
c*d)/(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-c*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(
1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(e+(-b*e+2*c*d)/
(-4*a*c+b^2)^(1/2))/(a*e^2-b*d*e+c*d^2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)/(2*c*d
-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)
```

Rubi [A]

time = 0.56, antiderivative size = 341, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {1186, 197, 1706, 385, 211}

$$-\frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{d+ex^2}}\right)}{\sqrt{b-\sqrt{b^2-4ac}}\;\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\;\left(ae^2-bde+cd^2\right)}-\frac{c\left(\frac{2cd-be}{\sqrt{b^2-4ac}}+e\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\;\sqrt{d+ex^2}}\right)}{\sqrt{\sqrt{b^2-4ac}+b}\;\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\;\left(ae^2-bde+cd^2\right)}+\frac{e^2x}{d\sqrt{d+ex^2}\;\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (e^2*x)/(d*(c*d^2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - (c*(e - (2*c*d - b*e)
/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqr
t[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sq
rt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(e + (2
*c*d - b*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])
*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(Sqrt[b + Sqrt[b^2 -
 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1186

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x^2)^q, x], x] + Dist[1
/(c*d^2 - b*d*e + a*e^2), Int[(d + e*x^2)^(q + 1)*((c*d - b*e - c*e*x^2)/(a
 + b*x^2 + c*x^4)), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c,
 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !IntegerQ[q] && LtQ[q, -1]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx = \frac{\int \frac{cd-be-cex^2}{\sqrt{d+ex^2}\,(a+bx^2+cx^4)}\,dx}{cd^2-bde+ae^2} + \frac{e^2\int \frac{1}{(d+ex^2)^{3/2}}\,dx}{cd^2-bde+ae^2}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} + \frac{\int \left(\frac{-ce-\frac{c(-2cd+be)}{\sqrt{b^2-4ac}}}{\left(b-\sqrt{b^2-4ac}+2cx^2\right)\sqrt{d+ex^2}} \cdots\right.}{cd^2-\cdots}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\int \frac{\cdots}{\left(b-\sqrt{b^2-\cdots}\right.}}{cd^2-bde+a\cdots}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{\left(c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\right)\mathrm{Subst}\left(\int \frac{\cdots}{b-\cdots}\right.}{\cdots}$$

$$= \frac{e^2x}{d\left(cd^2-bde+ae^2\right)\sqrt{d+ex^2}} - \frac{c\left(e-\frac{2cd-be}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2c\cdots}}{\sqrt{b\cdots}}\right.}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-\left(b-\sqrt{b^2\cdots}\right.}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 15.38, size = 2061, normalized size = 6.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (2*c*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
```

```
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2)))]]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
 + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*Sqrt[b^2 -
 4*a*c]*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 -
 4*a*c] - 2*c*x^2))))^(3/2)*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)*Sqrt[d + e*x^
2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] -
 2*c*x^2))]) - (2*c*x*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2))]*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d +
(b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2
*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqr
t[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^
2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*
c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 -
4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin
[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt
[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)
)]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(
d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 -
 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sq
rt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hyperg
eometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a
*c])*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)))^(3/2)*(d + e*x^2)^(3/2))
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.12, size = 240, normalized size = 0.70

method	result

default	$-64e^{\frac{3}{2}}\left(-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)}\frac{\left(c\_R^2+2(2eb-3cd)\_R+c\,d^2\right)\ln}{c\_R^3+3\_R^2be-3\_R^2cd+8\_}}{8(16a\,e^2-16deb+16c\,d^2)}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] -64*e^(3/2)*(-1/8/(16*a*e^2-16*b*d*e+16*c*d^2)*sum((c*_R^2+2*(2*b*e-3*c*d)*
_R+c*d^2)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*
d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4
*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c))-1
/2/(16*a*e^2-16*b*d*e+16*c*d^2)/(((e*x^2+d)^(1/2)-e^(1/2)*x)^2+d))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(x^2*e + d)^(3/2)), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 17065 vs. $2(309) = 618$.
time = 225.02, size = 17065, normalized size = 50.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] 1/4*(sqrt(1/2)*(c*d^4 + a*d*x^2*e^3 - (b*d^2*x^2 - a*d^2)*e^2 + (c*d^3*x^2
- b*d^3)*e)*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*e + 3*(b^3*c - 3*a
*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + ((a*b^2*c^3 - 4*a^2*c^4
)*d^6 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 3*(a*b^4*c - 3*a^2*b^2*c^2 - 4*
a^3*c^3)*d^4*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)*d^3*e^3 + 3*(a^2*b^
4 - 3*a^3*b^2*c - 4*a^4*c^2)*d^2*e^4 - 3*(a^3*b^3 - 4*a^4*b*c)*d*e^5 + (a^4
*b^2 - 4*a^5*c)*e^6)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5
)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c
^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^
```

6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/((a^2*b^2*c^6 - 4*a^3*c^7)*d^12 - 6*(a^
2*b^3*c^5 - 4*a^3*b*c^6)*d^11*e + 3*(5*a^2*b^4*c^4 - 18*a^3*b^2*c^5 - 8*a^4
*c^6)*d^10*e^2 - 10*(2*a^2*b^5*c^3 - 5*a^3*b^3*c^4 - 12*a^4*b*c^5)*d^9*e^3
+ 15*(a^2*b^6*c^2 - 15*a^4*b^2*c^4 - 4*a^5*c^5)*d^8*e^4 - 6*(a^2*b^7*c + 6*
a^3*b^5*c^2 - 30*a^4*b^3*c^3 - 40*a^5*b*c^4)*d^7*e^5 + (a^2*b^8 + 26*a^3*b^
6*c - 30*a^4*b^4*c^2 - 340*a^5*b^2*c^3 - 80*a^6*c^4)*d^6*e^6 - 6*(a^3*b^7 +
6*a^4*b^5*c - 30*a^5*b^3*c^2 - 40*a^6*b*c^3)*d^5*e^7 + 15*(a^4*b^6 - 15*a^
6*b^2*c^2 - 4*a^7*c^3)*d^4*e^8 - 10*(2*a^5*b^5 - 5*a^6*b^3*c - 12*a^7*b*c^2
)*d^3*e^9 + 3*(5*a^6*b^4 - 18*a^7*b^2*c - 8*a^8*c^2)*d^2*e^10 - 6*(a^7*b^3
- 4*a^8*b*c)*d*e^11 + (a^8*b^2 - 4*a^9*c)*e^12)))/((a*b^2*c^3 - 4*a^2*c^4)*
d^6 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 3*(a*b^4*c - 3*a^2*b^2*c^2 - 4*a^
3*c^3)*d^4*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)*d^3*e^3 + 3*(a^2*b^4
- 3*a^3*b^2*c - 4*a^4*c^2)*d^2*e^4 - 3*(a^3*b^3 - 4*a^4*b*c)*d*e^5 + (a^4*b
^2 - 4*a^5*c)*e^6))*log(-(b*c^5*d^4*x^2 - 2*a*c^5*d^4 + 4*(a*b^3*c^2 - 2*a^
2*b*c^3)*x^2*e^4 + 2*sqrt(1/2)*(2*(a*b^2*c^4 - 4*a^2*c^5)*d^4*x*e - 7*(a*b^
3*c^3 - 4*a^2*b*c^4)*d^3*x*e^2 + 3*(3*a*b^4*c^2 - 14*a^2*b^2*c^3 + 8*a^3*c^
4)*d^2*x*e^3 - (5*a*b^5*c - 27*a^2*b^3*c^2 + 28*a^3*b*c^3)*d*x*e^4 + (a*b^6
- 6*a^2*b^4*c + 8*a^3*b^2*c^2)*x*e^5 + (2*(a^2*b^2*c^5 - 4*a^3*c^6)*d^8*x
- 8*(a^2*b^3*c^4 - 4*a^3*b*c^5)*d^7*x*e + (13*a^2*b^4*c^3 - 48*a^3*b^2*c^4
- 16*a^4*c^5)*d^6*x*e^2 - (11*a^2*b^5*c^2 - 32*a^3*b^3*c^3 - 48*a^4*b*c^4)*
d^5*x*e^3 + 5*(a^2*b^6*c - a^3*b^4*c^2 - 12*a^4*b^2*c^3)*d^4*x*e^4 - (a^2*b
^7 + 6*a^3*b^5*c - 40*a^4*b^3*c^2)*d^3*x*e^5 + (3*a^3*b^6 - 9*a^4*b^4*c - 1
6*a^5*b^2*c^2 + 16*a^6*c^3)*d^2*x*e^6 - (3*a^4*b^5 - 16*a^5*b^3*c + 16*a^6*
b*c^2)*d*x*e^7 + (a^5*b^4 - 6*a^6*b^2*c + 8*a^7*c^2)*x*e^8)*sqrt((c^6*d^6 -
6*b*c^5*d^5*e + 3*(5*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c
^4)*d^3*e^3 + 3*(5*b^4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c -
3*a*b^3*c^2 + 2*a^2*b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/
((a^2*b^2*c^6 - 4*a^3*c^7)*d^12 - 6*(a^2*b^3*c^5 - 4*a^3*b*c^6)*d^11*e + 3*
(5*a^2*b^4*c^4 - 18*a^3*b^2*c^5 - 8*a^4*c^6)*d^10*e^2 - 10*(2*a^2*b^5*c^3 -
5*a^3*b^3*c^4 - 12*a^4*b*c^5)*d^9*e^3 + 15*(a^2*b^6*c^2 - 15*a^4*b^2*c^4 -
4*a^5*c^5)*d^8*e^4 - 6*(a^2*b^7*c + 6*a^3*b^5*c^2 - 30*a^4*b^3*c^3 - 40*a^
5*b*c^4)*d^7*e^5 + (a^2*b^8 + 26*a^3*b^6*c - 30*a^4*b^4*c^2 - 340*a^5*b^2*c
^3 - 80*a^6*c^4)*d^6*e^6 - 6*(a^3*b^7 + 6*a^4*b^5*c - 30*a^5*b^3*c^2 - 40*a
^6*b*c^3)*d^5*e^7 + 15*(a^4*b^6 - 15*a^6*b^2*c^2 - 4*a^7*c^3)*d^4*e^8 - 10*
(2*a^5*b^5 - 5*a^6*b^3*c - 12*a^7*b*c^2)*d^3*e^9 + 3*(5*a^6*b^4 - 18*a^7*b^
2*c - 8*a^8*c^2)*d^2*e^10 - 6*(a^7*b^3 - 4*a^8*b*c)*d*e^11 + (a^8*b^2 - 4*a
^9*c)*e^12)))*sqrt(x^2*e + d)*sqrt(-(b*c^3*d^3 - 3*(b^2*c^2 - 2*a*c^3)*d^2*
e + 3*(b^3*c - 3*a*b*c^2)*d*e^2 - (b^4 - 4*a*b^2*c + 2*a^2*c^2)*e^3 + ((a*b
^2*c^3 - 4*a^2*c^4)*d^6 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 3*(a*b^4*c -
3*a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)*d
^3*e^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*d^2*e^4 - 3*(a^3*b^3 - 4*a^4
*b*c)*d*e^5 + (a^4*b^2 - 4*a^5*c)*e^6)*sqrt((c^6*d^6 - 6*b*c^5*d^5*e + 3*(5
*b^2*c^4 - 2*a*c^5)*d^4*e^2 - 2*(10*b^3*c^3 - 11*a*b*c^4)*d^3*e^3 + 3*(5*b^
4*c^2 - 10*a*b^2*c^3 + 3*a^2*c^4)*d^2*e^4 - 6*(b^5*c - 3*a*b^3*c^2 + 2*a^2*
b*c^3)*d*e^5 + (b^6 - 4*a*b^4*c + 4*a^2*b^2*c^2)*e^6)/((a^2*b^2*c^6 - 4*a^3

```
*c^7)*d^12 - 6*(a^2*b^3*c^5 - 4*a^3*b*c^6)*d^11*e + 3*(5*a^2*b^4*c^4 - 18*a
^3*b^2*c^5 - 8*a^4*c^6)*d^10*e^2 - 10*(2*a^2*b^5*c^3 - 5*a^3*b^3*c^4 - 12*a
^4*b*c^5)*d^9*e^3 + 15*(a^2*b^6*c^2 - 15*a^4*b^2*c^4 - 4*a^5*c^5)*d^8*e^4 -
 6*(a^2*b^7*c + 6*a^3*b^5*c^2 - 30*a^4*b^3*c^3 - 40*a^5*b*c^4)*d^7*e^5 + (a
^2*b^8 + 26*a^3*b^6*c - 30*a^4*b^4*c^2 - 340*a^5*b^2*c^3 - 80*a^6*c^4)*d^6*
e^6 - 6*(a^3*b^7 + 6*a^4*b^5*c - 30*a^5*b^3*c^2 - 40*a^6*b*c^3)*d^5*e^7 + 1
5*(a^4*b^6 - 15*a^6*b^2*c^2 - 4*a^7*c^3)*d^4*e^8 - 10*(2*a^5*b^5 - 5*a^6*b^
3*c - 12*a^7*b*c^2)*d^3*e^9 + 3*(5*a^6*b^4 - 18*a^7*b^2*c - 8*a^8*c^2)*d^2*
e^10 - 6*(a^7*b^3 - 4*a^8*b*c)*d*e^11 + (a^8*b^2 - 4*a^9*c)*e^12)))/((a*b^2
*c^3 - 4*a^2*c^4)*d^6 - 3*(a*b^3*c^2 - 4*a^2*b*c^3)*d^5*e + 3*(a*b^4*c - 3*
a^2*b^2*c^2 - 4*a^3*c^3)*d^4*e^2 - (a*b^5 + 2*a^2*b^3*c - 24*a^3*b*c^2)*d^3
*e^3 + 3*(a^2*b^4 - 3*a^3*b^2*c - 4*a^4*c^2)*d^2*e^4 - 3*(a^3*b^3 - 4*a^4*b
*c)*d*e^5 + (a^4*b^2 - 4*a^5*c)*e^6)) - ((b^4*c...
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(d+ex^2)^{\frac{3}{2}}\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(1/((d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{(e\,x^2+d)^{3/2}\,(c\,x^4+b\,x^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/((d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.398 $\int \frac{1}{x^2\left(d+ex^2\right)^{3/2}\left(a+bx^2+cx^4\right)}\,dx$

Optimal. Leaf size=339

$$\frac{e(cd-be)x}{ad\left(cd^2+e(-bd+ae)\right)\sqrt{d+ex^2}}+\frac{-d-2ex^2}{ad^2x\sqrt{d+ex^2}}-\frac{2c^2\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\tan^{-1}\left(\frac{\sqrt{2cd-\left(b-\sqrt{b^2-4a\right.}}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d}}\right.}{a\sqrt{b-\sqrt{b^2-4ac}}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)}$$

```
[Out] e*(-b*e+c*d)*x/a/d/(c*d^2+e*(a*e-b*d))/(e*x^2+d)^(1/2)+(-2*e*x^2-d)/a/d^2/x
/(e*x^2+d)^(1/2)-2*c^2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x
^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(1+b/(-4*a*c+b^2)^(1/2))/a/(2*c*d
-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2)-2*c^2*arctan(
x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1
/2))^(1/2))*(1-b/(-4*a*c+b^2)^(1/2))/a/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/
2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 2.01, antiderivative size = 462, normalized size of antiderivative = 1.36, number of steps used = 12, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1315, 277, 197, 6860, 270, 1706, 385, 211}

$$-\frac{c\left(\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\left(ae^2-bde+cd^2\right)}-\frac{c\left(-\frac{2ace+b^2(-e)+bcd}{\sqrt{b^2-4ac}}-be+cd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\left(ae^2-bde+cd^2\right)}-\frac{e^2}{dx\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}-\frac{\sqrt{d+ex^2}(cd-be)}{adx\left(ae^2-bde+cd^2\right)}-\frac{2e^3x}{d^2\sqrt{d+ex^2}\left(ae^2-bde+cd^2\right)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -(e^2/(d*(c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2])) - (2*e^3*x)/(d^2*(c*d^
2 - b*d*e + a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(a*d*(c
*d^2 - b*d*e + a*e^2)*x) - (c*(c*d - b*e + (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b
^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*x)/(Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[2*
c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2)) - (c*(c*d - b*e -
 (b*c*d - b^2*e + 2*a*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqr
t[b^2 - 4*a*c])*e]*x)/(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a*Sq
rt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 -
b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(d + e*x
^2)^(q + 1)*(Simp[c*d - b*e - c*e*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && L
tQ[q, -1]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^2 (d+ex^2)^{3/2} (a+bx^2+cx^4)} dx = \frac{\int \frac{cd-be-cex^2}{x^2\sqrt{d+ex^2}(a+bx^2+cx^4)} dx}{cd^2-bde+ae^2} + \frac{e^2 \int \frac{1}{x^2(d+ex^2)^{3/2}} dx}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{d(cd^2-bde+ae^2)x\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd-be}{ax^2\sqrt{d+ex^2}} + \frac{-bcd+b^2e-ace}{a\sqrt{d+ex^2}}\right.}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{d(cd^2-bde+ae^2)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2(cd^2-bde+ae^2)\sqrt{d+ex^2}}$$

$$= -\frac{e^2}{d(cd^2-bde+ae^2)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2(cd^2-bde+ae^2)\sqrt{d+ex^2}}$$

$$= -\frac{e^2}{d(cd^2-bde+ae^2)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2(cd^2-bde+ae^2)\sqrt{d+ex^2}}$$

$$= -\frac{e^2}{d(cd^2-bde+ae^2)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2(cd^2-bde+ae^2)\sqrt{d+ex^2}}$$

$$= -\frac{e^2}{d(cd^2-bde+ae^2)x\sqrt{d+ex^2}} - \frac{2e^3x}{d^2(cd^2-bde+ae^2)\sqrt{d+ex^2}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 16.57, size = 2158, normalized size = 6.37

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((d + 2*e*x^2)/(a*d^2*x*Sqrt[d + e*x^2])) - ((c + (b*c)/Sqrt[b^2 - 4*a*c])
*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
```

```
)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2*(
d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt
[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] -
2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e
)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b + Sq
rt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)
*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*a*c]
 - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[-
(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (-b +
 Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/2)*
Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*
c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*
e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d + (-
b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5/
2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c] -
 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a*(b - Sqrt[b^
2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b
^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c]))*Sq
rt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2
 - 4*a*c] - 2*c*x^2))]) + ((-c + (b*c)/Sqrt[b^2 - 4*a*c])*x*(45*Sqrt[-(((b
+ Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/
(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*Sqrt[-(((b + Sqrt[b
^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d + e*x^2))/(d^2*(b
+ Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]] - (30*e*x^2*
ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*
a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[
Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30*e*(-2*c*d + (b + S
qrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x
^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + Sqrt[b^2 - 4*a*c] +
2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 -
 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b
 + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b
 + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))] + (4*e
*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] +
 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b
^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((2*c*d - (b + Sqrt[b
^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])/d))/(15*a*(b +
 Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sq
rt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]))*
Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(b + Sqrt[b^2
```

```
- 4*a*c] + 2*c*x^2))])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.17, size = 329, normalized size = 0.97

method	result
default	$\frac{16\sqrt{e}\left(-\frac{eb-cd}{2(4a\,e^2-4deb+4c\,d^2)\left(\left(\sqrt{e\,x^2+d}-\sqrt{e}\,x\right)^2+d\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2e\right.\right.}}{}\right.}{a}$
risch	$-\frac{\sqrt{e\,x^2+d}}{d^2ax}-\frac{e^2\sqrt{e\left(x+\frac{\sqrt{-de}}{e}\right)^2-2\sqrt{-de}\,\left(x+\frac{\sqrt{-de}}{e}\right)}}{2d^2(a\,e^2-deb+c\,d^2)\left(x+\frac{\sqrt{-de}}{e}\right)}-\frac{e^2\sqrt{e\left(x-\frac{\sqrt{-de}}{e}\right)^2+2\sqrt{}}}{2d^2(a\,e^2-deb+c\,d^2)\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 16/a*e^(1/2)*(-1/2*(b*e-c*d)/(4*a*e^2-4*b*d*e+4*c*d^2)/(((e*x^2+d)^(1/2)-e^
(1/2)*x)^2+d)+1/8/(4*a*e^2-4*b*d*e+4*c*d^2)*sum((c*(-b*e+c*d)*_R^2+2*(2*a*c
*e^2-2*b^2*e^2+3*b*c*d*e-c^2*d^2)*_R-d^2*e*b*c+c^2*d^3)/(_R^3*c+3*_R^2*b*e-
3*_R^2*c*d+8*_R*a*e^2-4*_R*b*d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1
/2)-e^(1/2)*x)^2-_R),_R=RootOf(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+
6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*d^3)*_Z+d^4*c)))+1/a*(-1/d/x/(e*x^2+d)^(1/2)-2
*e/d^2*x/(e*x^2+d)^(1/2))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(x^2*e + d)^(3/2)*x^2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^2 \left(d+ex^2\right)^{\frac{3}{2}} \left(a+bx^2+cx^4\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**2/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
[Out] Integral(1/(x**2*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^2/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] Timed out
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^2\,{\left(e\,x^2+d\right)}^{3/2}\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
[Out] int(1/(x^2*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.399 $\int \frac{1}{x^4(d+ex^2)^{3/2}(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=419

$$-\frac{1}{3adx^3\sqrt{d+ex^2}}+\frac{3bd+4ae}{3a^2d^2x\sqrt{d+ex^2}}+\frac{2e(3bd+4ae)x}{3a^2d^3\sqrt{d+ex^2}}-\frac{e(bcd-b^2e+ace)\,x}{a^2d\,(cd^2+e(-bd+ae))\,\sqrt{d+ex^2}}+\frac{2c^2\left(b+\frac{b}{\sqrt{b^2}}\right.}{a^2\sqrt{b-}}$$

```
[Out] -1/3/a/d/x^3/(e*x^2+d)^(1/2)+1/3*(4*a*e+3*b*d)/a^2/d^2/x/(e*x^2+d)^(1/2)+2/
3*e*(4*a*e+3*b*d)*x/a^2/d^3/(e*x^2+d)^(1/2)-e*(a*c*e-b^2*e+b*c*d)*x/a^2/d/(
c*d^2+e*(a*e-b*d))/(e*x^2+d)^(1/2)+2*c^2*arctan(x*(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))^(1/2)/(e*x^2+d)^(1/2)/(b-(-4*a*c+b^2)^(1/2))^(1/2))*(b+(-2*a*c+b^2)
/(-4*a*c+b^2)^(1/2))/a^2/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))^(3/2)/(b-(-4*a*c+
b^2)^(1/2))^(1/2)+2*c^2*arctan(x*(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(1/2)/(e*
x^2+d)^(1/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2
))/a^2/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))^(3/2)/(b+(-4*a*c+b^2)^(1/2))^(1/2)
```

Rubi [A]

time = 3.95, antiderivative size = 647, normalized size of antiderivative = 1.54, number of steps used = 15, number of rules used = 8, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.276$, Rules used = {1315, 277, 197, 6860, 270, 1706, 385, 211}

$$\frac{c\left(\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}}{\sqrt{b-\sqrt{b^2-4ac}}\sqrt{d+ex^2}}\right)}{a^2\sqrt{b-\sqrt{b^2-4ac}}\sqrt{2cd-e\left(b-\sqrt{b^2-4ac}\right)}\,(ae^2-bde+cd^2)}+\frac{c\left(-\frac{3abce-2ac^2d+b^3(-e)+b^2cd}{\sqrt{b^2-4ac}}+ace+b^2(-e)+bcd\right)\text{ArcTan}\left(\frac{x\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}}{\sqrt{\sqrt{b^2-4ac}+b}\sqrt{d+ex^2}}\right)}{a^2\sqrt{\sqrt{b^2-4ac}+b}\sqrt{2cd-e\left(\sqrt{b^2-4ac}+b\right)}\,(ae^2-bde+cd^2)}+\frac{\sqrt{d+ex^2}\,(ace+b^2(-e)+bcd)}{a^2dx\,(ae^2-bde+cd^2)}+\frac{2e\sqrt{d+ex^2}\,(cd-be)}{3ad^2x\,(ae^2-bde+cd^2)}-\frac{e^2}{3dx^3\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}-\frac{\sqrt{d+ex^2}\,(cd-be)}{3adx^3\,(ae^2-bde+cd^2)}+\frac{4e^3}{3d^2x\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}+\frac{8e^4x}{3d^3\sqrt{d+ex^2}\,(ae^2-bde+cd^2)}$$

Antiderivative was successfully verified.

```
[In] Int[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/3*e^2/(d*(c*d^2 - b*d*e + a*e^2)*x^3*Sqrt[d + e*x^2]) + (4*e^3)/(3*d^2*(
c*d^2 - b*d*e + a*e^2)*x*Sqrt[d + e*x^2]) + (8*e^4*x)/(3*d^3*(c*d^2 - b*d*e
 + a*e^2)*Sqrt[d + e*x^2]) - ((c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d*(c*d^2 -
b*d*e + a*e^2)*x^3) + (2*e*(c*d - b*e)*Sqrt[d + e*x^2])/(3*a*d^2*(c*d^2 - b
*d*e + a*e^2)*x) + ((b*c*d - b^2*e + a*c*e)*Sqrt[d + e*x^2])/(a^2*d*(c*d^2
- b*d*e + a*e^2)*x) + (c*(b*c*d - b^2*e + a*c*e + (b^2*c*d - 2*a*c^2*d - b^
3*e + 3*a*b*c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b - Sqrt[b^2 - 4*
a*c])*e]*x)/(Sqrt[b - Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b - S
qrt[b^2 - 4*a*c]]*Sqrt[2*c*d - (b - Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e +
a*e^2)) + (c*(b*c*d - b^2*e + a*c*e - (b^2*c*d - 2*a*c^2*d - b^3*e + 3*a*b*
c*e)/Sqrt[b^2 - 4*a*c])*ArcTan[(Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*x)/
(Sqrt[b + Sqrt[b^2 - 4*a*c]]*Sqrt[d + e*x^2])])/(a^2*Sqrt[b + Sqrt[b^2 - 4*
a*c]]*Sqrt[2*c*d - (b + Sqrt[b^2 - 4*a*c])*e]*(c*d^2 - b*d*e + a*e^2))
```

Rule 197

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x*((a + b*x^n)^(p + 1)
/a), x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

Rule 277

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Dist[b*((m + n*(p + 1) + 1)/(a*(m + 1
))), Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && IL
tQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 1315

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Dist[e^2/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(
d + e*x^2)^q, x], x] + Dist[1/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^m*(d + e*x
^2)^(q + 1)*(Simp[c*d - b*e - c*e*x^2, x]/(a + b*x^2 + c*x^4)), x], x] /; F
reeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4*a*c, 0] &&  !IntegerQ[q] && L
tQ[q, -1]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rule 6860

```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
```

```
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Rubi steps

$$\int \frac{1}{x^4 \left(d+ex^2\right)^{3/2} \left(a+bx^2+cx^4\right)} dx = \frac{\int \frac{cd-be-cex^2}{x^4\sqrt{d+ex^2}\,(a+bx^2+cx^4)} dx}{cd^2-bde+ae^2} + \frac{e^2 \int \frac{1}{x^4(d+ex^2)^{3/2}} dx}{cd^2-bde+ae^2}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{\int \left(\frac{cd-be}{ax^4\sqrt{d+ex^2}} + \frac{-bcd+b^2e-}{a^2x^2\sqrt{d+}}\right.}{}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+e}}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+e}}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+e}}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+e}}$$

$$= -\frac{e^2}{3d\left(cd^2-bde+ae^2\right)x^3\sqrt{d+ex^2}} + \frac{4e^3}{3d^2\left(cd^2-bde+ae^2\right)x\sqrt{d+e}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 16.16, size = 2218, normalized size = 5.29

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (b*(d + 2*e*x^2))/(a^2*d^2*x*Sqrt[d + e*x^2]) - (d^2 - 4*d*e*x^2 - 8*e^2*x^
4)/(3*a*d^3*x^3*Sqrt[d + e*x^2]) + ((b*c + (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a
*c])*x*(45*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*
x^2*Sqrt[-(((-b + Sqrt[b^2 - 4*a*c])*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x
^2*(d + e*x^2))/(d^2*(b - Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[
Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c
] - 2*c*x^2)))]] - (30*e*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/d - (45*(2*c*d + (-b
+ Sqrt[b^2 - 4*a*c])*e)*x^2*ArcSin[Sqrt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))]])/(d*(-b + Sqrt[b^2 - 4*
a*c] - 2*c*x^2)) - (30*e*(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sq
rt[-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2)))]])/(d^2*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)) + 4*(-(((2*c*d + (
-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(5
/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*c]
- 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4*a*
c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))] + (4*e*x^2*(-(((2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2))))
^(5/2)*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sqrt[b^2 - 4*a*
c] - 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, -(((2*c*d + (-b + Sqrt[b^2 - 4
*a*c])*e)*x^2)/(d*(-b + Sqrt[b^2 - 4*a*c] - 2*c*x^2)))])/d))/(15*a^2*(b - S
qrt[b^2 - 4*a*c])*d*(-(((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(-b +
Sqrt[b^2 - 4*a*c] - 2*c*x^2))))^(3/2)*(1 - (2*c*x^2)/(-b + Sqrt[b^2 - 4*a*c
]))*Sqrt[d + e*x^2]*Sqrt[((-b + Sqrt[b^2 - 4*a*c])*(d + e*x^2))/(d*(-b + Sq
rt[b^2 - 4*a*c] - 2*c*x^2))]) + ((b*c - (c*(b^2 - 2*a*c))/Sqrt[b^2 - 4*a*c]
)*x*(45*Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e
)*x^2*(d + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))] + (30*e*x^2*
Sqrt[-(((b + Sqrt[b^2 - 4*a*c])*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^2*(d
 + e*x^2))/(d^2*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)^2))])/d - 45*ArcSin[Sqrt[
((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x
^2))]] - (30*e*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d
*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/d + (45*(2*c*d - (b + Sqrt[b^2 - 4*a
*c])*e)*x^2*ArcSin[Sqrt[((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)) - (30
*e*(-2*c*d + (b + Sqrt[b^2 - 4*a*c])*e)*x^4*ArcSin[Sqrt[((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]])/(d^2*(b + S
qrt[b^2 - 4*a*c] + 2*c*x^2)) + 4*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)
/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])
*(d + e*x^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2
, 7/2, ((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c]
+ 2*c*x^2))] + (4*e*x^2*(((2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b +
Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(5/2)*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))]*Hypergeometric2F1[2, 2, 7/2, ((
2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2
```

```
))])/d))/(15*a^2*(b + Sqrt[b^2 - 4*a*c])*d*(((2*c*d - (b + Sqrt[b^2 - 4*a*c
])*e)*x^2)/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2)))^(3/2)*(1 + (2*c*x^2)/(b +
 Sqrt[b^2 - 4*a*c]))*Sqrt[d + e*x^2]*Sqrt[((b + Sqrt[b^2 - 4*a*c])*(d + e*x
^2))/(d*(b + Sqrt[b^2 - 4*a*c] + 2*c*x^2))])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.19, size = 431, normalized size = 1.03

method	result
default	$\frac{16\sqrt{e}\left(-\frac{\sum_{\_R=\mathrm{RootOf}\left(c\_Z^4+(4eb-4cd)\_Z^3+\left(16a\,e^2-8deb+6c\,d^2\right)\_Z^2+\left(4d^2eb-4c\,d^3\right)\_Z+d^4c\right)} \frac{\left(c\left(ace-b^2e+bcd\right)\_R^2+2\left(4abc\,e^2-3a\right.\right.}{c\_R}}{8\left(4a\,e^2-4deb+4c\,d^2\right)}\right.}{}$
risch	$-\frac{\sqrt{e\,x^2+d}\,\left(-5ae\,x^2-3bd\,x^2+ad\right)}{3d^3a^2x^3}+\frac{e^3\sqrt{e\left(x+\frac{\sqrt{-de}}{e}\right)^2-2\sqrt{-de}\,\left(x+\frac{\sqrt{-de}}{e}\right)}}{2d^3\left(a\,e^2-deb+c\,d^2\right)\left(x+\frac{\sqrt{-de}}{e}\right)}+\frac{e^3\sqrt{e\left(x-\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x,method=_RETURNVERBOSE)
```

```
[Out] 16/a^2*e^(1/2)*(-1/8/(4*a*e^2-4*b*d*e+4*c*d^2)*sum((c*(a*c*e-b^2*e+b*c*d)*_
R^2+2*(4*a*b*c*e^2-3*a*c^2*d*e-2*b^3*e^2+3*b^2*c*d*e-b*c^2*d^2)*_R+a*c^2*d^
2*e-b^2*c*d^2*e+b*c^2*d^3)/(_R^3*c+3*_R^2*b*e-3*_R^2*c*d+8*_R*a*e^2-4*_R*b*
d*e+3*_R*c*d^2+b*d^2*e-c*d^3)*ln(((e*x^2+d)^(1/2)-e^(1/2)*x)^2-_R),_R=RootO
f(c*_Z^4+(4*b*e-4*c*d)*_Z^3+(16*a*e^2-8*b*d*e+6*c*d^2)*_Z^2+(4*b*d^2*e-4*c*
d^3)*_Z+d^4*c))-1/2*(a*c*e-b^2*e+b*c*d)/(4*a*e^2-4*b*d*e+4*c*d^2)/(((e*x^2+
d)^(1/2)-e^(1/2)*x)^2+d))-b/a^2*(-1/d/x/(e*x^2+d)^(1/2)-2*e/d^2*x/(e*x^2+d)
^(1/2))+1/a*(-1/3/d/x^3/(e*x^2+d)^(1/2)-4/3*e/d*(-1/d/x/(e*x^2+d)^(1/2)-2*e
/d^2*x/(e*x^2+d)^(1/2)))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate(1/((c*x^4 + b*x^2 + a)*(x^2*e + d)^(3/2)*x^4), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{x^4 (d + ex^2)^{\frac{3}{2}} (a + bx^2 + cx^4)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x**4/(e*x**2+d)**(3/2)/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral(1/(x**4*(d + e*x**2)**(3/2)*(a + b*x**2 + c*x**4)), x)
```

Giac [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/x^4/(e*x^2+d)^(3/2)/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] Timed out
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{x^4 \left(e\,x^2 + d\right)^{3/2} \left(c\,x^4 + b\,x^2 + a\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)),x)
```

```
[Out] int(1/(x^4*(d + e*x^2)^(3/2)*(a + b*x^2 + c*x^4)), x)
```

3.400 $\int \frac{(fx)^m(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=243

$$\frac{2c(fx)^{1+m}\left(d+ex^2\right)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1+m}{2};1,-q;\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)}-\frac{2c(fx)^{1+m}\left(d+ex^2\right)^q\left(1+\right.}{\sqrt{b^2-4}}$$

```
[Out] 2*c*(f*x)^(1+m)*(e*x^2+d)^q*AppellF1(1/2+1/2*m,1,-q,3/2+1/2*m,-2*c*x^2/(b-(
-4*a*c+b^2)^(1/2)),-e*x^2/d)/f/(1+m)/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))
/(-4*a*c+b^2)^(1/2)-2*c*(f*x)^(1+m)*(e*x^2+d)^q*AppellF1(1/2+1/2*m,1,-q,3/2
+1/2*m,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/f/(1+m)/((1+e*x^2/d)^q)/(-
4*a*c+b^2)^(1/2)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.44, antiderivative size = 243, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {1319, 525, 524}

$$\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)}-\frac{2c(fx)^{m+1}\left(d+ex^2\right)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{m+1}{2};1,-q;\frac{m+3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{f(m+1)\sqrt{b^2-4ac}\left(\sqrt{b^2-4ac}+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (2*c*(f*x)^(1 + m)*(d + e*x^2)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*
c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b - Sqrt
[b^2 - 4*a*c])*f*(1 + m)*(1 + (e*x^2)/d)^q) - (2*c*(f*x)^(1 + m)*(d + e*x^2
)^q*AppellF1[(1 + m)/2, 1, -q, (3 + m)/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(b + Sqrt[b^2 - 4*a*c])*f*(1 + m)*(1 +
 (e*x^2)/d)^q)
```

Rule 524

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[a^p*c^q*((e*x)^(m + 1)/(e*(m + 1)))*AppellF1[(m
+ 1)/n, -p, -q, 1 + (m + 1)/n, (-b)*(x^n/a), (-d)*(x^n/c)], x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 525

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^
n/a))^FracPart[p]), Int[(e*x)^m*(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1319

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q, 1/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[b^2
- 4*a*c, 0] &&  !IntegerQ[q] &&  !IntegerQ[m]
```

Rubi steps

$$\begin{aligned}\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx &= \int \left(\frac{2c(fx)^m (d+ex^2)^q}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx^2\right)}-\frac{2c(fx)^m (d+ex^2)^q}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}+2cx^2\right)}\right.\\
&= \frac{(2c)\int \frac{(fx)^m (d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}-\frac{(2c)\int \frac{(fx)^m (d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}\\
&= \frac{\left(2c(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{(fx)^m\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}-\frac{\left(2c(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)\right.}{\sqrt{b^2}}\\
&= \frac{2c(fx)^{1+m}(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1+m}{2};1,-q;\frac{3+m}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}\right)f(1+m)}-\end{aligned}$$

Mathematica [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(fx)^m (e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)**m*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((f*x)^m*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^q*(f*x)^m/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(f\,x)^m\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int(((f*x)^m*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.401 $\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=313

$$-\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)}+\frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)}+\frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(}{2cd-\left(b-\sqrt{}\right.}\right.}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

```
[Out] -1/2*(b*e+c*d)*(e*x^2+d)^(1+q)/c^2/e^2/(1+q)+1/2*(e*x^2+d)^(2+q)/c/e^2/(2+q
)+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-
4*a*c+b^2)^(1/2))))*(a-b^2/c+b*(-3*a*c+b^2)/c/(-4*a*c+b^2)^(1/2))/c/(1+q)/(
2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q
],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(a-b^2/c-b*(-3*a*c+b^2)/c
/(-4*a*c+b^2)^(1/2))/c/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]
time = 0.65, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1265, 1642, 70}

$$\frac{\left(\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}+a-\frac{b^2}{c}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(be+cd)(d+ex^2)^{q+1}}{2c^2e^2(q+1)}+\frac{(d+ex^2)^{q+2}}{2ce^2(q+2)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*((c*d + b*e)*(d + e*x^2)^(1 + q))/(c^2*e^2*(1 + q)) + (d + e*x^2)^(2 +
 q)/(2*c*e^2*(2 + q)) + ((a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c
]))*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2)
)/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c
])*e)*(1 + q)) + ((a - b^2/c - (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*(d
+ e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*
d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(
1 + q))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\begin{aligned}\int \frac{x^7(d+ex^2)^q}{a+bx^2+cx^4}\,dx &= \frac{1}{2}\text{Subst}\left(\int \frac{x^3(d+ex)^q}{a+bx+cx^2}\,dx,x,x^2\right)\\ &= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(-cd-be)(d+ex)^q}{c^2e}+\frac{\left(\frac{b^2}{c^2}-\frac{a}{c}-\frac{b(b^2-3ac)}{c^2\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}+\frac{\left(\frac{b^2}{c^2}-\right.}{}\right.\right.\\ &= -\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)}+\frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)}-\frac{\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{}{b+\sqrt{b^2}}\right.}{2c}\\ &= -\frac{(cd+be)\left(d+ex^2\right)^{1+q}}{2c^2e^2(1+q)}+\frac{\left(d+ex^2\right)^{2+q}}{2ce^2(2+q)}+\frac{\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right)\left(d+ex^2\right)^{1+q}\,{}_2F}{2c\left(2cd-\left(b-\sqrt{b^2}\right.\right.}\end{aligned}$$

Mathematica [A]
time = 1.01, size = 272, normalized size = 0.87

$$\frac{\left(d+ex^2\right)^{1+q}\left(-\frac{cd+be}{e^2(1+q)}+\frac{c(d+ex^2)}{e^2(2+q)}+\frac{c\left(a-\frac{b^2}{c}+\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{\left(2cd+\left(-b+\sqrt{b^2-4ac}\right)e\right)(1+q)}+\frac{c\left(a-\frac{b^2}{c}-\frac{b(b^2-3ac)}{c\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}\right)}{2c^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(-((c*d + b*e)/(e^2*(1 + q))) + (c*(d + e*x^2))/(e^2*(
2 + q)) + (c*(a - b^2/c + (b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeo
metric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*
```

```
c])*e)])/((2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(a - b^2/c - (
b*(b^2 - 3*a*c))/(c*Sqrt[b^2 - 4*a*c]))*Hypergeometric2F1[1, 1 + q, 2 + q,
(2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/((2*c*d - (b + Sqrt
[b^2 - 4*a*c])*e)*(1 + q))))/(2*c^2)
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{x^7 (e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((x^2*e + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**7*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^7*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((x^2*e + d)^q*x^7/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^7\,{\left(e\,x^2+d\right)}^q}{c\,x^4+b\,x^2+a} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^7*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.402 $\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=256

$$\frac{(d+ex^2)^{1+q}}{2ce(1+q)}+\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4a\ldots}}\right.}{\ldots}$$

```
[Out] 1/2*(e*x^2+d)^(1+q)/c/e/(1+q)+1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],
2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)
^(1/2))/c/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))+1/2*(e*x^2+d)^(1+q)*hyperg
eom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(b+(-2*a
*c+b^2)/(-4*a*c+b^2)^(1/2))/c/(1+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]
time = 0.38, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1265, 1642, 70}

$$\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{(d+ex^2)^{q+1}}{2ce(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (d + e*x^2)^(1 + q)/(2*c*e*(1 + q)) + ((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c]
)*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/
(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*c*(2*c*d - (b - Sqrt[b^2 - 4*a*c])
*e)*(1 + q)) + ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*H
ypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2
- 4*a*c])*e)])/(2*c*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
```

```
gerQ[(m - 1)/2]
```

Rule 1642

```
Int[(Pq_)*((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*Pq*(a + b*x + c*x^2)^p, x
], x] /; FreeQ[{a, b, c, d, e, m}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rubi steps

$$\int \frac{x^5(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x^2(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(d+ex)^q}{c} + \frac{\left(-\frac{b}{c}+\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(-\frac{b}{c}-\frac{b^2-2ac}{c\sqrt{b^2-4ac}}\right)}{b+\sqrt{b^2-4ac}+\cdots}\right.\right.$$

$$= \frac{(d+ex^2)^{1+q}}{2ce(1+q)} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{2c} - \frac{\left(b+\cdots\right.}{\cdots}$$

$$= \frac{(d+ex^2)^{1+q}}{2ce(1+q)} + \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4\cdots}\right.}\right.}{2c\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

Mathematica [A]
time = 0.43, size = 211, normalized size = 0.82

$$\frac{(d+ex^2)^{1+q}\left(\frac{1}{e}+\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2c(1+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(e^(-1) + ((b + (-b^2 + 2*a*c)/Sqrt[b^2 - 4*a*c])*Hype
rgeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 -
4*a*c])*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e) + ((b + (b^2 - 2*a*c)/Sqr
t[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
```

```
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(2*c*
(1 + q))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^5 (e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((x^2*e + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**5*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^5*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^q*x^5/(c*x^4 + b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^5\,{\left(e\,x^2+d\right)}^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^5*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.403 $\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=210

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}-\frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)}{2\left(2cd-\right.}$$

```
[Out] -1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4
*a*c+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^
(1/2)))-1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e
*(b+(-4*a*c+b^2)^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/(1+q)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2)))
```

Rubi [A]
time = 0.23, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = $\{1265, 844, 70\}$

$$-\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/2*((1 - b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1
+ q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/((2*c*d
 - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - ((1 + b/Sqrt[b^2 - 4*a*c])*(d + e*
x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d -
(b + Sqrt[b^2 - 4*a*c])*e)])/(2*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)
)
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
```

```
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{x^3(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{x(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$
$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\right) dx, x$$
$$= \frac{1}{2}\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right) + \frac{1}{2}\left(1+\frac{b}{\sqrt{b^2-4a}}\right.$$
$$= -\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} - \frac{(1}{}$$

Mathematica [A]
time = 0.26, size = 183, normalized size = 0.87

$$-\frac{(d+ex^2)^{1+q}\left(\left(-bd+\sqrt{b^2-4ac}\,d+2ae\right)\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)+\left(bd+\sqrt{b^2-4ac}\,d-2ae\right)\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)\right)}{4\sqrt{b^2-4ac}\,(cd^2+e(-bd+ae))(1+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -1/4*((d + e*x^2)^(1 + q)*((-(b*d) + Sqrt[b^2 - 4*a*c]*d + 2*a*e)*Hypergeom
etric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c
])*e)] + (b*d + Sqrt[b^2 - 4*a*c]*d - 2*a*e)*Hypergeometric2F1[1, 1 + q, 2
+ q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]))/(Sqrt[b^2 - 4
*a*c]*(c*d^2 + e*(-(b*d) + a*e))*(1 + q))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^3(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

```
[Out] int(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**3*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^3*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^q*x^3/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^3\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^3*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.404 $\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=198

$$-\frac{c(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}+\frac{c(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b+\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

```
[Out] -c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a
*c+b^2)^(1/2))))/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)))/(-4*a*c+b^2)^(1/2)+
c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b+(-4*a*
c+b^2)^(1/2))))/(1+q)/(-4*a*c+b^2)^(1/2)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]

time = 0.26, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {1261, 725, 70}

$$\frac{c(d+ex^2)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{c(d+ex^2)^{q+1}\ {}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{(q+1)\sqrt{b^2-4ac}\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2
))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(Sqrt[b^2 - 4*a*c]*(2*c*d - (b - S
qrt[b^2 - 4*a*c])*e)*(1 + q))) + (c*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1
, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(Sq
rt[b^2 - 4*a*c]*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q))
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 725

```
Int[((d_.) + (e_.)*(x_))^(m_)/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol
] :> Int[ExpandIntegrand[(d + e*x)^m, 1/(a + b*x + c*x^2), x], x] /; FreeQ[
{a, b, c, d, e, m}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && NeQ[2*c*d - b*e, 0] &&  !IntegerQ[m]
```

Rule 1261

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(
p_.), x_Symbol] :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + b*x + c*x^2)^p, x],
 x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x]
```

Rubi steps

$$\int \frac{x(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{2c(d+ex)^q}{\sqrt{b^2-4ac}\left(b-\sqrt{b^2-4ac}+2cx\right)} - \frac{2c(d+ex)^q}{\sqrt{b^2-4ac}\left(b+\sqrt{b^2-4ac}\right.}\right.\right.$$

$$= \frac{c\text{Subst}\left(\int \frac{(d+ex)^q}{b-\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{\sqrt{b^2-4ac}} - \frac{c\text{Subst}\left(\int \frac{(d+ex)^q}{b+\sqrt{b^2-4ac}+2cx}\,dx, x, x^2\right)}{\sqrt{b^2-4ac}}$$

$$= -\frac{c(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c(d+ex^2)^{1+q}\,{}_2F_1\left(1,\right.}{\sqrt{b^2-4ac}\left(2cd\right.}$$

Mathematica [A]
time = 0.26, size = 168, normalized size = 0.85

$$\frac{c(d+ex^2)^{1+q}\left(-\frac{{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}+\frac{{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{\sqrt{b^2-4ac}\,(1+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] (c*(d + e*x^2)^(1 + q)*(-(Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^
2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)]/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])
*e)) + Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + S
qrt[b^2 - 4*a*c])*e)]/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)))/(Sqrt[b^2 - 4*a
*c]*(1 + q))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x(e\,x^{2}+d)^{q}}{c\,x^{4}+b\,x^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((x^2*e + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^q*x/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x\,(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.405 $\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=262

$$\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2}{2a\left(2cd-\right.}$$

```
[Out] -1/2*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^2/d)/a/d/(1+q)+1/2*c*(e
*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^
2)^(1/2))))*(1+b/(-4*a*c+b^2)^(1/2))/a/(1+q)/(2*c*d-e*(b-(-4*a*c+b^2)^(1/2)
))+1/2*c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b
+(-4*a*c+b^2)^(1/2))))*(1-b/(-4*a*c+b^2)^(1/2))/a/(1+q)/(2*c*d-e*(b+(-4*a*c
+b^2)^(1/2)))
```

Rubi [A]
time = 0.37, antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1265, 974, 67, 844, 70}

$$\frac{c\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}+\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}-\frac{(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{ex^2}{d}+1\right)}{2ad(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c*(1 + b/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q
, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*
d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*(d +
 e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d
 - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a*(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)*(1
 + q)) - ((d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2
)/d])/(2*a*d*(1 + q))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
```

```
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(d+ex^2)^q}{x(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^q}{x(a+bx+cx^2)}\,dx, x, x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax} + \frac{(-b-cx)(d+ex)^q}{a(a+bx+cx^2)}\right)\,dx, x, x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x}\,dx, x, x^2\right)}{2a} + \frac{\text{Subst}\left(\int \frac{(-b-cx)(d+ex)^q}{a+bx+cx^2}\,dx, x, x^2\right)}{2a}$$

$$= -\frac{(d+ex^2)^{1+q}\ {}_2F_1\left(1, 1+q; 2+q; 1+\frac{ex^2}{d}\right)}{2ad(1+q)} + \frac{\text{Subst}\left(\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex)}{b-\sqrt{b^2-4ac}+2cx} \cdots\right.\right.}{\cdots}$$

$$= -\frac{(d+ex^2)^{1+q}\ {}_2F_1\left(1, 1+q; 2+q; 1+\frac{ex^2}{d}\right)}{2ad(1+q)} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\text{Subst}\left(\int \cdots\right.}{2a}$$

$$= \frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\ {}_2F_1\left(1, 1+q; 2+q; \frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)} + \frac{c\left(\cdots\right.}{\cdots}$$

Mathematica [A]
time = 0.60, size = 218, normalized size = 0.83

$$\frac{(d+ex^2)^{1+q}\left(\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e} + \frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e} - \frac{{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{d}\right)}{2a(1+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*((c*(1 + b/Sqrt[b^2 - 4*a*c])*Hypergeometric2F1[1, 1 +
 q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d
+ (-b + Sqrt[b^2 - 4*a*c])*e) + (c*(1 - b/Sqrt[b^2 - 4*a*c])*Hypergeometric
2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)]
)/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) - Hypergeometric2F1[1, 1 + q, 2 + q,
1 + (e*x^2)/d]/d))/(2*a*(1 + q))
```

Maple [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^q}{x\,(c\,x^4+b\,x^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)^q/(c*x^5 + b*x^3 + a*x), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^q}{x\,(a+bx^2+cx^4)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x/(c*x**4+b*x**2+a),x)
```

```
[Out] Integral((d + e*x**2)**q/(x*(a + b*x**2 + c*x**4)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^q/((c*x^4 + b*x^2 + a)*x), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^q}{x\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x*(a + b*x^2 + c*x^4)), x)
```

3.406 $\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=322

$$-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\,{}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+e}{2a^2\left(2c\right.}$$

```
[Out] 1/2*b*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],1+e*x^2/d)/a^2/d/(1+q)+1/2*e
*(e*x^2+d)^(1+q)*hypergeom([2, 1+q],[2+q],1+e*x^2/d)/a/d^2/(1+q)-1/2*c*(e*x
^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/(2*c*d-e*(b-(-4*a*c+b^2)
^(1/2))))*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1+q)/(2*c*d-e*(b-(-4*a*c
+b^2)^(1/2)))-1/2*c*(e*x^2+d)^(1+q)*hypergeom([1, 1+q],[2+q],2*c*(e*x^2+d)/
(2*c*d-e*(b+(-4*a*c+b^2)^(1/2))))*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/(1
+q)/(2*c*d-e*(b+(-4*a*c+b^2)^(1/2)))
```

Rubi [A]
time = 0.48, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {1265, 974, 67, 844, 70}

$$-\frac{c\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(b-\sqrt{b^2-4ac}\right)\right)}-\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{2c(ex^2+d)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2a^2(q+1)\left(2cd-e\left(\sqrt{b^2-4ac}+b\right)\right)}+\frac{b(d+ex^2)^{q+1}\,{}_2F_1\left(1,q+1;q+2;\frac{ex^2}{d}+1\right)}{2a^2d(q+1)}+\frac{e(d+ex^2)^{q+1}\,{}_2F_1\left(2,q+1;q+2;\frac{ex^2}{d}+1\right)}{2ad^2(q+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -1/2*(c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeom
etric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d - (b - Sqrt[b^2 - 4*a*c]
)*e)])/(a^2*(2*c*d - (b - Sqrt[b^2 - 4*a*c])*e)*(1 + q)) - (c*(b - (b^2 - 2
*a*c)/Sqrt[b^2 - 4*a*c])*(d + e*x^2)^(1 + q)*Hypergeometric2F1[1, 1 + q, 2
+ q, (2*c*(d + e*x^2))/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e)])/(2*a^2*(2*c*d
- (b + Sqrt[b^2 - 4*a*c])*e)*(1 + q)) + (b*(d + e*x^2)^(1 + q)*Hypergeometr
ic2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a^2*d*(1 + q)) + (e*(d + e*x^2)^(
1 + q)*Hypergeometric2F1[2, 1 + q, 2 + q, 1 + (e*x^2)/d])/(2*a*d^2*(1 + q))
```

Rule 67

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)/(d*(n + 1)*(-d/(b*c))^m))*Hypergeometric2F1[-m, n + 1, n + 2, 1 +
 d*(x/c)], x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[m]
 || GtQ[-d/(b*c), 0])
```

Rule 70

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[(b
*c - a*d)^n*((a + b*x)^(m + 1)/(b^(n + 1)*(m + 1)))*Hypergeometric2F1[-n, m
 + 1, m + 2, (-d)*((a + b*x)/(b*c - a*d))], x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 844

```
Int[(((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_)))/((a_.) + (b_.)*(x_) +
(c_.)*(x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m, (f + g*x)/(a +
 b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ[b^2 - 4*a*c
, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&  !RationalQ[m]
```

Rule 974

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_.) + (b_.)*(x_)
 + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f + g
*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g}, x] && NeQ
[e*f - d*g, 0] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && (
IntegerQ[p] || (ILtQ[m, 0] && ILtQ[n, 0])) &&  !(IGtQ[m, 0] || IGtQ[n, 0])
```

Rule 1265

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_
)^4)^(p_.), x_Symbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a +
 b*x + c*x^2)^p, x], x, x^2], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && Inte
gerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{(d+ex^2)^q}{x^3(a+bx^2+cx^4)}\,dx = \frac{1}{2}\text{Subst}\left(\int \frac{(d+ex)^q}{x^2(a+bx+cx^2)}\,dx,x,x^2\right)$$

$$= \frac{1}{2}\text{Subst}\left(\int \left(\frac{(d+ex)^q}{ax^2} - \frac{b(d+ex)^q}{a^2x} + \frac{(b^2-ac+bcx)(d+ex)^q}{a^2(a+bx+cx^2)}\right)dx,x,x^2\right)$$

$$= \frac{\text{Subst}\left(\int \frac{(b^2-ac+bcx)(d+ex)^q}{a+bx+cx^2}\,dx,x,x^2\right)}{2a^2} + \frac{\text{Subst}\left(\int \frac{(d+ex)^q}{x^2}\,dx,x,x^2\right)}{2a} - \frac{b\text{Subst}\left(\int \right.}{}$$

$$= \frac{b(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2a^2d(1+q)} + \frac{e(d+ex^2)^{1+q}\ {}_2F_1\left(2,1+q;2+q;\right.}{2ad^2(1+q)}$$

$$= \frac{b(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{2a^2d(1+q)} + \frac{e(d+ex^2)^{1+q}\ {}_2F_1\left(2,1+q;2+q;\right.}{2ad^2(1+q)}$$

$$= -\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^{1+q}\ {}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b-\sqrt{b^2-4ac}\right)e}\right)}{2a^2\left(2cd-\left(b-\sqrt{b^2-4ac}\right)e\right)(1+q)}$$

Mathematica [A]
time = 0.36, size = 259, normalized size = 0.80

$$\frac{(d+ex^2)^{1+q}\left(-\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e}\right)}{2cd+\left(-b+\sqrt{b^2-4ac}\right)e} - \frac{c\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right){}_2F_1\left(1,1+q;2+q;\frac{2c(d+ex^2)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e}\right)}{2cd-\left(b+\sqrt{b^2-4ac}\right)e} + \frac{b\,{}_2F_1\left(1,1+q;2+q;1+\frac{ex^2}{d}\right)}{d} + \frac{ae\,{}_2F_1\left(2,1+q;2+q;1+\frac{ex^2}{d}\right)}{d^2}\right)}{2a^2(1+q)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x]
```

```
[Out] ((d + e*x^2)^(1 + q)*(-((c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*Hypergeome
tric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d + (-b + Sqrt[b^2 - 4*a*c]
)*e)])/(2*c*d + (-b + Sqrt[b^2 - 4*a*c])*e)) - (c*(b + (-b^2 + 2*a*c)/Sqrt[
b^2 - 4*a*c])*Hypergeometric2F1[1, 1 + q, 2 + q, (2*c*(d + e*x^2))/(2*c*d -
 (b + Sqrt[b^2 - 4*a*c])*e)])/(2*c*d - (b + Sqrt[b^2 - 4*a*c])*e) + (b*Hype
rgeometric2F1[1, 1 + q, 2 + q, 1 + (e*x^2)/d])/d + (a*e*Hypergeometric2F1[2
, 1 + q, 2 + q, 1 + (e*x^2)/d])/d^2))/(2*a^2*(1 + q))
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^q}{x^3\,(c\,x^4+b\,x^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

```
[Out] int((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)^q/(c*x^7 + b*x^5 + a*x^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x**3/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^3/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^q/((c*x^4 + b*x^2 + a)*x^3), x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^q}{x^3\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x^3*(a + b*x^2 + c*x^4)), x)
```

3.407 $\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=339

$$\frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{\left(b^2-ac+\frac{b(b^2}{\sqrt{b^2}}\right.}{}$$

```
[Out] -b*x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/c^2/((1+e*x^2/d)^q)+1/
3*x^3*(e*x^2+d)^q*hypergeom([3/2, -q],[5/2],-e*x^2/d)/c/((1+e*x^2/d)^q)+x*(
e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(b^2-a*c-b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/((1+e*x^2/d)^q)/(b-(-4*a*c+
b^2)^(1/2))+x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1
/2)),-e*x^2/d)*(b^2-a*c+b*(-3*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c^2/((1+e*x^2/d)
^q)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.41, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 8, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.296$, Rules used = {1317, 252, 251, 372, 371, 1706, 441, 440}

$$\frac{x\left(-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{x\left(\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}-ac+b^2\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c^2\left(\sqrt{b^2-4ac}+b\right)}-\frac{bx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2}+\frac{x^3(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(\frac{3}{2},-q;\frac{5}{2};-\frac{ex^2}{d}\right)}{3c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] ((b^2 - a*c - (b*(b^2 - 3*a*c))/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1
[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(
b - Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + ((b^2 - a*c + (b*(b^2 - 3*a*c))
/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b
 + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q) - (b*x*(d + e*x^2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d
)])/(c^2*(1 + (e*x^2)/d)^q) + (x^3*(d + e*x^2)^q*Hypergeometric2F1[3/2, -q,
 5/2, -((e*x^2)/d)])/(3*c*(1 + (e*x^2)/d)^q)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
```

```
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] && !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \left(-\frac{b(d+ex^2)^q}{c^2}+\frac{x^2(d+ex^2)^q}{c}+\frac{(ab+(b^2-ac)\,x^2)\,(d+ex^2)^q}{c^2\,(a+bx^2+cx^4)}\right)dx$$
$$= \frac{\int \frac{(ab+(b^2-ac)x^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx}{c^2} - \frac{b\int (d+ex^2)^q\,dx}{c^2} + \frac{\int x^2(d+ex^2)^q\,dx}{c}$$
$$= \frac{\int \left(\frac{\left(b^2-ac+\frac{b(-b^2+3ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}+\frac{\left(b^2-ac-\frac{b(-b^2+3ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{c^2} - \frac{\left(b(d+ex^2)^q\right.}{}$$
$$= -\frac{bx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2} + \frac{x^3(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(\frac{3}{2},\right.}{3c}$$
$$= -\frac{bx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c^2} + \frac{x^3(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}{}_2F_1\left(\frac{3}{2},\right.}{3c}$$
$$= \frac{\left(b^2-ac-\frac{b(b^2-3ac)}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{c^2\left(b-\sqrt{b^2-4ac}\right)}$$

Mathematica [F]
time = 1.71, size = 0, normalized size = 0.00

$$\int \frac{x^6(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^6(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((x^2*e + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**6*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^6*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((x^2*e + d)^q*x^6/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^6\,{\left(e\,x^2+d\right)}^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^6*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.408 $\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=273

$$-\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(\cdots}{}$$

```
[Out] x*(e*x^2+d)^q*hypergeom([1/2, -q],[3/2],-e*x^2/d)/c/((1+e*x^2/d)^q)-x*(e*x^
2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*(b+(
2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/c/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))-x*(
e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/c/((1+e*x^2/d)^q)/(b+(-4*a*c+b^2)^(1/2)
)
```

Rubi [A]

time = 0.27, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1317, 252, 251, 1706, 441, 440}

$$-\frac{x\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}-\frac{x\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(\sqrt{b^2-4ac}+b\right)}+\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c}$$

Antiderivative was successfully verified.

```
[In] Int[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -(((b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(c*(b - Sqrt[b^2
 - 4*a*c])*(1 + (e*x^2)/d)^q)) - ((b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(
d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]),
-((e*x^2)/d)])/(c*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (x*(d + e*x^
2)^q*Hypergeometric2F1[1/2, -q, 3/2, -((e*x^2)/d)])/(c*(1 + (e*x^2)/d)^q)
```

Rule 251

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, (-b)*(x^n/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 252

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^IntPart[p]*((a + b*x
^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(1 + b*(x^n/a))^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
```

```
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \left(\frac{(d+ex^2)^q}{c} - \frac{(a+bx^2)(d+ex^2)^q}{c(a+bx^2+cx^4)}\right) dx$$
$$= \frac{\int (d+ex^2)^q\,dx}{c} - \frac{\int \frac{(a+bx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx}{c}$$
$$= -\frac{\int \left(\frac{\left(b+\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(b-\frac{-b^2+2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right) dx}{c} + \frac{\left((d+ex^2)^q \left(1+\frac{ex^2}{d}\right.\right.}{}$$
$$= \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c} - \frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2}}{c}$$
$$= \frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(\frac{1}{2},-q;\frac{3}{2};-\frac{ex^2}{d}\right)}{c} - \frac{\left(\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(1+\right.\right.}{c}$$
$$= -\frac{\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{c\left(b-\sqrt{b^2-4ac}\right)}$$

Mathematica [F]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{x^4(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^4(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((x^2*e + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**4*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^4*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((x^2*e + d)^q*x^4/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{x^4\,{\left(e\,x^2+d\right)}^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
[Out] int((x^4*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.409 $\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=162

$$-\frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}+\frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\right.}{\sqrt{b^2-4ac}}$$

```
[Out] -x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2
/d)/((1+e*x^2/d)^q)/(-4*a*c+b^2)^(1/2)+x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,
-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/((1+e*x^2/d)^q)/(-4*a*c+b^2)^(1/2
)
```

Rubi [A]
time = 0.13, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {1317, 441, 440}

$$\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}-\frac{x(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}}$$

Antiderivative was successfully verified.

```
[In] Int[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
```

```
[Out] -((x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a
*c]), -((e*x^2)/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)) + (x*(d + e*x^2
)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)
/d)])/(Sqrt[b^2 - 4*a*c]*(1 + (e*x^2)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
```

```
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rubi steps

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \int \left(\frac{\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right) dx$$

$$= \left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx + \left(1+\frac{b}{\sqrt{b^2-4ac}}\right)\int \frac{(d-}{b+\sqrt{b^2-}}$$

$$= \left(\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx + \left(\left(\right.\right.$$

$$= -\frac{x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{\sqrt{b^2-4ac}} + \frac{x(d+ex^2)^q\left(1\right.}{}$$

Mathematica [F]
time = 0.19, size = 0, normalized size = 0.00

$$\int \frac{x^2(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{x^2(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x**2*(e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(x^2*(e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^q*x^2/(c*x^4 + b*x^2 + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{x^2\,{\left(e\,x^2+d\right)}^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((x^2*(d + e*x^2)^q)/(a + b*x^2 + c*x^4), x)
```

3.410 $\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx$

Optimal. Leaf size=190

$$-\frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}}-\frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-\right.}{b^2-4ac+b\sqrt{b^2}}$$

```
[Out] -2*c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e
*x^2/d)/((1+e*x^2/d)^q)/(b^2-4*a*c-b*(-4*a*c+b^2)^(1/2))-2*c*x*(e*x^2+d)^q*
AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)/((1+e*x^2/d
)^q)/(b^2-4*a*c+b*(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.12, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {1188, 441, 440}

$$-\frac{2cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{-b\sqrt{b^2-4ac}-4ac+b^2}-\frac{2cx(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b\sqrt{b^2-4ac}-4ac+b^2}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
```

```
[Out] (-2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 -
4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c - b*Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d
)^q) - (2*c*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[
b^2 - 4*a*c]), -((e*x^2)/d)])/((b^2 - 4*a*c + b*Sqrt[b^2 - 4*a*c])*(1 + (e*
x^2)/d)^q)
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1188

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> With[{r = Rt[b^2 - 4*a*c, 2]}, Dist[2*(c/r), Int[(d + e*x^2)^q/(b -
r + 2*c*x^2), x], x] - Dist[2*(c/r), Int[(d + e*x^2)^q/(b + r + 2*c*x^2), x
], x]] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2
- b*d*e + a*e^2, 0] &&  !IntegerQ[q]
```

Rubi steps

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx = \frac{(2c)\int \frac{(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{(2c)\int \frac{(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}}$$

$$= \frac{\left(2c(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)\int \frac{\left(1+\frac{ex^2}{d}\right)^q}{b-\sqrt{b^2-4ac}+2cx^2}\,dx}{\sqrt{b^2-4ac}} - \frac{\left(2c(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}\right)}{\sqrt{b^2-4a}}$$

$$= -\frac{2cx(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{b^2-4ac-b\sqrt{b^2-4ac}} - \frac{2cx(d+ex^2)^q}{}$$

Mathematica [F]
time = 0.03, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^q}{a+bx^2+cx^4}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4),x]
[Out] Integrate[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x]
```

Maple [F]
time = 0.04, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="maxima")
```

```
[Out] integrate((x^2*e + d)^q/(c*x^4 + b*x^2 + a), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="fricas")
```

```
[Out] integral((x^2*e + d)^q/(c*x^4 + b*x^2 + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/(c*x**4+b*x**2+a),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/(c*x^4+b*x^2+a),x, algorithm="giac")
```

```
[Out] integrate((x^2*e + d)^q/(c*x^4 + b*x^2 + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{(e\,x^2+d)^q}{c\,x^4+b\,x^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(a + b*x^2 + c*x^4),x)
```

```
[Out] int((d + e*x^2)^q/(a + b*x^2 + c*x^4), x)
```

3.411 $\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=264

$$-\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)}{}$$

```
[Out] -(e*x^2+d)^q*hypergeom([-1/2, -q],[1/2],-e*x^2/d)/a/x/((1+e*x^2/d)^q)-c*x*(
e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^2/d)*
(1+b/(-4*a*c+b^2)^(1/2))/a/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2)^(1/2))-c*x*(e*x^
2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/2)),-e*x^2/d)*(1-b
/(-4*a*c+b^2)^(1/2))/a/((1+e*x^2/d)^q)/(b+(-4*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.27, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1317, 372, 371, 1706, 441, 440}

$$-\frac{cx\left(\frac{b}{\sqrt{b^2-4ac}}+1\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(b-\sqrt{b^2-4ac}\right)}-\frac{cx\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a\left(\sqrt{b^2-4ac}+b\right)}-\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
```

```
[Out] -((c*(1 + b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-
2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b - Sqrt[b^2 - 4*a*c])
*(1 + (e*x^2)/d)^q)) - (c*(1 - b/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF
1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a*(b
 + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e*x^2)^q*Hypergeometric2F1
[-1/2, -q, 1/2, -((e*x^2)/d)])/(a*x*(1 + (e*x^2)/d)^q)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx = \int \left(\frac{(d+ex^2)^q}{ax^2} + \frac{(-b-cx^2)(d+ex^2)^q}{a(a+bx^2+cx^4)}\right)dx$$

$$= \frac{\int \frac{(d+ex^2)^q}{x^2}\,dx}{a} + \frac{\int \frac{(-b-cx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx}{a}$$

$$= \frac{\int \left(\frac{\left(-c-\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(-c+\frac{bc}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a} + \frac{\left((d+ex^2)^q\left(\right.\right.}{}$$

$$= -\frac{(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)\right)\int \frac{}{b+\sqrt{}}}{a}$$

$$= -\frac{(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{ax} - \frac{\left(c\left(1-\frac{b}{\sqrt{b^2-4ac}}\right)(d+ex\right.}{}$$

$$= -\frac{c\left(1+\frac{b}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\right.}{a\left(b-\sqrt{b^2-4ac}\right)}$$

Mathematica [**F**]
time = 0.33, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^q}{x^2(a+bx^2+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x]
[Out] Integrate[(d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x]
```

Maple [**F**]
time = 0.05, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^q}{x^2(c\,x^4+b\,x^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((x^2*e + d)^q/(c*x^6 + b*x^4 + a*x^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x**2/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^2/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((x^2*e + d)^q/((c*x^4 + b*x^2 + a)*x^2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^q}{x^2\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x^2*(a + b*x^2 + c*x^4)), x)
```

3.412 $\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx$

Optimal. Leaf size=328

$$\frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{c\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(}{}$$

```
[Out] -1/3*(e*x^2+d)^q*hypergeom([-3/2, -q],[-1/2],-e*x^2/d)/a/x^3/((1+e*x^2/d)^q
)+b*(e*x^2+d)^q*hypergeom([-1/2, -q],[1/2],-e*x^2/d)/a^2/x/((1+e*x^2/d)^q)+
c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b-(-4*a*c+b^2)^(1/2)),-e*x^
2/d)*(b+(-2*a*c+b^2)/(-4*a*c+b^2)^(1/2))/a^2/((1+e*x^2/d)^q)/(b-(-4*a*c+b^2
)^(1/2))+c*x*(e*x^2+d)^q*AppellF1(1/2,1,-q,3/2,-2*c*x^2/(b+(-4*a*c+b^2)^(1/
2)),-e*x^2/d)*(b+(2*a*c-b^2)/(-4*a*c+b^2)^(1/2))/a^2/((1+e*x^2/d)^q)/(b+(-4
*a*c+b^2)^(1/2))
```

Rubi [A]
time = 0.31, antiderivative size = 328, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.222$, Rules used = {1317, 372, 371, 1706, 441, 440}

$$\frac{cx\left(\frac{b^2-2ac}{\sqrt{b^2-4ac}}+b\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(b-\sqrt{b^2-4ac}\right)}+\frac{cx\left(b-\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b+\sqrt{b^2-4ac}},-\frac{ex^2}{d}\right)}{a^2\left(\sqrt{b^2-4ac}+b\right)}+\frac{b(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(-\frac{1}{2},-q;\frac{1}{2};-\frac{ex^2}{d}\right)}{a^2x}-\frac{(d+ex^2)^q\left(\frac{ex^2}{d}+1\right)^{-q}{}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3}$$

Antiderivative was successfully verified.

```
[In] Int[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
```

```
[Out] (c*(b + (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*x*(d + e*x^2)^q*AppellF1[1/2, 1, -
q, 3/2, (-2*c*x^2)/(b - Sqrt[b^2 - 4*a*c]), -((e*x^2)/d)])/(a^2*(b - Sqrt[b
^2 - 4*a*c])*(1 + (e*x^2)/d)^q) + (c*(b - (b^2 - 2*a*c)/Sqrt[b^2 - 4*a*c])*
x*(d + e*x^2)^q*AppellF1[1/2, 1, -q, 3/2, (-2*c*x^2)/(b + Sqrt[b^2 - 4*a*c]
), -((e*x^2)/d)])/(a^2*(b + Sqrt[b^2 - 4*a*c])*(1 + (e*x^2)/d)^q) - ((d + e
*x^2)^q*Hypergeometric2F1[-3/2, -q, -1/2, -((e*x^2)/d)])/(3*a*x^3*(1 + (e*x
^2)/d)^q) + (b*(d + e*x^2)^q*Hypergeometric2F1[-1/2, -q, 1/2, -((e*x^2)/d)]
)/(a^2*x*(1 + (e*x^2)/d)^q)
```

Rule 371

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p
*((c*x)^(m + 1)/(c*(m + 1)))*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1
, (-b)*(x^n/a)], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 372

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[a^I
ntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]), Int[(c*x)^
m*(1 + b*(x^n/a))^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 440

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, (-b)*(x^n/a), (-d)*(x^n/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 441

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[a^IntPart[p]*((a + b*x^n)^FracPart[p]/(1 + b*(x^n/a))^FracPart[p]),
 Int[(1 + b*(x^n/a))^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1317

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_))/((a_) + (b_.)*(x_)^2 +
(c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q, (f*x)^m/(a +
 b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, q}, x] && NeQ[b^2 - 4*a
*c, 0] &&  !IntegerQ[q] && IntegerQ[m]
```

Rule 1706

```
Int[(Px_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^
(p_.), x_Symbol] :> Int[ExpandIntegrand[Px*(d + e*x^2)^q*(a + b*x^2 + c*x^4
)^p, x], x] /; FreeQ[{a, b, c, d, e, q}, x] && PolyQ[Px, x^2] && NeQ[b^2 -
4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx = \int \left(\frac{(d+ex^2)^q}{ax^4} - \frac{b(d+ex^2)^q}{a^2x^2} + \frac{(b^2-ac+bcx^2)(d+ex^2)^q}{a^2(a+bx^2+cx^4)}\right)dx$$
$$= \frac{\int \frac{(b^2-ac+bcx^2)(d+ex^2)^q}{a+bx^2+cx^4}\,dx}{a^2} + \frac{\int \frac{(d+ex^2)^q}{x^4}\,dx}{a} - \frac{b\int \frac{(d+ex^2)^q}{x^2}\,dx}{a^2}$$
$$= \frac{\int \left(\frac{\left(bc+\frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b-\sqrt{b^2-4ac}+2cx^2} + \frac{\left(bc-\frac{c(b^2-2ac)}{\sqrt{b^2-4ac}}\right)(d+ex^2)^q}{b+\sqrt{b^2-4ac}+2cx^2}\right)dx}{a^2} + \frac{\Big((d+ex^2)^q\Big(1}{}$$
$$= -\frac{(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3} + \frac{b(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1}{a^2x}$$
$$= -\frac{(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1\left(-\frac{3}{2},-q;-\frac{1}{2};-\frac{ex^2}{d}\right)}{3ax^3} + \frac{b(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} {}_2F_1}{a^2x}$$
$$= \frac{c\left(b+\frac{b^2-2ac}{\sqrt{b^2-4ac}}\right)x(d+ex^2)^q\left(1+\frac{ex^2}{d}\right)^{-q} F_1\left(\frac{1}{2};1,-q;\frac{3}{2};-\frac{2cx^2}{b-\sqrt{b^2-4ac}},-\frac{ex}{d}\right.}{a^2\left(b-\sqrt{b^2-4ac}\right)}$$

Mathematica [F]
time = 1.28, size = 0, normalized size = 0.00

$$\int \frac{(d+ex^2)^q}{x^4(a+bx^2+cx^4)}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x]
[Out] Integrate[(d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x]
```

Maple [F]
time = 0.02, size = 0, normalized size = 0.00

$$\int \frac{(e\,x^2+d)^q}{x^4(c\,x^4+b\,x^2+a)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
[Out] int((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="maxima")
[Out] integrate((x^2*e + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="fricas")
[Out] integral((x^2*e + d)^q/(c*x^8 + b*x^6 + a*x^4), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x**2+d)**q/x**4/(c*x**4+b*x**2+a),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((e*x^2+d)^q/x^4/(c*x^4+b*x^2+a),x, algorithm="giac")
[Out] integrate((x^2*e + d)^q/((c*x^4 + b*x^2 + a)*x^4), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{(e\,x^2+d)^q}{x^4\,\left(c\,x^4+b\,x^2+a\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)),x)
[Out] int((d + e*x^2)^q/(x^4*(a + b*x^2 + c*x^4)), x)
```

3.413 $\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx$

Optimal. Leaf size=40

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{1-c^4x^4}}{c\sqrt{1+\frac{1}{c^2x^2}}\,x}\right)}{c}$$

[Out] -arctanh((-c^4*x^4+1)^(1/2)/c/x/(1+1/c^2/x^2)^(1/2))/c

Rubi [A]
time = 0.05, antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 5, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.179$, Rules used = {1462, 1266, 862, 65, 214}

$$-\frac{x\sqrt{\frac{1}{c^2x^2}+1}\,\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{c^2x^2+1}}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]

[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^2*x^2]])/Sqrt[1 + c^2*x^2])

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 862

```
Int[((d_) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))^(n_)*((a_) + (c_.)*(x_)^2
)^(p_.), x_Symbol] :> Int[(d + e*x)^(m + p)*(f + g*x)^n*(a/d + (c/e)*x)^p,
x] /; FreeQ[{a, c, d, e, f, g, m, n}, x] && NeQ[e*f - d*g, 0] && EqQ[c*d^2
+ a*e^2, 0] && (IntegerQ[p] || (GtQ[a, 0] && GtQ[d, 0] && EqQ[m + p, 0]))
```

Rule 1266

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_S
ymbol] :> Dist[1/2, Subst[Int[x^((m - 1)/2)*(d + e*x)^q*(a + c*x^2)^p, x],
x, x^2], x] /; FreeQ[{a, c, d, e, p, q}, x] && IntegerQ[(m + 1)/2]
```

Rule 1462

```
Int[((d_) + (e_.)*(x_)^(mn_.))^(q_)*((a_) + (c_.)*(x_)^(n2_.))^(p_.), x_Sym
bol] :> Dist[(e^IntPart[q]*((d + e*x^mn)^FracPart[q]/(1 + d*(1/(x^mn*e)))^F
racPart[q]))/x^(mn*FracPart[q]), Int[x^(mn*q)*(1 + d*(1/(x^mn*e)))^q*(a + c
*x^n2)^p, x], x] /; FreeQ[{a, c, d, e, mn, p, q}, x] && EqQ[n2, -2*mn] &&
!IntegerQ[p] &&  !IntegerQ[q] && PosQ[n2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{1-c^4x^4}}\,dx &= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\int \frac{\sqrt{1+c^2x^2}}{x\sqrt{1-c^4x^4}}\,dx}{\sqrt{1+c^2x^2}}\\
&= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+c^2x}}{x\sqrt{1-c^4x^2}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}}\\
&= \frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\operatorname{Subst}\left(\int \frac{1}{x\sqrt{1-c^2x}}\,dx,x,x^2\right)}{2\sqrt{1+c^2x^2}}\\
&= -\frac{\left(\sqrt{1+\frac{1}{c^2x^2}}\,x\right)\operatorname{Subst}\left(\int \frac{1}{\frac{1}{c^2}-\frac{x^2}{c^2}}\,dx,x,\sqrt{1-c^2x^2}\right)}{c^2\sqrt{1+c^2x^2}}\\
&= -\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\sqrt{1-c^2x^2}\right)}{\sqrt{1+c^2x^2}}
\end{aligned}
$$

Mathematica [A]
time = 0.47, size = 58, normalized size = 1.45

$$
-\frac{\sqrt{1+\frac{1}{c^2x^2}}\,x\tanh^{-1}\left(\frac{\sqrt{1-c^4x^4}}{\sqrt{1+c^2x^2}}\right)}{\sqrt{1+c^2x^2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + 1/(c^2*x^2)]/Sqrt[1 - c^4*x^4],x]
```

```
[Out] -((Sqrt[1 + 1/(c^2*x^2)]*x*ArcTanh[Sqrt[1 - c^4*x^4]/Sqrt[1 + c^2*x^2]])/Sq
rt[1 + c^2*x^2])
```

Maple [C] Result contains higher order function than in optimal. Order 9 vs. order 3.
time = 0.13, size = 101, normalized size = 2.52

method	result	size
default	$-\frac{\sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}\, x \sqrt{-c^{4} x^{4}+1}\, \mathrm{csgn}\left(\frac{1}{c}\right) \ln \left(\frac{2 \,\mathrm{csgn}\left(\frac{1}{c}\right) c \sqrt{-\frac{c^{2} x^{2}-1}{c^{2}}}+2}{c^{2} x}\right)}{\left(c^{2} x^{2}+1\right) \sqrt{-\frac{c^{2} x^{2}-1}{c^{2}}}\, c}$	101

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -((c^2*x^2+1)/c^2/x^2)^(1/2)*x*(-c^4*x^4+1)^(1/2)*csgn(1/c)*ln(2*(csgn(1/c)
*c*(-(c^2*x^2-1)/c^2)^(1/2)+1)/c^2/x)/(c^2*x^2+1)/(-(c^2*x^2-1)/c^2)^(1/2)/
c
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(1/(c^2*x^2) + 1)/sqrt(-c^4*x^4 + 1), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(36) = 72.
time = 0.37, size = 120, normalized size = 3.00

$$-\frac{\log\left(\frac{c^{2} x^{2}+\sqrt{-c^{4} x^{4}+1}\, c x \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}+1}{c^{2} x^{2}+1}\right)-\log\left(-\frac{c^{2} x^{2}-\sqrt{-c^{4} x^{4}+1}\, c x \sqrt{\frac{c^{2} x^{2}+1}{c^{2} x^{2}}}+1}{c^{2} x^{2}+1}\right)}{2\, c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(log((c^2*x^2 + sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 + 1)/(c^2*x^2)) +
 1)/(c^2*x^2 + 1)) - log(-(c^2*x^2 - sqrt(-c^4*x^4 + 1)*c*x*sqrt((c^2*x^2 +
 1)/(c^2*x^2)) + 1)/(c^2*x^2 + 1)))/c
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{1+\frac{1}{c^2x^2}}}{\sqrt{-(cx-1)(cx+1)(c^2x^2+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c**2/x**2)**(1/2)/(-c**4*x**4+1)**(1/2),x)
```

```
[Out] Integral(sqrt(1 + 1/(c**2*x**2))/sqrt(-(c*x - 1)*(c*x + 1)*(c**2*x**2 + 1))
, x)
```

Giac [**A**]
time = 5.06, size = 42, normalized size = 1.05

$$-\frac{\left(\log\left(\sqrt{-c^2x^2+1}+1\right)-\log\left(-\sqrt{-c^2x^2+1}+1\right)\right)|c|}{2\,c^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+1/c^2/x^2)^(1/2)/(-c^4*x^4+1)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(log(sqrt(-c^2*x^2 + 1) + 1) - log(-sqrt(-c^2*x^2 + 1) + 1))*abs(c)/c^
2
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sqrt{\frac{1}{c^2\,x^2}+1}}{\sqrt{1-c^4\,x^4}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2),x)
```

```
[Out] int((1/(c^2*x^2) + 1)^(1/2)/(1 - c^4*x^4)^(1/2), x)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

   #debug=False
   if debug:
       print ("Enter is_atom, expn=",expn)

   if not hasattr(expn, 'parent'):
      return False


   #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
   try:
      if expn.parent() is SR:
         return expn.operator() is None
      if expn.parent() in (ZZ, QQ, AA, QQbar):
         return expn in expn.parent() # Should always return True
      if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
         return expn in expn.parent().base_ring() or expn in expn.parent().gens()

      return False

   except AttributeError as error:
      print("Exception,AttributeError in  is_atom")
      print ("cought exception" , type(error).__name__ )
      return False


def expnType(expn):

   if debug:
      print (">>>>>Enter expnType, expn=", expn)
      print (">>>>>is_atom(expn)=", is_atom(expn))

   if is_atom(expn):
      return 1
   elif type(expn)==list:   #isinstance(expn,list):
      return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
         return 1
      else:
         return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
   elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
      if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
         return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
         if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
    m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
    return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
    return max(4,m1)   #max(4,m1)
elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(5,m1)   #max(5,m1)
elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(6,m1)   #max(6,m1)
elif str(expn).find("Integral") != -1: #this will never happen, since it
        #is checked before calling the grading function that is passed.
        #but kept it here.
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
else:
    return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```